Outline

- Introduction to Bose-Einstein condensation
- Long-range order
- Critical phenomena
- Measuring a critical exponent of the BEC-normal transition

L1

- Long-range interactions in quantum gases
- Cavity QED intro
- Realizing the superfluid – supersolid phase transition
- Studying critical properties of the phase transition
- (Realizing an extended Hubbard model: superfluid – Mott insulator – supersolid – charge density wave)

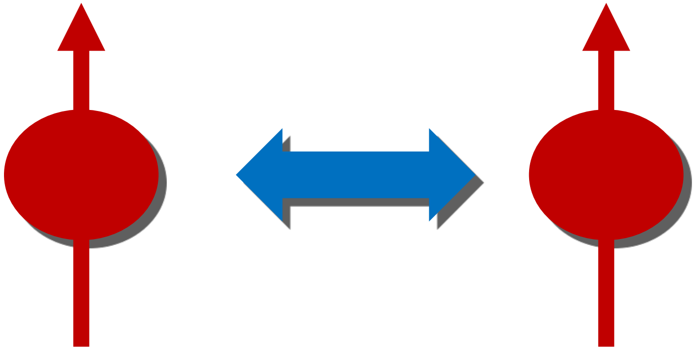
L2

Ritsch, H. et al.: Cold atoms in cavity-generated dynamical optical potentials. *Reviews of Modern Physics*, *85*(2), 553–601. (2013)

Other experiments:

- Andreas Hemmerich (Hamburg)
- Dan Stamper-Kurn (Berkeley)
- Benjamin Lev (Stanford)
- Claus Zimmermann (Tübingen)

Long range interactions

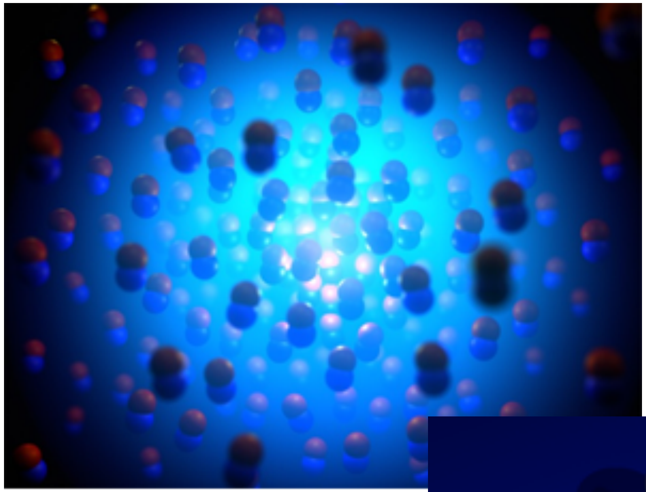


dipole-dipole interaction

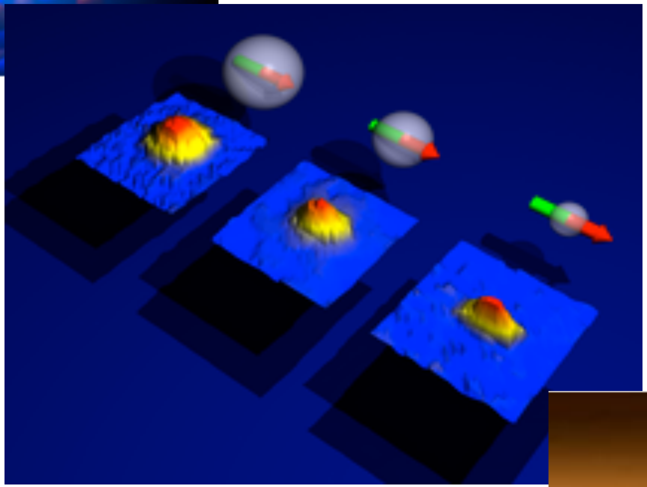
Long-range interacting quantum gases:

- Formation of new quasi-particles
- Roton-type mode softening
- Phase transition e.g. to supersolid phase at roton instability

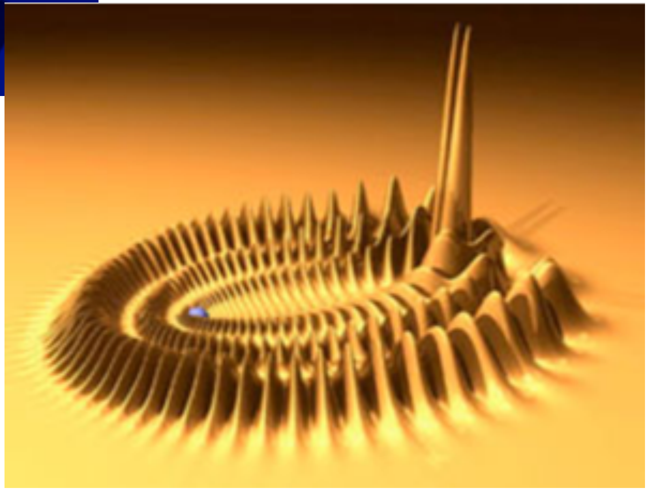
Long-range interacting quantum gases



Heteronuclear molecules

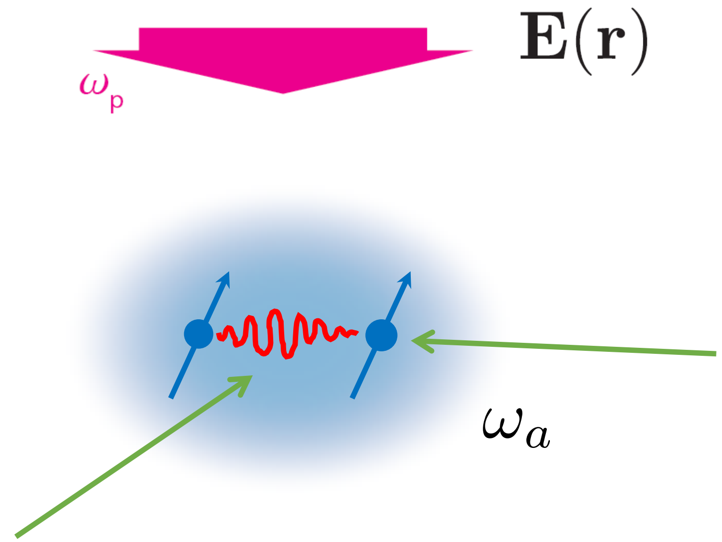


Strong magnetic dipole moments



Rydberg atoms

Long-range interactions between induced AC electric dipoles



Induced AC electric dipole moments:

$$\mathbf{d}(\mathbf{r}) \propto \mathbf{E}(\mathbf{r})$$

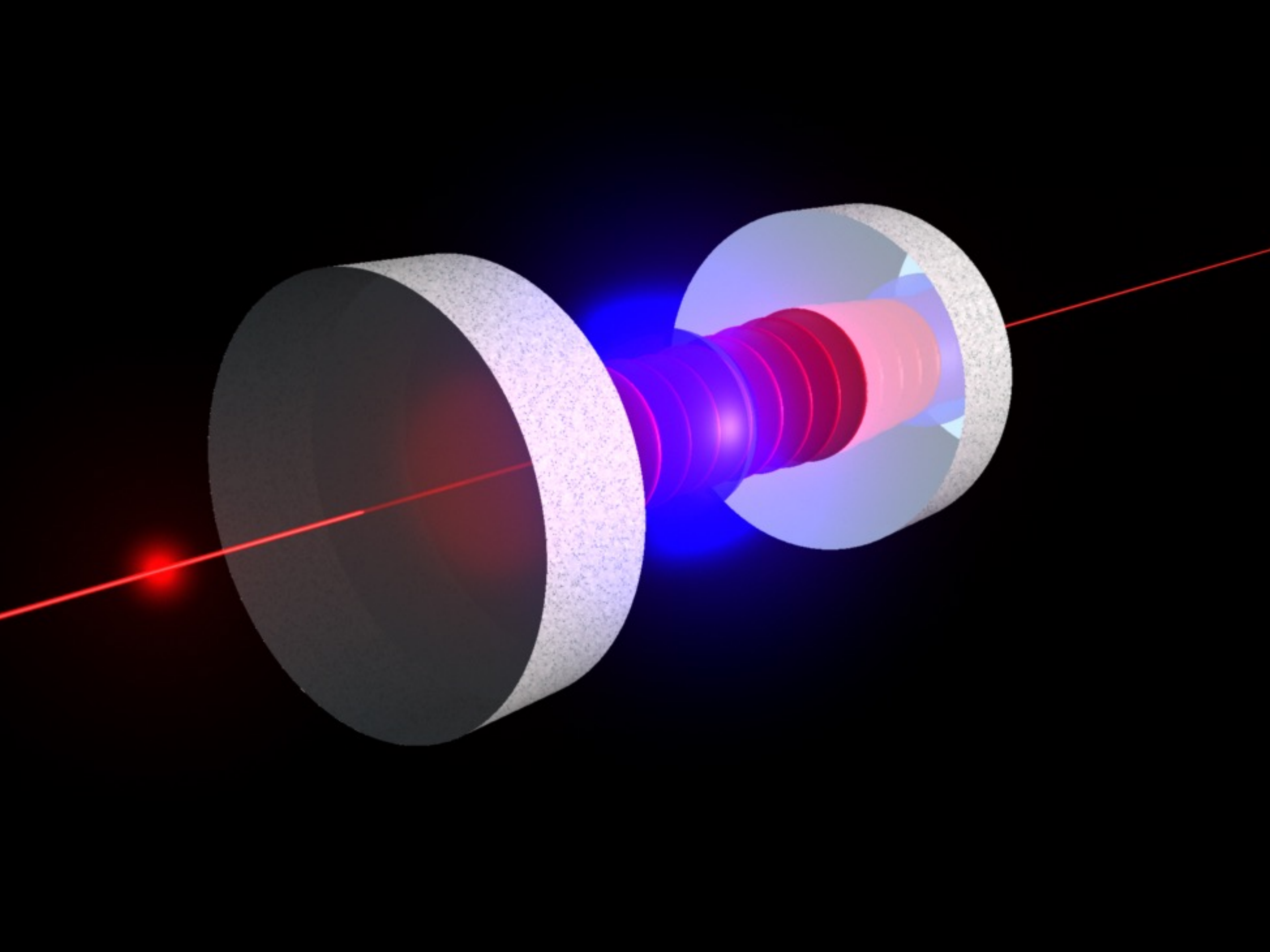
long-range interaction

$$V_{\mathrm{lr}} \propto \frac{I}{\Delta_a}$$

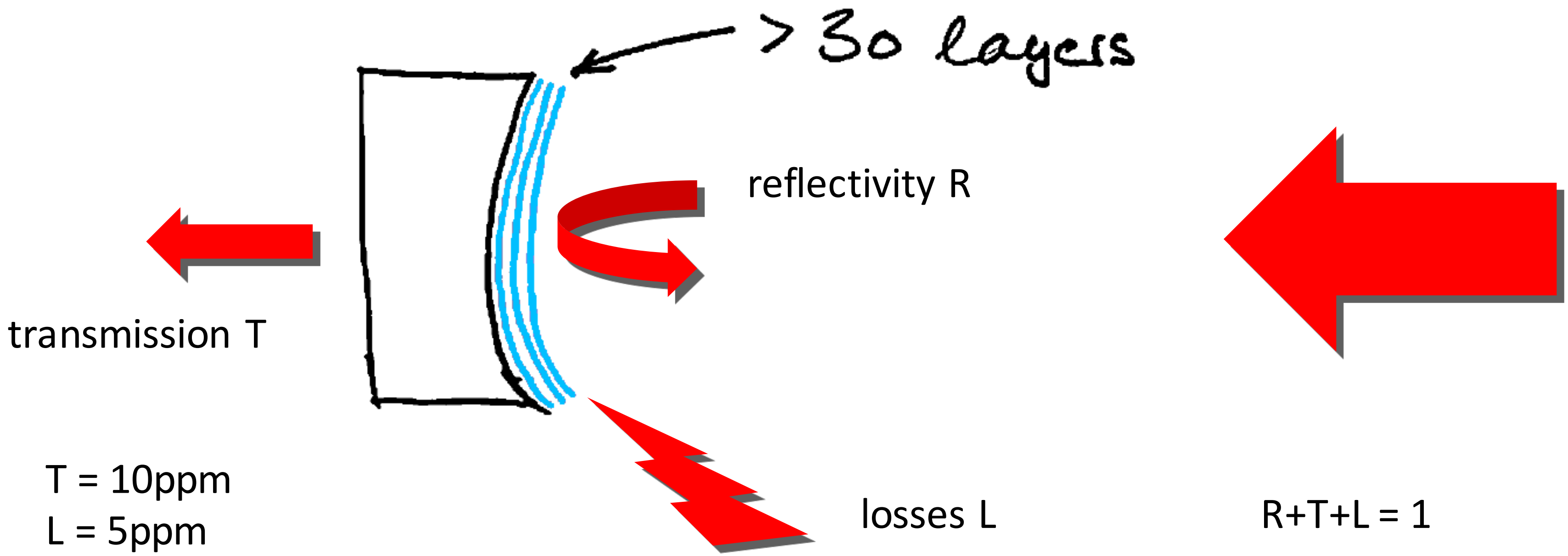
Strong spontaneous scattering

$$\Gamma \propto \frac{I}{\Delta_a}$$

D. O'Dell et al. PRL 84, 5687 (2000), D. O'Dell et al. PRL 90, 110402 (2003), C. Maschler and H. Ritsch PRL **95**, 260401 (2005)

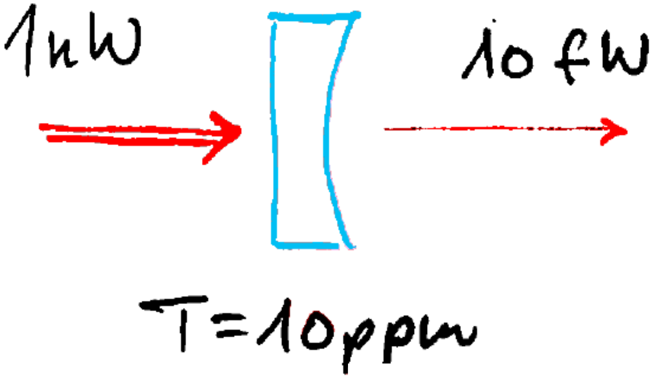


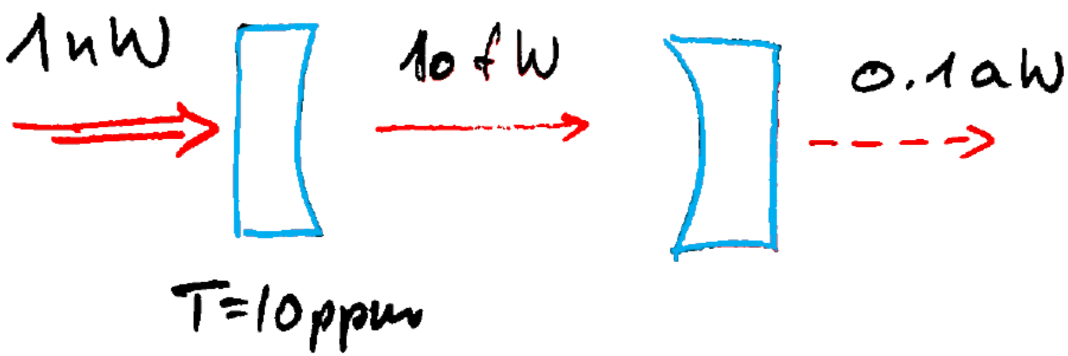
Introduction to cavities



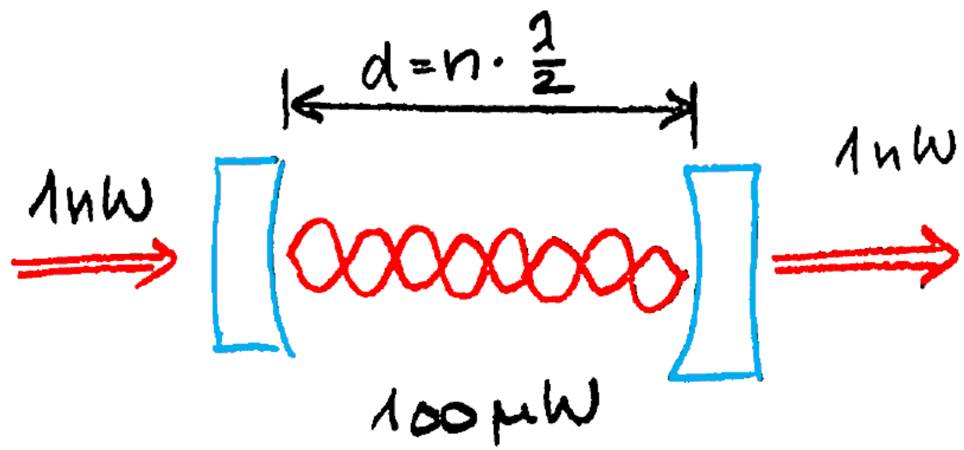
T = 10ppm
L = 5ppm
R = 99.999985% !!

R+T+L = 1





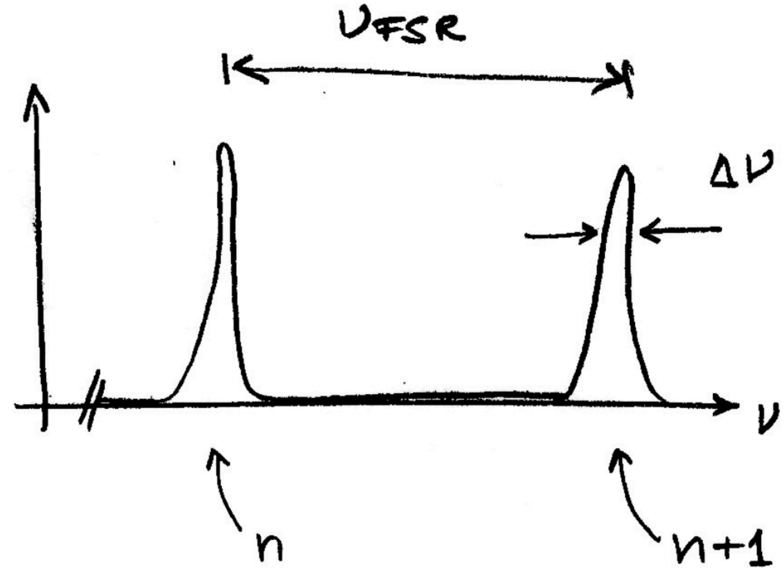
light field amplitudes sum up
Enhancement by 1/T.



Field enhancement

Free spectral range:

$$\nu_{FSR} = \frac{c}{2 \cdot d}$$



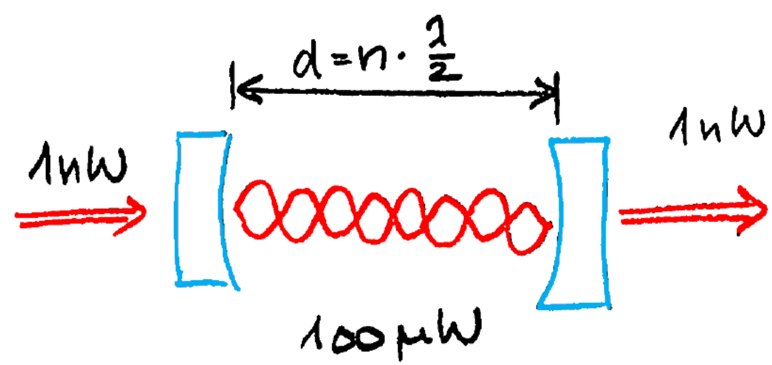
Finesse of a cavity:

$$\mathcal{F} = \frac{\nu_{FSR}}{\Delta\nu}$$

"ultra high Finesse":

$\Rightarrow$ Enhancement by $\frac{\mathcal{F}}{\pi}$.

typical $10^5 - 10^6$



Enhanced matter-light interaction

1 atom

probability to scatter:

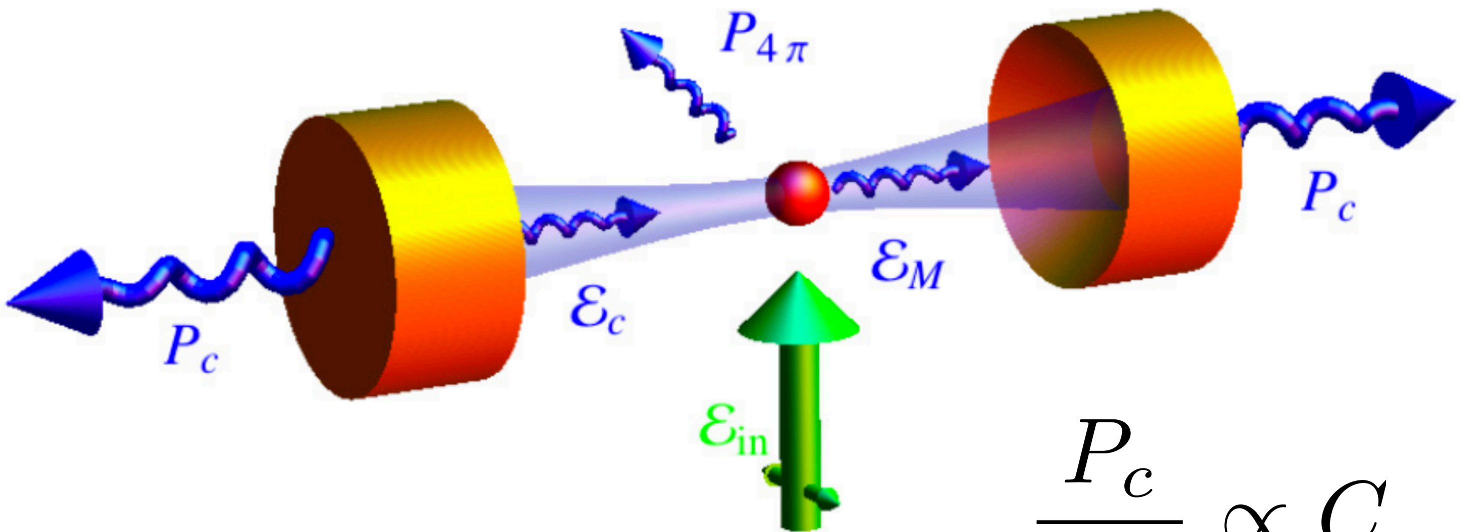
$$C = \frac{\sigma}{A} \sim \frac{\lambda^2}{A} \leq 1$$

λ^2

$$C = \frac{F}{\pi} \cdot \frac{\sigma}{A}$$

F/π roundtrips ; also cooperativity

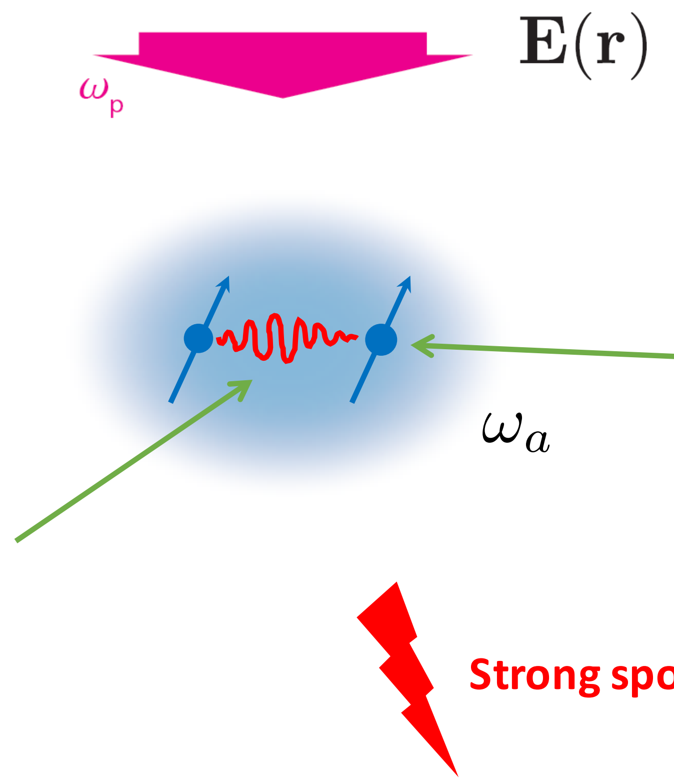
Scattering into the cavity



$$\frac{P_c}{P_{4\pi}} \propto C$$

Atomic ensemble instead of single atom: $C_N = N \times C$

Long-range interactions between induced AC electric dipoles



Induced AC electric dipole moments:

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long-range interaction

Strong spontaneous scattering

D. O'Dell et al. PRL 84, 5687 (2000), D. O'Dell et al. PRL 90, 110402 (2003), C. Maschler and H. Ritsch PRL **95**, 260401 (2005)

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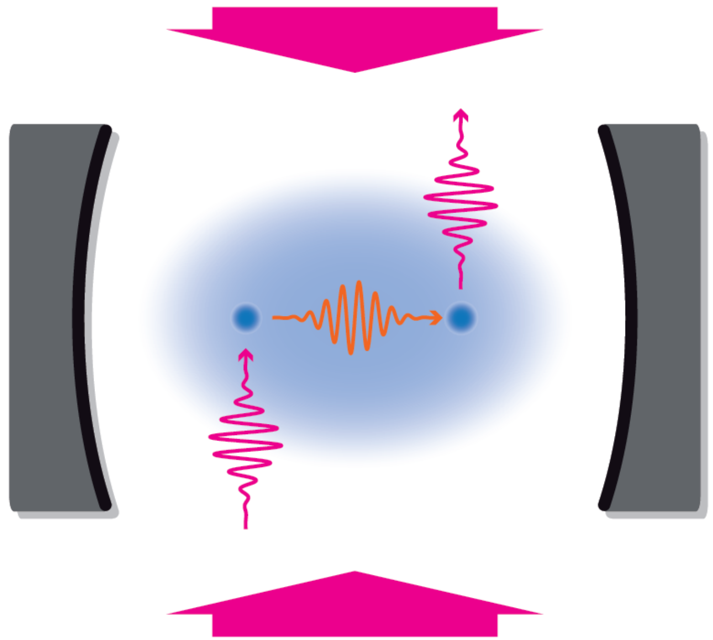


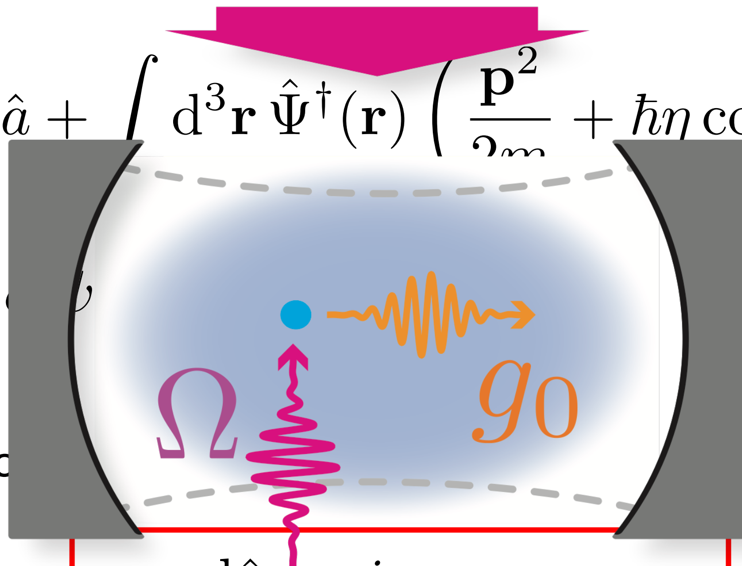
Coherent processes dominate over losses!

Hamiltonian description

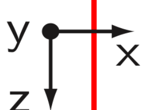
$$H^{(1)} = -\hbar\Delta_c \hat{a}^\dagger\hat{a} + \frac{\mathbf{p}^2}{2m} + \hbar\eta\cos(kx)\cos(kz)(\hat{a}+\hat{a}^\dagger) \qquad \eta = \frac{\Omega g_0}{\Delta_a}$$

$$H_{\mathrm{mb}} = -\hbar\Delta_c \hat{a}^\dagger\hat{a} + \int \mathrm{d}^3\mathbf{r}\,\hat{\Psi}^\dagger(\mathbf{r})\left(\frac{\mathbf{p}^2}{2m} + \hbar\eta\cos(kx)\cos(kz)(\hat{a}+\hat{a}^\dagger)\right)\hat{\Psi}(\mathbf{r})$$

$$\hat{\Psi}(\mathbf{r}) = \hat{c}_0\psi_0 + \ldots \quad \psi_1 \propto \cos(kx)\cos(kz)$$

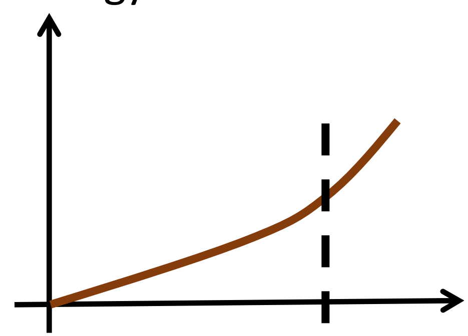


Equatio



$$\frac{\mathrm{d}\hat{a}}{\mathrm{d}t} = \frac{i}{\hbar}[H, \hat{a}]$$

$$\frac{\mathrm{d}\hat{\Psi}(\mathbf{r})}{\mathrm{d}t} = \frac{i}{\hbar}\left[H, \hat{\Psi}(\mathbf{r})\right]$$



$$\frac{\mathrm{d}\hat{a}}{\mathrm{d}t} = \frac{i}{\hbar}\left[H, \hat{a}\right]$$

$$\frac{\mathrm{d}\hat{\Psi}(\mathbf{r})}{\mathrm{d}t} = \frac{i}{\hbar}\left[H, \hat{\Psi}(\mathbf{r})\right]$$

Evolves with recoil (kHz)

Adiabatic elimination of light field:

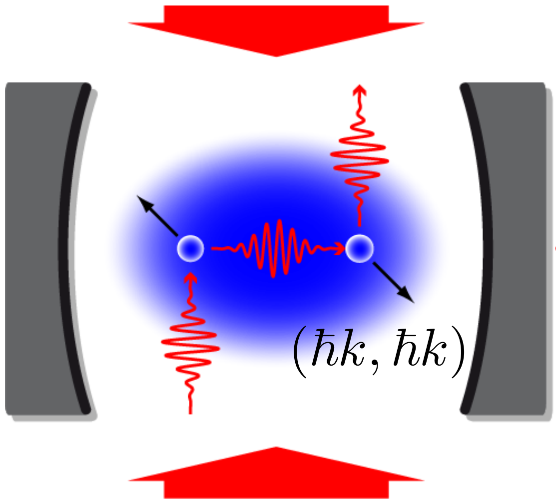
$$\frac{\mathrm{d}\hat{a}}{\mathrm{d}t} = 0 \quad \Rightarrow \quad \hat{a} = \frac{\eta}{\Delta_c}\int \mathrm{d}^3\mathbf{r}\, \hat{\Psi}^\dagger(\mathbf{r})\cos(kx)\cos(kz)\hat{\Psi}(\mathbf{r})$$

Put $\hat{a}$ into $\frac{\mathrm{d}\hat{\Psi}(\mathbf{r})}{\mathrm{d}t}$ and get effective Hamiltonian:

$$H_{\mathrm{eff}} = \int \mathrm{d}^3\mathbf{r}\, \hat{\Psi}^\dagger(\mathbf{r})\frac{\mathbf{p}^2}{2m}\hat{\Psi}(\mathbf{r}) + \int\int \mathrm{d}^3\mathbf{r}\mathrm{d}^3\mathbf{r}'\, \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}^\dagger(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}')$$

$$V(\mathbf{r},\mathbf{r}') = V\cos(kx)\cos(kz)\cos(kx')\cos(kz')$$

Cavity-mediated long-range interaction



Long-range interaction:

$$H_{lr} = \int \mathrm{d}^3\mathbf{r}\mathrm{d}^3\mathbf{r}'\hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}^\dagger(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}')$$

$$V(\mathbf{r},\mathbf{r}') = V\cos(kx)\cos(kz)\cos(kx')\cos(kz')$$

tunable via the pump power $V = \frac{\Omega^2}{\Delta_c} \propto P$

→ coherent exchange of *virtual* cavity photons by distant atoms

→ Momentum transfer to atoms

R. Mottl et al. Science 336, 1570 (2012)

Competing energy scales – again a phase transition?

$$H_{\text{eff}} = \int \mathrm{d}^3\mathbf{r}\, \hat{\Psi}^\dagger(\mathbf{r}) \frac{\mathbf{p}^2}{2m} \hat{\Psi}(\mathbf{r}) + \int\int \mathrm{d}^3\mathbf{r}\mathrm{d}^3\mathbf{r}'\, \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}^\dagger(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}')$$

Hamiltonian: kinetic vs interaction energy

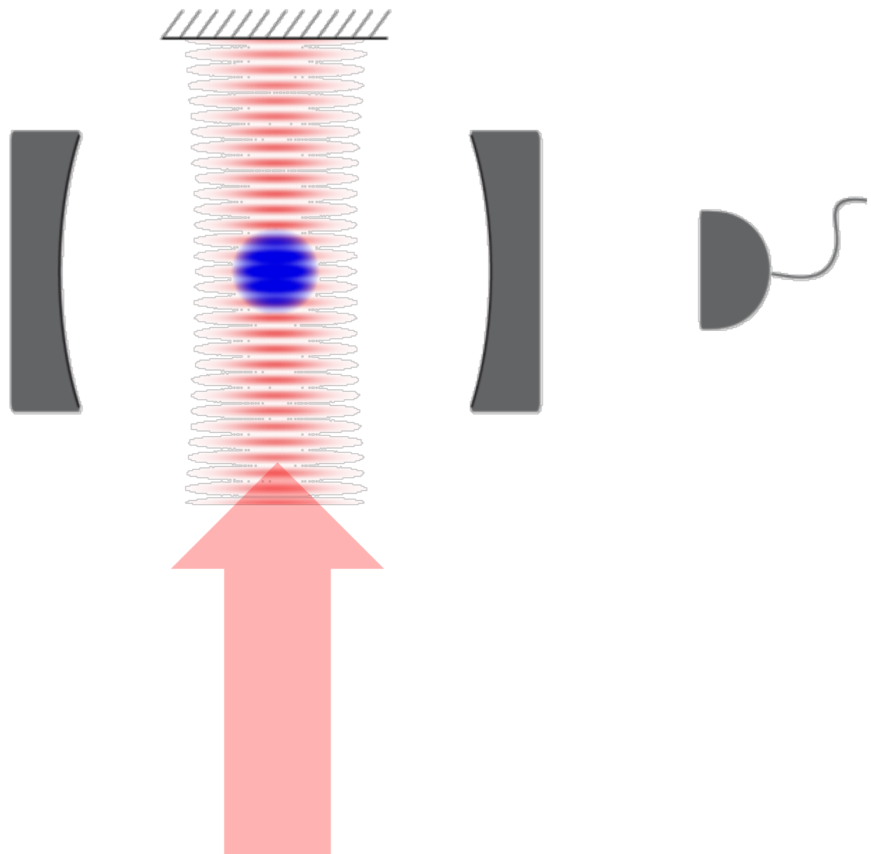
Wavefunction adapts to minimize free energy

$$\hat{\Psi}(\mathbf{r}) = \hat{c}_0\psi_0 + \hat{c}_1\psi_1$$

$$\psi_0 = 1; \qquad \psi_1 \propto \cos(kx)\cos(kz)$$

-> Modulation of atomic density distribution

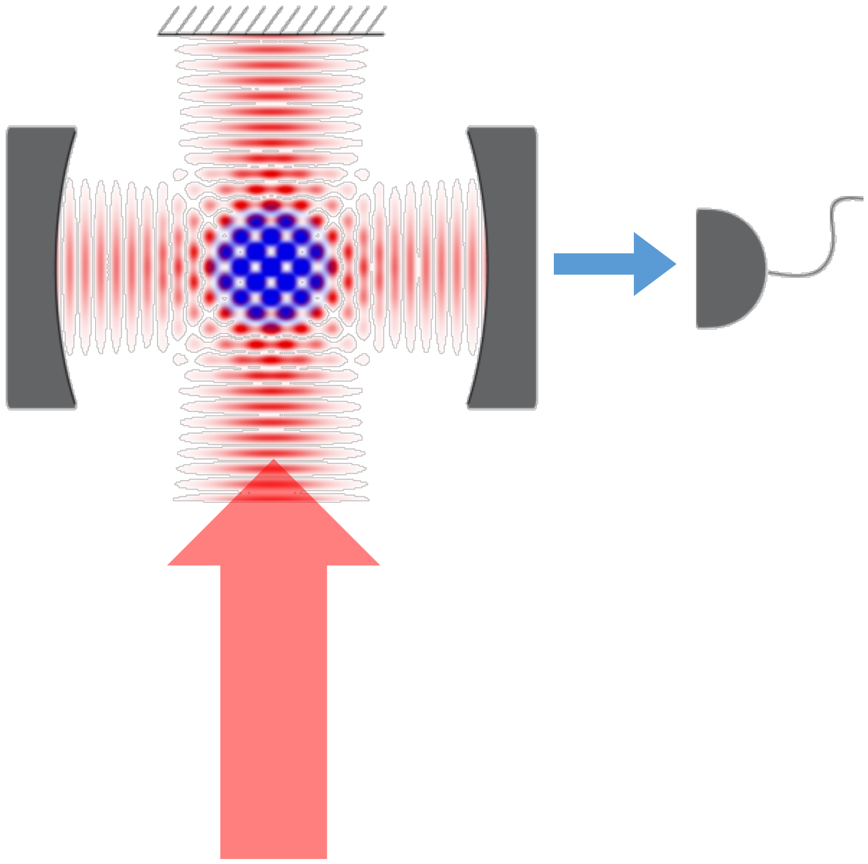
Transverse Pumping



Theory: H. Ritsch, P. Domokos, Exp. with thermal atoms: V. Vuletic

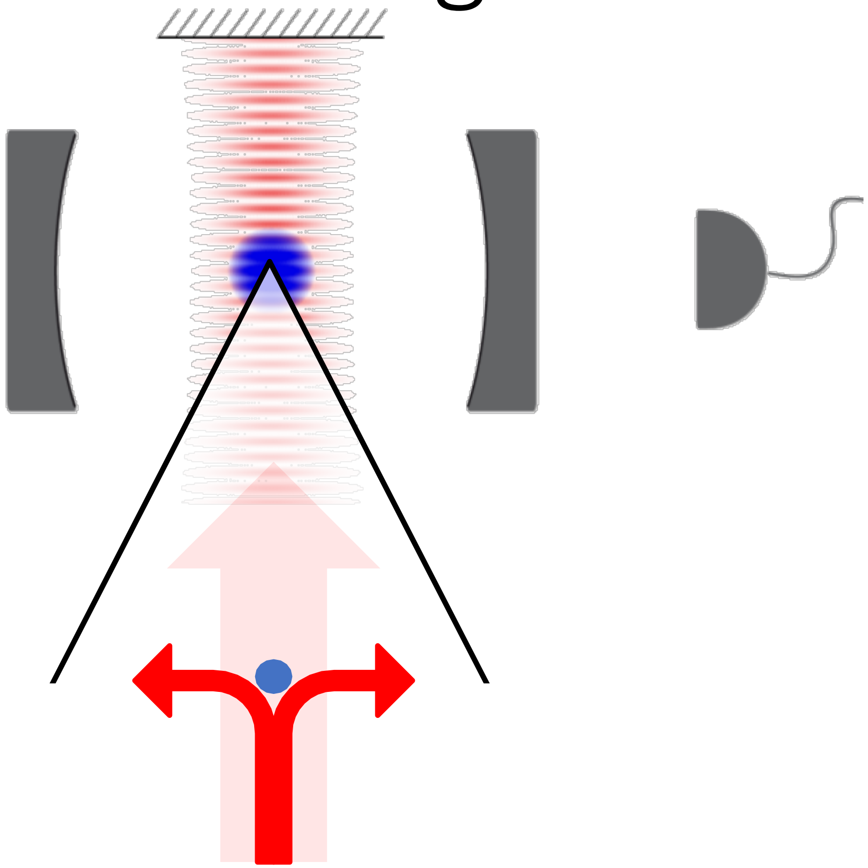
Phase Transition

Dynamical optical lattice!

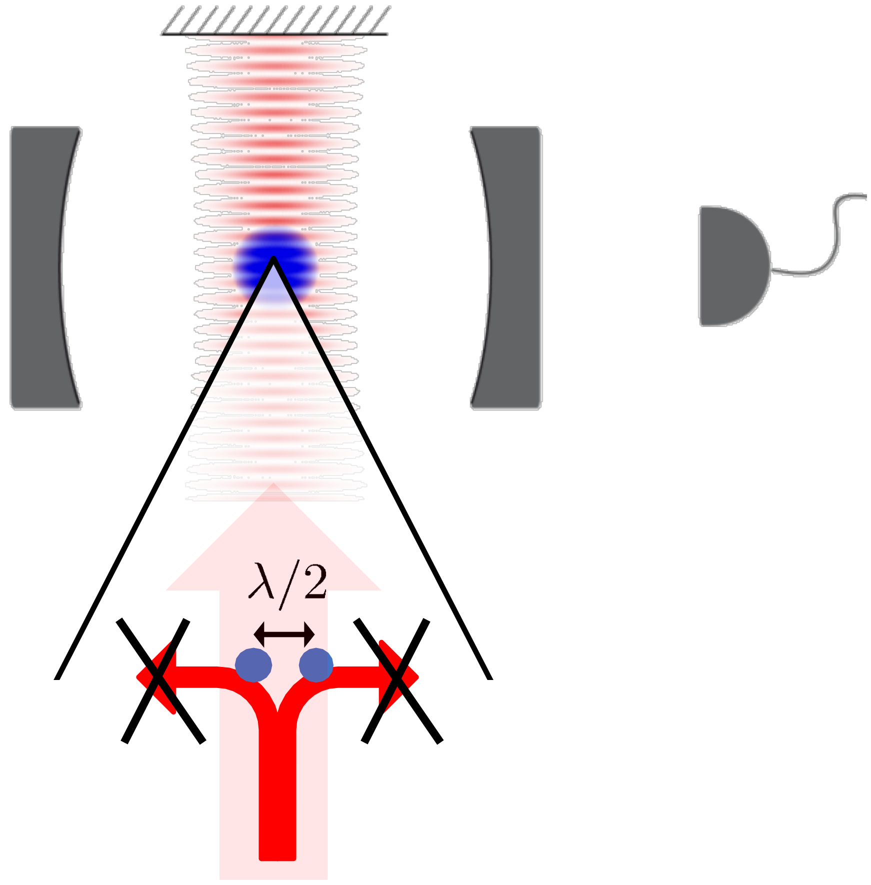


Theory: H. Ritsch, P. Domokos, Exp. with thermal atoms: V. Vuletic

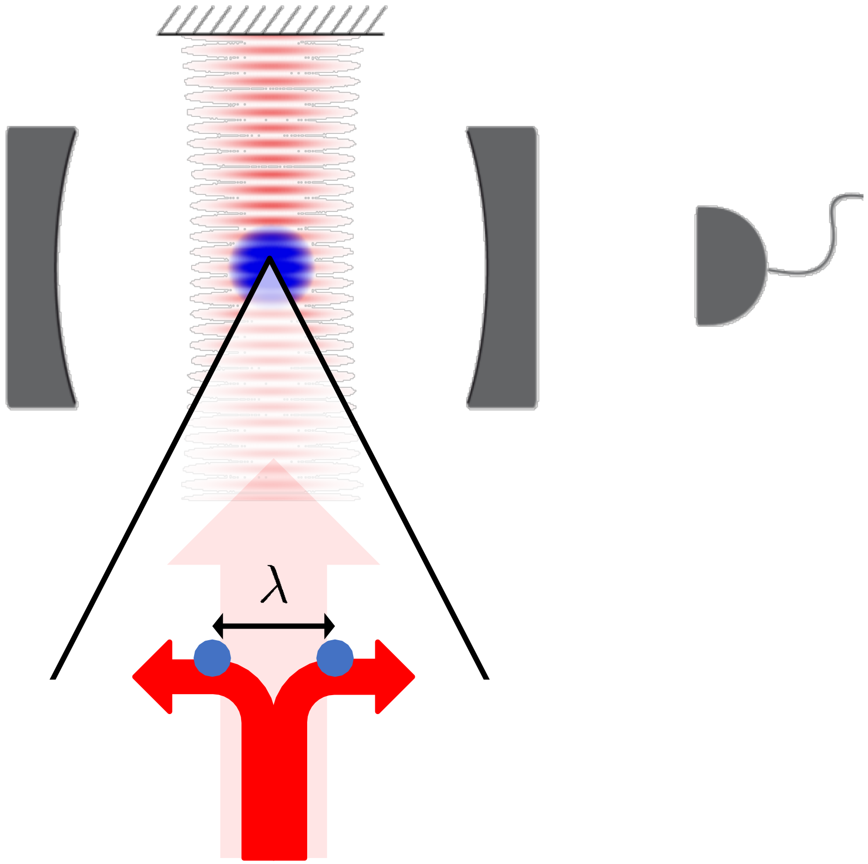
Scattering from a single atom



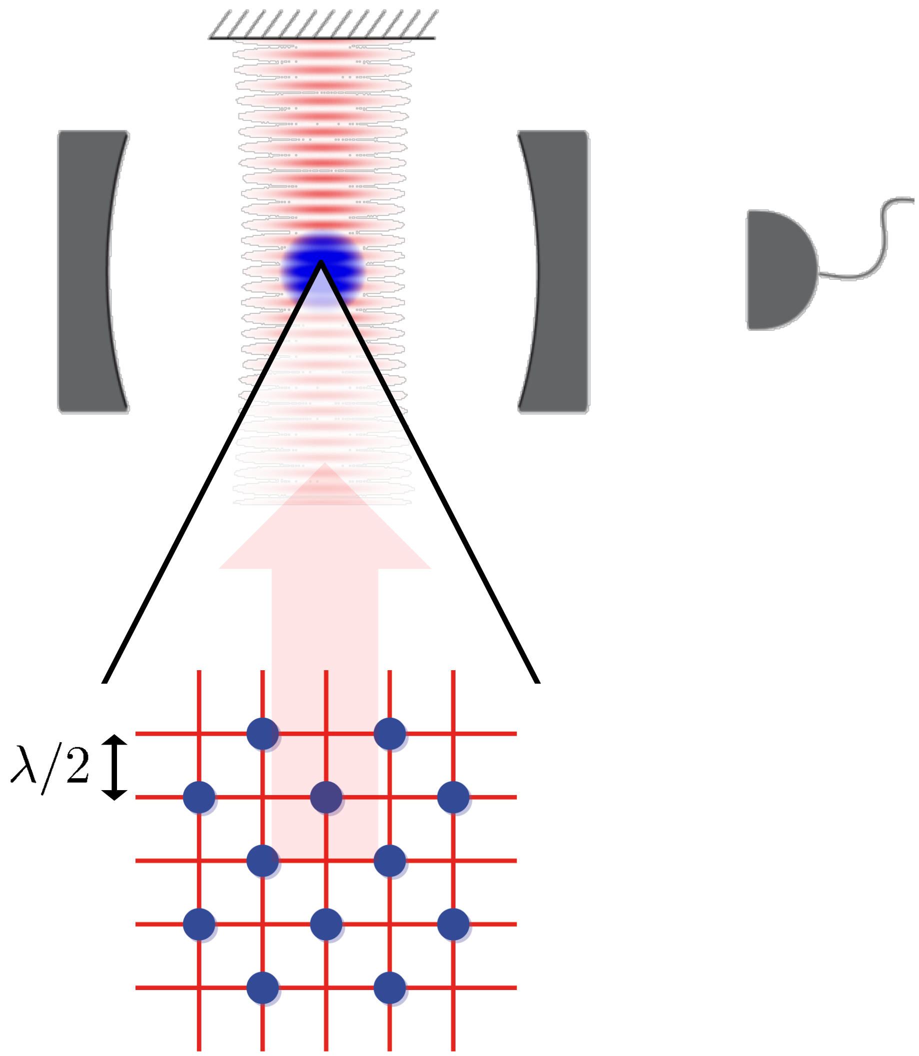
Scattering from two atoms: Interference



Scattering from two atoms: Interference

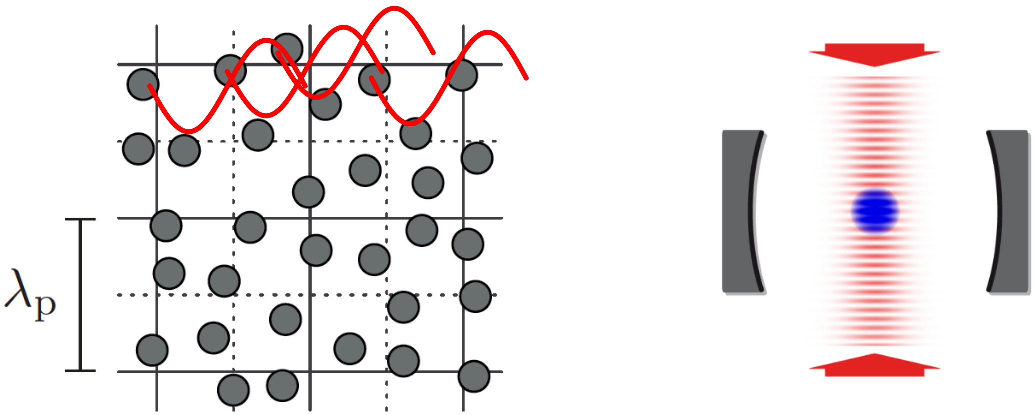


Self-organization

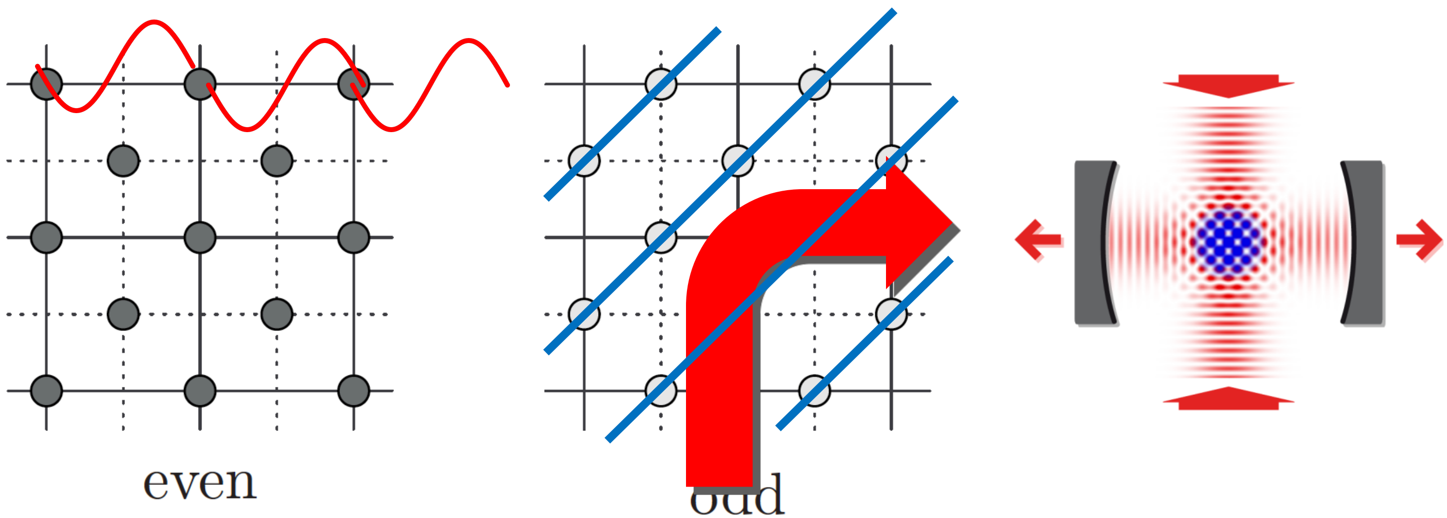


Atomic self-organization – coherent scattering into the cavity

$P < P_c$

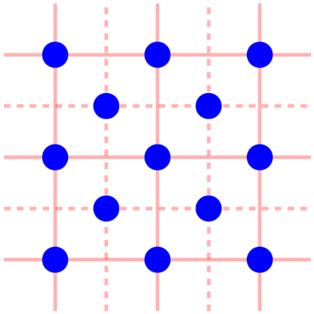


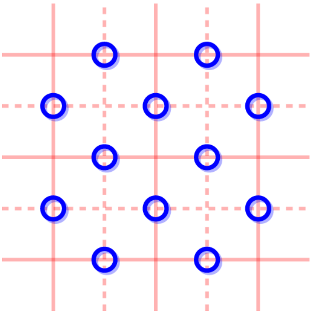
$P > P_c$

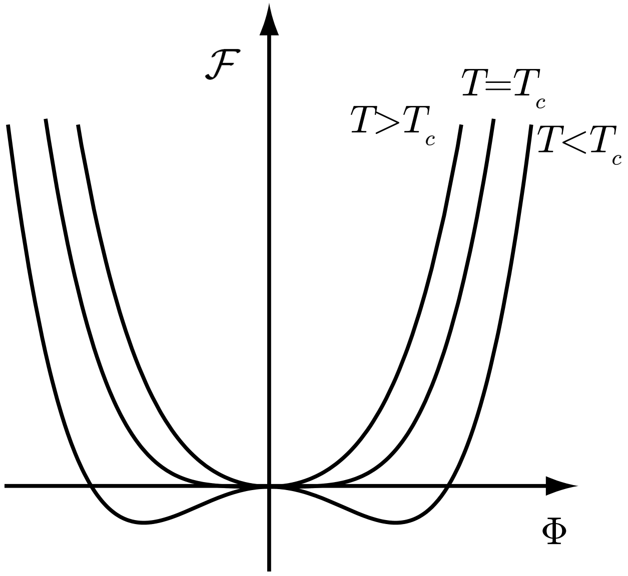


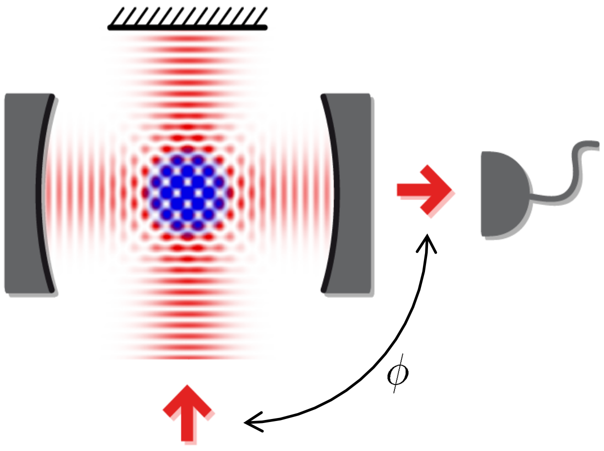
Broken discrete symmetry

Symmetry-breaking

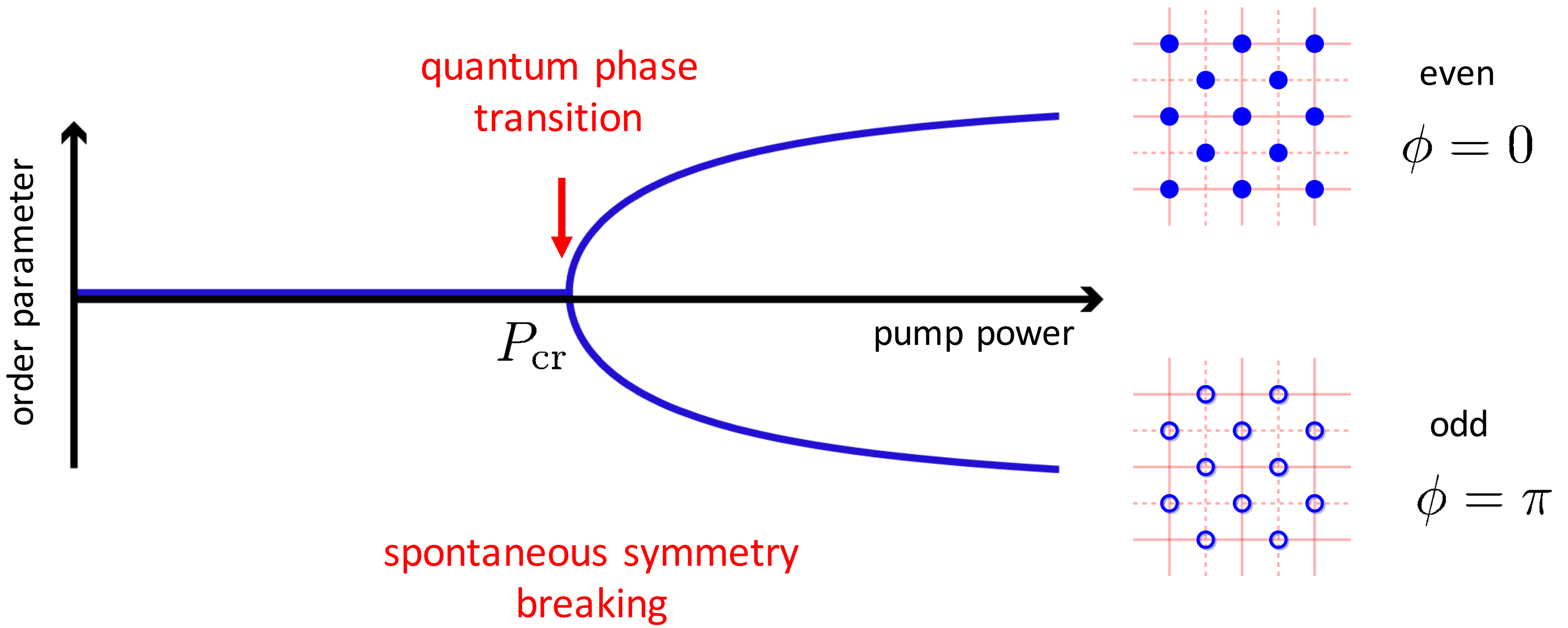




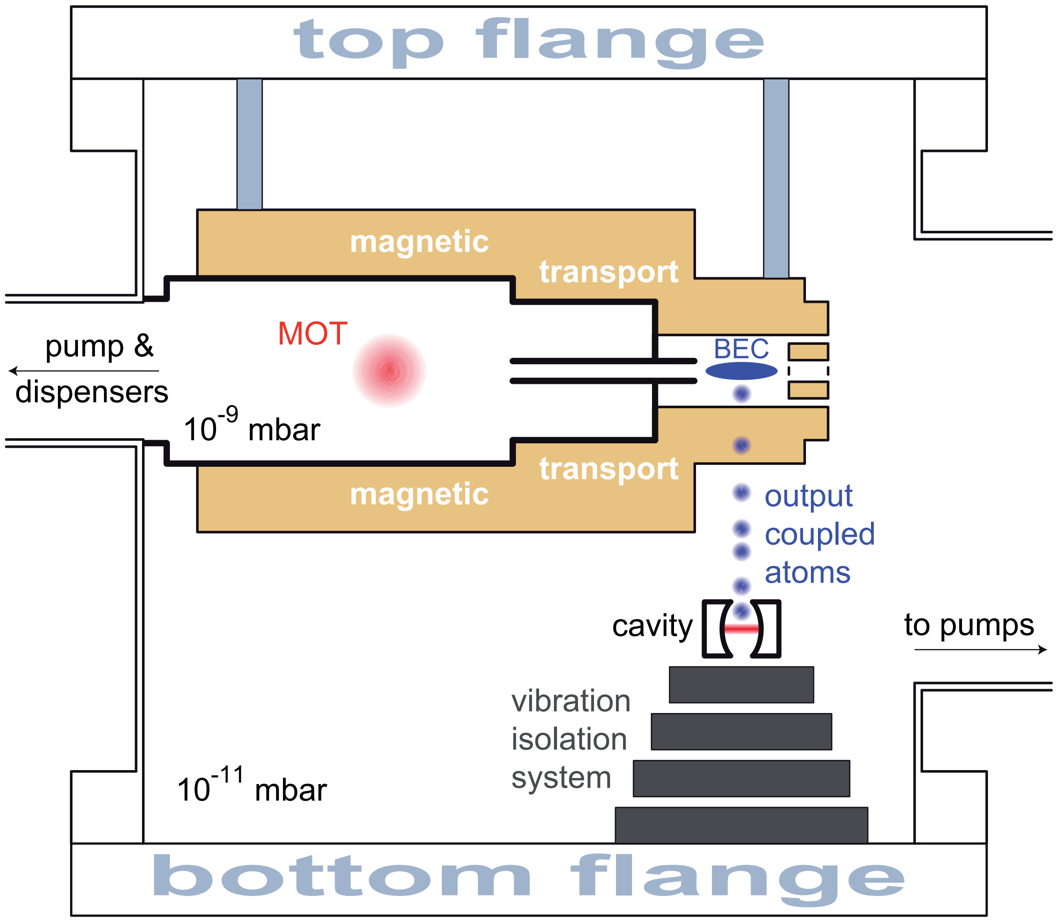




Self-organization

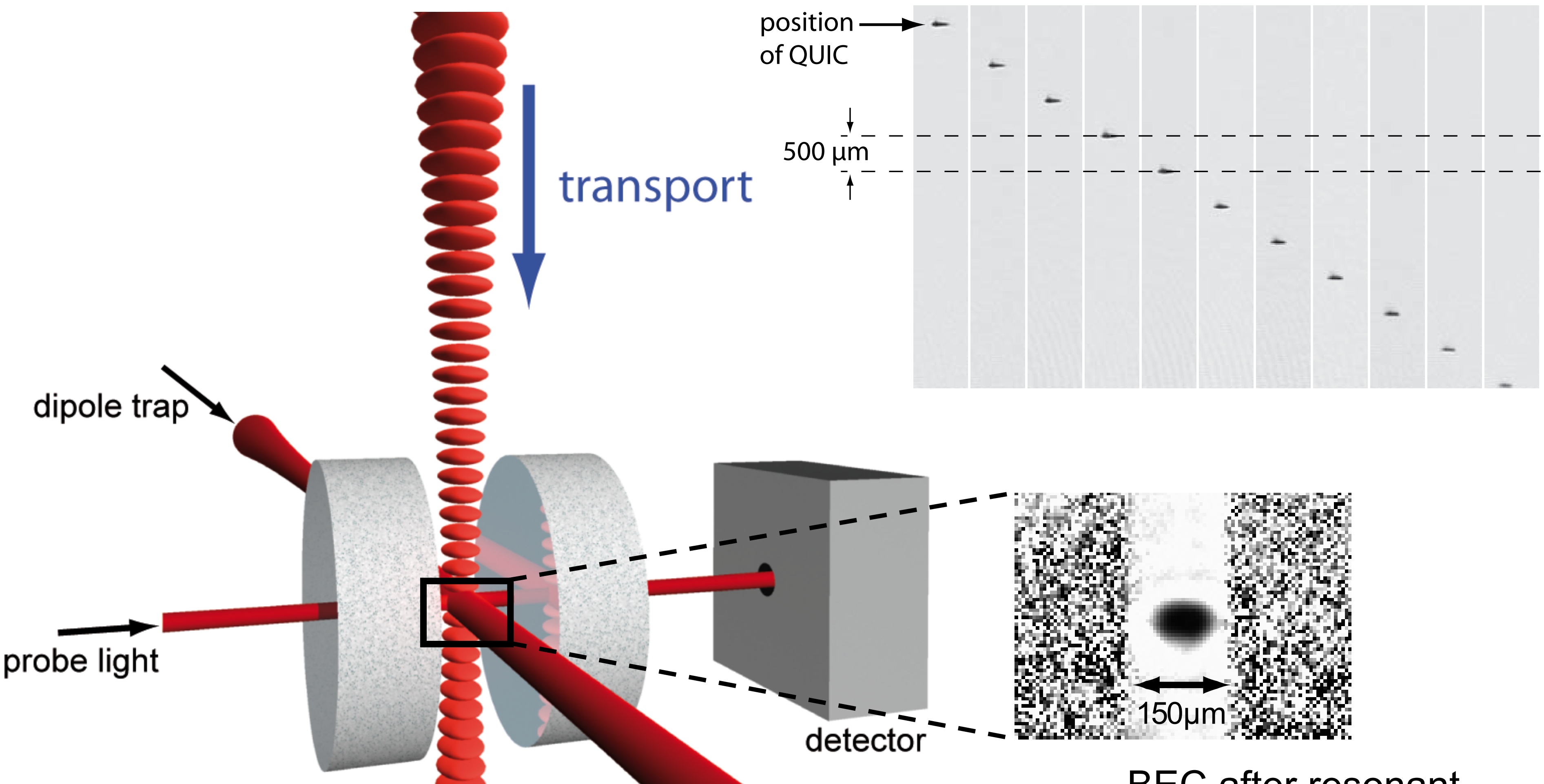


Apparatus



top flange
magnetic
transport
MOT
pump &
dispensers
BEC
10-9 mbar
transport
magnetic
output
coupled
atoms
cavity
to pumps
vibration
isolation
system
10-11 mbar
bottom flange

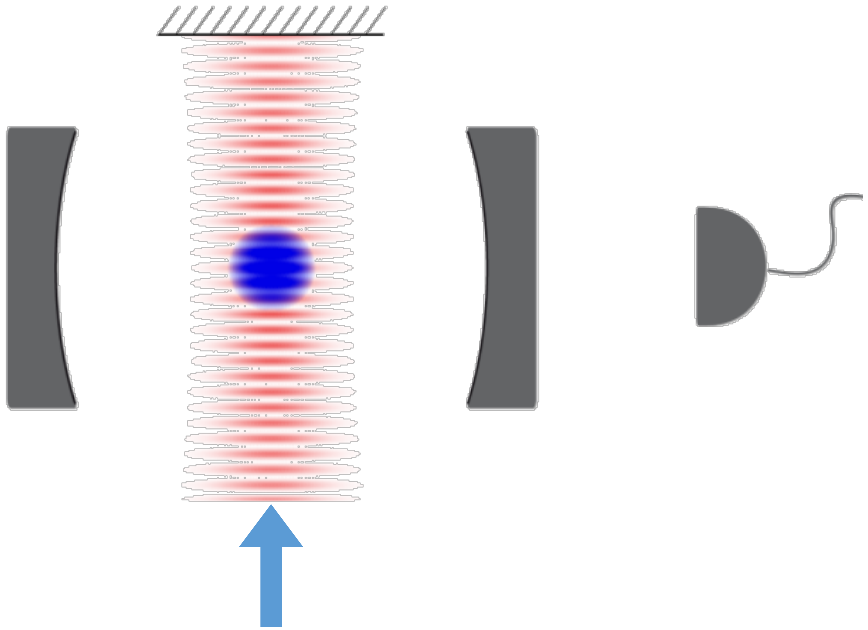
Experimental setup

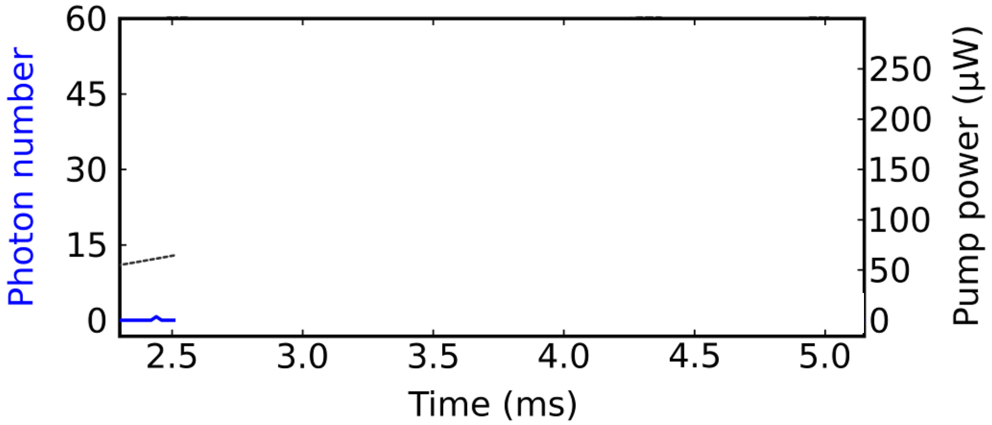


BEC after resonant probing and free expansion

See also: Zimmermann, Hemmerich, Stamper-Kurn, Reichel

Observing Self-Organization





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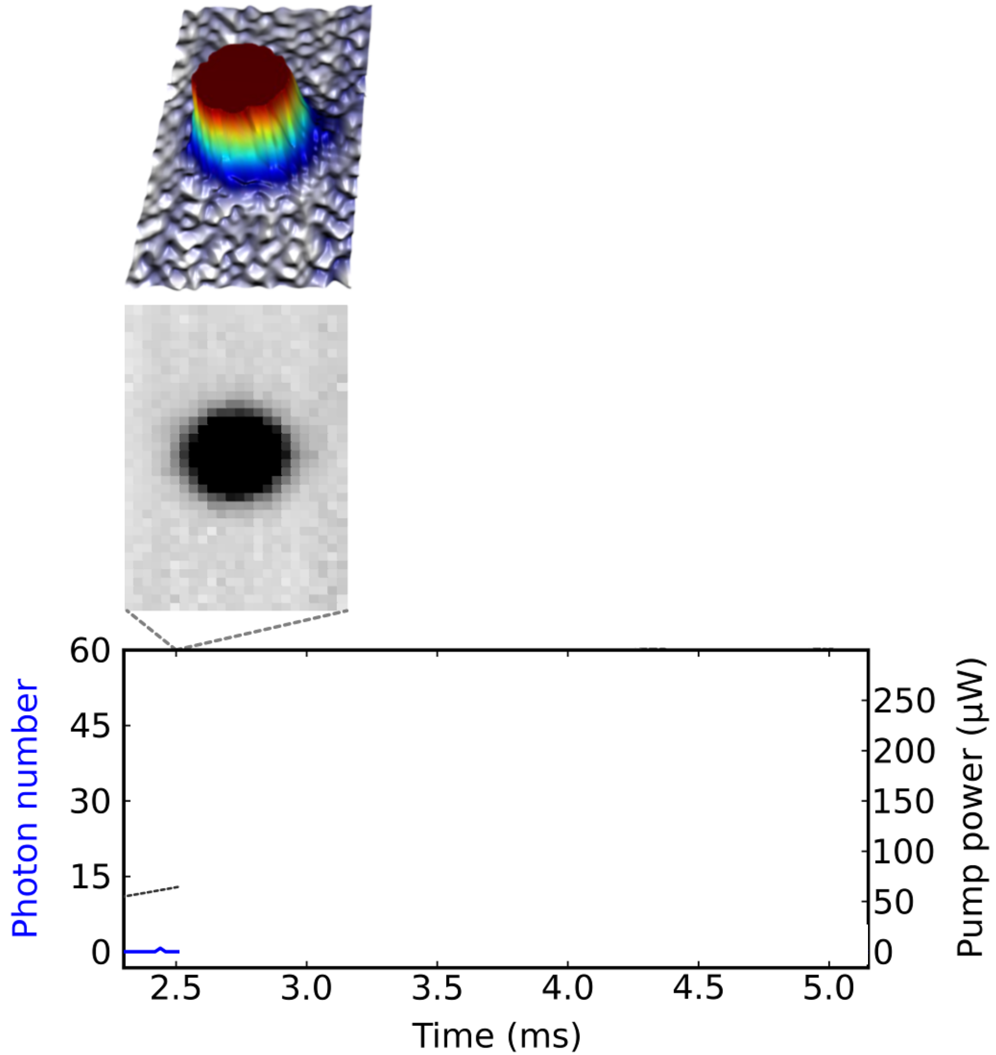

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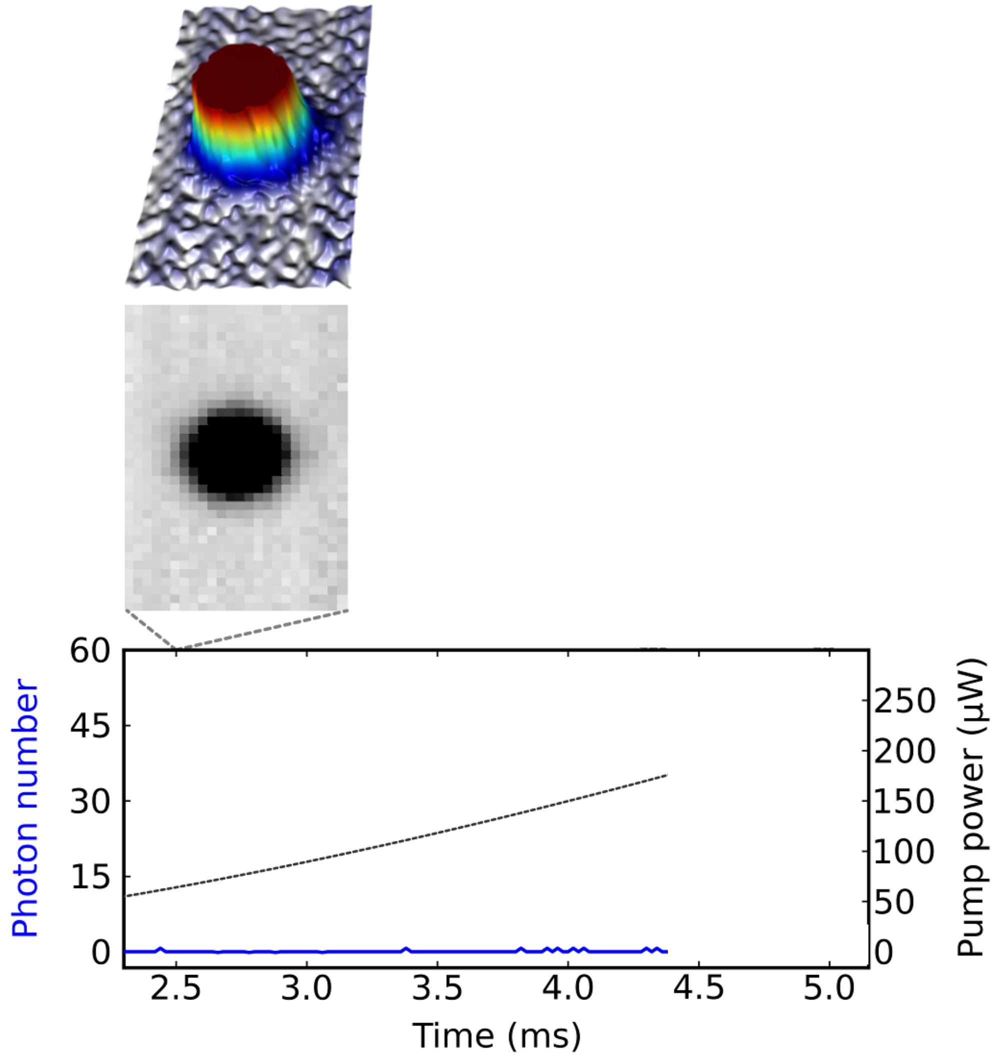

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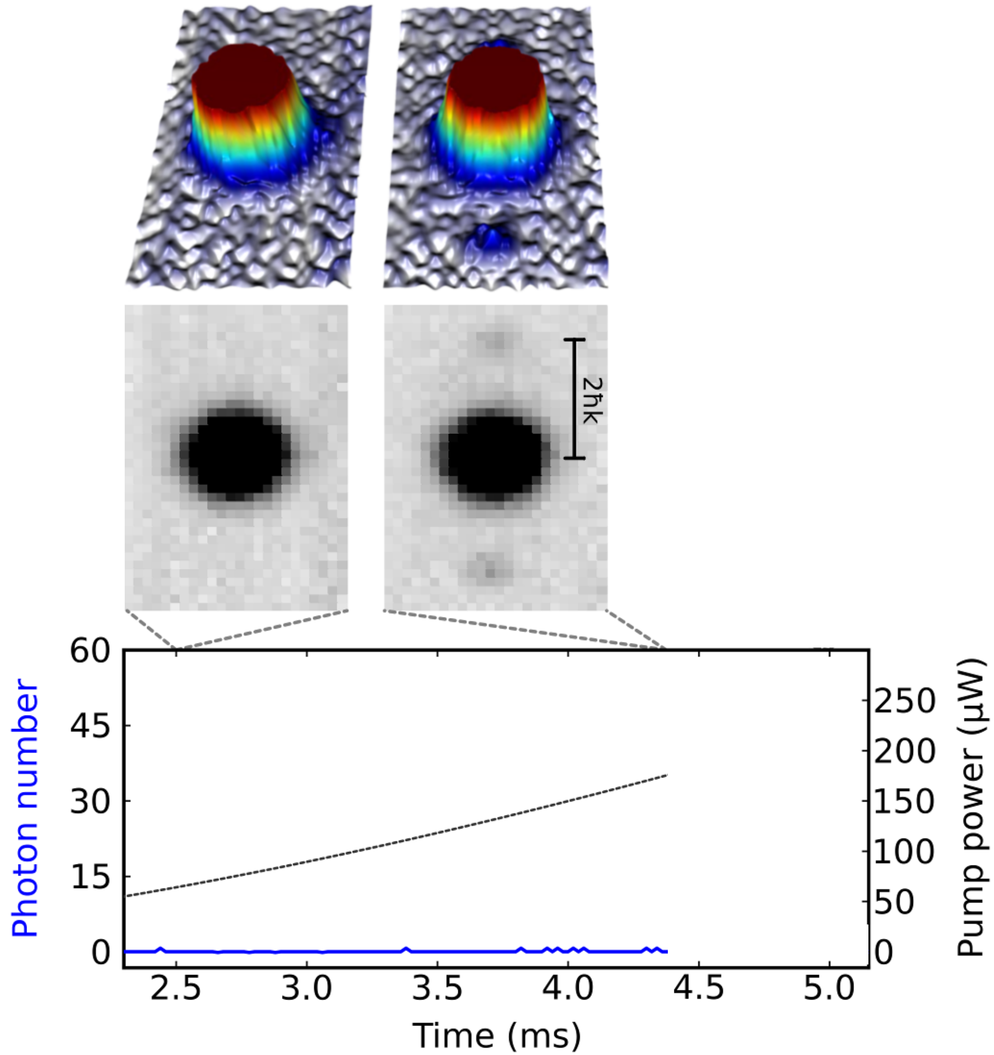

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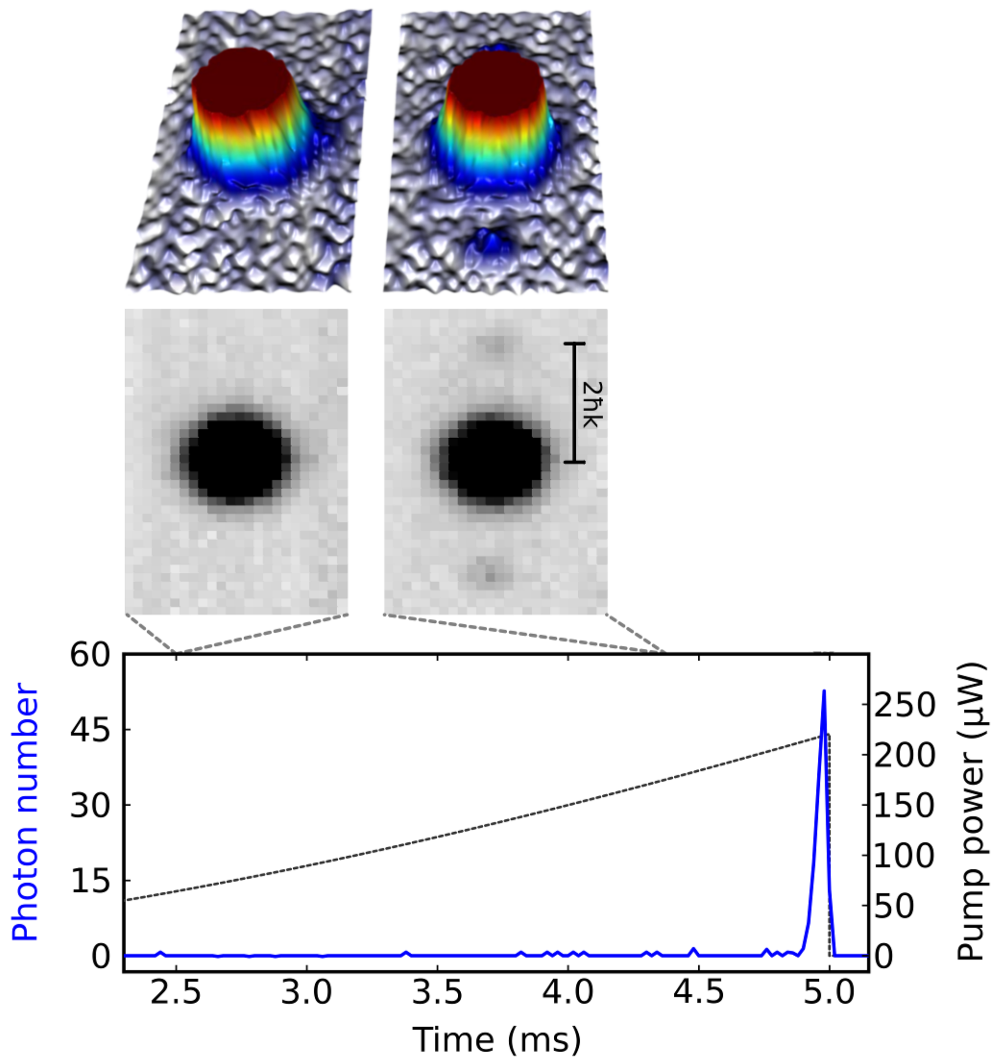

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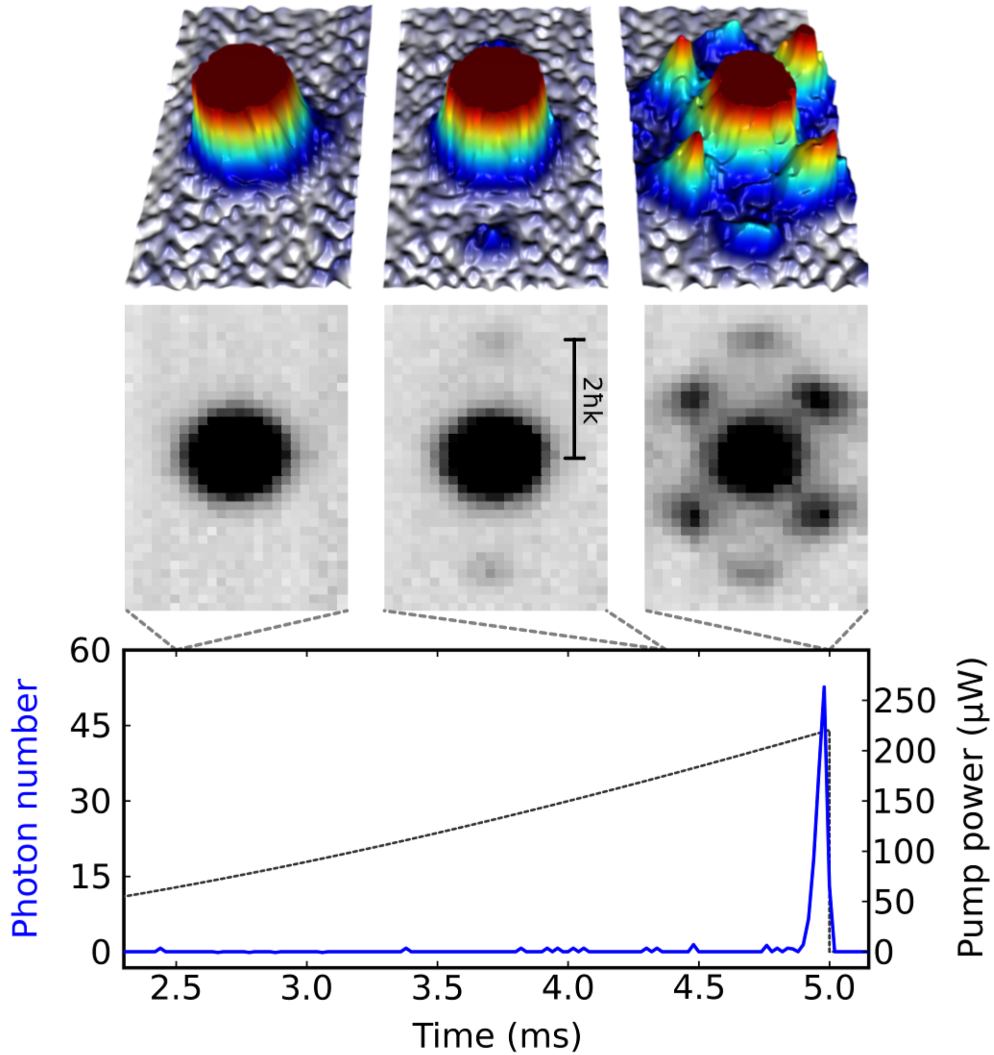

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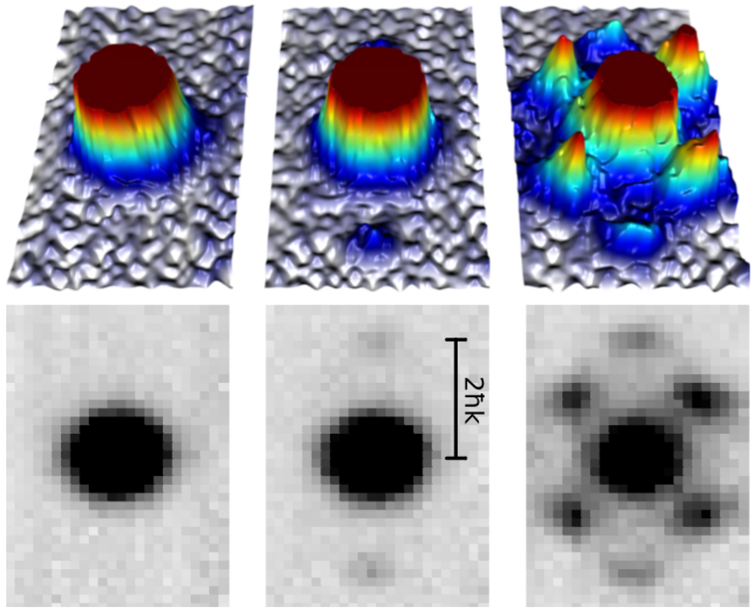
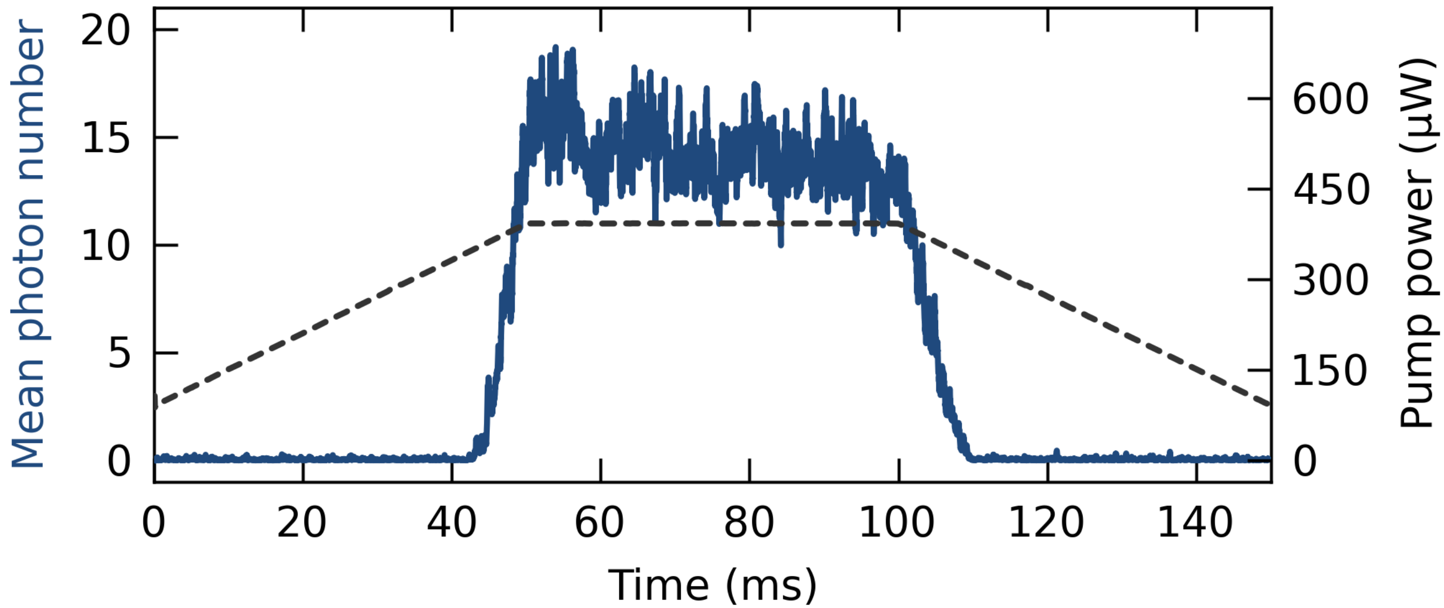


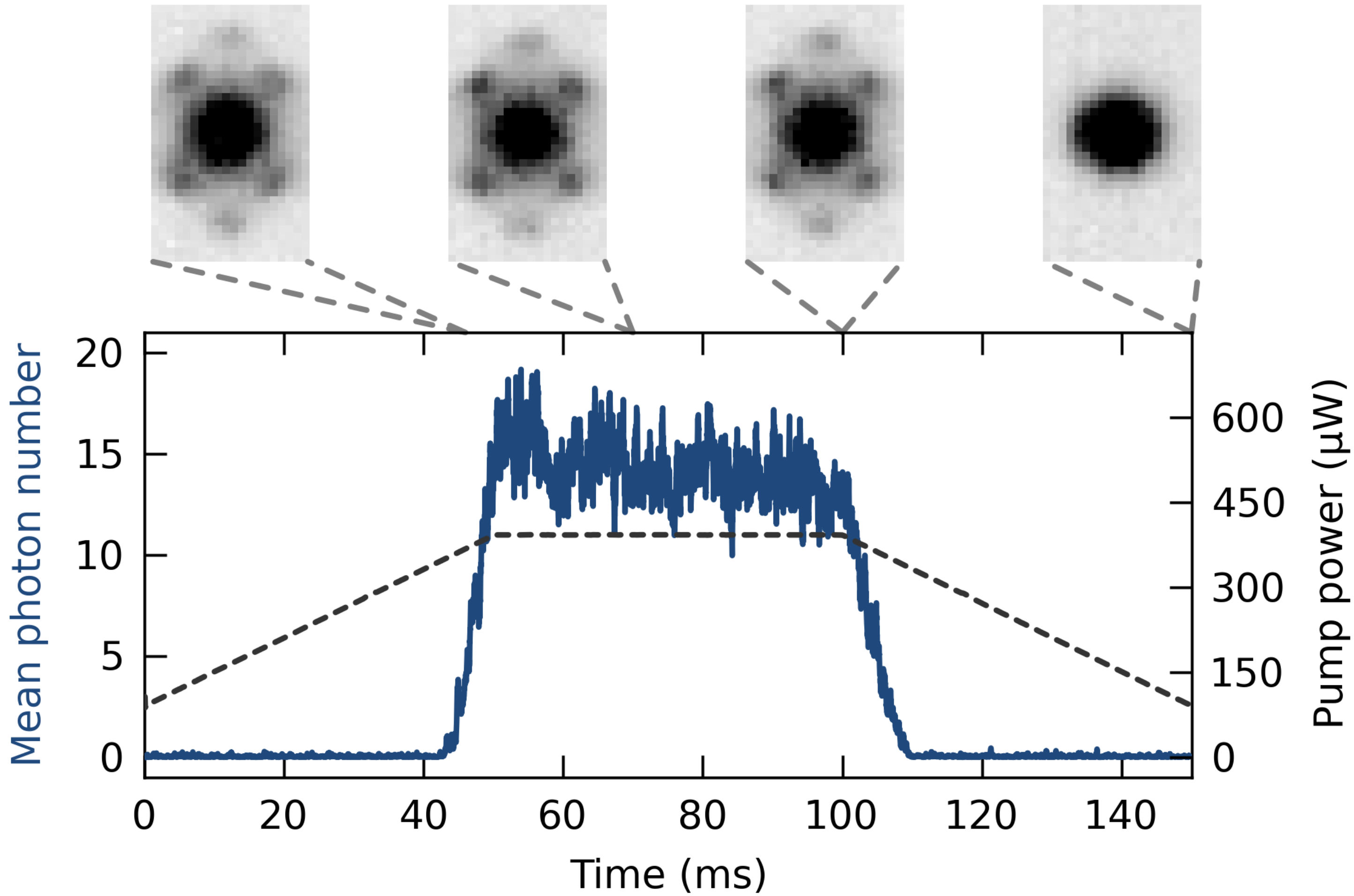
Coexistence of:
- non-trivial diagonal long-range order
- off-diagonal long-range order

The atoms can be regarded as a Supersolid

Stability

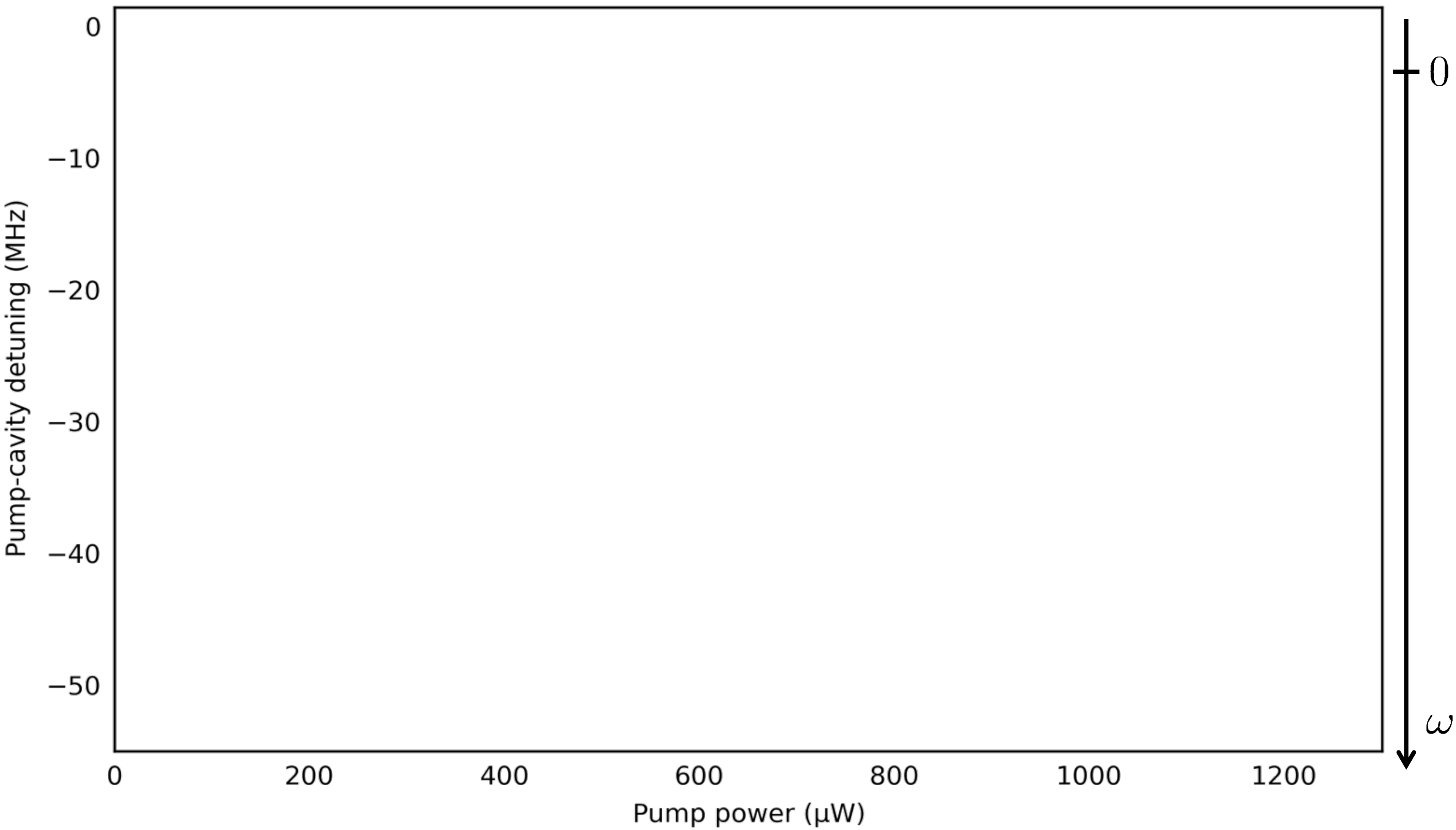

Mean photon number
20
15
10
5
0
Pump power (μW)
600
450
300
150
0
0
20
40
60
80
100
120
140
Time (ms)

Stability and Dephasing



Zero Temperature Phase Diagram

$$\lambda_{\mathrm{cr}} = \sqrt{\omega\omega_0}/2 \qquad P_{\mathrm{cr}} \propto \lambda_{\mathrm{cr}}^2$$



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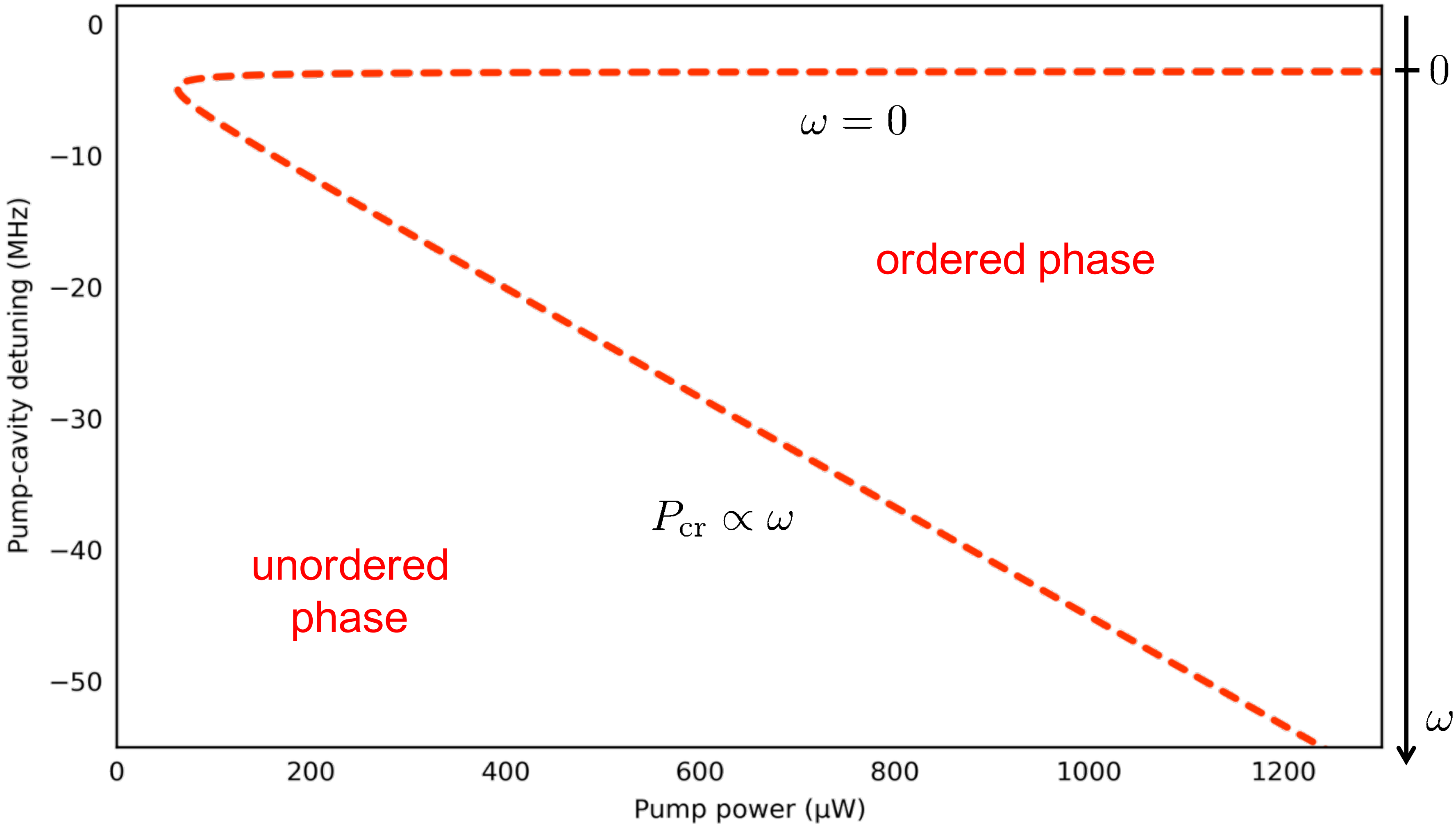

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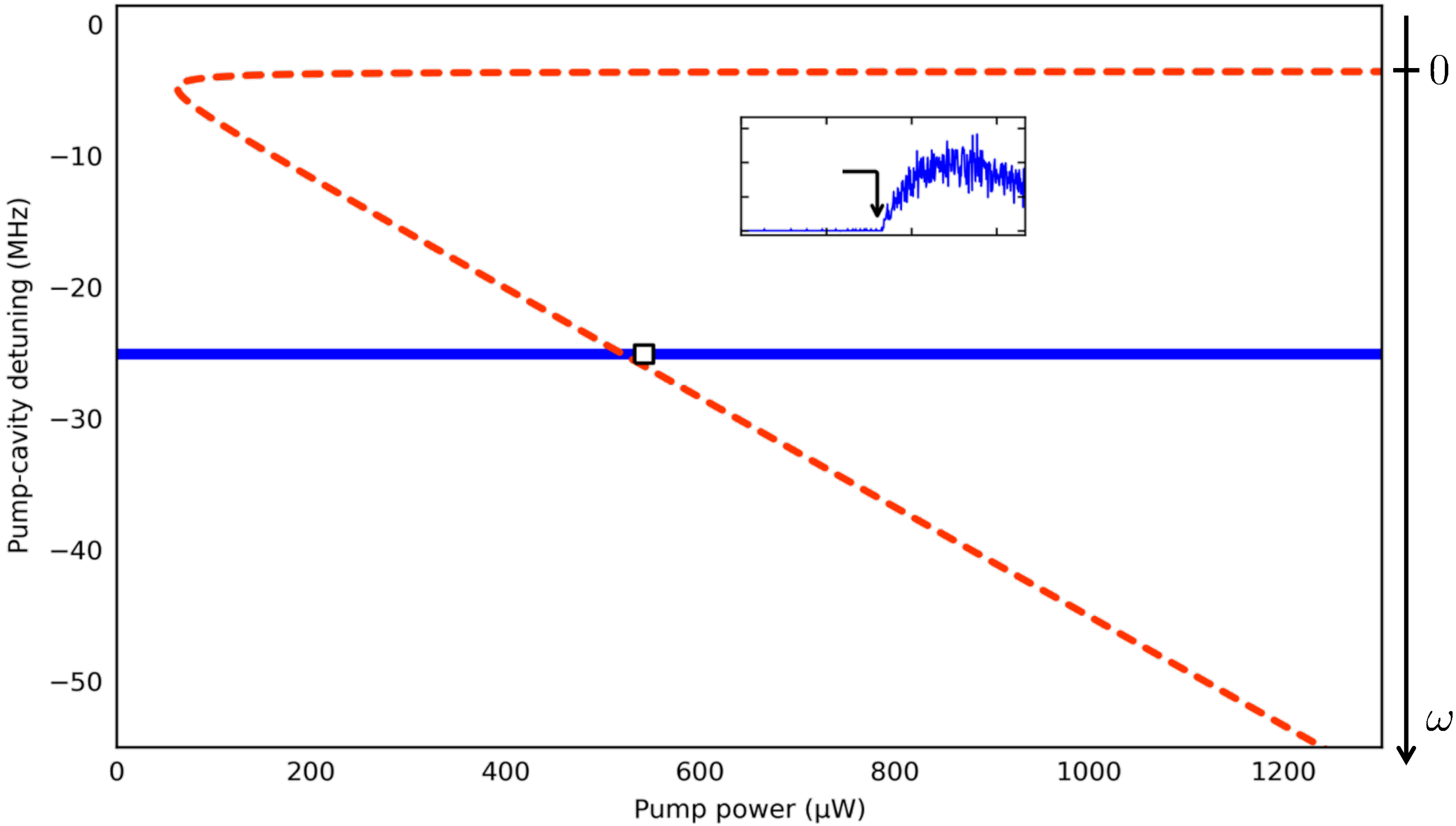
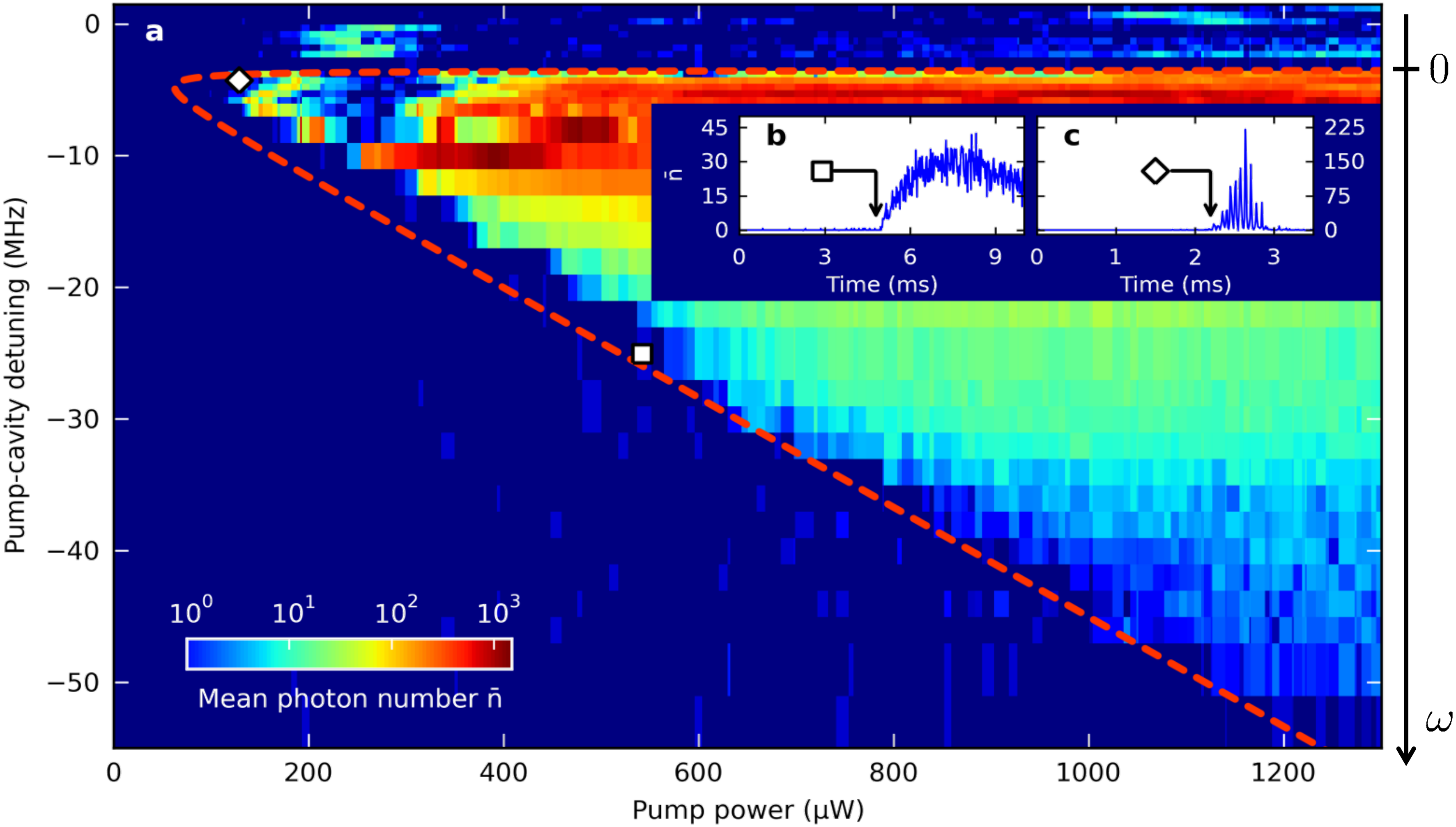
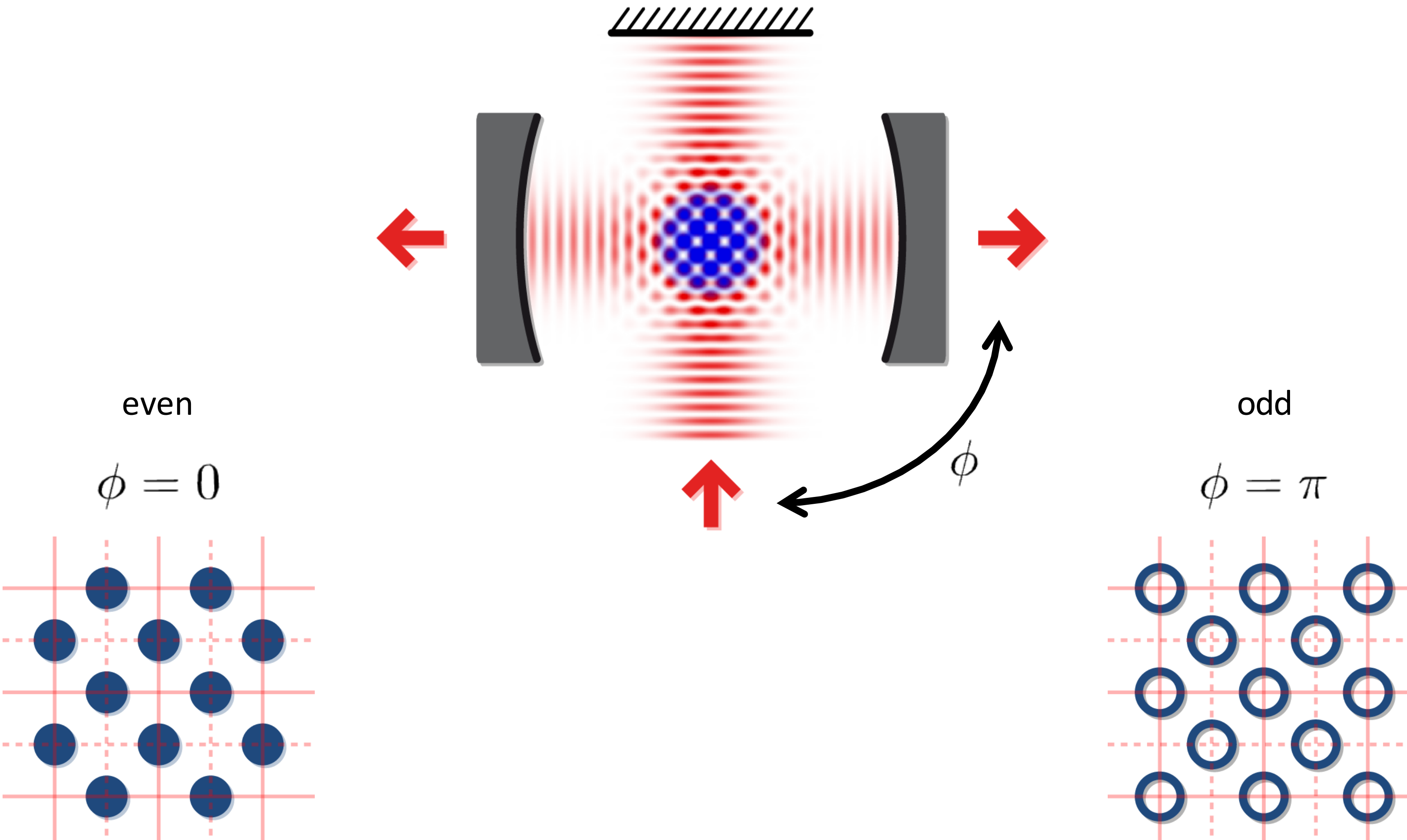


Zero Temperature Phase Diagram



K. Baumann, C. Guerlin, F. Brennecke, and T. Esslinger. Nature 464, 1301 (2010)

Phase Sensitive Detection



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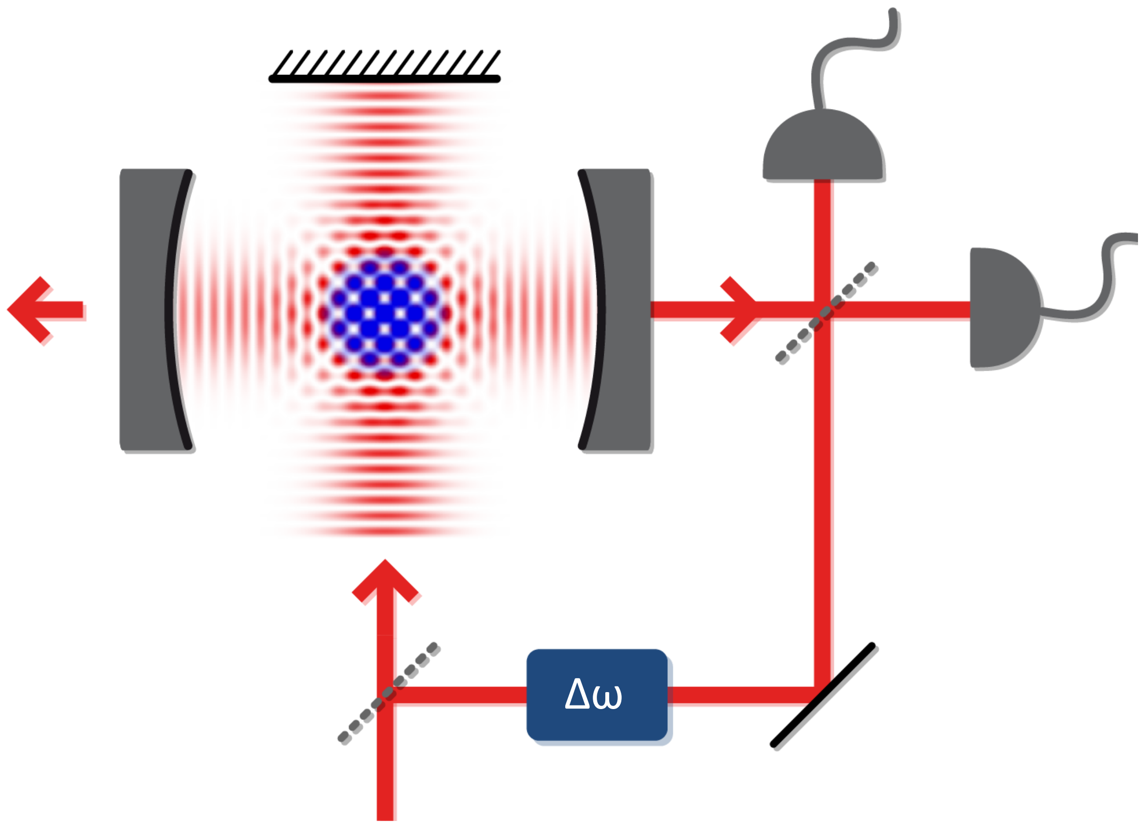
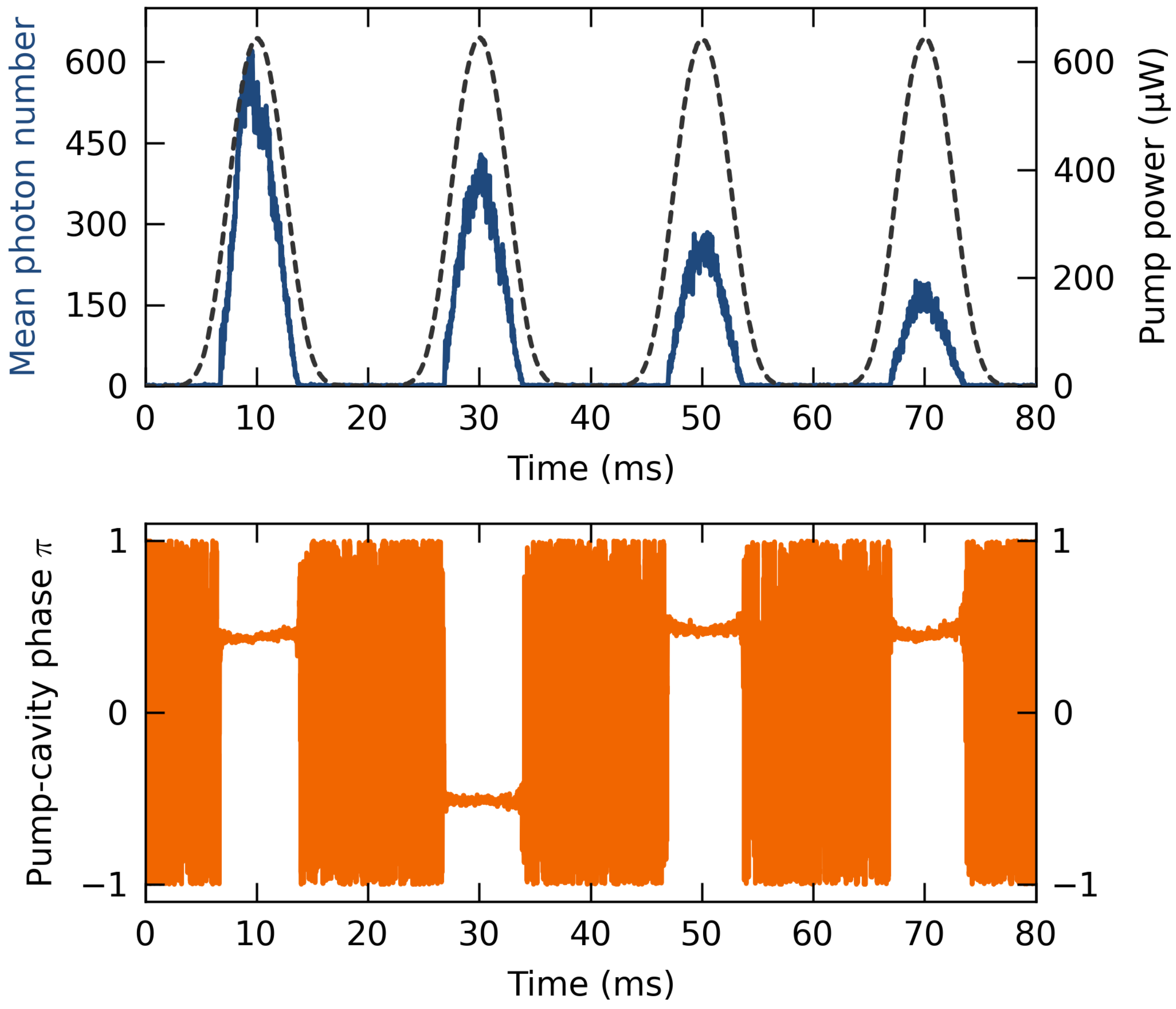


Phase Sensitive Detection



A Quantum phase transition?

…depends on whom you ask:

Common answer:
“Yes! Phase transition taking place at T=0 is a quantum phase transition. There, no thermal fluctuations exist, and the system is breaking the symmetry driven by quantum fluctuations”

S. Sachdev:
“No! This is a classical T=0 phase transition. You should compare how the fluctuations scale in the thermodynamic limit. In your case they disappear – it’s like a pencil on its tip and tipping to one side”

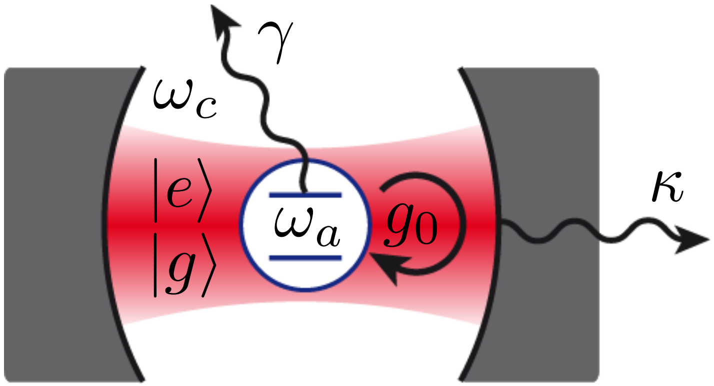
“Well, but in the finite system it’s still the quantum fluctuations driving the system, or not?”

“...there is no phase transition in a finite size system!”

Connection to other physics?

1. Dicke quantum phase transition

Cavity QED: Jaynes-Cummings model



Atom-light interaction:

$$\hat{H}_{\text{int}} = \hat{\mathbf{d}} \cdot \hat{\mathbf{E}} = \hbar g_0 (|g\rangle\langle e| + |e\rangle\langle g|)(\hat{a}^\dagger + \hat{a})$$

$$\hbar g_0 = \mathcal{D}\sqrt{\frac{\hbar\omega_c}{2\epsilon_0 V}}$$

cavity mode volume

Rotating-wave approximation: $g_0 \ll \omega_a, \omega_c$

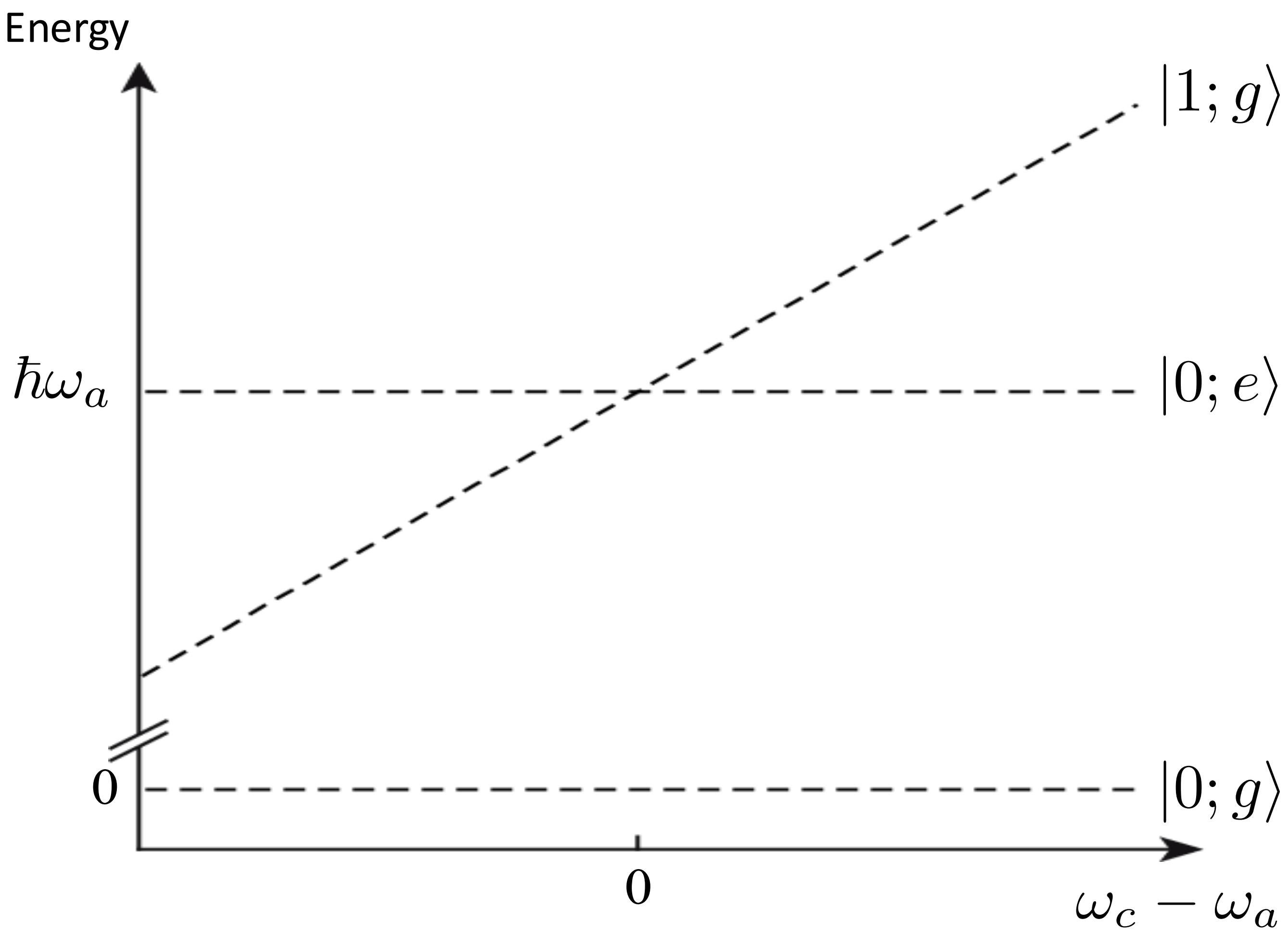
$$\hat{H}_{\text{JC}} = \hbar\omega_a |e\rangle\langle e| + \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar g_0 (\hat{a}^\dagger |g\rangle\langle e| + |e\rangle\langle g| \hat{a})$$

Strong-coupling regime: $g_0 \gg \kappa, \gamma$

Pioneering experiments in cavity QED: H. Walther, S. Haroche, J. Kimble, G. Rempe

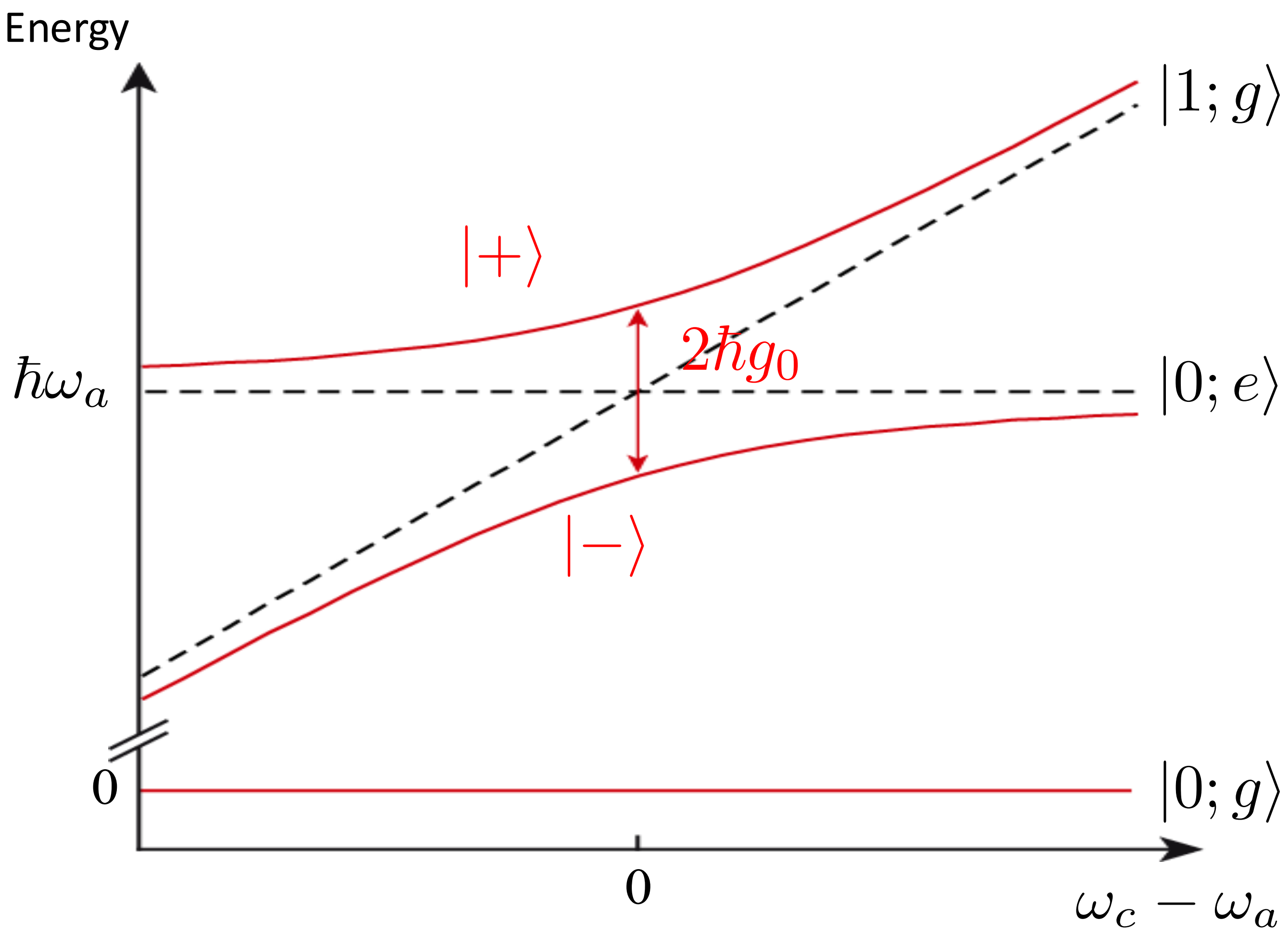
Normal mode splitting

$$\hat{H}_{\rm JC} = \hbar\omega_a|e\rangle\langle e| + \hbar\omega_c\hat{a}^\dagger\hat{a} + \hbar g_0(\hat{a}^\dagger|g\rangle\langle e| + |e\rangle\langle g|\hat{a})$$



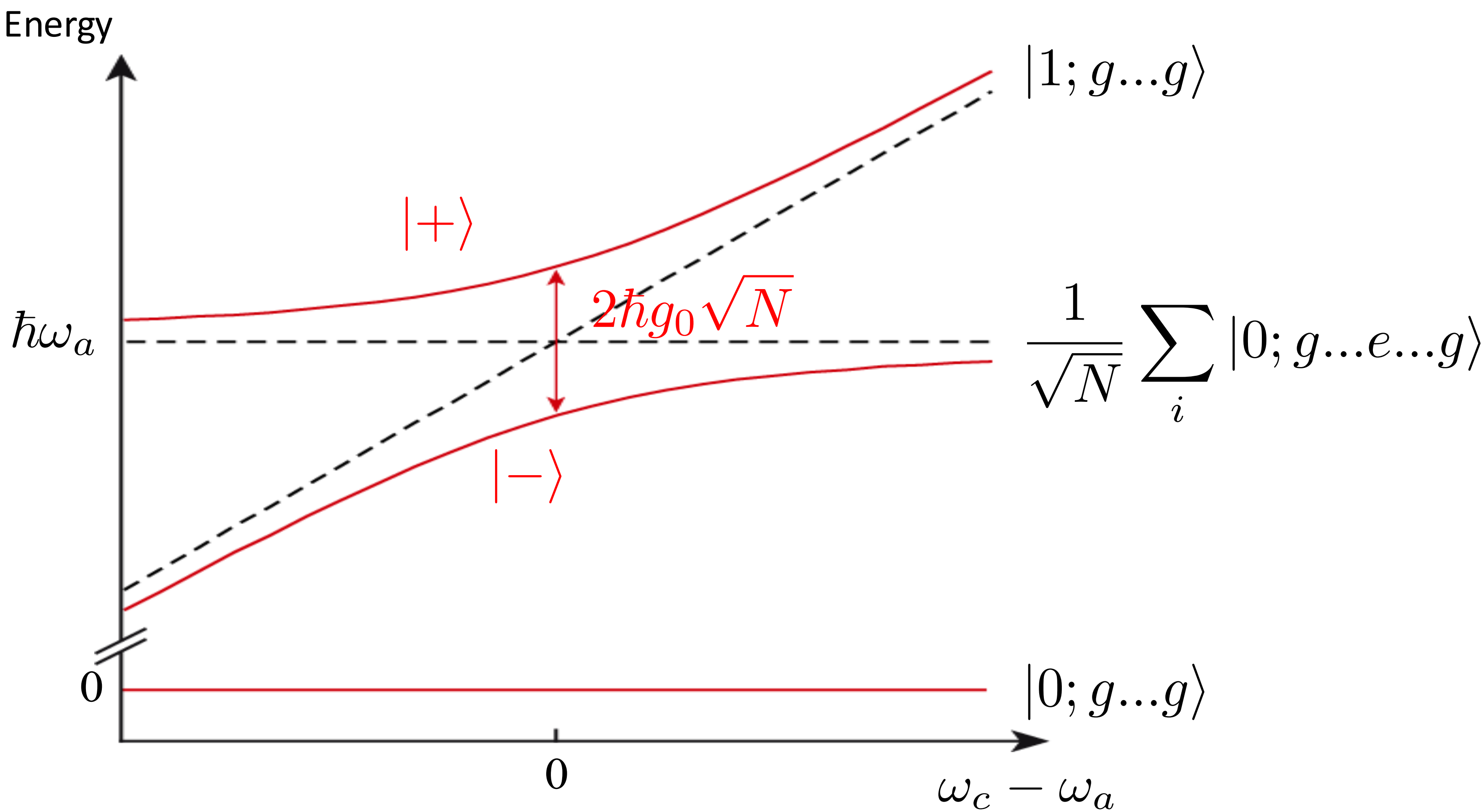
Normal mode splitting

$$\hat{H}_{\mathrm{JC}} = \hbar\omega_a|e\rangle\langle e| + \hbar\omega_c\hat{a}^\dagger\hat{a} + \hbar g_0(\hat{a}^\dagger|g\rangle\langle e| + |e\rangle\langle g|\hat{a})$$



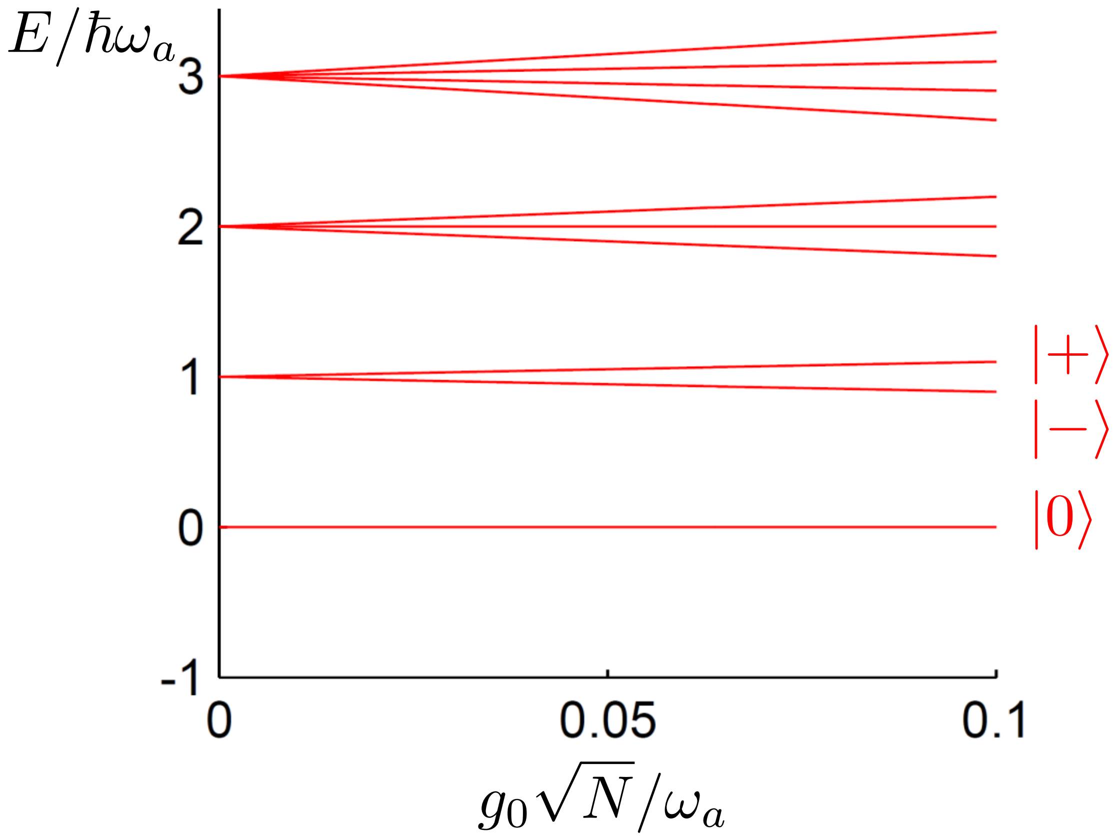
Extension to N atoms: Dicke model

$$\hat{H}_{\text{Dicke}} = \sum_i \hbar\omega_a |e\rangle_i\langle e| + \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar g_0 \sum_i (\hat{a}^\dagger |g\rangle_i\langle e| + |e\rangle_i\langle g|\hat{a})$$



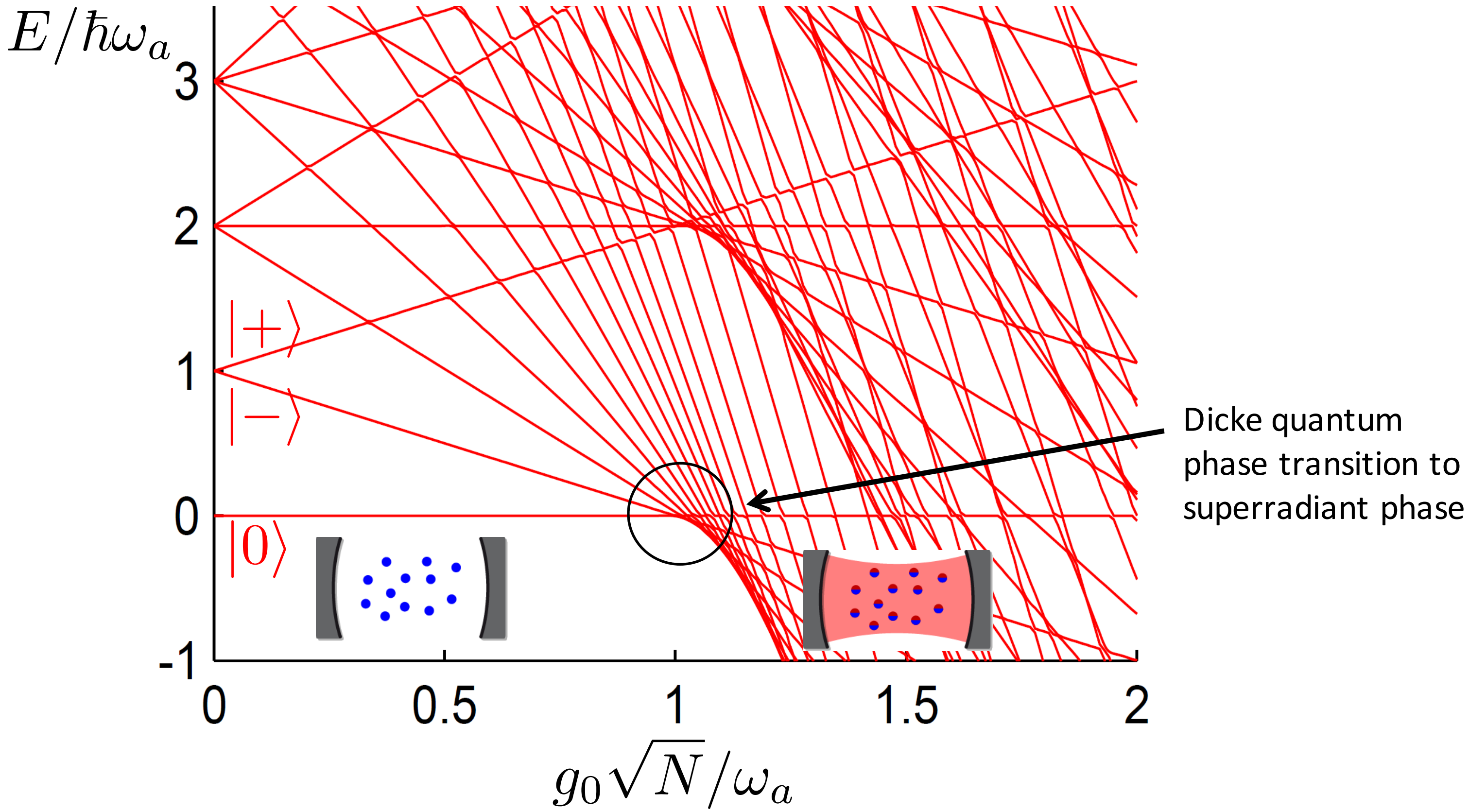
Ultrastrong coupling limit

What happens if $g_0\sqrt{N}$ reaches ω_a?



Ultrastrong coupling limit

What happens if $g_0\sqrt{N}$ reaches ω_a?



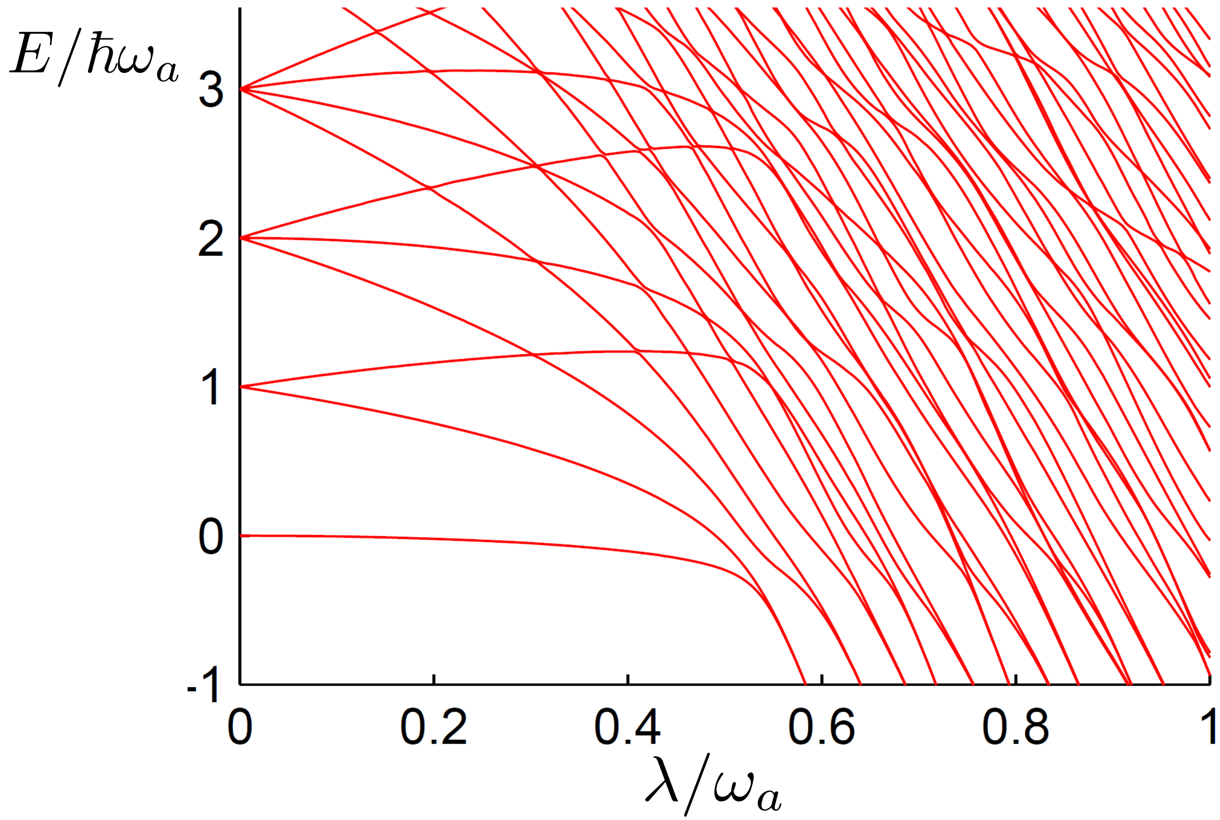
Hepp & Lieb. Ann. Phys. 76, 370 (1973), Wang & Hioe PRA 7, 831 (1973), Emary & Brandes PRE 67, 066203 (2003)

Ultrastrong coupling limit

… without rotating wave approximation…

$$\hat{H}_{\text{Dicke}}/\hbar = \omega_a \hat{J}_z + \omega_c \hat{a}^\dagger \hat{a} + \frac{\lambda}{\sqrt{N}}(\hat{a}^\dagger + \hat{a})(\hat{J}_+ + \hat{J}_-)$$

$$\hat{J}_z = \sum_i \frac{(|e\rangle_i\langle e| - |g\rangle_i\langle g|)}{2}$$
$$\hat{J}_+ = \sum_i |e\rangle_i\langle g|$$
$$\lambda = g_0\sqrt{N}$$



$$\lambda_{\text{cr}} = \frac{\sqrt{\omega_a \omega_c}}{2}$$

Hepp & Lieb. Ann. Phys. 76, 370 (1973), Wang & Hioe PRA 7, 831 (1973), Emary & Brandes PRE 67, 066203 (2003)

Reaching the Dicke phase transition

- critical atom number:

$$\lambda_{\rm cr} = \frac{\sqrt{\omega_a \omega_c}}{2} = g_0 \sqrt{N_{\rm cr}}$$

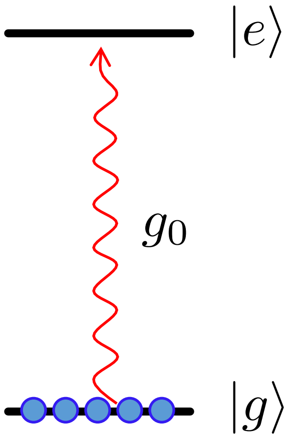
$$\Rightarrow N_{\rm cr} \approx 10^{14}$$

$$\omega_a/2\pi = 384\mathrm{THz}$$
$$g_0/2\pi = 10\mathrm{MHz}$$

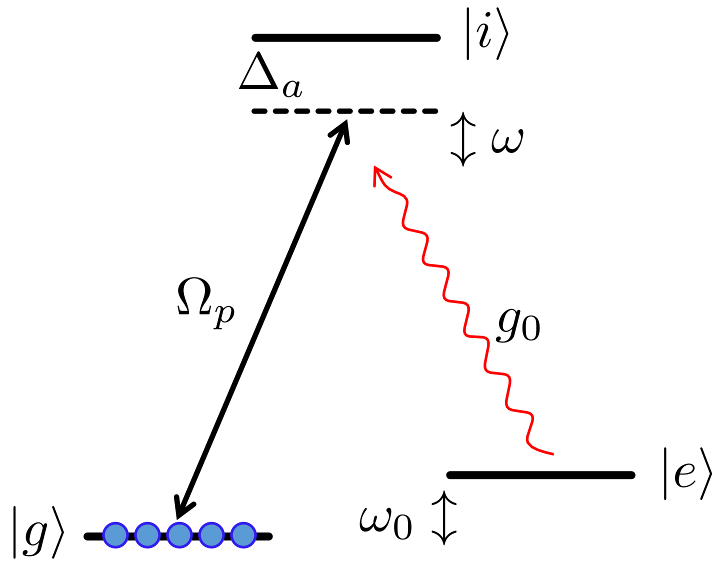
$$\hat{H}_{\rm Dicke}/\hbar = \omega_a \hat{J}_z + \omega_c \hat{a}^\dagger \hat{a} + \frac{\lambda}{\sqrt{N}}(\hat{a}^\dagger + \hat{a})(\hat{J}_+ + \hat{J}_-)$$

Rzazewski et al. PRL. 35 , 432 (1975), Vukics & Domokos PRA 86, 053807 (2012)

Effective realizations of the Dicke model



Effective realizations of the Dicke model



$$\lambda = \frac{\Omega_p g_0}{\Delta_a}\sqrt{N}$$
$$\lambda_{\mathrm{cr}} \approx \sqrt{\omega_0 \omega}$$

Theoretical proposal: Dimer et al. PRA 75, 013804 (2007)

PHYSICAL REVIEW A **75**, 013804 (2007)

Proposed realization of the Dicke-model quantum phase transition in an optical cavity QED system

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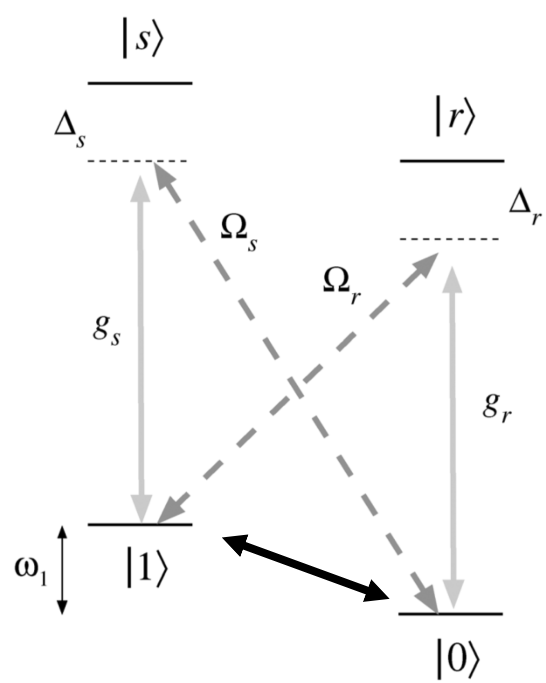
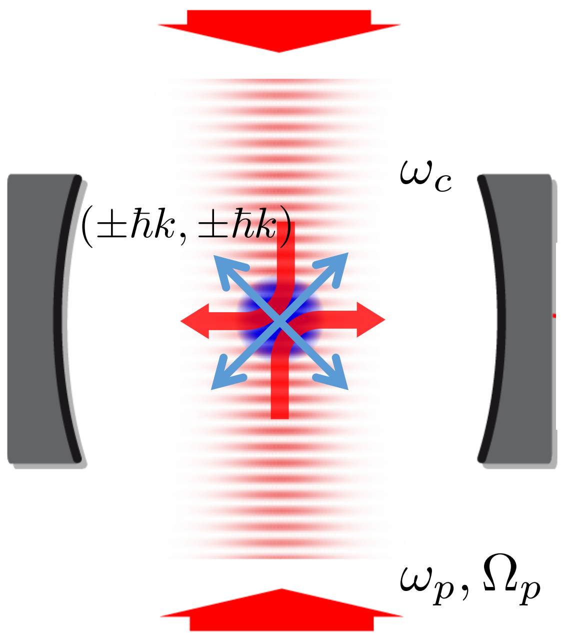
(Received 18 July 2006; published 8 January 2007)

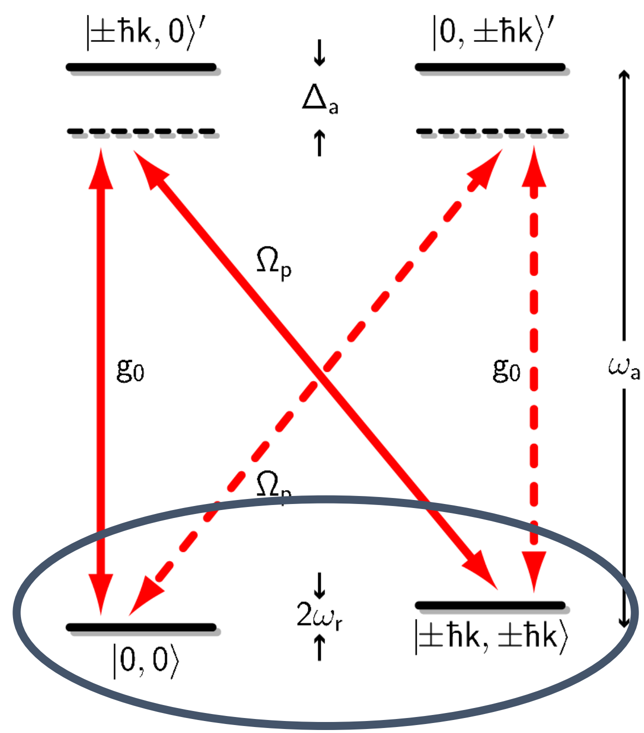


described by the Dicke Hamiltonian

Dicke model with motional states



Pump laser



$$\hat{H}/\hbar = \omega_0 \hat{J}_z + \omega \hat{a}^\dagger \hat{a} + \frac{\lambda}{\sqrt{N}}(\hat{a}^\dagger + \hat{a})(\hat{J}_+ + \hat{J}_-)$$

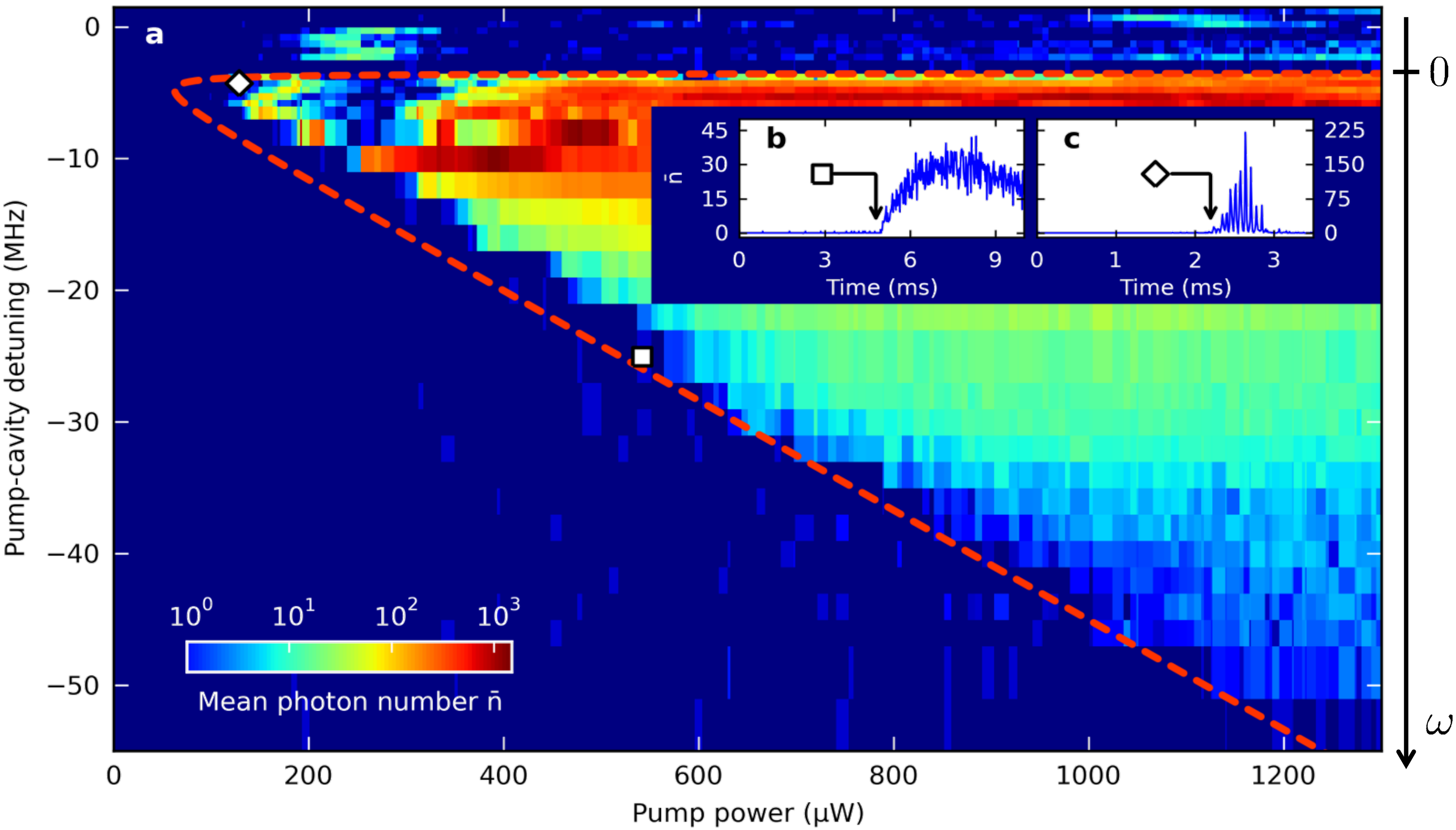
$$\omega_p - \omega_a \gg \gamma$$
$$\omega_p - \omega_c \simeq \kappa$$

$$\omega_0 = 2\omega_r = 2\pi \times 8.4\text{kHz}$$
$$\omega = \omega_p - \omega_c' \simeq 2\pi \times 10\text{MHz}$$
$$\lambda = \sqrt{N} g_0 \Omega_p / 2\Delta_a$$

} tunable!

K. Baumann et al., Nature 464, 1301 (2010), D. Nagy et al. PRL 104, 130401 (2010)

Zero Temperature Phase Diagram
OR
Dicke phase diagram



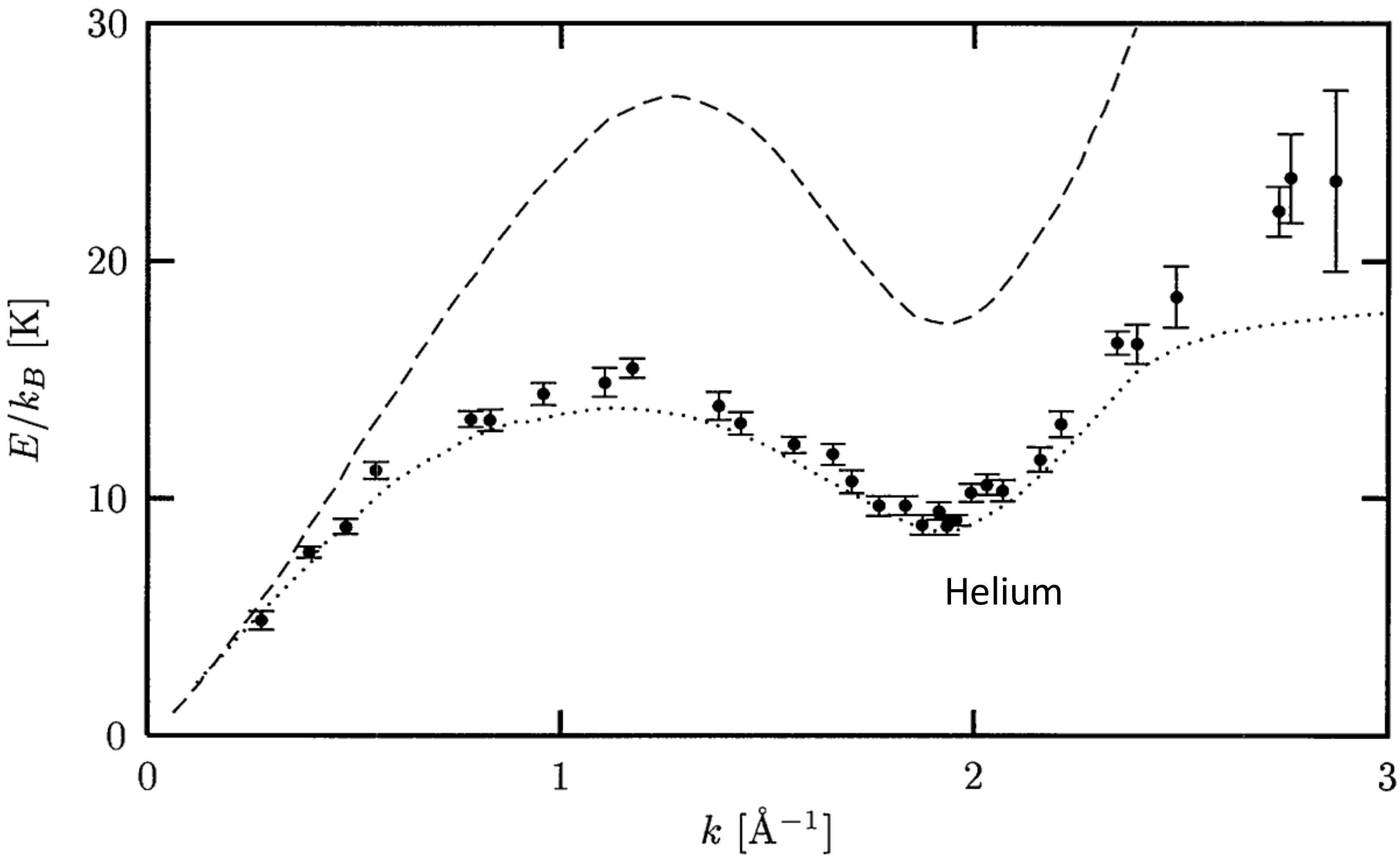
K. Baumann, C. Guerlin, F. Brennecke, and T. Esslinger. Nature 464, 1301 (2010)

Connection to other physics?

2. Roton minimum in SF 4He

Momentum dependent interaction - roton

For dipolar gases: L. Santos, G.V. Shlyapnikov, and M. Lewenstein, PRL 90, 250403 (2003)

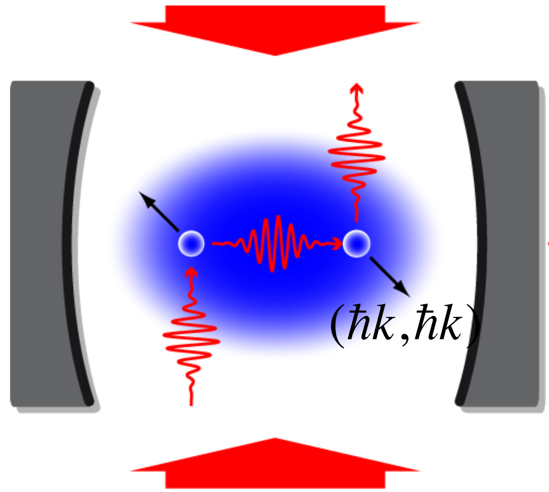


Momentum dependent interaction - roton

For dipolar gases: L. Santos, G.V. Shlyapnikov, and M. Lewenstein, PRL 90, 250403 (2003)

We emphasize that the roton-maxon spectrum finds its origin in the momentum dependence of the interparticle interaction. In this sense, it is a general physical phenomenon that should be present in any weakly interacting gas with a similar momentum dependence of the interparticle interaction (scattering amplitude).

Cavity-mediated atom-atom interaction



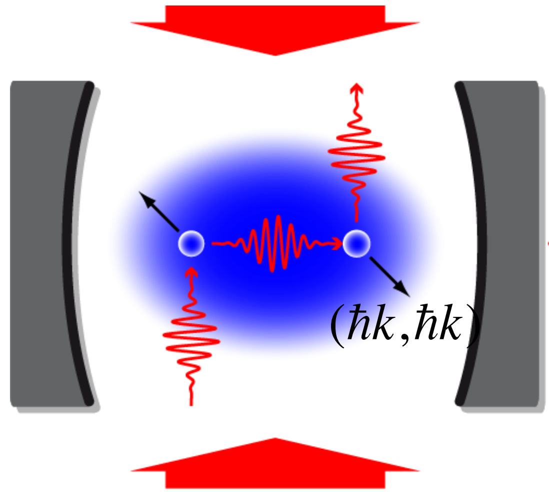
$$H_{\mathrm{aa}}^{\mathrm{eff}} = \int \Psi^\dagger(\mathbf{r})\Psi^\dagger(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\Psi(\mathbf{r}')\Psi(\mathbf{r})\mathrm{d}^3(r,r')$$

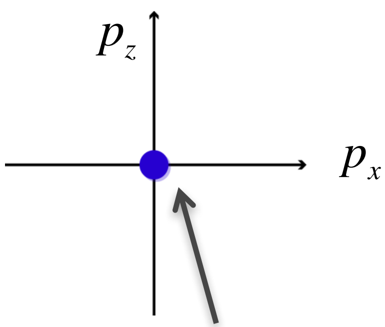
$$V(\mathbf{r},\mathbf{r}') = V\cos(kx)\cos(kz)\cos(kx')\cos(kz')$$

$$V = -\frac{\hbar\eta^2}{\omega} \propto P$$

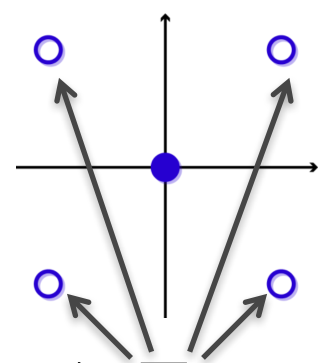
related: Münstermann et al. PRL 84, 4068 (2000), J. Asboth at al. PRA 70, 013414 (2004)

Two-Mode Picture



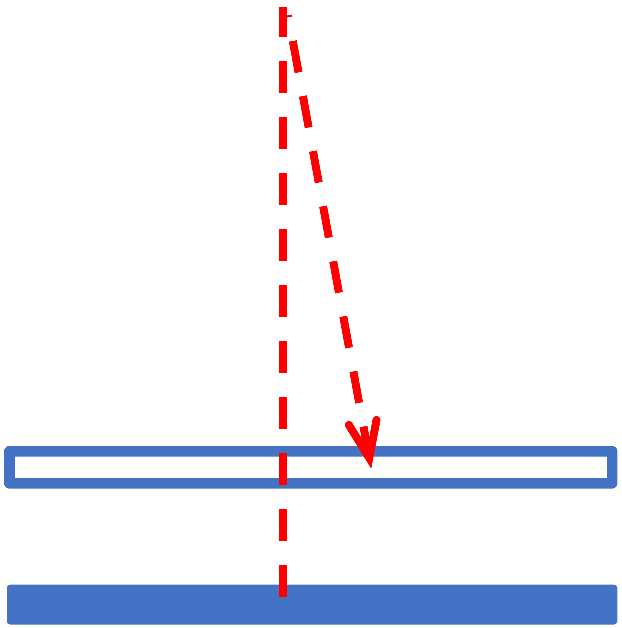


$$|p_x, p_z\rangle = |0,0\rangle$$

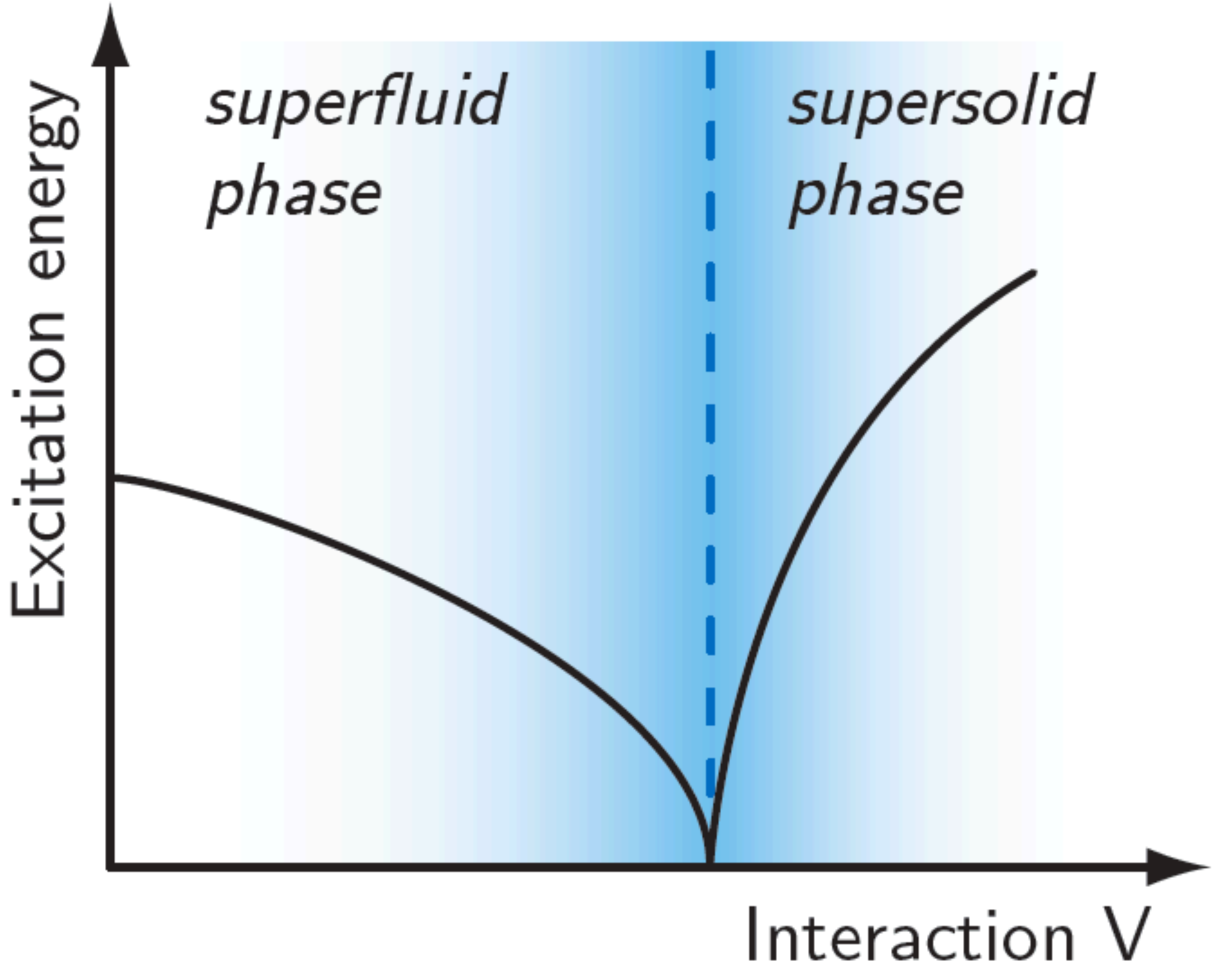


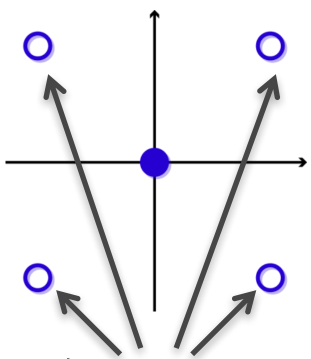
$$|p_x, p_z\rangle = \sum |\pm\hbar k, \pm\hbar k\rangle$$

Two-Mode Description



Excitation spectrum – mode softening

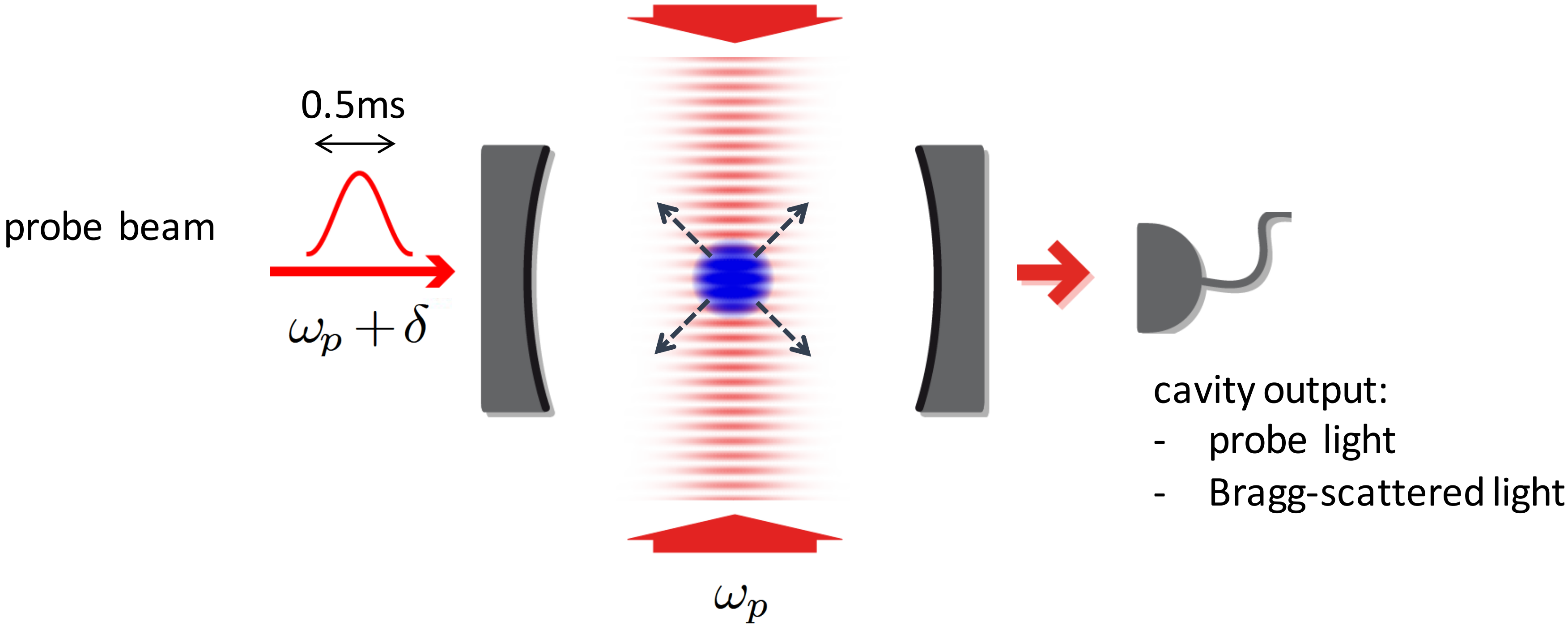




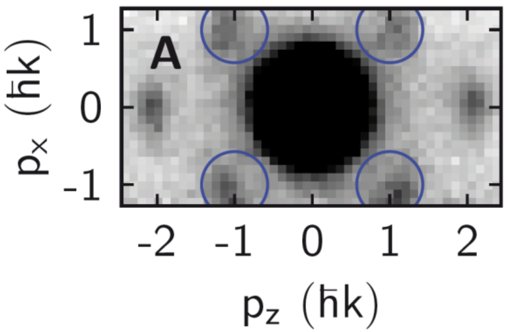
$$|p_x, p_z\rangle = \sum |\pm\hbar k, \pm\hbar k\rangle$$

see also: C. Emary et al. 90, 044101 (2003), D. Nagy et al. EPJD 48,127 (2008)…

Probing the excitation spectrum

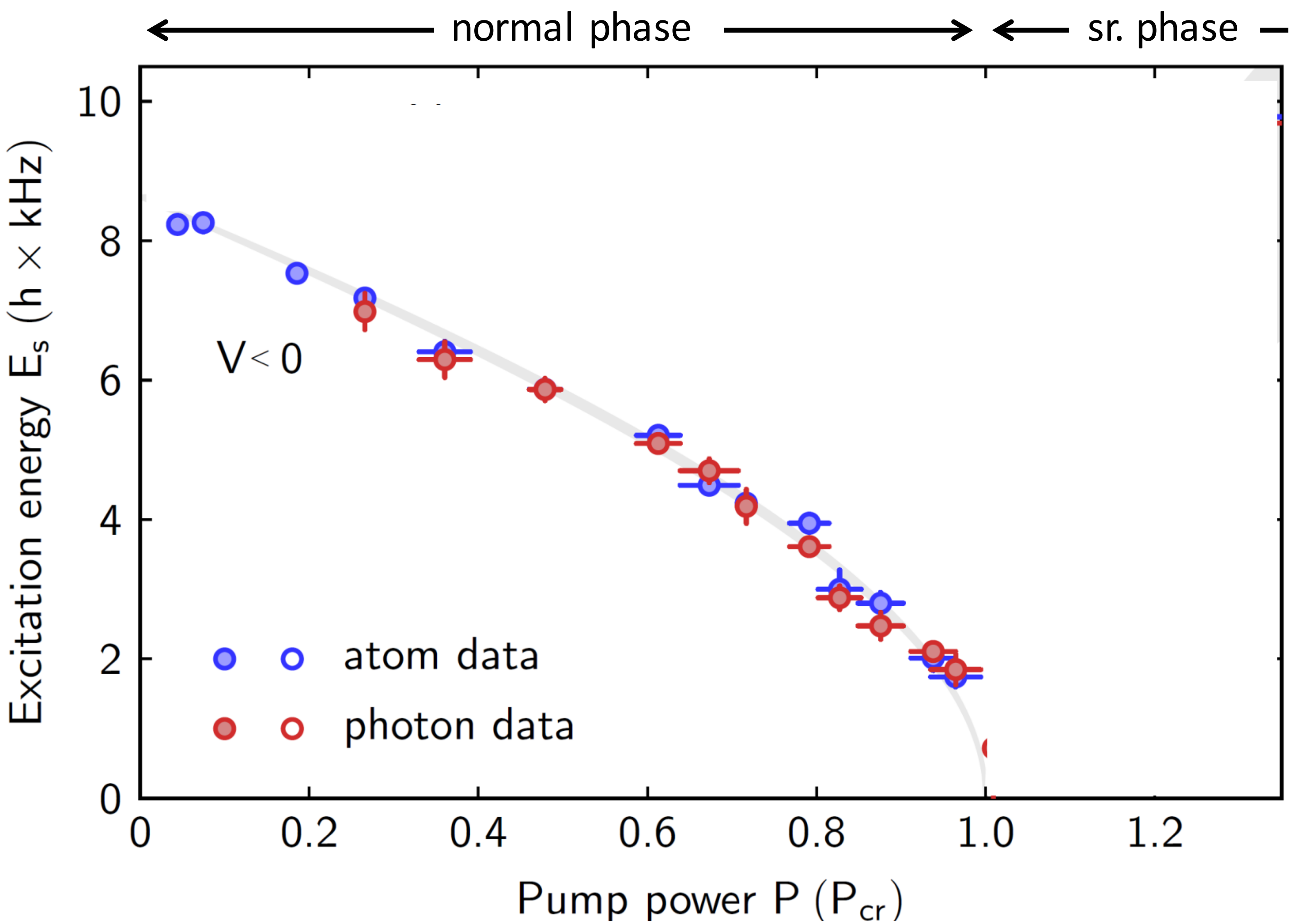


Probing the excitation spectrum



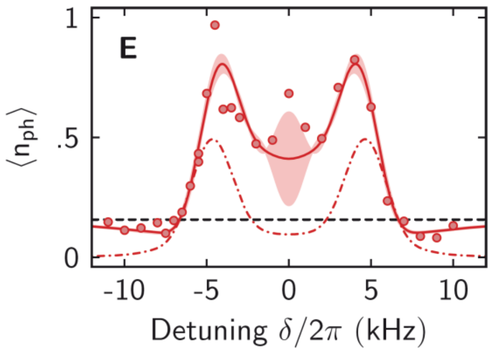
R. Mottl, F. Brennecke, K. Baumann, R. Landig, T. Donner, T. Esslinger, arXiv:1203.1322, Science Express, 17 May 2012

Excitation spectrum



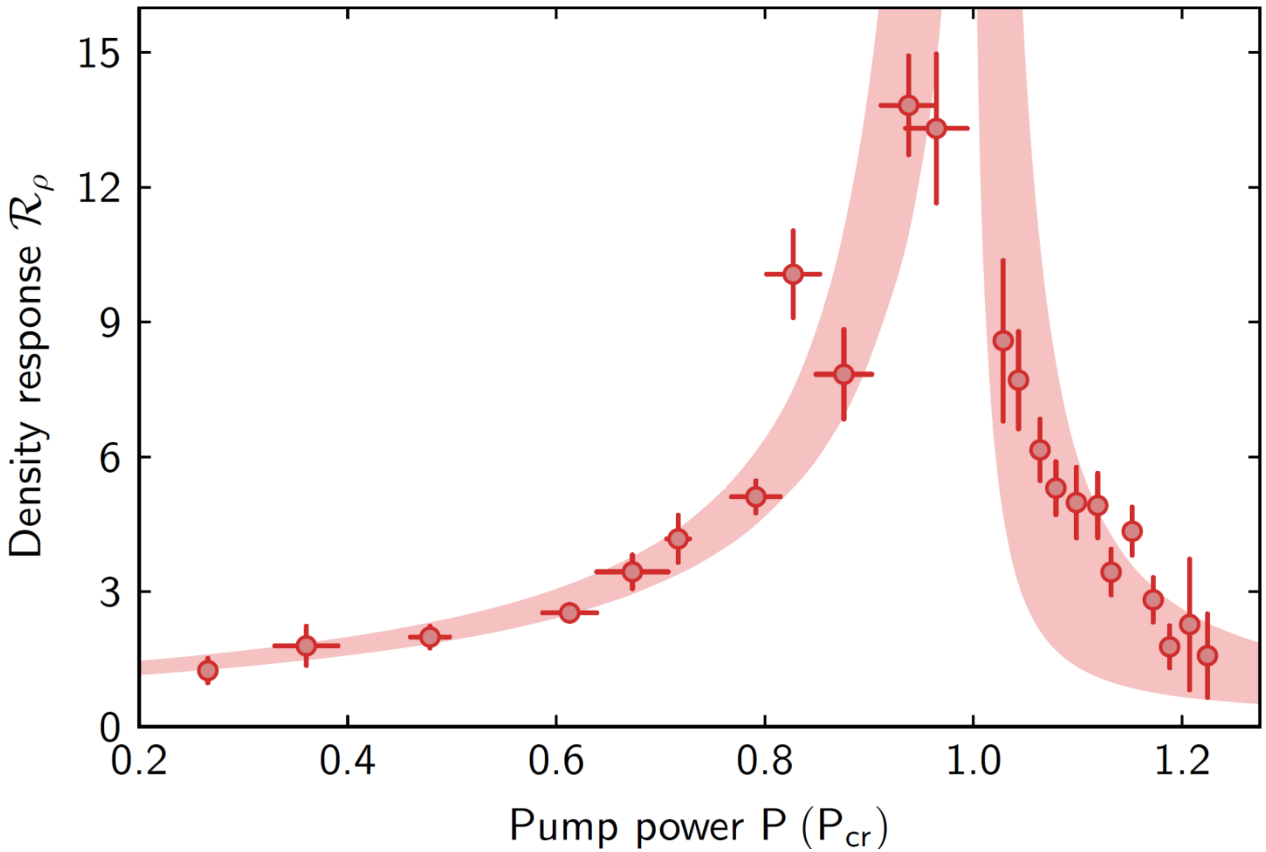
shading: ab-initio calculation including collisions and trapping

Susceptibility



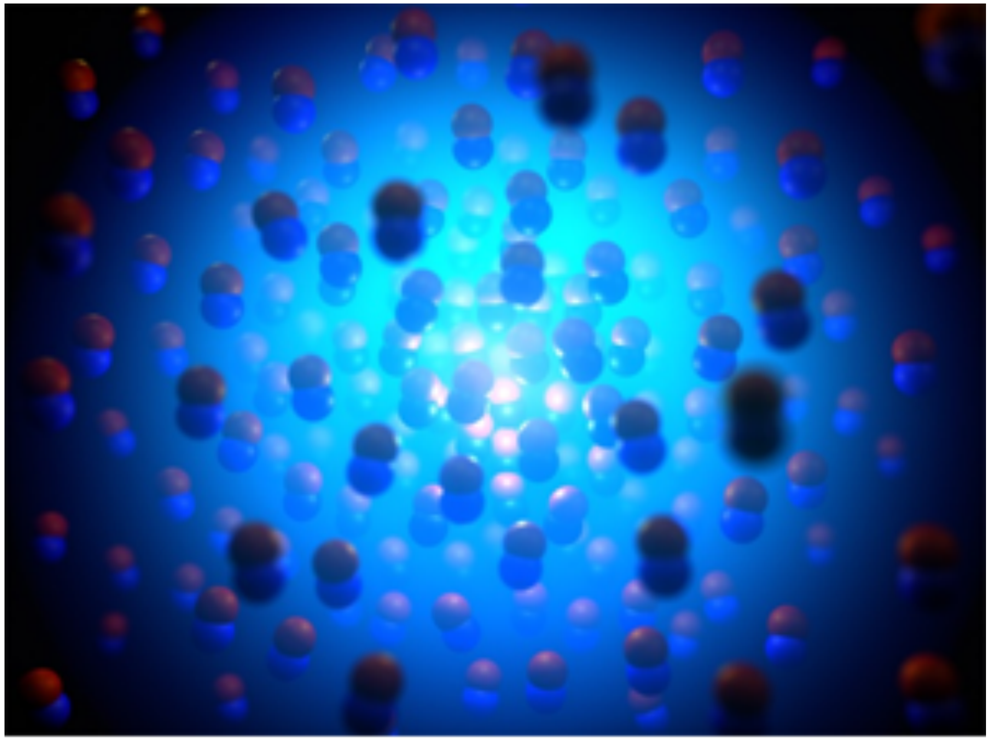
$$\mathcal{R}_\rho \propto \frac{\int d\delta \langle n_{\rm Bragg} \rangle}{P\langle n_{\rm pr}\rangle}$$

$$\mathcal{R}_{N_e} = \frac{\int d\delta N_e(\tau)}{P\langle n_{\rm pr}\rangle}$$



Critical properties of this phase transition

Characterization of many-body systems



study of the elementary excitations of a Bose condensate. The ultimate goal is a complete survey of the spectrum of collective excitations, including the lifetimes of the quasiparticles and the behavior at different temperatures and higher excitation energies. For excitation frequencies larger than the mean interaction energy

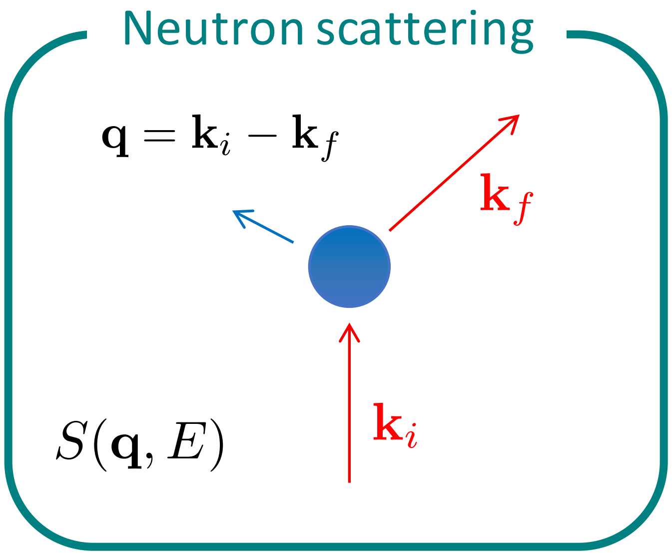
W. Ketterle et al., PRL 77, 988 (1996)

Dynamic structure factor

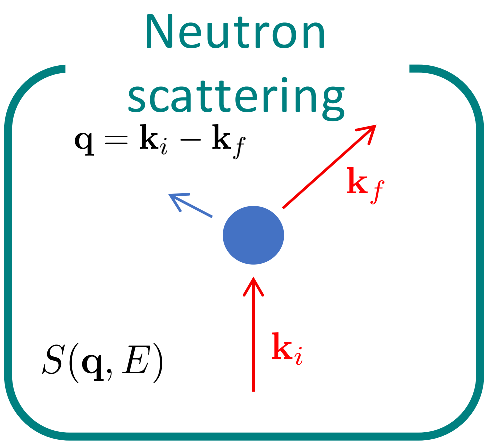
$$S(\mathbf{q},\omega) = \frac{V}{2\pi N}\int \mathrm{d}\mathbf{r}\mathrm{d}t e^{-i(\mathbf{q}\mathbf{r}-\omega t)}\langle\rho(\mathbf{r},t)\rho(0,0)\rangle$$

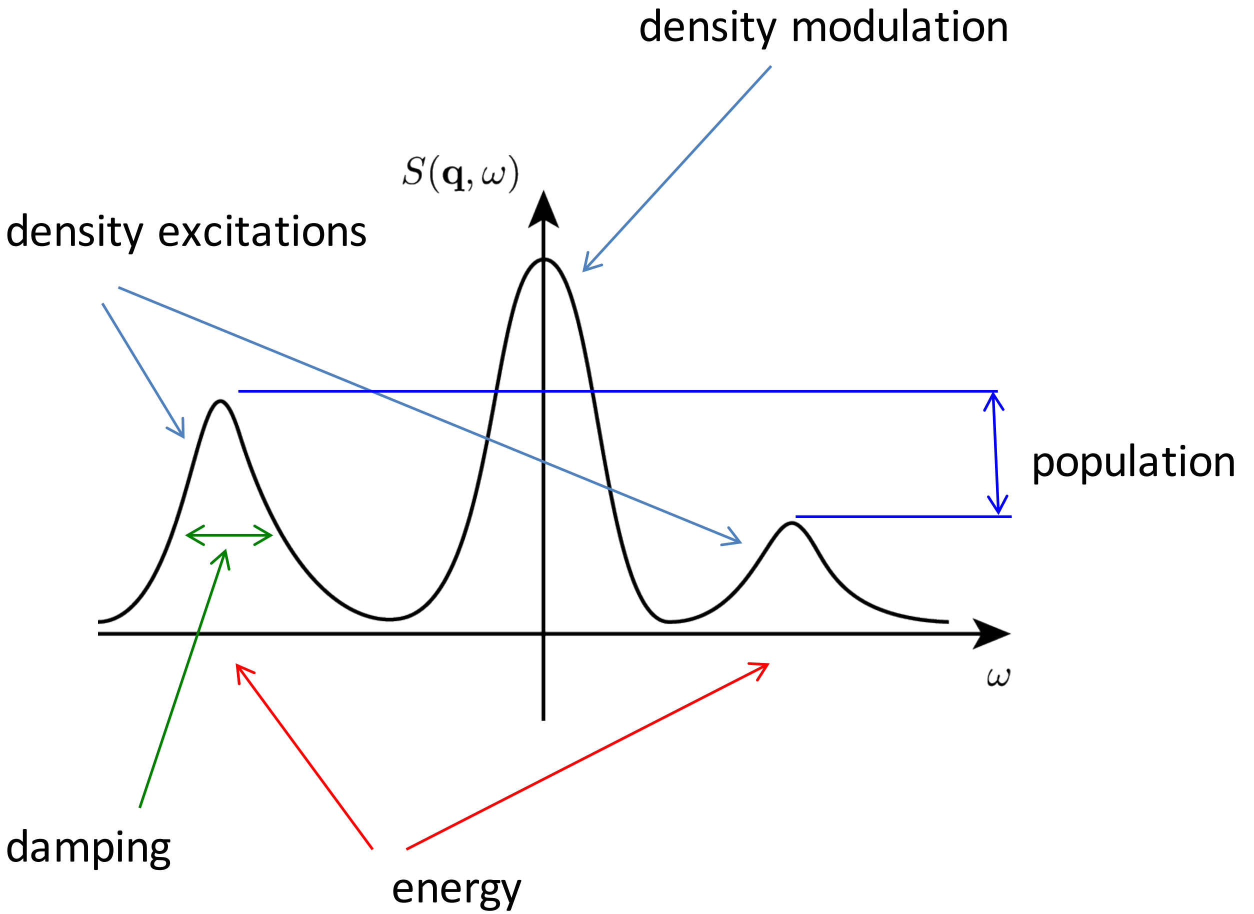
Used to extract:

- Structure of matter
- Quasiparticle modes in interacting systems
- Excitation spectrum
- Fluctuations and their correlations
- Quasiparticle mode occupation / temperature

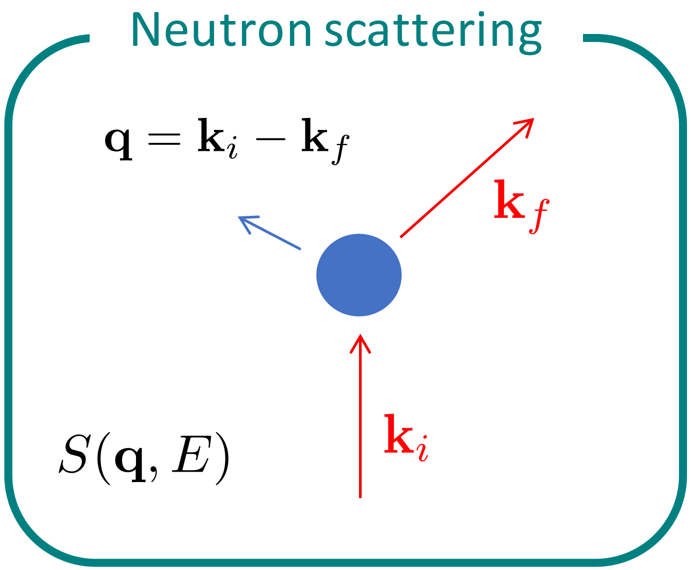


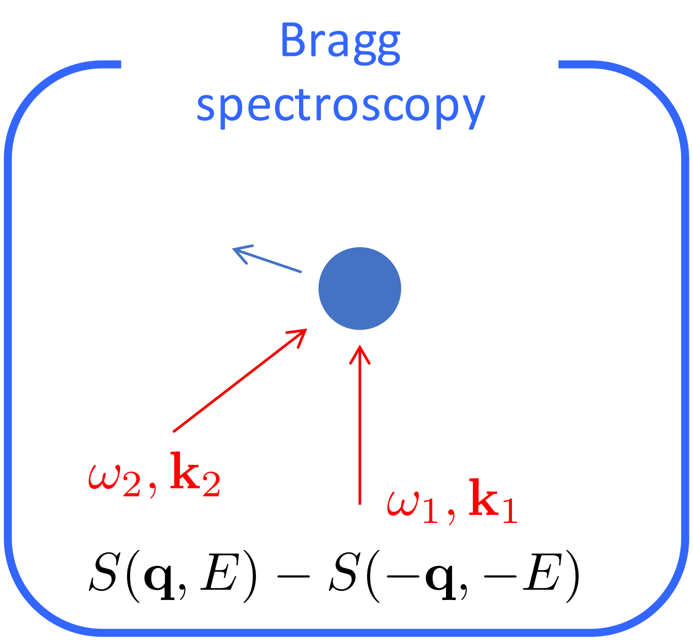
Dynamic structure factor

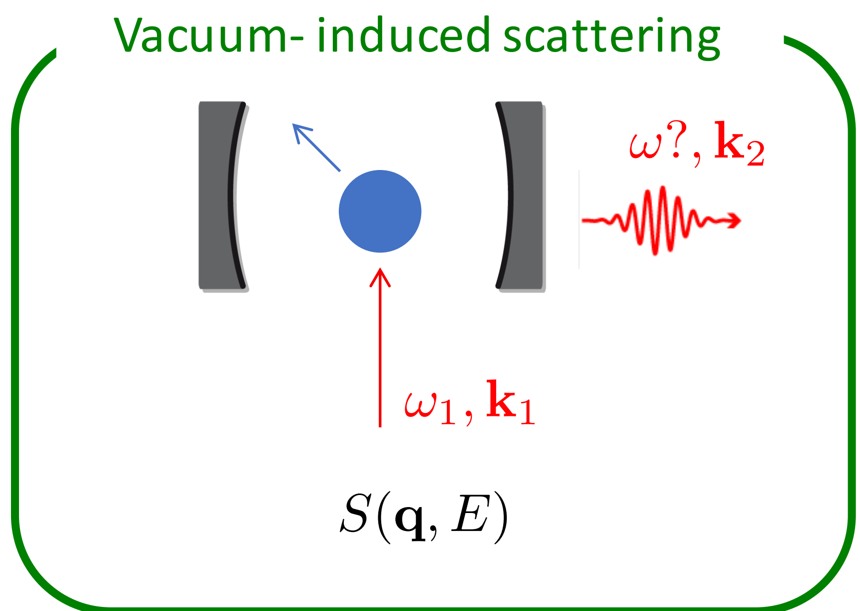




Vacuum-induced scattering



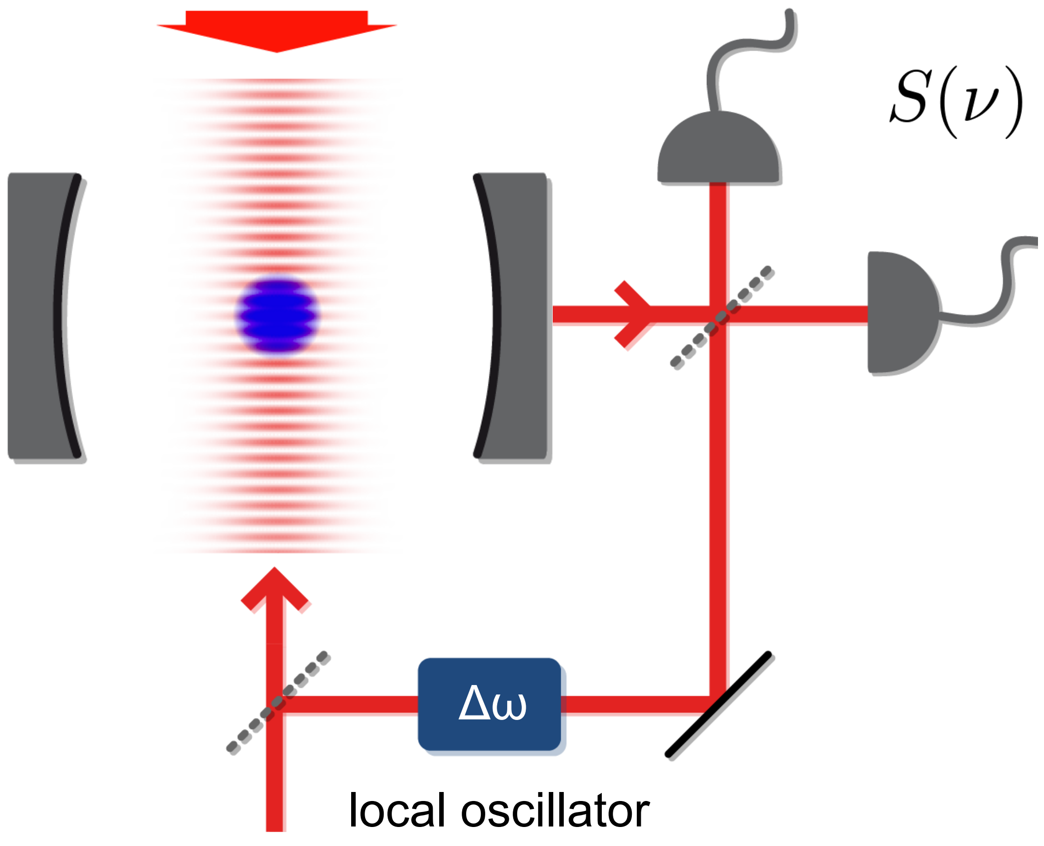




Measures dynamic structure factor at difference wave vector of pump mode and cavity mode.

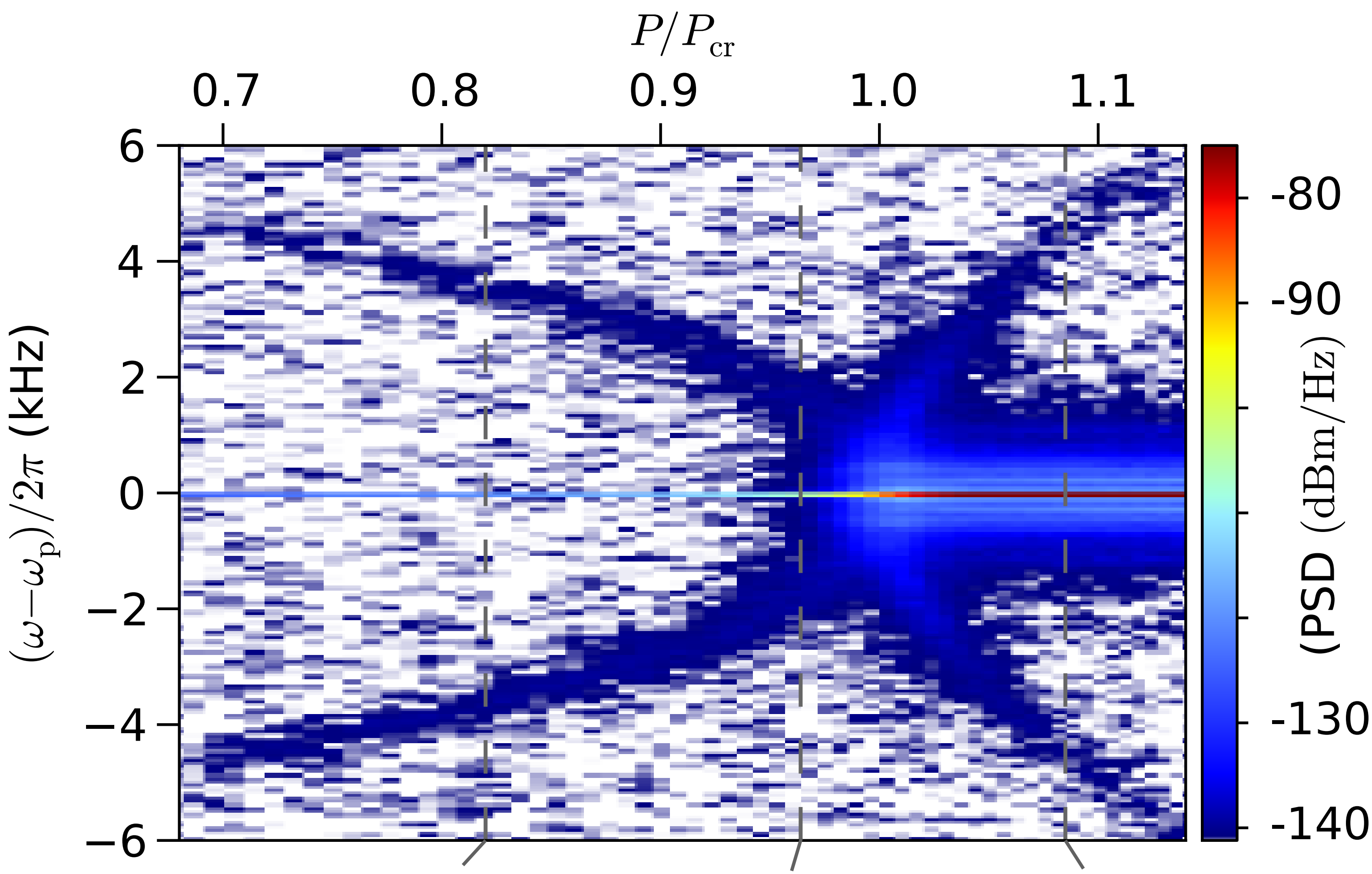
Real-time observation of critical fluctuations

Spectrum of cavity output field: extract coherent and incoherent cavity field component independently

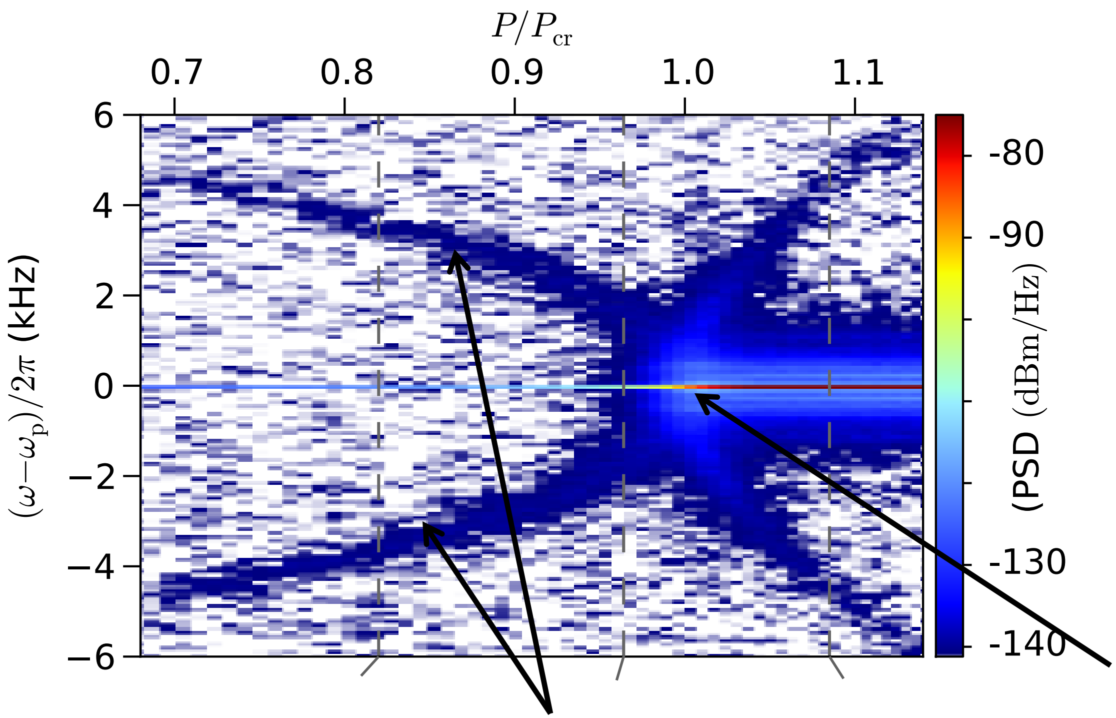


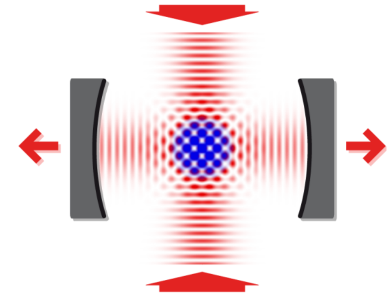
$$S(\nu) = \mathcal{F}\langle \hat{a}^\dagger(t)\hat{a}(0)\rangle$$

Power spectral density
(... or dynamic structure factor...)



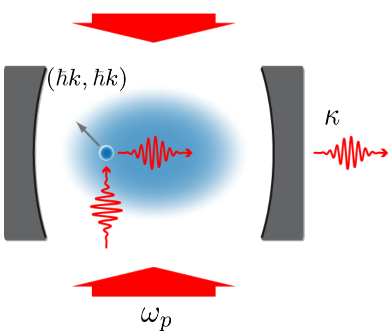
Dynamic structure factor – interpretation





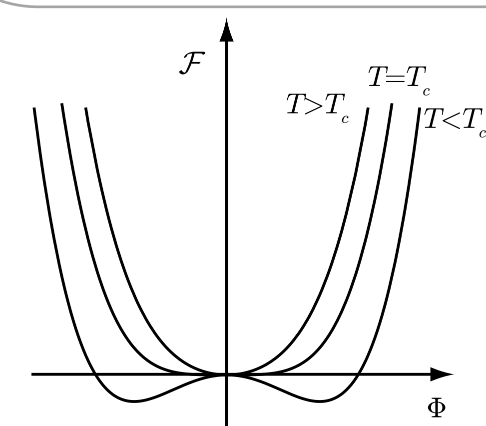
Carrier:

Elastic scattering at ***density modulation***

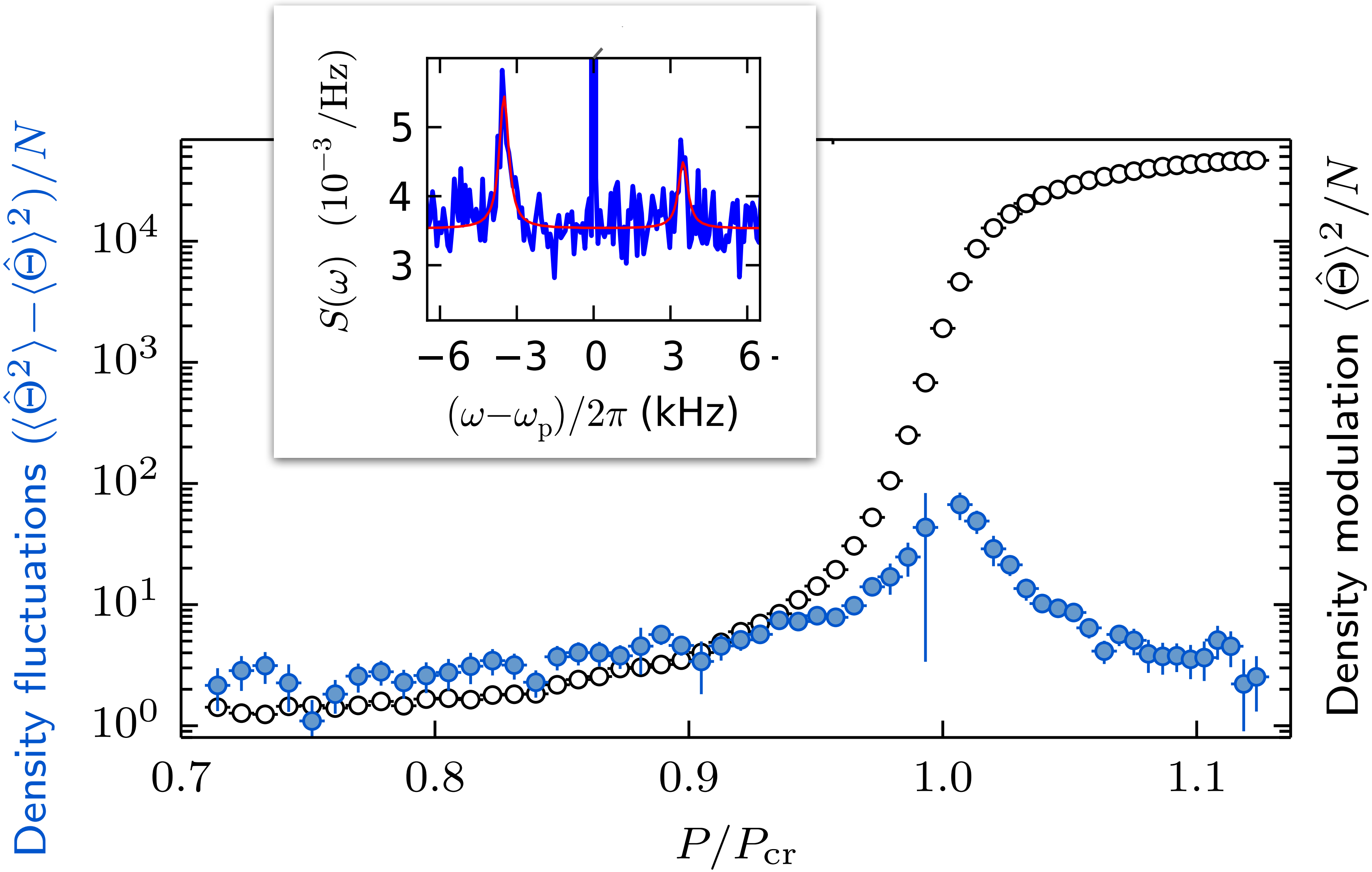


Sidebands:

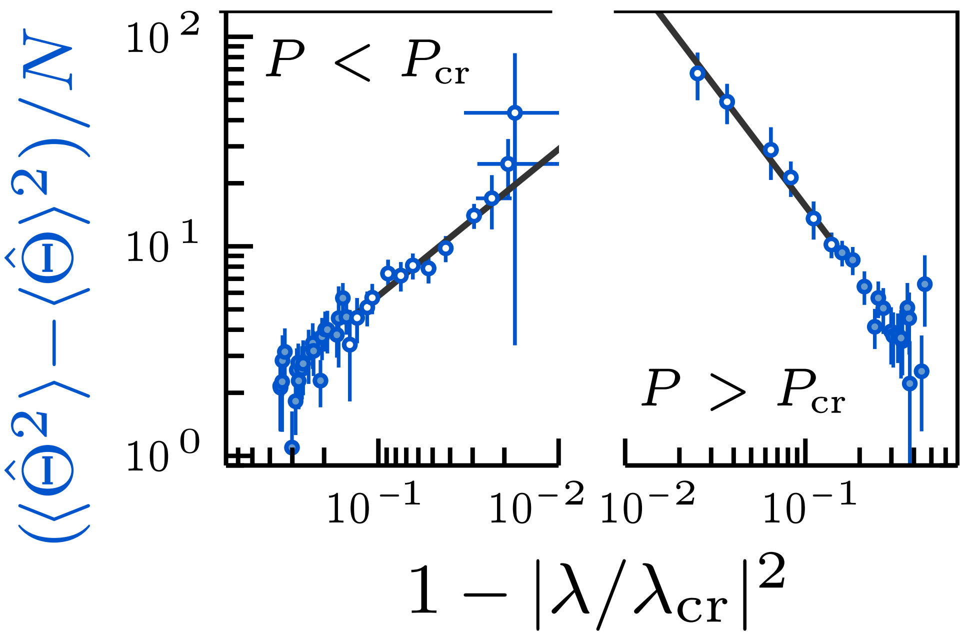
creation and annihilation of a ***density fluctuation, i.e. quasiparticle***



Static structure factor



Scaling of the fluctuations of the order parameter

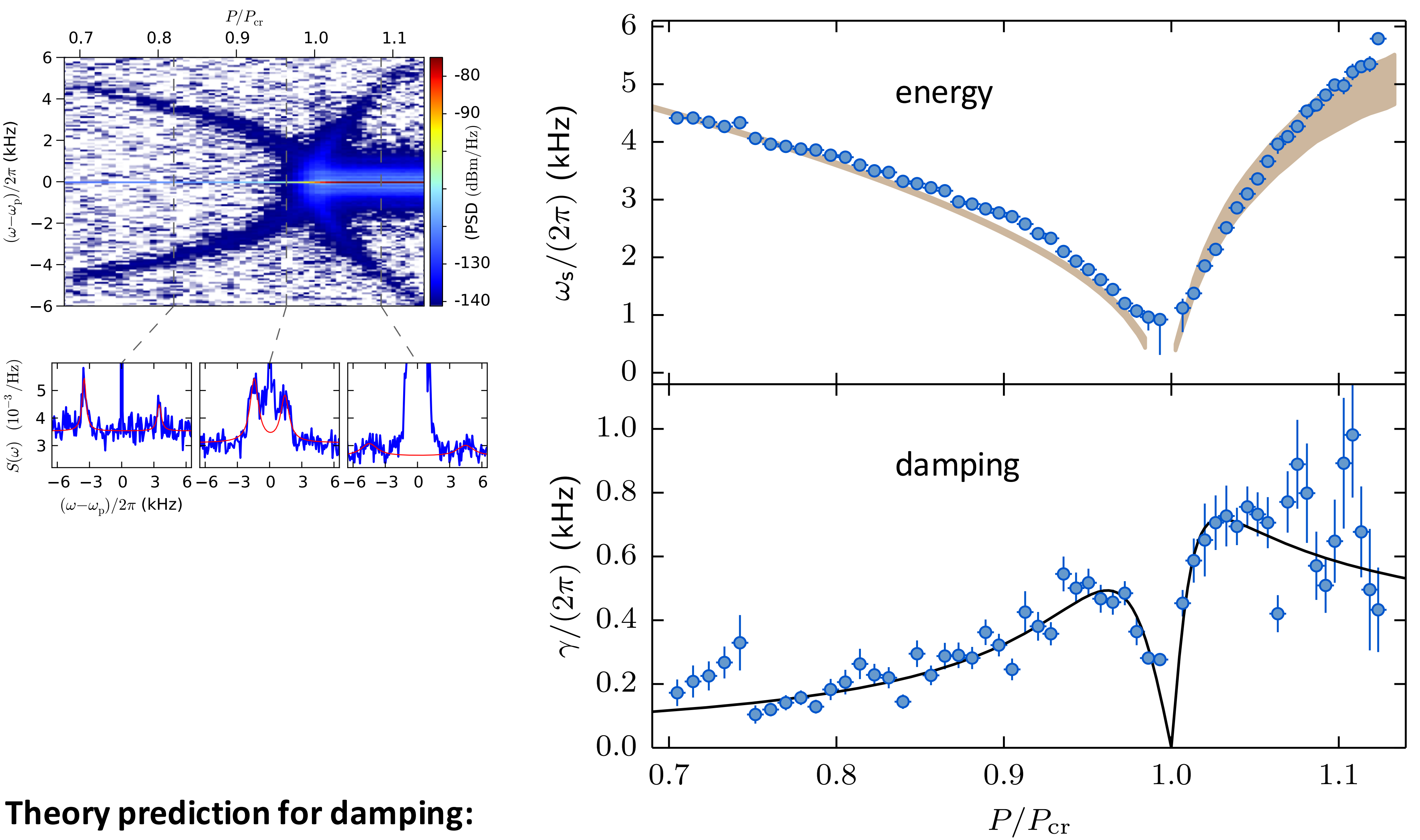


Exponents of density fluctuations:
0.7(0.1) in normal phase
1.1(0.1) in organized phase

- **Theory prediction for closed systems: 0.5**
- **Theory prediction for open systems: 1.0**

PRA 84, 043637 (2011) (P. Domokos)
New J Phys 14:085011 (2012) (H. Türeci)
PRL 111, 220408 (2013) (H. Türeci)

Quasi-particles: excitation energy and lifetime

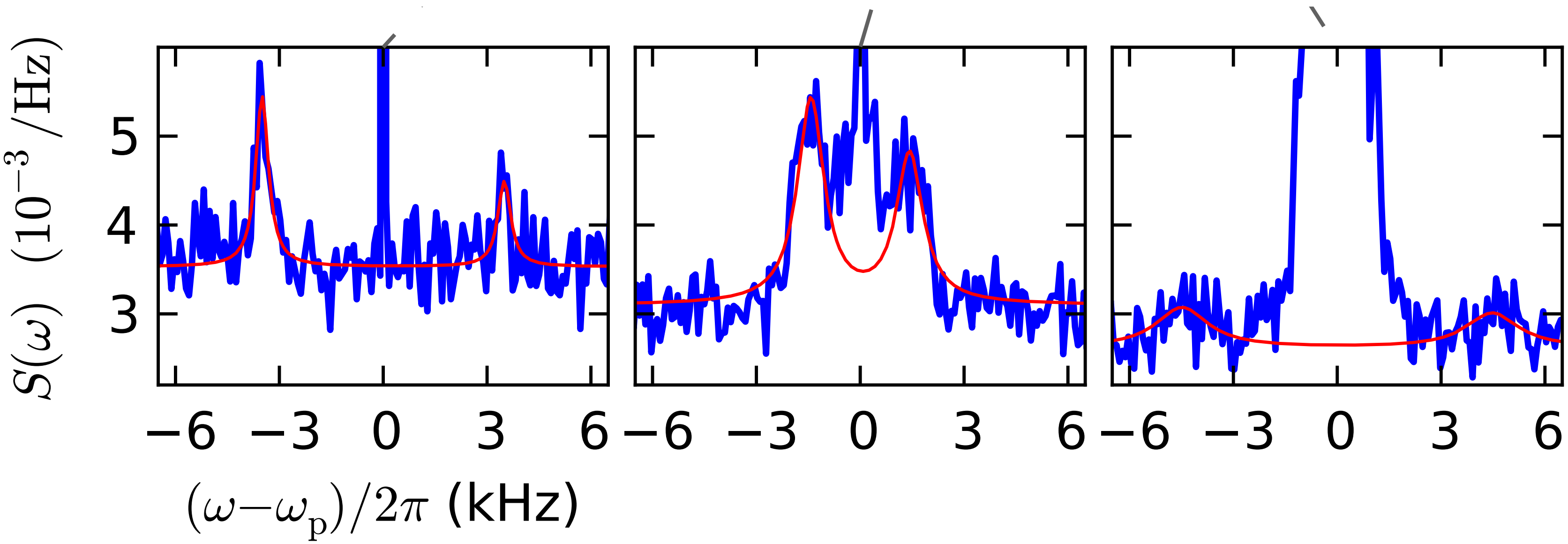


Theory prediction for damping:

Kulkarni, et al. PRL 111, 220408 (2013)

Konya et al. PRA 90, 013623 (2014)

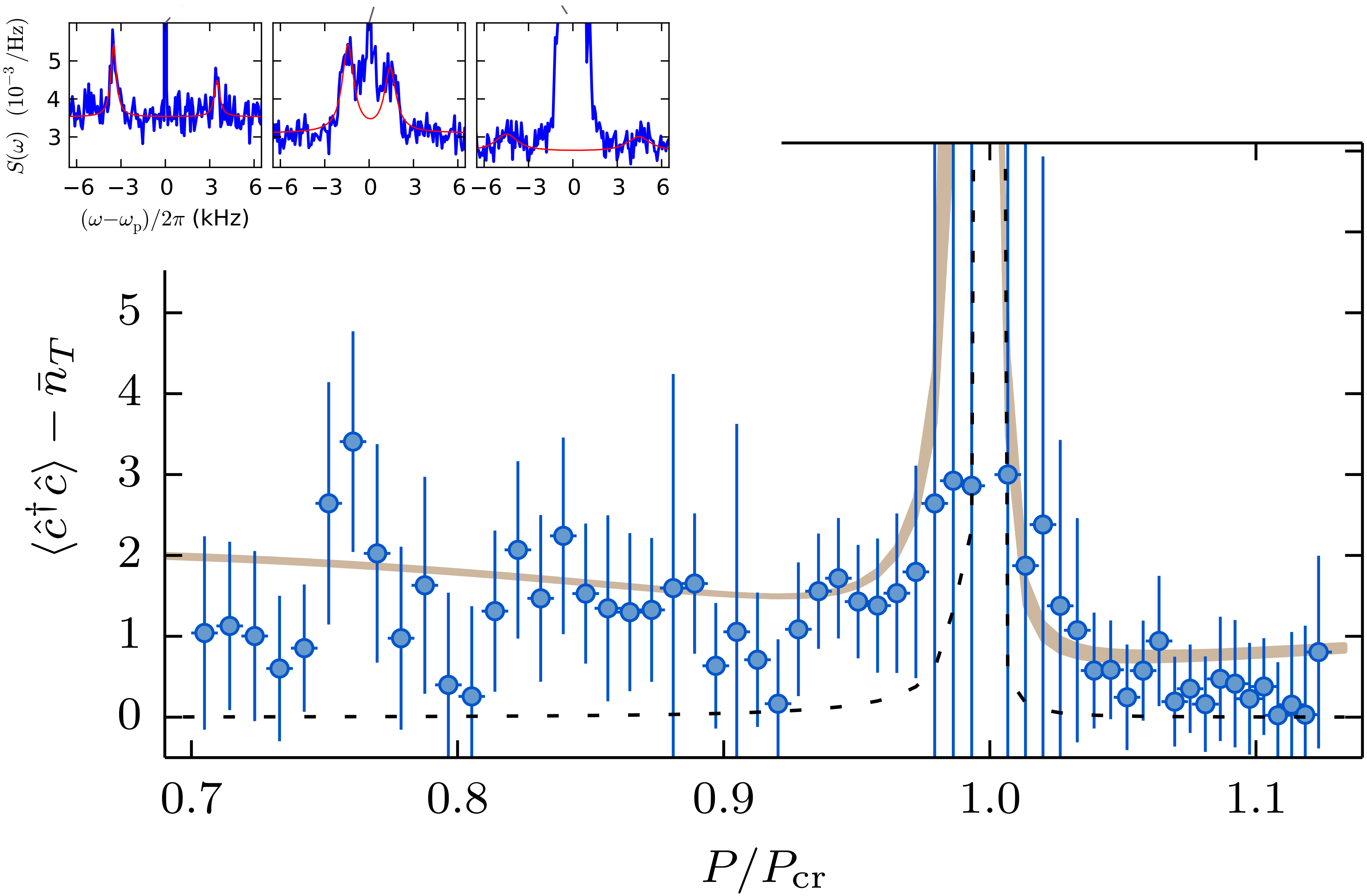
Sideband asymmetry



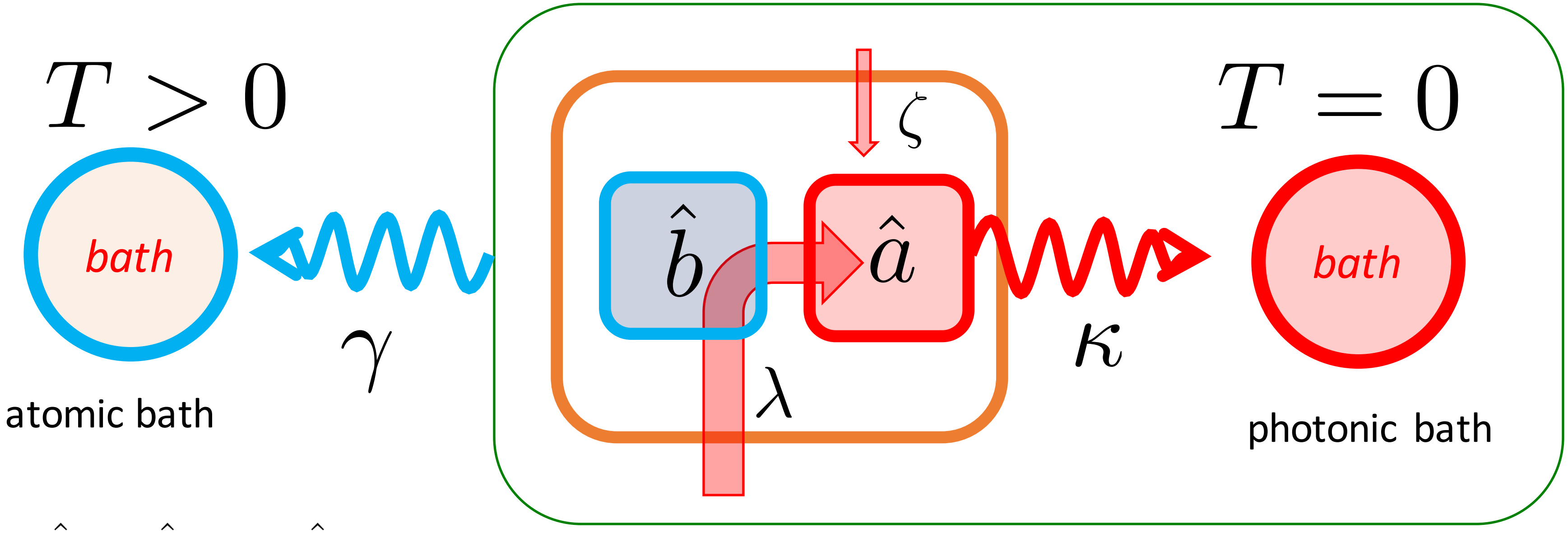
$$2\kappa(\langle\delta\hat{a}^\dagger\delta\hat{a}\rangle_- - \langle\delta\hat{a}^\dagger\delta\hat{a}\rangle_+) = 2\gamma(\langle\hat{c}^\dagger\hat{c}\rangle - n_T)$$

number of quasi-particles

Quasi-particles: mode occupation



Theoretical description – Langevin equations



$$\hat{H} = \hat{H}_{\rm TL} + \hat{H}_{\rm SB}$$

$$\hat{H}_{\rm TL} = \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\omega_0\hat{b}^\dagger\hat{b} + \hbar\lambda(\hat{a}^\dagger + \hat{a})(\hat{b}^\dagger + \hat{b})$$ (thermodynamic limit)

$$\hat{H}_{\rm SB} = \hbar\lambda\zeta(\hat{a}^\dagger + \hat{a})$$ (symmetry-breaking term)

Quantum Langevin equations:

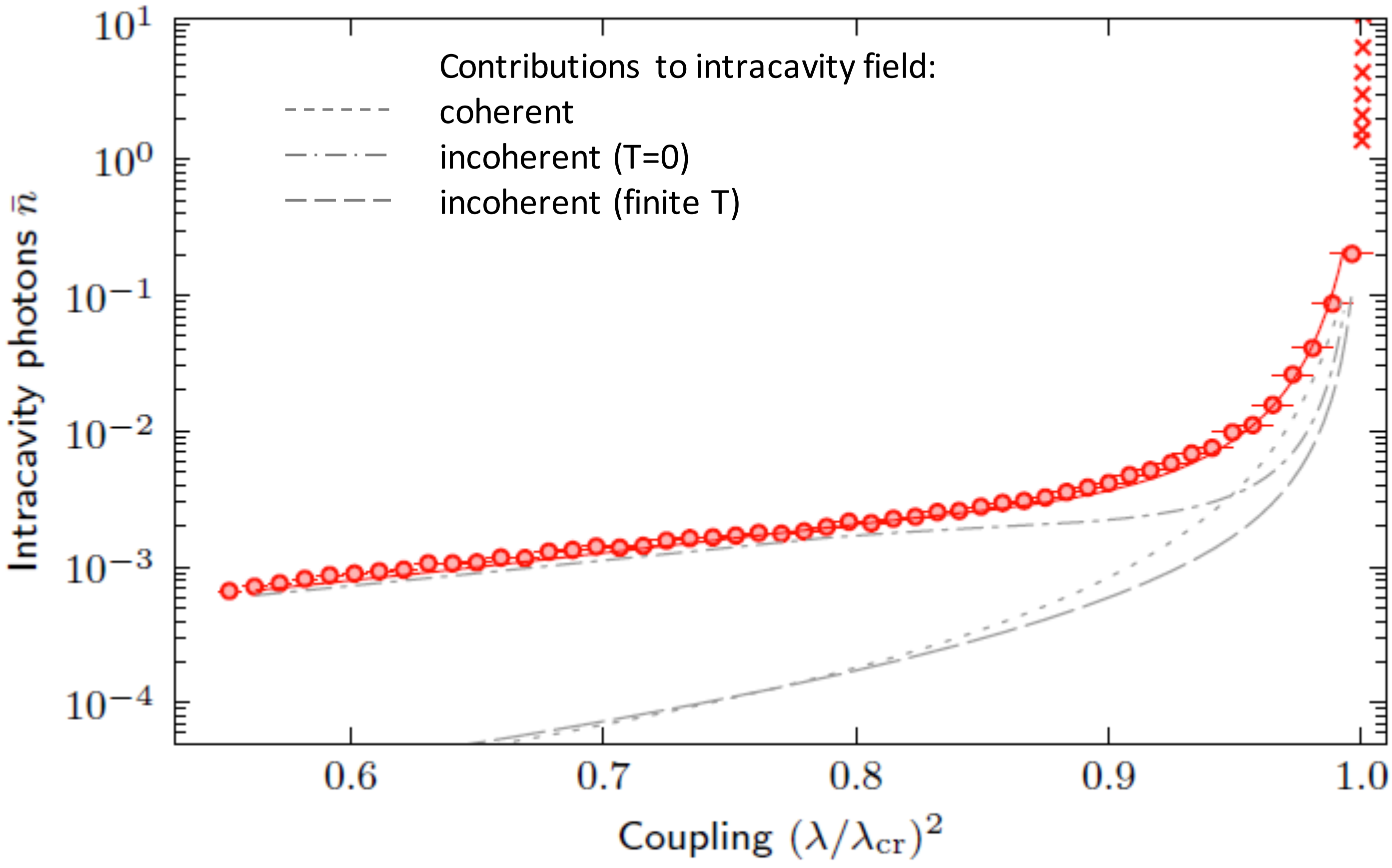
$$\dot{\hat{a}} = -i[\hat{a}, \hat{H}] - \kappa\hat{a} + \sqrt{2\kappa}\hat{a}_{\rm in}$$

$$\dot{\hat{b}} = -i[\hat{b}, \hat{H}] - \gamma\hat{b} + \sqrt{2\gamma}\hat{b}_{\rm in}$$

PNAS **110**, 11763 (2013)

see also Dimer et al. PRA **75**, 013804 (2007)

Theoretical description – normal phase

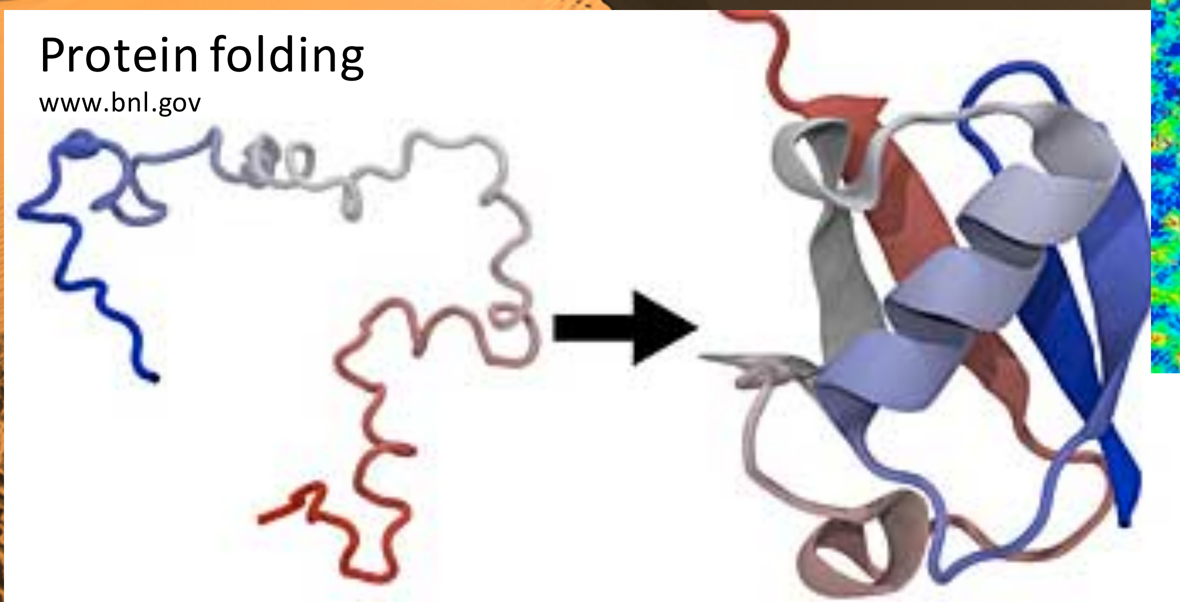


no additional free fit parameter!

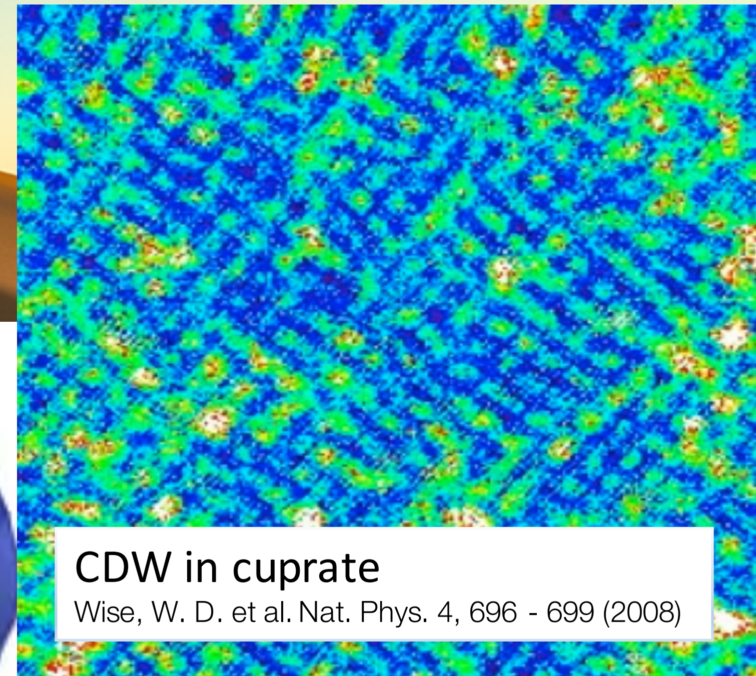
Competing long- and short range interactions

Structure formation

Competition between short- and long-range interactions

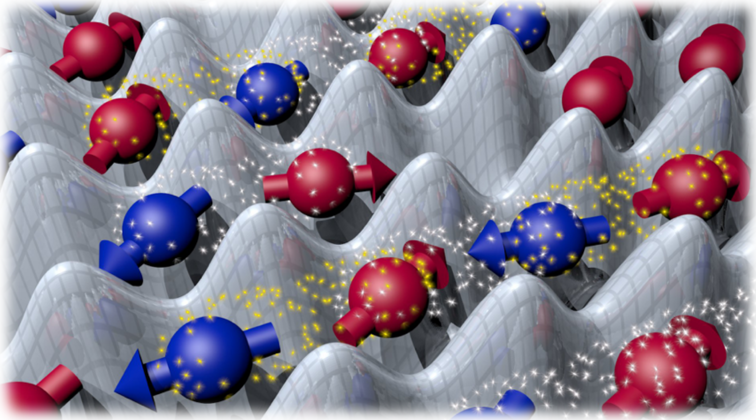


Protein folding
www.bnl.gov



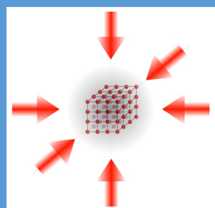
CDW in cuprate
Wise, W. D. et al. Nat. Phys. 4, 696 - 699 (2008)

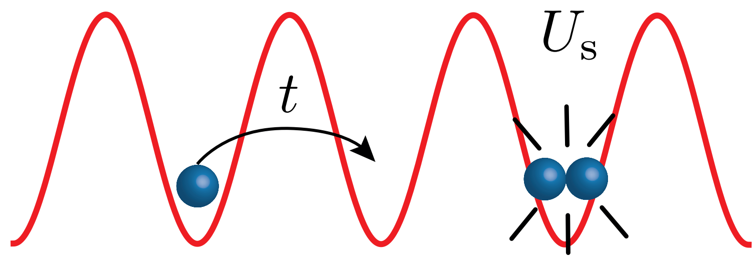
Quantum simulation with ultracold atoms



Bose-Hubbard model: onsite interactions

Optical lattice



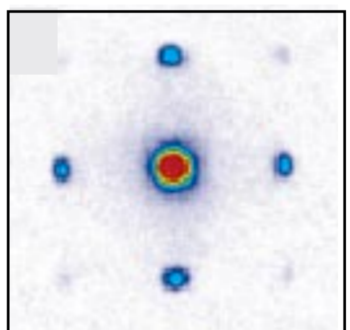


$$H = -t \sum_{<i,j>} (a_i^\dagger a_j + a_j^\dagger a_i) + \frac{U_s}{2} \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$

2 competing energy scales: t, U_s

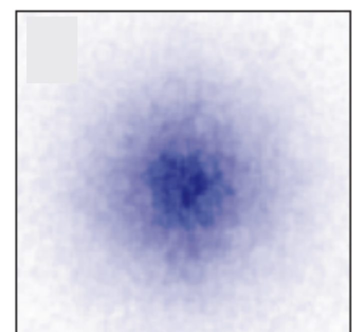
Momentum distribution of the atomic cloud

Shallow lattice: $t \gg U_s$



Superfluid

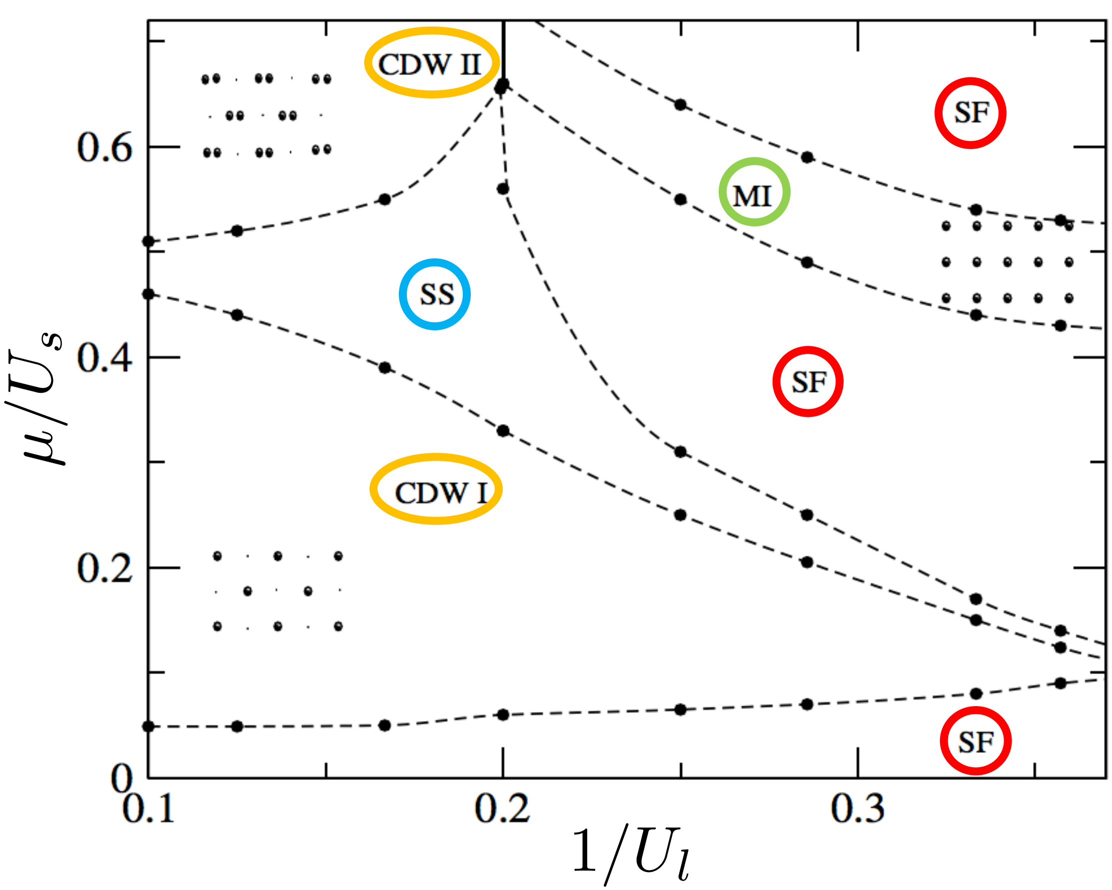
Deep lattice: $t \ll U_s$



Mott insulator

Extended Bose-Hubbard model

$$H = -t \sum_{<i,j>} (a_i^\dagger a_j + a_j^\dagger a_i) + \frac{U_s}{2} \sum_i n_i(n_i - 1) - \mu \sum_i n_i + U_l \sum_{<i,j>} n_i n_j$$



3 competing energy scales, 2 length scales:

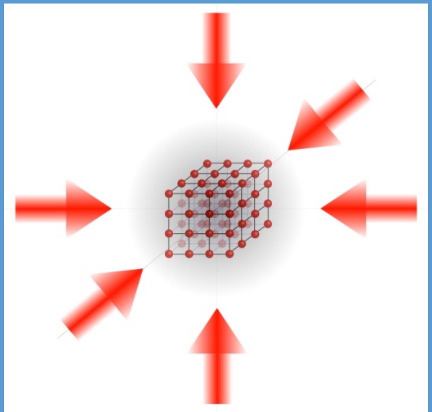
$$t, U_s, U_l$$

- Superfluid SF
- Mott insulator MI
- Supersolid SS
- Charge density wave CDW

Sengupta et al., PRL 94, 207202 (2005)

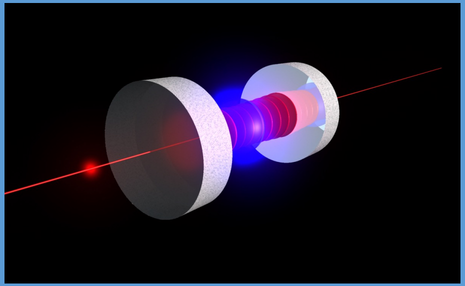
Lattice model with competing long- and short-range interactions: realization

Optical lattice



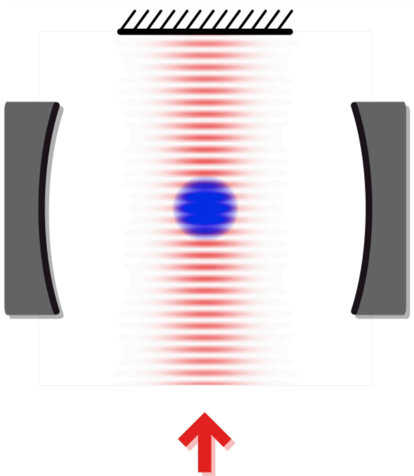
➕

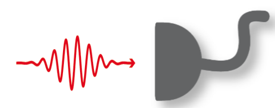
Optical cavity

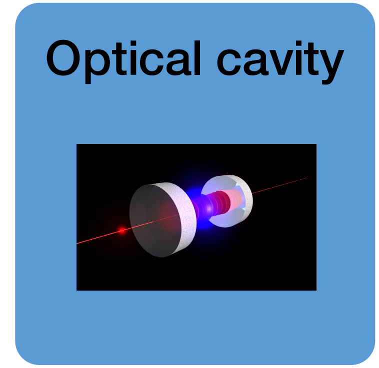


Infinite-range interactions

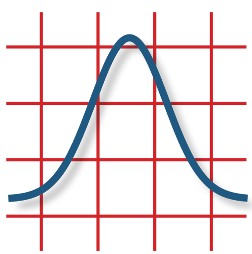
U_1/t



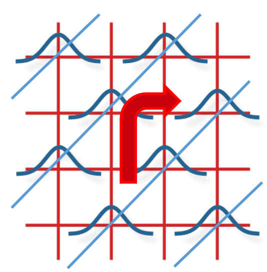




$$-t\sum_{\langle i,j\rangle}\left(\hat{b}_i^\dagger\hat{b}_j + \text{h.c.}\right)$$



Superfluid



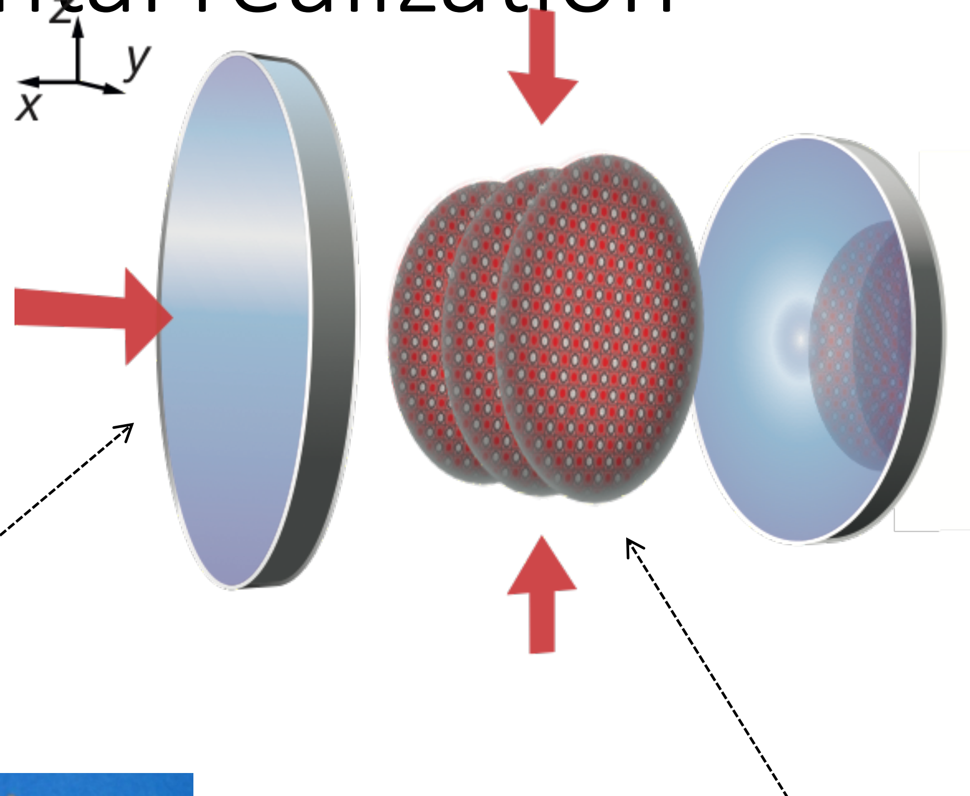
Bragg-scattered cavity light field

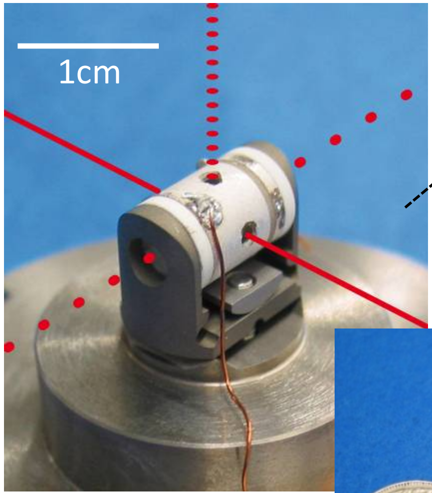
→ signaling even-odd imbalance

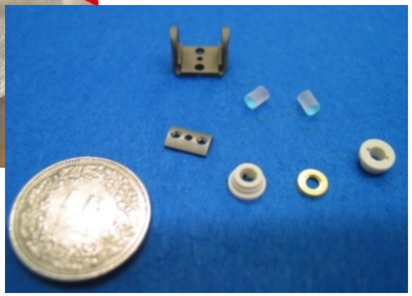
roken Z_2 symmetry

→ even-odd imbalance

Experimental realization





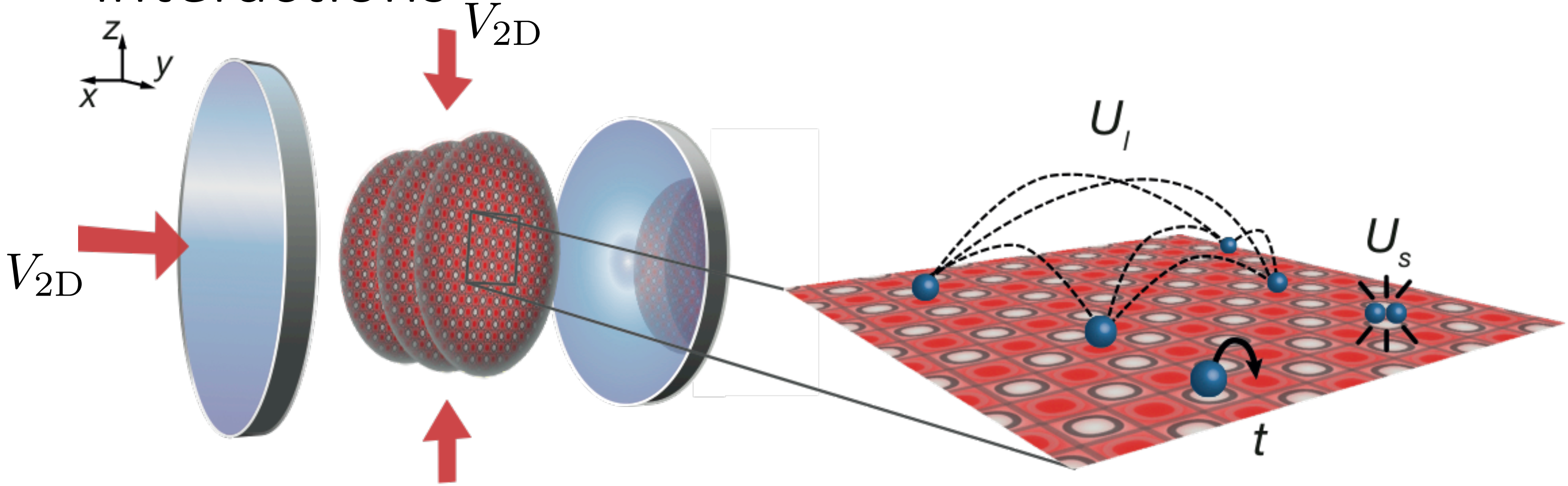


42’000 ^{87}Rb atoms

Stack of **60** 2D Layers

Max. **2.8** atoms / lattice site

Lattice model with on-site and infinite-range interactions



$$\hat{H} = -t \sum_{\langle e,o \rangle} \left(\hat{b}_e^\dagger \hat{b}_o + \text{h.c.} \right) + \frac{U_\text{s}}{2} \sum_{i \in e,o} \hat{n}_i (\hat{n}_i - 1) - U_\text{l} \left(\sum_e \hat{n}_e - \sum_o \hat{n}_\text{o} \right)^2 - \sum_{i \in e,o} \mu_i \hat{n}_i$$

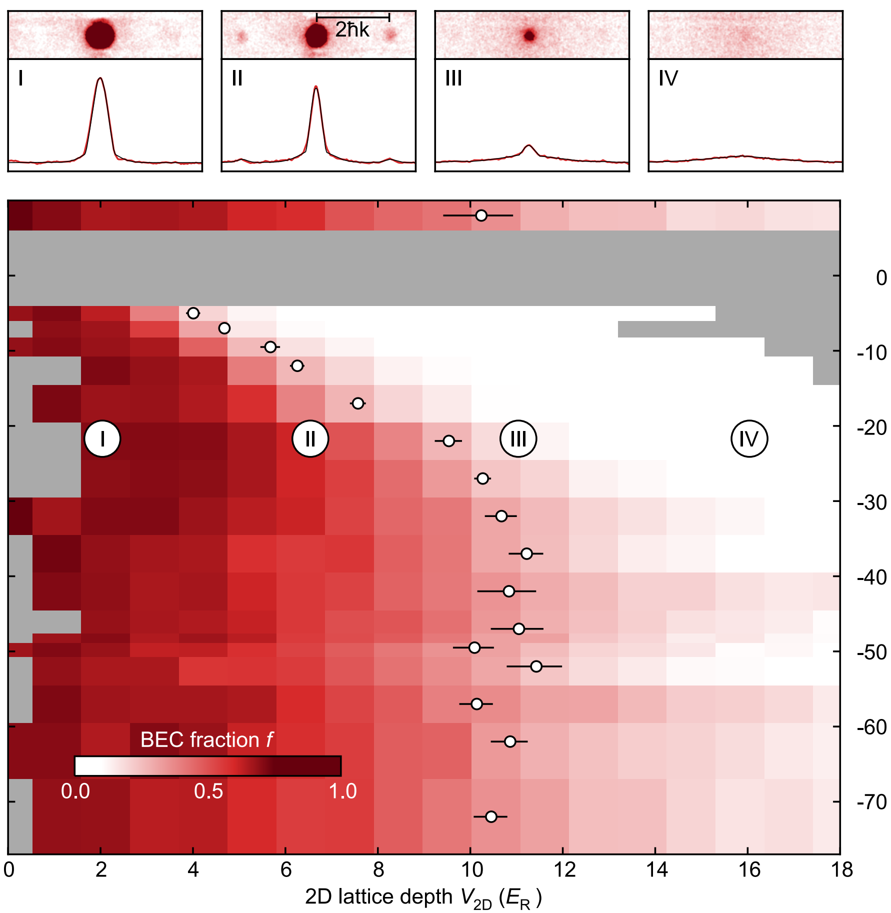
Tuning interactions:

$$U_\text{s} \propto V_\text{2D}$$

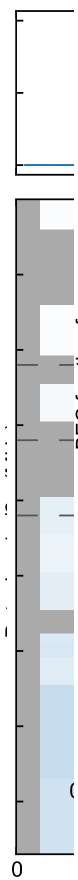
$$U_\text{l} \propto \frac{V_\text{2D}}{\Delta_\text{c}}$$

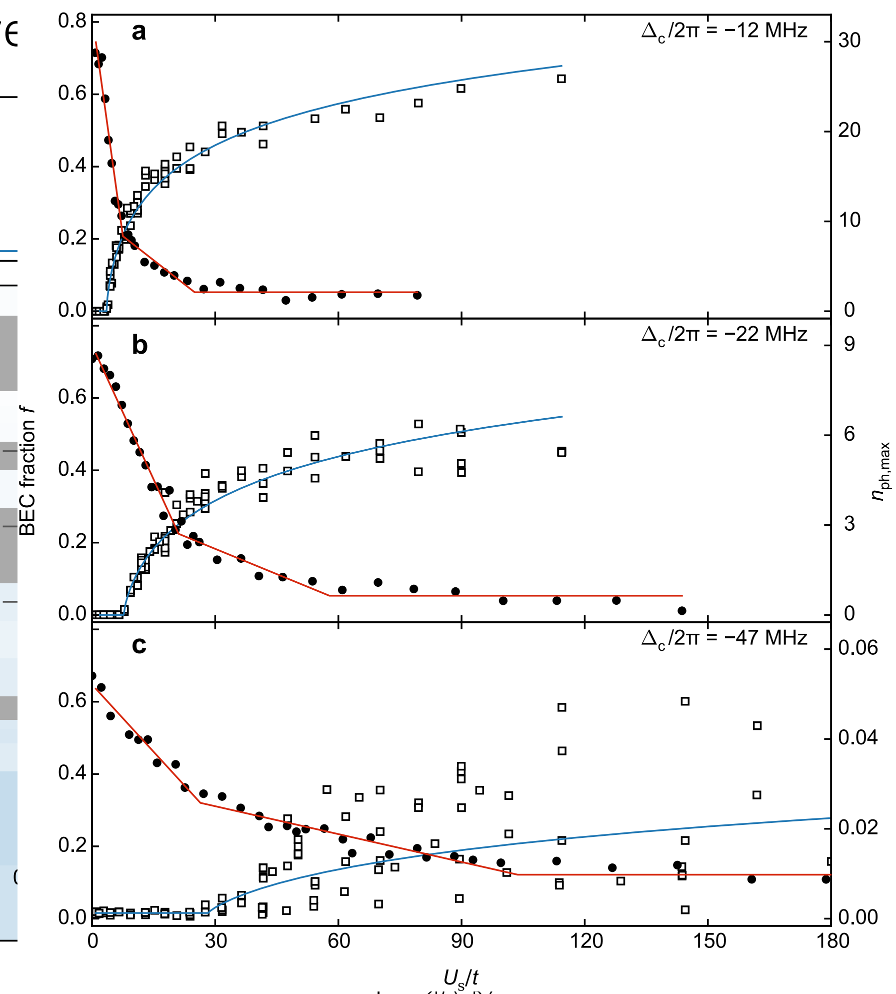
Exploring the phase diagram…

Spatial coherence: BEC fraction



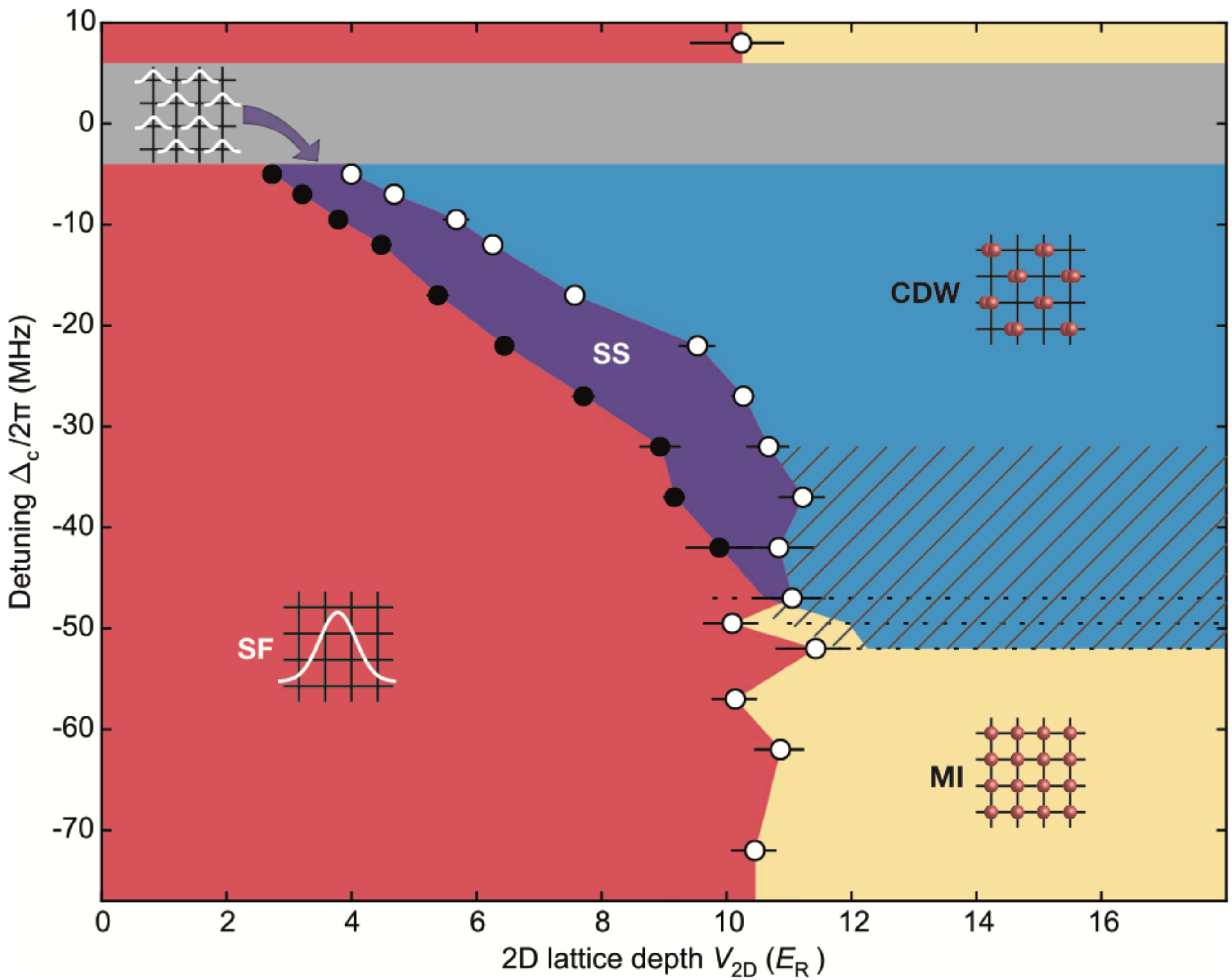
Eve





$$\Theta = \left| \frac{n_e - n_o}{n_e + n_o} \right| \propto \sqrt{n_{ph}}$$

Exploring the phase diagram…

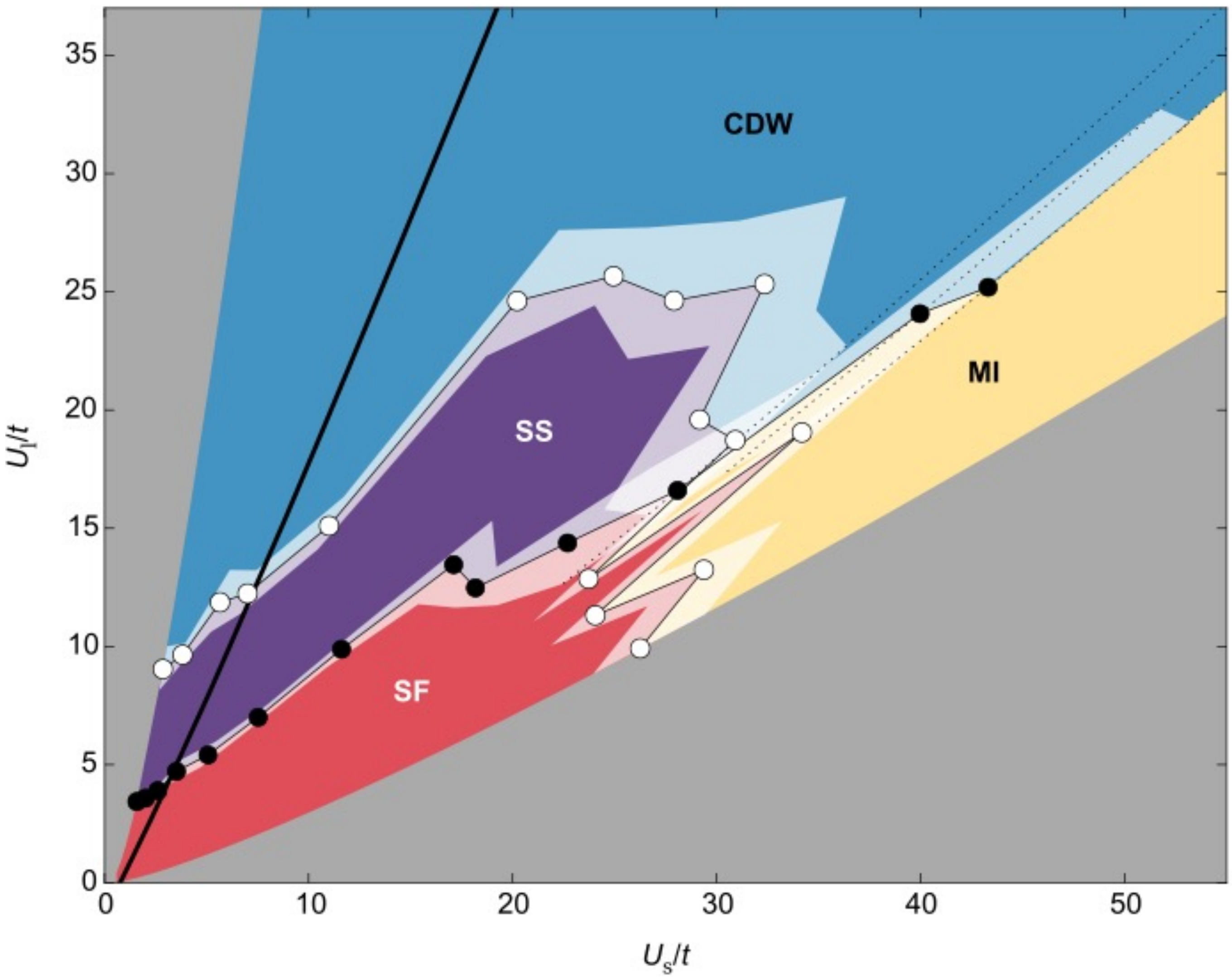


○ spatial coherence
● even-odd imbalance

$$U_{\mathrm{s}} \propto V_{\mathrm{2D}}$$

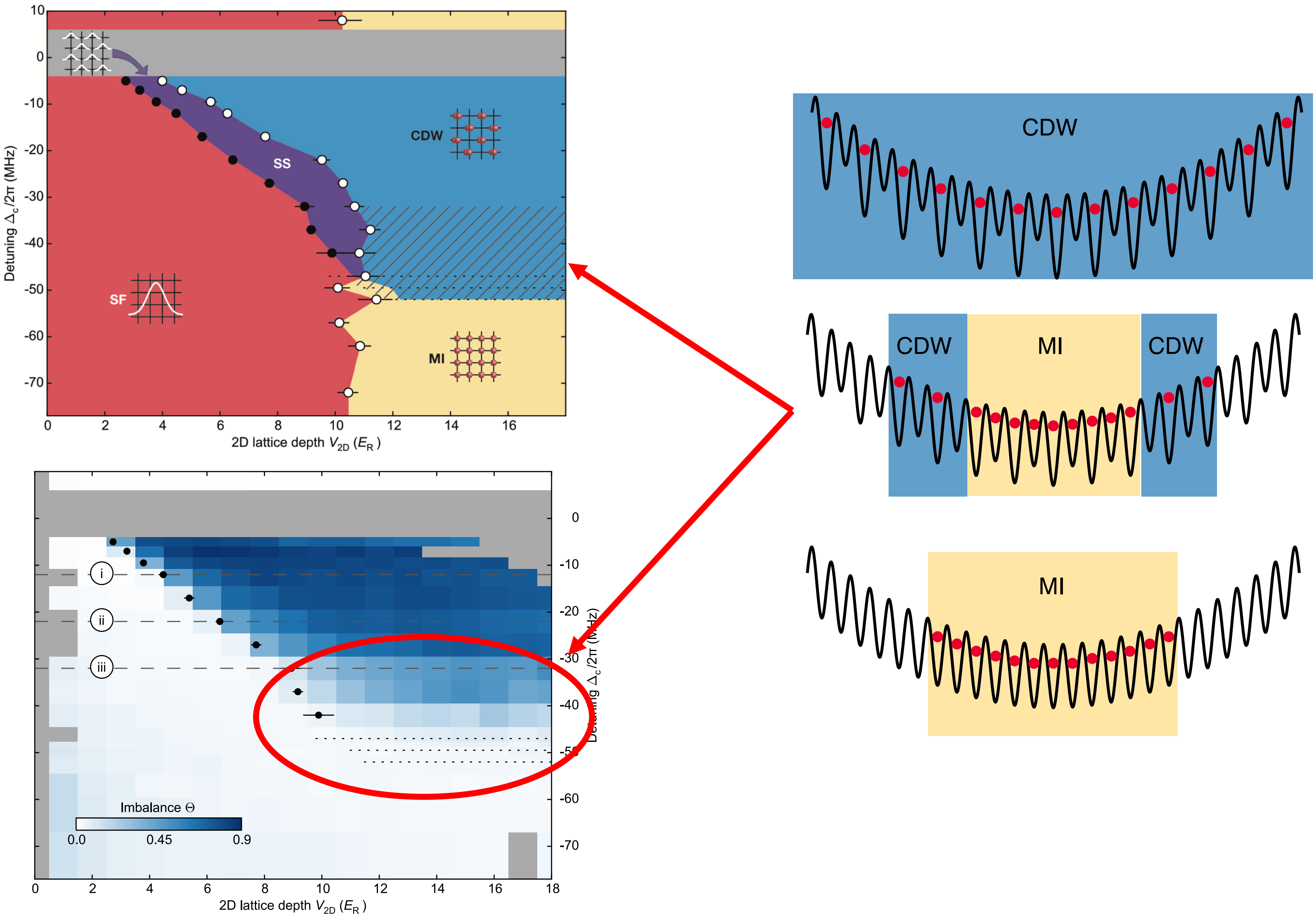
$$U_{\mathrm{l}} \propto \frac{V_{\mathrm{2D}}}{\Delta_{\mathrm{c}}}$$

Phase diagram in Hamiltonian parameters

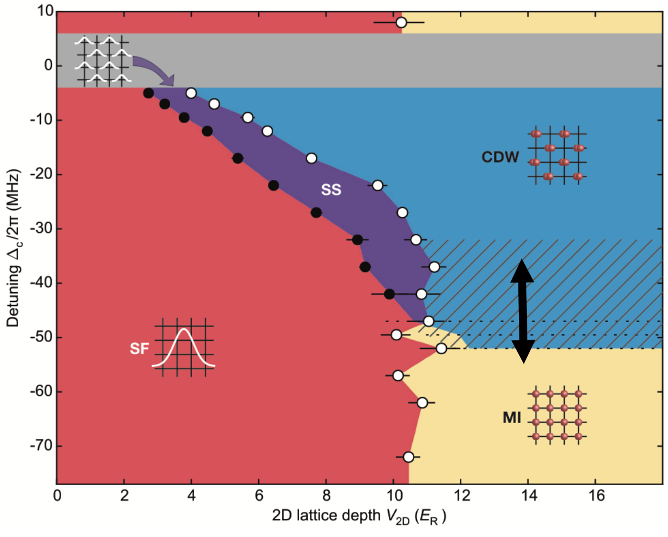


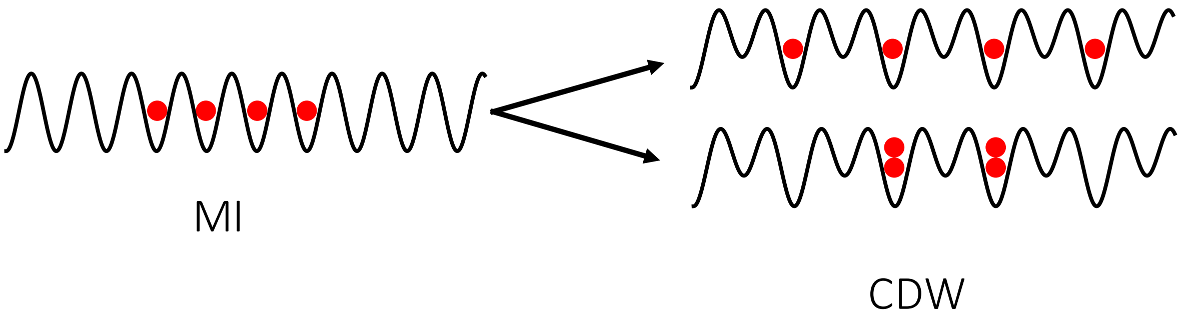
○ spatial coherence
● even-odd imbalance

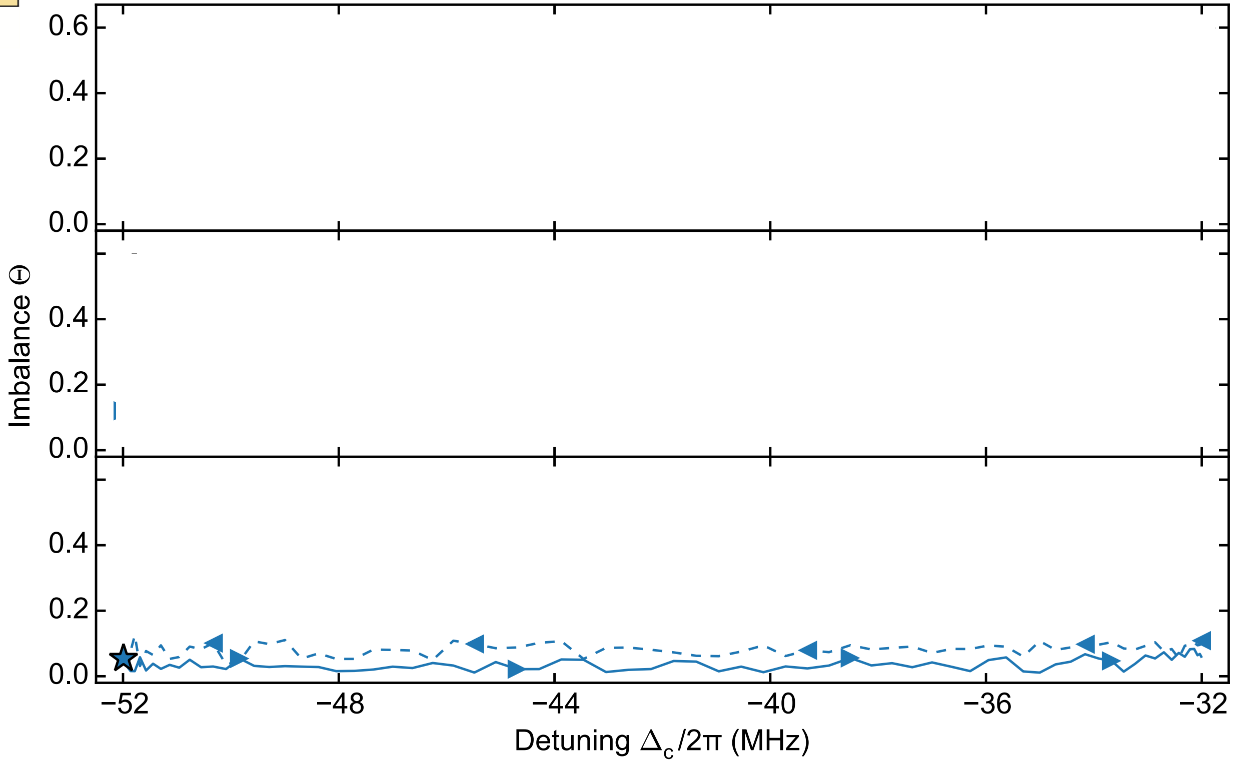
Coexistence of phases?



Transition from MI to CDW

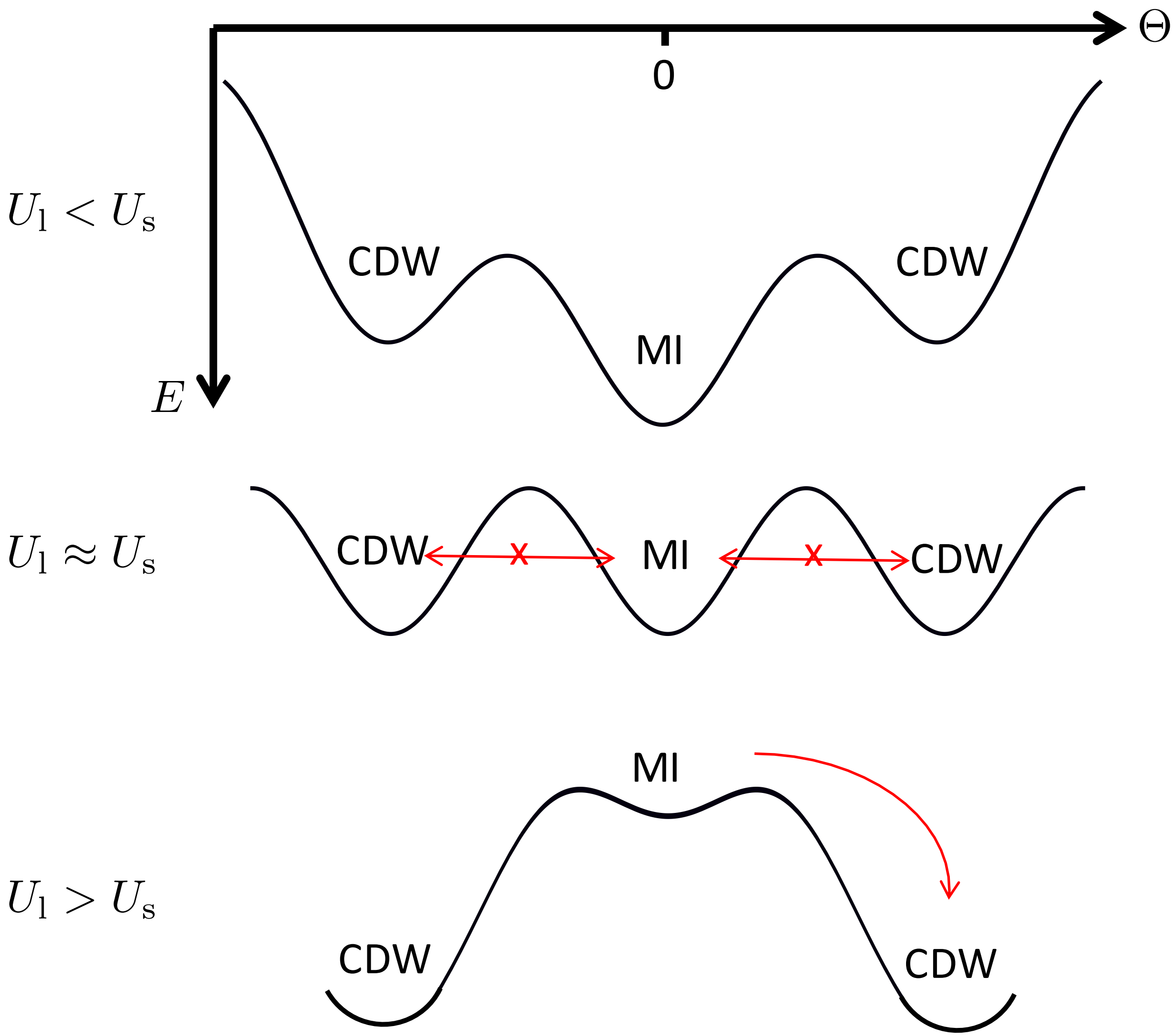


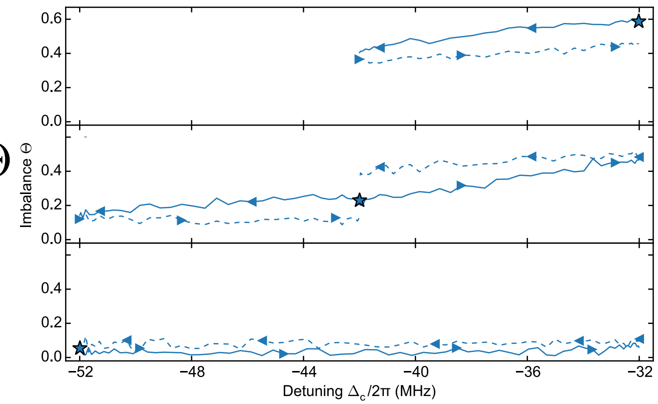




Hysteresis points towards a first order phase transition

First order phase transition?





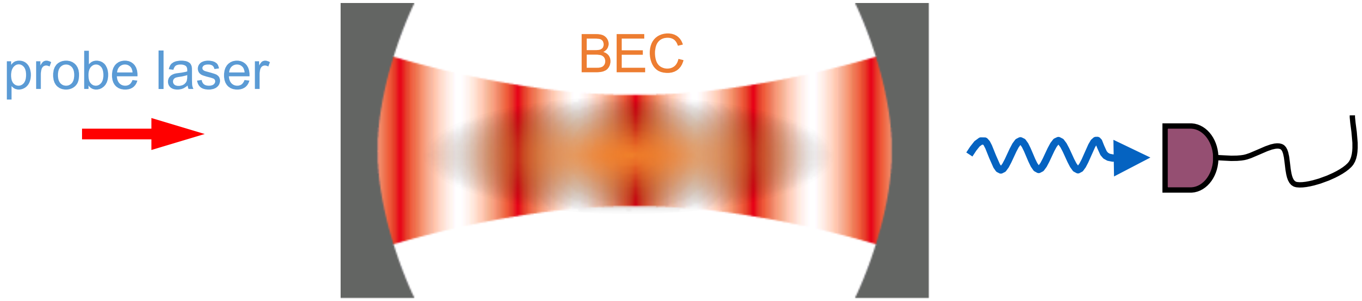
CDW energetically favorable, but blocked by energy barrier

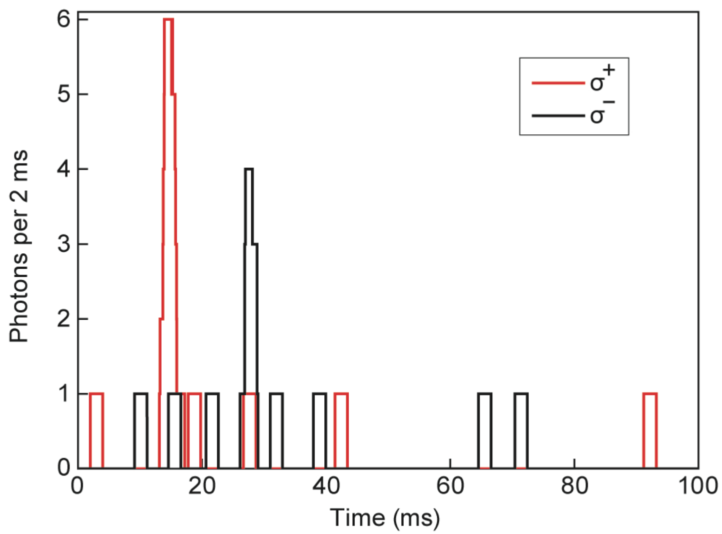
System driven out of metastable state

Thanks:

- Tilman Esslinger
- Anton Öttl
- Stephan Ritter
- Michael Köhl
- Thomas Bourdel
- Ferdinand Brennecke
- Christine Guerlin
- Kristian Baumann
- Rafael Mottl
- Renate Landig
- Lorenz Hruby
- Nishant Dogra
- Manuele Landini

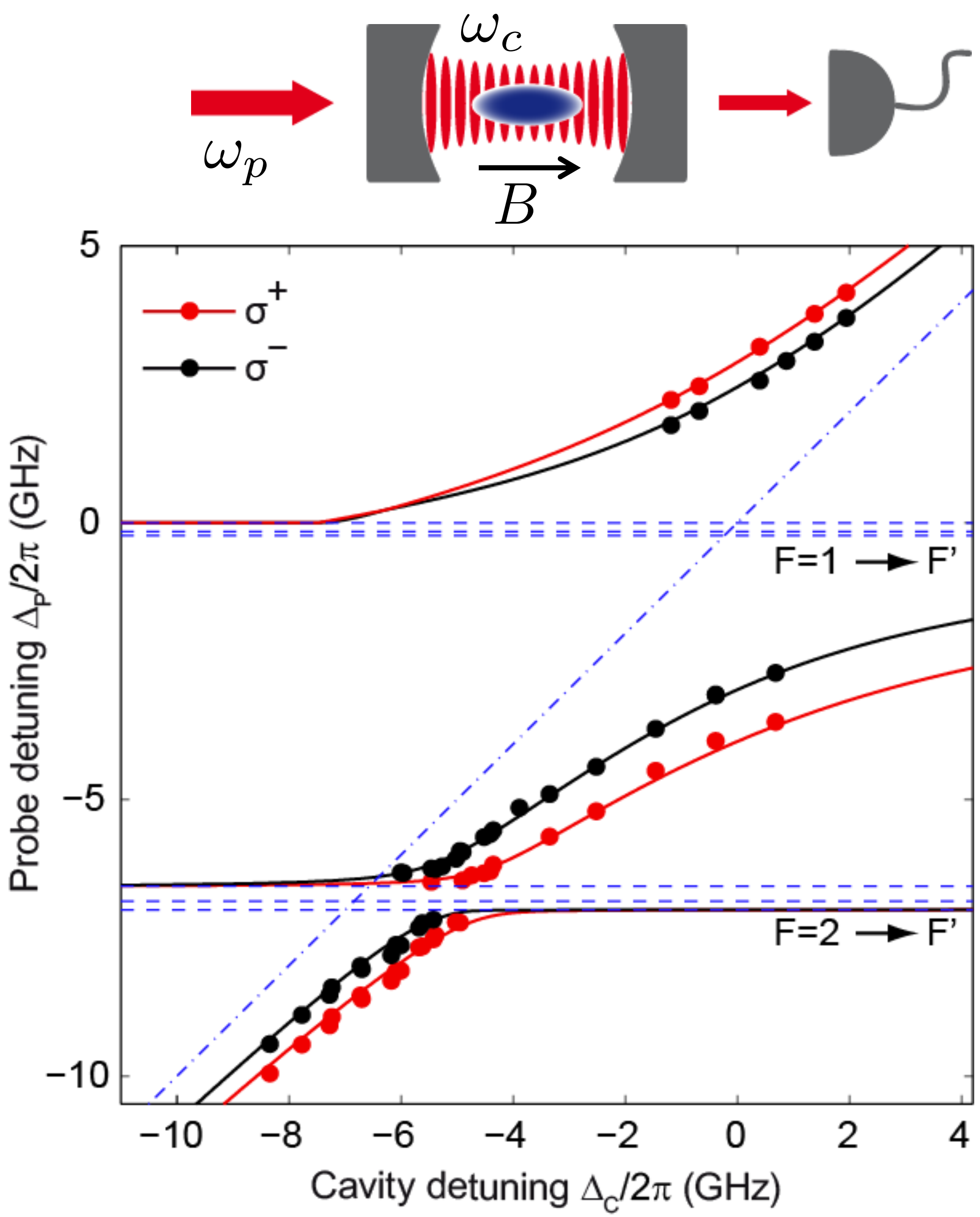
Spectroscopy of Cavity-BEC





Probe frequency scan over 2.5 GHz at fixed cavity detuning

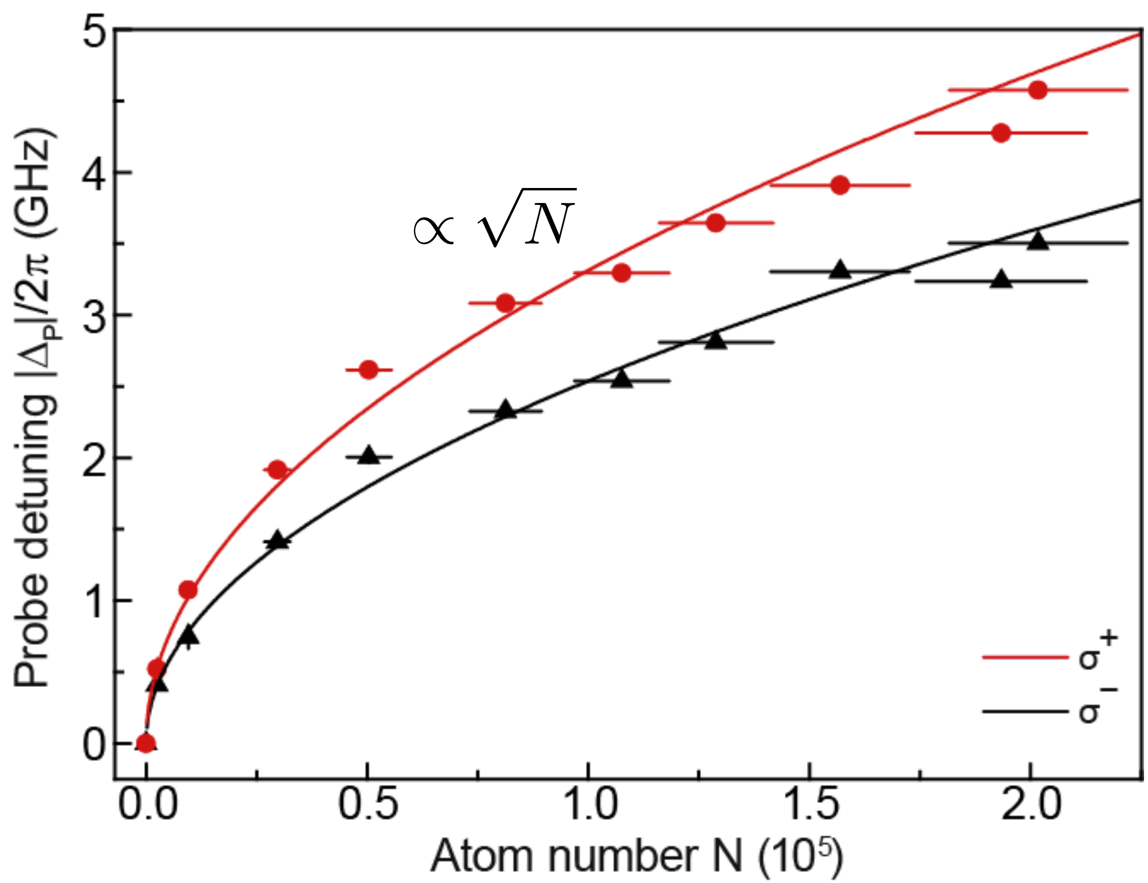
Probing the internal excitation spectrum



$$N_{F=1} = 1.5 \times 10^5$$

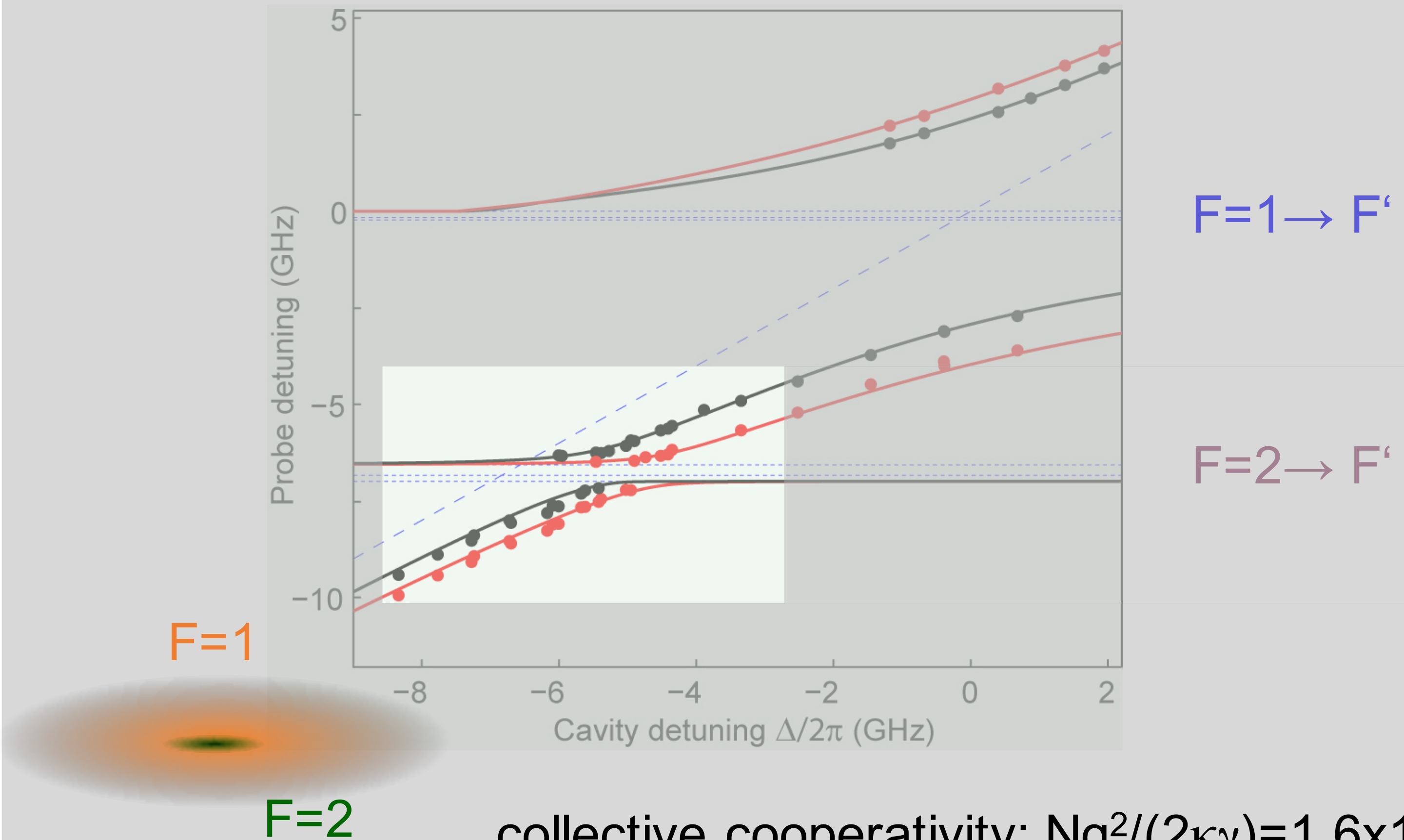
$$N_{F=2} = 2.7 \times 10^3$$

Scaling with atom number



F. Brennecke et al. Nature 450, 268 (2007) ; see also: Y. Colombe et al. Nature 450, 272 (2007)

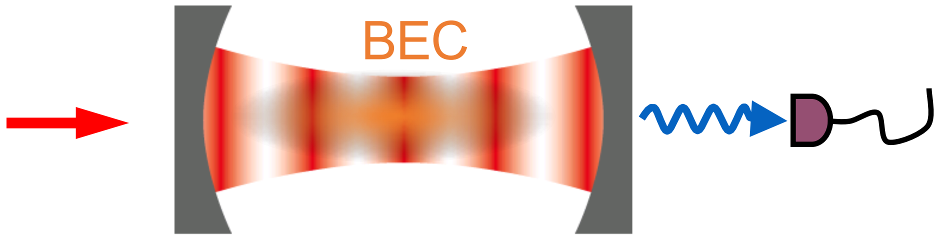
Cavity-BEC Energy Spectrum



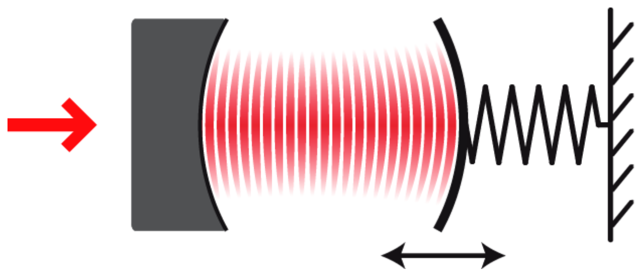
collective cooperativity: $Ng^2/(2\kappa\gamma)=1.6\text{x}10^6$

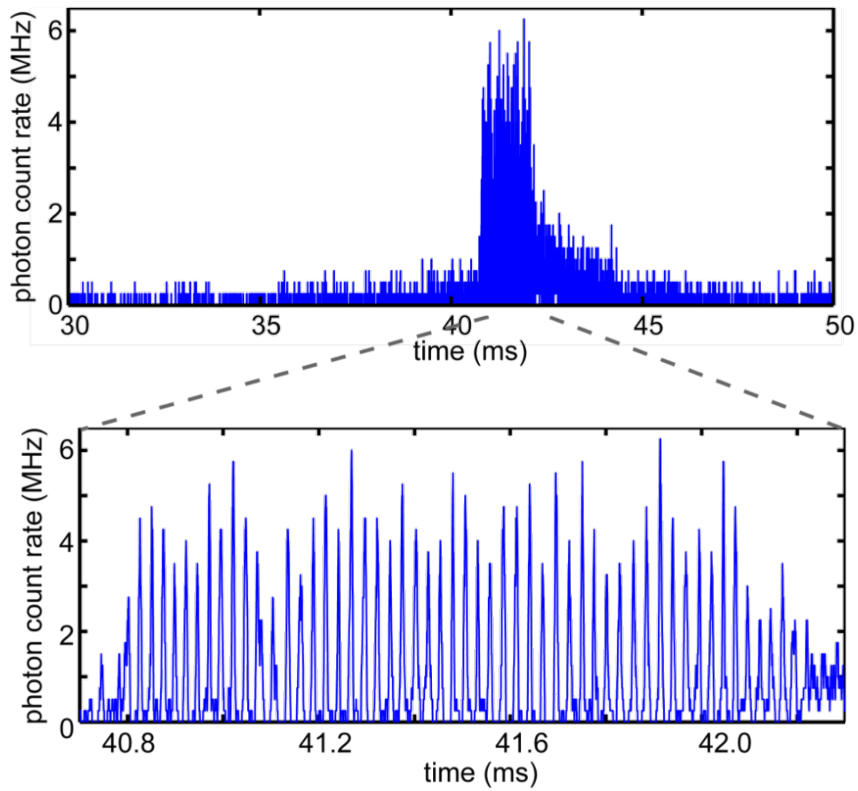
*F. Brennecke, T. Donner, S.Ritter, T. Bourdel, M. Köhl, T. Esslinger, Nature **450**, 268 (2007)*
*Y. Colombe, T. Steinmetz, G. Dubois, F. Linke, D. Hunger, J. Reichel, Nature **450**, 272 (2007)*

Outlook: Cavity opto-mechanics

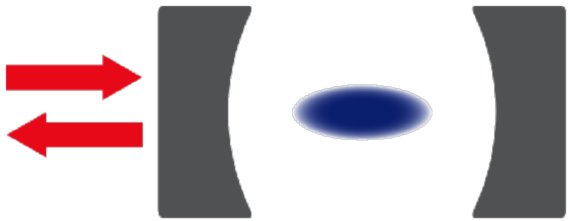


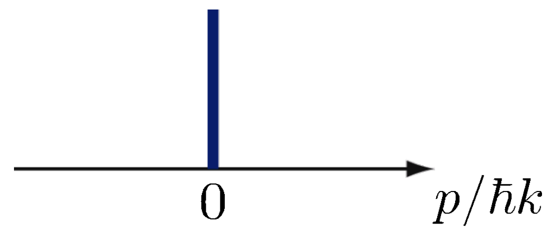
External state dynamics:
Backaction of BEC on the light field!

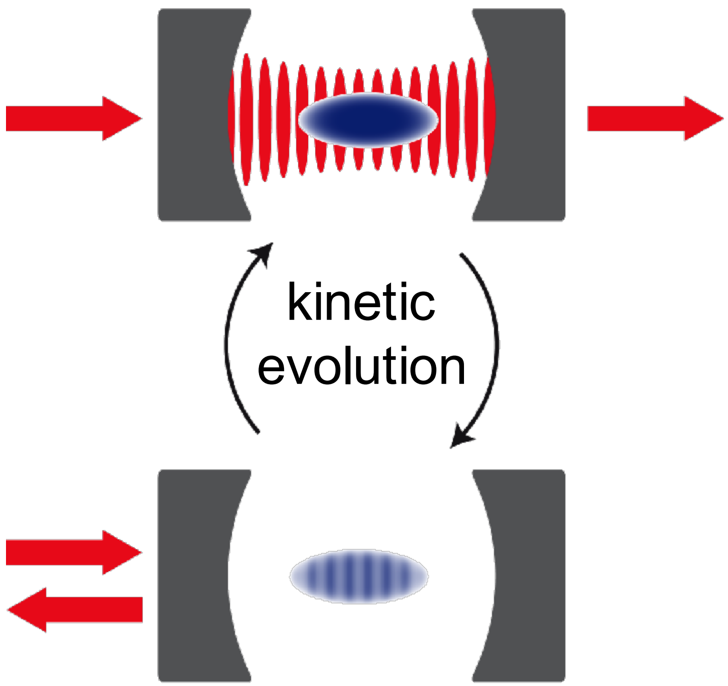




External State Dynamics







$$\psi(t) = |0\rangle + \epsilon\, e^{-4i\omega_{\mathrm{rec}}t} |\pm 2\hbar k\rangle$$

$$(\epsilon \ll 1)$$