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Ultracold quantum gases in optical lattices

solid-state physics with atoms in crystals made of light

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International school on phase transitions, Bavaria

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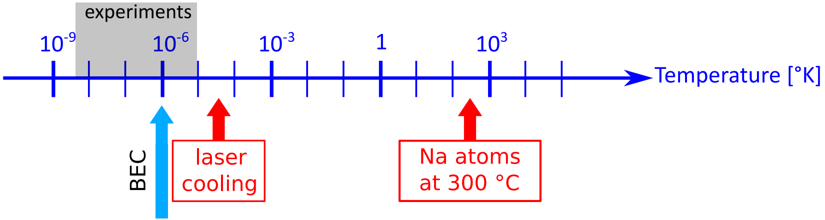
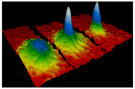
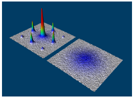
Laser cooling and trapping successes in the 80's-90's lead to ever colder and denser atomic samples [Chu, Cohen-Tannoudhi, Phillips : Nobel 1997]





Quantum degenerate gases:

- Bose-Einstein condensation in 1995 [Cornell, Wieman, Ketterle : Nobel 2001]
- Degenerate Fermi gases in 2001 [JILA]
- Superfluid-Mott insulator transition for bosons in 2002 [Munich]



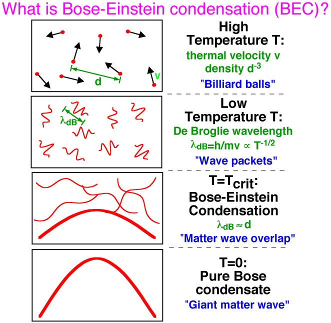
Quantum degeneracy :

phase space density $n\lambda_{\mathrm{dB}}^3 > 1$

n :spatial density

$\lambda_{\mathrm{dB}} = \sqrt{\frac{2\pi\hbar^2}{mk_BT}}$: thermal De Broglie wavelength

From W. Ketterle group website, http://www.cua.mit.edu/

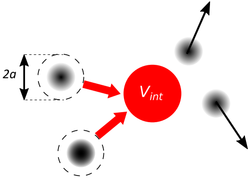


Interacting atoms, but dilute gas: $na^3 \ll 1$

a : scattering length for $s-$wave interactions

$8\pi a^2$: scattering cross-section (bosons)

$$a \ll n^{-1/3} \ll \lambda_{\mathrm{dB}}$$



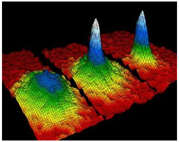
Typical values (BEC of ^{23}Na) : $a \sim 2\,\mathrm{nm}$, $n^{-1/3} \sim 100\,\mathrm{nm}$, $\lambda_{\mathrm{dB}} \sim 1\,\mu\mathrm{m}$ at $T = 100\,\mathrm{nK}$

Ultracold atomic gases as model systems for many-body physics :

- dilute but *interacting* gases
- tunability (trapping potential, interactions, density, ...) and experimental flexibility
- microscopic properties well-characterized
- well-isolated from the external world

Bose-Einstein condensates :

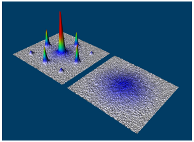
Superfluid gas
"Atom laser"



JILA, MIT, Rice (1995)

Optical lattices :

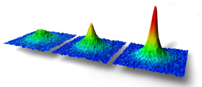
Superfluid-Mott insulator transition



Munich 2002

BEC-BCS crossover :

Condensation of fermionic pairs

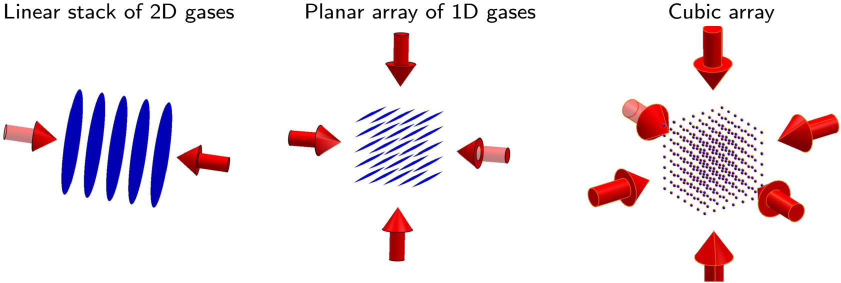


JILA, MIT, ENS (2003-2004)

Many other examples :

- gas of impenetrable bosons in 1D,
- non-equilibrium many-body dynamics,
- disordered systems, ...

A coherent superposition of waves with different wavevectors results in interferences.
The resulting interference pattern can be used to trap atoms in a periodic structure.



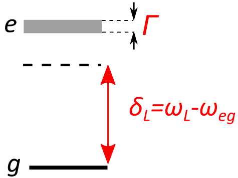
Additional, weaker trapping potentials provide overall confinement of the atomic gas.
Many more geometries are possible by “playing” with the interference patterns.

Why is this interesting ?

1. Connection with solid-state physics (band structure and related phenomenon)
2. A tool for atom optics and atom interferometry: coherent manipulation of external degrees of freedom
3. Path to realize strongly correlated gases and new quantum phases of matter

Two-level atom interacting with a monochromatic laser field:

- Electric field : $\boldsymbol{E} = \frac{1}{2}\boldsymbol{\mathcal{E}}(\boldsymbol{r})e^{-i\omega_L t+i\phi} + \mathrm{c.c.}$
- Γ: transition linewidth
- d: electric dipole matrix element
- $\delta_L = \omega_L - \omega_{\mathrm{eg}}$: detuning from atomic resonance



For large detuning, the atom-laser interaction can be pictured using the Lorentz model of an elastically bound electron.

The electric field induces an oscillating electric dipole moment $\boldsymbol{d} \propto \boldsymbol{\mathcal{E}}$.

The (time-averaged) potential energy of the induced dipole is

$$V(\boldsymbol{r}) = -\frac{1}{2}\langle \boldsymbol{d}\cdot\boldsymbol{E}\rangle = \frac{d^2|\boldsymbol{\mathcal{E}}(\boldsymbol{r})|^2}{4\hbar\delta_L}$$

- $\delta_L < 0$ (red detuning) : $V < 0$, atoms attracted to intensity maxima
- $\delta_L > 0$ (blue detuning) : $V > 0$, atoms repelled from intensity maxima

Potential energy :

$$V(\boldsymbol{r}) = \frac{d^2}{4\hbar\delta_L}|\boldsymbol{\mathcal{E}}(\boldsymbol{r})|^2$$

Take ^{87}Rb atoms for definiteness :

- $\Gamma/2\pi \approx 6\,$MHz,
- $\lambda_{eg} \approx 780\,$nm [$\omega_L/2\pi \approx 4 \times 10^{14}$ Hz]
- $\lambda_0 \approx 1064\,$nm, $\delta_L/2\pi \approx -3 \times 10^{13}$ Hz,
- Laser parameters : power $P = 200\,$mW, beam size $100\,\mu$m]

One finds a potential depth $|V| \sim h \times 20\,$kHz $\sim k_B \times 1\,\mu$K.

Trapping atoms in such a potential requires sub-μK temperatures, or equivalently degenerate (or almost degenerate) gases.

NB : trap depths up to mK are possible using tightly focused lasers.

Superposition of mutually coherent plane waves :

$$|\boldsymbol{\mathcal{E}}(\boldsymbol{r})|^2 = \left|\sum_n \boldsymbol{\mathcal{E}}_n e^{i\boldsymbol{k}_n\cdot\boldsymbol{r}}\right|^2 = \left(\sum_n |\boldsymbol{\mathcal{E}}_n|^2 + \sum_{n\neq n'} \boldsymbol{\mathcal{E}}_n^* \cdot \boldsymbol{\mathcal{E}}_{n'} e^{i(\boldsymbol{k}_{n'}-\boldsymbol{k}_n)\cdot\boldsymbol{r}}\right)$$

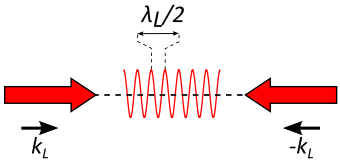
Intensity (and dipole potential) modulations with wavevectors $\boldsymbol{k}_{n'} - \boldsymbol{k}_n$

Simplest example :

- Standing wave with period $d = \pi/k_L$
- Trapping potential of the form (red detuning):

$$V(x) = -2V_0\,(1+\cos(2k_L x))$$

Atoms trapped near the antinodes where $V \approx -2V_0$.



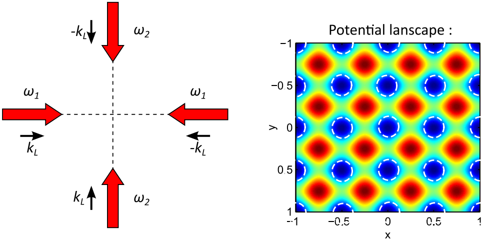
N.B. : $V_0 = \dfrac{d^2 E_0^2}{4|\hbar\delta_L|}$

- d: electric dipole matrix element,
- E_0: electric field amplitude,
- Detuning from atomic resonance :$\delta_L = \omega_L - \omega_{\rm eg} < 0$.

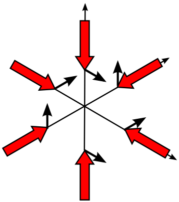
Two and three-dimensional optical lattices

Two dimensions :

$$|\boldsymbol{\mathcal{E}}(\boldsymbol{r})|^2 \approx |2E_0\cos(k_L x)|^2 + |2E_0\cos(k_L y)|^2$$



Two-dimensional square potential

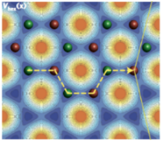


Three-dimensional cubic potential

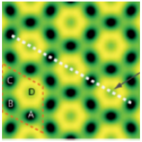
Square or cubic lattices : $V_{\rm lat}(\mathbf{r}) = \sum_{\nu=1,\cdots,d} -V_\nu \cos(k_\nu x_\nu)^2$

- Separable potentials : sufficient to analyze the 1D case

Other lattice geometries are realizable as well:



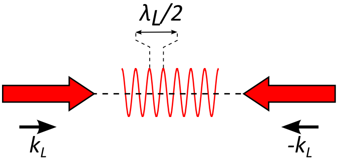
Soltan-Panahi *et al.*, Nature Phys. (2011)



Jo *et al.*, PRL (2012)

- Standing wave with period $d = \pi / k_L$
- Trapping potential :

$$V(x) = -2V_0 \cos^2(k_L x) \\ = -2V_0 \left(1 + \cos(2k_L x)\right)$$



Natural units:

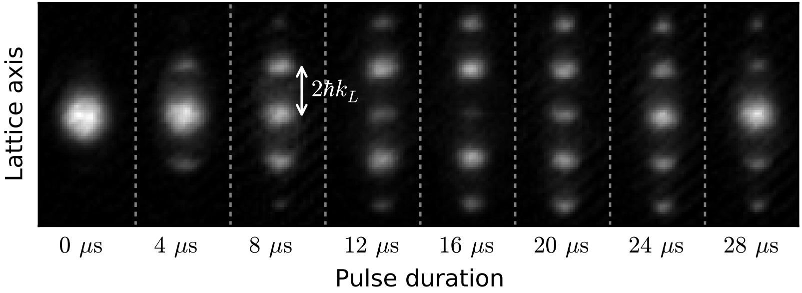
- lattice spacing $d = \lambda_L/2 = \pi/k_L$
- recoil momentum $\hbar k_L$
- recoil energy $E_R = \frac{\hbar^2 k_L^2}{2M_a}$

^{87}Rb, $\lambda_L = 1064$ nm:

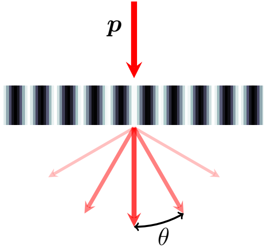
- $d \approx 532$ nm
- $E_R \approx h \times 2$ kHz (100 nK)

Diffraction from a pulsed lattice

Pulsing a lattice potential on a cloud of ultracold atoms (BEC) :

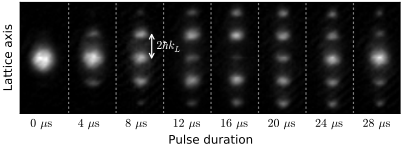


Diffraction of a matter wave from a light grating : $n\lambda_{\mathrm{dB}} = 2\left(\frac{\lambda_L}{2}\right)\sin(\theta_n)$



$$\left.\begin{array}{l} \text{Incoming : } p_x = 0 \\ \text{Outgoing : } p'_x = p\sin(\theta_n) \\ \text{De Broglie wavelength : } \lambda_{\mathrm{dB}} = \frac{h}{p} \end{array}\right\} \rightarrow p'_x = 2n\hbar k_L$$

Pulsing a lattice potential on a cloud of ultracold atoms (BEC) :



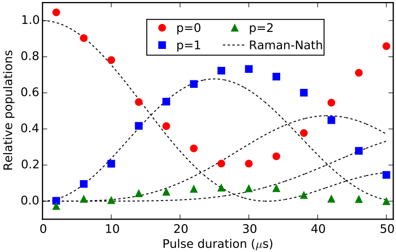
Raman-Nath approximation:

- BEC → plane wave with $\boldsymbol{k} = 0$
- neglect atomic motion in the potential during the diffraction pulse :

$$\Psi(x,t) \approx e^{-i\frac{V_0 \cos(2k_L x)t}{\hbar}} \Psi(x,0)$$

$$\approx \sum_{p=-\infty}^{+\infty} J_p(V_0 t/\hbar) e^{i2pk_L x}.$$

- Analogous to phase modulation of light wave by a *thin* phase grating



Raman-Nath approximation valid only for short times

Hamiltonian :

$$\hat{H} = \frac{\hat{p}^2}{2M} + V_{\text{lat}}(\hat{x}), V_{\text{lat}}(x) = -V_0 \sin^2(k_L x)$$

Lattice translation operator :

- definition : $\hat{T}_d = \exp(i\hat{p}d/\hbar)$
- $\langle x|\hat{T}_d|\phi\rangle = \phi(x+d)$ for any $|\phi\rangle$
- $[\hat{T}_d, \hat{H}] = 0.$

Bloch theorem : Simultaneous eigenstates of $\hat{H}$ and $\hat{T}_d$ (*Bloch waves*) are of the form

$$\phi_{n,q}(x) = e^{iqx} u_{n,q}(x),$$

where the $u_{n,q}$'s (*Bloch functions*) are periodic in space with period d.

- q : *quasi-momentum*
- n : band index

Bloch waves :

$$\phi_{n,q}(x) = e^{iqx} u_{n,q}(x),$$

where the $u_{n,q}$'s (*Bloch functions*) are periodic in space with period d.

- q : *quasi-momentum*
- n : band index

Quasi-momentum is defined from the eigenvalue of $\hat{T}_d$:

$$\hat{T}_d \phi_{n,q}(x) = e^{iqd} \phi_{n,q}(x).$$

For $Q_p = 2pk_L$ with p integer (a vector of the *reciprocal lattice*),

$$\hat{T}_d \phi_{n,q+Q_p}(x) = e^{i(q+Q_p)d} \phi_{n,q+Q_p}(x) = e^{iqd} \phi_{n,q+Q_p}(x).$$

To avoid double-counting, restrict q to the

first Brillouin zone: $\mathrm{BZ1} = [-k_L, k_L]$.

Bloch waves :

$$\phi_{n,q}(x) = e^{ikx} u_{n,q}(x)$$

The Bloch function $u_{n,q}$ is periodic with period d : Fourier expansion with harmonics $Q_m = 2mk_L$ of $2\pi/d = 2k_L$.

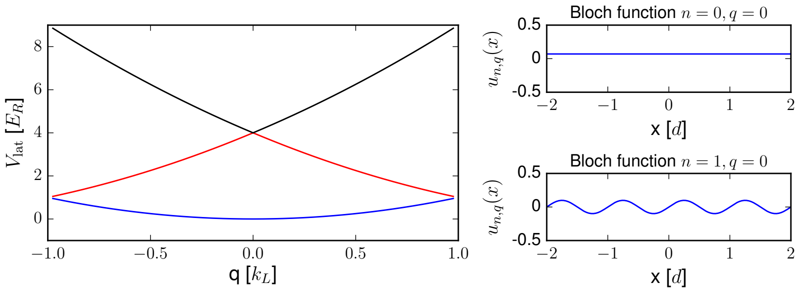
$$u_{n,q}(x) = \sum_{m\in\mathbb{Z}} \tilde{u}_{n,q}(m) e^{iQ_m x},$$

$$V_{\text{lat}}(x) = \sum_{m\in\mathbb{Z}} \tilde{V}_{\text{lat}}(m) e^{iQ_m x} = -\frac{V_0}{2} + \frac{V_0}{4}\left(e^{iQ_{-1}x} + e^{iQ_1 x}\right)$$

- the Bloch functions are superpositions of all harmonics of the fundamental momentum $2k_L$.
- the lattice potential couples momenta p and $p \pm 2k_L$.

Useful to solve Schrödinger equation : reduction to band-diagonal matrix equation for the Fourier coefficients $\tilde{u}_{n,q}(m)$ (tridiagonal for sinusoidal potential)

Spectrum and a few Bloch states, $V_0 = 0E_R$

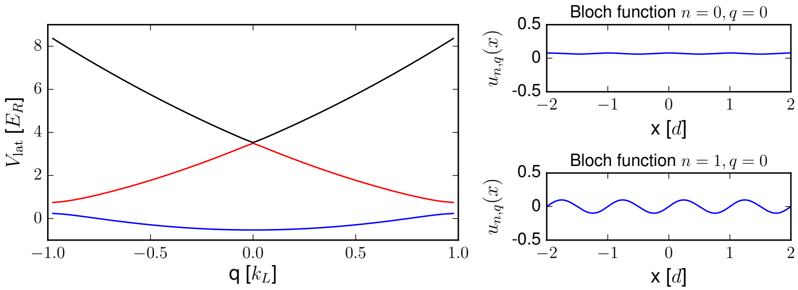


Free particle spectrum : $\epsilon_n(q) = \frac{\hbar^2 (q+2nk_L)^2}{2M}$

Momentum : $k = q + 2nk_L$

Degeneracy at the edges of the Brillouin zone : $E_n(\pm k_L) = E_{n+1}(\pm k_L)$

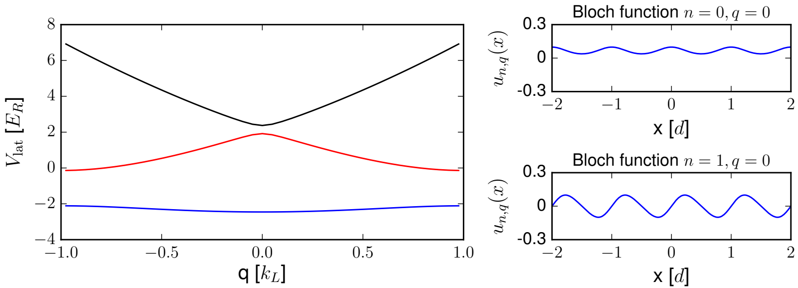
Spectrum and a few Bloch states, $V_0 = 1E_R$



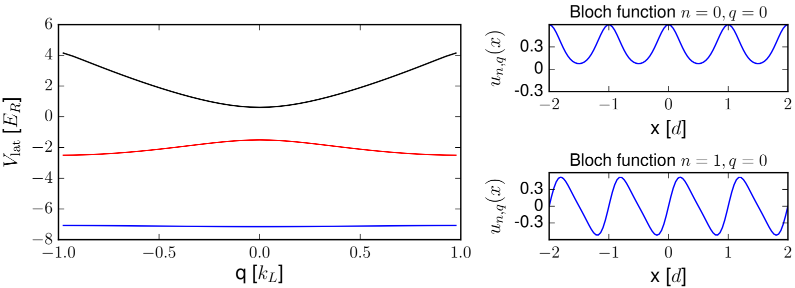
Gap opening near the edges of the Brillouin zones ($q \approx \pm k_L$)

Lifting of free particle degeneracy by the periodic potential

Spectrum and a few Bloch states, $V_0 = 4E_R$



Spectrum and a few Bloch states, $V_0 = 10E_R$



Very deep lattices

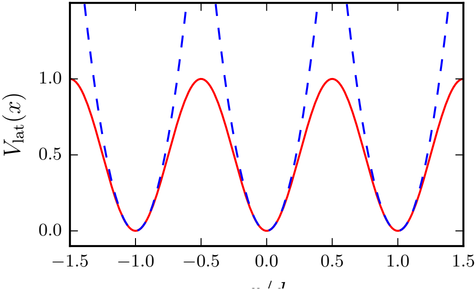
In a deep lattice potential, atoms are tightly trapped around the potential minima.

Harmonic approximation for each well :

$$V_{\rm lat}(x \approx x_i) \approx \frac{1}{2} M\omega_{\rm lat}^2 (x - x_i)^2,$$

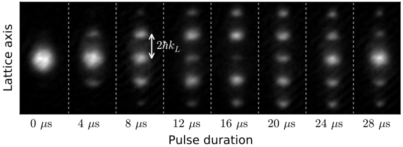
$$\hbar\omega_{\rm lat} = 2\sqrt{V_0 E_R}.$$

The bands are centered around the energy $\overline{E}_n \approx (n + 1/2)\hbar\omega_{\rm lat}$.



First correction : quantum tunneling across the potential barriers, as in tight-binding methods used in solid-state physics (Linear Combination of Atomic Orbitals)

Pulsing a lattice potential on a cloud of ultracold atoms (BEC) :



Bloch wave treatment:

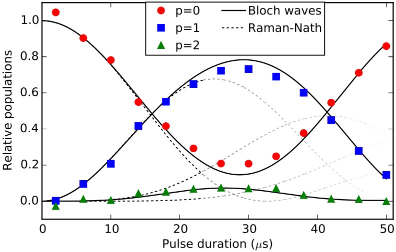
$$|\phi_{n,q}\rangle = \sum_{m=-\infty}^{\infty} \tilde{u}_{n,q}(m)|q + 2mk_L\rangle$$

Initial state :

$$|\Psi(t=0)\rangle = |k=0\rangle = \sum [\tilde{u}_{n,q=0}(m)]^* \, |\phi_{n,q=0}\rangle$$

Evolution in lattice potential :

$$|\Psi(t)\rangle = \sum [\tilde{u}_{n,q=0}(m)]^* \, e^{-i\frac{E_{n,q=0}t}{\hbar}} |\phi_{n,q=0}\rangle$$



Raman-Nath approximation valid only for short times

The total population of excited states is bounded from above :

$$N' \leq N'_{\text{max}} = \sum_{\epsilon_i > \epsilon_{\text{min}}} \frac{1}{e^{\beta(\epsilon_i - \epsilon_{\text{min}})} - 1}$$

Saturation of excited states and macroscopic accumulation of particles in the lowest energy state when $N \geq N'_{\text{max}}$ for fixed T

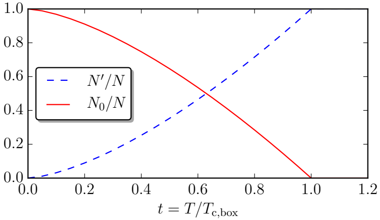
For a uniform gas :

$$N'_{\text{max}} \approx 2.612 \left(\frac{L}{\lambda_{\text{th}}} \right)^3$$

$$\lambda_{\text{th}} = \sqrt{\frac{2\pi\hbar^2}{M k_B T}} \; : \text{ thermal De Broglie wavelength}$$

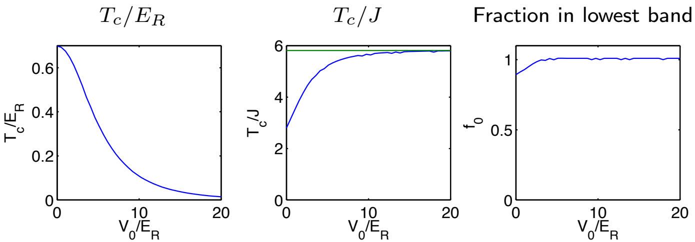
Fixing $n = N/V$:

$$T_{\text{c,box}} \approx \frac{2\pi\hbar^2}{M} \left(\frac{n}{2.612} \right)^{2/3}$$



Thermodynamics of an ideal Bose gas in a cubic lattice

We do the same calculation (numerically) for a cubic lattice. We introduce the filling fraction $\overline{n} = \frac{N}{N_s} = \rho d^3$, *i.e.* the mean number of atoms per lattice site.

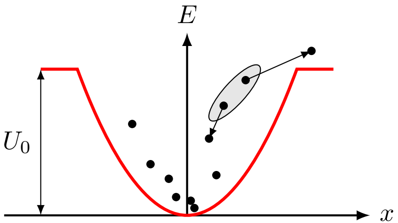


Here $J \equiv$ energy with of the lowest Bloch band.

- T_c decreases quickly as V_0 increases,
- Atoms accumulate quickly in the lowest band as V_0 increases.
- tight-binding limit : $k_B T_c \approx 6J$ for $\overline{n} = 1$, reached for $V_0 \geq 10\,E_R$,

Principle of evaporative cooling :
Atoms trapped in a potential of depth U_0, undergoing collisions :

- two atoms with energy close to U_0 collide
- result: one "cold" atom and a "hot" one with energy $> U_0$
- rethermalization of the $N-1$ atoms remaining in the trap results in a lower mean energy per atom.



Experimental procedure to prepare a cold atomic gas in a lattice :

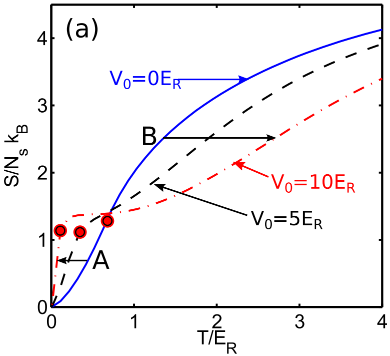
- prepare a quantum gas using evaporation in an auxiliary trap,
- transfer it to the lattice by increasing the lattice potential from zero and simultaneously removing the auxiliary trap.

Why not cool atomic gases directly in the periodic potential ? .

- evaporative cooling no longer works due to the band structure as soon as $V_0 \sim$ a few E_R.

The best one can do is to transfer the gas adiabatically, *i.e.* **at constant entropy**.

- Increase lattice depth from 0 to 10 E_R a constant entropy.
- The three curves indicate the entropy *vs* lattice depth curves for $V_0 = 0, 5, 10\,E_R$.
- Isentropic path goes horizontally from the blue curve to the red one.
- Red dots mark the location of T_c for each case.



- A: adiabatic cooling path
- B: adiabatic heating path

- Increase lattice depth from 0 to 10 E_R a constant entropy.
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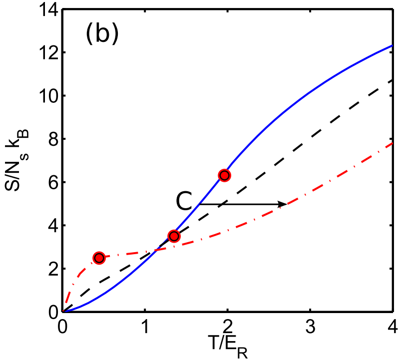


- C: adiabatic decondensation

Quantum adiabatic theorem :

Slowly evolving quantum system, with Hamiltonian $\hat{H}(t)$.

Instantaneous eigenbasis of $\hat{H}$: $\hat{H}(t)|\phi_n(t)\rangle = \varepsilon_n(t)|\phi_n(t)\rangle$.

Time-dependent wave function in the $\{|\phi_n(t)\rangle\}$ basis:

$$|\Psi(t)\rangle = \sum_n a_n(t) e^{-\frac{i}{\hbar}\int_0^t \varepsilon_n(t')dt'} |\phi_n(t)\rangle,$$

From Schrödinger equation, one gets $[\omega_{mn} = \varepsilon_m - \varepsilon_n]$:

$$\dot{a}_n = -\langle\phi_n|\dot{\phi}_n\rangle a_n(t) - \sum_{m\neq n} e^{-\frac{i}{\hbar}\int_0^t \omega_{mn}(t')dt'} \langle\phi_n|\dot{\phi}_m\rangle a_m(t),$$

- Berry phase : $\langle\phi_n|\dot{\phi}_n\rangle = -i\gamma_B$ is a pure phase. Wavefunction unchanged up to a phase evolution after a cyclic change.
- The adiabatic theorem : for *arbitrarily slow* evolution starting from a particular state n_0 ($a_n(0) = \delta_{n,n_0}$), and in the *absence of level crossings*, $a_n(t) \to \delta_{n,n_0}$ (up to a global phase).

Adiabatic approximation for slow evolutions and initial condition $a_n(0) = \delta_{n,n_0}$:

$$a_n(t) \to e^{i\phi(t)}\delta_{n,n_0}$$

Validity criterion :

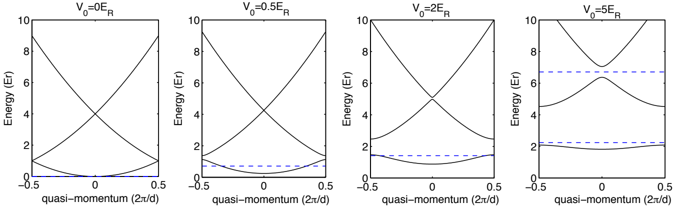
$$\langle \phi_n | \dot{\phi}_m \rangle = \frac{\langle \phi_n | \dot{H} | \phi_m \rangle}{\varepsilon_m - \varepsilon_n} \implies \left| \langle \phi_n | \dot{H} | \phi_m \rangle \right| \ll \frac{\left(\varepsilon_m - \varepsilon_n \right)^2}{\hbar}.$$

Time-dependent lattice potential :

$$V_{\rm lat} = V_0(t) \sum_\alpha \sin(k_\alpha x_\alpha)^2$$

$V_0(t)$ increases from 0 to some final value.

Quasi-momentum = good quantum number: only band-changing transitions



Adiabaticity criterion for a system prepared in a Bloch state $(n, \boldsymbol{q})$:

$$\left| \hbar \dot{V}_0 \right| \ll \left(\varepsilon_m(\boldsymbol{q}) - \varepsilon_n(\boldsymbol{q}) \right)^2$$

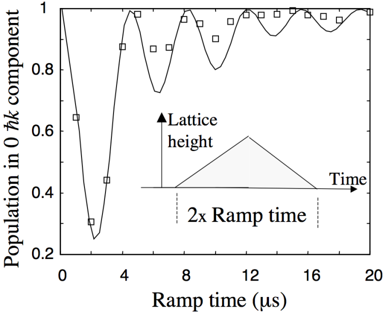
Adiabatic loading of a condensate

Adiabaticity criterion for a system prepared in a Bloch state $(n, \boldsymbol{q})$:

$$V_{\rm lat} = V_0(t) \sum_\alpha \sin(k_\alpha x_\alpha)^2$$

Quasi-momentum = good quantum number: $\left|\hbar \dot{V}_0\right| \ll \Big(\varepsilon_m(\boldsymbol{q}) - \varepsilon_n(\boldsymbol{q})\Big)^2$

Sodium atoms, $E_R/h \approx 20\,\mathrm{kHz}$ [Denschlag *et al.*, J. Phys. B 2002]:

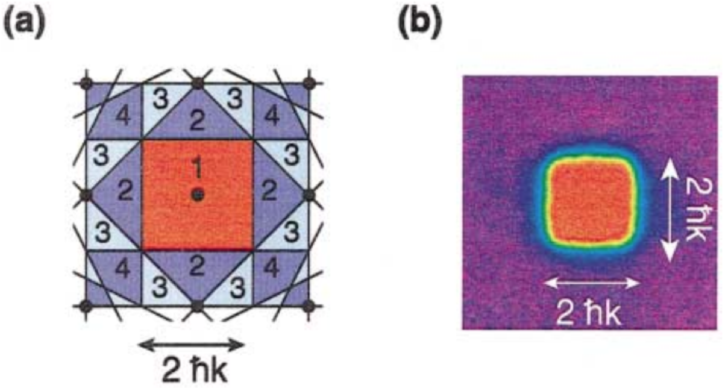


Adiabaticity most sensitive for small depths :

- Near band center $n = 0, \boldsymbol{q} = 0$: $|\varepsilon_1 - \varepsilon_0| \geq 4E_R$
- Near band edge $n = 0, q_\alpha = \pi/d$: $|\varepsilon_1 - \varepsilon_0| \geq 0$

Caution: for real systems interactions and tunneling within the lowest band are the limiting factors, not the band structure. Adiabaticity requires ramp-up times in excess of 100 ms.

- Thermal Bose gas, $J_0 \ll k_B T \ll \hbar\omega_{\rm lat}$: almost uniform quasi-momentum distribution.
- Mapping by releasing slowly the band structure before time of flight (instead of suddenly)– typically a few ms.
- Qualitative value only if band edges matter.



Greiner *et al.*, PRL 2001

Time-of-flight experiment : suddenly switch off the trap potential at $t = 0$ and let the cloud expand for a time t.

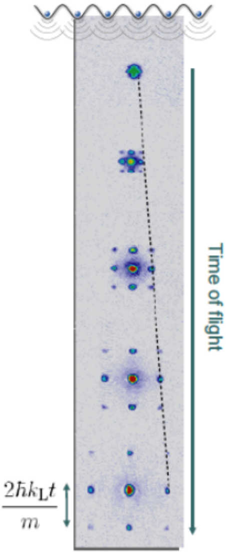
Time of flight (tof) expansion reveals momentum distribution (if interactions can be neglected).

Classical version : $\boldsymbol{r}(t) = \boldsymbol{r}(0) + \frac{\boldsymbol{p}(0)t}{M}$

Quantum version: wave-function after tof mirrors the initial *momentum distribution* $\mathcal{P}_0(\mathbf{p})$ with $\boldsymbol{p} = \frac{M\boldsymbol{r}}{t}$.

$$n_{\rm tof}(\boldsymbol{r}, t) = |\psi(\boldsymbol{r}, t)|^2 \approx \left(\frac{M}{t}\right)^3 \mathcal{P}_0\left(\mathbf{p} = \frac{M\boldsymbol{r}}{t}\right)$$

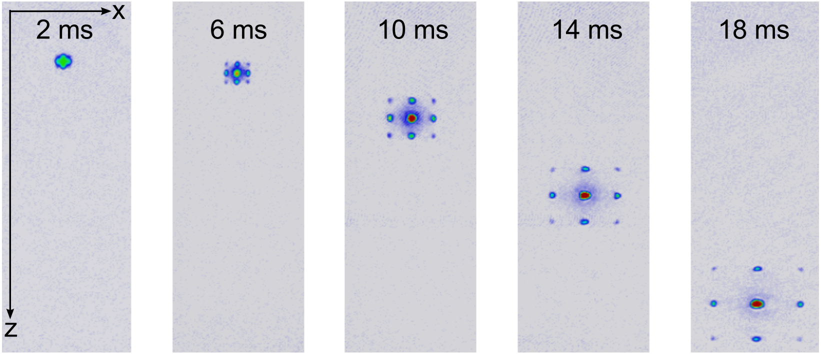
- Analogous to Fraunhofer regime of optical diffraction, requires $\frac{\Delta p_0 t}{M} \gg \Delta x_0$ with $\Delta x_0, \Delta p_0$ the spread of ψ_0 in real and in momentum space.
- for a condensate : N atoms behaving identically, density profile $n_{\rm tof}(\boldsymbol{r}, t) \propto N|\tilde{\psi}(\boldsymbol{p}, t)|^2$ with $\tilde{\psi}$ the Fourier transform of the condensate wavefunction.



Non-interacting condensate: Atoms condense in the lowest band $n = 0$ at quasi-momentum $\boldsymbol{q} = 0$:

$$\tilde{\phi}_{0,0}(\boldsymbol{p}) \propto \sum_{\boldsymbol{m}\in\mathbb{Z}^3} \tilde{u}_{0,0}(\mathbf{m})\delta(\mathbf{p} - \hbar\mathbf{Q}_{\mathbf{m}}),$$

$\mathbf{Q}_{\mathbf{m}} = 2k_L\mathbf{m}$ ($\mathbf{m} \in \mathbb{Z}^3$) is a vector of the reciprocal lattice.
Time-of-flight distribution: comb structure with peaks mirroring the reciprocal lattice ("Bragg peaks").

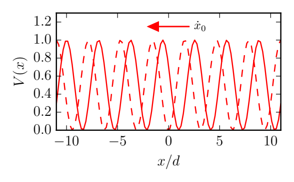


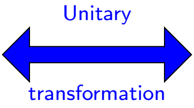
Bloch oscillations

Uniformly accelerated lattice : $V_{\text{lat}}[x - x_0(t)]$ with $x_0 = -\frac{Ft^2}{2m}$

Lab frame:

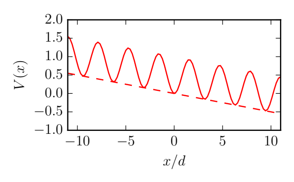
$$H_{\text{lab}} = \frac{p^2}{2m} + V_{\text{lat}}[x - x_0(t)]$$





Moving frame:

$$H_{\text{mov}} = \frac{p^2}{2m} + V_{\text{lat}}[x] - Fx$$



Bloch theorem still applies : H_{lab} invariant by lattice translations

Upon acceleration (moving frame):

$$|n, q_0\rangle \rightarrow |n, q(t)\rangle : q(t) = q_0 - m\dot{x}_0 = q_0 + \frac{Ft}{\hbar}$$

Quasi-momentum scans linearly accross the Brillouin zone.

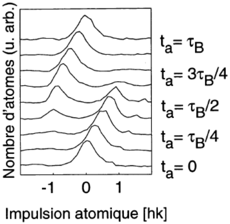
Bloch oscillations

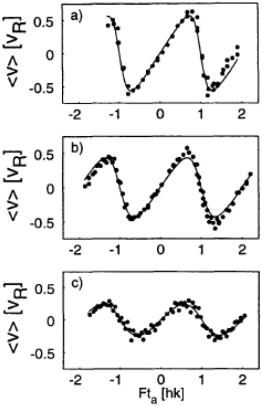
Quasi-momentum scan accross the Brillouin zone : $q(t) = q_0 - m\dot{x}_0 = q_0 + \frac{Ft}{\hbar}$

When $q = +k_L$, either non-adiabatic transfer to higher bands or, if adiabatic, Bragg reflection to $q = -k_L$.

Bloch oscillations of quasi-momentum with period $T_B = \frac{2\hbar k_L}{F}$

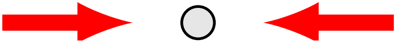
Experimental observation with cold Cs atoms [Ben Dahan *et al.*, PRL 1995, also in Raizen's group at UT Austin]:





Moving frame

$\omega_L, +k_L$ $\omega_L + \delta, -k_L$

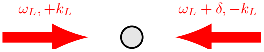


Accelerated lattice :

$V_{\rm lat} = V_0 \cos^2(k_L x - \delta t)$

Consider an atomic wavepacket narrow in momentum space (typical velocities $v \ll \hbar k_L/M$).

Accelerated lattice : $V_{\rm lat} = V_0 \cos^2(k_L x - \delta t)$



Standing wave traveling at velocity $v = \delta/k_L$

Quasi-momentum remains a good quantum number

Lattice frame : $q \to q - \frac{mv}{\hbar}$

For slow (adiabatic) acceleration, a Bloch state $|q\rangle$ evolves to $|q - mv\rangle$.

A wavepacket built from Bloch states propagates with the group velocity :
$v_g = \frac{d\varepsilon(q)}{dq}|_{q=-mv} = \frac{-M}{M^*} v$, with $M^* = \frac{d^2\varepsilon(q)}{dq^2}$ the effective mass (for $v \ll \hbar k_L/M$).

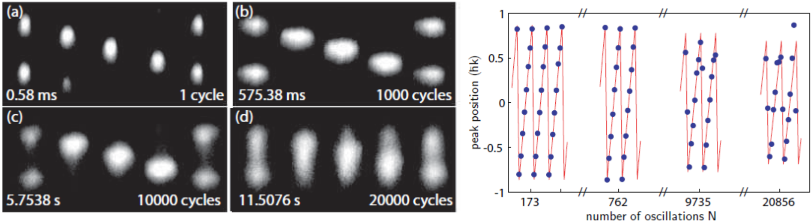
Lab frame : group velocity : $v_{\rm BEC} = v + v_g = \left(1 - \frac{M}{M^*}\right) v$

- shallow lattice, $V_0 \lesssim E_R$: $M^* \approx M$, atoms stand still
- deep lattice, $V_0 \gg E_R$: $M^* \ll M$, atoms dragged by the moving lattice

Quasi-momentum scan accross the Brillouin zone : $q(t) = q_0 - m\dot{x}_0 = q_0 + \frac{Ft}{\hbar}$

Bloch oscillations of quasi-momentum with period $T_B = \frac{2\hbar k_L}{F}$

Experimental observation with non-interacting BEC [Gustavsson *et al.*, PRL 2008]:

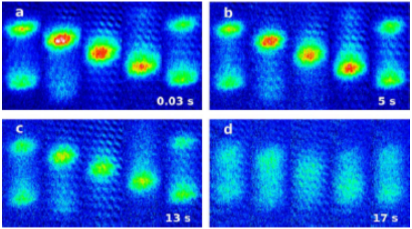


N.B.: Assume atoms are prepared in a given band $n = 0$, and do not make a transition to higher bands (adiabatic approximation).

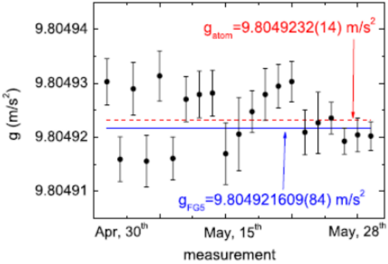
Bloch oscillations in a vertical optical lattice :

$$T_B = \frac{2\hbar k_L}{Mg}$$

Poli *et al.*, PRL 2011, Tino group at LENS (Florence)



approx. 10^4 periods



Measurement uncertainty : 10^{-6} g

Further application :

- measurement of G at the 10^{-5} level [Rosi *et al.*, Nature 2014, LENS Florence]

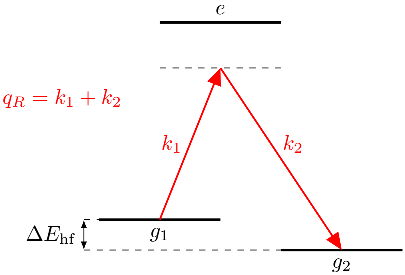
Measurement of the fine structure constant α :

$$\alpha^2 = \frac{4\pi R_\infty}{c} \times \frac{M}{m_e} \times \frac{\hbar}{M}$$

R_∞: Rydberg constant
m_e: electron mass
M: atomic mass

- possible window on physics beyond QED : interactions with hadrons and muons, constraints on theories postulating an internal structure of the electron, ...

Measurement of $\frac{\hbar}{M}$: Experiment in the group of F. Biraben (LKB, Paris)

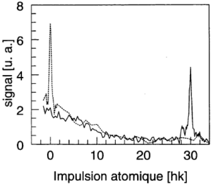


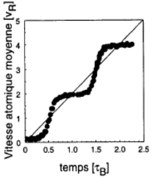
Doppler-sensitive Raman spectroscopy :

$$\hbar\omega_{\rm res} = \Delta E + \frac{\hbar^2}{2M}\left(p_i + \Delta k + q_R\right)^2$$

$$\Longrightarrow \frac{\hbar}{M} = \frac{\omega_{\rm res}(p_i + \Delta k) - \omega_{\rm res}(p_i)}{q_R \Delta k}$$

Large momentum beamsplitter using Bloch oscillations :



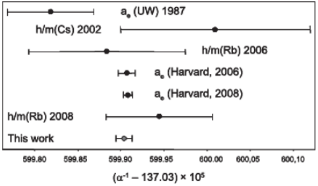


After N Bloch oscillations, momentum transfer of $\Delta k = 2N\hbar k_L$ to the atoms in the lab frame.

This transfer is perfectly coherent and enables beamsplitters where part of the wavepacket remains at rest while the other part is accelerated.

Measurement of $\frac{\hbar}{M}$:

- $N \sim 10^3$: Comparable uncertainty as current best measurement (anomalous magnetic moment of the electron – Gabrielse group, Harvard).
- Independent of QED calculations



[Bouchendira *et al.*, PRL 2011]

- Other applications in precision measurements: measurement of weak forces, *e.g.* Casimir-Polder [Beaufils *et al.*, PRL 2011].