Problem Set 3
Theoretical Solid State Physics (SoSe2017)

Due: Due Thursday, May 11, 2017; **at the beginning of class**

Problem 1: Critical exponent in Landau theory

Consider Landau theory for an Ising magnet, i.e., with a scalar (and real) order parameter S. Compute the following critical exponents in mean field approximation:

(a) dependence of the order parameter on temperature below T_c

$$S \sim (T_c - T)^{\beta} \tag{1}$$

with critical exponent β;

(b) dependence of the order parameter on a symmetry breaking magnetic field h at T_c

$$S \sim h^{1/\delta} \tag{2}$$

with critical exponent δ;

(c) susceptibility of the order parameter to changes in the magnetic field as a function of temperature below and above T_c

$$\chi = \frac{\partial S}{\partial h} \sim |T_c - T|^{\gamma} \tag{3}$$

with critical exponent γ.

In addition, we had already seen the critical exponent $\nu = 1/2$ for the correlation length as a function of temperature, $\xi \sim |T - T_c|^{-\nu}$.

Problem 2: Landau levels

Consider (noninteracting) electrons moving in two dimensions in a perpendicular magnetic field B. Compute the spectrum and the wavefunctions.

(a) The easiest way to compute the spectrum is to compute the commutator of the kinetic momenta π_x and π_y, where

$$\pi = \mathbf{p} - e\mathbf{A}. \tag{4}$$

Show that when combined with the Hamiltonian

$$H = \frac{1}{2m}\left(\pi_x^2 + \pi_y^2\right), \tag{5}$$

this immediately yields the spectrum by analogy with a well-known problem.

(b) Compute the spectrum and wavefunctions by explicitly considering the Schrodinger equation for the Landau gauge

$$\mathbf{A} = (0, Bx, 0). \tag{6}$$

(c) Determine the degeneracy of the resulting Landau levels for a sample of area $L_x L_y$. Compare the degeneracy to the number of flux quanta $\phi_0 = h/e$ threading the sample. For a given electron density, determine the magnetic fields for which the so called Landau level filling factor ν takes on an integer value, i.e., for which an integer number of Landau levels is exactly filled, and the remaining Landau levels are empty.

Problem 3: Upper critical field of type-II superconductors

Near the upper critical field of a type-II superconductor, the superconducting order parameter is suppressed to near zero. Moreover, superconductivity is so weak that screening currents can be neglected. As a result, we can ignore the nonlinear term in the Ginzburg-Landau equation and take the vector potential to be equal to the one for the externally applied uniform magnetic field.

(a) Show that this implies that the order parameter ψ satisfies the equation

$$\left[(-i\nabla - 2\pi\mathbf{A}/\Phi_0)^2 + \frac{1}{\xi^2}\right]\psi = 0. \tag{7}$$

Use your result to the previous problem to show that this implies that the upper critical field of a type-II superconductor is equal to

$$H_{c2} = \frac{\Phi_0^2}{2\pi\xi^2}, \tag{8}$$

where temperature enters through the coherence length ξ. Note that we are now considering a 3D problem and Φ_0 is the superconducting flux quantum.

(b) What would you have to do to check Abrikosov's famous result that the vortices enter the sample in a triangular lattice? (Read!)