Problem Set 7
Theoretical Solid State Physics (SoSe2017)

Due: Due Thursday, June 8, 2017; **at the beginning of class**

Problem 1: Time reversal symmetry

In the following lectures, the concept of time reversal symmetry will repeatedly play an important role. Hence, this problem is meant to encourage you to go back and study the treatment of time reversal in quantum mechanics. Read your favorite quantum mechanics textbook and explain the following points:

(a) Why is time reversal effected by an antiunitary (rather than a unitary) symmetry? (What is an antiunitary operator?)

(b) Find the time reversal operator T for spinless and spinful systems and show that T squares to $+1$ for integer spin systems, but to -1 for half integer spin systems.

(c) Explain why time reversal symmetry implies doubly degenerate eigenvalues (Kramers degeneracy) for half integer spin systems, but no such degeneracy for integer spin systems.

Problem 2: Chern number for generalized two-band model

In the lecture, we quoted the equation

$$C = \frac{1}{4\pi} \int \mathrm{d}k_x \mathrm{d}k_y \hat{\mathbf{d}} \cdot \left(\frac{\partial \hat{\mathbf{d}}}{\partial k_x} \times \frac{\partial \hat{\mathbf{d}}}{\partial k_y} \right) \tag{1}$$

for the Chern number of one of the bands of the generalized two-band model

$$H = d_1(\mathbf{k})\tau_1 + d_2(\mathbf{k})\tau_2 + d_3(\mathbf{k})\tau_3. \tag{2}$$

Proof this equation by computing the explicit Bloch functions, then the Berry connection, finally the Berry curvature, and comparing with this equation. (For help, see Bernevig's book).

You find an alternative derivation in the lecture notes.

Problem 3: Two-band model of a Chern insulators

Consider the two-band model

$$H = d_1(\mathbf{k})\tau_1 + d_2(\mathbf{k})\tau_2 + d_3(\mathbf{k})\tau_3 \tag{3}$$

with $d_1 = \sin k_x$, $d_2 = \sin k_y$, and $d_3 = m + \cos k_x + \cos k_y$ discussed in the lecture

(a) Compute the band structure.

(b) Plot the bandstructure for a set of characteristic values of m, making sure that all phases as well as the gapless points are included in the set of m's.

(c) If you have the energy: Compute the Chern number of one of the bands directly using the result of Problem 2, possibly using some numerics.