Problem Set 6
Theoretical Solid State Physics (SoSe2017)

Due: Due Thursday, June 1, 2017; **at the beginning of class**

Problem 1: Hall conductivity

(a) Compute the conductivity tensor from the resistivity tensor for a two-dimensional electron system subject to a perpendicular magnetic field as discussed in class. Plot the longitudinal and the Hall conductivity as a function of magnetic field and electron density, assuming that the scattering time is independent of density or magnetic field. Explain in which sense quantum Hall fluids are simultaneously analogous to superfluids and insulators.

(b) The conductivity or resistivity tensor as derived in class and in part (a) apply to isotropic systems. When discussing a general anisotropic sample, there are some changes when looking at these tensors in arbitrary coordinate systems. Explain why more generally the dissipative conductivity is described by the symmetric part of the conductivity tensor, while the Hall conductivity is the antisymmetric part.

Problem 2: Tight-binding models

To continue our discussion of tight-binding models, consider a simple tight-binding model on the square lattice, with one orbital per site. Derive the bandstructure and plot the Fermi surface (or rather line) for different fillings of the band (including half filling). Explain the meaning of nesting and why systems are prone to density-wave instabilities in its presence.

Problem 3: Graphene

One of the most popular and well-funded research fields of the past decade or so has been graphene, a single sheet of graphite which contains carbon atoms arranged in a honeycomb lattice. Using a tight-binding description of graphene with one orbital at each lattice site of the honeycomb lattice and assuming only a nearest-neighbor hopping amplitude t, derive the graphene band structure. Notice that the honeycomb lattice has two sublattices so that each unit cell contains two atoms, and that hopping is between atoms in different unit cells only. Frequently used terms in this context are Dirac points and valley degeneracy. Explain what these refer to. (This problem is done in many places so you will easily find appropriate help in the literature if needed.)

If you have the energy, you can go and think about the following aspects: (i) Explicitly derive the description of the spectrum in terms of a Dirac equation from your tight-binding solution. (ii) How stable are the Dirac points? After all, our model neglects many aspects present in real graphene, e.g., next-nearest-neighbor hopping. As is familiar from quantum mechanics, perturbations have a tendency to lift degeneracies (von Neumann-Wigner theorem). Which perturbations would lift the degeneracy of the Dirac point, which would leave it intact? You might want to think in terms of symmetries such as inversion and time-reversal symmetry.