

# Microwave photoconductivity of a modulated two-dimensional electron gas due to intra-Landau-level transitions

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Motivated by a recent experiment, we study the microwave-induced photoconductivity of a two-dimensional electron gas arising from intra-Landau-level transitions within a model where the electrons are subject to a unidirectional periodic potential in addition to a weaker impurity potential. With appropriate identifications, our results can be compared to experiment and allow us to explain the sign of the photocurrent, its dependence on magnetic field and microwave frequency, as well as the microwave-induced suppression of the Shubnikov-deHaas oscillations.

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## I. INTRODUCTION

Recently, magneto-oscillations of the microwave photoconductivity were discovered in ultra-high-mobility two-dimensional (2D) electron systems in the presence of a weak magnetic field.<sup>1-5</sup> For sufficiently strong microwave intensity, the longitudinal photoconductivity can become close to zero in certain magnetic field regions, with seemingly activated temperature dependence. Specifically, such so-called zero-resistance states occur whenever the microwave frequency  $\omega$  is related to the cyclotron frequency  $\omega_c$  as  $\omega = (k + \alpha)\omega_c$  ( $k=1, 2, 3, \dots$ ;  $\alpha$  is a constant phase shift).

Several mechanisms have been proposed to account for the photocurrent oscillations.<sup>6-19</sup> One mechanism is based on the observation that disorder-assisted microwave absorption is accompanied by a real-space displacement which, depending on the magnetic field, is preferentially along or against the applied dc electric field.<sup>8-12</sup> We refer to this mechanism as *displacement mechanism* (DP). A second contribution to the photoconductivity arises from the microwave-induced change in the electronic distribution function.<sup>4,13,14</sup> It turns out that typically, this *distribution-function mechanism* (DF) tends to dominate the magneto-oscillations of the photoconductivity in realistic samples,<sup>14</sup> although there are exceptions to this.<sup>17</sup> Zero-resistance states are expected to occur once the microwave-induced oscillations become so strong that the *microscopic* longitudinal conductivity is negative within certain magnetic-field regions.<sup>6</sup>

Recent experiments<sup>20,21</sup> focus on the regime  $\omega \ll \omega_c$ . Unlike in previous experiments, the microwave irradiation can then only induce *intra-Landau-level* (LL) transitions. According to Dorozhkin *et al.*,<sup>20</sup> a considerable microwave-induced reduction of the diagonal conductivity is observed in this regime, but it appears that no zero-resistance states were found. This overall reduction of the diagonal conductivity is accompanied by a significant suppression of the Shubnikov-deHaas oscillations.

In this paper, we investigate the effect of microwave-induced intra-Landau-level transitions on the photoconductivity within a model in which the two-dimensional electron gas (2DEG) is subjected to a unidirectional and *static* periodic potential. As shown in Ref. 17, this model allows one to

compute the photoconductivity using Fermi's golden rule which, in the appropriate geometry, leads to results which are parametrically consistent with those for disorder-broadened Landau levels. We find that our results are consistent with the principal experimental findings and predict a periodic-potential induced anisotropy of the intra-LL photocurrent.

We note that numerical results on the intra-LL photoconductivity, based on the theory of Ref. 14, were obtained in one of the experimental papers.<sup>20</sup> Other theoretical works on the intra-LL photoconductivity<sup>22,23</sup> focus on different scenarios for the photoconductivity.

This paper is organized as follows. In Sec. II we introduce the model and discuss the basic processes which contribute to the photoconductivity for  $\omega \ll \omega_c$ . In Sec. III, we explicitly compute the photoconductivity in this regime, including both the displacement and the distribution-function mechanism. Our results are compared to experiment in Sec. III D and summarized in Sec. IV.

## II. MODEL AND BASIC PROCESSES

We consider a two-dimensional electron gas subjected to a perpendicular magnetic field  $B$  and a unidirectional, static modulation potential  $V(\mathbf{r}) = V \cos(Qx)$  of period  $a = 2\pi/Q$ . We assume that the modulation potential  $V(\mathbf{r})$  exceeds the residual disorder potential  $U(\mathbf{r})$ , whose correlator  $W(\mathbf{r})$  falls off isotropically on the scale of the correlation length  $\xi$ . As appropriate for a high-mobility 2DEG, we assume a smooth disorder potential with correlation length  $\xi \gg \lambda_F$ . Our results for the photoconductivity will be obtained for modulation periods  $a$  satisfying the inequality  $a \ll \ell_B^2/\xi$ . ( $\lambda_F$  denotes the zero-field Fermi wavelength and  $\ell_B = (\hbar/eB)^{1/2}$  the magnetic length.)

The 2DEG is irradiated by microwaves described by the electric potential

$$\phi(\mathbf{r}, t) = -\frac{e}{2} \mathbf{r} \cdot (\mathbf{E}^* e^{i\omega t} + \mathbf{E} e^{-i\omega t}) = \phi_+ e^{-i\omega t} + \phi_- e^{i\omega t}, \quad (1)$$

where  $\phi_+ = [\phi_-]^* = -e\mathbf{E}\mathbf{r}/2$  and  $\omega > 0$ . We first consider linearly polarized microwaves whose polarization vector  $\mathbf{E} = E\hat{x}$  points along the  $x$  direction, i.e., parallel to the direction

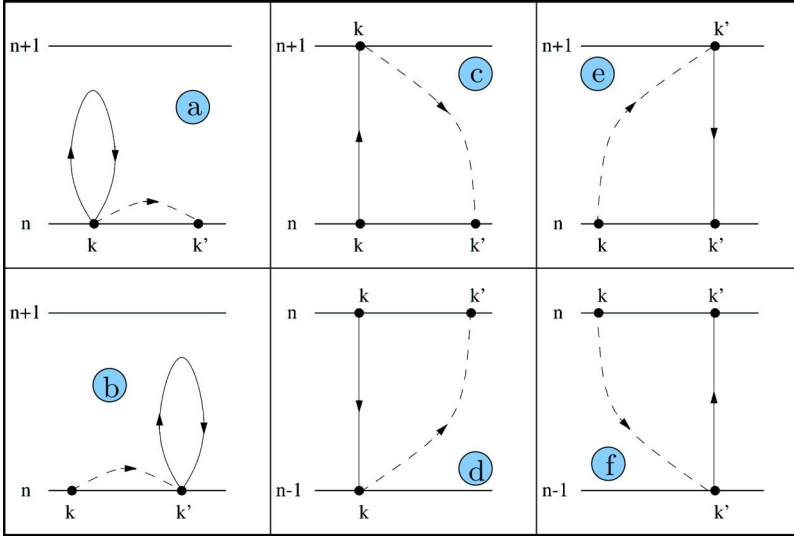


FIG. 1. (Color online) Relevant processes in the regime  $\omega \ll \omega_c$ . Solid lines represent microwave absorption ( $\phi_+$ ) or emission ( $\phi_-$ ) and dashed lines disorder scattering ( $U$ ). In processes (a) and (b), the intermediate states are in the same (valence) LL as the initial and final states. As shown in the text, these processes dominate the photocurrent. For processes (c)–(f), the LL index of the intermediate states differs by one from the LL index of initial and final states. Their contribution is smaller by a factor  $\omega/\omega_c$  than (a) and (b).

of modulation. Results for more general polarizations will also be given.

In the absence of disorder, the Landau level states  $|nk\rangle$  in the Landau gauge remain good eigenstates if the amplitude of the periodic modulation is small compared to the LL spacing  $\hbar\omega_c$ . ( $n$  denotes the LL index and  $k$  the momentum in  $y$  direction.) The corresponding eigenenergies are approximately given by  $\epsilon_{nk}^0 \approx \hbar\omega_c(n + \frac{1}{2}) + V_n \cos(Qk\ell_B^2)$ , where the modulation amplitude  $V_n$  is given by  $V_n \approx VJ_0(QR_c)$  for large LL index  $n$ . [ $R_c$  is the cyclotron radius and  $J_n(z)$  denotes a Bessel function.]

To identify the relevant microwave-induced processes, we discuss the matrix elements of  $U$  and  $\phi$ . For microwaves linearly polarized in the  $x$  direction, the matrix elements for absorption and emission are the same and given by

$$\langle n'k' | \phi_{\pm} | nk \rangle = -\frac{eE}{2} k \ell_B^2 \delta_{n,n'} \delta_{k,k'} + \frac{eER_c}{4} (\delta_{n,n'-1} + \delta_{n,n'+1}) \delta_{k,k'}. \quad (2)$$

Thus, microwaves leave the LL index unchanged or couple neighboring LLs. By contrast, the disorder potential has nonzero matrix elements between arbitrary LLs,

$$|\langle n'k' | U | nk \rangle|^2 \approx \int \frac{d^2q}{(2\pi)^2} \delta_{q_y, k'-k} [J_{|n'-n|}(qR_c)]^2 \tilde{W}(\mathbf{q}). \quad (3)$$

This expression is valid in the limit of large LL indices  $n, n' \gg 1$ . The dominant microwave-induced processes contributing to the photocurrent arise from the contributions

$$T_{\pm} = UG_0\phi_{\pm} + \phi_{\pm}G_0U \quad (4)$$

to the general  $T$  matrix

$$T = U + \phi + (U + \phi)G_0(U + \phi) + \dots \quad (5)$$

of the system.<sup>17</sup> Here,  $G_0$  denotes the retarded Green's function of the unperturbed system ( $U = \phi = 0$ ). We refer to  $T_+$  ( $T_-$ ) as disorder-assisted microwave absorption (emission).  $T_+$  and  $T_-$  can be considered separately, since they contribute

incoherently. The matrix elements of  $T_-$  can be shown to equal those of  $T_+$  up to a phase.

We now specialize to the regime  $\omega \ll \omega_c$  for well-separated LLs, which is the focus of this paper. In this case, only scattering processes with initial and final state in the same LL are relevant. Disorder-assisted microwave absorption and emission then proceeds via intermediate states either in the same LL (with amplitude  $M_0$ ) or in neighboring LLs (with amplitude  $M_1$ ), so that  $\langle nk' | T_{\pm} | nk \rangle = M_0 + M_1$ . These processes are depicted in Figs. 1(a), 1(b) and 1(c)–1(f) respectively. We find that the amplitude  $M_1$  is smaller than  $M_0$  in the parameter  $\omega/\omega_c$ . Thus, we first turn to the contribution  $M_0$  which, using Eq. (2), is equal to<sup>25</sup>

$$M_0 = \langle nk' | U | nk \rangle G_{0,nk}(\epsilon_{nk} + \omega) \langle nk | \phi_{\pm} | nk \rangle + \langle nk' | \phi_{\pm} | nk' \rangle G_{0,nk'}(\epsilon_{nk'}) \langle nk' | U | nk \rangle = \frac{eE}{2\omega} (k' - k) \ell_B^2 \langle nk' | U | nk \rangle. \quad (6)$$

Here we used that  $G_{0,nk}(\epsilon_{nk} + \omega) = (\epsilon_{nk} + \omega - \epsilon_{nk})^{-1} = \omega^{-1}$  and  $G_{0,nk'}(\epsilon_{nk'}) = (\epsilon_{nk} - \epsilon_{nk'})^{-1} = -\omega^{-1}$ . With Eq. (3), we obtain

$$|M_0|^2 \approx \left(\frac{eE}{2\omega}\right)^2 \int \frac{d^2q}{(2\pi)^2} \delta_{q_y, k'-k} [q_y \ell_B^2 J_0(qR_c)]^2 \tilde{W}(\mathbf{q}), \quad (7)$$

valid in the limit of high Landau levels.

We now turn to an estimate of the contribution  $M_1$ . The processes depicted in Figs. 1(c)–1(f) lead to

$$M_1 = \langle nk' | U | n+1k \rangle G_{0,n+1k}(\epsilon_{nk} + \omega) \langle n+1k | \phi_{\pm} | nk \rangle + \langle nk' | U | n-1k \rangle G_{0,n-1k}(\epsilon_{nk} + \omega) \langle n-1k | \phi_{\pm} | nk \rangle + \langle nk' | \phi_{\pm} | n-1k' \rangle G_{0,n-1k'}(\epsilon_{nk'}) \langle n-1k' | U | nk \rangle + \langle nk' | \phi_{\pm} | n+1k' \rangle G_{0,n+1k'}(\epsilon_{nk'}) \langle n+1k' | U | nk \rangle = \frac{eER_c}{4} \left[ \frac{\langle nk' | U | n+1k \rangle}{\omega - \omega_c} + \frac{\langle nk' | U | n-1k \rangle}{\omega + \omega_c} - \frac{\langle n-1k' | U | nk \rangle}{\omega - \omega_c} - \frac{\langle n+1k' | U | nk \rangle}{\omega + \omega_c} \right]. \quad (8)$$

At first sight, the ratio  $M_1/M_0$  is of order  $(R_c/q\ell_B^2)(\omega/\omega_c)$ . For smooth disorder,  $q \sim 1/\xi$ , so that  $M_1/M_0 \sim (k_F\xi)(\omega/\omega_c)$ , where  $k_F\xi \gg 1$ . This would imply that  $M_1$  can actually dominate over  $M_0$ . However, this estimate turns out to be too simplistic. The reason is that for  $\omega \ll \omega_c$ , we can write Eq. (8) as

$$M_1 = \frac{eER_c}{4\omega_c} \{ [\langle n-1k' | U | nk \rangle - \langle nk' | U | n+1k \rangle] + [\langle nk' | U | n-1k \rangle - \langle n+1k' | U | nk \rangle] \}. \quad (9)$$

We observe that the square brackets involve differences of matrix elements which differ by a uniform shift by one Landau level. This leads to a partial cancellation which reduces our previous estimate of  $M_1$  by  $q/k_F \sim 1/k_F\xi$ . As a result, we find that  $M_1/M_0 \sim \omega/\omega_c$  as claimed above.

### III. PHOTOCURRENT

#### A. Mechanisms

In perturbation theory, disorder-assisted microwave absorption and emission leads to two contributions to the photocurrent. First, it changes the electron momentum from  $k$  to  $k'$ , which effectively corresponds to real-space jumps in the  $x$  direction of length  $(k'-k)\ell_B^2$ . Due to the applied dc electric field, these jumps occur preferentially in a fixed direction. Generalizing the approach of Titeica<sup>24</sup> to the present situation, this displacement contribution to the longitudinal photocurrent can be expressed as<sup>17</sup>

$$j_x^{\text{DP}} = \frac{\pi e}{L_x L_y} \sum_{\sigma=\pm} \sum_n \sum_{k,k'} (k'-k)\ell_B^2 |\langle nk' | T_\sigma | nk \rangle|^2 [f_{nk}^0 - f_{nk'}^0] \times \delta(\epsilon_{nk} - \epsilon_{nk'} + \sigma\omega). \quad (10)$$

Here  $f_{nk}^0$  is the equilibrium electron distribution function and  $\epsilon_{nk} = \epsilon_{nk}^0 - eE_{dc}k\ell_B^2$  is the Landau level energy including the effect of the dc electric field.

Secondly, the microwaves change the electronic distribution function away from equilibrium. The resulting distribution-function contribution to the longitudinal photocurrent is<sup>17</sup>

$$j_x^{\text{DF}} = \frac{\pi e}{L_x L_y} \sum_n \sum_{k,k'} (k'-k)\ell_B^2 |\langle nk' | U | nk \rangle|^2 [\delta f_{nk} - \delta f_{nk'}] \times \delta(\epsilon_{nk} - \epsilon_{nk'}), \quad (11)$$

where  $\delta f_{nk} = f_{nk} - f_{nk}^0$  is the deviation of the nonequilibrium electron distribution function  $f_{nk}$  from the equilibrium distribution  $f_{nk}^0$ . The distribution function  $f_{nk}$  can be obtained from the kinetic equation

$$\frac{\partial f_{nk}}{\partial t} = \left( \frac{\partial f_{nk}}{\partial t} \right)_{\text{dis}} + \left( \frac{\partial f_{nk}}{\partial t} \right)_{\text{mw}} - \frac{f_{nk} - f_{nk}^0}{\tau_{\text{in}}}. \quad (12)$$

This kinetic equation includes collision integrals for disorder scattering,  $(\partial f_{nk}/\partial t)_{\text{dis}} = \sum_{n',k'} 2\pi |\langle n'k' | U | nk \rangle|^2 [f_{n'k'} - f_{nk}] \delta(\epsilon_{nk} - \epsilon_{n'k'})$ , and for disorder-assisted microwave absorption and emission,  $(\partial f_{nk}/\partial t)_{\text{mw}}$

$= \sum_{n',k'} \sum_{\sigma=\pm} 2\pi |\langle n'k' | T_\sigma | nk \rangle|^2 [f_{n'k'} - f_{nk}] \delta(\epsilon_{nk} - \epsilon_{n'k'} + \sigma\omega)$ . These collision integrals involve the electron energies including the effects of the dc electric field. Finally, we include inelastic relaxation within the relaxation-time approximation, with a phenomenological relaxation time  $\tau_{\text{in}}$ .

#### B. Longitudinal photocurrent

In this section, we compute the photocurrent for dc electric fields applied along the modulation direction. We first turn to the distribution-function mechanism which gives the dominant contribution in the (experimentally relevant) limit of slow inelastic relaxation. The microwave-induced change in the distribution function, as obtained from the kinetic equation (12), equals

$$\delta f_{Nk} = \tau_{\text{in}} \sum_{k'} \sum_{\sigma} 2\pi |\langle Nk' | T_\sigma | Nk \rangle|^2 (f_{Nk'}^0 - f_{Nk}^0) \times \delta(\epsilon_{Nk}^0 - \epsilon_{Nk'}^0 + \sigma\omega), \quad (13)$$

where  $N$  denotes the valence Landau level in which the Fermi energy is situated.  $\delta f_{nk}$  vanishes for all other Landau levels  $n \neq N$ . In the temperature regime  $\omega_c \gg T \gg V$ , the distribution function changes only weakly within the Landau level. Exploiting the  $\delta$  function, we can thus write  $f_{Nk'}^0 - f_{Nk}^0 \approx -\sigma\beta\omega n_F(\epsilon_{Nk}^0)[1 - n_F(\epsilon_{Nk'}^0)]$ , where  $\beta = 1/k_B T$  and  $n_F(\epsilon)$  denotes the Fermi-Dirac distribution. Noting that to leading order  $n_F(\epsilon_{Nk}^0)$  is just the partial filling factor  $\nu_N^*$  of the valence Landau level, we obtain the relation

$$f_{Nk'}^0 - f_{Nk}^0 \approx -\sigma\beta\omega\nu_N^*(1 - \nu_N^*). \quad (14)$$

Thus, the change in the distribution function is maximal for half-filled (valence) Landau levels and falls off to zero for empty and completely occupied Landau levels. Inserting the expression (7) for the matrix element and performing the sum over  $k'$ , we obtain

$$\delta f_{Nk} = -2\pi\tau_{\text{in}}\beta\omega\nu_N^*(1 - \nu_N^*) \left( \frac{eE}{2\omega} \right)^2 \sum_{\sigma} \int \frac{d\mathbf{q}}{(2\pi)^2} \times [q_y \ell_B^2 J_0(qR_c)]^2 \tilde{W}(\mathbf{q}) \delta(\epsilon_{Nk}^0 - \epsilon_{Nk+q}^0 + \sigma\omega). \quad (15)$$

The  $\mathbf{q}$  integration simplifies significantly in the limit  $\lambda_F \ll a \ll \ell_B^2/\xi$ , where it factorizes into an average over the  $\delta$  function and an integral over the remaining integrand. The average over the  $\delta$  function can be expressed through the Landau-level density of states,

$$\langle \delta(\epsilon_{Nk}^0 - \epsilon_{Nk+q}^0 + \sigma\omega) \rangle_{q_y} = 2\pi\ell_B^2 \nu^*(\epsilon_{Nk}^0 + \sigma\omega) \times \theta(V - |\epsilon_{Nk}^0 + \sigma\omega|). \quad (16)$$

Here, we introduced the LL density of states (DOS)  $\nu^*(\epsilon) = \nu^* \tilde{\nu}^*(\epsilon)$  where  $\nu^* = (1/2\pi\ell_B^2)(1/\pi V_N)$  denotes the DOS at the LL center and  $\tilde{\nu}^*(\epsilon) = 1/\{1 - [(\epsilon - E_N)/V_N]^2\}^{1/2}$  a normalized density of states. Exploiting the fact that the integrand of the remaining  $q$  integration is cut off by the correlator  $\tilde{W}(q)$  at large  $q$ , we can replace the Bessel function by an asymptotic expression for large argument. In this way, we

can relate the integral to the zero-field transport time, defined by

$$\frac{1}{\tau_{\text{tr}}} = \frac{1}{\pi v_F} \int_0^\infty dq (q^2 / 2k_F^2) \tilde{W}(q). \quad (17)$$

This yields for the change in the distribution function

$$\begin{aligned} \delta f_{Nk} = & -\beta \omega \nu_N^* (1 - \nu_N^*) \left( \frac{eER_c}{2\omega} \right)^2 \sum_\sigma \sigma \frac{\tau_{\text{in}}}{\tau_{\text{tr}}^*(\epsilon_{Nk}^0 + \sigma\omega)} \\ & \times \theta(V - |\epsilon_{Nk}^0 + \sigma\omega|). \end{aligned} \quad (18)$$

This expression should be inserted into Eq. (11). Here, we introduced the transport time  $\tau_{\text{tr}}^*(\epsilon) = \tau_{\text{tr}} \nu / \nu^*(\epsilon)$  in the presence of the magnetic field. ( $\nu$  is the DOS in the absence of the  $B$  field.) In a smooth random potential, this should be distinguished from the single-particle times  $\tau_s$  and  $\tau_s^*(\epsilon) = \tau_s \nu / \nu^*(\epsilon)$  in the absence and presence of  $B$ , respectively.

By a sequence of steps very similar to those for the evaluation of  $\delta f_{Nk}$  just described, we can rewrite Eq. (11) as

$$\sigma_{xx}^{\text{DF}} = \left[ e^2 \frac{R_c^2}{2\tau_{\text{tr}}^*} \nu^* \right] \frac{2\pi\ell_B^2}{L_x L_y} 2\pi V \sum_k \left( -\frac{\partial \delta f_{Nk}}{\partial \epsilon_{Nk}^0} \right) \tilde{\nu}^*(\epsilon_{Nk}^0). \quad (19)$$

Finally performing the sum over  $k$ , we obtain the result

$$\begin{aligned} \sigma_{xx}^{\text{DF}} = & -2\beta \omega \nu_N^* (1 - \nu_N^*) \left[ e^2 \frac{R_c^2}{2\tau_{\text{tr}}^*} \nu^* \right] \left( \frac{eER_c}{2\omega} \right)^2 \frac{\tau_{\text{in}}}{\tau_{\text{tr}}^*} \\ & \times B_1(\omega/2V_N), \end{aligned} \quad (20)$$

where<sup>17</sup>

$$B_1(x) = \frac{1}{16} \frac{1-2x}{(x-x^2)^{3/2}} \ln\left(\frac{V_N}{\Delta}\right). \quad (21)$$

Here,  $\Delta$  denotes an effective broadening in energy of the LL edge due to disorder or the dc electric field, both of which cut off the logarithmic divergence of the integral.

The sign and frequency dependence of the longitudinal distribution function contribution are therefore determined by the function  $-xB_1(x)$ . This function is plotted in Fig. 2. We find a negative photoconductivity in the frequency range  $\omega < V_N$ . For larger frequencies  $V_N < \omega < 2V_N$ , the sign of the photoconductivity changes. This sign change is a specific feature of our model, arising from the singular density of states at the band edge for the static periodic modulation potential. While this sign change is an interesting feature of our model and may be helpful in distinguishing between the displacement and the distribution function mechanism in an appropriate experiment, it is not expected to occur in a more generic situation without a singularity at the Landau level edge. Specifically, one expects a negative photoconductivity for all  $\omega < 2V_N$  for the case of disorder-broadened Landau levels, relevant to current experiments.

Up to numerical factors, the result Eq. (20) for the longitudinal distribution-function photocurrent differs from the corresponding result for *inter*-LL transitions (cf. Ref. 17) in two ways: The detuning  $\Delta\omega = \omega_c - \omega$  is replaced by  $-\omega$ , and there is an additional prefactor  $\beta\omega\nu_N^*(1-\nu_N^*)$ . The first change directly reflects the fact that initial, final, and inter-

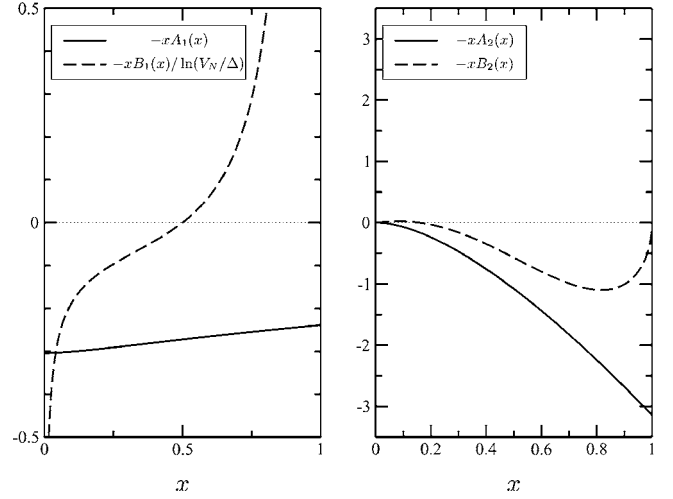


FIG. 2. Sign and frequency dependence of the photocurrent. Left panel: functions  $-xA_1(x)$  (solid line) and  $-xB_1(x)/\ln(V_N/\Delta)$  (dashed) determining the sign and frequency dependence of the displacement and distribution-function contributions to the longitudinal photoconductivities, respectively (with  $x = \omega/2V_N$ ). Right panel: the corresponding functions  $-xA_2(x)$  (solid line) and  $-xB_2(x)$  (dashed) for the transverse photocurrent.

mediate states are *all* in the same Landau level. The additional prefactor stems from the difference in the thermal populations of initial and final state (which was equal to unity for inter-LL transitions). Apart from these changes, the magnitude of the photocurrent can be estimated as explained in Ref. 17. Interestingly, we find that this provides a universal prescription to relate our results for the *inter*-LL photoconductivity to the *inter*-LL results of Ref. 17.

In accord with these remarks, starting from Eq. (10), we obtain

$$\sigma_{xx}^{\text{DP}} \propto -\beta \omega \nu_N^* (1 - \nu_N^*) \left[ e^2 \frac{R_c^2}{2\tau_{\text{tr}}^*} \nu^* \right] \left( \frac{eER_c}{2\omega} \right)^2 \frac{\tau_s^*}{\tau_{\text{tr}}^*} A_1(\omega/2V_N) \quad (22)$$

for the longitudinal displacement photoconductivity. We note in passing that the precise numerical prefactor of  $\sigma_{xx}^{\text{DP}}$  depends on the details of the smooth-disorder model. Clearly, this result is parametrically smaller than the distribution-function mechanism by a factor  $\tau_s^*/\tau_{\text{in}}^*$ , where  $\tau_s^*$  denotes the single-particle scattering time in the presence of the magnetic field. The function  $A_1$  appearing in Eq. (22) is given by<sup>17</sup>

$$A_1(x) = -\frac{3}{\pi^2} \frac{\partial}{\partial x} K(\sqrt{1-x^2}) \quad (23)$$

and the resulting frequency dependence is plotted in Fig. 2. Here,  $K(x)$  denotes a complete elliptic function.

The dominant distribution-function contribution to the longitudinal photocurrent is clearly independent of the microwave polarization. This is different from the cyclotron-resonance case,<sup>17</sup> where the photocurrent depends on the type of circular polarization. The (subdominant) displacement photocurrent does exhibit a polarization dependence.

Specifically, we find that for linear polarization along the  $y$  direction (i.e., perpendicular to the modulation direction), it includes an additional numerical factor of  $1/3$  compared to Eq. (22). For circularly polarized microwaves (of identical microwave power), the corresponding factor is  $2/3$ , irrespective of the type of circular polarization.

### C. Transverse photocurrent

It was shown in Ref. 17 that within our model, the contributions of the displacement and the distribution-function mechanisms to the transverse photocurrent can be of the same order of magnitude for the cyclotron resonance  $\omega \simeq \omega_c$ . We find the same conclusion to hold for intra-LL transitions.

The methods used in Ref. 17 can be readily extended to intra-LL transitions. The essential new ingredient in computing the distribution-function contribution is a drift term  $-eE_{dc}(\partial f_{nk}/\partial k)$  which enters the right-hand side of the kinetic equation, Eq. (12). In this way, we arrive at the result

$$\sigma_{yy}^{\text{DF}} = -4\beta\omega\nu_N^*(1 - \nu_N^*) \left[ e^2(v_y^2\tau_s^*)\tilde{v}^* \right] \left( \frac{eER_c}{2\omega} \right)^2 \frac{\tau_{\text{in}}^*}{\tau_{\text{tr}}^*} \times B_2(\omega/2V_N), \quad (24)$$

where  $B_2(\omega/2V_N)$  is given by<sup>26</sup>

$$B_2(x) = \left[ 4x \left( \arcsin(1 - 2x) + \frac{\pi}{2} \right) - 4\sqrt{x - x^2} \right]. \quad (25)$$

Computing the displacement contribution to the transverse photoconductivity requires one to evaluate transition rates between quantum states corresponding to the ‘‘meander’’ equipotential lines in the presence of both static periodic modulation and dc electric field. Following the relevant formalism developed in Ref. 17, we obtain

$$\sigma_{yy}^{\text{DP}} = -2\beta\omega\nu_N^*(1 - \nu_N^*) \left[ e^2(v_y^2\tau_s^*)\tilde{v}^* \right] \left( \frac{eER_c}{2\omega} \right)^2 \frac{\tau_{\text{in}}^*}{\tau_{\text{tr}}^*} \times A_2(\omega/2V_N) \left( \frac{E_{\text{dc}}^*}{E_{\text{dc}}} \right)^2. \quad (26)$$

The frequency dependence of the photocurrent is described by the function  $A_2$  given by

$$A_2(x) = 2xK(\sqrt{1 - x^2}). \quad (27)$$

A plot of the frequency dependence implied by the functions  $B_2$  and  $A_2$  is provided in Fig. 2.

Note the singular dependence of the displacement contribution to the transverse photoconductivity on the dc electric field  $E_{dc}$ . This singularity is cut off for small dc electric fields by inelastic processes when  $E_{dc} \sim E_{\text{dc}}^*$ , where  $E_{\text{dc}}^* = Ba/2\pi\sqrt{\tau_{\text{in}}\tau_s}$ .<sup>17</sup> For  $E_{dc} \ll E_{\text{dc}}^*$ , the photoconductivity crosses over to Ohmic behavior, matching with Eq. (26) for  $E_{dc} \sim E_{\text{dc}}^*$ .<sup>17</sup> This implies that the contributions by displacement and distribution mechanisms are of the same order of magnitude in the transverse case.

We finally remark that both displacement and distribution-function contribution to the intra-LL transverse photocurrent are independent of the type of polarization.

### D. Comparison with experiment

Strictly speaking, our model is different from the experimental system, due to the assumption of a static periodic potential. However, previous work<sup>17</sup> shows that the magnitude of the longitudinal photocurrent obtained within our model is parametrically identical to that for disorder-broadened Landau levels. Specific features arise within our model due to its anomalous density of states at the LL edge which leads to additional sign changes of the photocurrent.

Keeping these caveats in mind, we compare our results to the experiment of Ref. 20. Our main results relevant to experiment are: (i) When ignoring effects of the singular density of states at the LL edge, the sign of the photocurrent due to intra-LL transitions is negative, leading to a reduction of the experimentally observed resistivity. (ii) Comparing the longitudinal photoconductivity in Eq. (20) to the dark conductivity  $\sigma_{xx}^{\text{dark}} \sim e^2(R_c^2/2\tau_{\text{tr}}^*)\nu^*(\Gamma/T)$ , we find that their ratio depends on magnetic field as  $\sigma_{xx}^{\text{DF}}/\sigma_{xx}^{\text{dark}} \sim R_c^2/\Gamma\tau_{\text{tr}}^* \sim 1/B^2$  at fixed  $\omega$ . This magnetic-field dependence actually also holds for inter-LL processes.<sup>17</sup> Here, we used that both for disorder-broadened LLs as well as modulation-broadened LLs, the LL width  $\Gamma$  scales with magnetic field as  $\Gamma \sim \sqrt{B}$ . (iii) The amplitude of the photoconductivity due to intra-LL transitions scales as  $1/\omega$  with the microwave frequency, see Eq. (20). (iv) Due to the factor  $\nu_N^*(1 - \nu_N^*)$ , the magnitude of the effect is strongest for half-filled LLs and falls off to zero when the valence LL is either empty or completely occupied. We note that this filling-factor dependence is specific to intra-LL transitions and does not occur for inter-LL transitions near the cyclotron resonance or its harmonics.<sup>17</sup>

These results are in good agreement with the central experimental observations. (i) explains the sign of the effect. (ii) is in agreement with the observations that over the magnetic-field range where intra-LL processes dominate, the relative microwave-induced suppression of the conductivity decreases as the magnetic field increases (see Fig. 1 of Ref. 20). In addition, this magnetic-field scaling explains why experimentally, it is apparently harder to reach zero resistance in the regime of intra-LL transitions which occur at higher magnetic fields compared to the cyclotron resonance or its harmonics. (iii) is in accordance with the observation that the microwave-induced reduction of the diagonal resistivity decreases with increasing microwave frequency (see Fig. 2 of Ref. 20). Finally, (iv) implies that the photoconductivity suppresses the Shubnikov-deHaas oscillations, an effect which was very pronounced experimentally.

## IV. SUMMARY

We have studied the microwave photoconductivity of a 2DEG in a perpendicular magnetic field with additional unidirectional static periodic modulation in the regime of intra-LL transitions. We identify the dominant disorder-assisted microwave absorption and emission processes for this regime and compute both the longitudinal and transverse photocurrents.

We find that the distribution-function mechanism dominates for the longitudinal photocurrent while both distribution-function and displacement mechanism contrib-

ute to the same order to the transverse photocurrent. Except for subdominant contributions, we find that the photoconductivity due to intra-LL processes is polarization independent. The singular density of states of our model near the LL edges leads to interesting sign changes, making the photoconductivity positive in certain frequency ranges. With the exception of these model-specific predictions, our results are in good agreement with experiment. Specifically, we can ex-

plain the microwave-induced suppression of the Shubnikov-deHaas oscillations in the regime of intra-LL transitions.

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- <sup>25</sup>The divergence at  $\omega \rightarrow 0$  is an artifact of low-order perturbation theory in the disorder potential. In a more accurate treatment, it would be cut off by the disorder broadening.
- <sup>26</sup>The corresponding result in Ref. 17 erroneously contained an additional overall factor of 2.