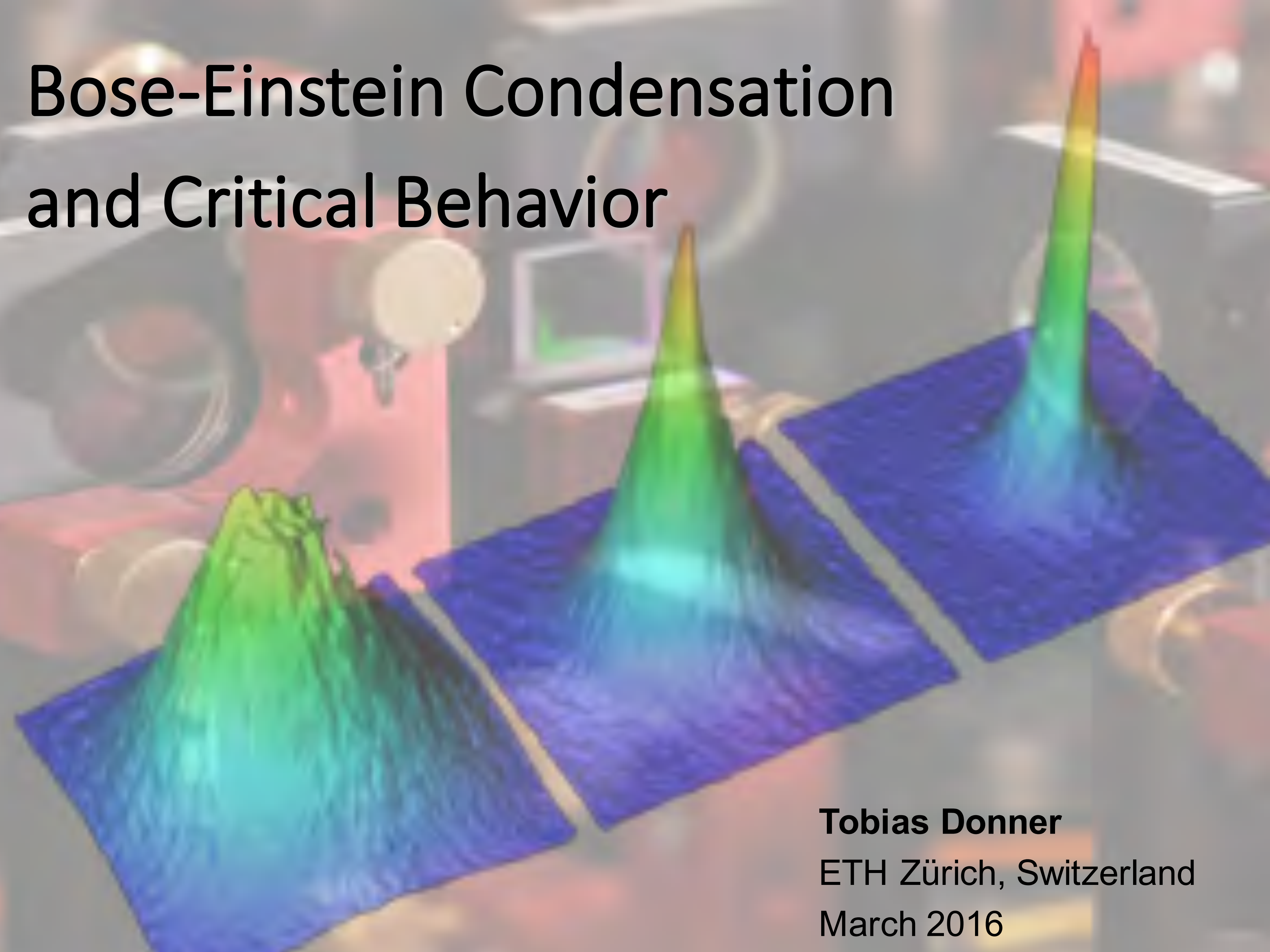
Bose-Einstein Condensation and Critical Behavior



Tobias Donner
ETH Zürich, Switzerland
March 2016

Outline

- Introduction to Bose-Einstein condensation
- Long-range order
- Critical phenomena
- Measuring a critical exponent of the BEC-normal transition

L1

- Long-range interactions in quantum gases
- Cavity QED intro
- Realizing the superfluid – supersolid phase transition
- Studying critical properties of the phase transition
- (Realizing an extended Hubbard model: superfluid – Mott insulator – supersolid – charge density wave

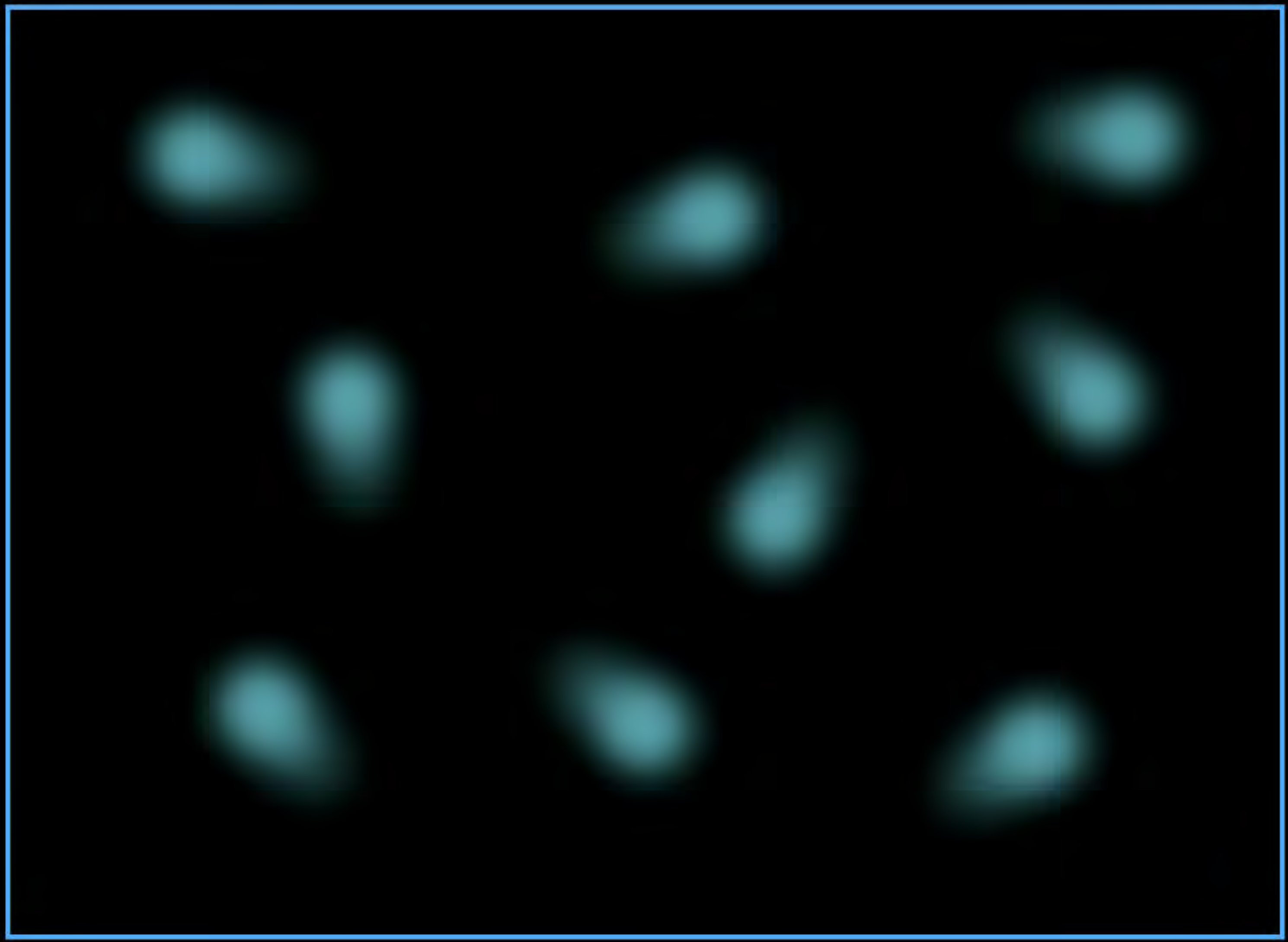
L2

A gas at room temperature

$T >> T_c$

An ultracold gas

$T > T_c$ $\lambda_{db} = h/mv \propto T^{-1/2}$



Condensation

$T < T_c$ λ_{db}

Pure condensate

(1) I 7040

Quantentheorie des einatomigen idealen Gases

Zweite Abhandlung.

In einer neulich (in diesen Berichten (XXII 1924. S. 261)) erschienenen Abhandlung wurde unter Anwendung einer von Herrn D. Bose zur Ableitung der Planck'schen Strahlungsformel erdachten Methode eine Theorie der „Entartung" idealer Gase angegeben. Das Interesse dieser Theorie liegt darin, dass sie auf die Hypothese einer weitgehenden formalen Verwandtschaft zwischen Strahlung und Gas gegründet ist. Nach dieser Theorie weicht das entartete Gas von dem Gas der mechanischen Statistik in analoger Weise ab wie die Strahlung gemäss dem Planck'schen Gesetze von der Strahlung gemäss dem Wien'schen Gesetze. Wenn die Bose'sche Ableitung der Planck'schen Strahlungsformel ernst genommen wird, so wird man auch an dieser Theorie des idealen Gases nicht vorbeigehen dürfen; denn wenn es gerechtfertigt ist, die Strahlung als Quantengas aufzufassen, so muss die Analogie zwischen Quantengas und Molekülgas eine vollständige sein. Im Folgenden sollen die früheren Überlegungen durch einige neue ergänzt werden, die mir das Interesse an dem Gegenstande zu steigern scheinen. Der Bequemlichkeit halber schreibe ich das Folgende formal als Fortsetzung der zitierten Abhandlung.

§6. Das gesättigte ideale Gas.

Bei der Theorie des idealen Gases scheint es eine selbstverständliche Forderung, dass Volumen und Temperatur einer Gasmenge willkürlich gegeben werden können. Die Theorie bestimmt dann die Energie bezw. den Druck des Gases. Das Studium der in den Gleichungen (18), (19), (20), (21) enthaltenen Zustandsgleichung zeigt aber, dass bei gegebener Molekülzahl n und gegebener Temperatur T das Volumen nicht beliebig klein gemacht werden kann. Gleichung (18) verlangt nämlich, dass für alle s $\alpha^s \geqq 0$ sei, was gemäss (20) bedeutet, dass $A \geqq 0$ sein muss. Dies bedeutet, dass in der in diesem Falle gültigen Gleichung (18b) λ $(=e^{-A})$ zwischen 0 und 1 liegen muss. Aus (18b) folgt demnach, dass die Zahl der Moleküle in einem solchen Gas bei gegebenem Volumen V nicht grösser sein kann als

$$n = \frac{(2\pi m \kappa T)^{\frac{3}{2}} V}{h^3} \sum_{1}^{\infty} \tau^{-\frac{3}{2}} \ldots (24)$$

Bose-Einstein statistics

(grand-canonical ensemble)

Bose-Einstein distribution function:

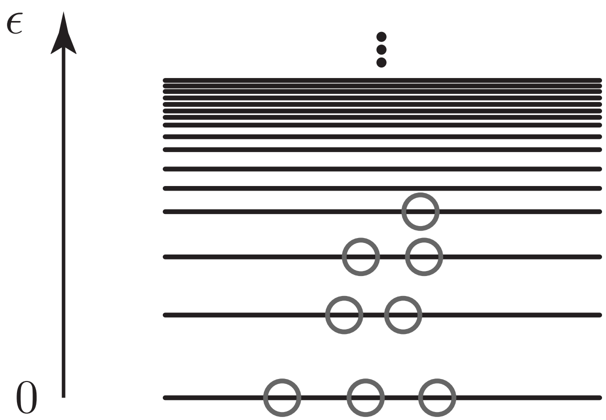
$$f(\epsilon_\nu) = \langle n_\nu \rangle = \frac{1}{e^{\frac{\epsilon_\nu - \mu}{k_B T}} - 1}$$

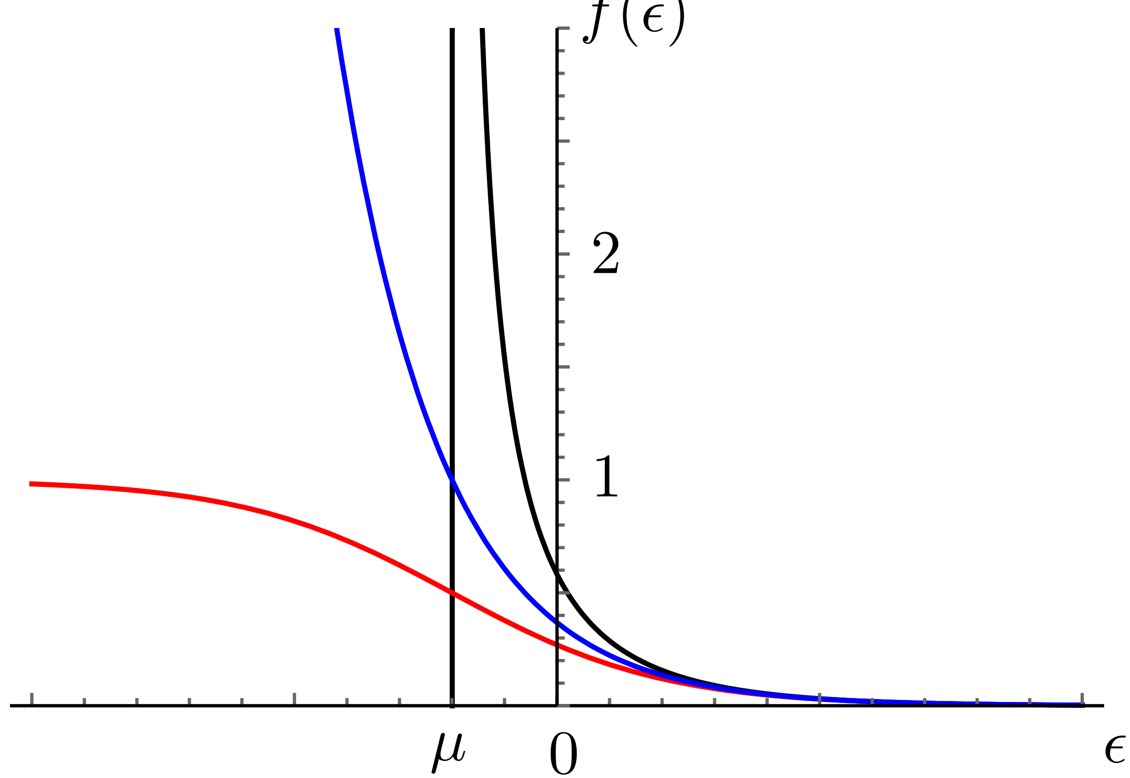
Chemical potential :

$$\mu = \left(\frac{\partial E}{\partial N} \right)_{S,V}$$

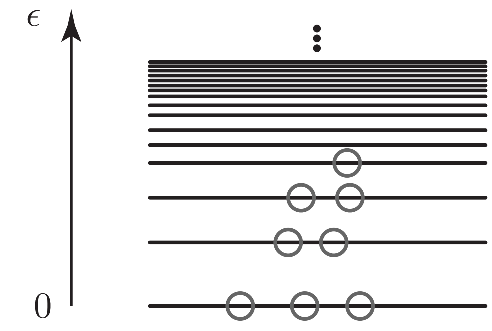
$\mu \leq 0$

or smaller ϵ_{min}





Bose-Einstein statistics



$$N = \sum_\nu f(\epsilon_\nu)$$ Total number of particles

$$N = \sum_\nu f(\epsilon_\nu) \simeq \int_0^\infty \mathrm{d}\epsilon f(\epsilon) g(\epsilon)$$

Good approximation for large density of states

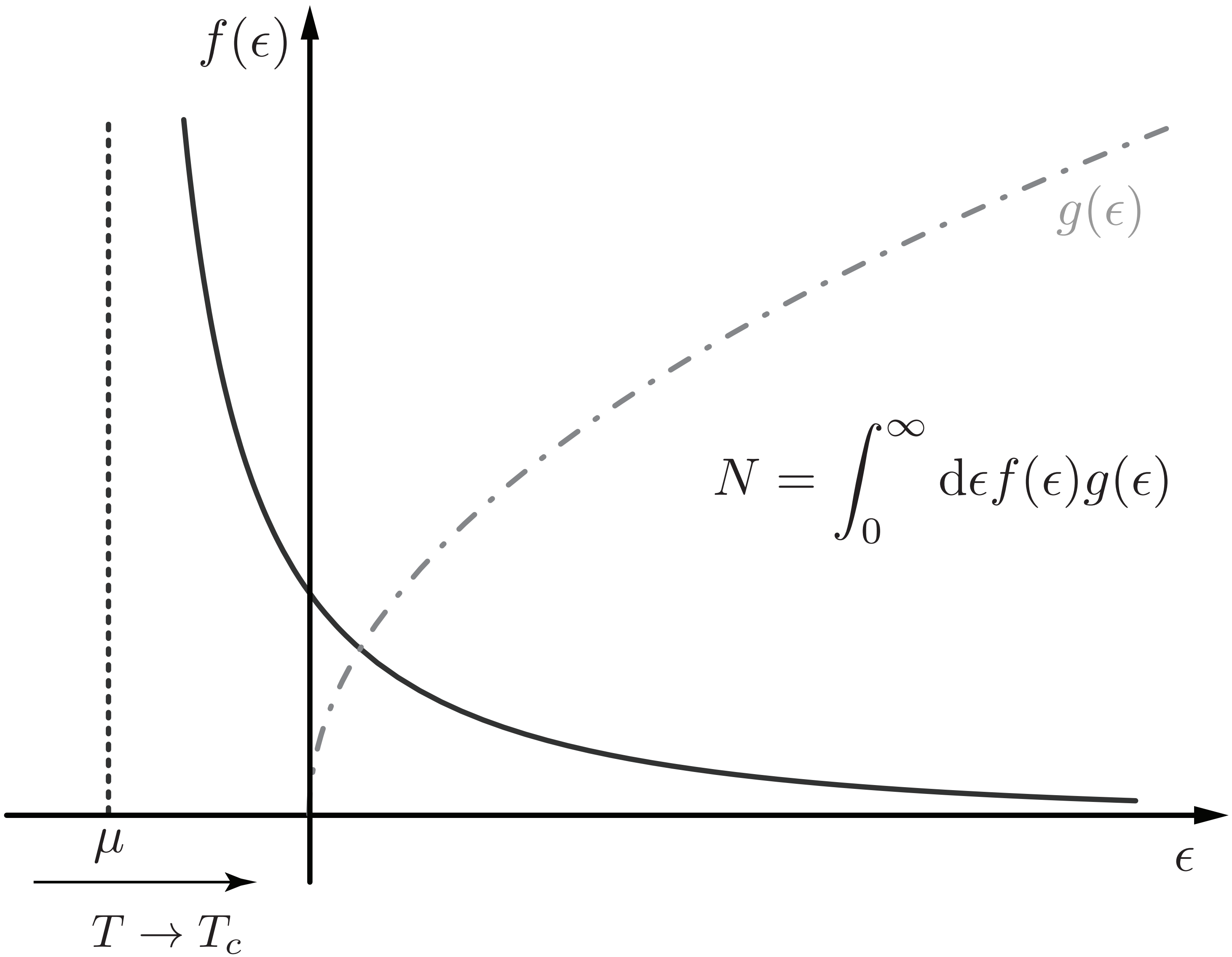
Density of states in 3D: $g(\epsilon) = C\sqrt{\epsilon}$

$$N = C \int_0^\infty \mathrm{d}\epsilon \frac{\sqrt{\epsilon}}{e^{\frac{\epsilon-\mu}{k_B T}} - 1}$$

Keeping T fixed add particles: increase μ however, $\mu \leq 0$

$$N_c = N(\mu = 0) = C \int_0^\infty \mathrm{d}\epsilon \frac{\sqrt{\epsilon}}{e^{\frac{\epsilon}{k_B T}} - 1} = \tilde{C} \left(k_B T\right)^{3/2}$$

Bose-Einstein statistics



Bose-Einstein statistics

Here we made a mistake: $$N = \sum_{\nu} f(\epsilon_\nu) \simeq \int_0^\infty \mathrm{d}\epsilon f(\epsilon) g(\epsilon)$$

Our way out is to single out the ground state:

$$N = N_0 + N_{exc} = \frac{1}{e^{\frac{\mu}{k_B T}} - 1} + C \int_0^\infty \mathrm{d}\epsilon \frac{\sqrt{\epsilon}}{e^{\frac{\epsilon - \mu}{k_B T}} - 1}$$

Now any additional particle is accommodated in the ground state!

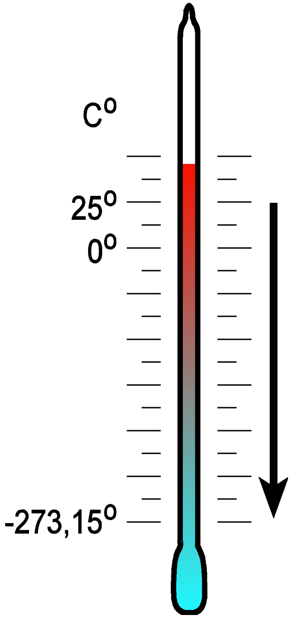
At the critical point:

$$N = N_{exc} \quad \Rightarrow \quad k_B T_c \simeq 3.31 \frac{\hbar^2 n^{2/3}}{m}$$

$$n = 10^{14} \mathrm{cm}^{-3} \qquad m = 1.4 \times 10^{-25} \mathrm{kg} \qquad T_c = 400 \mathrm{nK}$$

$\sim 100 nK$

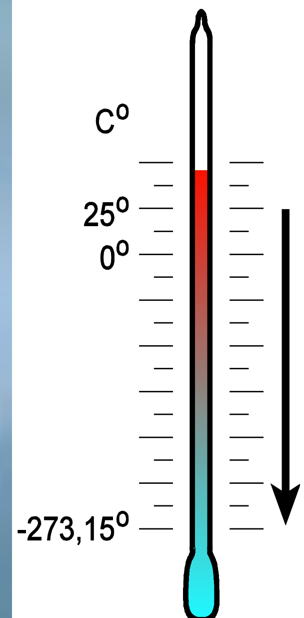
Getting cold: Liquid helium?



Room temperature

Liquid He: 4K

Getting cold: Dilution fridge??



Room temperature

Liquid He: 4K

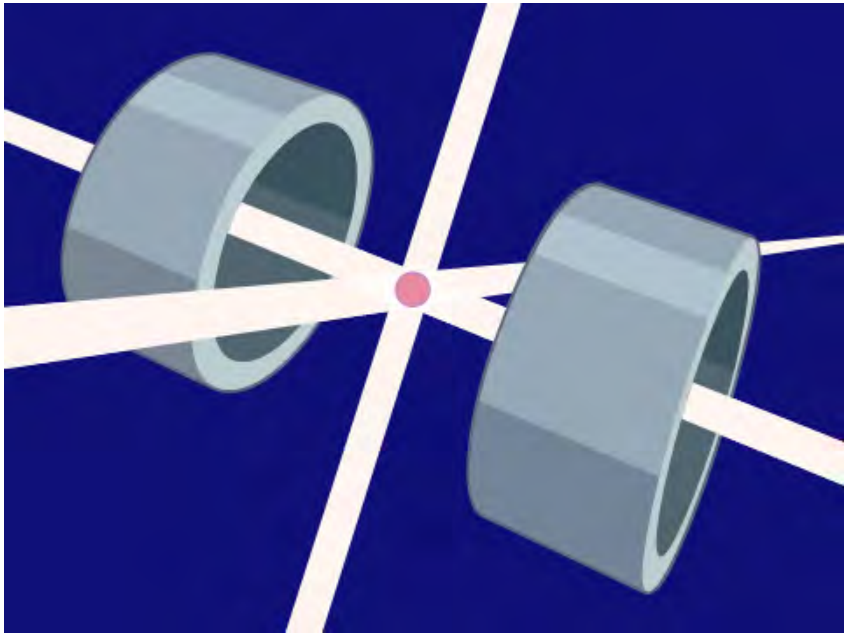
Dilution fridge: 10 mK

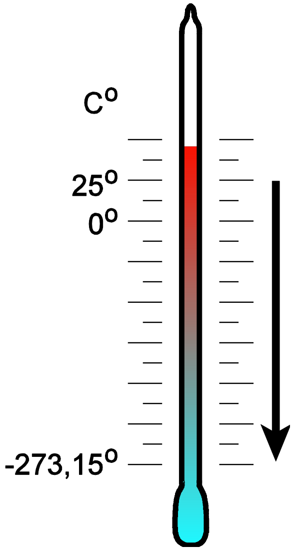
300K

…

100uK

Laser Cooling

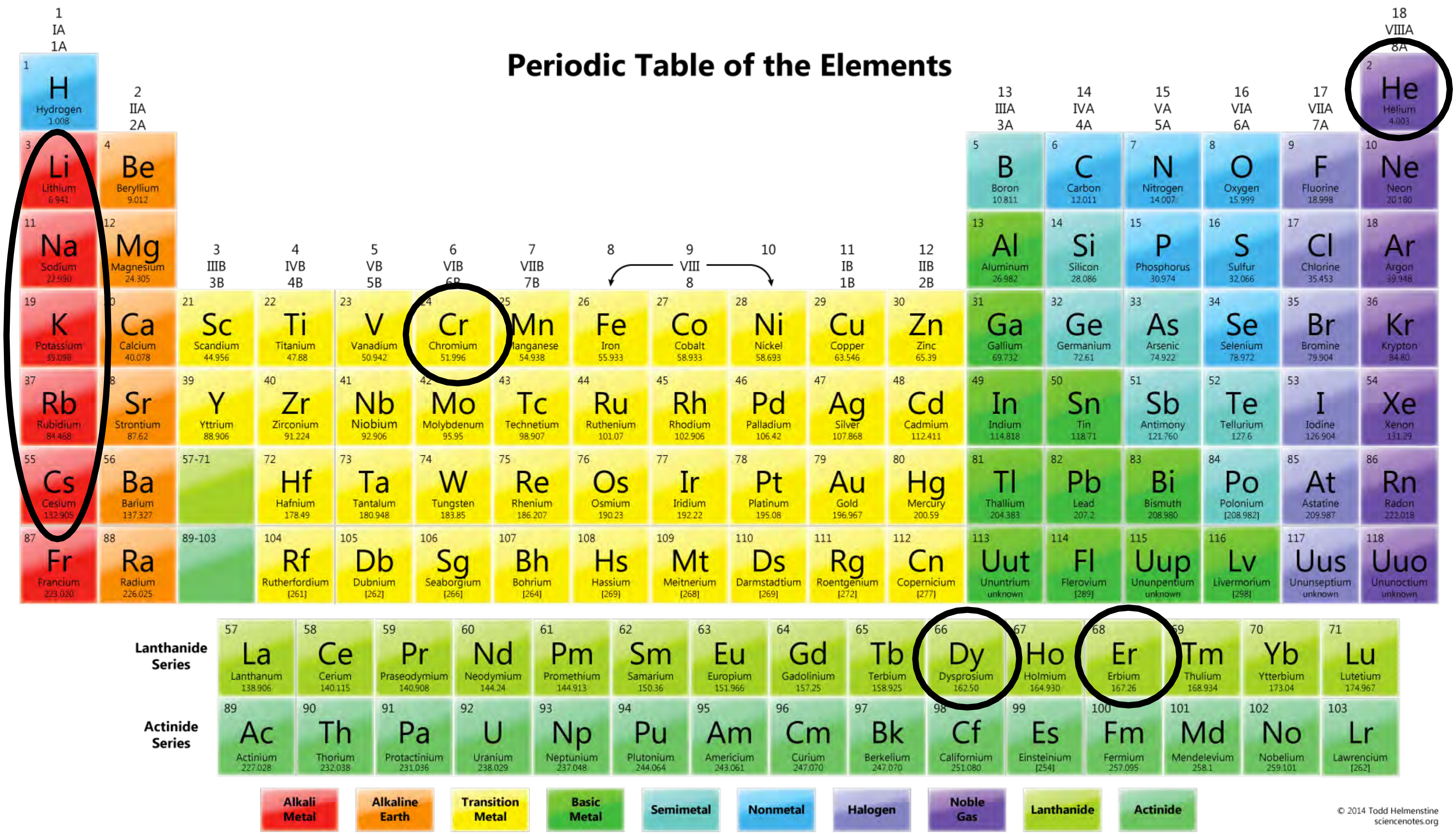




300 K

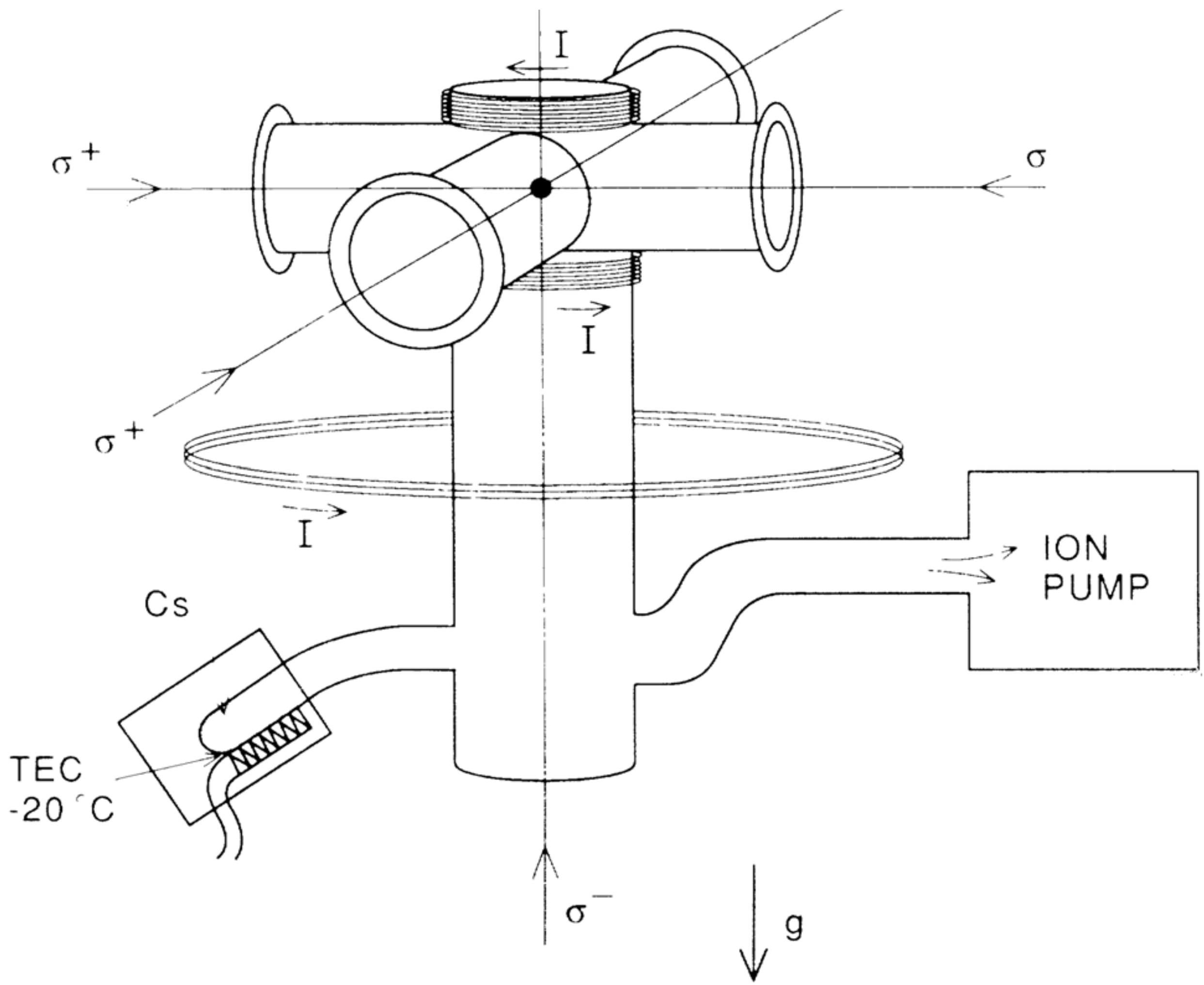
0.1 mK

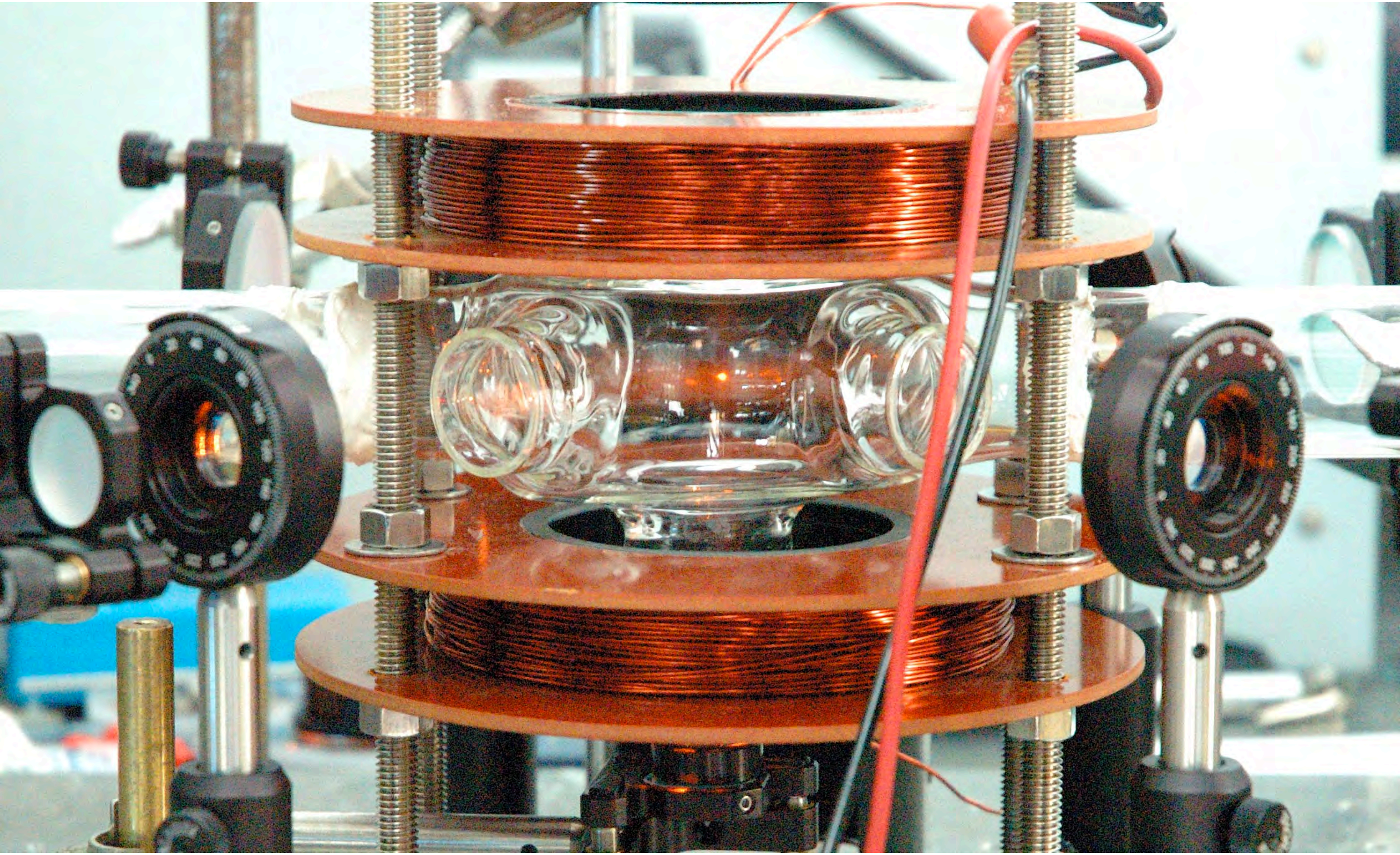
Periodic Table of the Elements

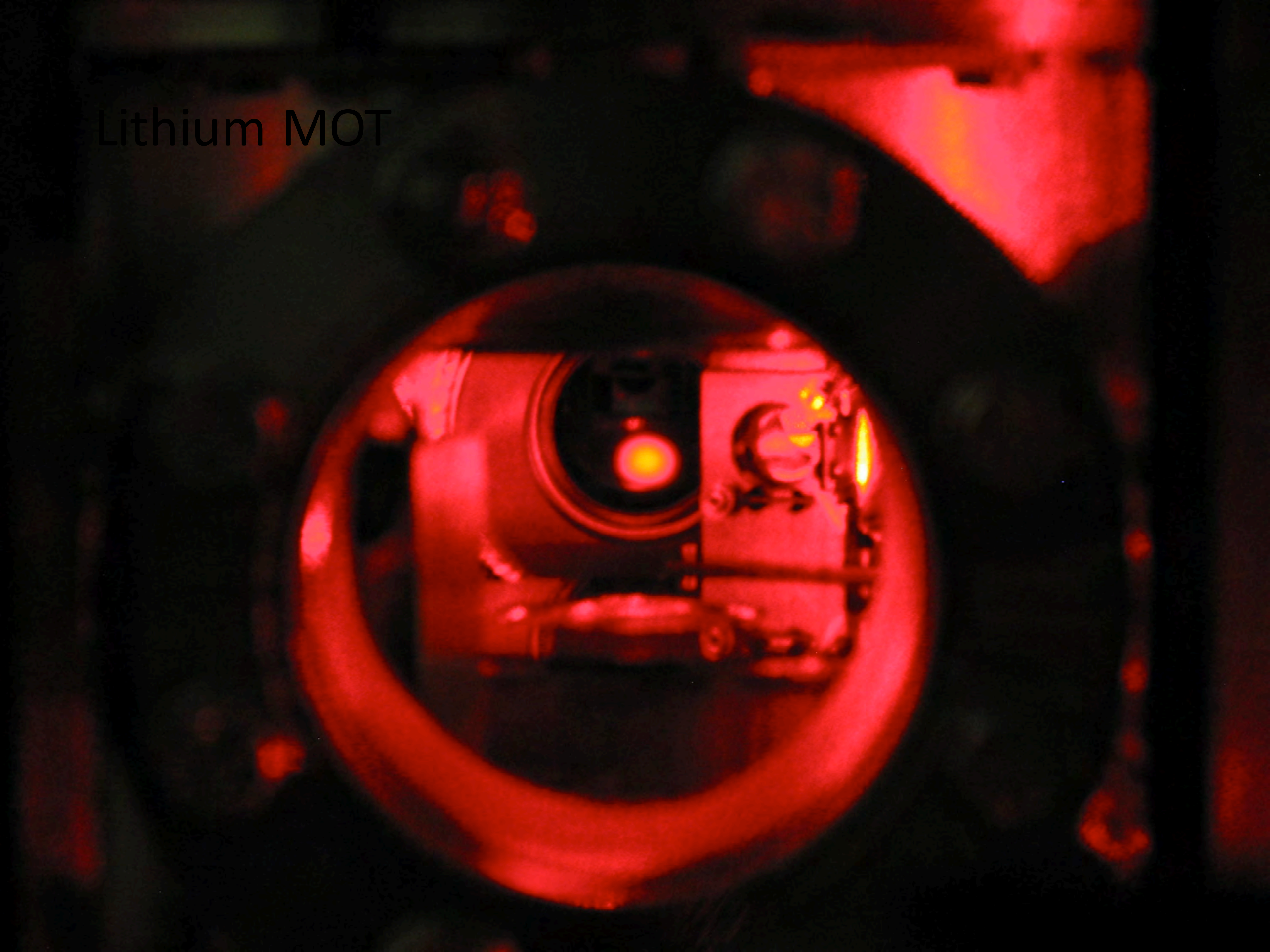


Very Cold Trapped Atoms in a Vapor Cell

C. Monroe, W. Swann, H. Robinson,[a] and C. Wieman
Joint Institute for Laboratory Astrophysics, University of Colorado and National Institute of Standards and Technology, and Department of Physics, University of Colorado, Boulder, Colorado 80309-0440
(Received 31 May 1990)





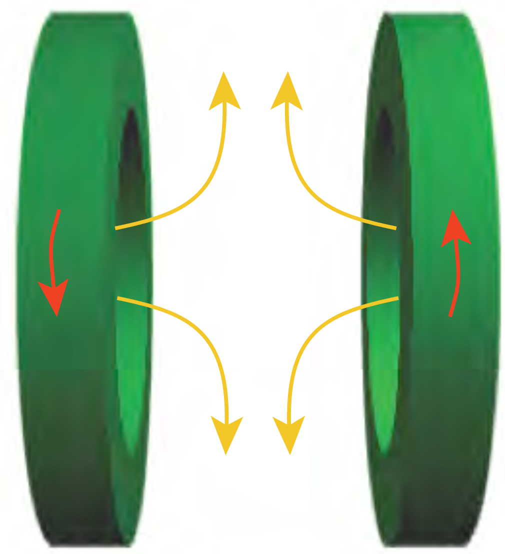

Lithium MOT

100uK

…

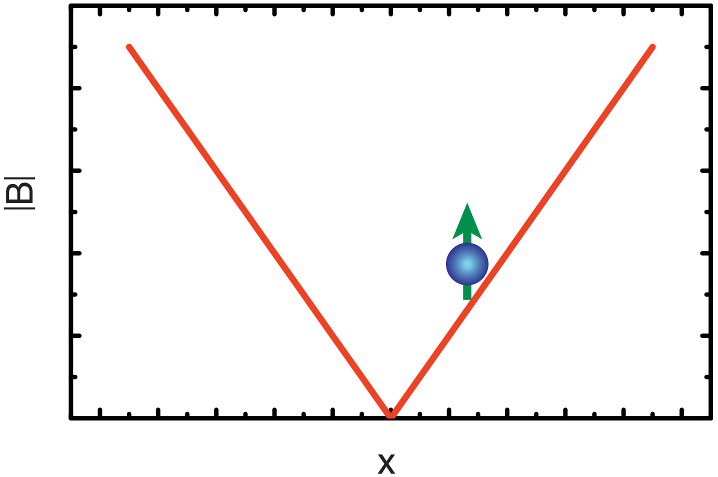
100nK

Magnetic Trapping



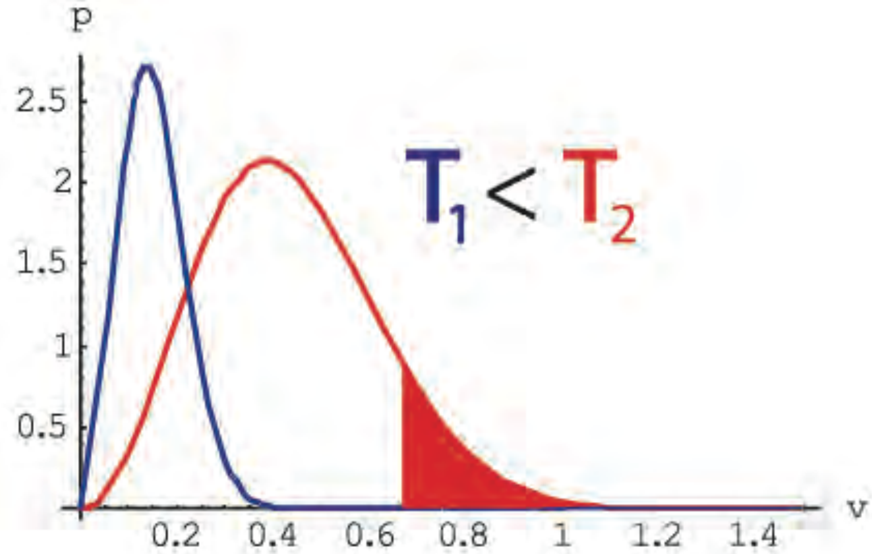
magnetic quadrupole trap

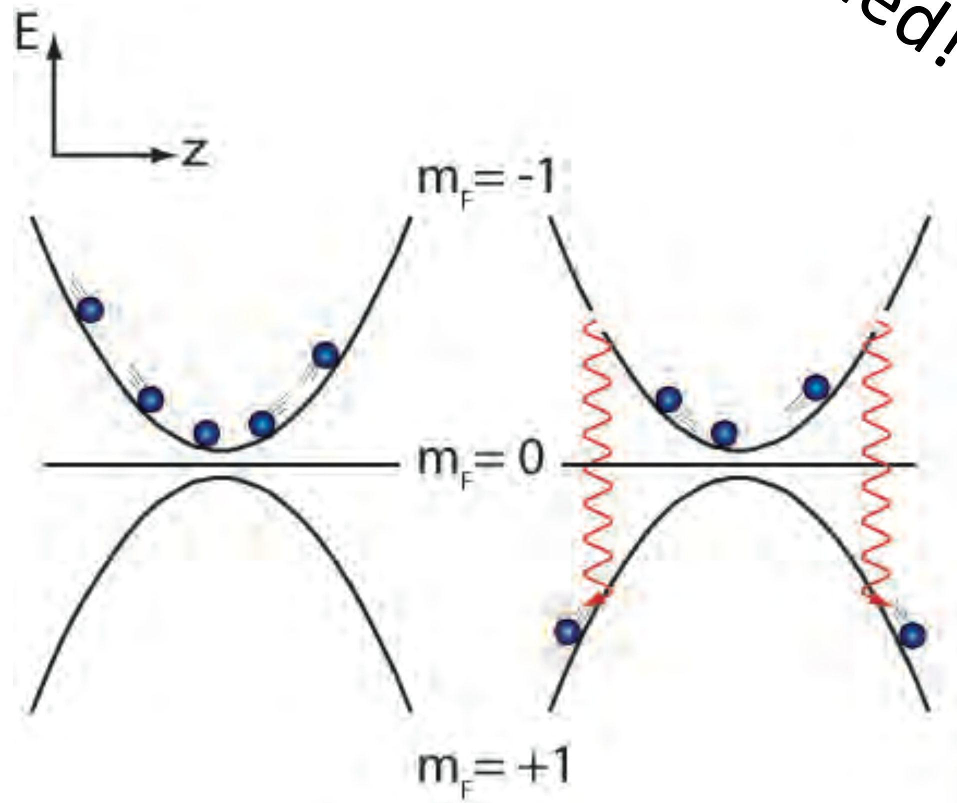
$V = \mu|B|$



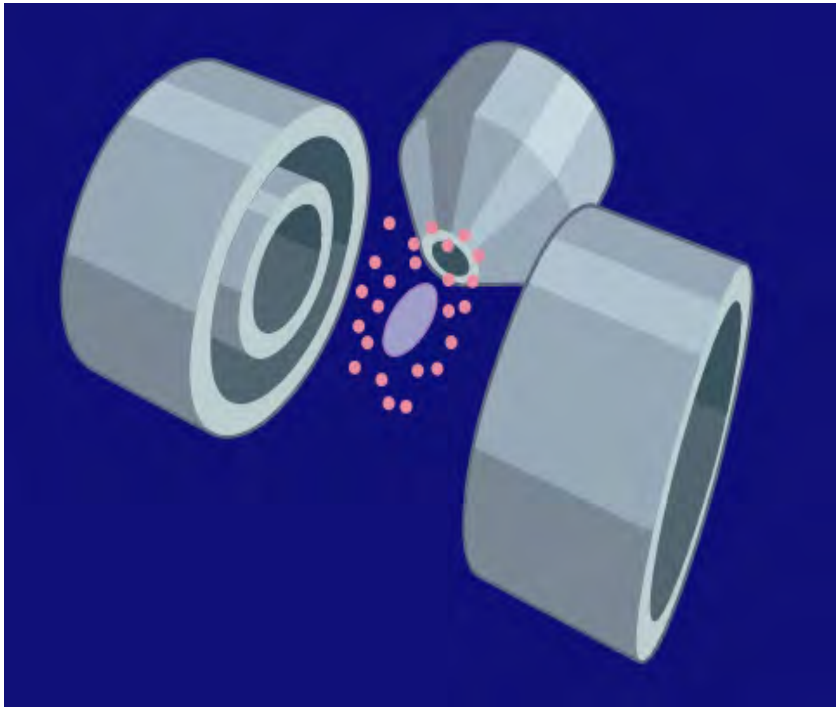
Evaporative cooling

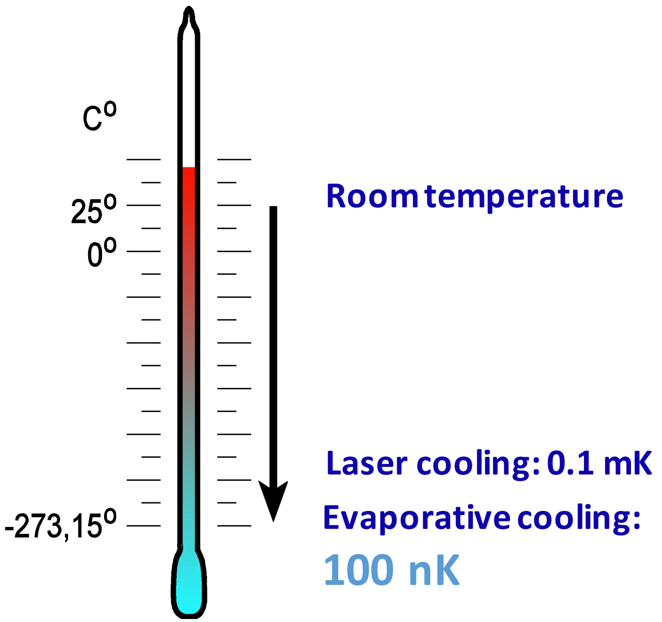
Interactions needed!





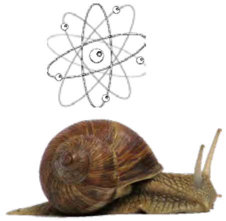
Getting cold: Evaporative cooling



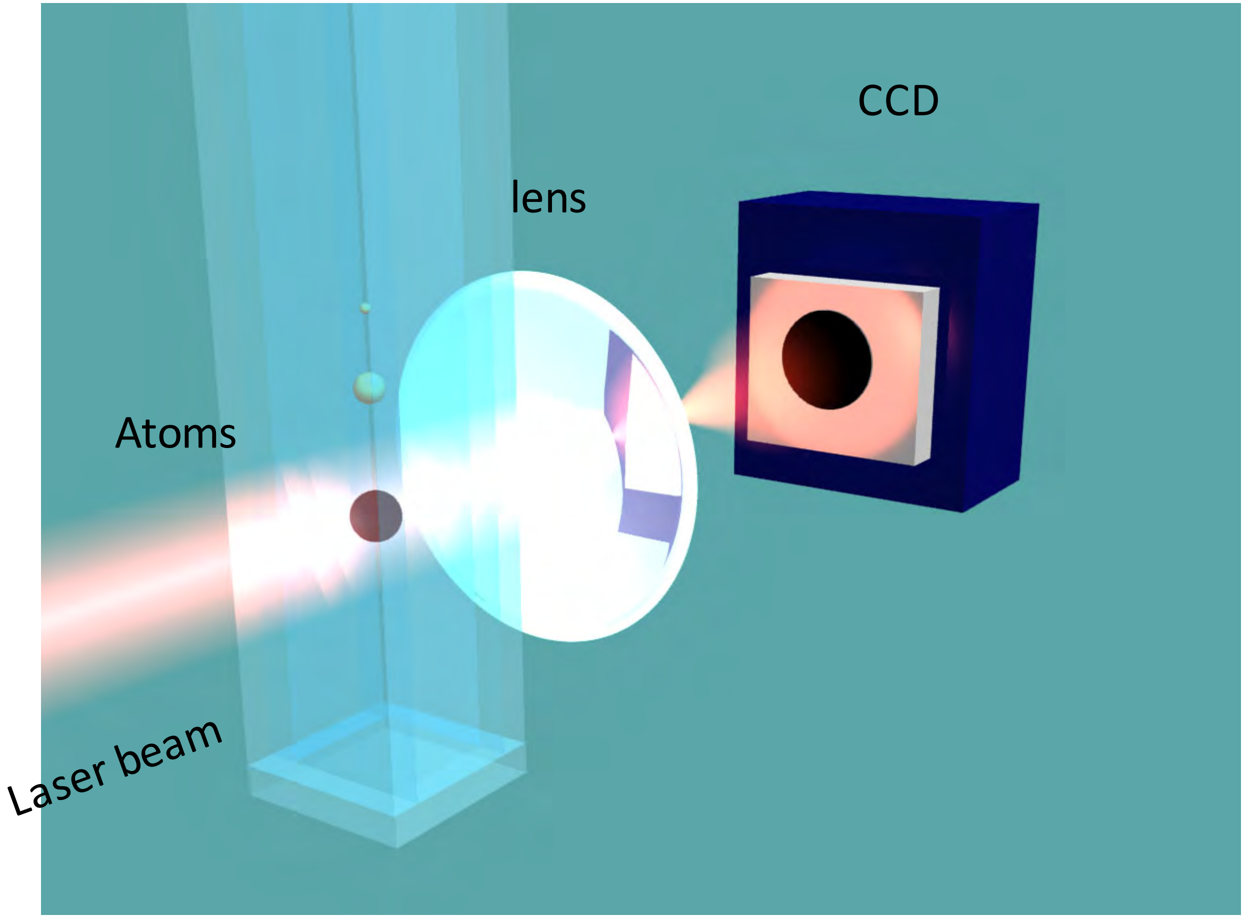


How cold is 100 nK ?

$$\frac{3}{2}mv^2 = k_B T \quad \rightarrow \quad v \approx 2\,\mathrm{mm/s}$$

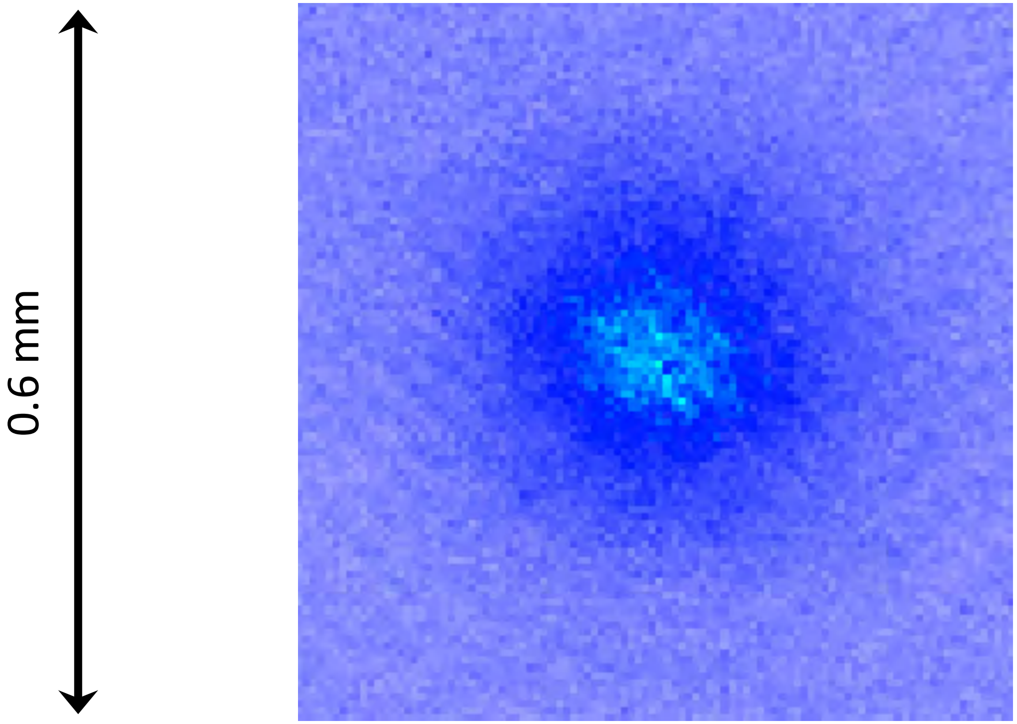


Absorption Imaging



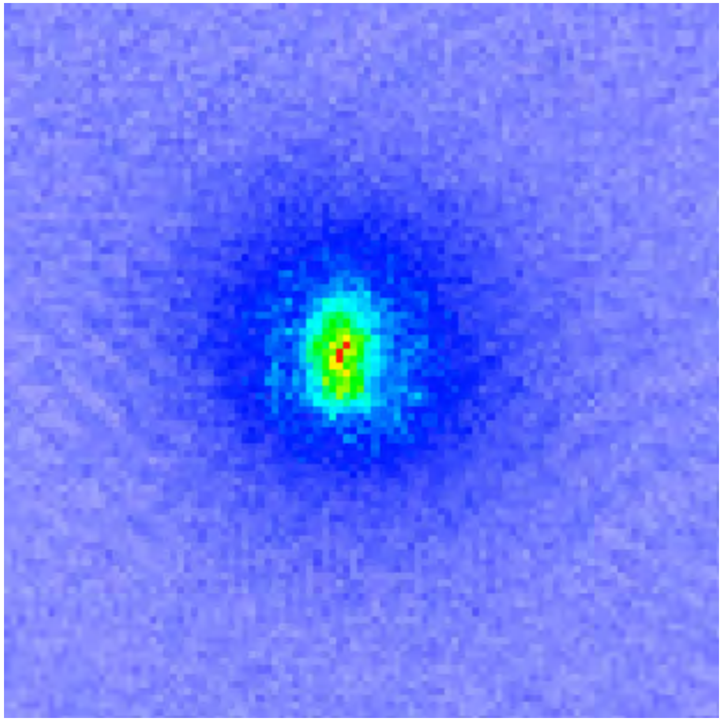
UHV glass cell

Expansion of a cold gas



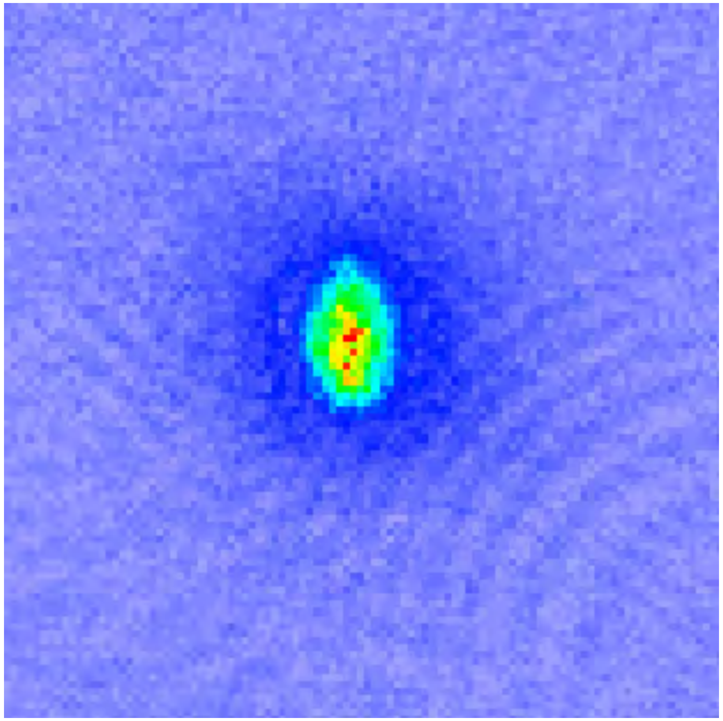
500 nK

Condensation



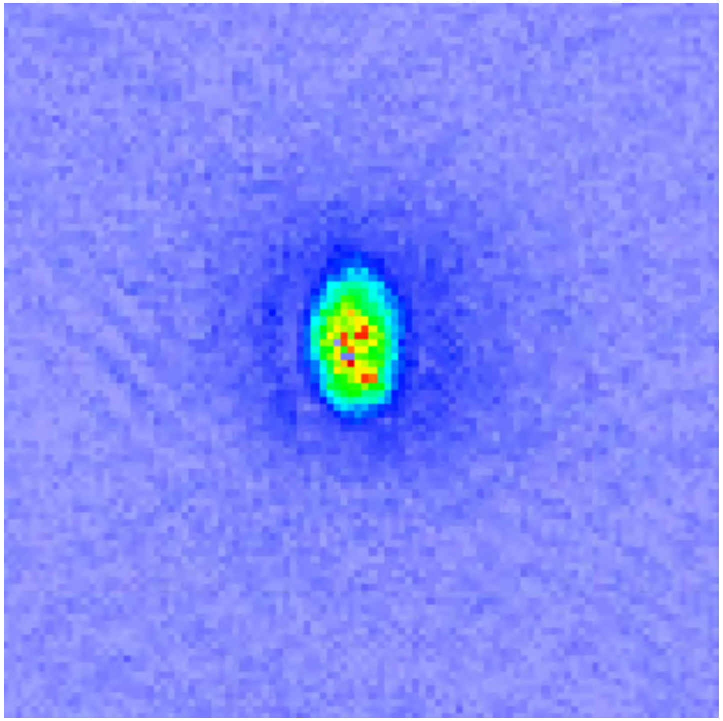
430 nK

Condensation



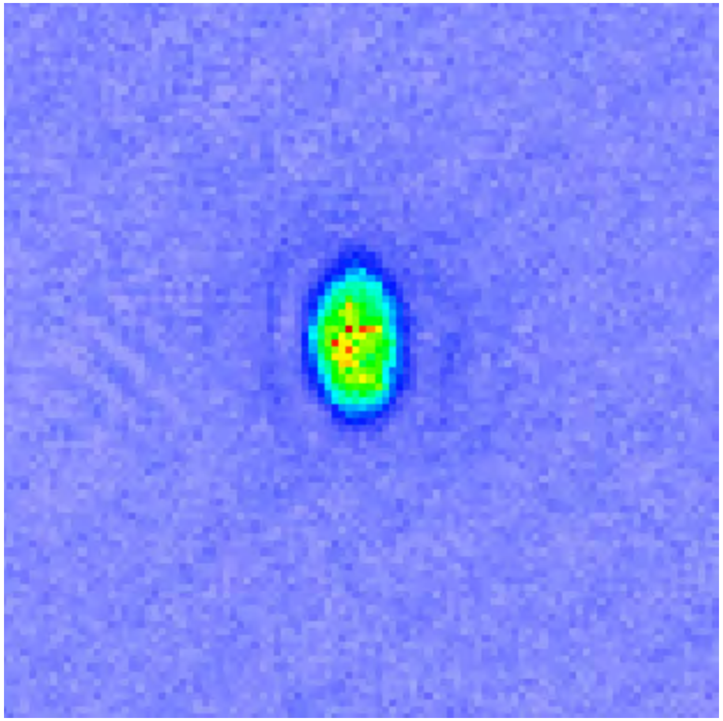
300 nK

Condensation



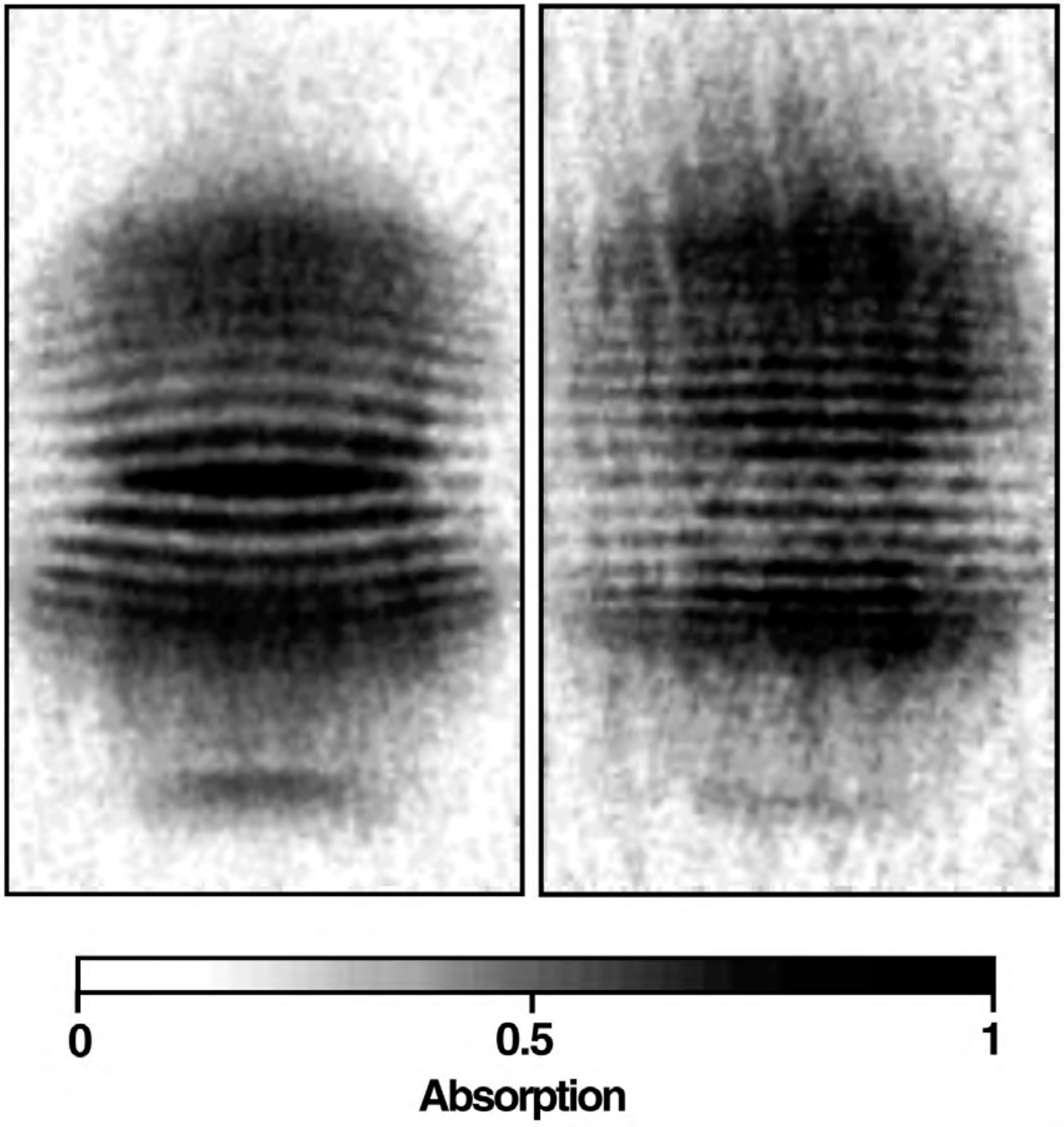
200 nK

Pure Condensate



100 nK

Characteristics of BEC?



(MIT 1997)

Off-diagonal long range order

Many-body correlations can get complicated.

Single-particle density matrix however captures most relevant physics:

$$\rho(\mathbf{r},\mathbf{r}') = \langle \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r}')\rangle$$

$\hat{\Psi}(\mathbf{r})$ annihilates particle at r

with

$$\rho(\mathbf{r},\mathbf{r}') = N\int \mathrm{d}\mathbf{r}_2\mathrm{d}\mathbf{r}_3\ldots\mathrm{d}\mathbf{r}_N \Psi^*(\mathbf{r},\mathbf{r}_2,\mathbf{r}_3,\ldots\mathbf{r}_N)\Psi(\mathbf{r}',\mathbf{r}_2,\mathbf{r}_3,\ldots\mathbf{r}_N)$$

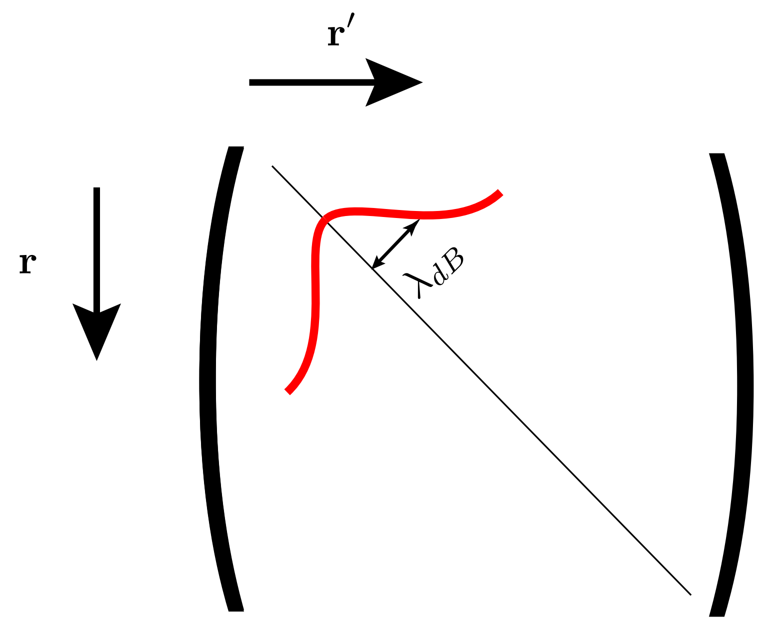
e.g. density of gas at position r:

$$n(\mathbf{r}) = \rho(\mathbf{r},\mathbf{r}) = \langle \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r})\rangle$$

also captures momentum distribution
(that's where BEC takes place!):

$$n(\mathbf{p}) = \langle \hat{\Psi}^\dagger(\mathbf{p})\hat{\Psi}(\mathbf{p})\rangle$$

$$\hat{\Psi}(\mathbf{p}) = (2\pi\hbar)^{-3/2}\int \mathrm{d}\mathbf{r}e^{-i\mathbf{p}\mathbf{r}/\hbar}\hat{\Psi}(\mathbf{r})$$

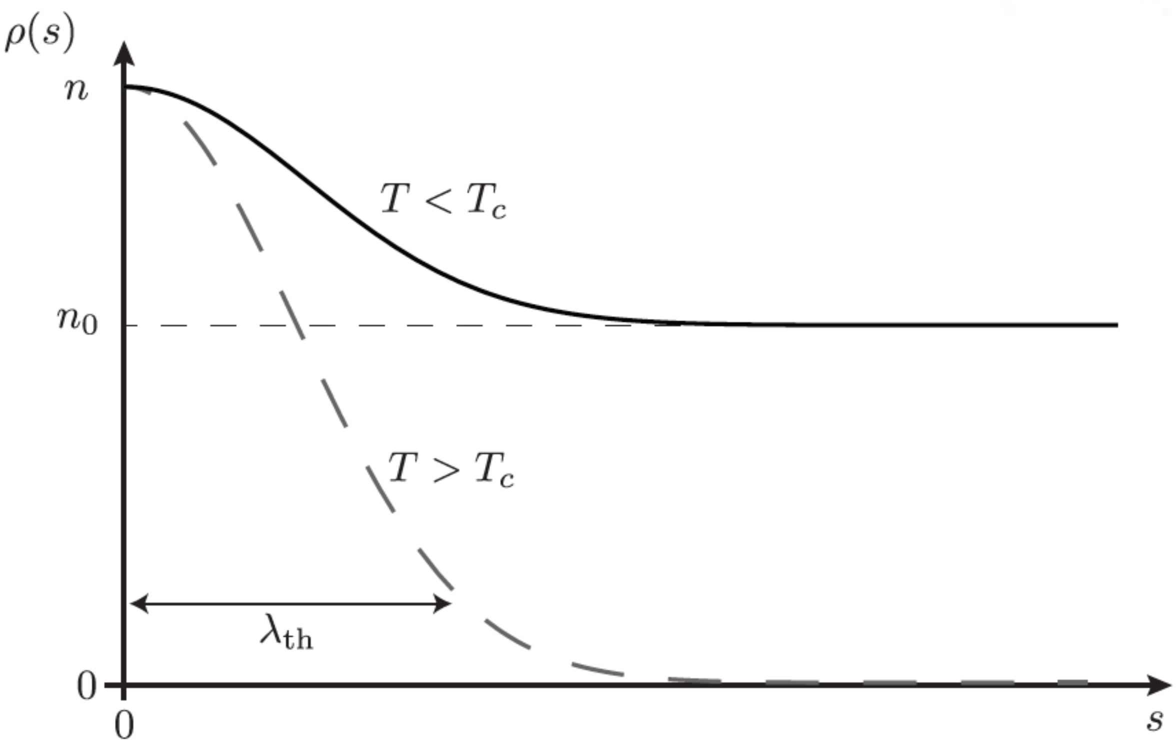


Off-diagonal long range order

Consider homogeneous gas in the thermodynamic limit: $N, V \to \infty \quad n = \frac{N}{V} = \text{const.}$

$\Rightarrow$ translational invariant system $\rho(\mathbf{r}, \mathbf{r}') = \rho(|\mathbf{r} - \mathbf{r}'|) = \rho(s)$

BEC takes place in momentum space, thus $\rho(s) = \frac{1}{V} \int \mathrm{d}\mathbf{p} e^{-i\mathbf{ps}/\hbar} n(\mathbf{p})$



$T > T_c$

$$\lim_{s \to \infty} \rho(s) = 0$$

$T < T_c$

$$n(\mathbf{p}) = N_0 \delta(\mathbf{p}) + \tilde{n}(\mathbf{p})$$

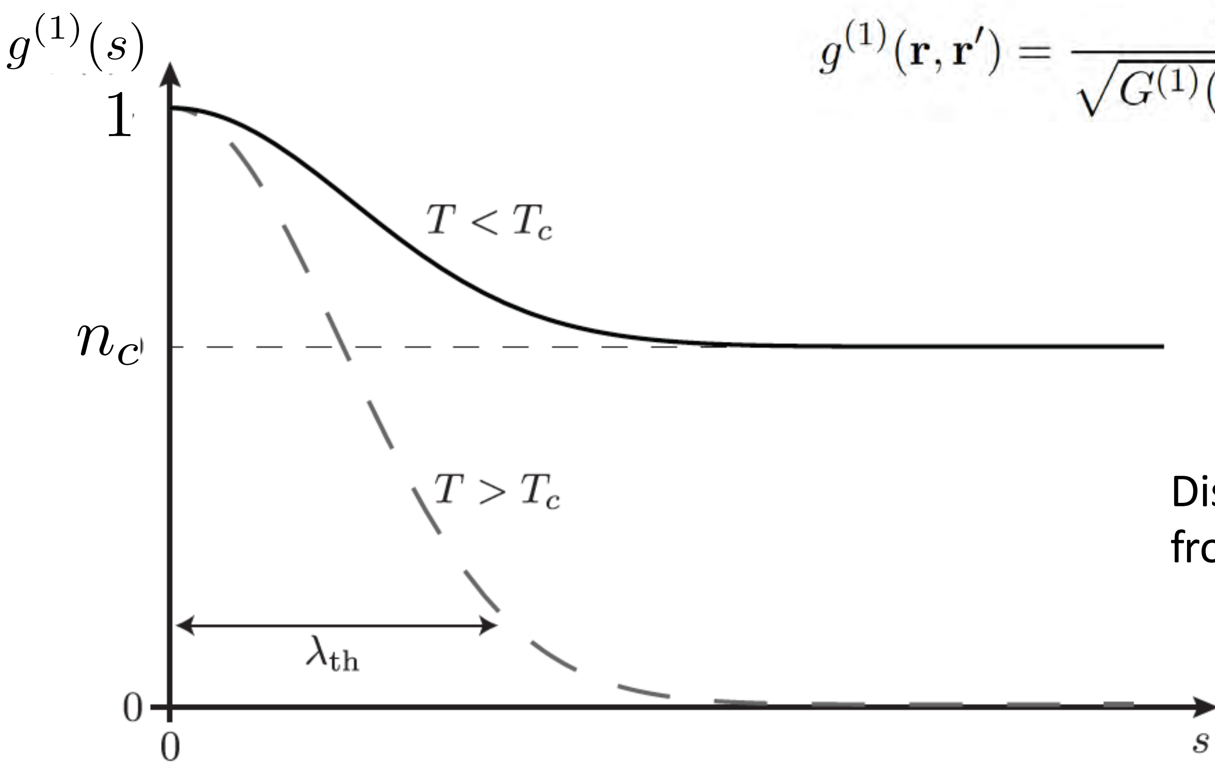
$$\lim_{s \to \infty} \rho(s) = n_0 = \frac{N_0}{V}$$

Correlation function

Penrose + Onsager criterion for BEC: $$\lim_{s\to\infty} \rho(s) = n_0 = \frac{N_0}{V}$$

First order correlation function: $$G^{(1)}(\mathbf{r}, \mathbf{r}') = \rho(\mathbf{r}, \mathbf{r}')$$

$$g^{(1)}(\mathbf{r}, \mathbf{r}') = \frac{G^{(1)}(\mathbf{r}, \mathbf{r}')}{\sqrt{G^{(1)}(\mathbf{r}, \mathbf{r})}\sqrt{G^{(1)}(\mathbf{r}', \mathbf{r}')}}$$



Distinguish coherence length from correlation length!

Field operator

Field operator: $\hat{\Psi}(\mathbf{r}) = \sum_i \phi_i(\mathbf{r})\hat{a}_i = \phi_0(\mathbf{r})\hat{a}_0 + \sum_{i\neq 0} \phi_i(\mathbf{r})\hat{a}_i$

BEC excitations

Bogoliubov approximation: $\hat{a}_0, \hat{a}_0^\dagger \to \sqrt{N_0}$

$$\Rightarrow \hat{\Psi}(\mathbf{r}) = \psi(\mathbf{r}) + \delta\hat{\Psi}(\mathbf{r})$$

with

$$\psi(\mathbf{r}) = \sqrt{N_0}\phi_0(\mathbf{r})$$

$$\delta\hat{\Psi}(\mathbf{r}) = \sum_{i\neq 0} \phi_i(\mathbf{r})\hat{a}_i$$

BEC wave function: $\psi(\mathbf{r}) = |\psi(\mathbf{r})|e^{i\varphi(\mathbf{r})}$

Phase ("off-diagonal elements")

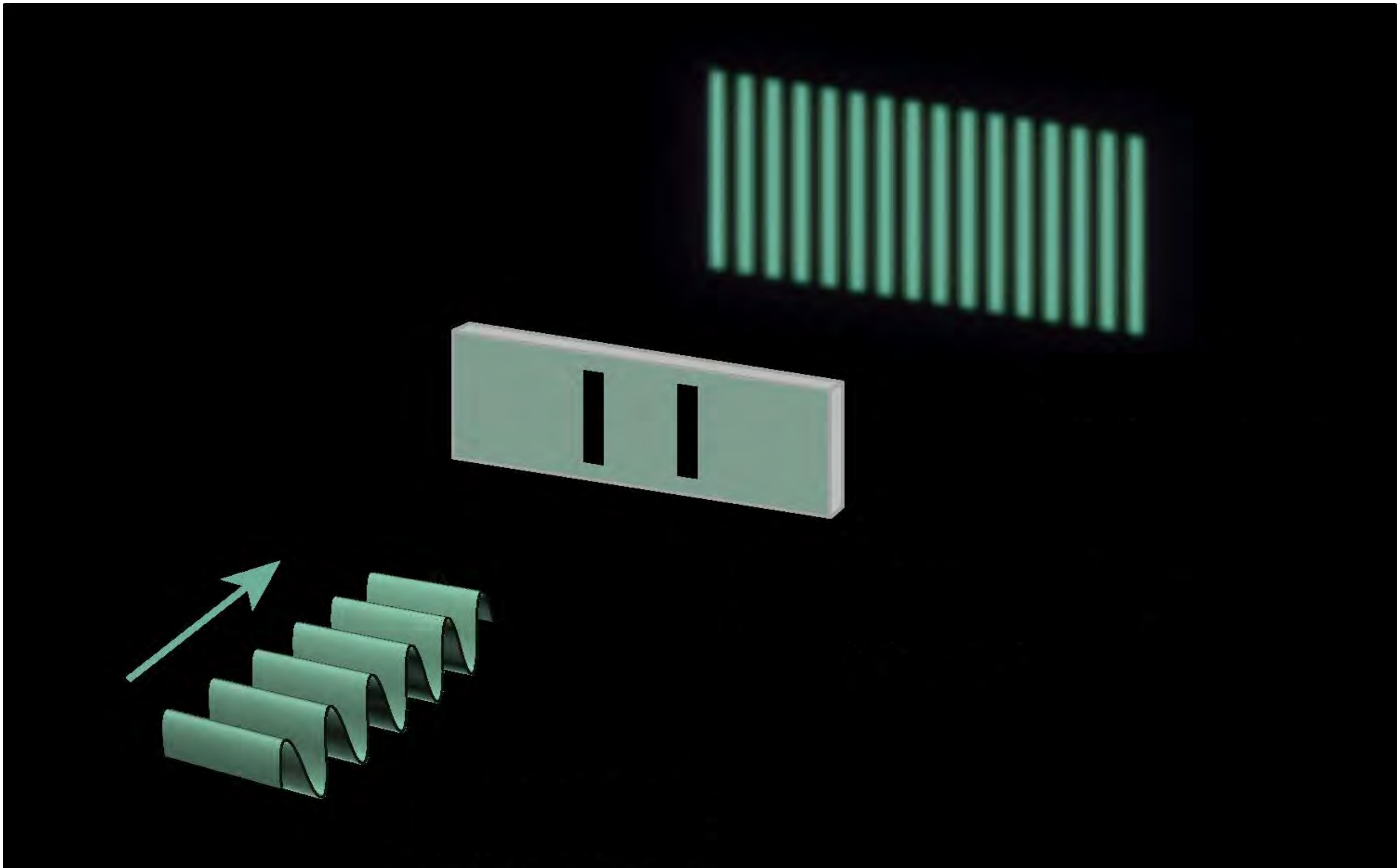
density ("diagonal density")

Correlation function:

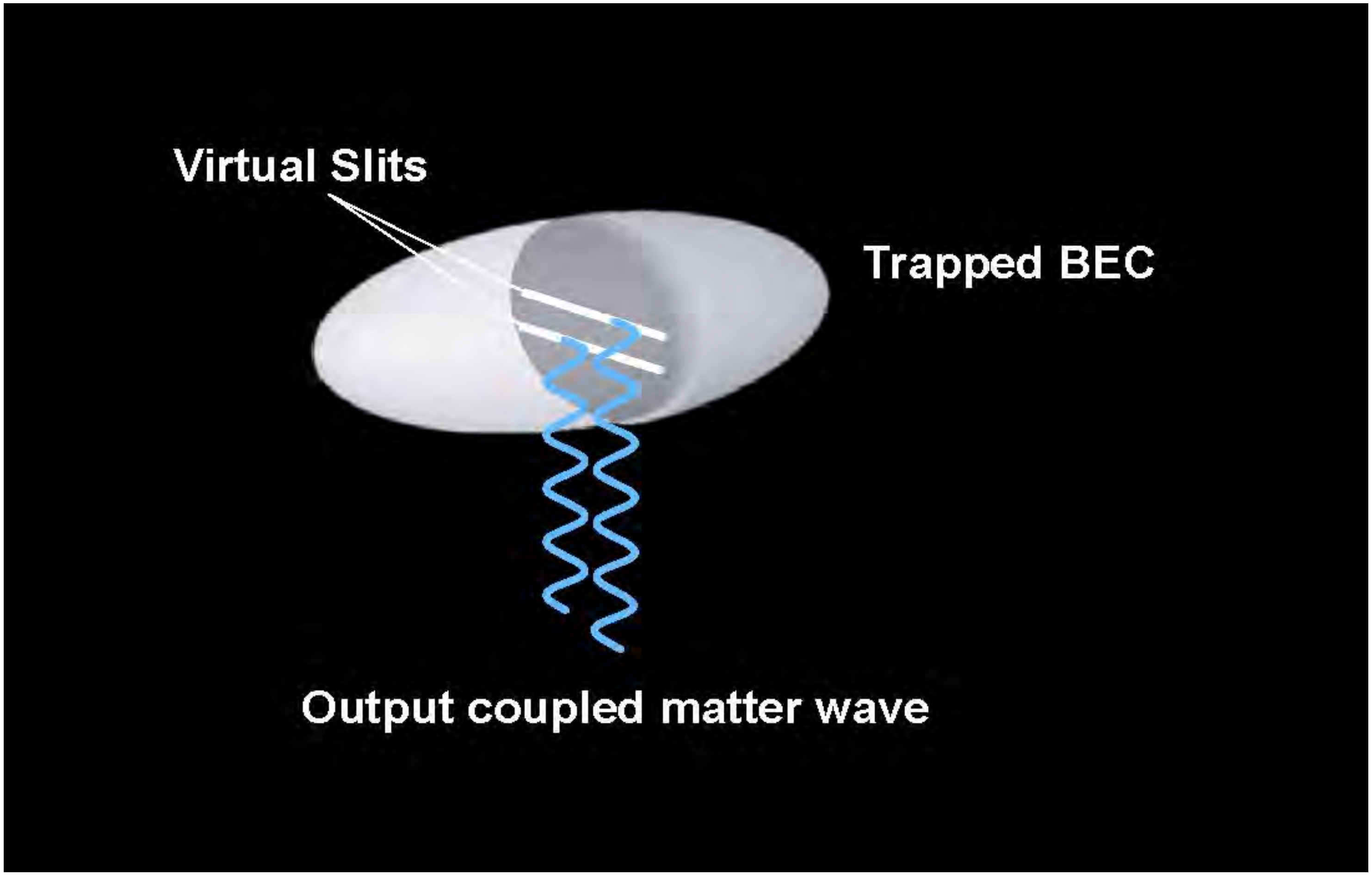
$$\rho(\mathbf{r},\mathbf{r}') = \langle\hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r}')\rangle = \psi^*(\mathbf{r})\psi(\mathbf{r}') + \langle\delta\hat{\Psi}^\dagger(\mathbf{r})\delta\hat{\Psi}(\mathbf{r}')\rangle$$

condensate fluctuations

Optics: Youngs Double Slit Experiment

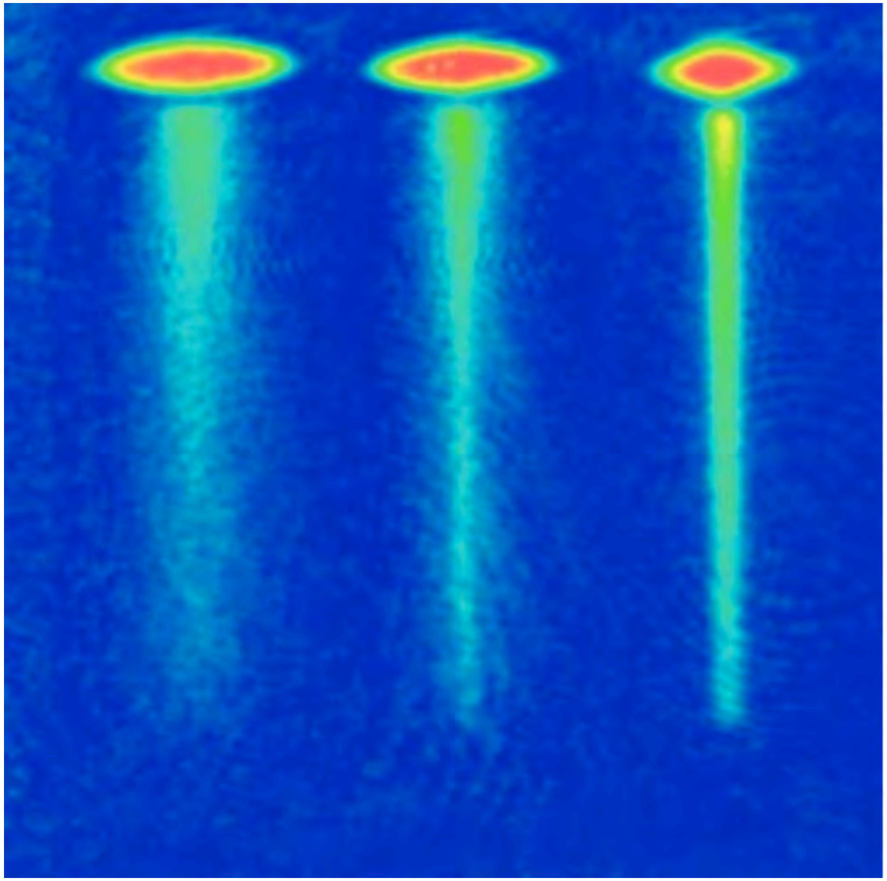


Measuring the Coherence of a BEC



Atom Laser with a cw Output Coupler

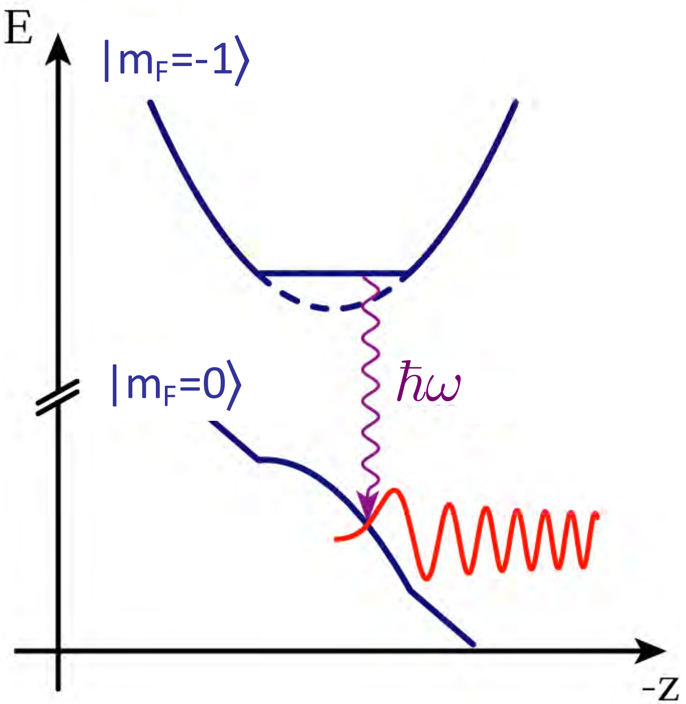
Immanuel Bloch, Theodor W. Hänsch, and Tilman Esslinger
Sektion Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4/III, D-80799 Munich, Germany
and Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany
(Received 3 December 1998)



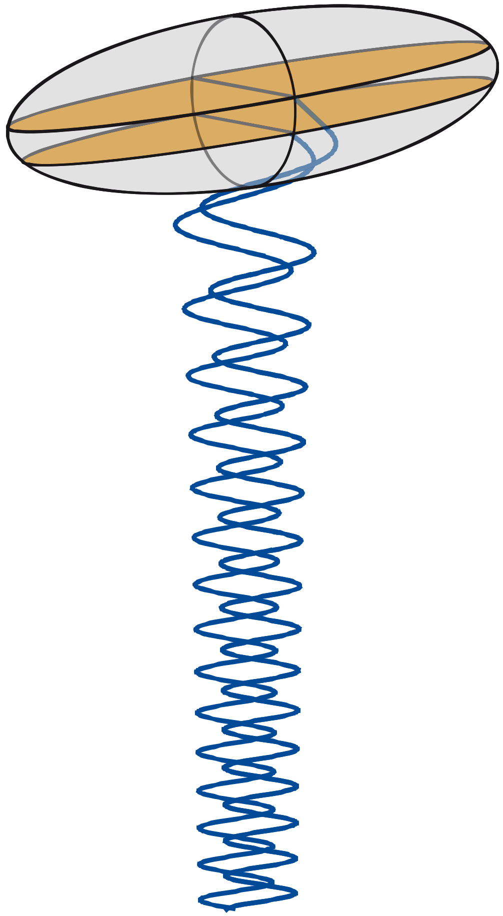
Radio-frequency output coupling

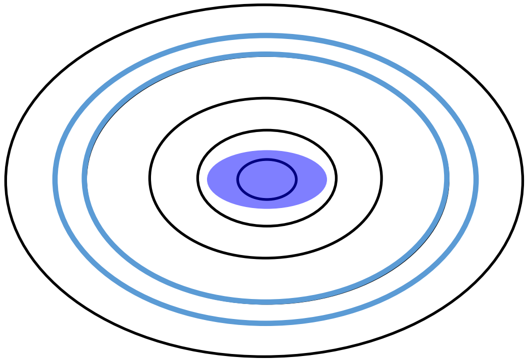
^{87}Rb condensate

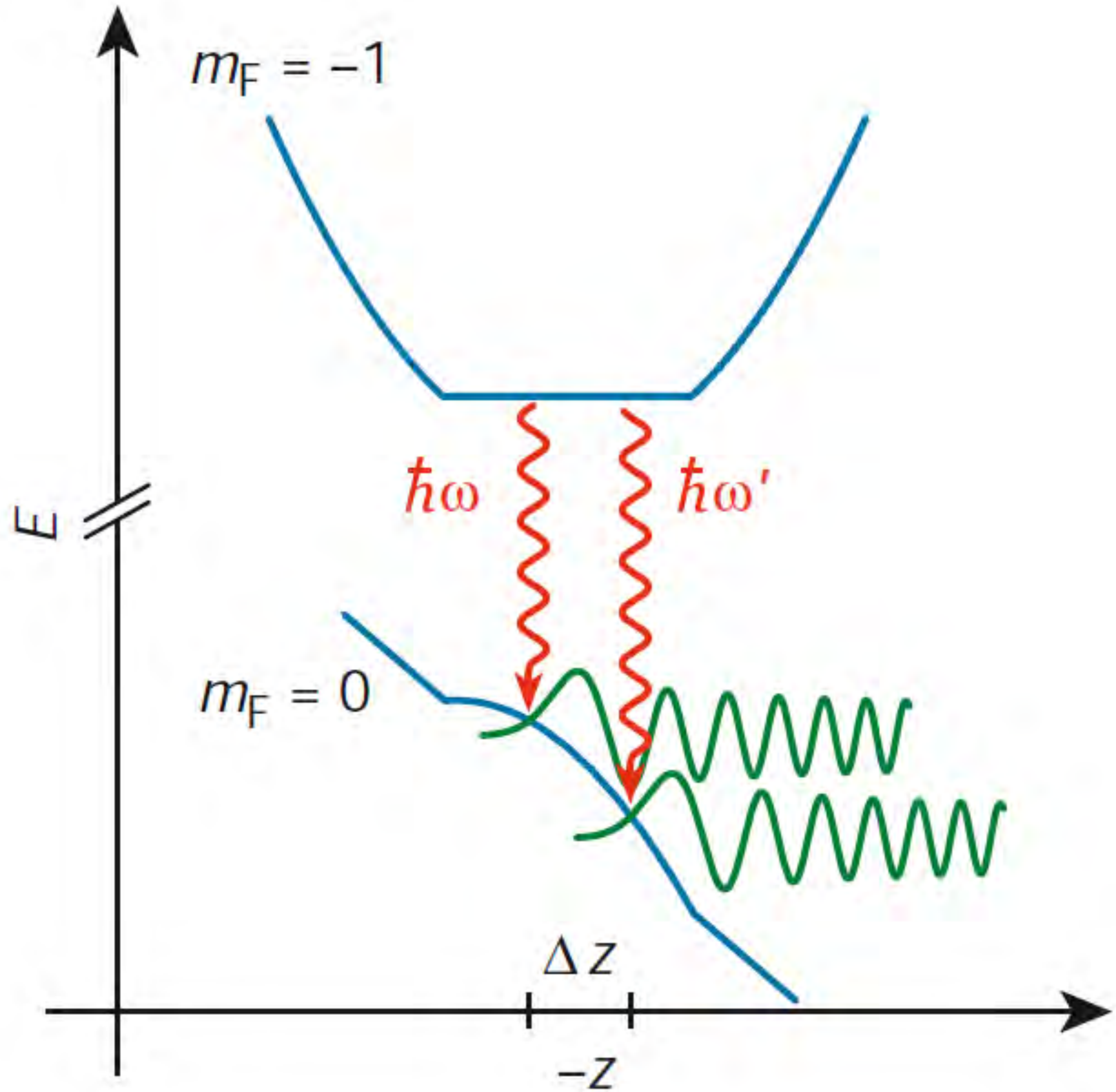
2×10^6 atoms



Radio-frequency output coupling

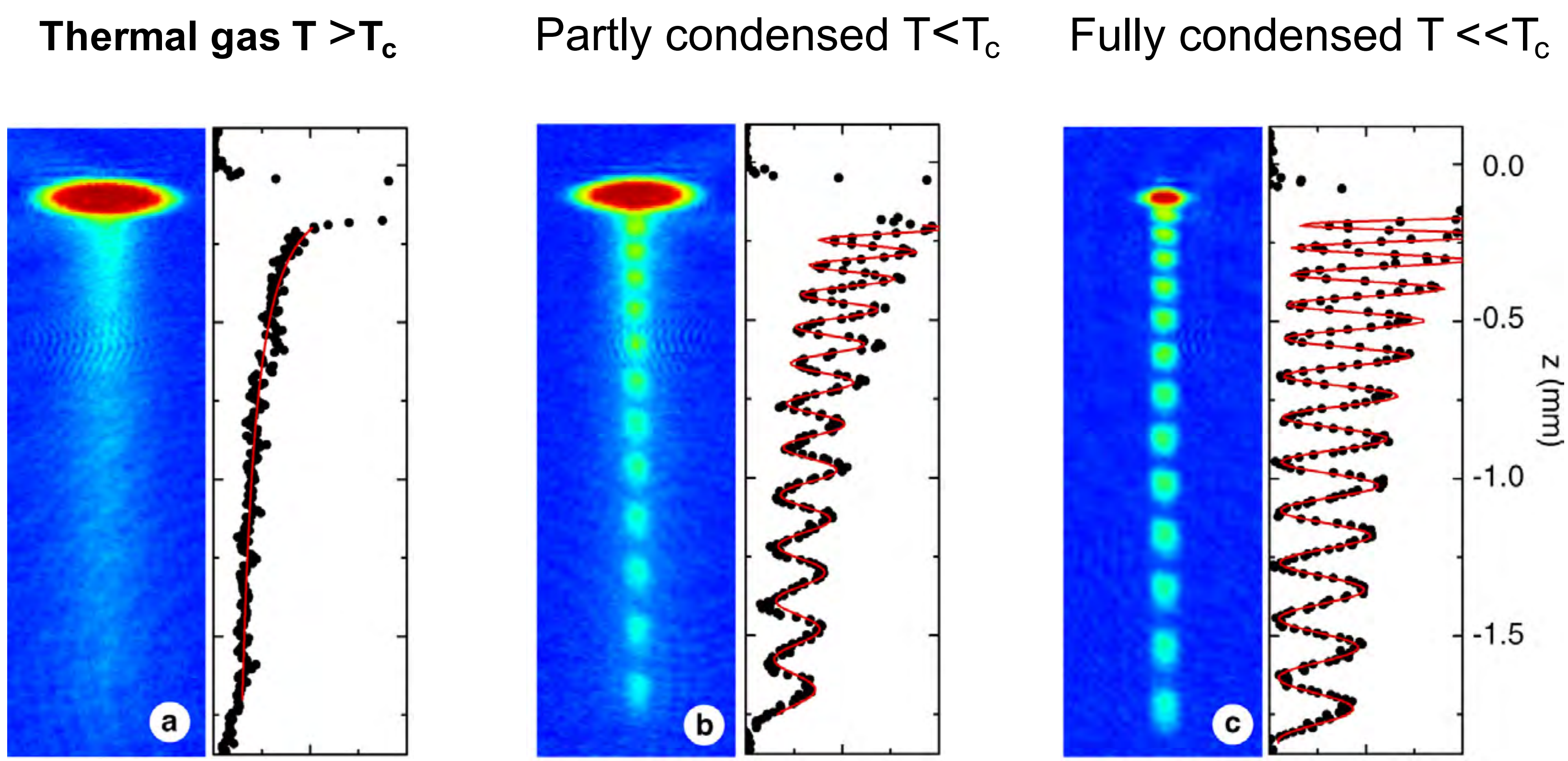




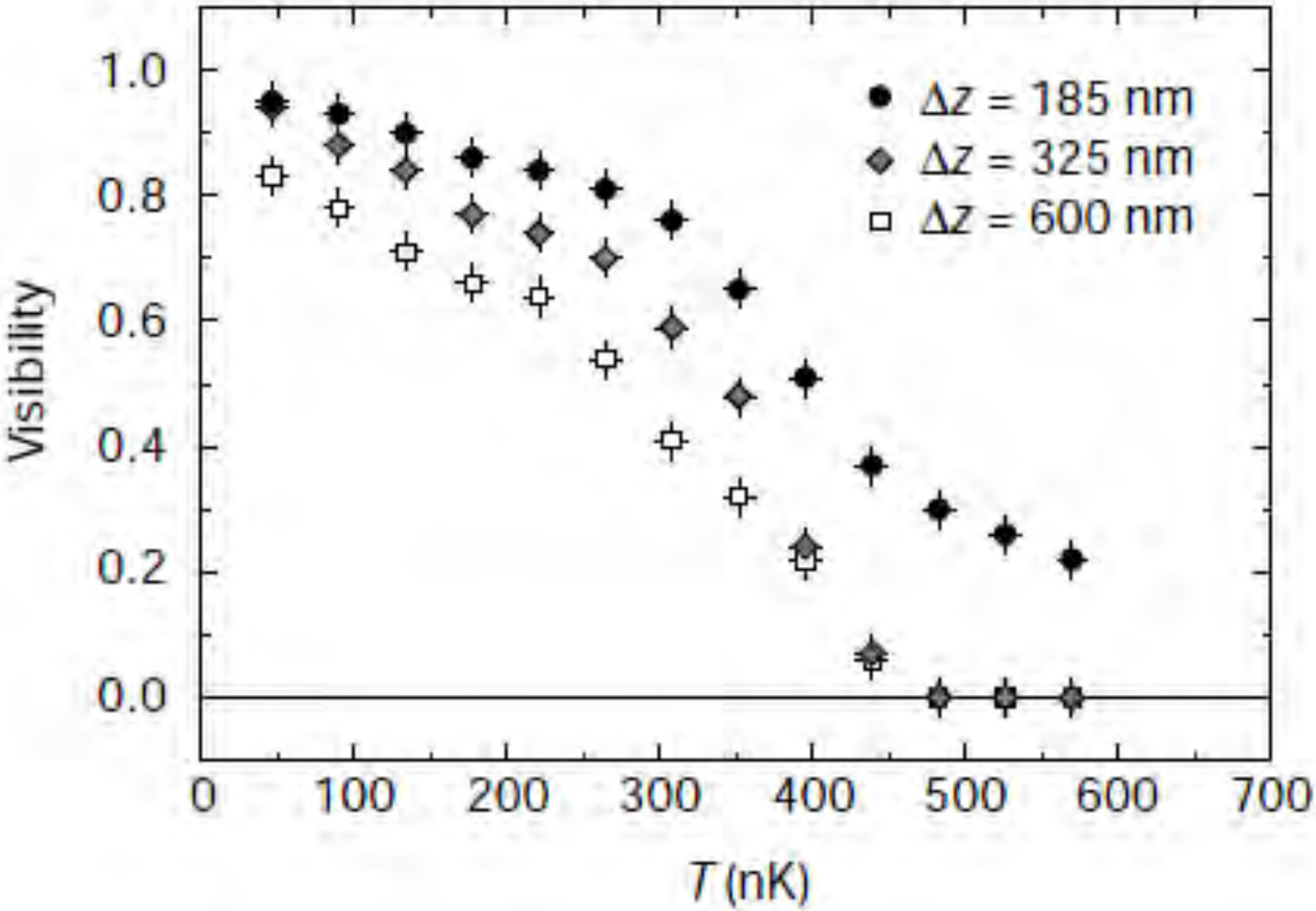


$m_F = -1$
$\hbar\omega$
$\hbar\omega'$
E
$m_F = 0$
Δz
$-z$

Probing First Order Coherence (Experiment)

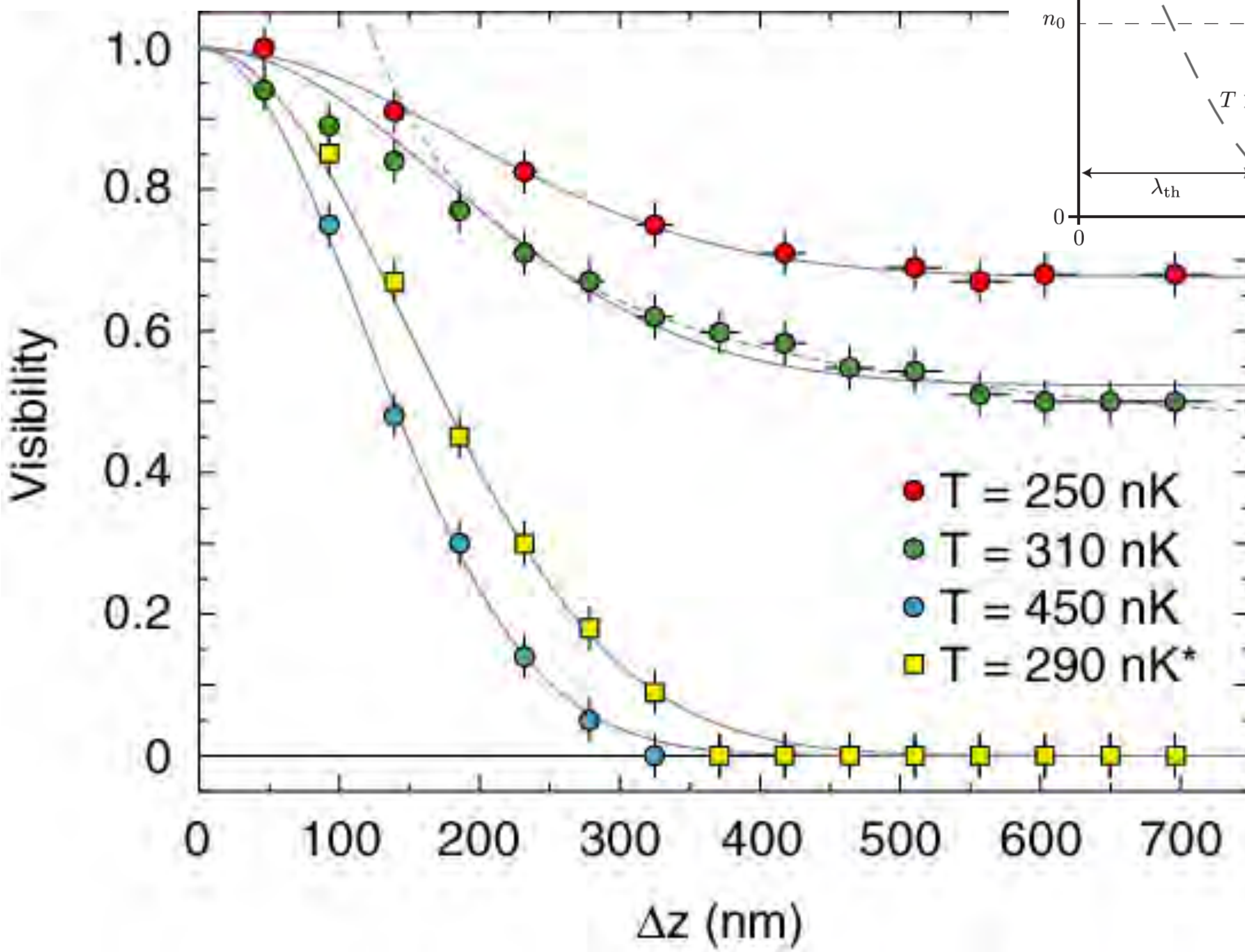


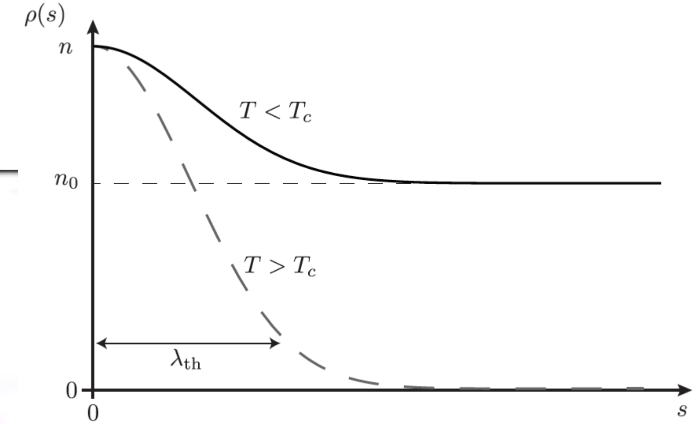
$\longrightarrow$ **V(Δz,T)**



Δz = 185 nm
Δz = 325 nm
Δz = 600 nm
Visibility
1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500
600
700
T (nK)

Spatial Correlation Function of a Trapped Bose Gas



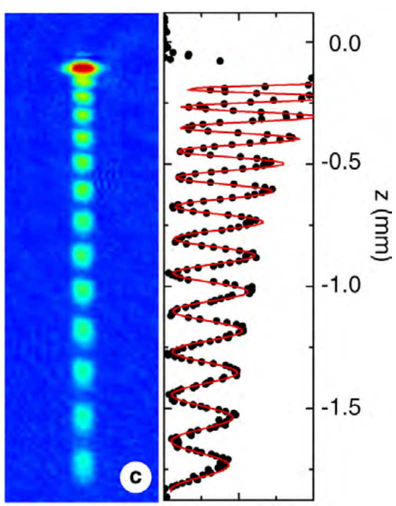


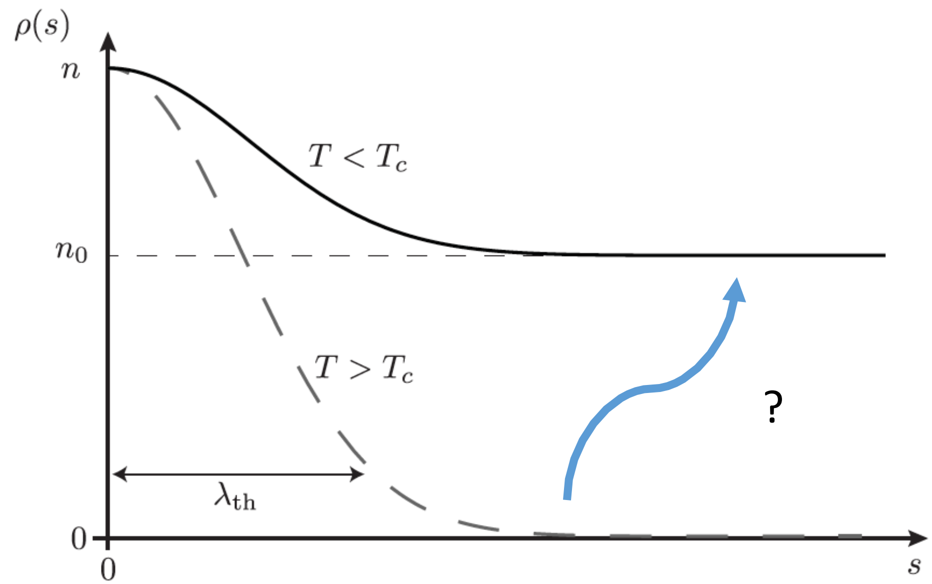
Constant correlation function indicates the presence of long-range phase coherence !

I. Bloch, T. W. Hänsch, and T. Esslinger, Nature 403, 166-170 (2000)

But what happens right at the critical point of the phase transition?

Change T and measure?





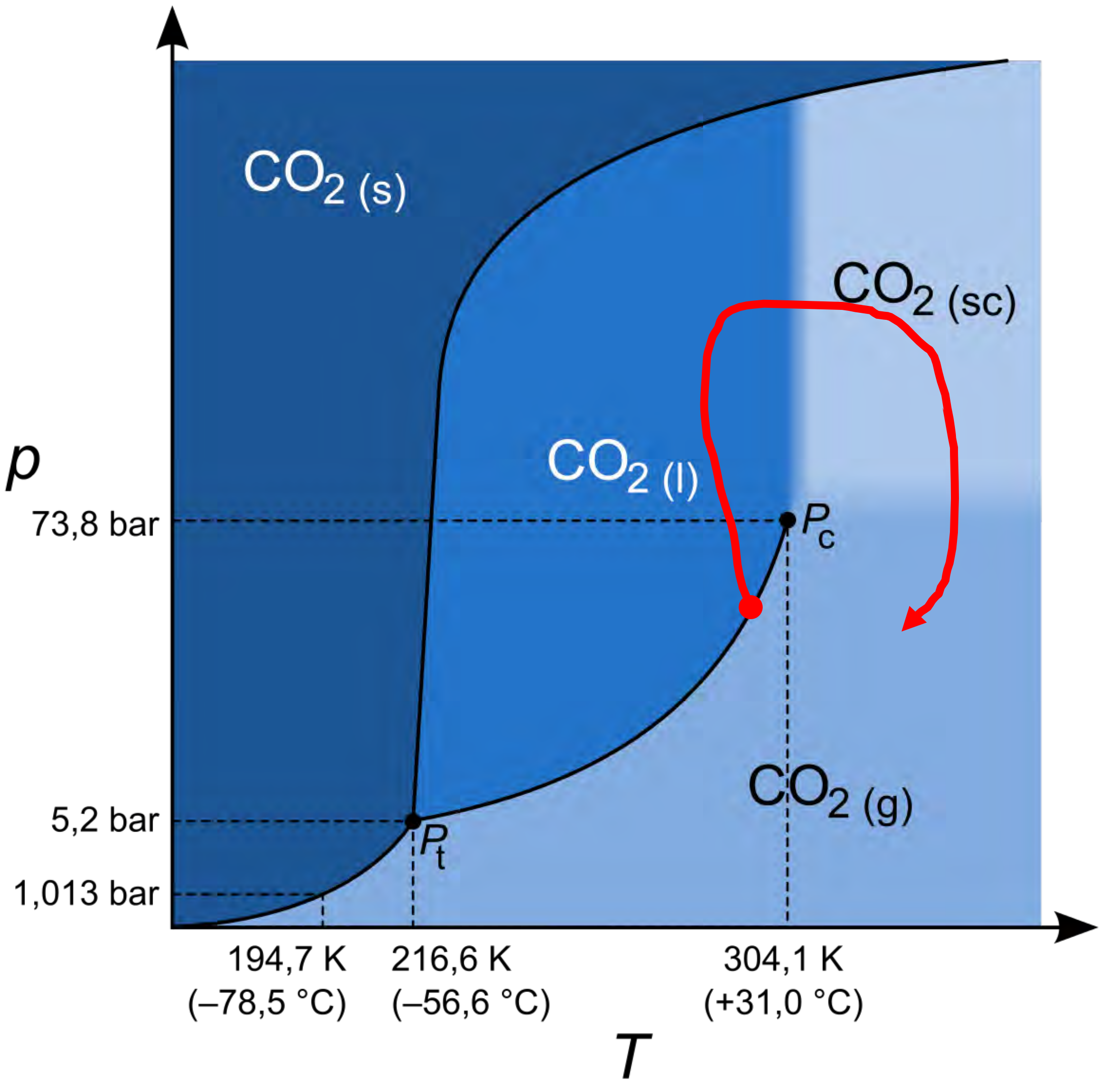
- ☹ Influence of output coupling 30% of cloud
- ☹ Signal-to-noise of visibility

Critical behavior at a phase transition

History of phase transitions

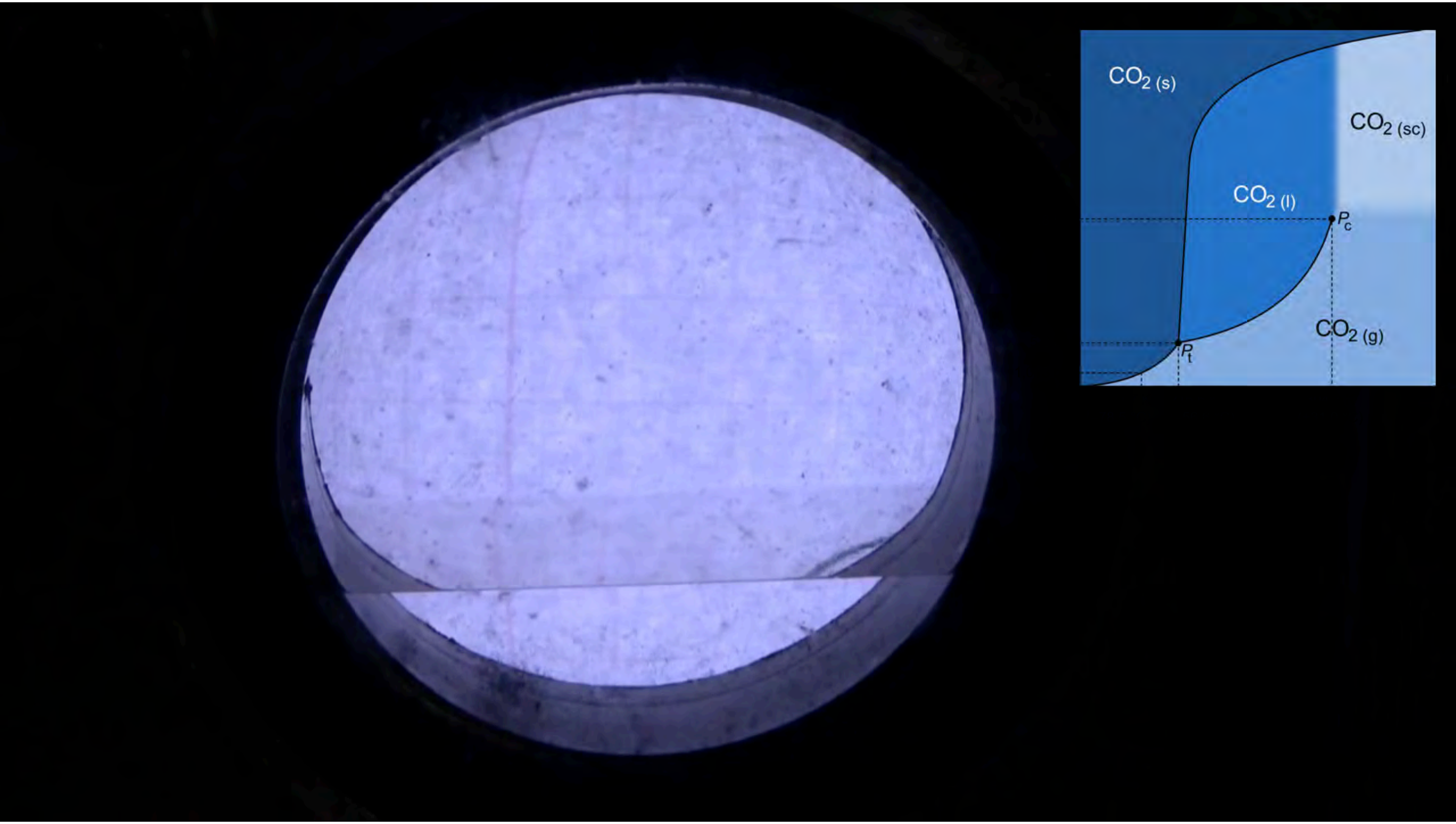
- 1869: Andrews observes critical opalescence in CO2

Phase diagram of carbon dioxide

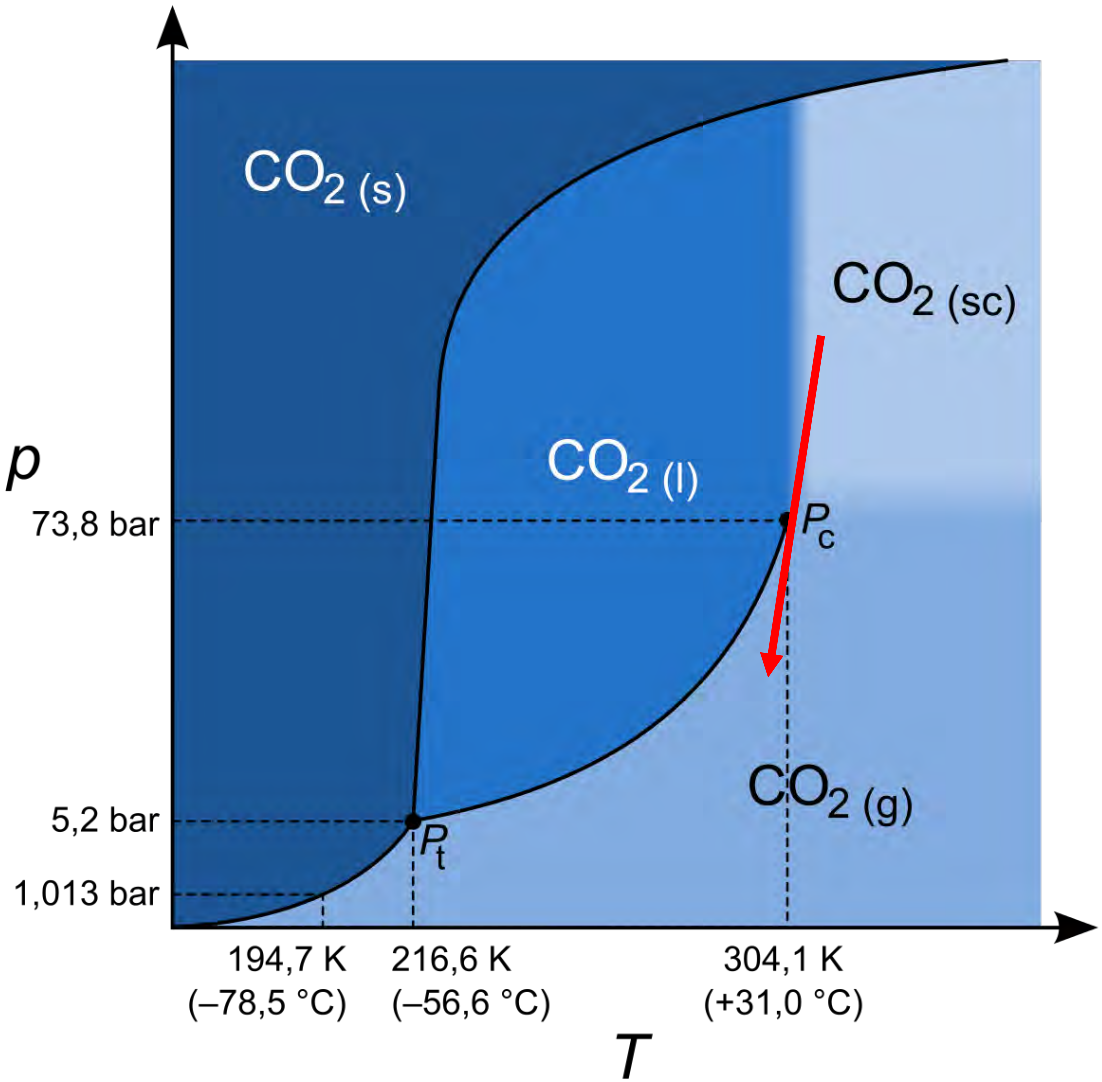


wikipedia

Critical point dryer (MEMS, spices, vaccines, SEM samples, decaffination)

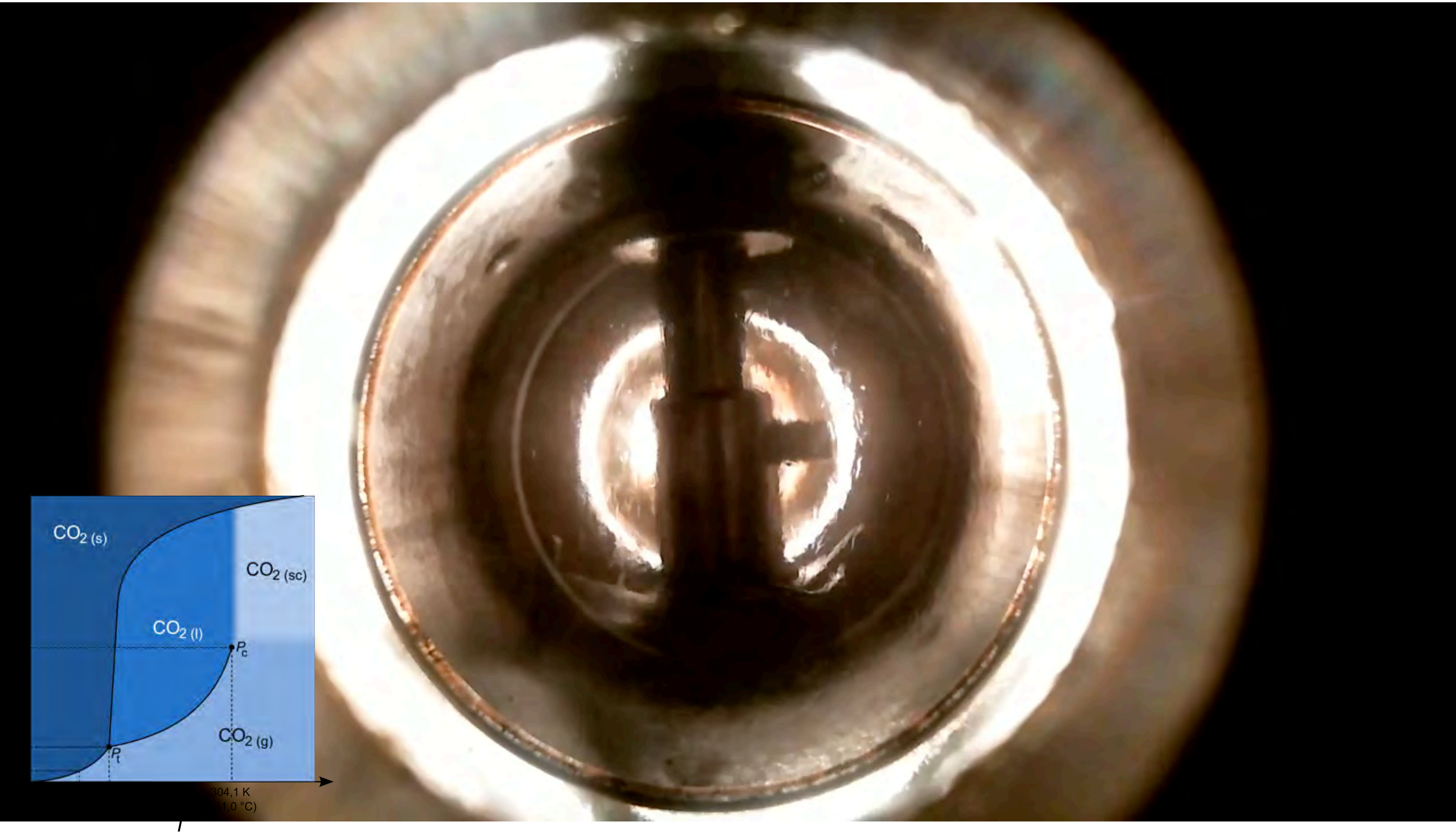


Phase diagram of carbon dioxide



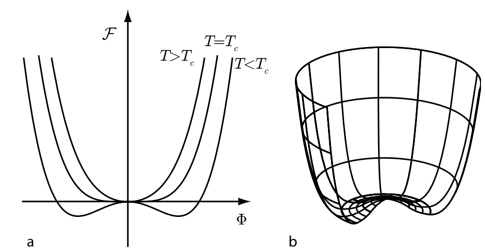
wikipedia

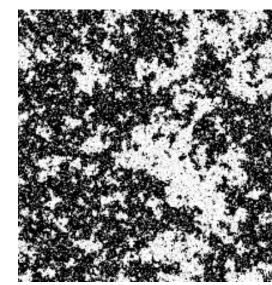
Passing the critical point (Andrews 1869)



History of phase transitions

- 1869: Andrews observes critical opalescence in CO_2
- 1873: van der Waals formulates microscopic theory
- 1937: Landau theory
  - Global order parameter Φ
  - Expand free energy in potentials of Φ
  - -> predicts critical exponents describing divergences at 2nd order phase transitions
- 1944: Onsager solves 2D Ising model exactly
  - Critical exponent differs from Landau theory
  - Experiments also show different exponents
- 1950: Landau-Ginzburg theory
  - Extends Landau theory by space- and time-varying order parameter $\Phi(\mathbf{r}, t)$
  - Now fluctuations can be captured!
  - Close to the critical point, fluctuations dominate, i.e. Landau-theory breaks down
  - Ginzburg criterion predicts size of critical region



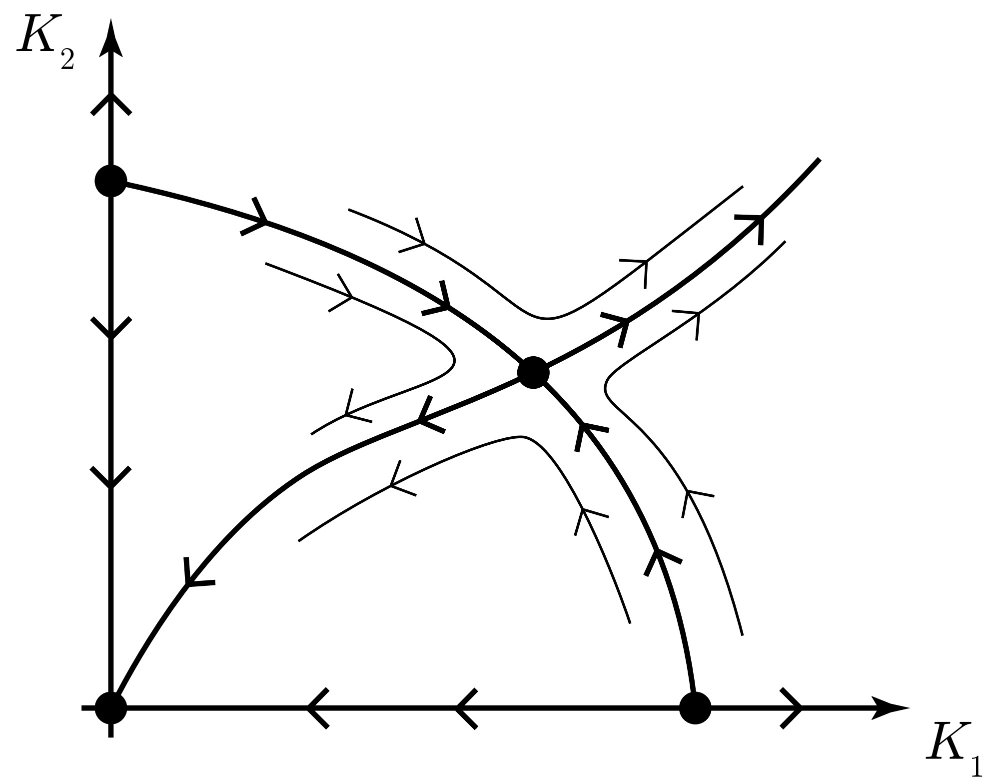


www.math.duke.edu

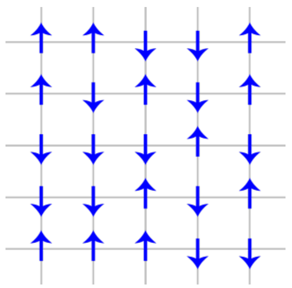
$$\Phi(\mathbf{r}, t)$$

History of phase transitions

- 1950: Still hard to calculate physics in the critical regime
- 1971: Wilson renormalization group theory
  - See lectures of Zinn-Justin
  - Theory predicts critical exponents matching to experiments



Order parameter



System undergoing a 2nd order phase transition from T>Tc to T<Tc reduces its symmetry.

Examples?

e.g. Ising magnet: direction of magnetization

e.g. BEC: phase of wave function

-> extra parameter needed to describe state of system: **order parameter**

$$\Phi(\mathbf{r}, t) = 0 \quad \text{for} \quad T > T_c$$

$$\Phi(\mathbf{r}, t) \neq 0 \quad \text{for} \quad T < T_c$$

Choice of order parameter is not always obvious, has to be done for every system afresh. The order parameter has to reflect the symmetries of the system.

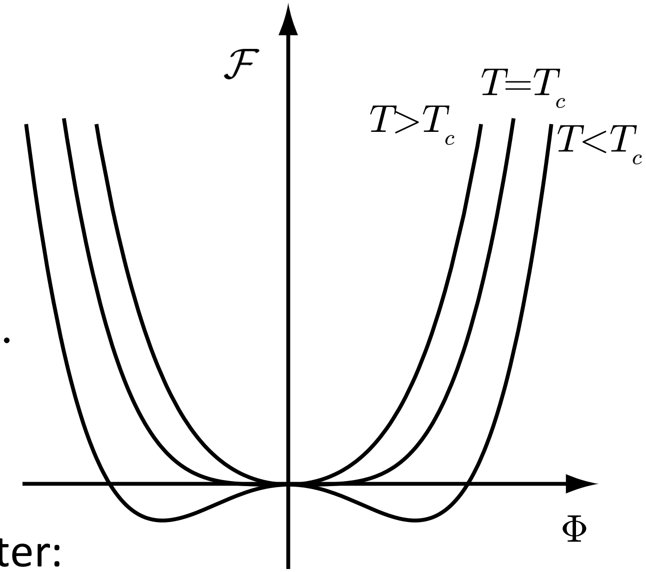
Magnetization is order parameter of Ising model

BEC wave function $\psi(\mathbf{r}) = |\psi(\mathbf{r})| e^{i\varphi(\mathbf{r})}$

is order parameter of the normal-to-superfluid phase transition:

$\psi(\mathbf{r}) = 0$ for $T > T_c$ $\qquad$ $\psi(\mathbf{r}) \neq 0$ for $T < T_c$

Landau theory



Expand free energy in powers of the order parameter:

$$\mathcal{F}(T,\Phi,h) = \mathcal{F}_0 - h\Phi + \frac{1}{2}a(T)\Phi^2 + \frac{1}{4}b(T)\Phi^4 + \ldots$$

External field h (=0 for BEC).

Find minimum in free energy with respect to order parameter:

$$\frac{\partial\mathcal{F}(T,\Phi)}{\partial\Phi} = 0\,,\ \frac{\partial^2\mathcal{F}(T,\Phi)}{\partial\Phi^2} > 0\,.$$

One global minimum for $a(T),\ b(T) > 0$ $\qquad$ $\Phi_0 = 0$

Two minima for $a(T) < 0,\ b(T) > 0$ $\qquad$ $\Phi_0^2 = -a(T)/b(T)\,.$

Parameterize

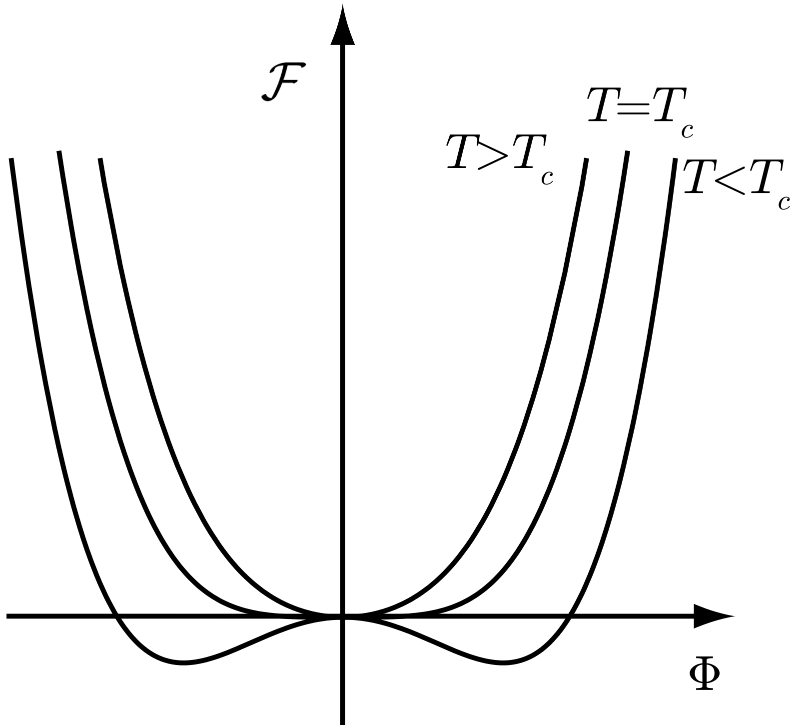
$a(T) = \alpha_0 t \qquad b = const.$

$$\Phi_0 = \pm\left(-\frac{\alpha_0 t}{b}\right)^{1/2}, \quad \text{for}\ t < 0$$

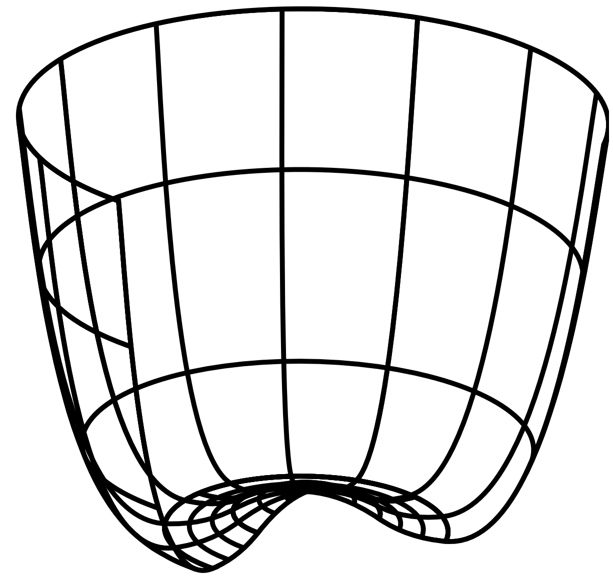
Critical exponent of order parameter:

$$\beta = 1/2$$

Landau theory: Mexican hat



1D order parameter



2D order parameter

Landau-Ginzburg theory

Take fluctuations into account: $\Phi(\mathbf{r})$

$$\mathcal{F}_{\mathrm{LG}}(\Phi(\mathbf{r})) = \int \mathcal{F}_0 - h\Phi(\mathbf{r}) + \frac{1}{2}\alpha_0 t\Phi(\mathbf{r})^2 + \frac{1}{4}b(T)\Phi(\mathbf{r})^4 + c\left(\nabla\Phi(\mathbf{r})\right)^2 \mathrm{d}\mathbf{r}$$

" Phi-4 model" makes pretty accurate predictions. Increasingly higher orders have to be included when approaching Tc

Prediction for decay of correlation function $g^{(1)}(r) \sim \frac{e^{-r/\xi}}{r^p}$

Ginzburg criterion for size of critical region:

Compare magnitude of fluctuations of order parameter with mean value of order parameter.

$$\langle(\delta\Phi(\mathbf{r}))^2\rangle \ll \langle\Phi(\mathbf{r})^2\rangle$$

For an interacting BEC: $t < 0.08$ or $\xi > \frac{\lambda_{dB}^2}{\sqrt{128}\pi^2 a} \simeq 0.4\mu\mathrm{m}$

(from microscopic arguments)

Critical exponents

At critical points, some properties can diverge:

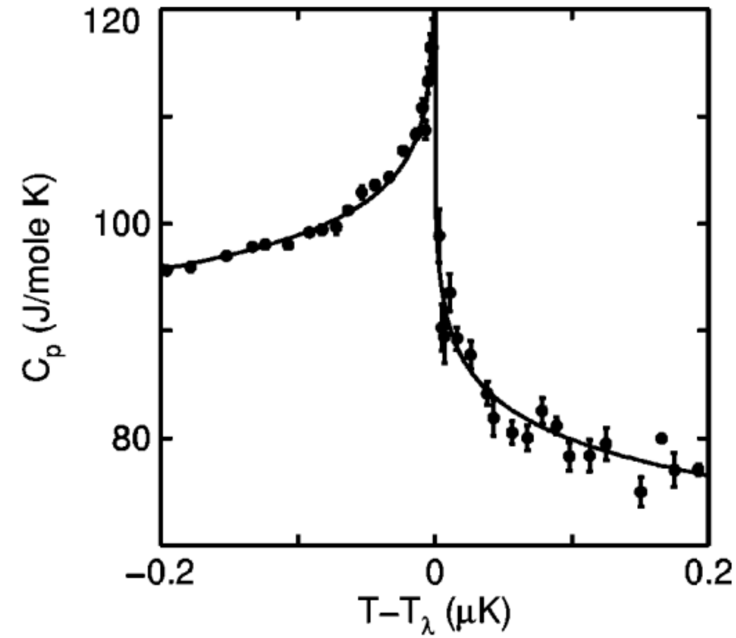
$$\sim |\frac{T-T_c}{T_c}|^{-c} = |t|^{-c}$$

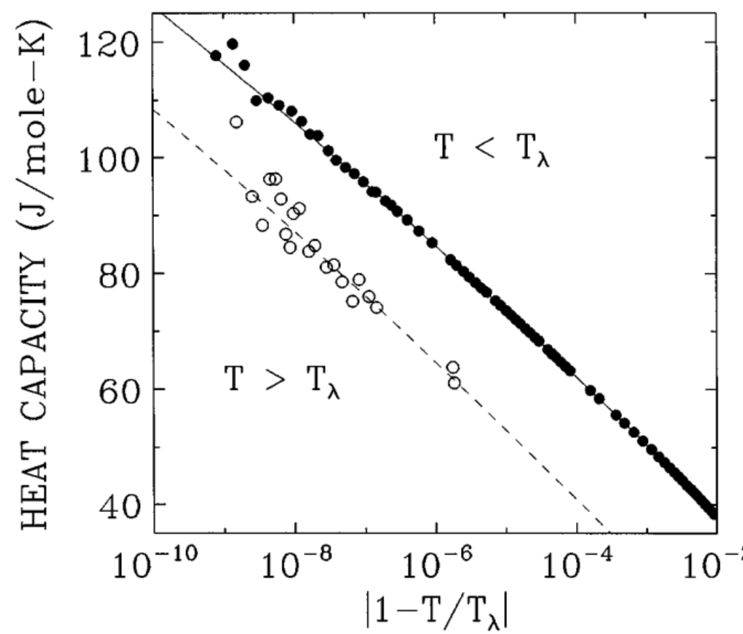
- Correlation length of fluctuations (critical opalescence) $\xi \sim |t|^{-\nu}$

  LG: $\nu = \frac{1}{2}$

- Heat capacity of liquid helium diverges at the critical point $C \sim |t|^{-\alpha}$

Most precise measurements of heat capacity exponent done in a space mission (Lipa et al. PRL 76, 944 (1996))

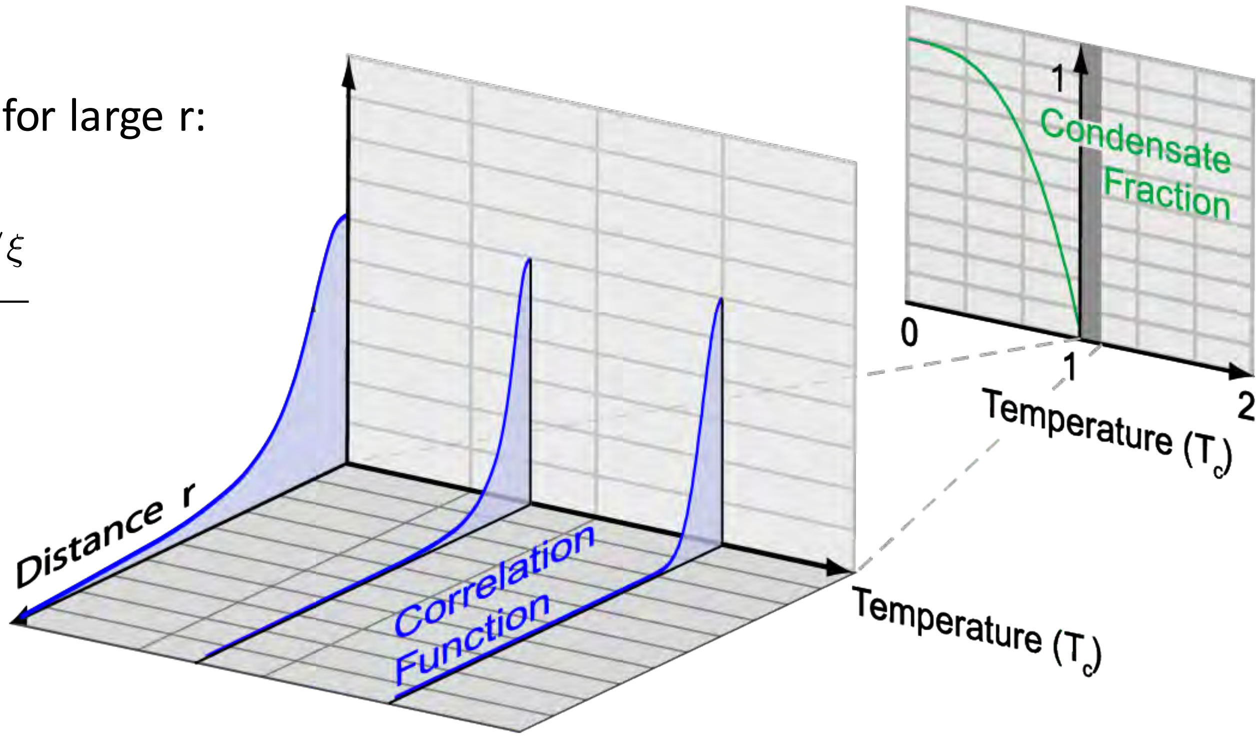




Divergence of correlation length

correlation function for large r:

$$g^{(1)}(r) \sim \frac{e^{-r/\xi}}{r^p}$$



For $T \to T_c$

$$\xi \to \infty$$

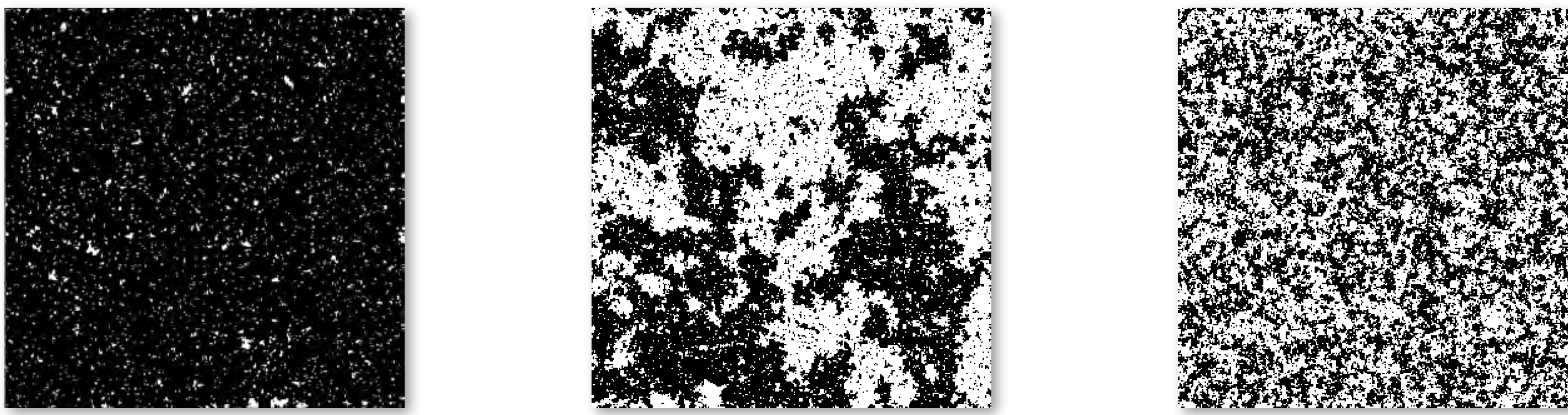
$$g^{(1)}(r) \to \frac{1}{r^p}$$

Scaling hypothesis (Griffiths):

At the critical point, the only relevant length scale diverges. Thus the system is invariant under scale transformations!

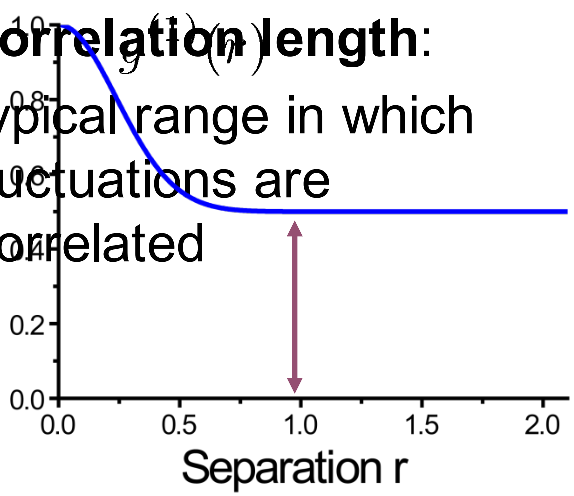
Diverging Correlation Length

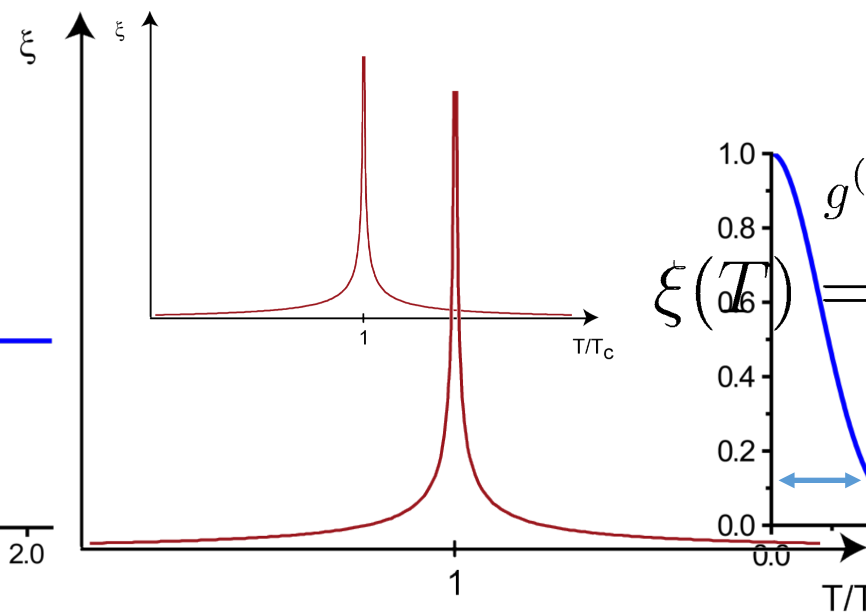
$T<T_c$ **$T=T_c$** **$T>T_c$**

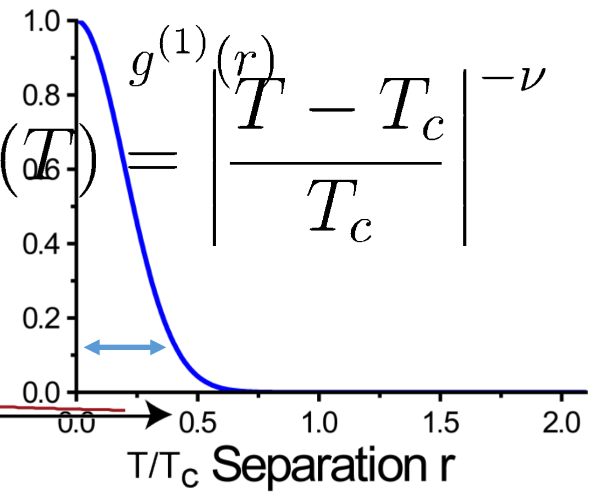


correlation length:
typical range in which fluctuations are correlated

$$\xi(T) = \left|\frac{T-T_c}{T_c}\right|^{-\nu}$$







Scaling hypothesis

Scale invariance also means self-similarity.

$$g^{(1)}(r) \sim \frac{e^{-r/\xi}}{r^p}$$

Far away from Tc:

$$g^{(1)}(r) \sim e^{-r/\xi} \qquad g^{(1)}(r/a) \sim e^{-r/(a\xi)}$$

In the critical region:

$$g^{(1)}(r) \sim \frac{1}{r^p} \qquad g^{(1)}(r/a) \sim a^p \frac{1}{r^p} = a^p g^{(1)}(r)$$

Critical exponents are not independent, but are related via **scaling relations**:

$$\begin{aligned} 2 - \alpha &= \nu d \\ \alpha + 2\beta + \gamma &= 2 \\ \gamma &= \nu(2 - \eta) \\ \gamma &= \beta(\delta - 1) \end{aligned}$$

Universality classes

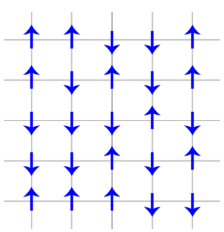
At the critical point, the correlation length diverges:

- the only length scale in the system is correlation length
- microscopic properties of the system become irrelevant

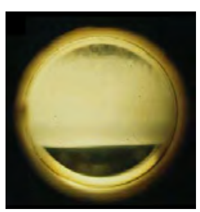
Systems can be sorted into universality classes given by symmetry and dimensionality!

- Each universality class is described by the same set of critical exponents.

Ising-model:

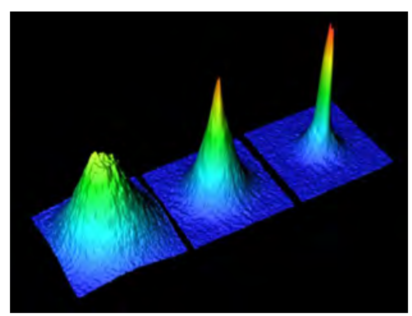


uniaxial magnet



critical opalescence

XY-model



BEC

superconductor

superfluid He

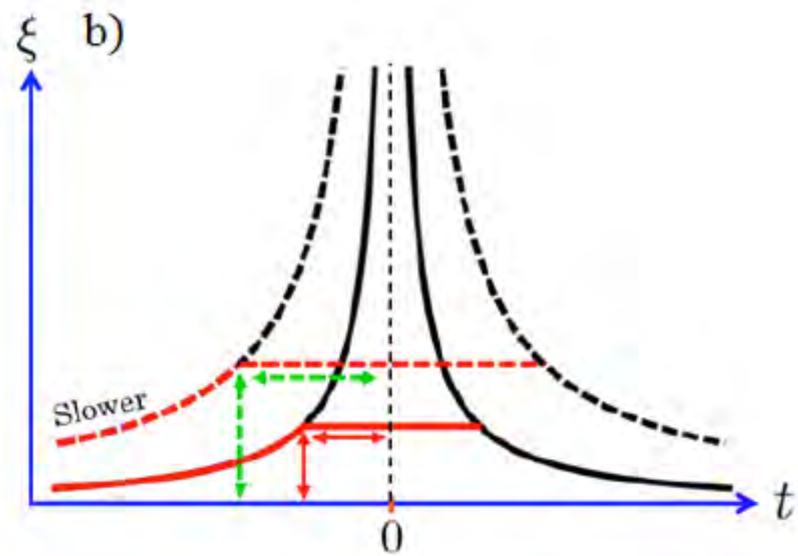
Critical slow down and finite size

Finite size:

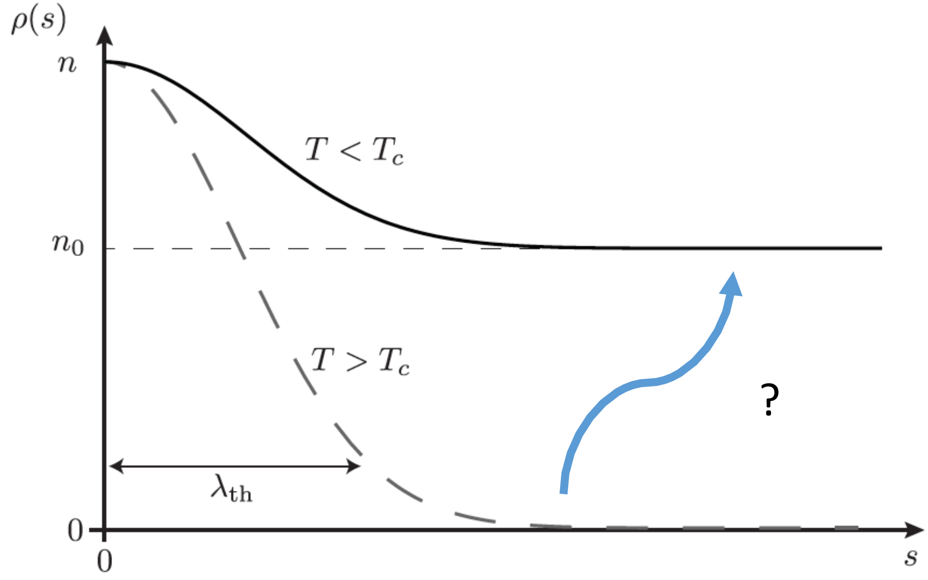
- Thermodynamic limit is out of experimental reach
- Finite size will round off divergences (think of correlation length)
- Unclear if trapped system belongs to different universality class

Critical slow down:

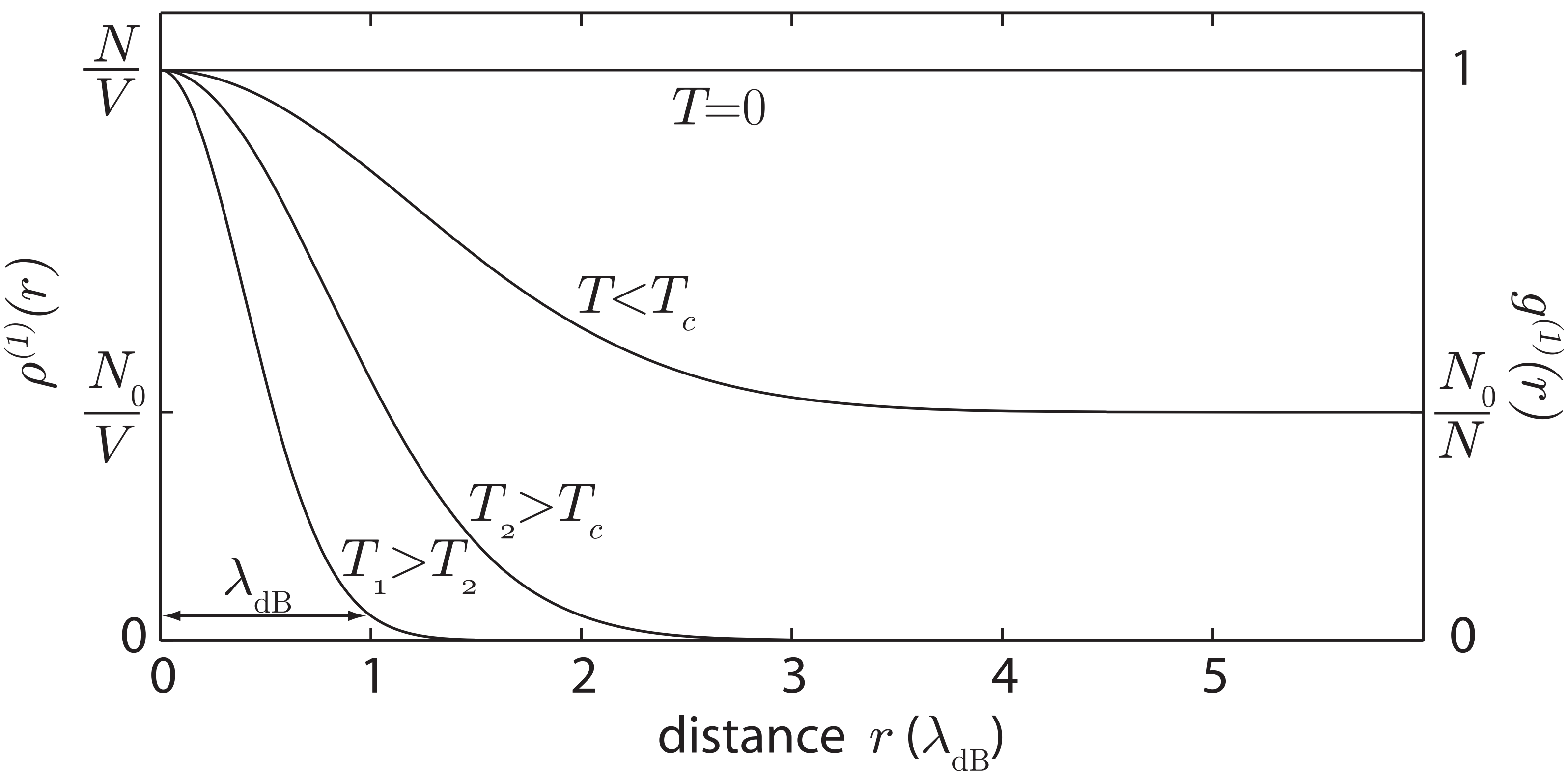
- Correlations over infinite distances also need infinite time! Also the relaxation time diverges: **"critical slow down"**
- Critical slow down can be experienced both experimentally and numerically.
- Depending on quench speed, correlations will "freeze out" at a certain point and remain as excitation in the system: Kibble-Zurek mechanism



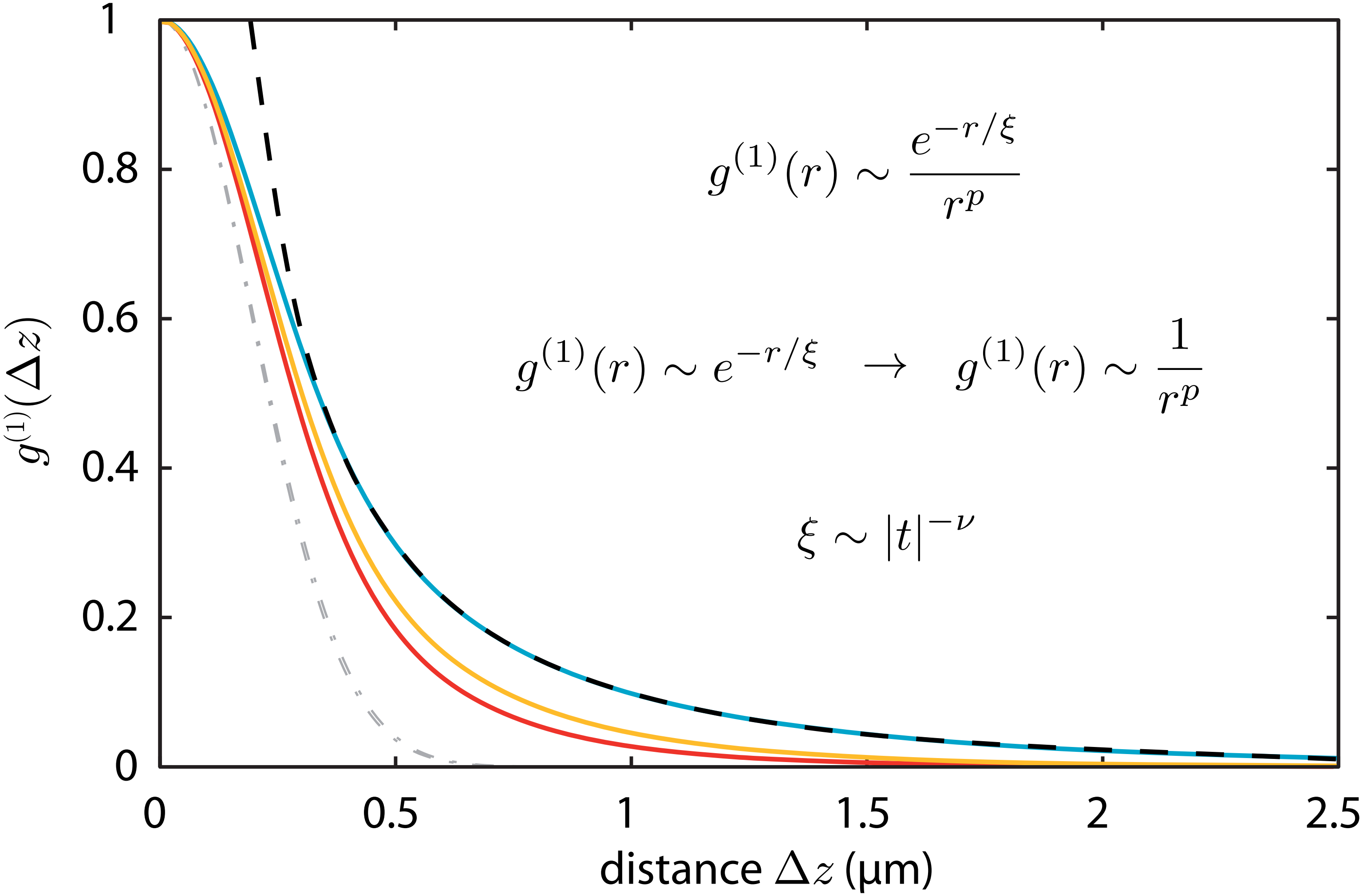
But what happens right at the critical point of the phase transition?



Correlation function

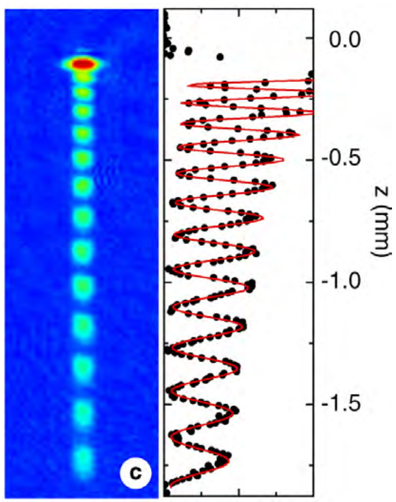


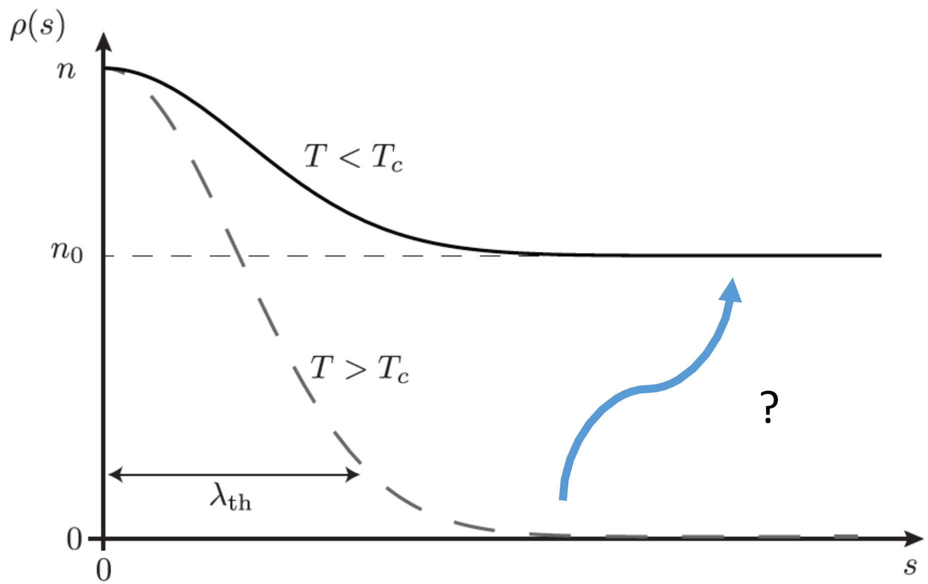
Correlation function in critical region



But what happens right at the critical point of the phase transition?

Change T and measure?





☹ Influence of output coupling 20% of cloud

☹ Signal-to-noise of visibility

☹ "Size of critical region is problem of experimentalists" (Zinn Justin)

RF output coupling

- ☹ Influence of output coupling 20% of cloud
- ☹ Signal-to-noise of visibility
- ☹ “Size of critical region is problem of experimentalists” (Zinn Justin)

Output couple only few atoms (<1%)

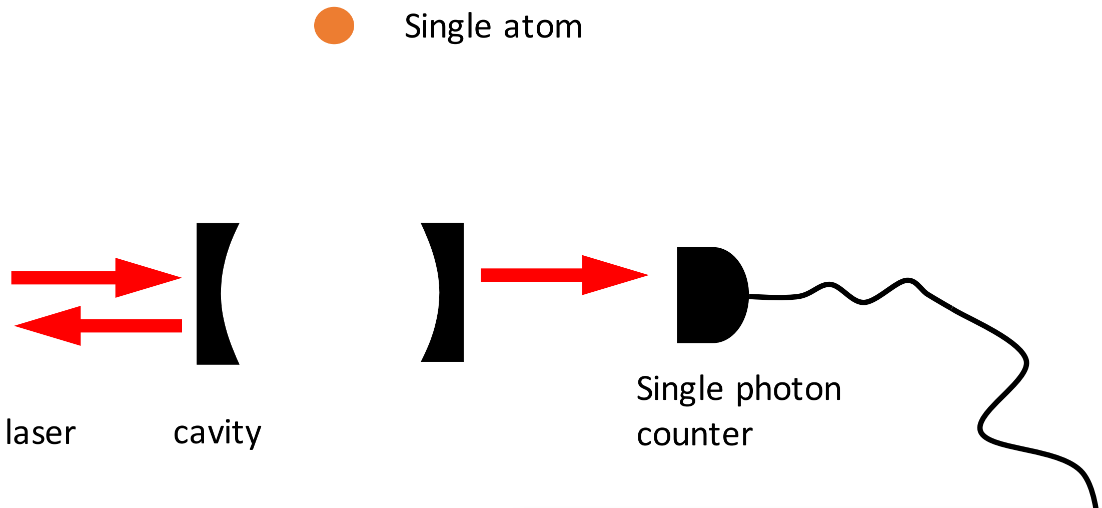
Detect with single-atom efficiency

Having luck: critical region

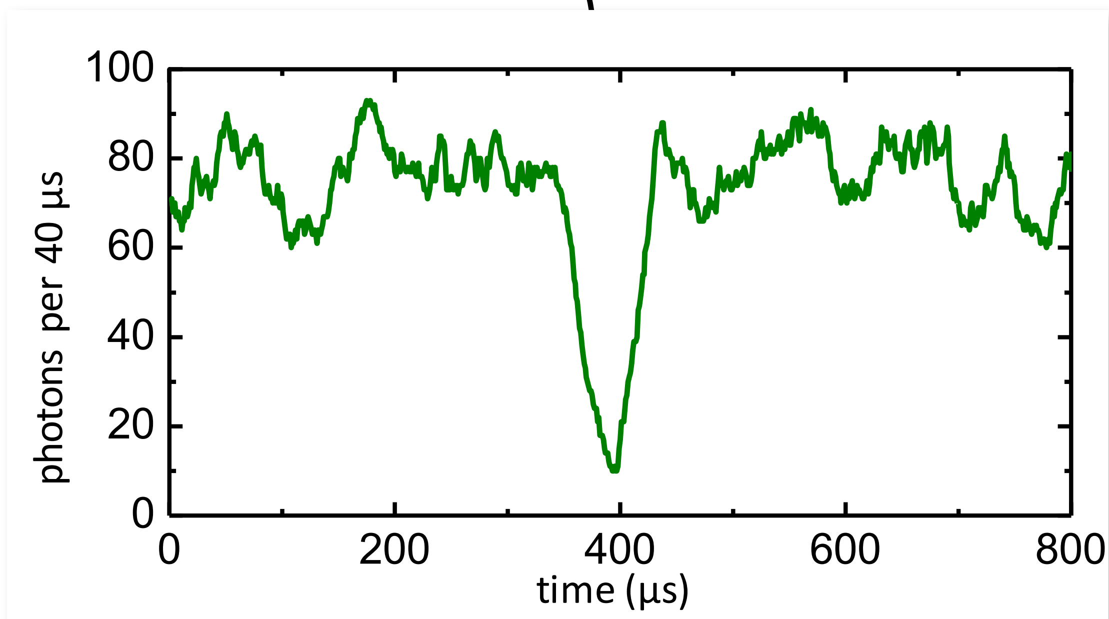
$$t < 0.08 \quad \text{or} \quad \xi > \frac{\lambda_{dB}^2}{\sqrt{128\pi^2 a}} \simeq 0.4\mu\text{m}$$

Single atom detector

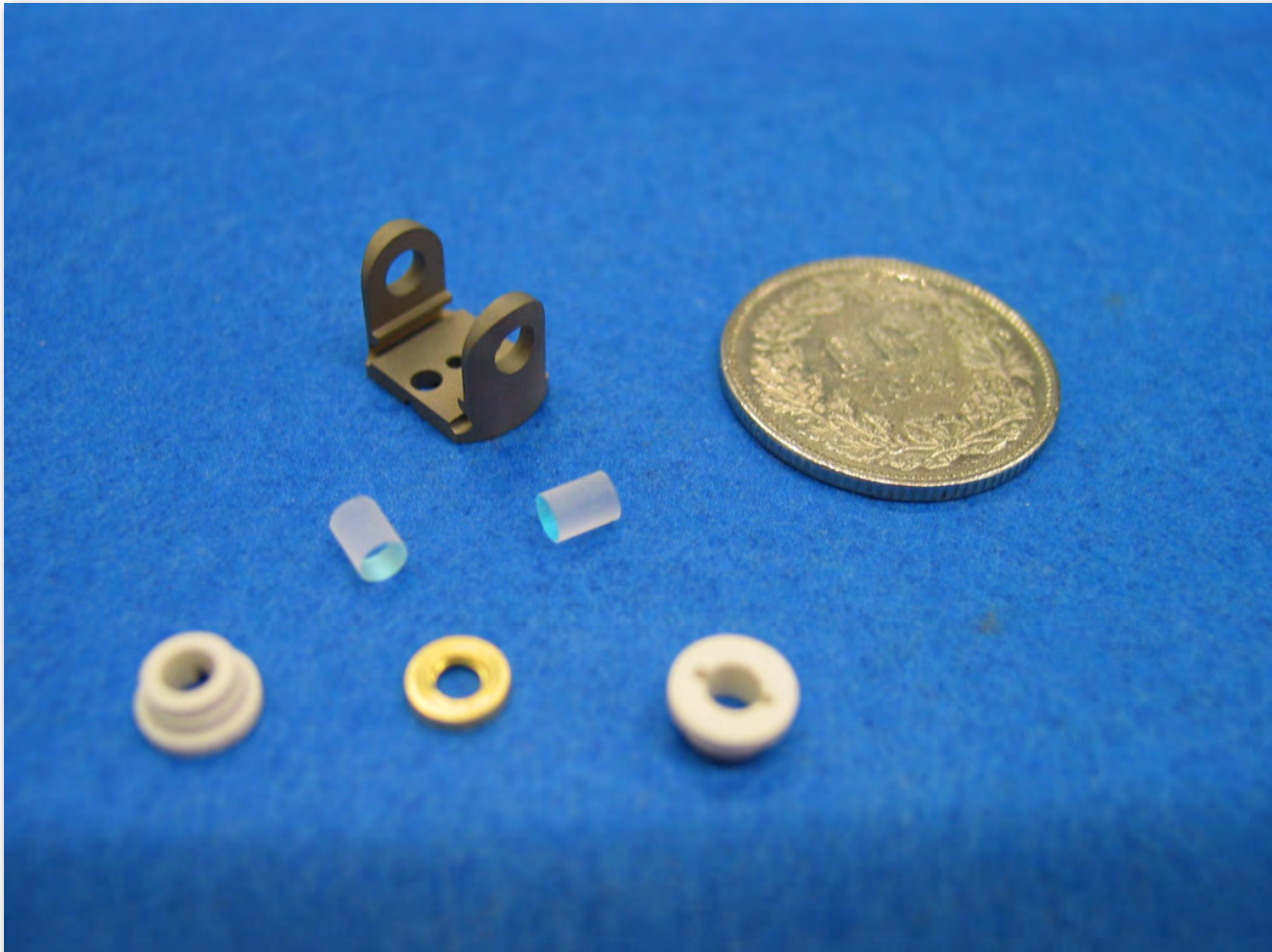
Detecting single atoms



Cavity Transmission:

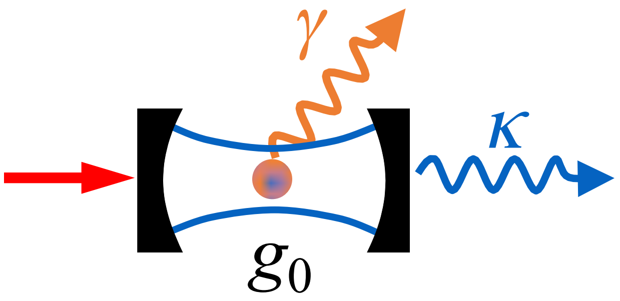


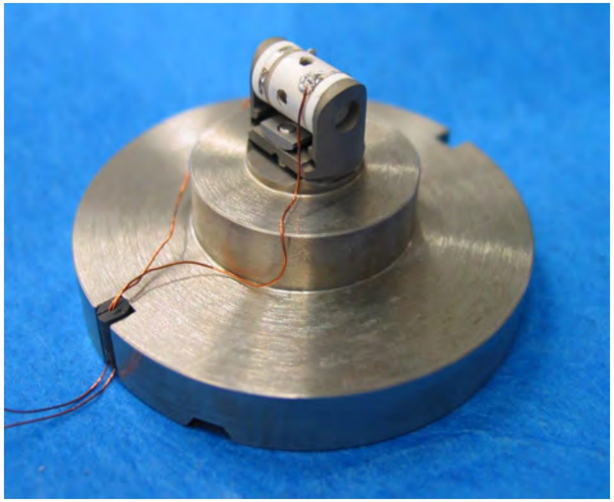
first observation by H. Mabuchi et al., Opt. Lett. **21**, 1393 (1996)



Optische Hoch-Finesse-Cavity

length	= 178 µm
Mode waist	= 25 µm
Finesse	≈ 300.000



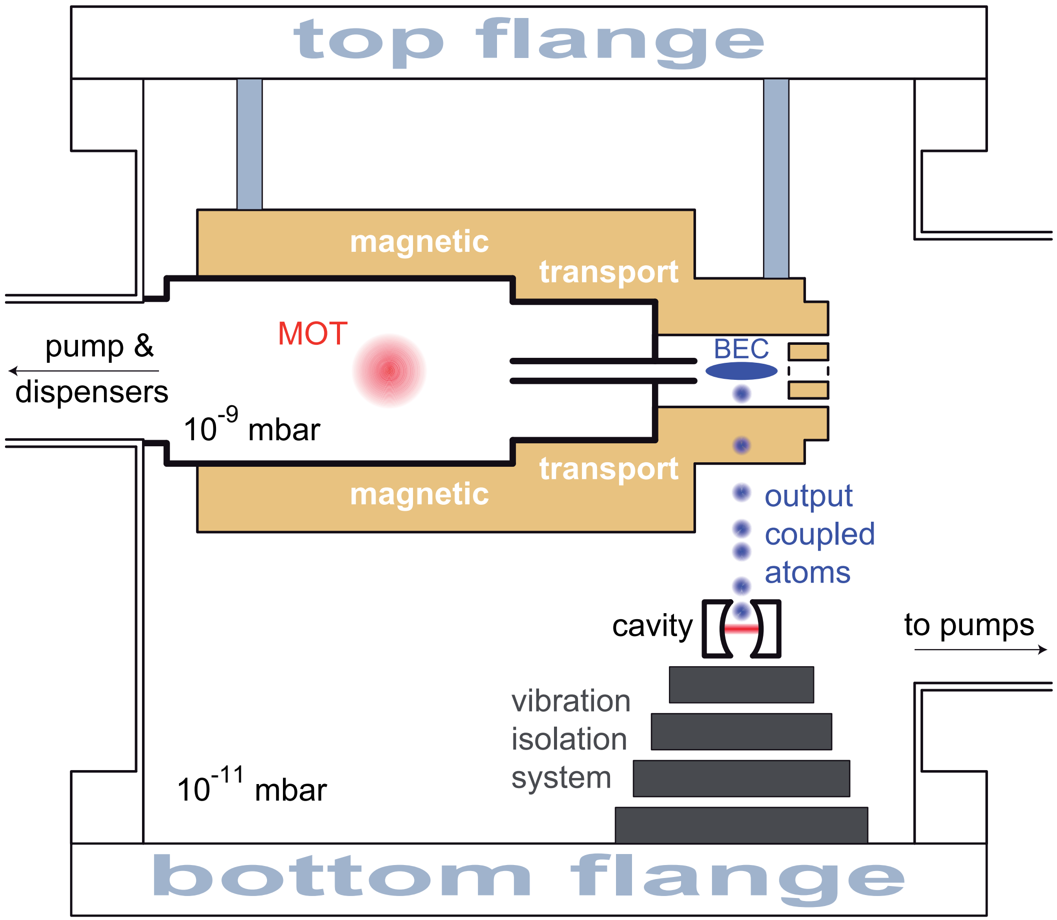


Atom light coupling	g_0= 2π 10 MHz
Atomic decay rate	γ = 2π 6.0 MHz
Cavity decay rate	κ = 2π 1.4 MHz

$$g_0 > \gamma, \kappa$$ ✓

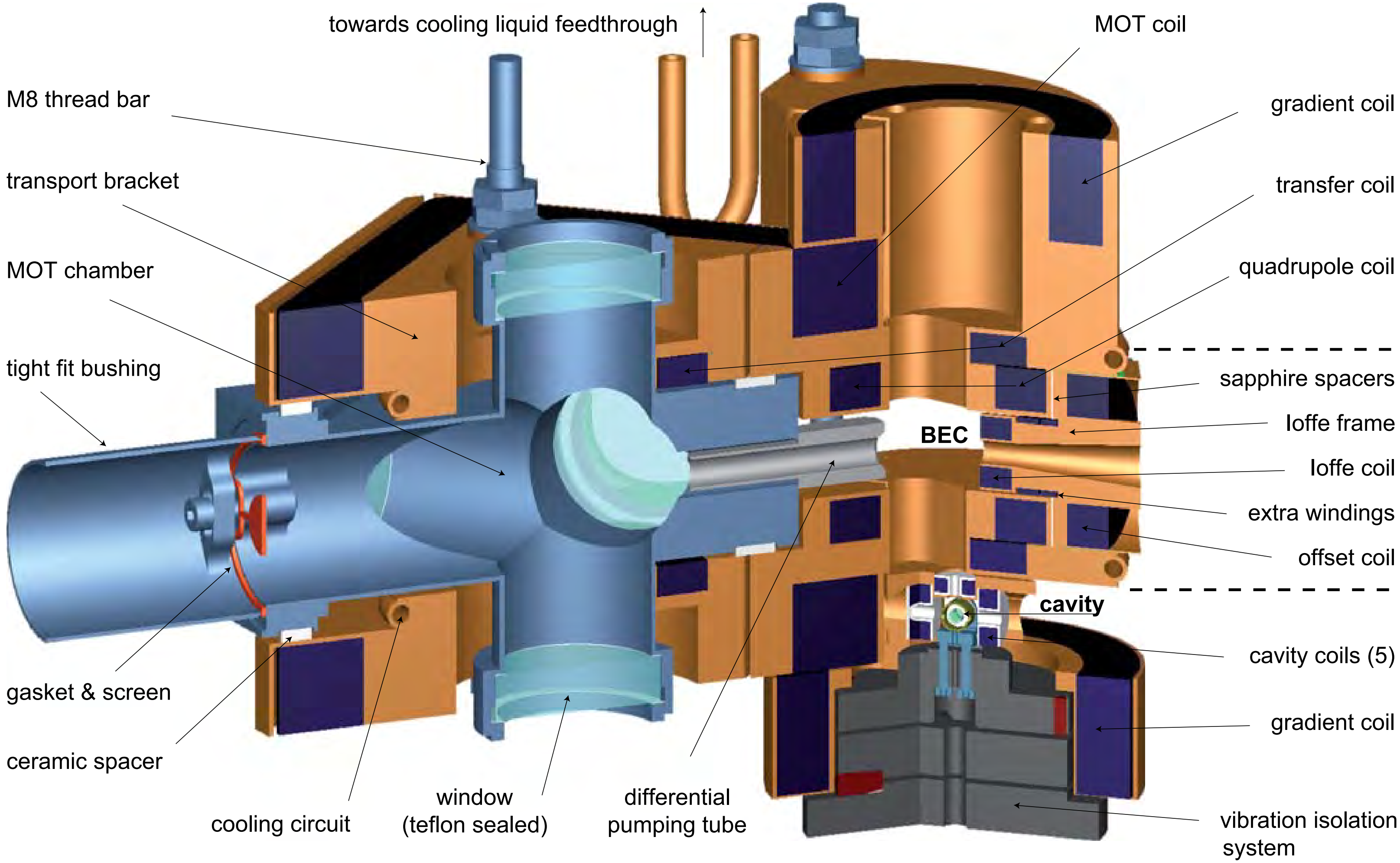
Strong coupling regime of cQED

Apparatus

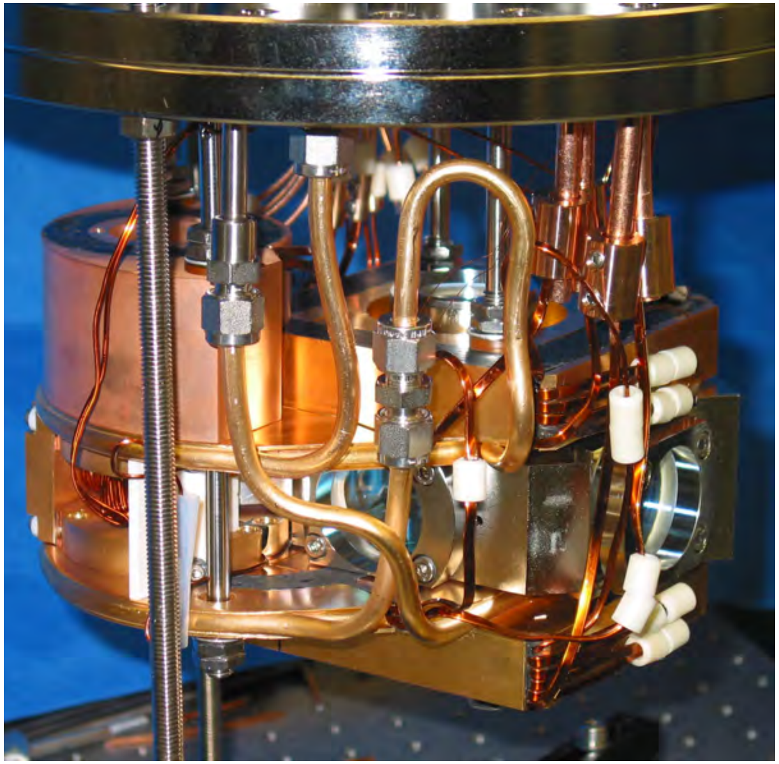


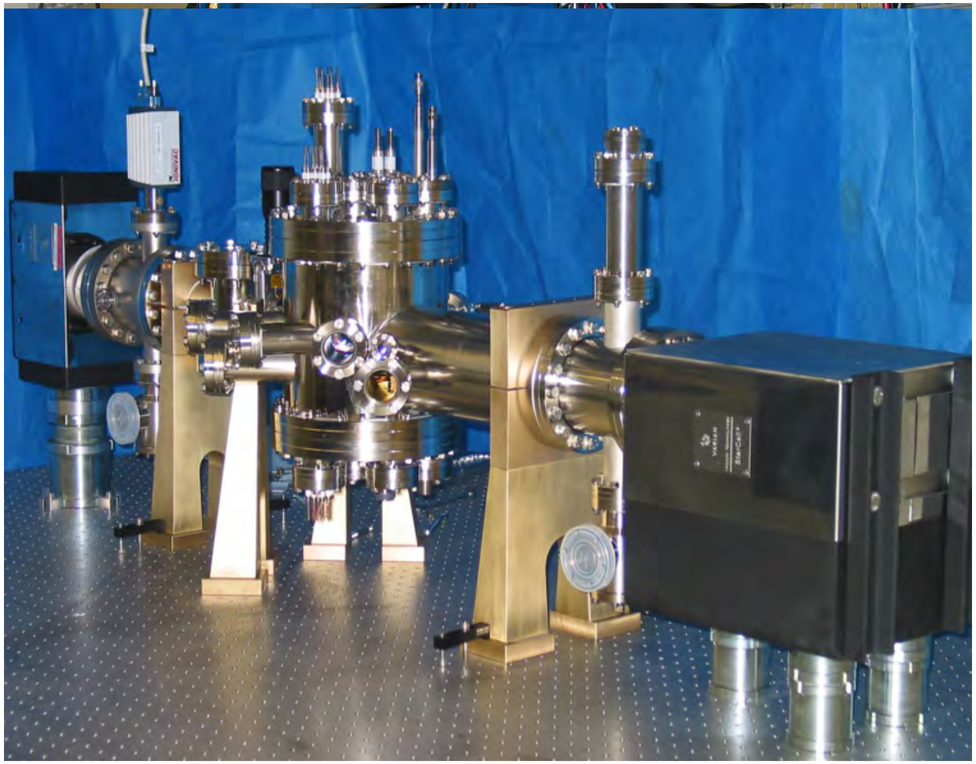
top flange
magnetic
transport
MOT
pump &
dispensers
10^-9 mbar
BEC
transport
magnetic
output
coupled
atoms
cavity
to pumps
vibration
isolation
system
10^-11 mbar
bottom flange

Apparatus


towards cooling liquid feedthrough
MOT coil
M8 thread bar
gradient coil
transport bracket
transfer coil
MOT chamber
quadrupole coil
tight fit bushing
sapphire spacers
BEC
Ioffe frame
Ioffe coil
extra windings
offset coil
cavity
cavity coils (5)
gasket & screen
gradient coil
ceramic spacer
cooling circuit
window
(teflon sealed)
differential
pumping tube
vibration isolation
system

Apparatus



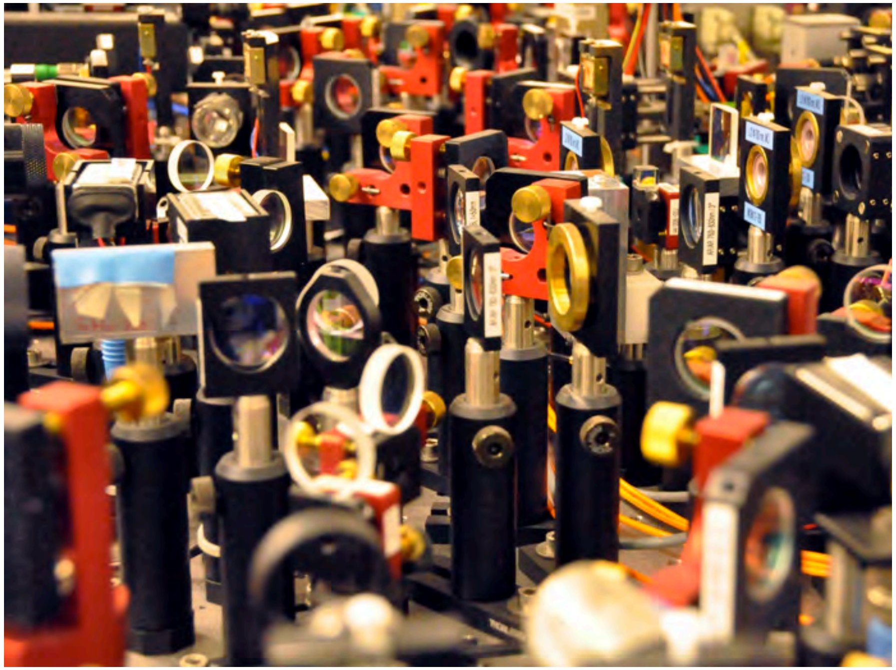


Apparatus

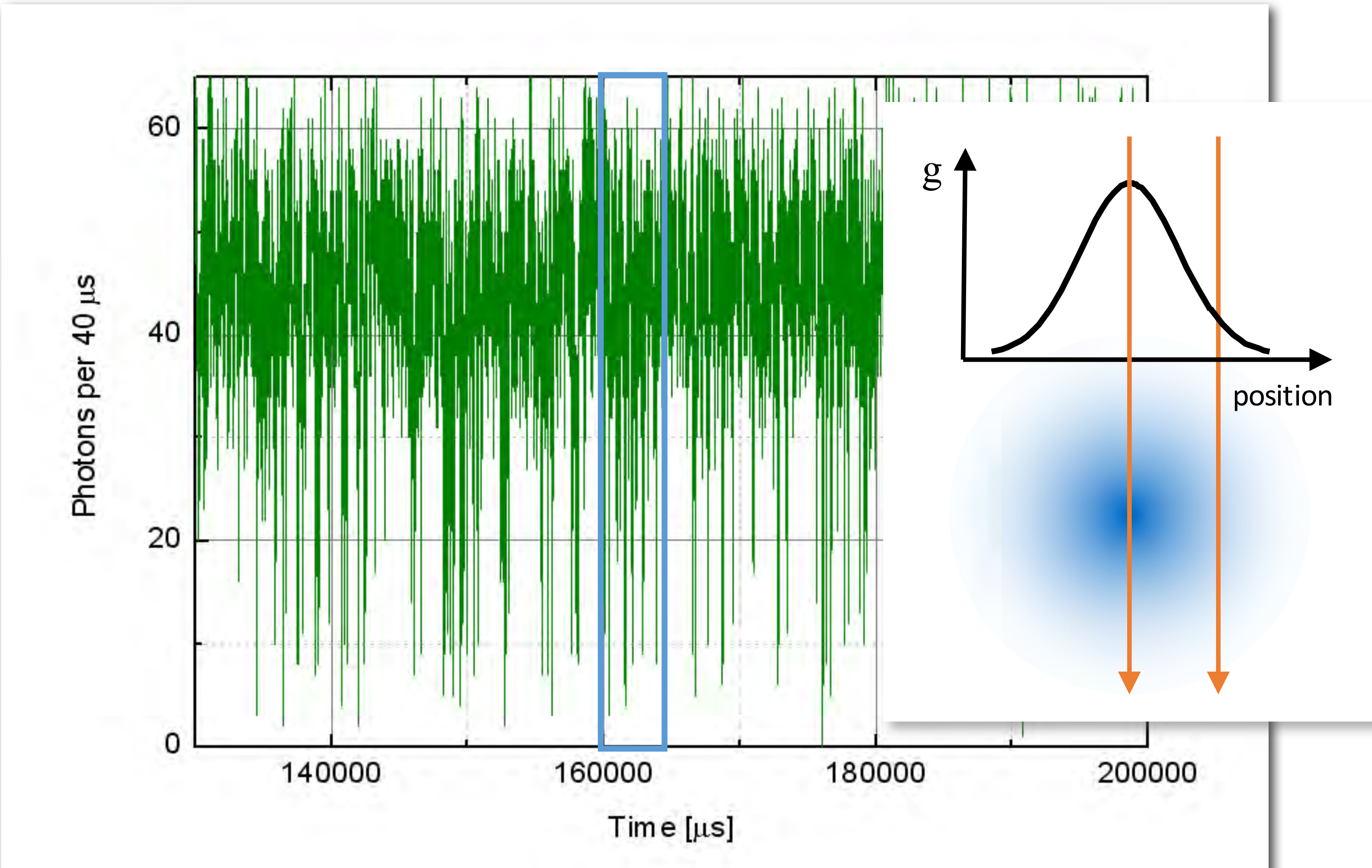
15 stabilized lasers

> 1000 optical elements

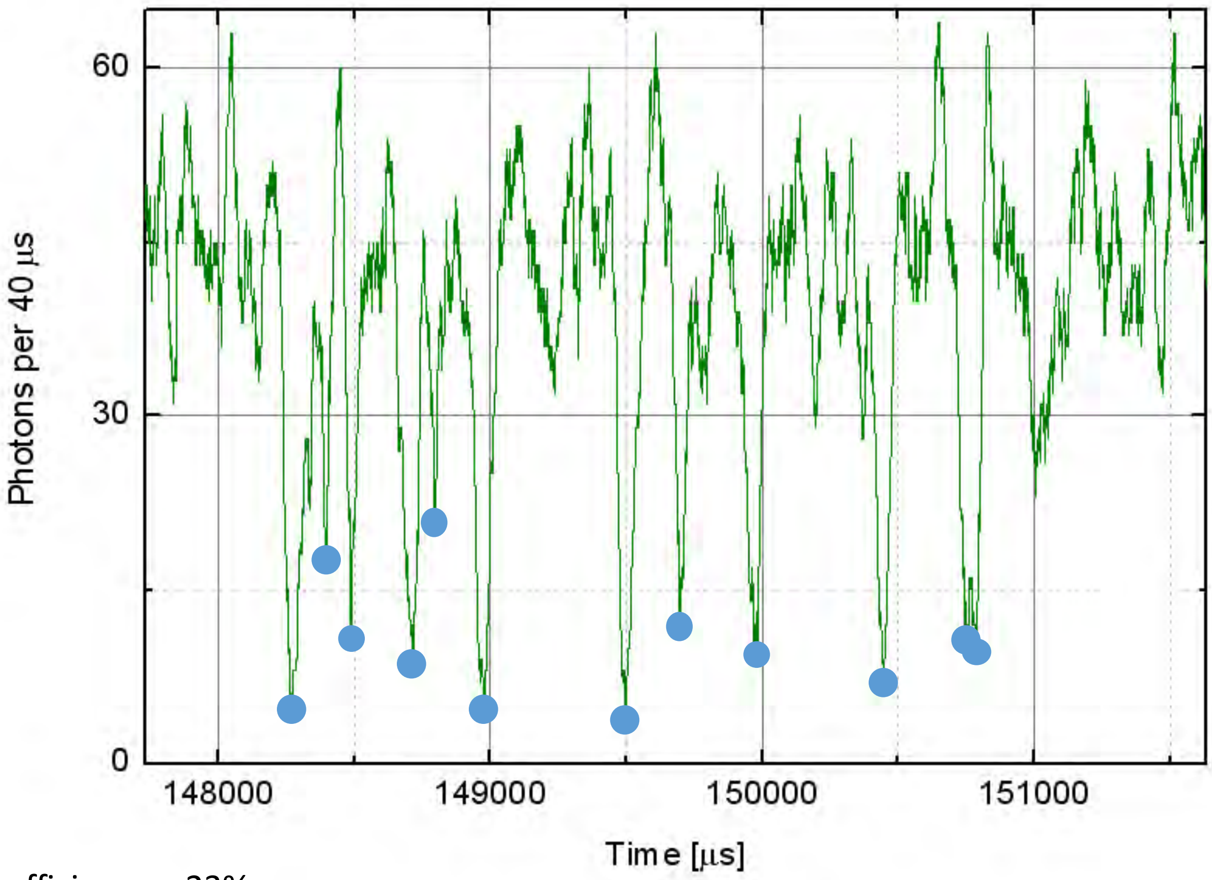
> 200m optical fibers



Detecting single atoms

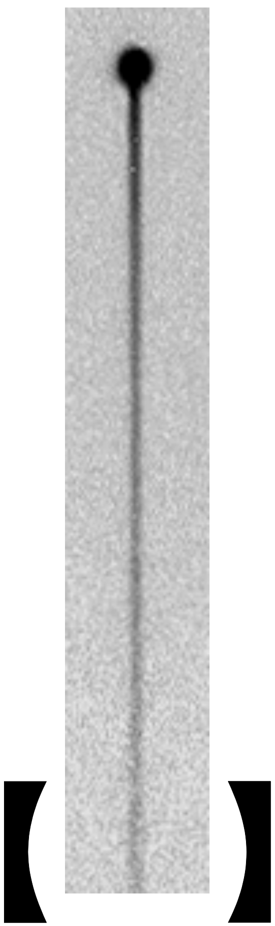


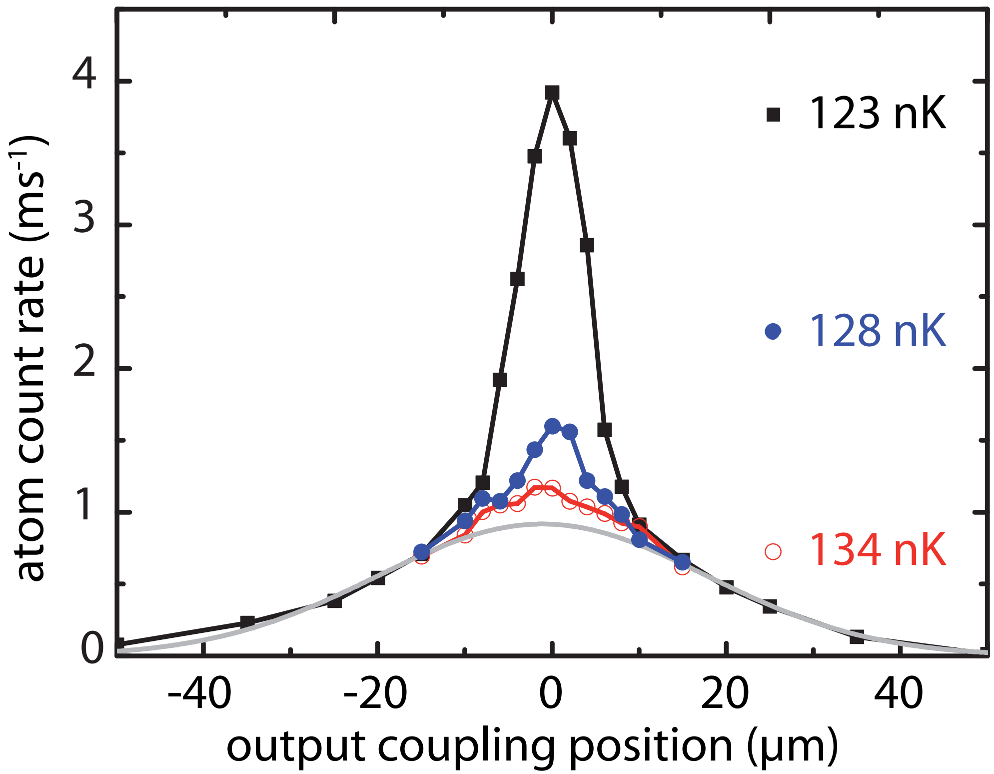
Detecting single atoms

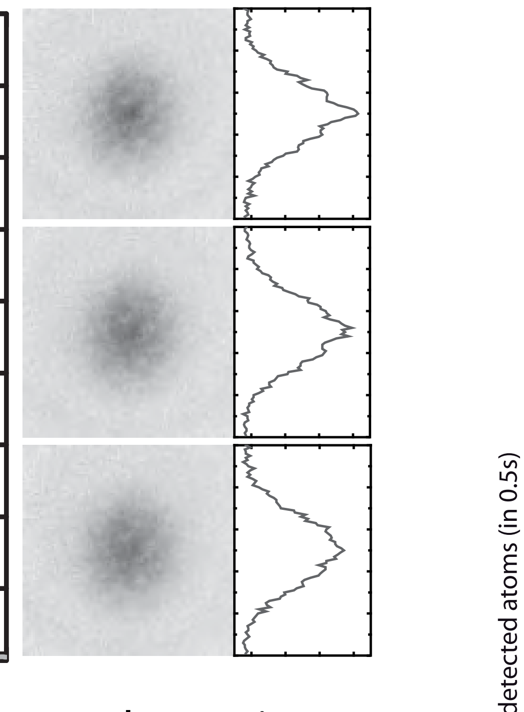


Detection efficiency > 23%

Detecting single atoms

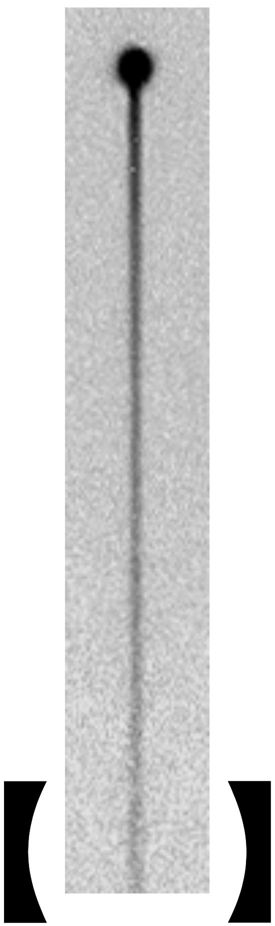


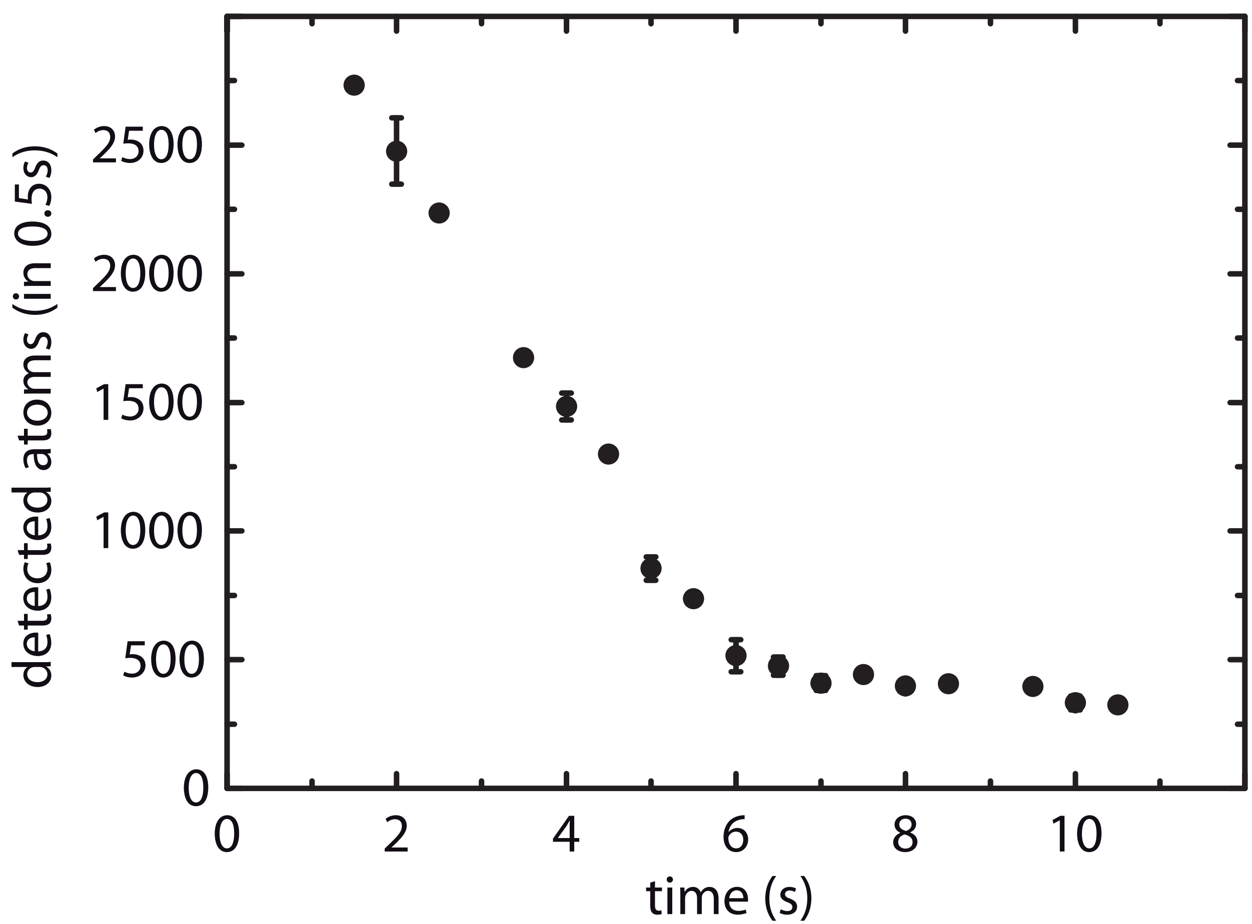




absorption images

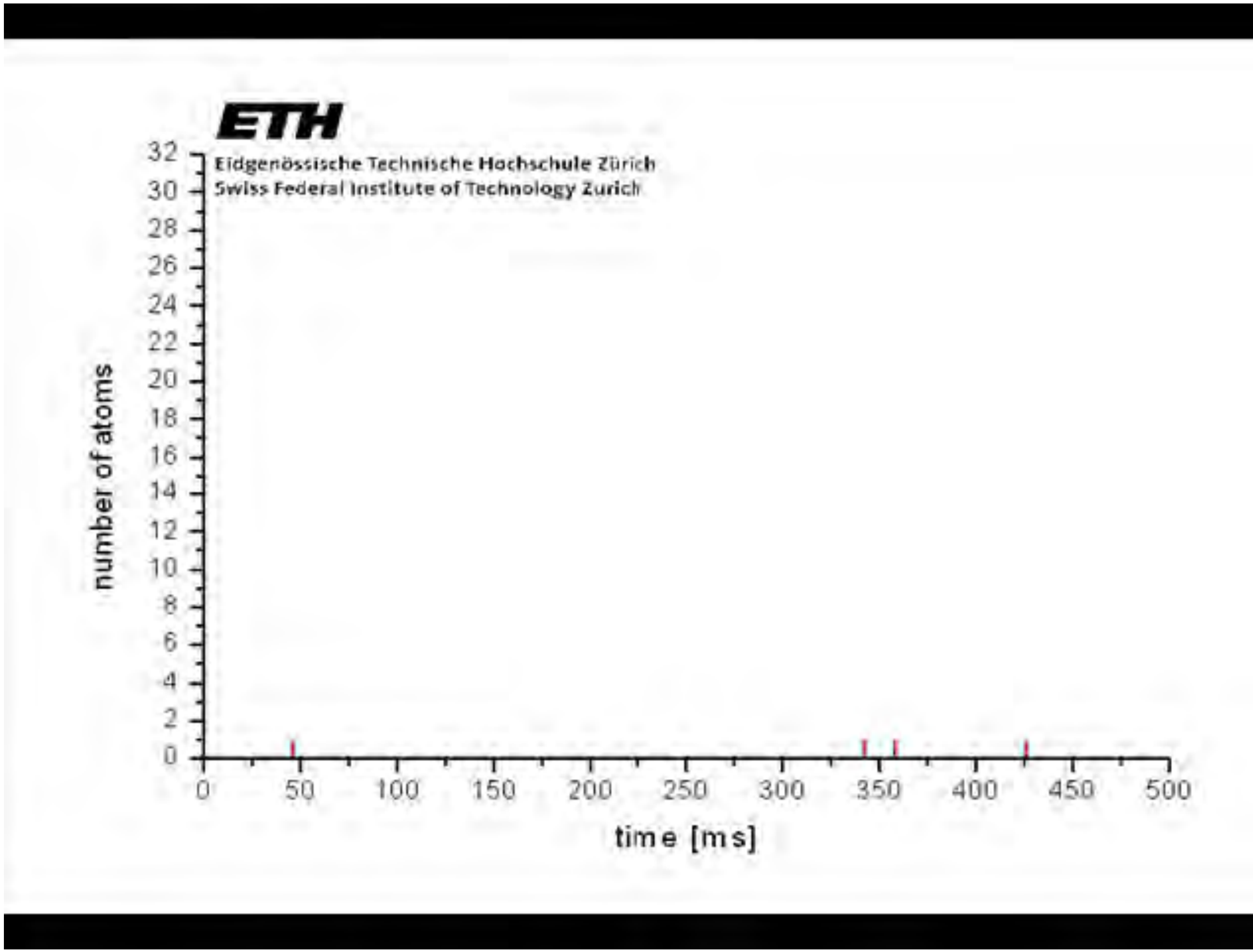
“Ramping” the temperature



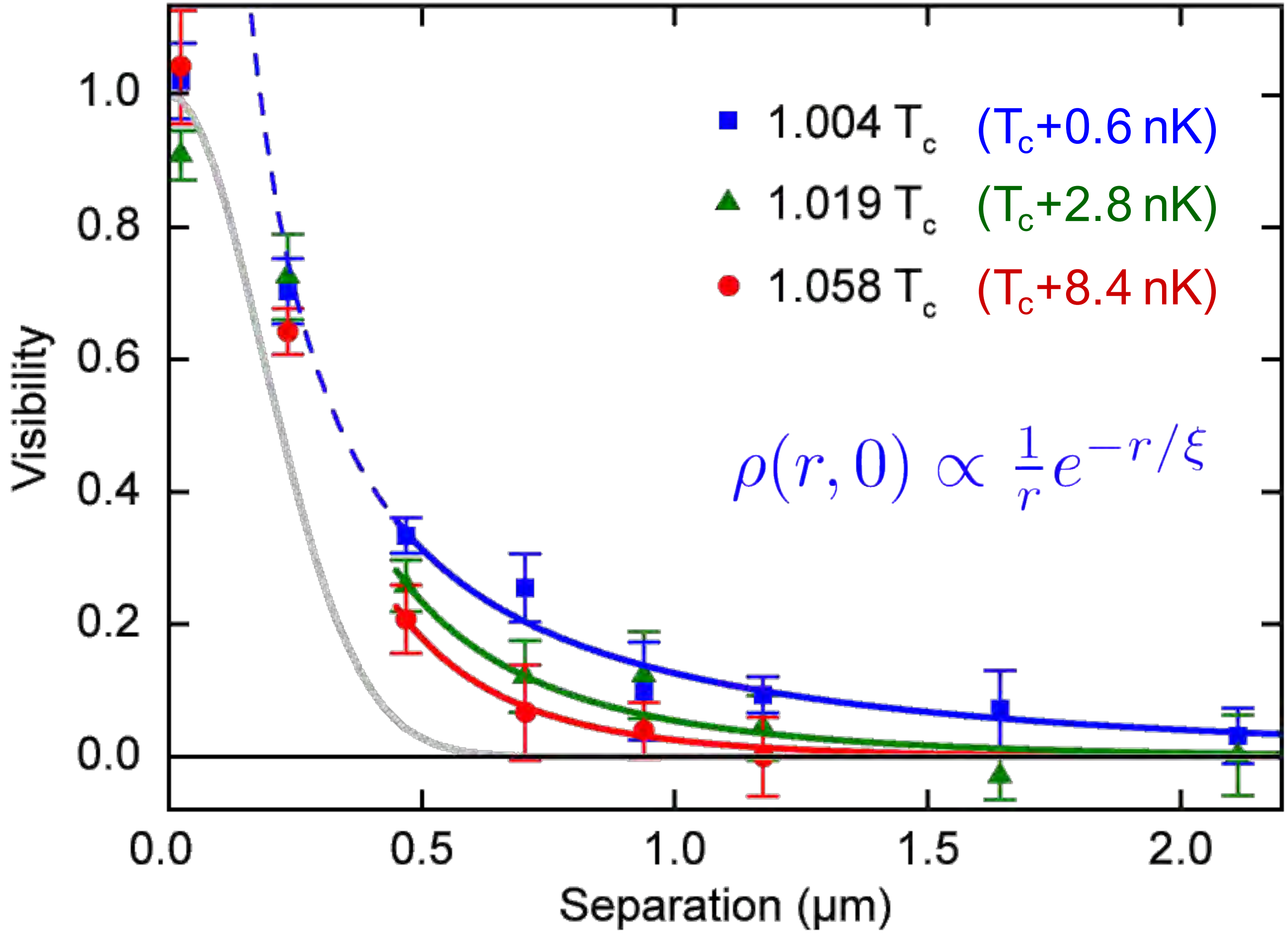


-> change temperature by waiting: heating rate 4nK/s

Detecting interference on the single atom level

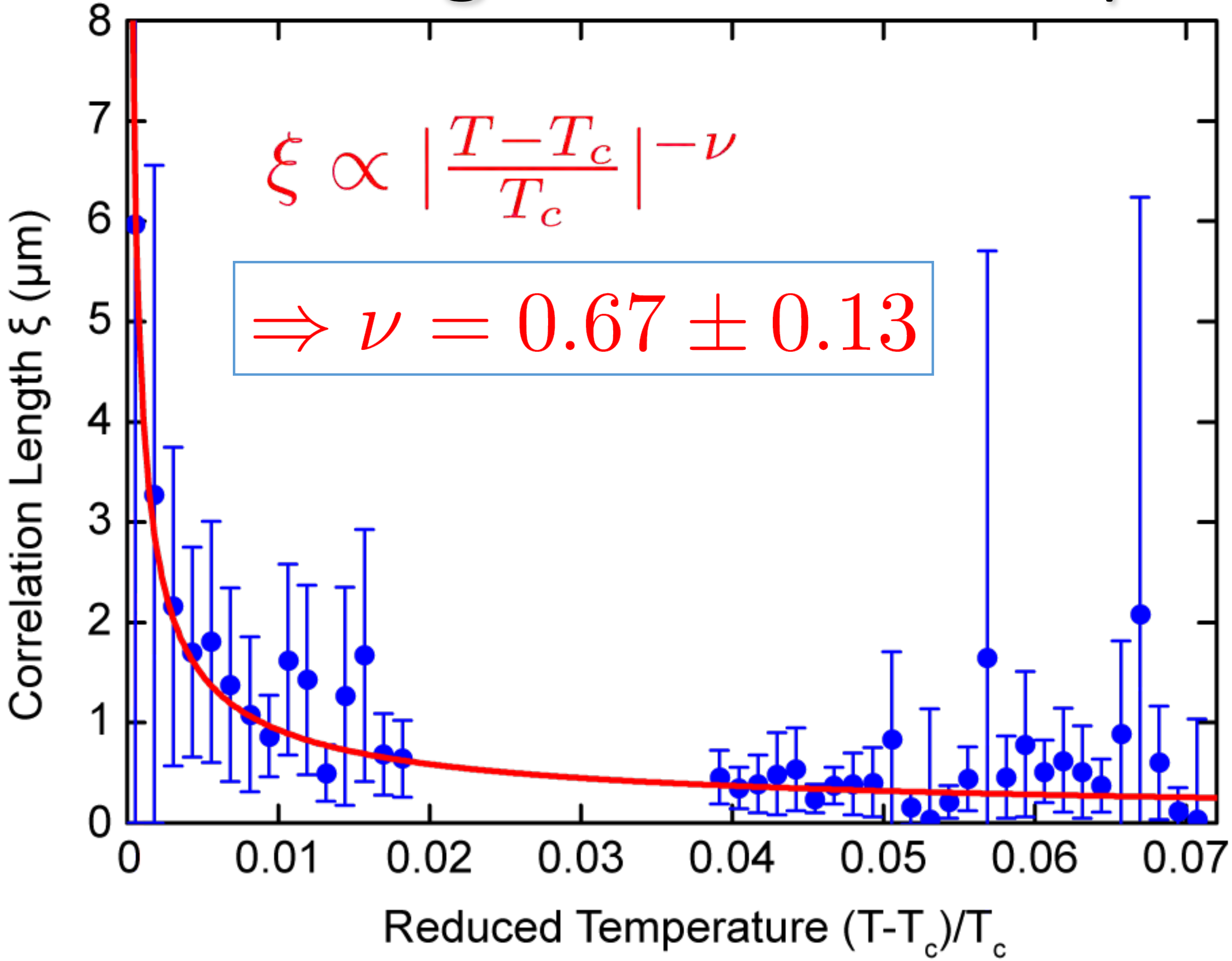


A Close Look at the Phase Transition



T_c=146 nK   temperature resolution: 300 picokelvin

Determining the Critical Exponent



T. Donner , S. Ritter, T. Bourdel, A. Öttl, M. Köhl, T. Esslinger, Science **315**, 1556 (2007)

Result

$$\nu = 0.67 \pm 0.13$$

Non-interacting system:

homogeneous	ν =0.5
trapped	ν =1

Landau theory homogeneous system — ν =0.5

Renormalization group theory — ν =0.6717(1)

Liquid Helium — spaceborne experiment (scaling relation $\alpha = 2 - 3\nu$) — ν =0.67056(6)

ten orders of magnitude difference in density !

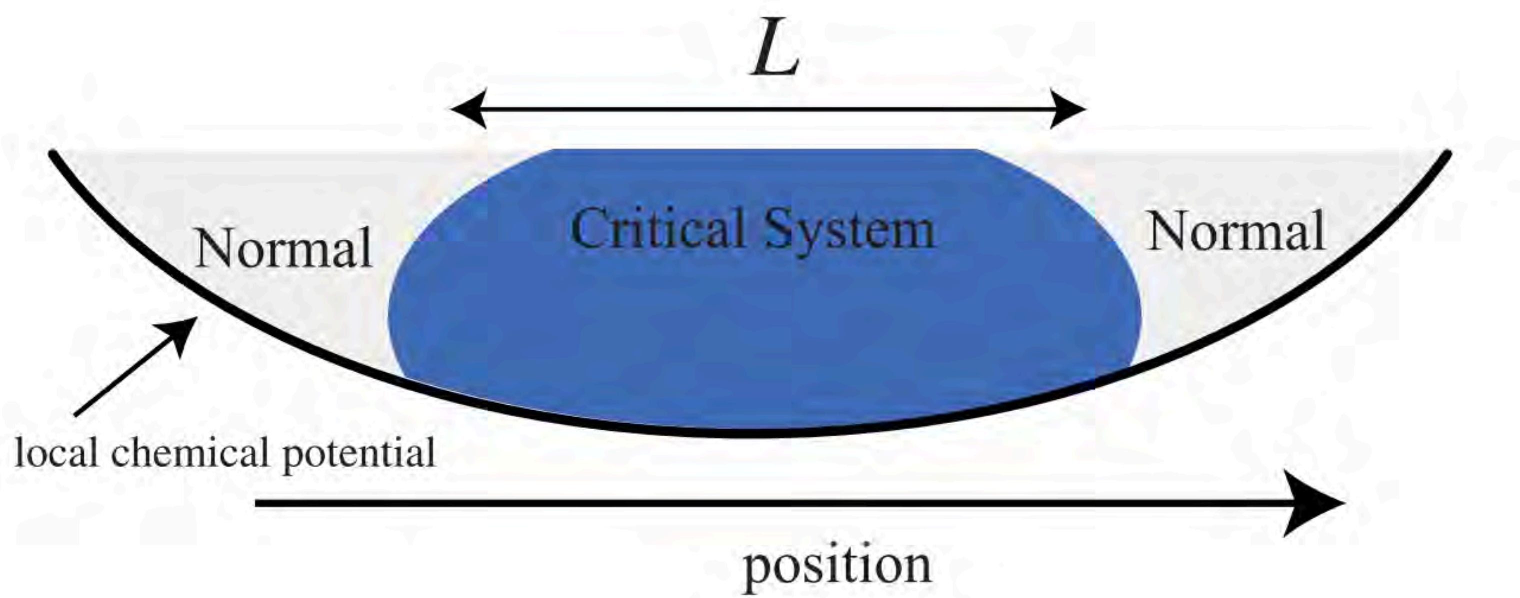
Liquid Helium: Lipa et al., Phys. Rev. B **68**, 174518 (2003)
Theory: Campostrini et al., Phys. Rev. B **74**, 144506 (2006)

Influence of trapping potential

Length scales:

- Thermal de Broglie wavelength 0.4 um
- Critical region in space (via local chemical potential): 50 um

- Experiment probes between 0.4um and 2.2um – expect to observe homogeneous results (?!)
- Experimental probe symmetric with respect to trap center!

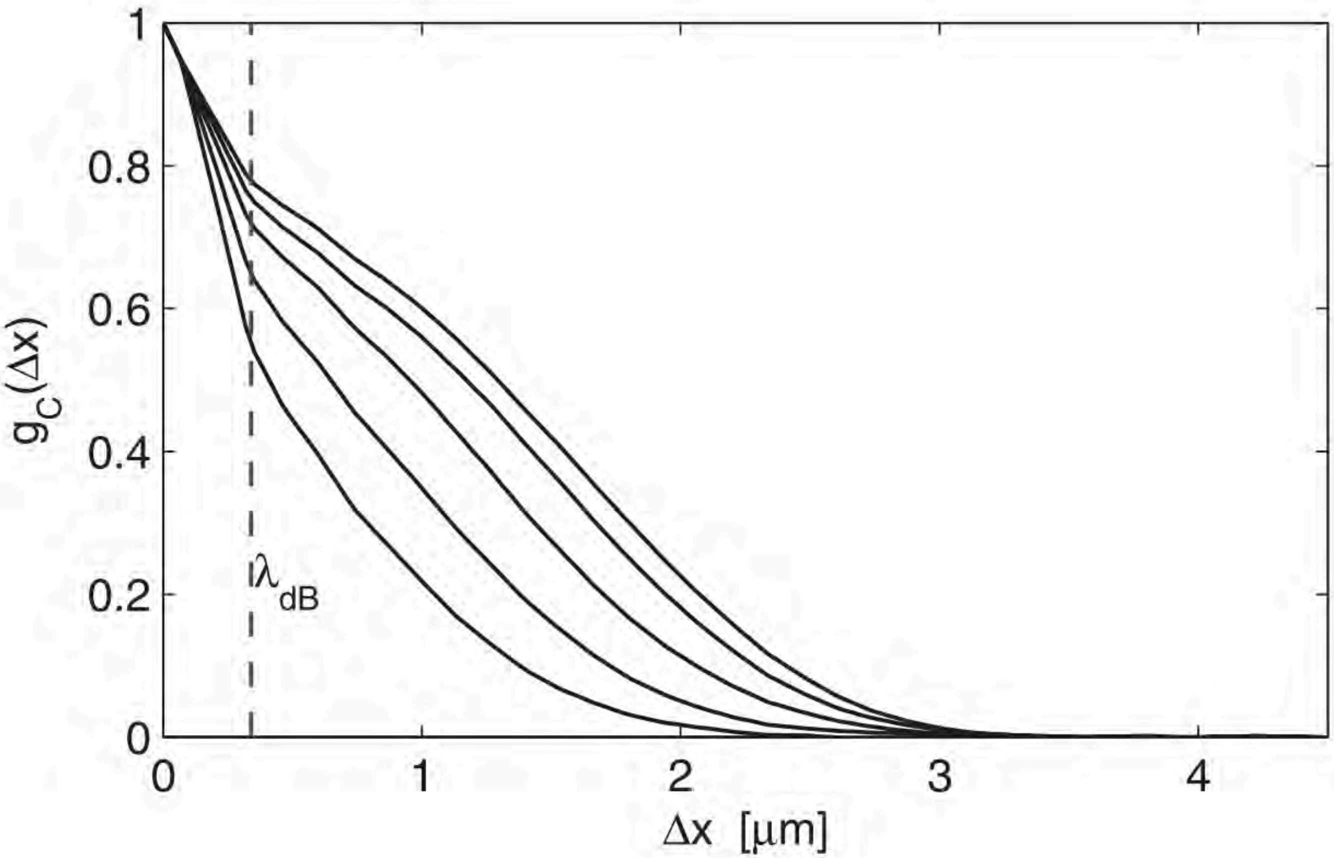


PRA 79, 033611 (2009)

PHYSICAL REVIEW A **79**, 033611 (2009)

Critical properties of a trapped interacting Bose gas

A. Bezett and P. B. Blakie

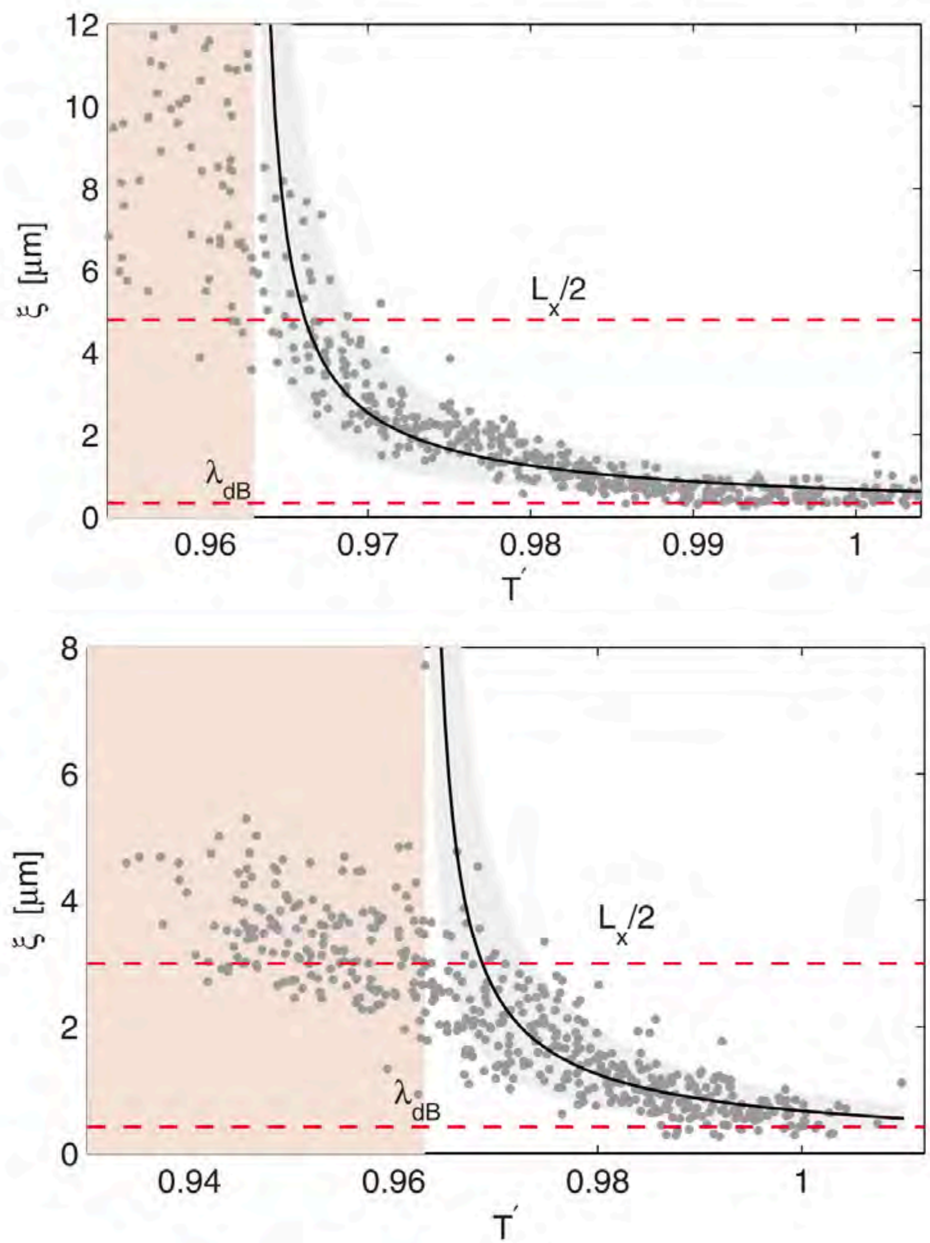


PHYSICAL REVIEW A **79**, 033611 (2009)

Critical properties of a trapped interacting Bose gas

A. Bezett and P. B. Blakie

System	λ_{dB} (T_c)	Δx_{max} (μm)	L_x (μm)	ν
Expt.	0.5 μm	2.2	20	0.67 ± 0.13
Theor. 1	0.34 μm	2.2	9	0.8 ± 0.12
Theor. 2	0.42 μm	2.2	6	0.8 ± 0.12 [a]



Smaller system:

Rounding off due to finite size effects