Problem Set 10
Theoretical Solid State Physics (SoSe2017)

Due: Thursday, June 29, 2017; **at the beginning of class**

Problem 1: Pumping

Consider a quantum spin Hall edge with counterpropagating edge states of spin-up and spin-down electrons. Coupling the edge to a magnetic insulator, this system is described by the Hamiltonian

$$H = v_F p\sigma_z + M_x\sigma_x + M_y\sigma_y, \tag{1}$$

where we assume that the magnetization $\mathbf{M} = (M_x, M_y, 0)$ is lying in the plane perpendicular to the quantization axis of the helical edge.

(a) Show that the coupling to the magnetization opens a gap in the edge spectrum.

(b) Discuss what happens when the magnetization is rotated adiabatically in the plane by an angle of 2π. (Hint: Essentially no calculation needed; problem can be solved by analogy.)

Problem 2: Phase diagram of the BHZ model

Consider the Bloch Hamiltonian of the Bernevig-Hughes-Zhang model which is used to describe the topological insulator state in HgTe quantum wells. Defined on a square lattice with two orbitals per site (Pauli matrices in the orbital space are denoted by τ_j), the Bloch Hamiltonian reads

$$H_{\mathrm{BHZ}}(k_x, k_y) = \begin{pmatrix} h(\mathbf{k}) & 0 \\ 0 & h^*(-\mathbf{k}) \end{pmatrix}, \tag{2}$$

where $h(\mathbf{k}) = \mathbf{d}\cdot\tau$ with $d_1(\mathbf{k}) = \sin k_x$, $d_2(\mathbf{k}) = \sin k_y$, and $d_3(\mathbf{k}) = 2B + M - B[\cos k_x + \cos k_y]$. Determine the phase diagram of the BHZ model depending on the parameter M/B.

Problem 3: Quantum spin Hall experiment

Read the experimental paper by Konig et al. in Science **318**, 766 (2007) and explain why the appropriate conductance measured in this paper is quantized to $2e^2/h$ and why this is a signature of the quantum spin Hall effect. Discuss the conditions for observing this quantized value.