FROM INFINITIES IN QED TO THE GENERAL RENORMALIZATION GROUP

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Jean Zinn-Justin

IRFU/CEA,
Paris-Saclay University

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Without a minimal understanding of quantum (or statistical) field theory and renormalization group, the theoretical basis of a notable part of the physics of the second half of the twentieth century remains incomprehensible.

Indeed, quantum field theory, in its various forms, describes completely the physics of fundamental interactions at the microscopic scale, the singular properties of continuous phase transitions (like liquid–vapour, ferromagnetism, superfluidity, binary mixtures...) near a transition point, the properties of dilute quantum gases beyond the model of Bose–Einstein condensation, the statistical properties of long polymer chains (or self-avoiding random walk), percolation...

In fact, quantum field theory (QFT) provides, up to now, the most powerful framework in which the large scale properties of physical systems characterized by many strongly interacting, fluctuating degrees of freedom can be discussed.

However, at its birth, QFT has been confronted with a somewhat unexpected problem, the appearance of infinities: the calculation of physical processes was yielding, beyond leading order, infinite results. An empirical recipe, called renormalization, was eventually discovered, which allowed deriving finite predictions from divergent expressions.

The procedure would hardly have been convincing if the corresponding predictions would not have been confirmed with increasing precision by experiments.

A new concept, Renormalization Group (RG), first abstracted from formal properties of QFT, but whose deep meaning, in a more general form, was fully appreciated only in the general framework of continuous, macroscopic phase transitions (a process in which Wilson's contribution was essential), has led, later, to a satisfactory interpretation of renormalizable QFT and of the origin and meaning of the renormalization method.

Moreover, the emergence of renormalizable QFTs as a tool to describe large distance properties of critical phenomena, has led to the modern concept that QFT's, even in particle physics, are only effective large distance, low energy theories.

QED: a local quantum relativistic field theory

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QED, which describes, in a quantum relativistic framework, interactions between charged particles is not a theory of individualized particles, like in non-relativistic quantum mechanics, but a Quantum Field Theory. It is a quantum extension of a classical relativistic field theory: the dynamic variables are fields, like the electric and magnetic fields.

Such a theory differs drastically from a theory of particles in the sense that fields have an infinite number of coupled, fluctuating degrees of freedom, the values of fields at each point in space. The non-conservation of particles in high-energy scattering is a manifestation of this property.

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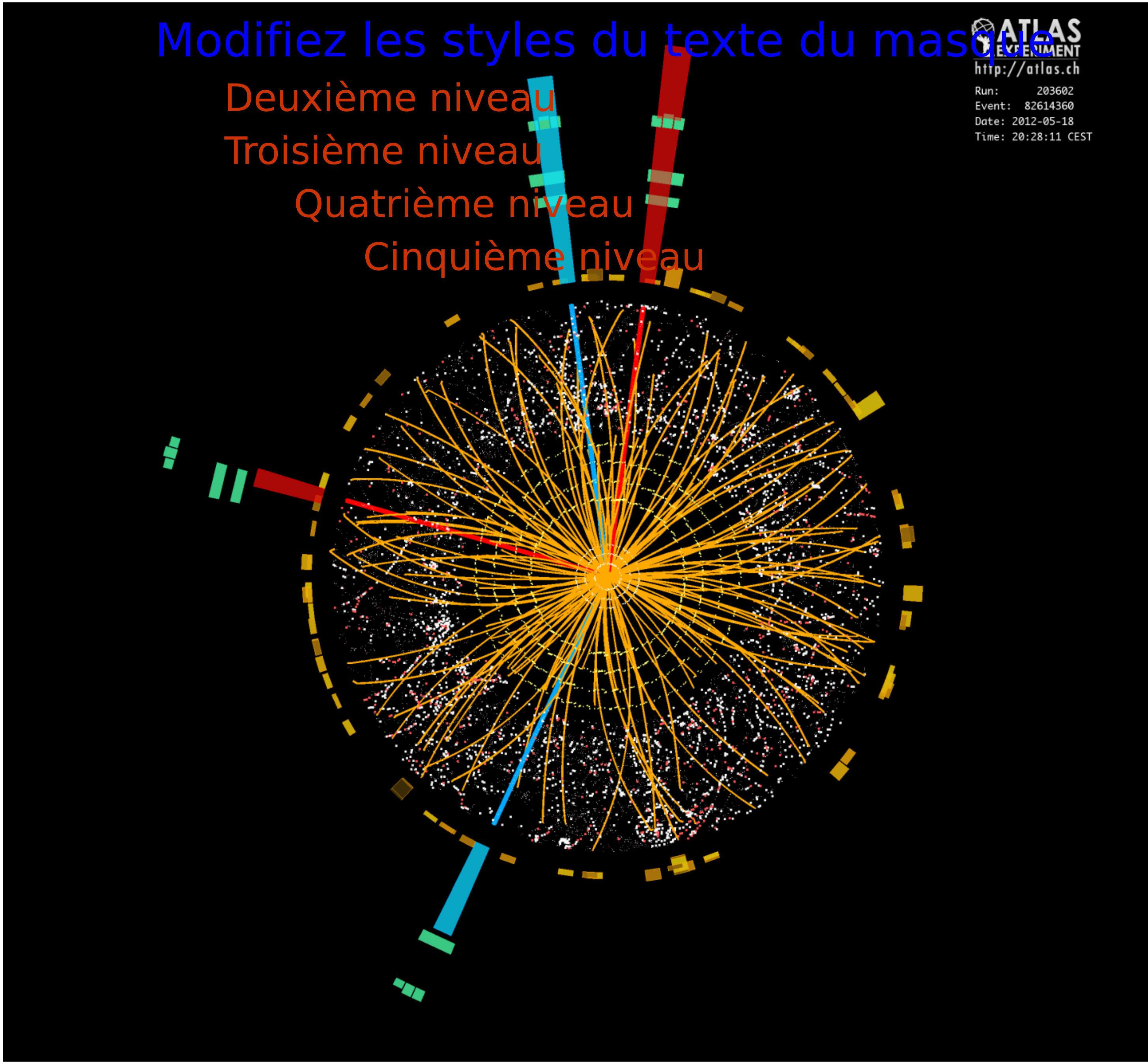
Higgs particle in the 4 e channel

Modifiez les styles du texte du masque

- Deuxième niveau
  - Troisième niveau
    - Quatrième niveau
      - Cinquième niveau



QED: a local quantum relativistic field theory

The quantum field theories that describe microscopic physics are local, an essential property and a generalization of the notion of point-like particles with contact interactions. In a straightforward interpretation, they do not possess any intrinsic short-distance structure.

The infinite number of fluctuating degrees of freedom combined with locality are the basic reasons why QFT's have somewhat unexpected new properties.

First calculations: the problem of infinities

Shortly after the work of Dirac, Heisenberg and Pauli, the first, but wrong, calculations of the order

$$\alpha = e^2/4\pi\hbar c \approx 1/137$$

(the fine structure constant) correction to the electron propagation in the photon field were published (Oppenheimer, Waller 1930). (The electric charge e being defined in terms of the Coulomb potential written as e2/R.)

One motivation: Cure the disease of the 'classical relativistic model' of the point-like electron.

If the electron is represented by a charged sphere of radius R, the contribution to its mass coming from the Coulomb self-energy diverges as

e2/R when R → 0.

The first (correct) QED calculation

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The first correct calculation was published by Weisskopf (1934) after in an erratum a last mistake pointed out by Furry was corrected.

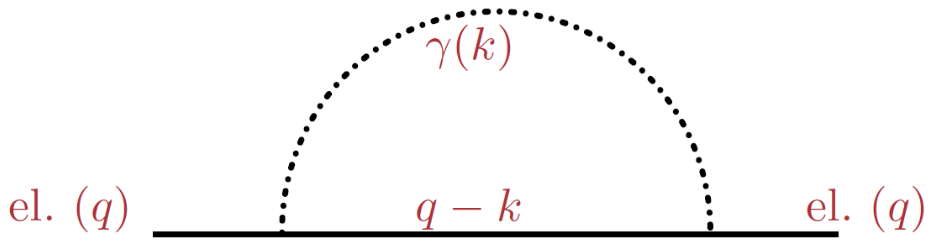

Figure 1: Electron propagation: dotted line for the photon and full line for the electron.

In terms of Feynman diagrams (a representation imagined only much later), the relevant physical process consists in the emission and re-absorption of a virtual photon of energy-momentum $\boldsymbol{k}$ by an electron of energy-momentum $\boldsymbol{q}$, (Fig. 1).

The problem of infinities

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Even though the linear classical divergence was replaced by a softer logarithmic UV divergence, the contribution to the electron mass was still infinite,

$$\delta m_{\mathrm{el.\,QED}} = -3\frac{\alpha}{2\pi} m_{\mathrm{el.}} \ln(m_{\mathrm{el.}} Rc/\hbar) \quad \text{with} \quad R = \hbar/c\Lambda_\gamma$$

It became slowly clear that the problem was very deep; these divergences seemed unavoidable consequences of locality (point-like particles with contact interactions) and unitarity (conservation of probabilities).

Indeed,

(i) one must sum over the contribution of virtual photons with arbitrarily high energies because there is no short-distance structure.

(ii) Due to conservation of probabilities, all processes contribute additively.

The problem of infinities

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These divergences seemed to indicate that QED was an incomplete theory, but it was hard to figure out how to modify it without giving up some fundamental physical principle.

Dirac (1942) proposed to abandon unitarity, but physical consequences seemed hardly acceptable.

A non-local extension (which would have given an inner structure to all particles) was hard to imagine in a relativistic context, though Heisenberg (1938) proposed the introduction of a fundamental length. In fact, only in the eighties were plausible candidates proposed in the form of string theories.

The problem of infinities

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Even more drastic: Wheeler (1937) and Heisenberg (1943) proposed to completely abandon QFT in favour of a theory of physical observables (scattering data): the so-called S-matrix theory, a somewhat desperate idea that nevertheless became very popular in the 1960's in the theory of Strong Interactions (responsible for strong nuclear forces).

Infinities and charged scalar bosons

More pragmatic physicists in the meantime explored the nature and form of divergences in quantum corrections, calculating different physical quantities.

An intriguing remark (Weisskopf 1939):

while in the case of charged fermions, the found logarithmic divergences are numerically acceptable if some reasonable momentum cut-off Λ can be found (a proposed candidate was the range of nuclear forces, about 100 MeV) because quantum corrections are proportional to $\alpha\ \mathbf{ln}(m/\Lambda)$,

but charged scalar bosons lead to large quadratic divergences, which are totally unacceptable because they would spoil the classical results.

Removing these divergences implies a fine-tuning of a parameter of the theory, equivalent to tuning the temperature to the critical temperature.

Infinities and charged scalar bosons

Thus, can scalar bosons be fundamental particles?

The problem remains very much relevant because the Standard Model of interactions at the microscopic scale involves a scalar particle, the Higgs boson, and, indeed, recently a boson with a 125 GeV mass has been discovered at the Large Hadron Collider (CERN) with the right properties.

It becomes even more acute when one contemplates the possibility that the Standard Model could be valid up to a possible grand unification (~ 1015 Gev) or to the gravitation (Planck's mass) (1019 GeV) scale.

Solving the fine tuning problem has been one motivation for introducing supersymmetry (a symmetry relating bosons and fermions). Supersymmetric particles are thus intensively searched at LHC, but with little success up to now.

The renormalization method

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An empirical observation. It was eventually noticed that, in some combinations of physical observables, divergences cancel (see, for instance, Weisskopf 1936) but the meaning of this observation remained obscure.

An essential experimental input. In 1947 Lamb et Retheford measured precisely the splitting between the levels 2s1/2-2p1/2 of the hydrogen atom, Rabi’s group in Columbia measured the anomalous magnetic moment of the electron.

The first QED results. Remarkably enough, it was possible to organize the calculation of the Lambshift in such a way that all infinities cancel (first approximate calculation by Bethe) and the result agreed beautifully with experiment. Shortly after, Schwinger obtained the leading contribution to the anomalous magnetic moment of the electron.

The renormalization procedure

Soon, the idea of divergence subtraction was generalized to the concept of renormalization (building up on work of Kramers) and in 1949 Dyson,

following work by Feynman, Schwinger and Tomonaga, gave the first proof that, after renormalization, divergences cancel to all orders of the perturbative expansion. The principles of renormalization theory were thus established.

The renormalization procedure

A momentum cut-off $c\Lambda$ is introduced, which modifies, in an unphysical (but not totally arbitrary) way the theory at a short distance of order $\hbar/c\Lambda$ and renders it finite.

One then calculates physical quantities as functions of the parameters of the Lagrangian, the `bare' charge e_0 and mass m_0 of the electron, as series in $\alpha_0 = e_0^2/4\pi\hbar c$.

In particular the physical charge e and mass m are given by

$$e^2/4\pi\hbar c \equiv \alpha = \alpha_0 - \beta_2\alpha_0^2 \ln(\Lambda/m_0) + \cdots,$$

$$m = m_0 - \gamma_1\, m_0\alpha_0 \ln(\Lambda C_1/m_0) + \cdots$$

Invert ... observables as functions of e and m.

Remarkably enough, the expansions of all other physical observables in powers of α have then a finite limit when Λ goes to infinity.

The renormalization procedure

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This *a priori* somewhat strange renormalization procedure, which in particular implies tuning the parameters of the Lagrangian as functions of the cut-off, had led to QED predictions that agree, with unprecedented precision, with experiment.

Moreover, the success of renormalization theory has led to the very important concept of renormalizable quantum field theories.

Since the renormalization procedure works only for a limited number of theories, looking for renormalizable field theories has strongly constrained the possible structure of new theories.

The nature of divergences and the meaning of renormalization

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Renormalized QED was obviously the right theory because predictions agreed with experiment, but why? A number of speculative answers were proposed.

- QED was an incomplete theory. The cut-off had a real physical meaning, being generated by additional interactions beyond QED (like Strong Interactions), but then the meaning of renormalizability, which reflected some form of short-distance insensitivity, had still to be understood.

This viewpoint is the closest to modern thinking, except that the cut-off is no longer linked to Strong Interactions but to a as yet unknown much higher energy scale.

The nature of divergences and the meaning of renormalization

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But a quite successful assumption, for some time, was:

QFT is only defined by renormalized perturbation theory: the procedure that generates the perturbative expansion has to be modified in order to generate automatically finite renormalized quantities (BPHZ).

The initial bare theory, based on a Lagrangian with divergent coefficients, is physically meaningless. It provided a simple book-keeping device to generate the physical, renormalized, perturbative expansion.

QFT and renormalization group

An intriguing consequence of renormalization in massless QED [Peterman–Stückelberg (1953), Gell-Mann–Low (1954), Bogoliubov–Shirkov (1955)]:

In a QED with massless electrons, the renormalized charge cannot be defined as usual in terms of the interaction between non-existing static electrons since massless particles propagate at the speed of light.

One must introduce some arbitrary mass or energy or momentum-scale μ and defines the renormalized charge e in terms of the strength of the e.m. interaction at scale μ : it characterizes the effective charge at scale μ. But then the same physics can be parametrized by the effective charge e' at another scale μ '. The set of transformations of physical parameters associated with this change of scale and required to keep physics constant has been called Renormalization Group (RG).

QFT and renormalization group

Moreover, one could show that in an infinitesimal scale change, the variation of the effective charge satisfies a differential (flow) equation of the form

$$\mu \frac{\mathrm{d}\alpha(\mu)}{\mathrm{d}\mu} = \beta\big(\alpha(\mu)\big), \quad \beta(\alpha) = \beta_2 \alpha^2 + O(\alpha^3) \text{ with } \beta_2 > 0\,,$$

(with β calculable in powers of α). In fact, even in a massive theory such a definition can be used.

Interpretation: At large distance, the strength of the electromagnetic interaction remains constant at the value measured through the Coulomb force. However, at distances much smaller that the wave length associated with the electron (one explore in some way the 'interior' of the particle), one observes screening effects. What is remarkable is that these short-distance screening effects are related to renormalization.

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QFT and renormalization group

Gell-Mann and Low’s initial hope was to determine the bare charge as the large momentum limit of the effective charge. It failed because the sign $\beta_2 > 0$ implies that the effective charge increases at short distance or large momentum (a phenomenon verified experimentally at the Z boson mass at CERN) until perturbation theory becomes useless.

A related issue: Landau’s ghost. A leading log summation of high energy contributions to the electron propagator exhibits an unphysical (a ghost) pole (Landau and Pomeranchuk (1955)) at a mass

$$M \propto m_{el} \exp(1/\beta_2 \alpha) \approx 1030 \mathrm{GeV}.$$

For Landau, this was a sign of QED inconsistency, but Bogoliubov and Shirkov noticed that this amounted to solving RG flow equation for α small and using it for α large.

The triumph of renormalizable QFT: the Standard Model

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The principle of renormalizability led, at the beginning of the 70s, to the construction of the Standard Model describing all interactions (but gravity) at the microscopic scale, based on non-Abelian gauge theories and spontaneous symmetry breaking.

In the sector of Strong Interactions, the negative sign of the RG β-function in Quantum Chromodynamics (QCD) (Gross–Wilczek, Politzer 1973), explained the weakness of interactions between quarks at short distance as seen in deep inelastic experiments, in a way consistent with quark confinement.

One remaining outstanding problem:

Failure up to now to include gravitation into the framework of renormalizable theories, has led to non-field theoretical extensions like String theories.

Critical phenomena: other infinities

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Second order or continuous macroscopic phase transitions, with short-range interactions, are characterized by a collective behaviour leading to correlations on large scales near the critical temperature.

The scale of these dynamically generated correlations is characterized by the correlation length, which diverges at the critical temperature. At the (large) scale of the correlation length, non-trivial macroscopic physics is observed due non-vanishing correlations.

The usual idea of scale decoupling then leads to expect that macroscopic properties could be described by a small number of well-chosen effective parameters, without explicit reference to the initial microscopic interactions.

Mean Field Theory and universality

This idea leads to Mean Field Theory and, in its more general form, to Landau's theory of critical phenomena (1937). Such a theory can also be called quasi-Gaussian (or perturbed Gaussian) with reference to the central limit theorem of probabilities.

Mean Field Theory leads to a number of super-universal predictions for the singular behaviour of thermodynamic quantities near T_c. For example, in all ferromagnetic systems, the magnetization vanishes like

$$M \propto \sqrt{T_c - T},$$

The correlation length diverges like

$$\xi \propto 1/\sqrt{T - T_c}$$

independently of the dimension of space, of symmetries, and the detailed form of the microscopic dynamics.

Beyond Mean Field Theory: infinities

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However, it came slowly apparent that these predictions disagreed with more precise experiments and lattice model calculations. They also disagreed with the exact partial solution of the 2D Ising model (Onsager 1944). Moreover, an attempt to calculate corrections to the Gaussian theory leads to infinities when the correlation length diverges, for all space dimensions $d \le 4$.

In fact, numerical investigations seemed to indicate that some universality survived but in a more limited form, the critical behaviour depending on the dimension of space as well as some general qualitative properties of models like symmetries but not on the detailed form of interactions.

The idea of universality classes emerged.

Scale Decoupling in Physics

A fundamental paradigm of physics: The decoupling of physics phenomena corresponding to very different length scales.

Classical example: From dimensional analysis, the period of the pendulum has the form

$$\tau \propto \sqrt{\ell/g}\ ,$$

ℓ being the size of the pendulum and g the gravitation constant.

Implicit assumption: sizes very different from the pendulum length like the size of atoms or the radius of the earth do not affect the period.

Similarly, orbits of planets can be determined quite precisely by replacing planets and the sun by point-like objects, and by neglecting all other stars in the galaxy.

Provided one is able to identify the relevant degrees of freedom and parameters, one can construct effective models adapted to the scale of the phenomena.

Scale Decoupling in Physics

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This empirical observation is essential for the predictivity of models. Indeed, if physics would be sensitive to all scales, prediction would be impossible since it would require a complete knowledge of all laws of nature.

However, in the 20th century, in two very different domains, this commonly accepted idea has been challenged:

The theory of fundamental microscopic interactions

The theory of continuous phase transitions (liquid-vapour, ferromagnetic, superfluid...).

In both situations, the INFINITE NUMBER of STRONGLY COUPLED MICROSCOPIC DEGREES of FREEDOM

cannot be replaced, in general, by a

SMALL NUMBER of EFFECTIVE MACROSCOPIC DEGREES of FREEDOM

Renormalization group

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To explain the compatibility between this non-decoupling of scales and, nevertheless, the relative insensitivity of large distance physics to microscopic properties, a new tool had to be invented:

- the RENORMALIZATION GROUP

RG has allowed not only understanding such a property, but also has inspired number of precise calculation methods.

RG also has provided a new interpretation of Quantum Field Theories as effective large distance or low energy theories

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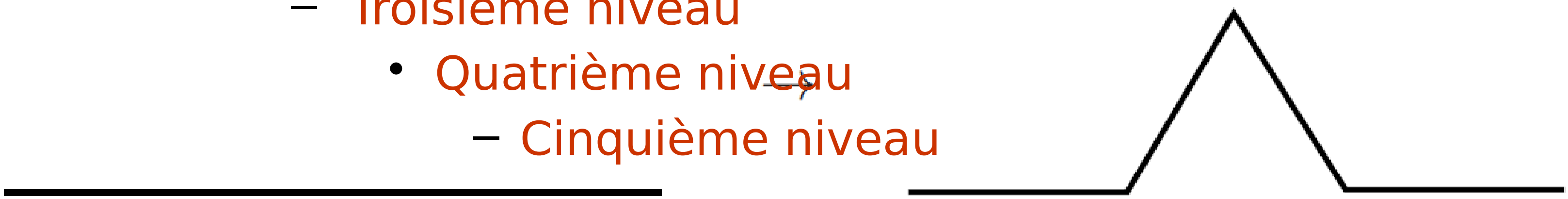
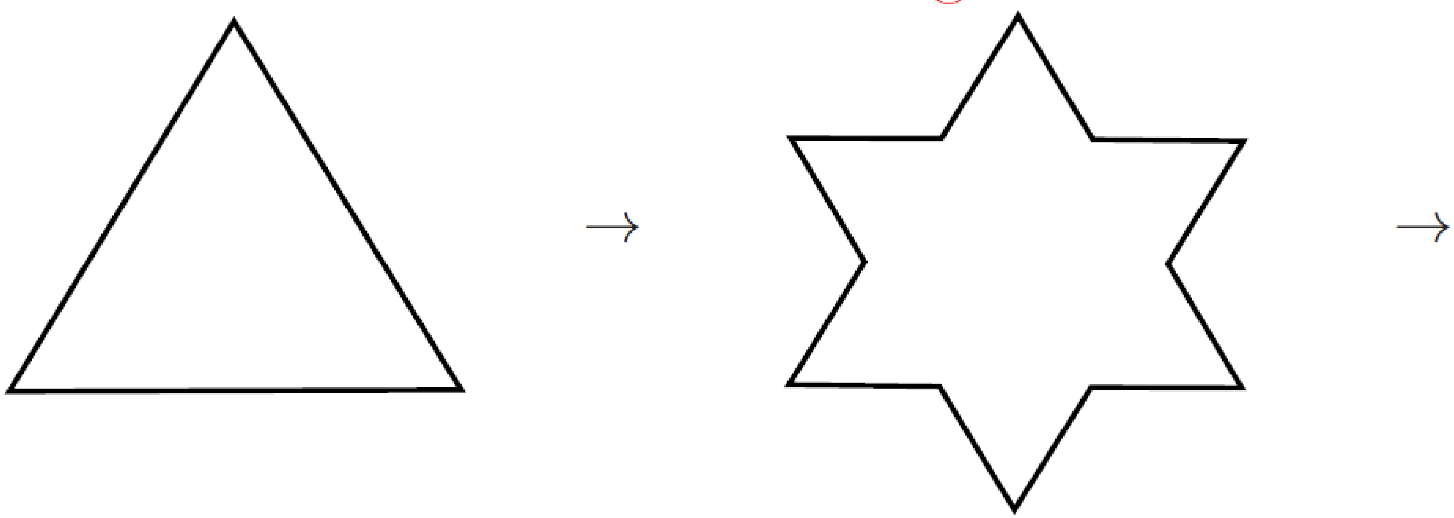
Fixed points, geometric analogies: straight line and fractals

- Modifiez les styles du texte du masque
  - Deuxième niveau
  - Troisième niveau
    - Quatrième niveau
      - Cinquième niveau

A Gaussian fixed point analogue: the straight line.



A RG transformation analogue.



A geometric fractal: a non-trivial fixed point analogue.

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The RG idea: example of ferromagnetic systems

We consider a partition function given by (ϕ here is an effective classical spin)

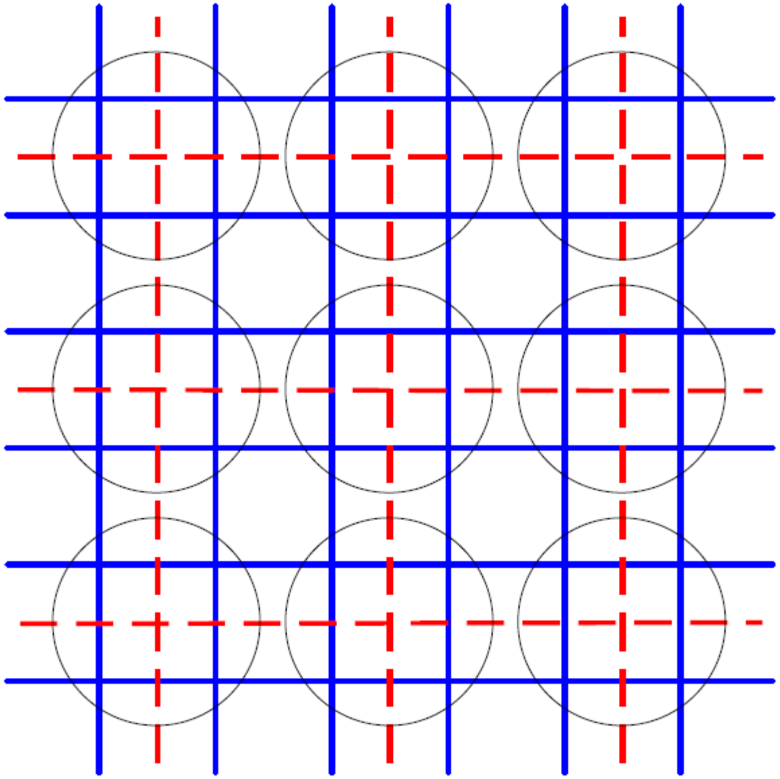
$$\mathcal{Z} = \sum_{\{\phi(\mathbf{n}a)\},\ \mathbf{n}\in\mathbb{Z}^d} \exp\left[-\mathcal{H}(\phi)\right].$$

To study large scale properties, as suggested by Kadanoff and Wilson one sums iteratively over short distance degrees of freedom, for example, over the initial spins with the constraint that the average value in a cell is fixed:

$$\phi(\mathbf{n}a) \mapsto \phi'(\mathbf{n}a) = \frac{\sqrt{Z}}{2d} \sum_{a\mathbf{n}' \ \text{neighbours of} \ 2a\mathbf{n}} \phi(a\mathbf{n}'),$$

where Z is a spin renormalization factor. This transformation leads to a new Hamiltonian, function on the average spin on a double size lattice.

The RG idea



Initial (blue) lattice with lattice size a and (red) lattice with size $2a$.

The iteration generates a renormalization group:

$$\mathcal{H}(\phi; 2^n a) = \mathcal{T}\left[\mathcal{H}(\phi; 2^{n-1} a)\right].$$

Fixed points

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One then looks for fixed points, solution of (this requires adjusting the value of the spin renormalization *Z*)

$$\mathcal{H}^*(\phi) = \mathcal{T}\left[\mathcal{H}^*(\phi)\right].$$

If attractive fixed points can be found, then

$$\mathcal{H}(\sqrt{Z}\phi; 2^n a) \underset{n\to\infty}{\to} \mathcal{H}^*(\phi).$$

The existence of attractive fixed points of an RG allows understanding universality of large distance physics, within universality classes, when scales do not decouple.

Fixed points (short range interactions)

Even if initially the spin variables take only discrete values and space is a discrete lattice, after a large number of iterations the effective spins can be replaced by continuous variables and the lattice by continuum space:

a fixed point Hamiltonian, if it exists, thus corresponds to a local statistical field theory (Wilson 1971) (a quantum field theory in imaginary time), the partition function being given by a field integral of the form

$$\mathcal{Z} = \int [\mathrm{d}\phi(x)] \exp\left[-\mathcal{H}(\phi)\right].$$

The Gaussian fixed point and the role of dimension 4

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The massless scalar field is a fixed point: the Gaussian fixed point, which corresponds to mean field theory:

$$\mathcal{Z} = \int [\mathrm{d}\phi(x)] \exp\left[-\mathcal{H}_{\mathrm{G}}(\phi)\right], \quad \mathcal{H}_{\mathrm{G}}(\phi) = \frac{1}{2}\int \mathrm{d}^d x \left(\nabla_x \phi(x)\right)^2,$$

It becomes unstable when the dimension ***d*** of space is lower than 4. The first instability is induced by the perturbation

$$\int \mathrm{d}^d x\, \phi^4(x).$$

Near dimension 4, ϕ^4tical phenomena can thus be described by a renormalizable Qfield theory with a natural cut-off, reflection of the initial short distance structure.

General Renormalization Group in the continuum

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In the continuum, general RG flow equations for the effective Hamiltonian at scale $\boldsymbol{\Lambda}$ can be written in the form

$$\Lambda \frac{\mathrm{d}}{\mathrm{d}\Lambda}\mathcal{H}(\phi,\Lambda) = \frac{1}{2}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\tilde{D}_\Lambda(k)\left[\frac{\delta^2\mathcal{H}}{\delta\tilde{\phi}(k)\delta\tilde{\phi}(-k)} - \frac{\delta\mathcal{H}}{\delta\tilde{\phi}(k)}\frac{\delta\mathcal{H}}{\delta\tilde{\phi}(-k)}\right]$$

$$+\int \frac{\mathrm{d}^d k}{(2\pi)^d}\tilde{L}_\Lambda(k)\frac{\delta\mathcal{H}}{\delta\tilde{\phi}(k)}\tilde{\phi}(k).$$

The renormalization group of QFT then appears, in the general RENORMALIZATION GROUP framework, as an asymptotic renormalization group in the neighbourhood of the Gaussian fixed point.

QFT and general Renormalization Group

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In the neighbourhood of the Gaussian fixed point, all interactions corresponding to stable directions in Hamiltonian space at the Gaussian fixed point can be neglected (irrelevant interactions).

The initial general RG with its infinite number of parameters is reduced to an RG with the finite number of renormalizable couplings.

It is possible to search for new fixed points close to the Gaussian fixed point.

Dimensional continuation allows working in $d=4-\varepsilon$ dimension. RG fixed points of order ε have been found leading to the famous $\varepsilon=4-d$ expansion (Wilson-Fisher).

Fixed dimension perturbative expansion has also be used, relying on an additional assumption.

Critical phenomena: explicit calculations

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The QFT renormalization group has led to precise determinations of various physical quantities like critical exponents (Le Guillou and Zinn-Justin, Guida and Zinn-Justin) and equation of state (Guida and Zinn-Justin) for Ising-like systems, in very good agreement with values extracted from lattice models and experiments.

This success, in turn, has confirmed that, remarkably enough, large distance physics, near a continuous phase transition in systems with short range interactions, can be described by local renormalizable quantum field theories.

Critical exponents from the $O(N)$ symmetric $(\phi^2)^2_3$ field theory

N	0	1	2	3
$\tilde{g}^*$	1.413 ± 0.006	1.411 ± 0.004	1.403 ± 0.003	1.390 ± 0.004
g^*	26.63 ± 0.11	23.64 ± 0.07	21.16 ± 0.05	19.06 ± 0.05
γ	1.1596 ± 0.0020	1.2396 ± 0.0013	1.3169 ± 0.0020	1.3895 ± 0.0050
ν	0.5882 ± 0.0011	0.6304 ± 0.0013	0.6703 ± 0.0015	0.7073 ± 0.0035
η	0.0284 ± 0.0025	0.0335 ± 0.0025	0.0354 ± 0.0025	0.0355 ± 0.0025
β	0.3024 ± 0.0008	0.3258 ± 0.0014	0.3470 ± 0.0016	0.3662 ± 0.0025
α	0.235 ± 0.003	0.109 ± 0.004	-0.011 ± 0.004	-0.122 ± 0.010
ω	0.812 ± 0.016	0.799 ± 0.011	0.789 ± 0.011	0.782 ± 0.0013
$\omega\nu$	0.478 ± 0.010	0.504 ± 0.008	0.529 ± 0.009	0.553 ± 0.012

Reference: R. Guida and J. Zinn-Justin, *J. Phys. A* 31 (1998) 8103, cond-mat/9803240, an improvement over the results published in

J.C. Le Guillou and J. Zinn-Justin, *Phys. Rev. Lett.* 39 (1977) 95; *Phys. Rev.* B21 (1980) 3976.

Effective Quantum Field Theories

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The condition that microscopic physics should be describable by a renormalizable QFT has been one of the basic principles that have led to the Standard Model of microscopic interactions. From the success of the program, one might have concluded that renormalizability was a new law of nature. The implication would have been that all interactions including gravity should be describable by renormalizable QFT. The failure to exhibit a renormalizable version of quantum gravity has shed some doubt on such a viewpoint.

Indeed, if the Standard Model and its envisaged QFT extensions are only low energy approximations, it becomes difficult to understand why they should obey such an abstract principle.

Effective Quantum Field Theories

By contrast, the theory of critical phenomena shows that a dynamical generation of a large scale may generate a non-trivial large distance physics, which can be described by a renormalizable QFT. This provides a simpler and more natural explanation for the appearance of renormalizable QFT's in physics. One can thus speculate that fundamental interactions are described at some more microscopic scale (like the Planck length) by a finite theory that has no longer the nature of a local quantum field theory. Although such a theory involves only some short microscopic scale, for reasons that can also only be a matter of speculation, it generates strong correlations between a large number of degrees of freedom and a large distance physics with very light particles.

From this viewpoint, the fine tuning of the Higgs bare mass becomes an essential physical issue.

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A few useful dates

1925 Heisenberg formulates the basis of Quantum Mechanics as a mechanics of matrices

1926 Schrödinger publishes his famous equation, which bases Quantum Mechanics on the solution of a non-relativistic wave equation.

Indeed, for accidental reasons, the spectrum of the hydrogen atom is better reproduced by the non-relativistic Schrödinger equation than by the relativistic spinless Klein-Gordon equation.

1928 Dirac introduces his famous equation, a relativistic wave equation that incorporates the spin 1/2 of the electron, and leads to a spectrum of the hydrogen atom in much better agreement with experiment, and this opens the way for a relativistic quantum theory.

1929--1930, Heisenberg and Pauli establish the general principles of Quantum Field Theory.

A few useful dates

1934 First correct calculation of a Quantum Electrodynamics (QED, the theory of charged particles with electromagnetic interactions) correction by Weisskopf and confirmation of the appearance of infinities, called UV divergences (since due, in this case, to very short wave length photons).
1937 Landau publishes his general theory of phase transitions.
1944 Exact (partial) solution of the two-dimensional Ising model by Onsager.
1947 Measurement of the Lambshift and surprising agreement with the QED prediction, after cancellation of infinities between physical observables.
1947--1949 Development of a general empirical strategy to eliminate divergences called Renormalization.
1954--1956 Discovery of a formal property of massless QED, called renormalzation group, whose deeper meaning is not fully understood Peterman--Stückelberg, Gellman--Low, Bogoliubov--Shirkov

A few useful dates

1967-1975 The Standard Model (Glashow, Weinberg, Salam), a renormalizable Quantum Field Theory based on the concept of non-Abelian gauge symmetry (Yang-Mills 1954) and spontaneous symmetry breaking (Higgs... 1964), is formulated, quantized (Faddeev--Popov, DeWitt) and shown to be consistent ('t Hooft-Veltman, Lee-Zinn-Justin). With minor modifications (neutrino masses and oscillations), it still describes with remarkable precision all fundamental interactions, except gravitation.

1971--1972 Inspired by some premonitory ideas of Kadanoff, Wilson, Wegner... develop a more general Renormalization Group, based on the iterative integration over short-distance degrees of freedom, which includes the field theory RG in some limit, and which is able to explain universal non mean-field (or non quasi-Gaussian)-like properties of continuous phase transitions} (like liquid--vapour, binary mixtures, superfluidity, ferromagnetism) or statistical properties of long polymeric chains.

A few useful dates

1972-1975 Several groups, including Brézin, Le Guillou and Zinn-Justin, develop efficient quantum field techniques to prove universality and calculate universal quantities.
1973 Politzer, Gross-Wilczek establish the asymptotic freedom of a class of non-Abelian gauge theories, which provides a RG explanation to the free particle behaviour of quarks at short distance inside nucleons.
1975-1976 Additional insight in the universal properties of phase transitions is provided by the study of non-linear \sigma model} and the (d-2) expansion (Polyakov, Brézin-Zinn-Justin)
1977-The calculation of the RG functions by field theory techniques, using a perturbative expansion within the Callan-Symanzik scheme, as suggested by Parisi, is initiated by Nickel and first precise estimates of critical exponents, based on large order behaviour estimates of perturbation theory, Borel summation and conformal mapping, are published by Le Guillou and Zinn-Justin.
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The nature of divergences and the meaning of renormalization

Renormalized QED was obviously the right theory because predictions agreed with experiment, but why? Several speculative answers were proposed, for example:

- Divergences were a disease of the perturbative expansion in $\alpha 0$ and a proper mathematical handling of the theory with non-perturbative input would free it from infinities.
- More drastic, the problem was fundamental: QFT was only defined by perturbation theory but the procedure that generated the perturbative expansion had to be modified in order to generate automatically finite renormalized quantities (BPHZ). The initial bare theory, based on a Lagrangian with divergent coefficients, was physically meaningless. It provided a simple book-keeping device to generate the physical, renormalized, perturbative expansion.