Problem Set 1
Theoretical Solid State Physics (SoSe2017)

Due: In class exercise, Monday, April 24, 2017

Problem 1: Gauge transformations in quantum mechanics

In class, it was claimed that a gauge transformation generated by $\Lambda(\mathbf{r}, t)$ changes the quantum wavefunction according to

$$\psi \to \psi' = \psi e^{ie\Lambda/\hbar}. \tag{1}$$

Prove this statement.

Problem 2: Persistent currents in normal-metal rings

Consider a charged particle on a 1D ring of length L, threaded by an Aharonov-Bohm flux ϕ. Treat the ring as a 1D system of length L with periodic boundary conditions.

(a) Find an explicit expression for the vector potential.

(b) Solve the Schrodinger equation for the spectrum and the eigenfunctions. Show explicitly that the spectrum is periodic in flux with period $\phi_0 = h/e$.

(c) Calculate the current I_n associated with each of these eigenstates. Show that the current can be computed as the derivative of the energy with respect to the flux, $I_n = -\partial E_n/\partial\phi$.

(d) Calculate the current flowing around the ring at zero temperature and fixed number of particles. Show that the direction in which the current is flowing around the ring changes when adding a single electron to the ring. (The latter statement is true only in strictly one-dimensional rings.) Show that the magnitude is given by the electron charge divided by the time it takes an electron at the Fermi energy to propagate once around the ring.

(e) Read the theoretical and experimental papers quoted below.

This current is known as mesoscopic persistent current in normal metals. It is a current which flows in thermodynamic equilibrium and does not decay as a function of time. Its origin is the Aharonov-Bohm effect and there has been a long history of attempts to measure it experimentally. Notice that it cannot be measured directly by an Ampere-meter, but instead, one has to measure the magnetic field generated by this current. The best experiment to date is: Ania Bleszynski-Jayich et al., Science **326**, 272 (2009). Theoretically, its modern history starts with a paper by Buttiker, Imry, and Landauer, Phys. Lett. A **96**, 365 (1983), but there is a much earlier paper by Friedrich Hund (of Hund's rules etc.) which already contains some of the basic ideas [available at http://myweb.rz.uni-augsburg.de/ eckern/adp/history/historic-papers/].