

Einführungsvorlesung:

Optische Wellenleiter

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1. Strahlenoptik

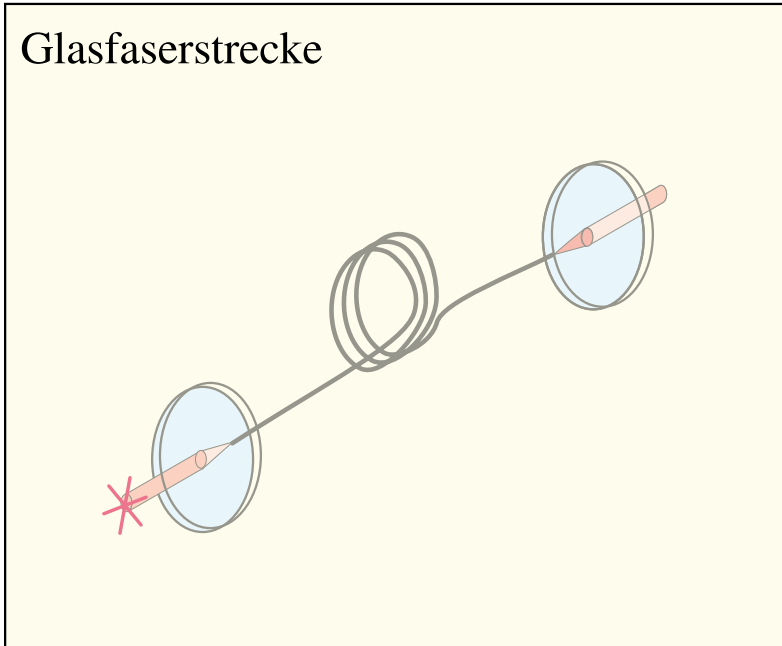
2. Wellenoptik

2.1. Dielektrischer Wellenleiter

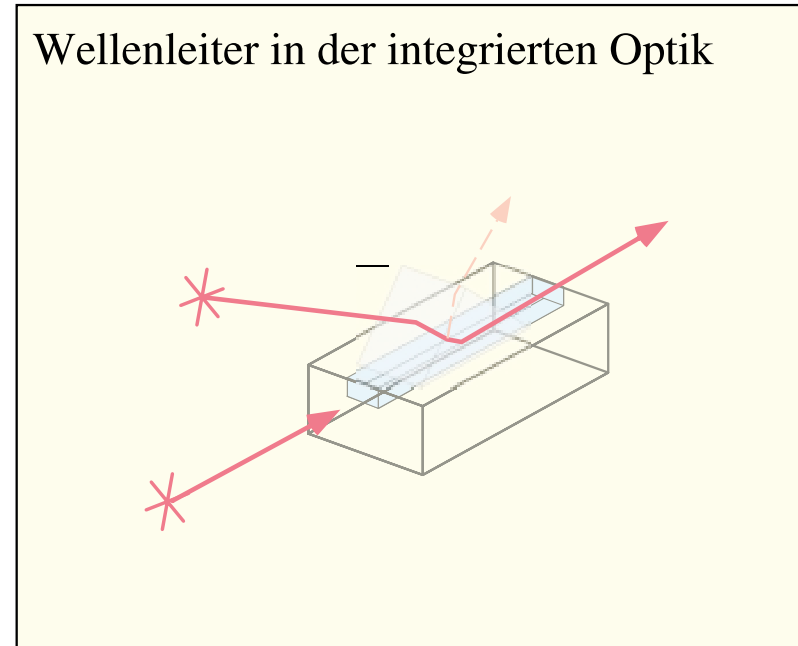
2.2. Stufenfaser

Lichtwellenleiter

Glasfaserstrecke



Wellenleiter in der integrierten Optik



(siehe D. Meschede: Optik, Licht und Laser)

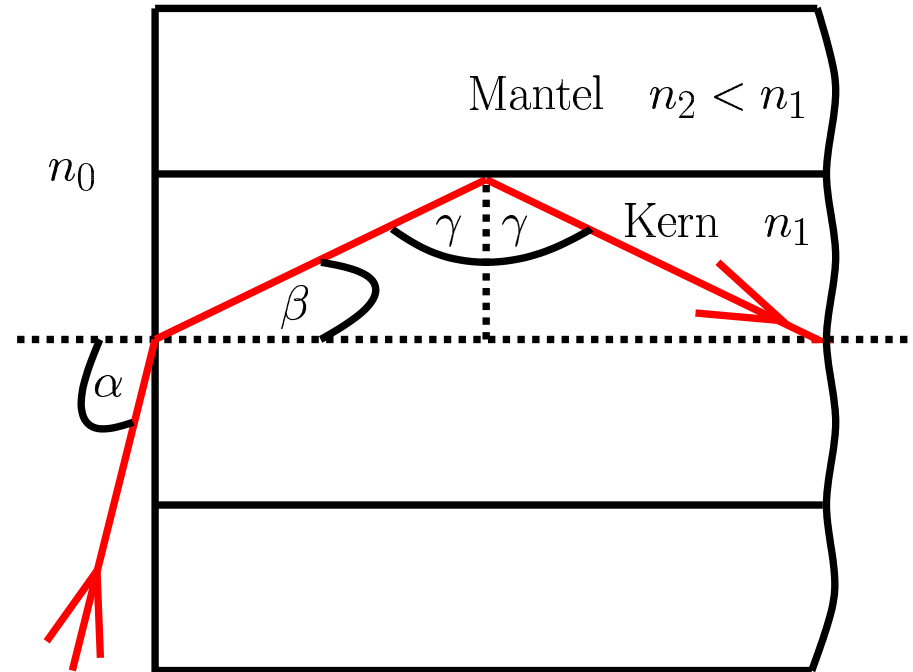
Grundprinzip

Totalreflexion:

$$\gamma > \gamma_c = \arcsin(n_2/n_1)$$

Maximaler Einfallswinkel:

$$n_0 \sin \alpha_c = \sqrt{n_1^2 - n_2^2} = \text{NA}$$



Typische Zahlenwerte:

$$n_0 = 1 \quad , \quad n_1 = n_2 + 0.015 \quad , \quad n_2 = 1.45 \quad \implies \quad \gamma_c = 81.8^\circ \quad , \quad \alpha_c = 12.1^\circ$$

Frequenzbandbreite

Abschätzung:

axialer Strahl:

$$s_a = ct$$

total reflektierter Strahl:

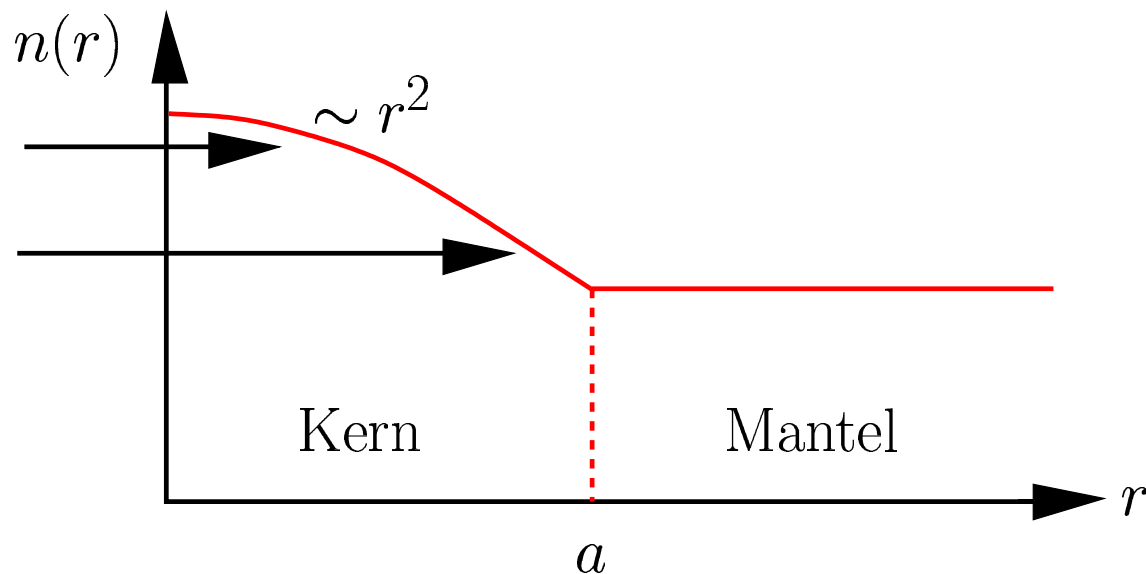
$$s_t = \frac{s_a}{\sin \beta_c} = c(t + \Delta t) > s_a$$

$$\Rightarrow \Delta\nu = \frac{1}{\Delta t} = \frac{cn_2}{s_a(n_1 - n_2)} \quad s_a \approx 1 \text{ km} \quad \Delta\nu = 30 \text{ MHz}$$

Gradientenindex-Wellenleiter: $\Delta\nu = 10^6 \text{ MHz}$

größeres n ,
kleineres c_0/n

kleineres n ,
größeres c_0/n



Maxwell-Gleichungen in Materie

gegeben: $\rho(\mathbf{r}, t), \mathbf{j}(\mathbf{r}, t)$

gesucht: $\mathbf{D}(\mathbf{r}, t), \mathbf{E}(\mathbf{r}, t); \mathbf{B}(\mathbf{r}, t), \mathbf{H}(\mathbf{r}, t)$

$$(M1) \quad \operatorname{div} \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t)$$

$$(M2) \quad \operatorname{div} \mathbf{B}(\mathbf{r}, t) = 0$$

$$(M3) \quad \operatorname{rot} \mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t}$$

$$(M4) \quad \operatorname{rot} \mathbf{H}(\mathbf{r}, t) = \mathbf{j}(\mathbf{r}, t) + \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t}$$

Materialgleichungen:

$$\mathbf{D}(\mathbf{r}, t), \mathbf{H}(\mathbf{r}, t) \iff \mathbf{E}(\mathbf{r}, t), \mathbf{B}(\mathbf{r}, t)$$

Maxwell-Gleichungen in Dielektrika

Keine Ladungen und Ströme: $\rho(\mathbf{r}, t) = 0$, $\mathbf{j}(\mathbf{r}, t) = \mathbf{0}$

Materialgleichungen:

$$\mathbf{D}(\mathbf{r}, t) = \epsilon_0 \epsilon_r(\mathbf{r}) \mathbf{E}(\mathbf{r}, t) \quad , \quad \mathbf{B}(\mathbf{r}, t) = \mu_0 \mu_r(\mathbf{r}) \mathbf{H}(\mathbf{r}, t) \quad , \quad \mu_r(\mathbf{r}) = 1$$

$$(M1) \quad \operatorname{div} \mathbf{E}(\mathbf{r}, t) = -\frac{\operatorname{grad} \epsilon_r(\mathbf{r})}{\epsilon_r(\mathbf{r})} \mathbf{E}(\mathbf{r}, t)$$

$$(M2) \quad \operatorname{div} \mathbf{H}(\mathbf{r}, t) = 0$$

$$(M3) \quad \operatorname{rot} \mathbf{E}(\mathbf{r}, t) = -\mu_0 \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t}$$

$$(M4) \quad \operatorname{rot} \mathbf{H}(\mathbf{r}, t) = \epsilon_0 \epsilon_r(\mathbf{r}) \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t}$$

Elektromagnetische Wellen?

Wellengleichungen in Dielektrika

$$\text{rot rot } \mathbf{E}(\mathbf{r}, t) = \text{grad} \left[\text{div } \mathbf{E}(\mathbf{r}, t) \right] - \left[\text{div grad} \right] \mathbf{E}(\mathbf{r}, t)$$

$$\stackrel{\text{(M1)}}{=} -\text{grad} \left[\frac{\text{grad } \epsilon_r(\mathbf{r})}{\epsilon_r(\mathbf{r})} \mathbf{E}(\mathbf{r}, t) \right] - \Delta \mathbf{E}(\mathbf{r}, t)$$

$$\stackrel{\text{(M3),(M4)}}{=} -\frac{\epsilon_r(\mathbf{r})}{c_0^2} \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2}, \quad c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$\implies \left[\frac{\epsilon_r(\mathbf{r})}{c_0^2} \frac{\partial^2}{\partial t^2} - \Delta \right] \mathbf{E}(\mathbf{r}, t) = \text{grad} \left[\frac{\text{grad } \epsilon_r(\mathbf{r})}{\epsilon_r(\mathbf{r})} \mathbf{E}(\mathbf{r}, t) \right]$$

**Analoge
Herleitung:**

$$\left[\frac{\epsilon_r(\mathbf{r})}{c_0^2} \frac{\partial^2}{\partial t^2} - \Delta \right] \mathbf{H}(\mathbf{r}, t) = \frac{\text{grad } \epsilon_r(\mathbf{r})}{\epsilon_r(\mathbf{r})} \times \mathbf{H}(\mathbf{r}, t)$$

\implies **Vernachlässigung der Korrektur zur Wellengleichung**

Dielektrische Wellenleiter

Propagierende Welle:

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(x, y) e^{i(kz - \omega t)}$$

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(x, y) e^{i(kz - \omega t)}$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{c_0^2} \epsilon_r(x, y) - k^2 \right] \begin{pmatrix} \mathbf{E}(x, y) \\ \mathbf{H}(x, y) \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

Zerlegung der Felder:

$$\mathbf{E}_\perp(x, y) = \begin{pmatrix} E_x(x, y) \\ E_y(x, y) \\ 0 \end{pmatrix}, \quad \mathbf{E}_\parallel(x, y) = \begin{pmatrix} 0 \\ 0 \\ E_z(x, y) \end{pmatrix}$$
$$\mathbf{H}_\perp(x, y) = \begin{pmatrix} H_x(x, y) \\ H_y(x, y) \\ 0 \end{pmatrix}, \quad \mathbf{H}_\parallel(x, y) = \begin{pmatrix} 0 \\ 0 \\ H_z(x, y) \end{pmatrix}$$

Transversale Felder

Konsequenz von (M3), (M4):

$$\mathbf{E}_\perp(x, y) = \frac{ik\nabla E_z(x, y) - i\omega\mu_0 (\mathbf{e}_z \times \nabla) H_z(x, y)}{k_0^2 \epsilon_r(x, y) - k^2}$$

$$\mathbf{H}_\perp(x, y) = \frac{ik\nabla H_z(x, y) + i\omega\epsilon_0\epsilon_r(x, y) (\mathbf{e}_z \times \nabla) E_z(x, y)}{k_0^2 \epsilon_r(x, y) - k^2}$$

(siehe J.D. Jackson: Klassische Elektrodynamik)

Helmholtz-Gleichungen für z -Komponenten:

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_0^2 n^2(x, y) - k^2 \right] \begin{pmatrix} E_z(x, y) \\ H_z(x, y) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Betrag des Wellenvektors im Vakuum: $k_0 = \omega/c_0$

Brechungsindex: $n(x, y) = \sqrt{\epsilon_r(x, y)}$

Analogie zur Quantenmechanik

Dielektrischer Wellenleiter	Quantenmechanik in $D = 2$
$[\Delta_{\perp} + k_0^2 n^2(x, y) - k^2] E_z(x, y) = 0$	$\left\{ \Delta_{\perp} - \frac{2M}{\hbar^2} [V(x, y) - E] \right\} \psi(x, y) = 0$
$k_0^2 n^2(x, y)$	$-\frac{2M}{\hbar^2} V(x, y)$
$-k^2$	$\frac{2M}{\hbar^2} E$
ausbreitungsfähige Mode: k reell	gebundene Lösung: $E < 0$
gedämpfte Mode: k imaginär	Streulösung: $E > 0$

\implies **Übertragung von Lösungsmethoden**

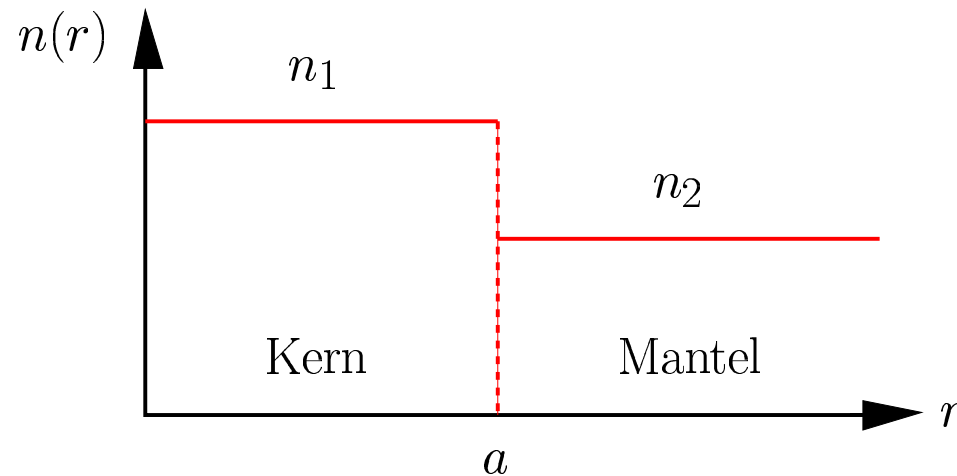
Stufenfaser

Zylindersymmetrie:

$$E_z(r, \varphi) = e_l(r) e^{il\varphi}, \quad H_z(r, \varphi) = h_l(r) e^{il\varphi}$$

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{l^2}{r^2} + k_0^2 n^2(r) - k^2 \right] \begin{pmatrix} e_l(r) \\ h_l(r) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Spezialisierung:



$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{l^2}{r^2} + k_1^2 - k^2 \right] \begin{pmatrix} e_l(r) \\ h_l(r) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad 0 \leq r \leq a, \quad k_1 = k_0 n_1$$

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{l^2}{r^2} + k_2^2 - k^2 \right] \begin{pmatrix} e_l(r) \\ h_l(r) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad r \geq a, \quad k_2 = k_0 n_2$$

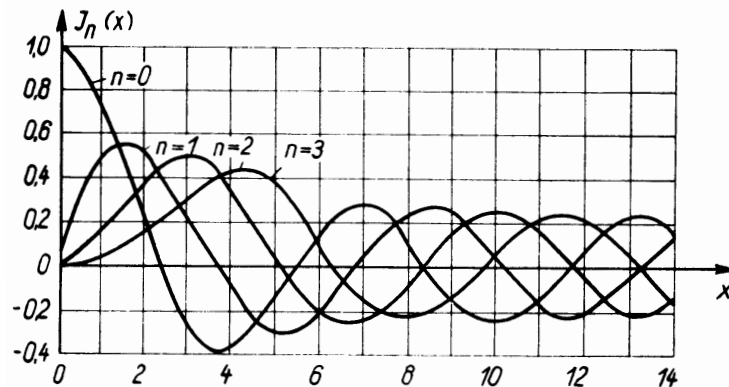
Helmholtz-Gleichung im Kern

Bedingung für geführte Welle: $k_{\perp}^2 = k_1^2 - k^2 > 0$

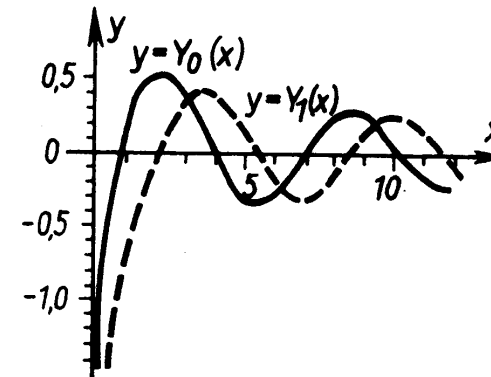
Dimensionslose Koordinate: $X = k_{\perp} r$

$$\left[\frac{\partial^2}{\partial X^2} + \frac{1}{X} \frac{\partial}{\partial X} + 1 - \frac{l^2}{X^2} \right] \begin{pmatrix} e_l(X) \\ h_l(X) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad 0 \leq X \leq k_{\perp} a$$

⇒ **Besselsche Differentialgleichung erster Gattung**



Bessel-Funktionen



Weber-Funktionen

$$\begin{pmatrix} e_l(X) \\ h_l(X) \end{pmatrix} = \begin{pmatrix} A_l \\ B_l \end{pmatrix} J_l(X), \quad 0 \leq X \leq k_{\perp} a$$

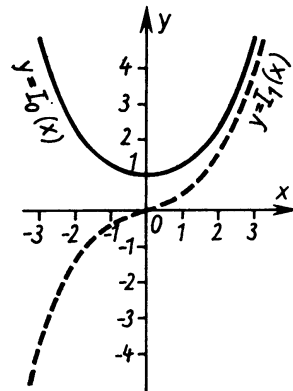
Helmholtz-Gleichung im Mantel

Bedingung für evaneszente Welle: $\kappa^2 = k^2 - k_2^2 > 0$

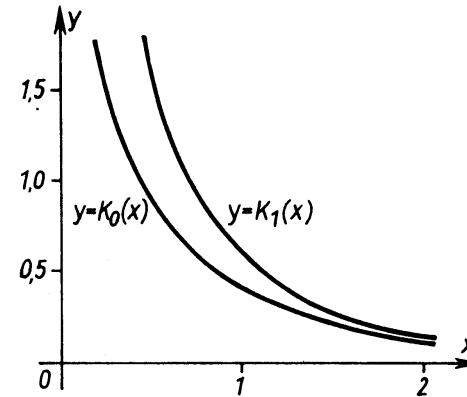
Dimensionslose Koordinate: $Y = \kappa r$

$$\left[\frac{\partial^2}{\partial Y^2} + \frac{1}{Y} \frac{\partial}{\partial Y} - 1 - \frac{l^2}{Y^2} \right] \begin{pmatrix} e_l(Y) \\ h_l(Y) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad Y \geq \kappa a$$

⇒ **Besselsche Differentialgleichung zweiter Gattung**



Modifizierte Bessel-Funktionen

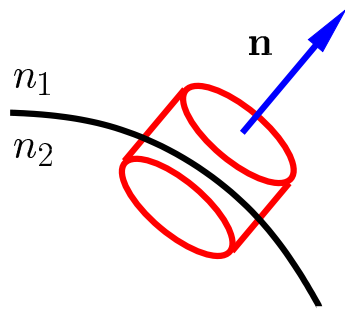


Macdonald-Funktionen

$$\begin{pmatrix} e_l(Y) \\ h_l(Y) \end{pmatrix} = \begin{pmatrix} C_l \\ D_l \end{pmatrix} K_l(Y), \quad Y \geq \kappa a$$

Randbedingungen

Dielektrische Wellenleiter:

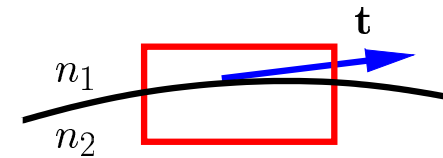


$$(M1) \quad \mathbf{n} (n_2^2 \mathbf{E}_2 - n_1^2 \mathbf{E}_1) = 0$$

$$(M2) \quad \mathbf{n} (\mathbf{H}_2 - \mathbf{H}_1) = 0$$

$$(M3) \quad \mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = \mathbf{0}$$

$$(M4) \quad \mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{0}$$



Zylindersymmetrie: $\mathbf{n} = \mathbf{e}_r$

$$(M1) \quad n_2^2 E_{2,r} = n_1^2 E_{1,r}$$

$$(M2) \quad H_{2,r} = H_{1,r}$$

$$(M3) \quad E_{2,\varphi} = E_{1,\varphi}, \quad E_{2,z} = E_{1,z}$$

$$(M4) \quad H_{2,\varphi} = H_{1,\varphi}, \quad H_{2,z} = H_{1,z}$$

Einarbeitung der Randbedingungen

Stetigkeit der z -Komponente:

$$e_l(r) = \begin{cases} A_l \frac{J_l(X)}{J_l(X_a)}, & 0 \leq X = k_{\perp} r \leq X_a = k_{\perp} a \\ A_l \frac{K_l(Y)}{K_l(Y_a)}, & Y = \kappa r \geq Y_a = \kappa a \end{cases}$$

$$h_l(r) = \begin{cases} B_l \frac{J_l(X)}{J_l(X_a)}, & 0 \leq X = k_{\perp} r \leq X_a = k_{\perp} a \\ B_l \frac{K_l(Y)}{K_l(Y_a)}, & Y = \kappa r \geq Y_a = \kappa a \end{cases}$$

Stetigkeit der φ -Komponente:

$$\begin{pmatrix} -il \left[\frac{1}{X_a^2} + \frac{1}{Y_a^2} \right] & \frac{\mu_0 \omega}{k} \left[\frac{J'_l(X_a)}{X_a J_l(X_a)} + \frac{K'_l(Y_a)}{Y_a K_l(Y_a)} \right] \\ \frac{\epsilon_0 \omega}{k} \left[\frac{n_1^2 J'_l(X_a)}{X_a J_l(X_a)} + \frac{n_2^2 K'_l(Y_a)}{Y_a K_l(Y_a)} \right] & il \left[\frac{1}{X_a^2} + \frac{1}{Y_a^2} \right] \end{pmatrix} \begin{pmatrix} A_l \\ B_l \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

\implies Charakteristische Gleichung

Schwach Führende Wellenleiter

Vereinfachung:

$$\Delta = \frac{n_1 - n_2}{n_1} \ll 1 \quad \Longrightarrow \quad k_1 \approx k \approx k_2$$
$$\Longrightarrow \quad \frac{J'_l(X_a)}{X_a J_l(X_a)} + \frac{K'_l(Y_a)}{Y_a K_l(Y_a)} = \pm l \left[\frac{1}{X_a^2} + \frac{1}{Y_a^2} \right] \quad (1)$$

Nebenbedingung:

$$X_a^2 + Y_a^2 = a^2 k_0^2 (n_1^2 - n_2^2) = \left(a k_0 \text{NA} \right)^2 = V^2 \quad (2)$$

Bilanz:

$$(1) + (2): \quad \Longrightarrow \quad X_{a,lm}, \quad Y_{a,lm}; \quad l = 0, 1, 2, \dots; \quad m = 1, 2, 3, \dots$$

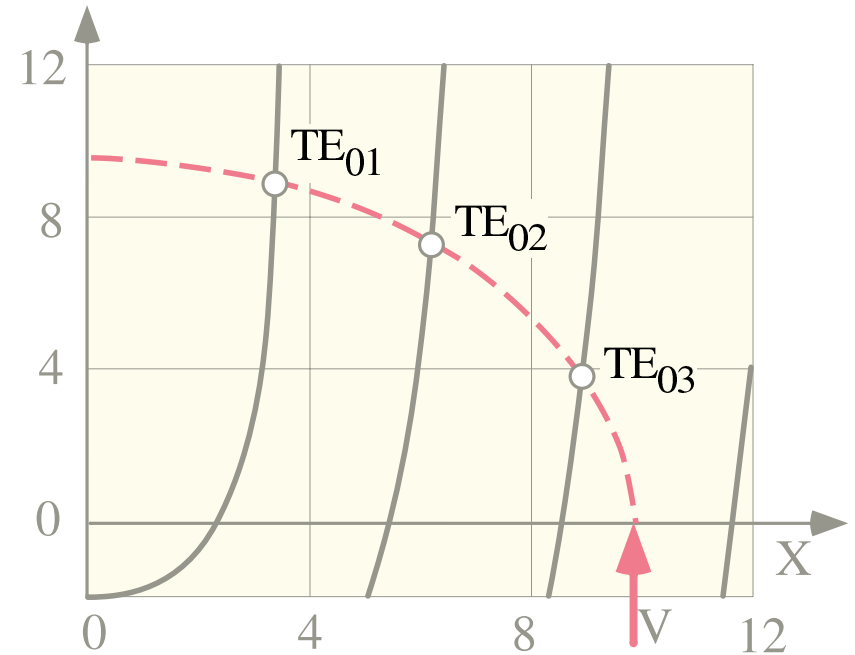
$$\Longrightarrow \quad \text{Betrag des Wellenvektors:} \quad k_{lm}$$

Transversale Moden ($l = 0$)

TM-Mode: $A_0 \neq 0, \quad B_0 = 0$

TE-Mode: $A_0 = 0, \quad B_0 \neq 0$

$$\underbrace{-\frac{X J_0(X)}{J_1(X)}}_{\text{---}} = \underbrace{\frac{\sqrt{V^2 - X^2} K_0(\sqrt{V^2 - X^2})}{K_1(\sqrt{V^2 - X^2})}}_{\text{---}}$$



Bedingung:

$$V = ak_0 \text{ NA} \geq V_G = X_1 \approx 2.4048, \quad J_0(X_1) = 0$$

$$\implies \omega \geq \omega_G = \frac{2.4048 c_0}{a \text{ NA}}, \quad \lambda_0 \leq \lambda_G = \frac{2\pi \text{ NA}}{2.4048} a$$

Hybrid-Moden ($l \geq 1$)

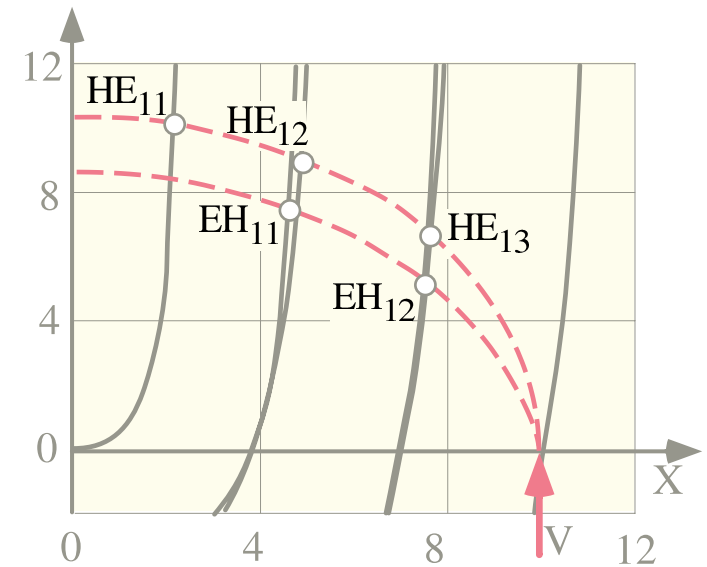
HE_l -Moden	EH_l -Moden
$\frac{X J_l(X)}{J_{l-1}(X)} - \frac{Y K_l(Y)}{K_{l-1}(Y)} = 0$	$\frac{X J_l(X)}{J_{l+1}(X)} + \frac{Y K_l(Y)}{K_{l+1}(Y)} = 0$

$HE_1 :$

$$\underbrace{\frac{X J_1(X)}{J_0(X)}} = \underbrace{\frac{\sqrt{V^2 - X^2} K_1(\sqrt{V^2 - X^2})}{K_0(\sqrt{V^2 - X^2})}}$$

$EH_1 :$

$$-\underbrace{\frac{X J_1(X)}{J_2(X)}} = \underbrace{\frac{\sqrt{V^2 - X^2} K_1(\sqrt{V^2 - X^2})}{K_2(\sqrt{V^2 - X^2})}}$$



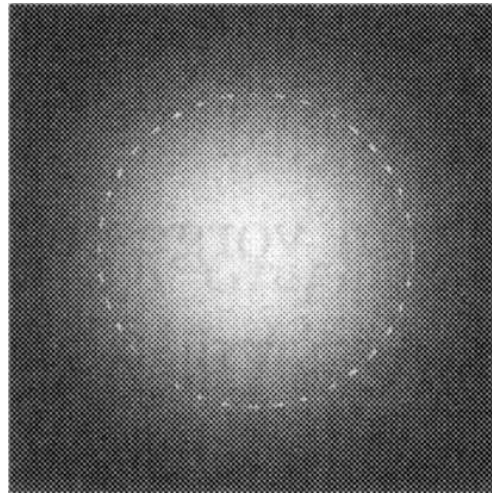
Monomodens-Wellenleiter (HE_{11}):

$$\lambda_0 \geq \lambda_G = \frac{2\pi NA}{2.4048} a$$

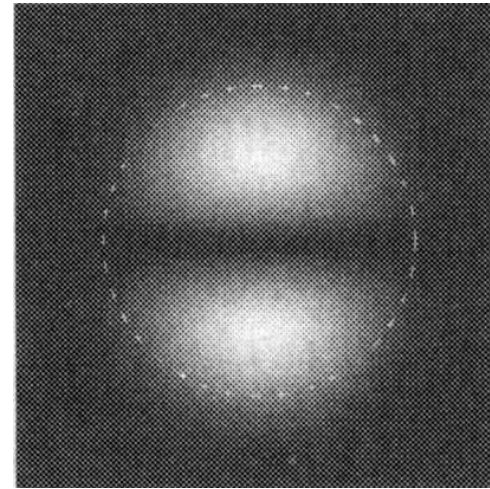
$$NA = 0.12, \quad \lambda_G = 800 \text{ nm} \quad \Rightarrow \quad a = \frac{2.4048}{2\pi NA} \lambda_G = 2.55 \mu\text{m}$$

Intensitätsverteilung (E-Feld)

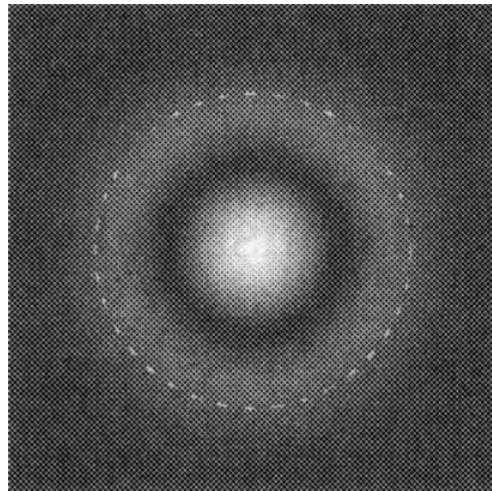
HE_{11}



HE_{21}



HE_{12}



TE_{01}

