Phase Transitions and Renormalization Group: From Theory to Numbers

Jean Zinn-Justin
Nuclear Research Centre, Saclay
jean.zinn-justin@cea.fr
http://www-centre-saclay.cea.fr/en

The large distance behaviour, near the critical temperature, of continuous phase transitions with short range interactions can be reproduced by an effective local field theory. Within this framework, the universal properties of the large distance can be determined by finding fixed points of general, functional renormalization group (RG) equations (Wilson, Wegner). For dimension $D > 4$, the large scale behavior is governed by the Gaussian fixed point (related to mean field theory), while for $D < 4$ the Gaussian fixed point is unstable. Near $D = 4$, the RG equations can be solved in the form of an $\epsilon = 4 - D$ expansion, a new stable fixed point is found and this leads to the famous Wilson-Fisher expansion for critical exponents and other universal quantities. Moreover, one realizes that, in this framework, the effective field theory can be reduced to a renormalizable field theory and, at leading order in the critical domain, the much simpler quantum field theory renormalization group can be used. A number of physicists, including the Saclay group (Brézin, Le Guillou, Zinn-Justin) have developed sophisticated field theory techniques, to prove general scaling laws and to calculate exponents to higher orders [1].

After the discovery that perturbative series and, thus, epsilon expansions are divergent, it has been understood that mathematical summation techniques have to devised to calculate precise and reliable numbers. Moreover, with an additional assumption, as an alternative to the epsilon expansion, one can use perturbation series directly in fixed dimension $D = 3$ (Parisi). The summation of $D = 3$ series obtained by Nickel have led to the most precise estimates (Le Guillou and Zinn-justin, Guida Zinn-Justin) for exponents and the equation of state of the Ising universality class.

[1] J. Zinn-Justin, *Phase Transitions and Renormalization Group*, Oxford University Press (2007)

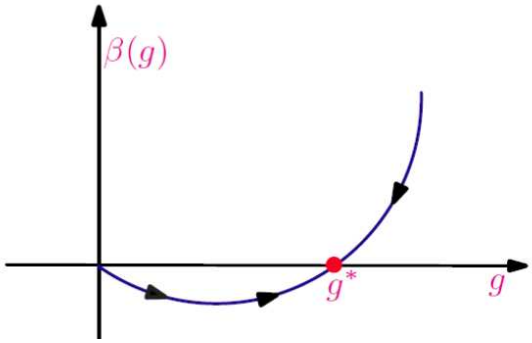

Figure 1: The renormalization group β-function for $D < 4$.