# Fractional charges on an integer quantum Hall edge

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We propose ways to create and detect fractionally charged excitations in *integer* quantum Hall edge states. The charge fractionalization occurs due to the Coulomb interaction between electrons propagating on different edge channels. The fractional charge of the soliton-like collective excitations can be observed in time resolved or frequency dependent shot noise measurements.

*Introduction–* Fractionalization in low dimensional systems is a striking example of emergent behavior caused by strong correlations. Well known examples include fractionally charged excitations in one-dimensional charge density wave systems [1], and in the fractional quantum Hall (FQH) effect [2]. In particular, the detection of fractional charge by measuring shot noise in the point contact scattering current between FQH edge states [3, 4] made the latter a celebrated example.

Pham *et. al.* have predicted [5] that an electron injected into an interacting wire will be fractionalized into right and left moving excitations, each carrying a non-integer charge that depends on the Luttinger parameter $g$. The observation of this effect is a considerable challenge, because it occurs inside the interacting wire, while most measurements are made in the Fermi liquid leads. Strong evidence for electron fractionalization has recently been given in GaAs quantum wires [6, 7] by a clever analysis of transport measurements. However, a direct detection of the fractional charge is desirable.

In this Letter, we propose ways to create and detect excitations with well-defined fractional charges by injecting electrons into integer quantum Hall (IQH) edge states. Unlike the FQH case [3], the fractional charge of these collective excitations is not associated with fractional quasiparticles in the bulk, but rather results from Coulomb interactions between electrons on the edges [8, 9]. The role of the bulk integer QH state is to provide edge states which would form a chiral Fermi liquid [10] in the absence of Coulomb interactions. An important advantage of our IQH setting is the spatial separation of the edge states of opposite chirality, which allows separate access to each edge. For instance, the current can be injected into one edge, while the backscattered current is collected on the other.

We propose to detect the charge fractionalization by specific time resolved or finite frequency [8, 11, 12] shot noise experiments, which can directly measure the charge of the elementary carriers. To demonstrate this, we calculate explicitly the shot noise in the two proposed geometries shown in Fig. 1. We will now discuss these geometries in detail.

$\nu = 1$ *geometry-* (Fig. 1a) This geometry consists of a pair of counter propagating IQH edge states, which are close enough for significant inter-edge interactions in the center region, and a lead which injects electrons into one of the edge states via tunneling. For simplicity, we assume that the electron spins are completely polarized along the magnetic field. The pair of edge states can be modelled as a non-chiral Luttinger liquid (LL) with position dependent interaction parameter $g(x)$, which varies smoothly (on the scale of the magnetic length $\ell_B$) from $g_1 = 1$ to $g_2 < 1$, and back to $g_1$. The value of $g_2$ is determined by the strength of the inter-edge interaction in the center region. We assume that the inter-edge separation in the center region is large enough so that inter-edge tunneling is negligible, while small enough to allow for significant inter-edge interactions. This is possible in principle, since the tunneling is suppressed exponentially with the inter-edge distance $d$ [13], while the interaction decays only as a power law. The lead is biased with voltage $V_{\rm lead}$, relative to the upper (right moving) edge.

Fractionalization due to interactions in the central region manifests itself through the reflection of a well-defined fractional charge $q^* = re$, with $r = (1-g_2)/(1+g_2) < 1$, in the lower edge each time an injected electron hits the $x = 0$ boundary between the non-interacting and interacting regions (see Fig. 1). This is a consequence of the fact that the right moving eigenmode of the interacting region consists of electrons of *both* chiralities [5, 14]. In this region, the injected electron in the upper edge induces a "mirror" charge $-q^*$ on the lower edge [15]. Since charge is conserved on each edge separately (due to the absence of inter-edge tunneling), this requires a simultaneous reflection of charge $q^*$ in the lower edge.

It is important to emphasize that $r$ is *not* a quantum amplitude for electron reflection [7]. A fractional charge $q^*$ is reflected to the lower edge *each time* an electron tunnels in from the lead to the upper edge. In fact, $r$ is the reflection coefficient of the edge plasmon modes in the infinite wavelength limit.

The propagating mode in the interacting region is later partially reflected from the $x = \ell$ boundary in a similar process. This repeats alternately at the $x = 0$ and $x = \ell$ boundaries. Eventually, the net reflected charge in the lower edge is zero, and the net transmitted charge on the

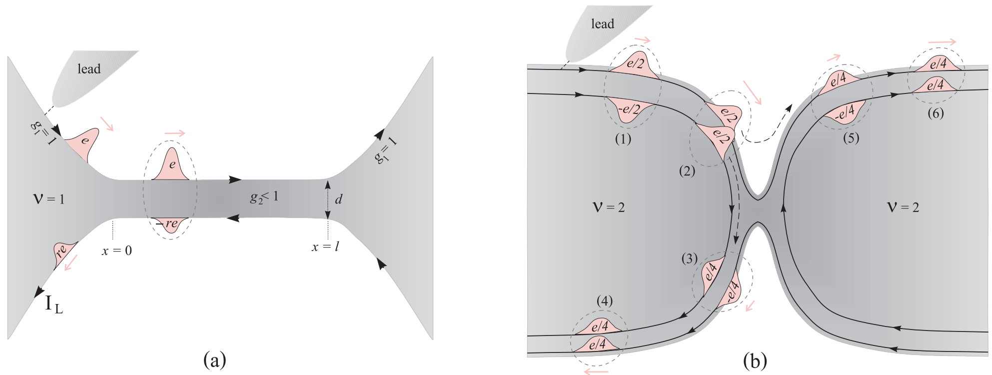


FIG. 1: (Color online.) Geometries of the proposed experiments. See text for details. (a) $\nu = 1$ geometry. The shaded region is an IQH bar. In the central narrow region inter-edge interactions are significant, leading to an interaction parameter $g < 1$, while everywhere else $g = 1$. (b) $\nu = 2$ geometry. In the narrow region, the inner edge mode is reflected, while the other is transmitted.

upper edge is $e$. This follows from the separate charge conservation laws on the two edges.

Clearly, in order to detect the excitations with a well defined charge $q^*$ created in the first reflection, it is necessary to avoid later reflections that carry a different charge. This can be done formally by sending $\ell$, the length of the interacting wire, to infinity, hence absorbing all the transmitted charge. We consider the noise in the reflected current at frequency $\omega$, defined as

$$S_L(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \left[ \langle \{ I_L(t), I_L(0) \} \rangle - 2 \langle I_L(0) \rangle^2 \right], \tag{1}$$

where $I_L$ is the left moving current and $\{...\}$ represents an anti-commutator. Taking the limit $\ell \to \infty$ first and then $\omega \to 0$, one expects the noise at temperature $T$ to take the form

$$S_\ell(\ell \to \infty, \omega \to 0) = S_{\rm tun}(\omega \to 0) + S_0(\omega \to 0). \tag{2}$$

Here, the noise due to the tunneling from the lead $S_{\rm tun}(\omega \to 0)$ has the form (assuming uncorrelated tunneling events)

$$S_{\rm tun}(\omega \to 0) = 2q^* \coth\left(\frac{eV_{\rm lead}}{T}\right) \langle I_L \rangle, \tag{3}$$

which depends explicitly on $q^* = re$. $S_0(\omega) = \frac{e^2}{2\pi}\omega \coth\left(\frac{\omega}{2T}\right)$ is the LL noise in the absence of tunneling [18]. We have set $\hbar = k_B = 1$. The main steps in the derivation of Eq. (3) will be outlined below.

In a finite size system, we propose two ways to observe the charge fractionalization. First, the fractionalization has imprints in the finite frequency noise of reflected current, similar to the case of an impurity in a LL [11]. Second, we propose a scheme for recovering Eq. (3) even for a finite system. The measurement is divided into cycles. In each cycle, $V_{\rm lead}$ is turned on for a time interval $\Delta T_0 \lesssim 2\ell/u$, where $u$ is the charge velocity in the interacting region, and then turned off. The backscattered current and noise are then measured over a time window which extends from $t = T_0$ to $t = T_0 + \Delta T_0$, where $T_0$ is the time interval between the tunneling of an electron from the lead and the arrival of a reflected charge to the detector in the lower edge. This ensures that only reflections from the $x = 0$ boundary are detected. The measurement is then stopped for a time interval of a few times $\Delta T_0$, during which the excess charge in the interacting region decays to a negligible value. The measurement cycle is then repeated. The noise averaged over many cycles should satisfy Eq. (3), from which the fractional charge $q^*$ can be extracted.

We now derive Eq. (3), as well as a general formula for the frequency dependent noise in the backscattered current. The system is described by the Hamiltonian

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_{\rm lead} + \mathcal{H}_{\rm tun}, \tag{4}$$

where $\mathcal{H}_1$ is the LL Hamiltonian

$$\mathcal{H}_1 = \int dx \Big\{ v\left[ (\partial_x \phi_R)^2 + (\partial_x \phi_L)^2 \right] + 2V(x)\, \partial_x \phi_L \partial_x \phi_R \Big\}. \tag{5}$$

Here $\phi_R$, $\phi_L$ are bosonic fields describing the two chiral edge modes, satisfying $\left[\phi_{R/L}(x), \phi_{R/L}(x')\right] = \pm \frac{i}{4} {\rm sgn}(x - x')$, where the upper (lower) sign corresponds to the right (left) moving field, and $[\phi_R(x), \phi_L(x')] = \frac{i}{4}$. $v = v_F + U$ where $v_F$ is the "bare" (noninteracting) Fermi velocity, and $U$, $V(x)$ are the intra-edge and inter-edge interaction strengths, respectively. We assume, for simplicity, that $U$ is position independent, and that $V(x) = 0$ for $x < 0$ and $x > \ell$. The Luttinger parameter $g(x)$ and the velocity $u(x)$ are related to the parameters in Eq. (5) by

$$g(x) = \sqrt{\frac{v - V(x)}{v + V(x)}}, \tag{6}$$

$$u(x) = \sqrt{v^2 - [V(x)]^2}. \tag{7}$$

$\mathcal{H}_{\text{lead}}$ is the lead Hamiltonian

$$\mathcal{H}_{\text{lead}} = \sum_k \varepsilon_k c_k^\dagger c_k, \tag{8}$$

and $\mathcal{H}_{\text{tun}}$ is the tunneling Hamiltonian between the lead and the upper (right moving) edge:

$$\mathcal{H}_{\text{tun}} = -\gamma \psi_R^\dagger (x_0) c(x_0) + h.c. \tag{9}$$

$c_k^\dagger$ is a creation operator of an electron in the lead, and $\psi_R^\dagger$ is the creation operator of a right moving electron. The tunnel junction is located at $x_0 < 0$.

In order to calculate the backscattered current $\langle I_L \rangle$ and the frequency dependent noise $S_t(\omega)$, we use the standard non-equilibrium Keldysh formalism [16]. Both the current and the noise are calculated to second order in the tunneling amplitude $\gamma$, assuming it is small (which is necessary to ensure Poisson distributed tunneling events). We omit the details of this calculation, which are similar to those of [18], and state only the results below. The backscattered current is

$$\langle I_L(x_1) \rangle = r(\omega \to 0) I_{\text{tun}}. \tag{10}$$

$r(\omega)$ is the frequency-dependent reflection coefficient

$$r(\omega) = -2iv_F \int_0^\infty dt e^{i\omega t} \langle [\phi_L(x,t), \phi_R(x_0,0)] \rangle. \tag{11}$$

$r(\omega)$ defined in Eq. (11) coincides with the reflection coefficient of the edges plasmon modes [8]. $I_{\text{tun}} = \frac{2\pi e^2 |\gamma|^2}{v_F} N(0) V_{\text{lead}}$ is the tunneling current [17], where $N(0)$ is the density of states of the lead at the Fermi energy.

The shot noise in the reflected current [after subtracting the thermal contribution $S_0(\omega)$] is

$$S_t(\omega) = 2er(\omega) r(-\omega) \coth\left(\frac{eV_{\text{lead}}}{T}\right) I_{\text{tun}} \tag{12}$$

where $r(\omega)$ is given by Eq. (11). We see that $S_t(\omega \to 0)$ satisfies Eq. (3).

As we noted before, for a finite length of the interacting region, $r(\omega \to 0) = 0$ and therefore both $\langle I_L(x_1) \rangle$ and $S_t(\omega \to 0)$ vanish. We demonstrate the signatures of charge fractionalization in the finite frequency noise by calculating the noise explicitly from Eq. (12) for the case of a "step" variation of the inter-edge interaction strength, *i.e.* $g(x) = g < 1$ for $0 < x < \ell$ and $g(x) = 1$ elsewhere. In this case, the reflection coefficient can be found analytically by considering the infinite sequence of reflections from the two boundaries. The time dependent reflection coefficient is [19]

$$r(t) = r_0 \delta(t) + t_0 r_0' t_0' \sum_{n=0}^\infty (r_0')^{2n} \delta[t - (n+1)\Delta T] \tag{13}$$

where $r_0 = \frac{1-g}{1+g}$ ($r_0' = \frac{g-1}{g+1}$) and $t_0 = \frac{2g}{1+g}$ ($t_0' = \frac{2}{1+g}$) are the reflection and transmission coefficients from the non-interacting to the interacting (interacting to non-interacting) boundary, respectively, and $\Delta T = \frac{2\ell}{u}$. Fourier transforming Eq. (13), we get

$$r(\omega) = r_0 \frac{1 - e^{i\omega \Delta T}}{1 - r_0^2 e^{i\omega \Delta T}}. \tag{14}$$

The resulting shot noise from Eq.(12) is peaked at $\omega = \frac{\pi}{\Delta T}$, and both its height and width depend on $g$.

In the above expression, the characteristic frequency of the noise spectrum is $\frac{1}{\Delta T} = u/\ell$. This sets the required time resolution for detecting charge fractionalization. Assuming that $\ell$ is as large as a few mm and $u \sim 10^5 - 10^6$m/s [21], the above characteristic frequency is of the order of $10^2$-$10^3$MHz.

We roughly estimate the typical values of the interaction parameter $g$, and hence the reflected fractional charge $q^*$. From Eq. (6), we need to estimate the intra-edge interaction $U$ and inter-edge interaction $V$. Measurements of the magnetoplasmon frequency on a single ($V = 0$) IQH edge [22] indicate that $U \geq v_F$. In order to estimate the value of $V/U$, we model the pair of edge states as cylindrical wires of radius $a \sim \ell_B$, at a distance $d$ apart. Assuming a screened Coulomb interaction with screening length $l_{sc} > d$, $U \sim \frac{e^2}{\varepsilon\pi} \ln \frac{l_{sc}}{a}$ and $V \sim \frac{e^2}{\varepsilon\pi} \ln \frac{l_{sc}}{d}$, where $\varepsilon$ is the dielectric constant of the surrounding semiconductor. For a rough feeling on the typical values of $g$, we use $l_{sc} = 2d$ and $d = 10a$, for which $\frac{V}{U} \simeq 0.3$. This gives (assuming that $U = v_F$, which yields an upper bound on $g$) $g \simeq 0.86$, and therefore $q^* = \frac{1-g}{1+g} e \simeq 0.075e$. Due to the logarithmic dependence of $U$ and $V$ on the geometrical parameters, $g$ is not extremely sensitive to the geometry as long as $l_{sc}$ is large enough. Similar estimates of $g$ were obtained in Ref. [23].

$\nu = 2$ *geometry-* This geometry, shown in Fig. 1b, consists of a $\nu = 2$ IQH liquid with two chiral edge modes of opposite spin. A constriction in the middle reflects only the inner edge mode, while the outer one is transmitted. We assume that the single-particle inter-channel scattering is negligible [24]. The two chiral edge modes are described by the Hamiltonian

$$\mathcal{H}_2 = \int dx \left[ \sum_{i=1,2} v_i (\partial_x \phi_i)^2 + 2V \partial_x \phi_1 \partial_x \phi_2 \right], \tag{15}$$

where $\phi_i$ ($i = 1, 2$) are the (chiral) bosonic fields for the outer and inner edge mode, respectively, $v_i = v_{F,i} + U_i$ where $v_{F,i}$ and $U_i$ are their Fermi velocity and intra-edge mode interaction, and $V$ is the interaction between the two modes.

As in the $\nu = 1$ case, electrons tunnel into the IQH edge from a lead. We assume that the electrons couple only to the outer ($i = 1$) edge mode. However, due to the inter-mode interaction, the eigenmodes of $\mathcal{H}_2$ are combinations of charge excitations on *both* edge modes. Therefore, the injected electron is decomposed into two eigenmodes [indicated as (1) and (2) in Fig. 1b]. For simplicity, let us consider the case $v_1 = v_2 \equiv v$. In this case, the charges of the two eigenmodes are $Q_\pm = (q^*, \pm q^*)$ where $q^* = \frac{e}{2}$ (the two components of $Q_\pm$ are the charges on the outer and inner edge modes, respectively), moving at velocities $u_\pm = v \pm V$. The faster even (+) mode reaches the point contact first. It then splits into two $\frac{e}{2}$ packets, one moving to the right and the other reflected to the lower, left moving edge. Both charge packets then split again, as indicated in Fig. 1b(3-6), into even (charge $\frac{e}{2}$) and odd (charge 0) modes, moving at velocities $u_\pm$.

The odd ($Q_-$) mode reaches the point contact later, and splits into a $-q^*$ packet scattered to the left and a $q^*$ packet transmitted to the right. Thus, as in the $\nu = 1$ case, the *net* effect (after a sufficiently long time) is the transmission of a single electron (charge $e$) to the right. The intermediate charge fractionalization can be detected by measuring either the finite frequency noise spectrum of the transmitted or reflected currents, or by performing a time resolved measurement, similar to the one described in the $\nu = 1$ case.

The finite frequency noise spectra in the transmitted and reflected currents $S_{r,t}(\omega)$ can be calculated very similarly to Eq. (12). The result is

$$S_{t,r}(\omega) = 2e\,|\alpha_{t,r}(\omega)|^2 \coth\left(\frac{eV_{\text{lead}}}{T}\right) I_{\text{tun}}, \qquad (16)$$

where $I_{\text{tun}}$ is the tunneling current from the lead, $\alpha_t(\omega) = \cos\left(\frac{\omega\Delta\tilde{T}}{2}\right)$ and $\alpha_r(\omega) = \sin\left(\frac{\omega\Delta\tilde{T}}{2}\right)$ are the Fourier transformed transmission and reflection coefficients. $\Delta\tilde{T} = \ell\left(\frac{1}{u_-} - \frac{1}{u_+}\right)$ where $\ell$ is the total length of the IQH edge from the lead to the detector.

In the more general case where $v_1 \neq v_2$, other values of $q^*$ can be obtained. The analysis in this case is straightforward, but slightly more involved, and will be presented elsewhere.

*Conclusions-* We propose ways to create and detect fractional charges on chiral edges of IQH liquids. The main advantage of using IQH edges for this purpose is their high controllability. In the proposed experiments, electrons are injected into IQH edges and “split” due to Coulomb interactions into fractionally charged packets. In the $\nu = 1$ setup, this occurs as a result of interactions between counter-propagating edge modes. In the $\nu = 2$ case, it occurs due to interactions between modes of the same chirality. (The latter case generalizes naturally to any $\nu \geq 2$ [25].) In all cases, the fractionalization is temporary, and after a sufficiently long time, a charge unity object is recovered. However, the fractional charges can be measured directly by using time resolved or finite frequency measurements.

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