Spin liquids

SFB/TR 49 student seminar
Hamburg, October 2013

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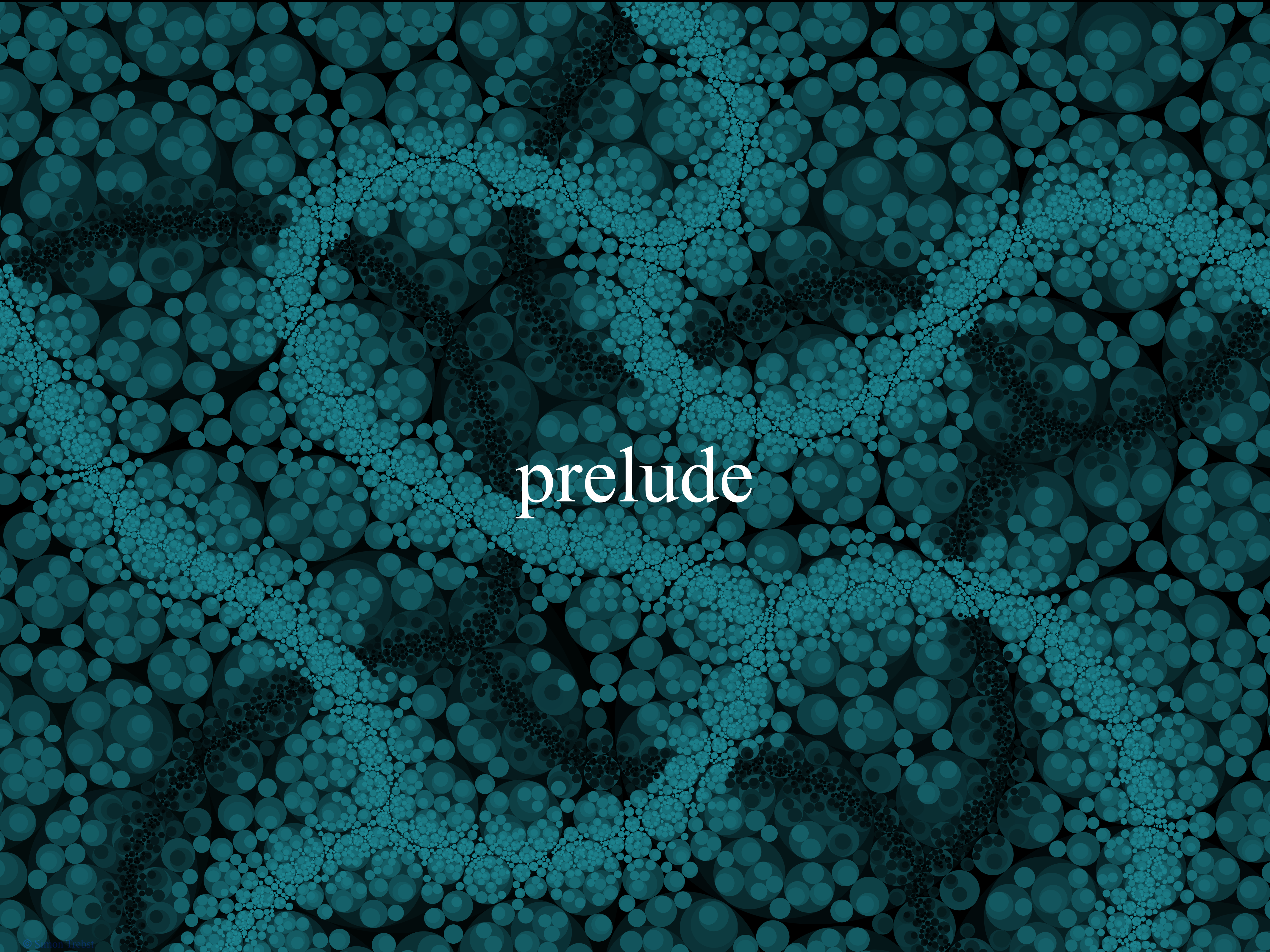
Today’s menu

classical spin liquids

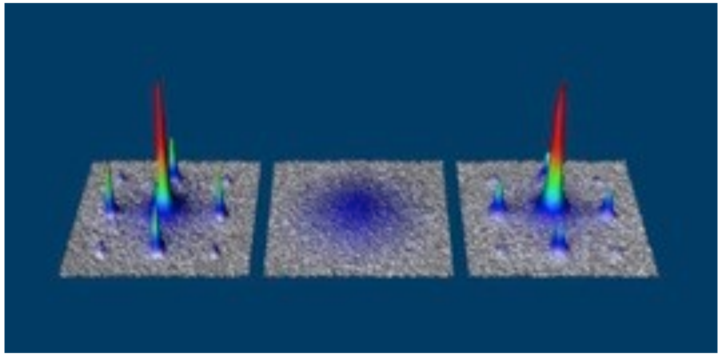
quantum spin liquids


prelude

Matter – a collective phenomenon

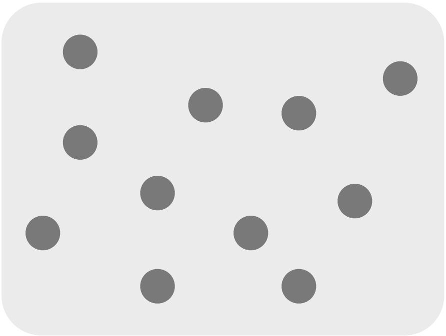
water ice

superconductor



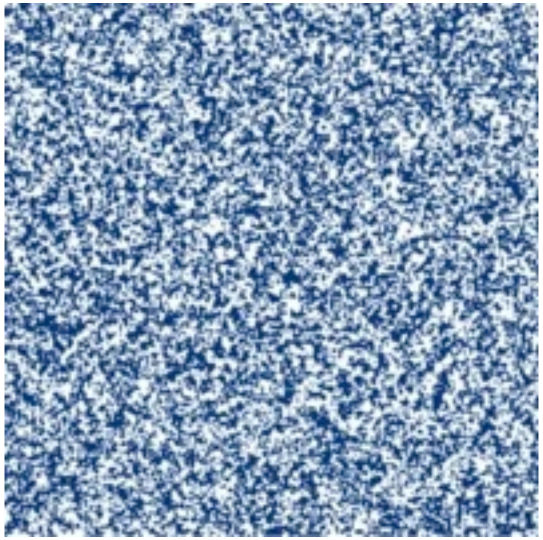
Bose-Einstein condensate

Motivation – a paradigm

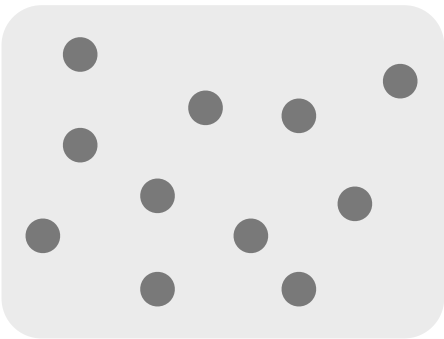


interacting
many-body system

$$\mathcal{H} = -\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$$



Motivation – a paradigm



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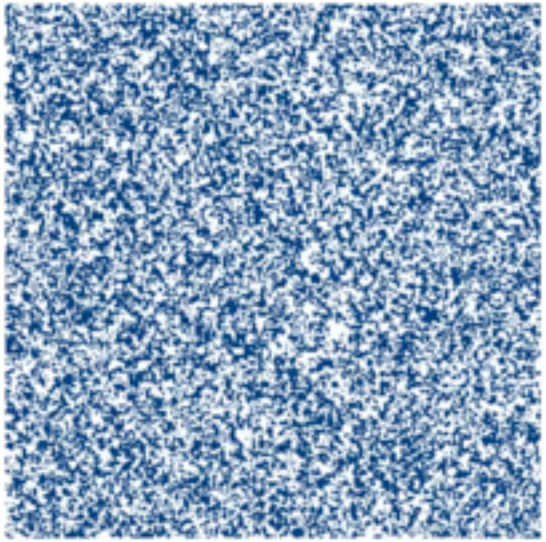
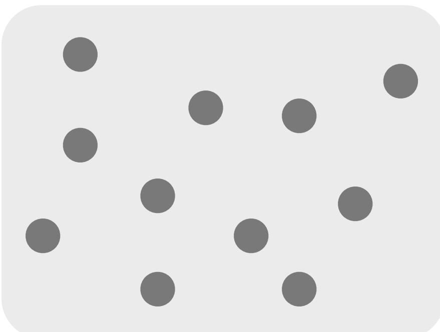


Motivation – a paradigm



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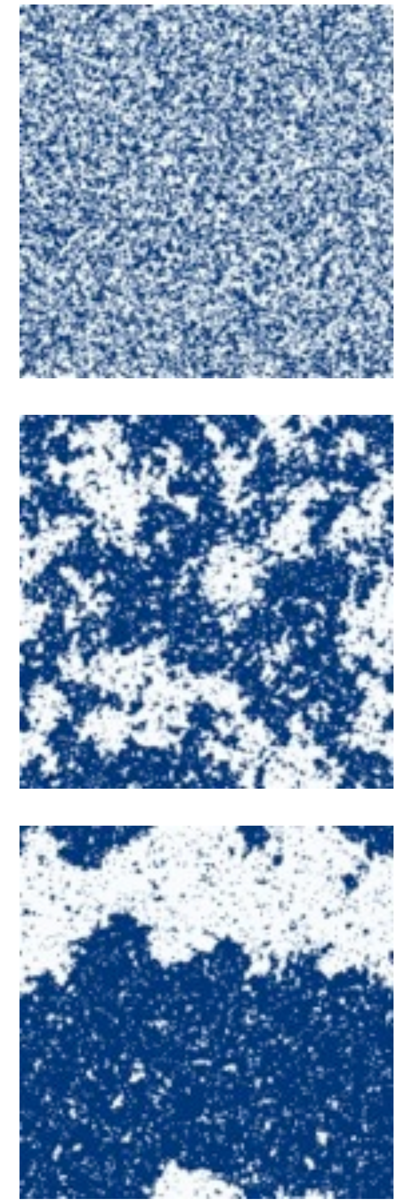
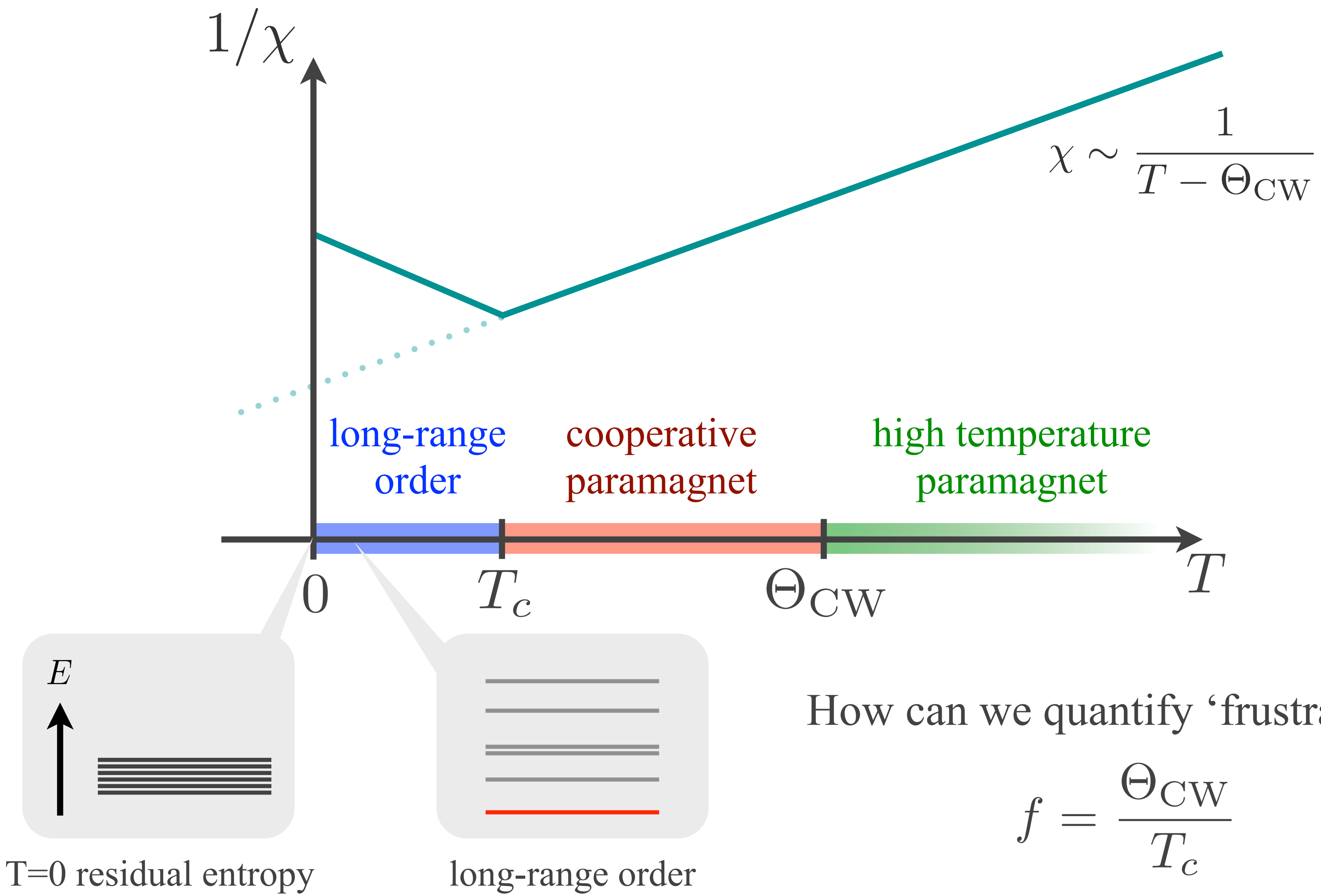


Spontaneous symmetry breaking

- ground state has **less** symmetry than Hamiltonian
- **local** order parameter
- **phase transition** / Landau-Ginzburg-Wilson theory

Beyond the paradigm – frustrated magnets

Insulating magnets with competing interactions.

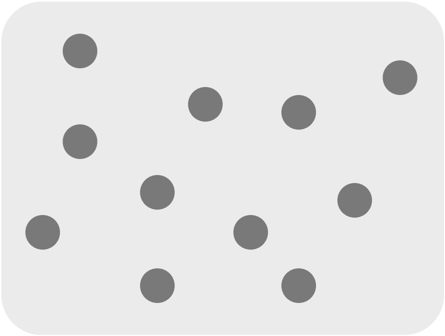


How can we quantify ‘frustration’?

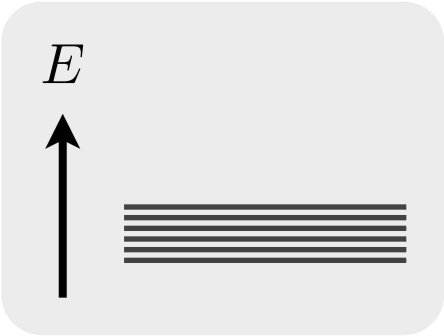
$$f = \frac{\Theta_{\mathrm{CW}}}{T_c}$$

Why we should look for the misfits

Some of the most intriguing phenomena in condensed matter physics arise from the splitting of ‘accidental’ degeneracies.



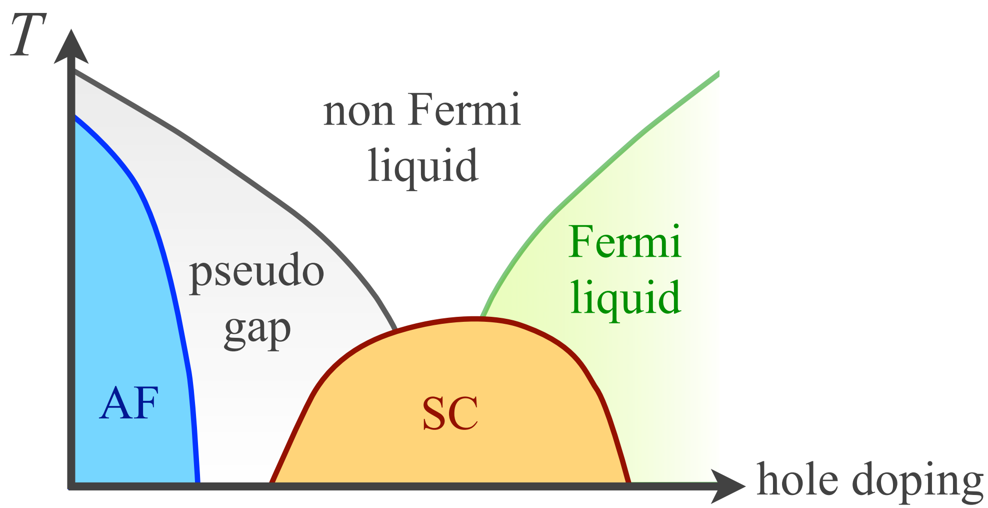
interacting
many-body system



‘accidental’
degeneracy

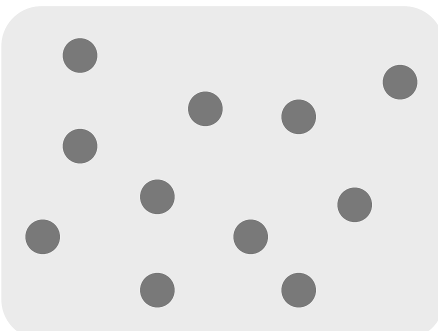
residual effects
select ground state

phase diagram of
cuprate superconductors

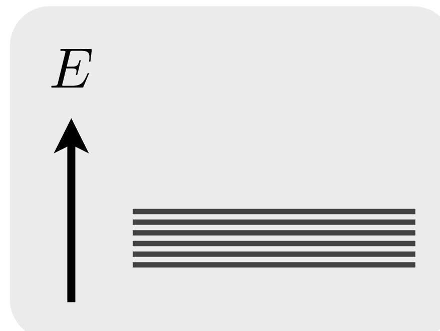


Why we should look for the misfits

Some of the most intriguing phenomena in condensed matter physics arise from the splitting of ‘accidental’ degeneracies.



interacting
many-body system



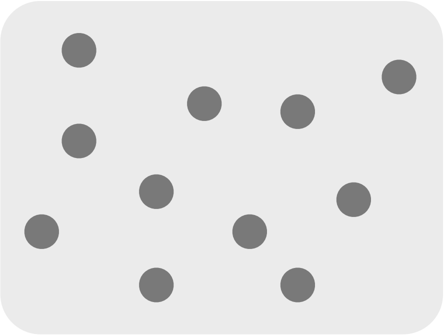
‘accidental’
degeneracy

residual effects
select ground state

But they are also notoriously difficult to handle analytically, due to

- multiple energy scales
- complex energy landscapes / slow equilibration
- strong coupling

Examples in this talk



interacting
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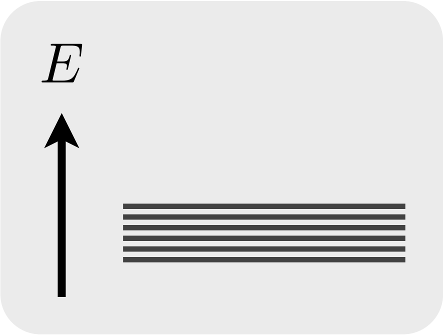
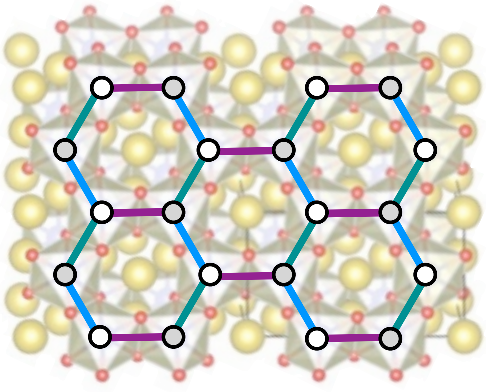


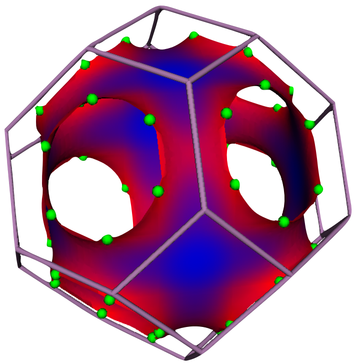
'accidental'
degeneracy

residual effects
select ground state

classical spin liquids
in layered Iridates



spiral spin liquids
in spinel compounds

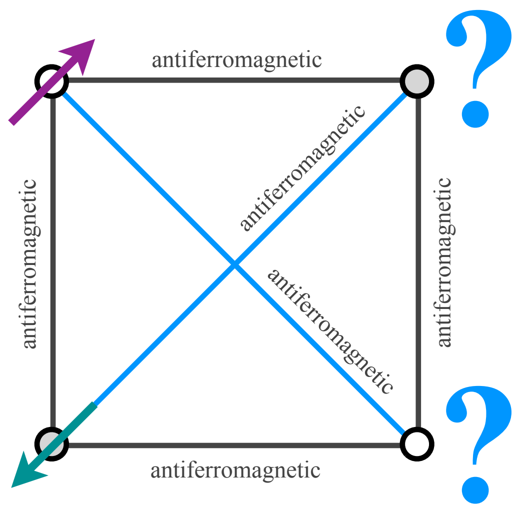


classical spin liquids

– Kitaev model –

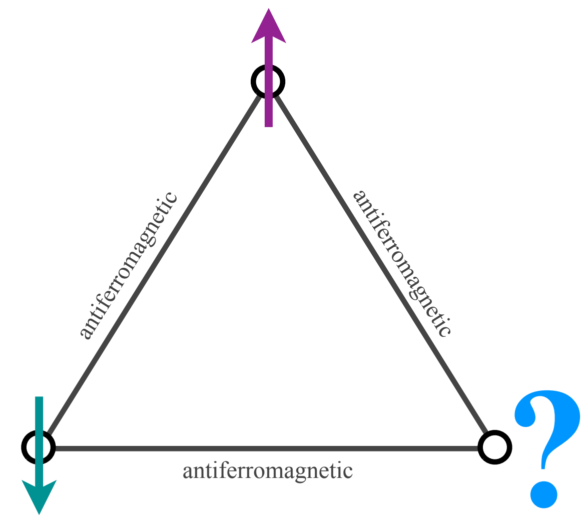
Frustration

Competing interactions lead to frustration.
We will see that frustration can originate interesting spin liquid behavior.



exchange frustration

classical Kitaev model



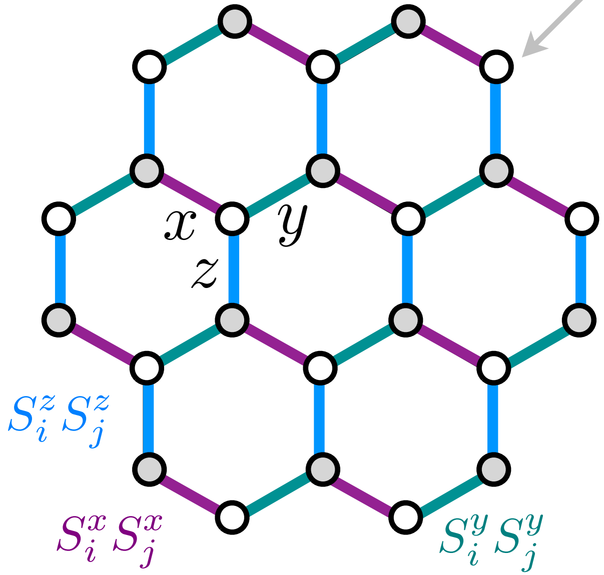
geometric frustration

triangular lattice antiferromagnet
diamond lattice antiferromagnet

The Kitaev model

A. Kitaev, Ann. Phys. **321**, 2 (2006)

$\vec{S}_i$ O(3) Heisenberg spins



$$H_{\text{Kitaev}} = \sum_{\gamma-\text{links}} J_\gamma \boxed{S_i^\gamma S_j^\gamma}$$

Ising-like* interaction

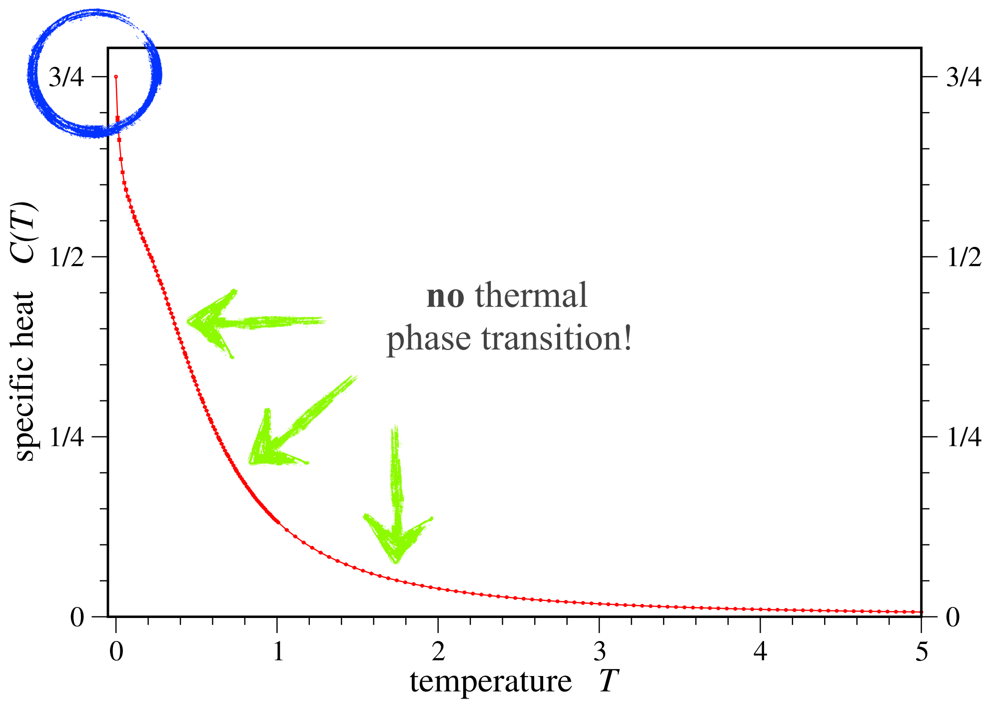
* preferred direction of spin alignment depends on spatial direction of bond

Its quantum mechanical cousin (see also next lecture) is well known for its rare combination of a model of fundamental conceptual importance and an *exact* analytical solution.

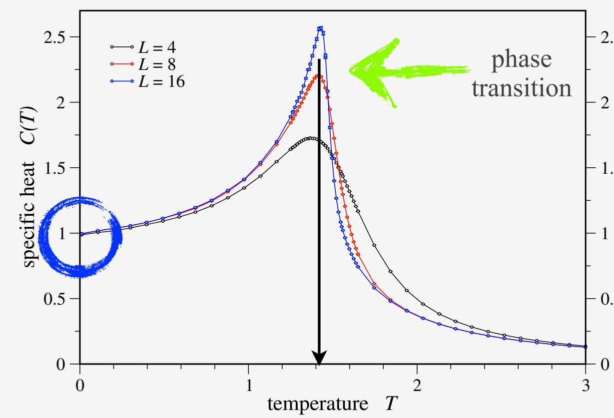
But to a good extent this is also true for the classical model (though much less known).

A first step – numerical simulation

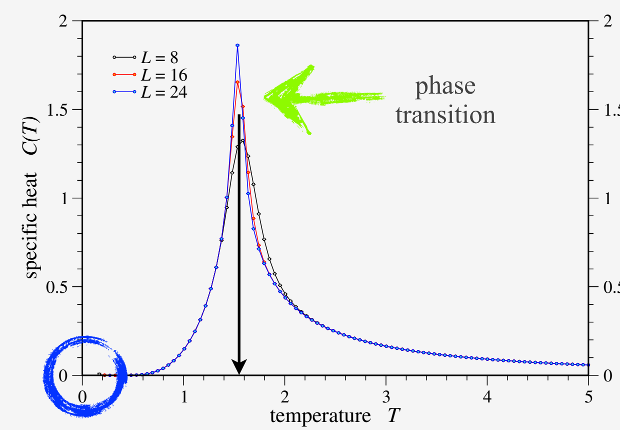
$$H_{\text{Kitaev}} = \sum_{\gamma-\text{links}} J_\gamma S_i^\gamma S_j^\gamma$$



$$H_{\text{Heisenberg}} = \sum_{\gamma-\text{links}} J_\gamma \vec{S}_i \vec{S}_j$$

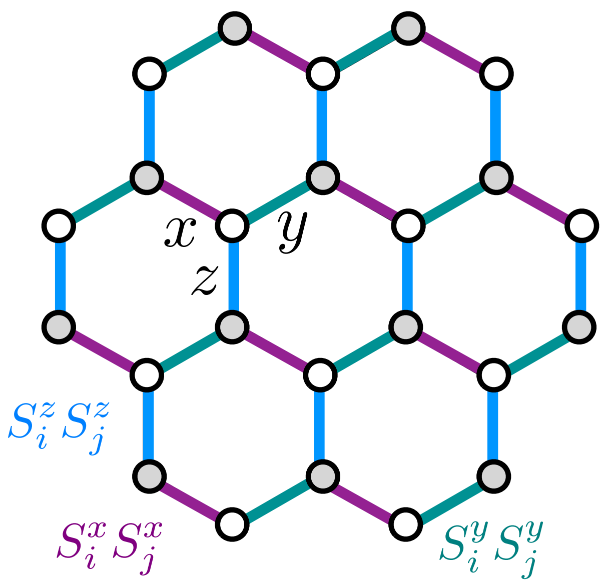


$$H_{\text{Ising}} = \sum_{\gamma-\text{links}} J_\gamma S_i^z S_j^z$$

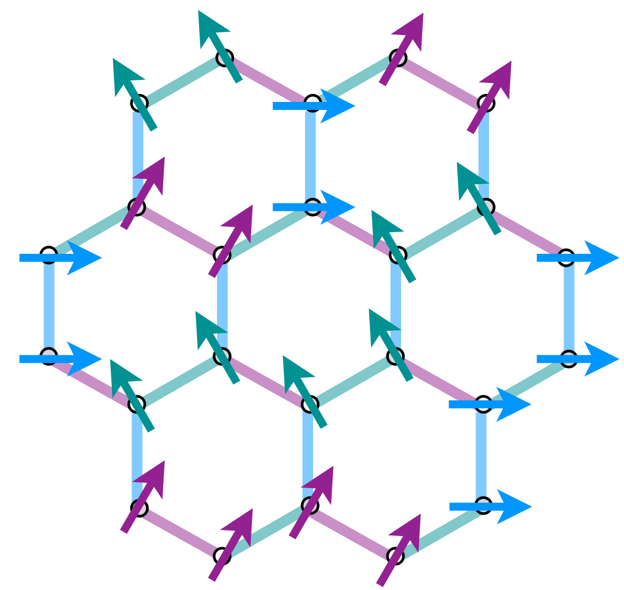


Frustration in the Kitaev model

Observation: no spin configuration can simultaneously satisfy all exchange terms.



T=0 **spin configuration**



$$H_{\text{Kitaev}} = \sum_{\gamma-\text{links}} J_\gamma S_i^\gamma S_j^\gamma$$

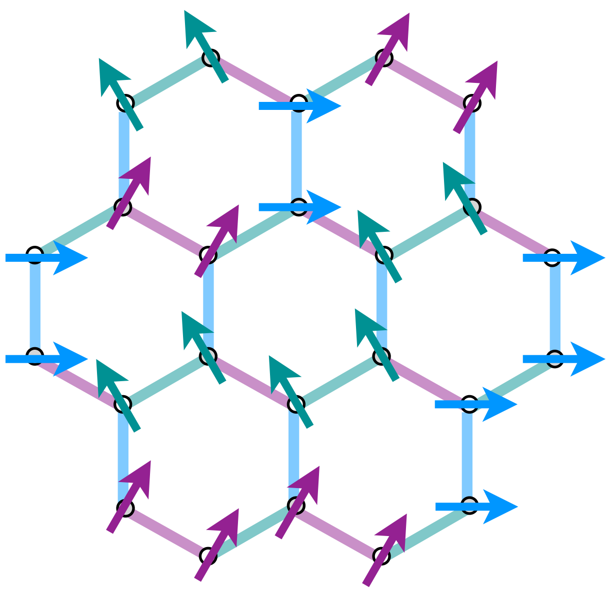
Ising-like* interaction

* preferred direction of spin alignment depends on spatial direction of bond

Every spin can minimize its energy by pointing parallel to precisely one neighbor.

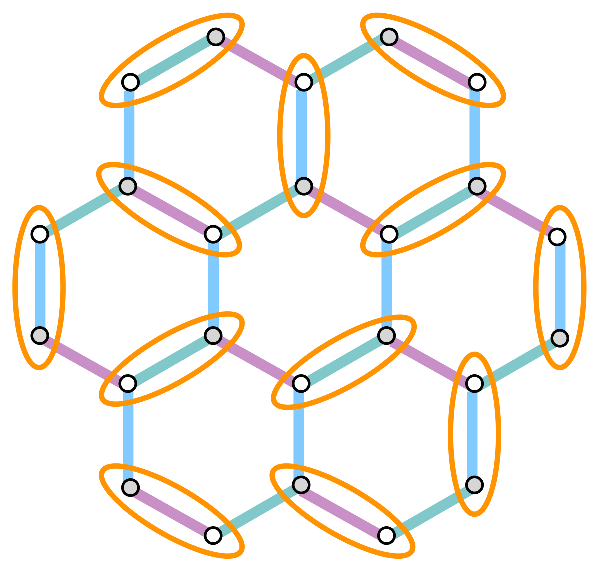
Emergent magnetostatics

T=0 **spin configuration**



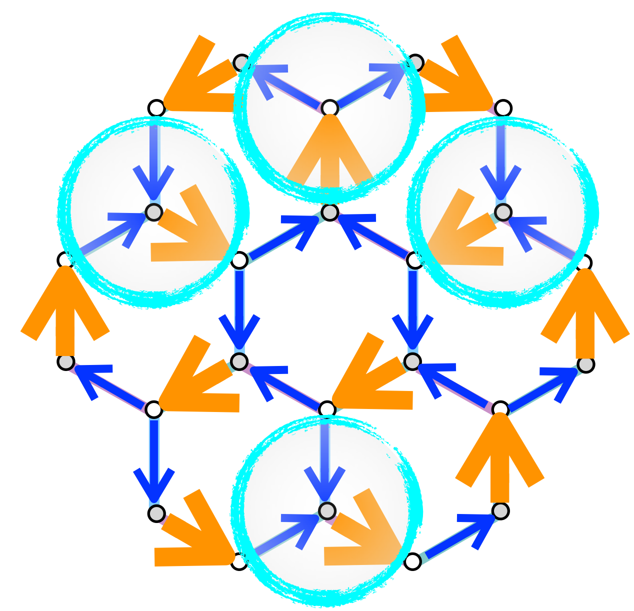
every spin is parallel to *precisely* one neighbor

dimer covering



every site is part of *precisely* one dimer

divergence-free field



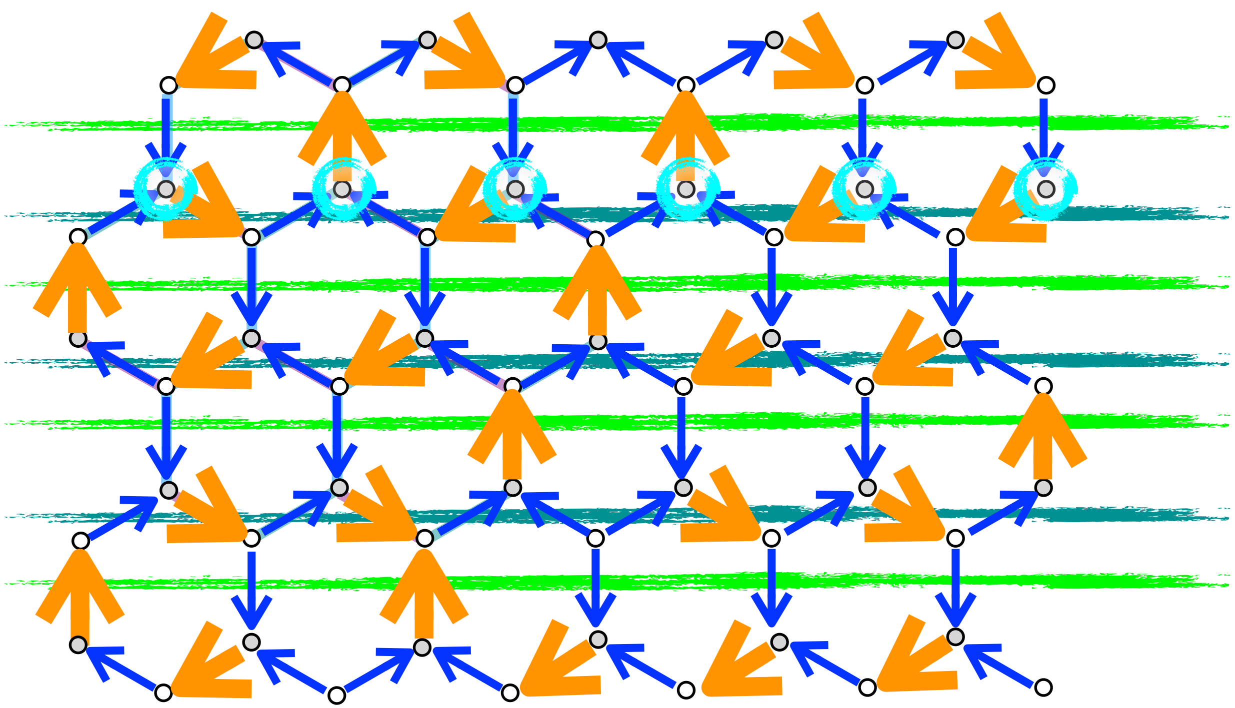
$$\text{div } \vec{B} = 0$$

 $= \hat{e}_{ij}$

 $= -\hat{e}_{ij}/2$

Long-range correlations

divergence-free field



$$\sum_i \vec{b}_i = \vec{M}$$

$$\sum_i \vec{b}_i = -\vec{M}$$

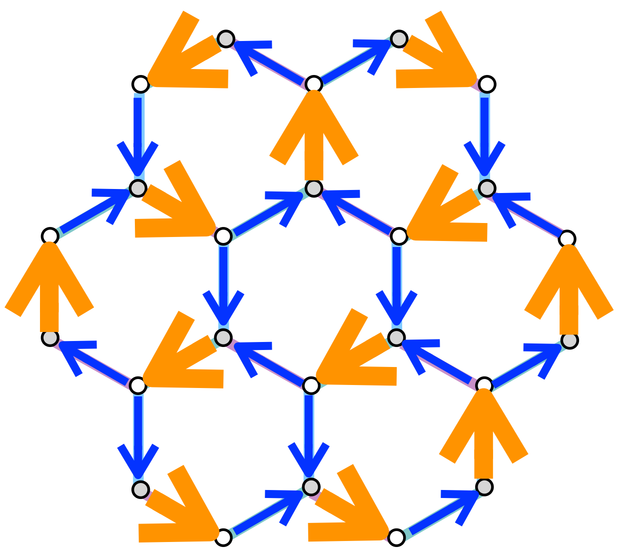
$\text{div } \vec{B} = 0$

An immediate consequence from the strictly enforced **local constraint** of a divergence-free field is the emergence of **long-range correlations**.

Emergent magnetostatics – Coulomb phase

look also at D.A. Huse *et al.*, Phys. Rev. Lett. **91**, 167004 (2003)

divergence-free field



$$\text{div } \vec{B} = 0$$

$= \hat{e}_{ij}$

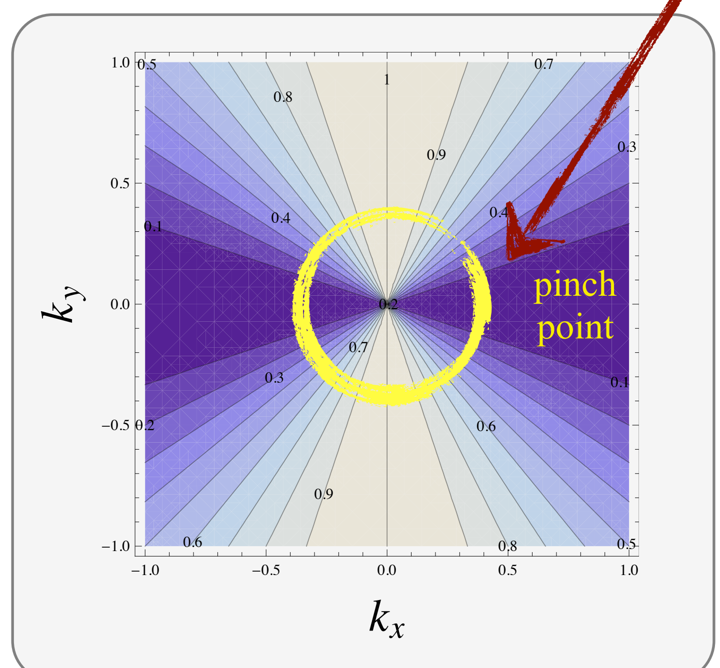
$= -\hat{e}_{ij}/2$

dimer-dimer correlations

$$\langle n(\vec{r})n(0)\rangle \propto \frac{1}{r^2}$$

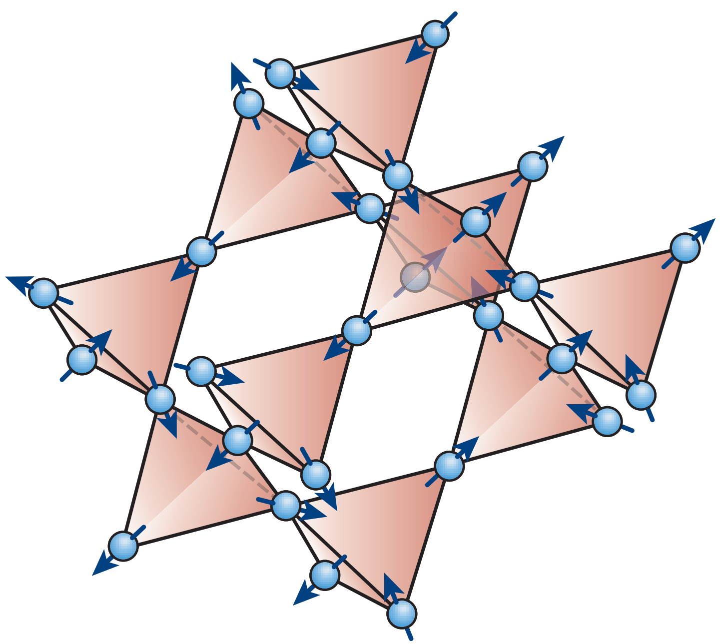
Coulomb phase

... and in Fourier space



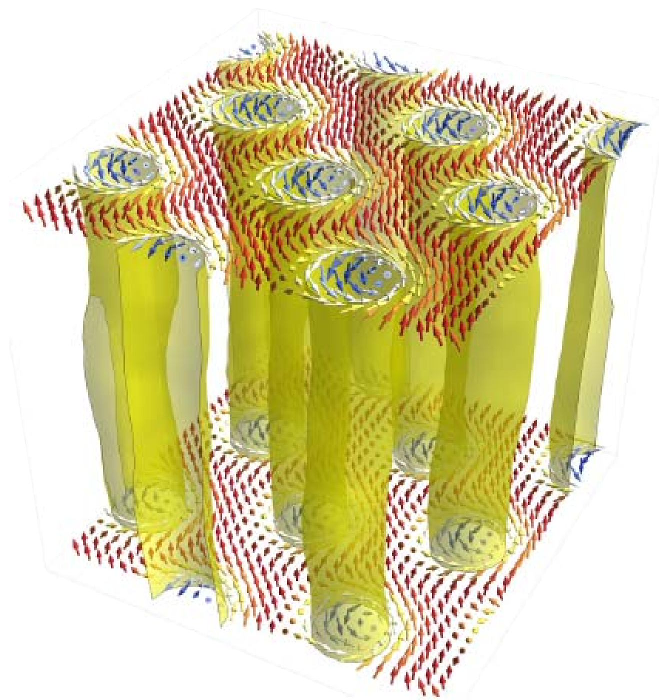
Emergent magnetostatics – Coulomb phase

Such analogies to electromagnetism have also been exploited to discuss the frustrated magnetism in **spin ice** materials and the physics of **skyrmion lattices** in chiral magnets.



spin ice on the pyrochlore lattice

Moessner group
MPI-PKS Dresden

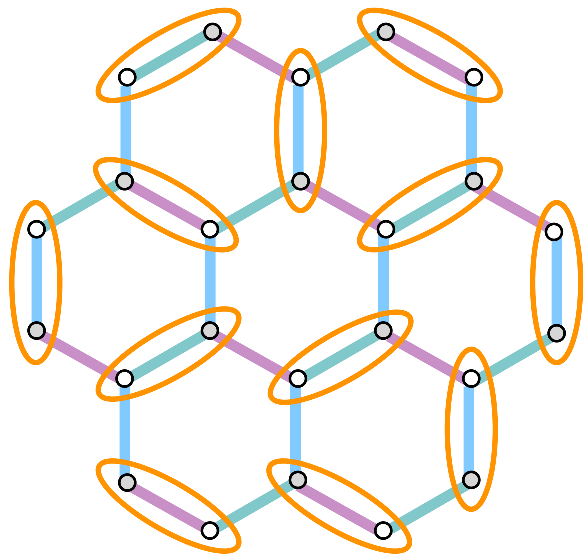


skyrmion lattice in MnSi

Rosch group
University of Cologne

degeneracy – the imprint of frustration

dimer covering



every site is part of
precisely one dimer

The number of dimer coverings
for the hexagonal lattice grows as

$$Z \propto 0.338314^N$$

degeneracy @ $T = 0$

(for periodic boundary conditions)

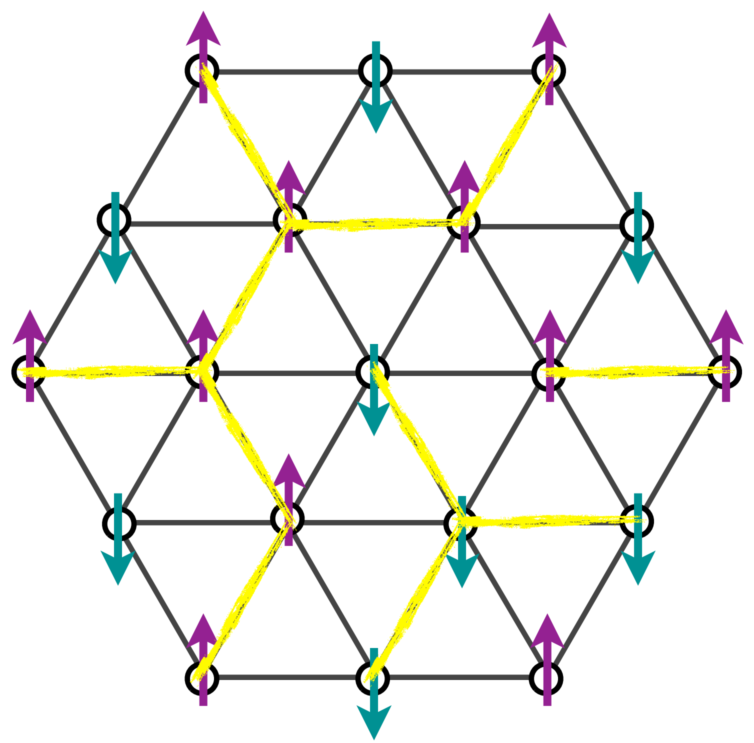
G.H. Wannier, Phys. Rev. **79**, 357 (1950)
P.W. Kasteleyn, J. Math. Phys. **4**, 287 (1963)
V. Elser, J. Phys. A: Math. Gen **17**, 1509 (1984)

At **finite temperature**
this degeneracy will be immediately lifted.
Monomer defects are introduced (and screened).

→ screened Coulomb phase
= high-temperature paramagnet

Triangular lattice Ising model

G.H. Wannier, Phys. Rev. **79**, 357 (1950)



$$H_{\text{Ising}} = \sum_{\gamma-\text{links}} J_\gamma S_i^z S_j^z$$

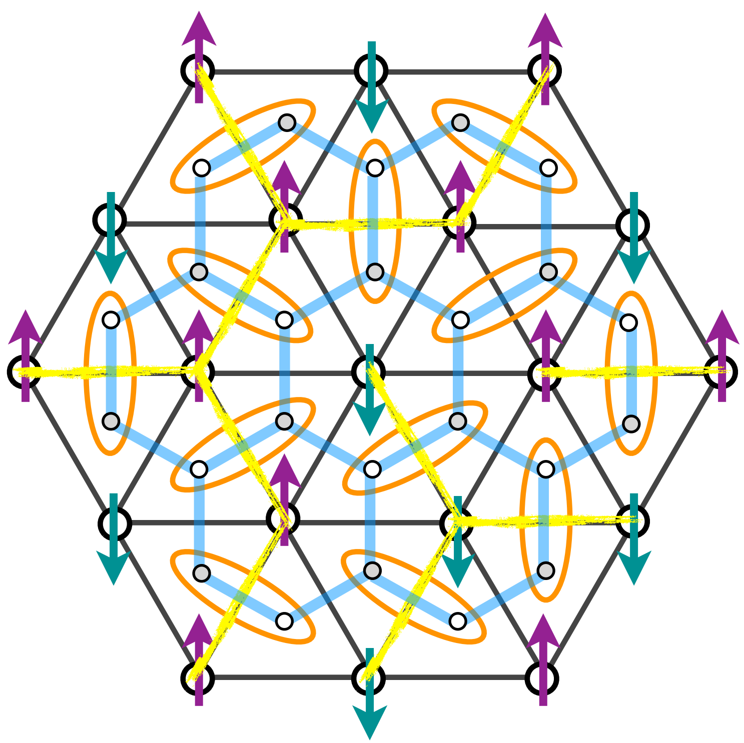
antiferromagnetic

T=0 **spin configuration**

precisely one frustrated bond per triangle

Triangular lattice Ising model

G.H. Wannier, Phys. Rev. **79**, 357 (1950)



$$H_{\text{Ising}} = \sum_{\gamma-\text{links}} J_\gamma S_i^z S_j^z$$

antiferromagnetic

T=0 **spin configuration**

precisely one frustrated bond per triangle

T=0 **dual dimer configuration**

precisely one dimer per site on dual honeycomb lattice

$Z \propto 0.338314^N$
degenerate spin configurations

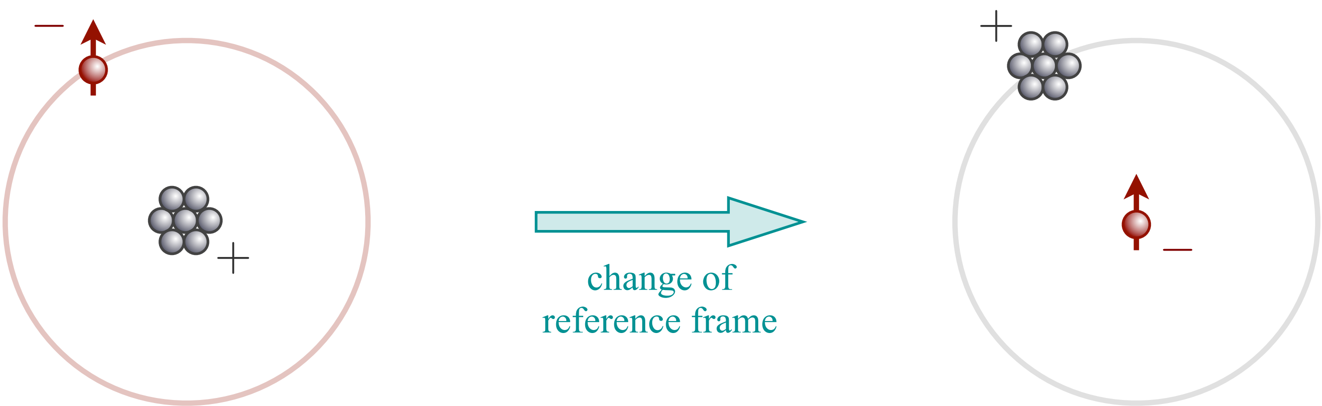
Coulomb correlations
$\langle S^z(\vec{r}) S^z(0) \rangle \propto \frac{1}{r^2}$

Where do we find this physics?

– an excursion to spin-orbit entangled Mott insulators –

Spin-orbit coupling

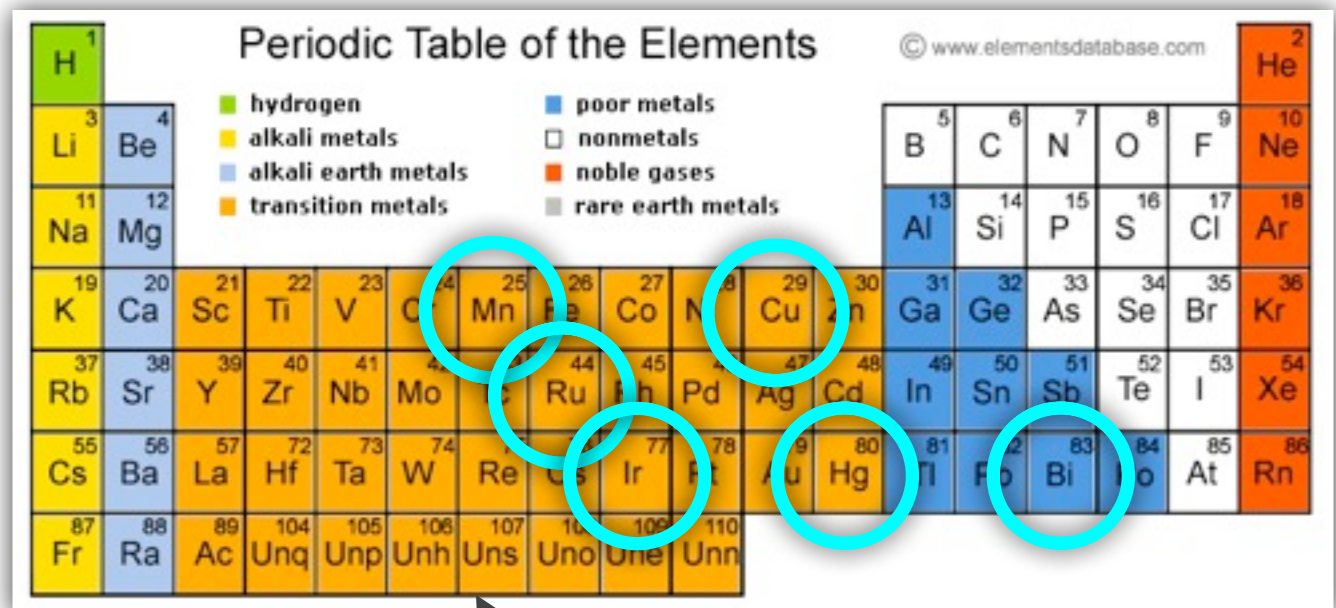
Spin-orbit coupling 101 – quantum mechanics lecture



$$\Delta E = \frac{\lambda}{\hbar^2}\,\vec{l}\cdot\vec{s} = \frac{\lambda}{2}\left[j(j-1) - l(l-1) - s(s-1)\right]$$

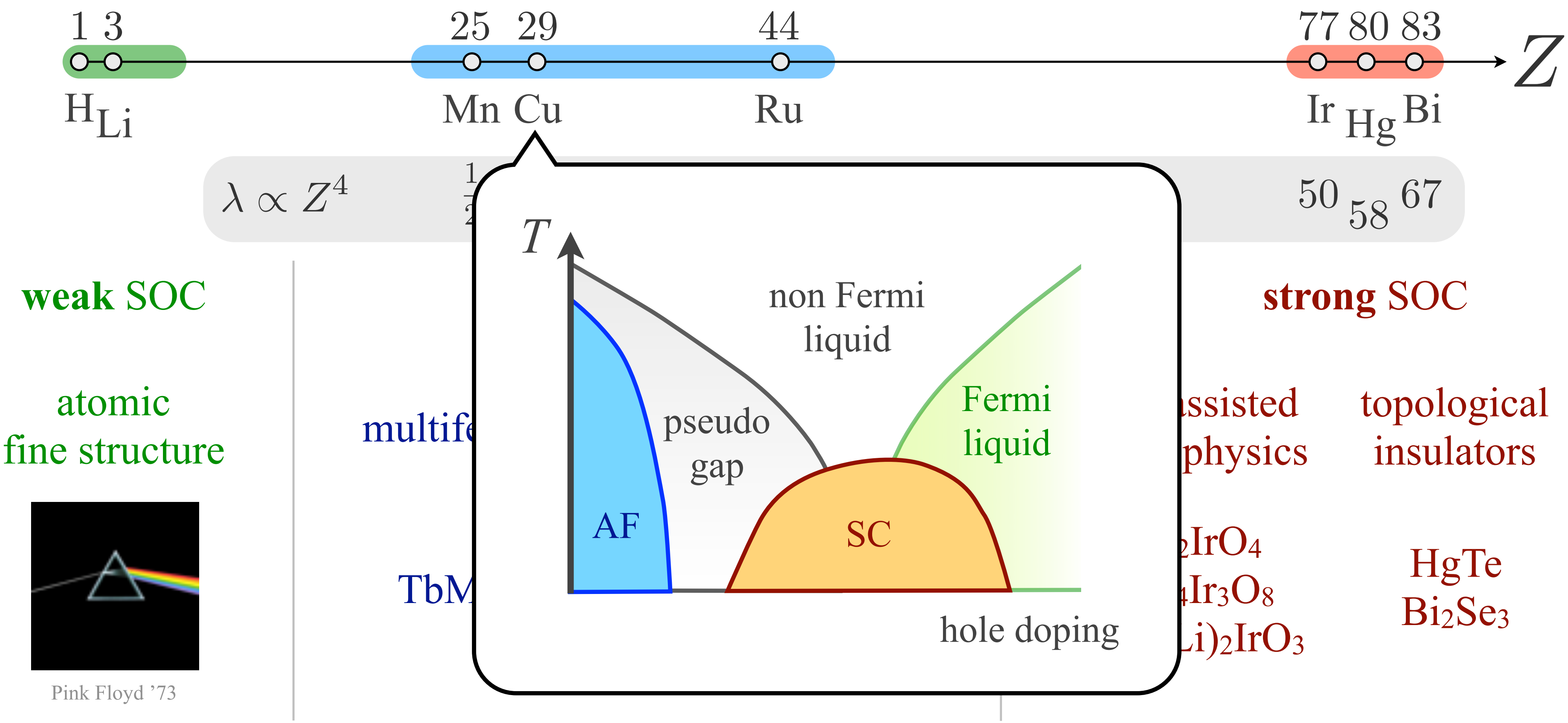
$$\lambda = \frac{Ze^2\mu_0\hbar^2}{8\pi m_e^2 r^3} \qquad r \propto 1/Z \qquad \lambda \propto Z^4$$

Spin-orbit coupling in condensed matter



typically we will be interested
in transition metal oxides

Spin-orbit coupling in condensed matter



Moderate SOC versus magnetic exchange

Microscopic interactions

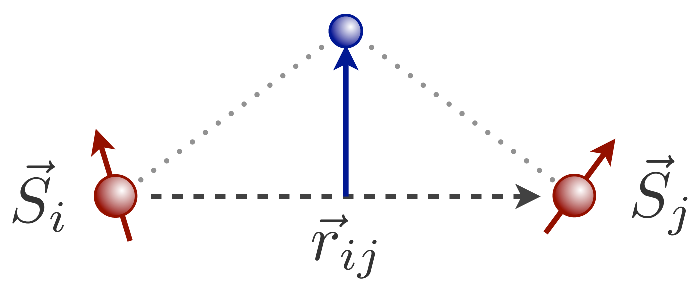
$$H_{\text{Hubbard}} = -t \sum_{\langle ij \rangle, \sigma} c^{\dagger}_{j\sigma} c_{i\sigma} + \text{h.c.} + U \sum_j n_{j\uparrow} n_{j\downarrow} - \mu \sum_j (n_{j\uparrow} + n_{j\downarrow})$$

Mott regime $t \ll U$ $\longrightarrow$

$$H_{\text{Heisenberg}} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \qquad J = -t^2/U$$

Dzyaloshinsky-Moriya interaction

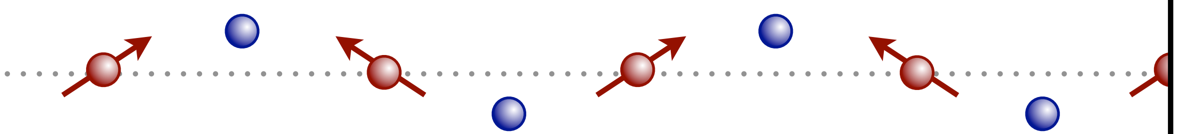
$$H_{\text{DM}} = \sum_{\langle ij \rangle} \vec{D}_{ij} \cdot \left(\vec{S}_i \times \vec{S}_j \right)$$

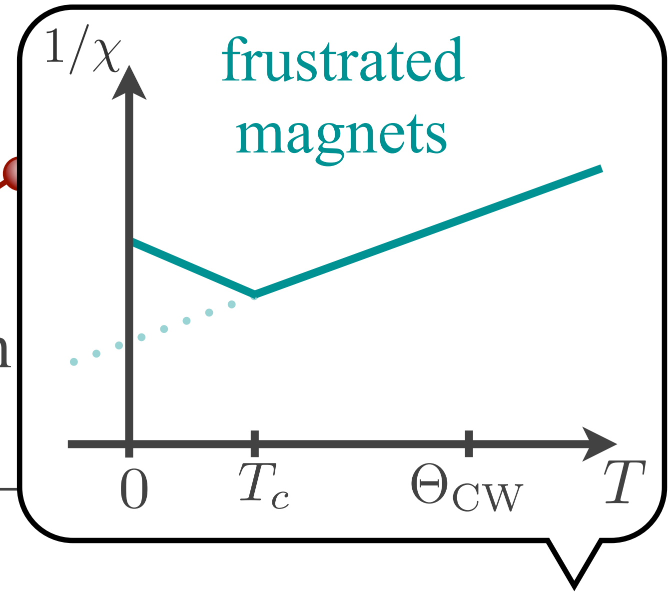


$$\vec{D}_{ij} \propto \lambda \, \vec{x} \times \vec{r}_{ij}$$

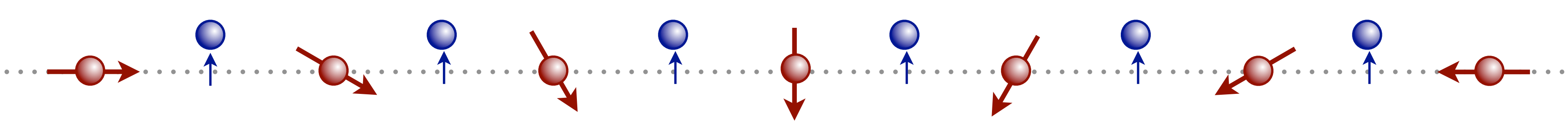
SOC induced ferroics

Dzyaloshinsky-Moriya interaction favors **non-collinear** ordering.





Weak **ferromagnetism** induced in



For **spin spiral** $\vec{S}_i \times \vec{S}_j$ points in the same direction for all pairs.

Weak **ferroelectricity** induced in $TbMnO_3$.

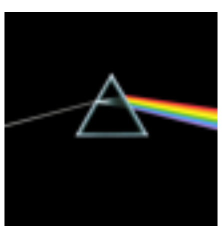
$\longrightarrow$ magnetoelectric effect, multiferroics

Spin-orbit coupling in condensed matter

Z	1	3	25	29	44	77	80	83
	H	Li	Mn	Cu	Ru	Ir	Hg	Bi
$\lambda \propto Z^4$			$\frac{1}{2}$	1	5	50	58	67

weak SOC

atomic
fine structure



Pink Floyd '73

moderate SOC

multiferroics

unconventional
superconductor

$TbMnO_3$

Sr_2RuO_4

SOC induced DM interaction
competes with **magnetic exchange**

strong SOC

SO-assisted
Mott physics

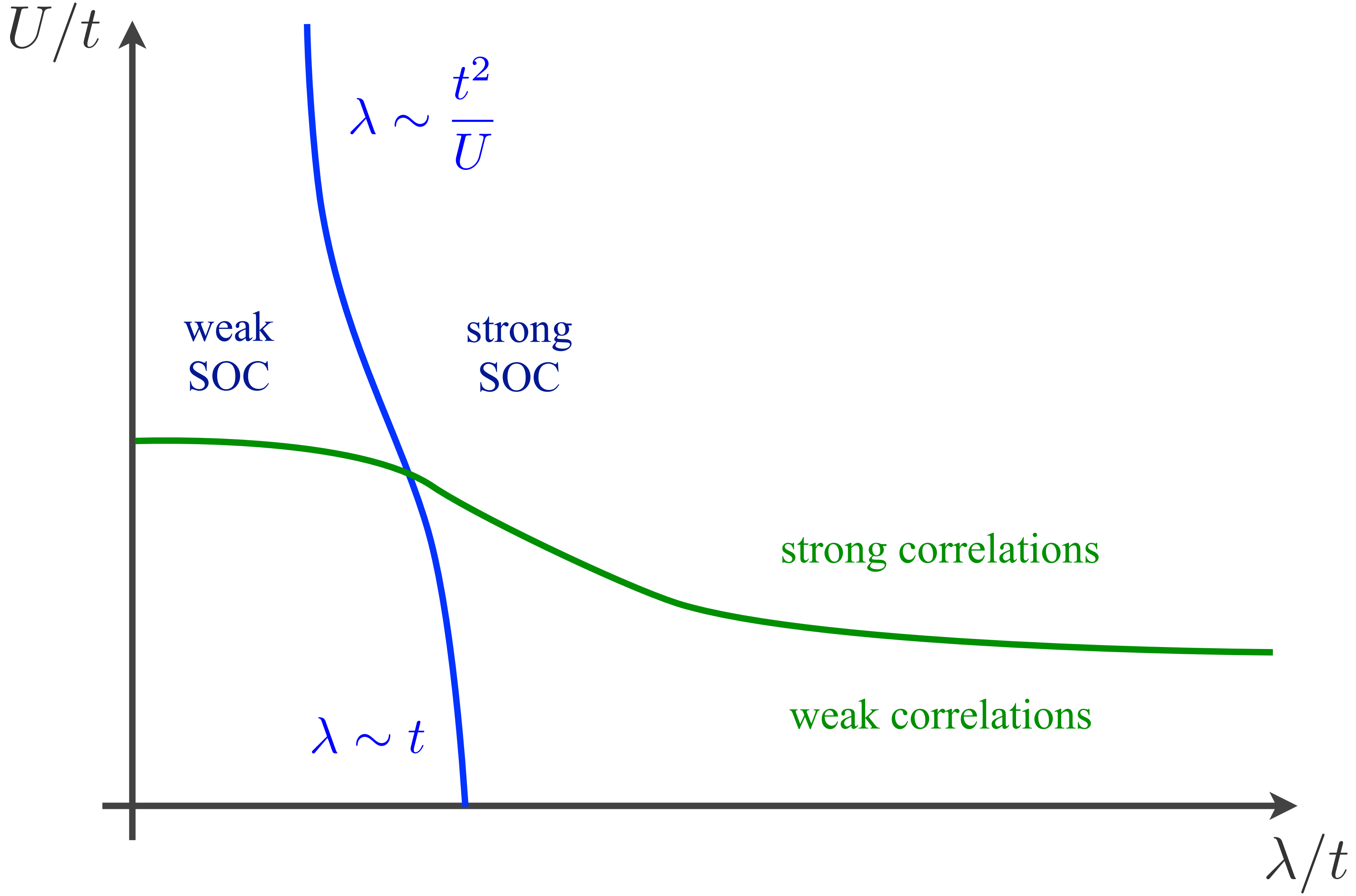
topological
insulators

Sr_2IrO_4
$Na_4Ir_3O_8$
$(Na,Li)_2IrO_3$

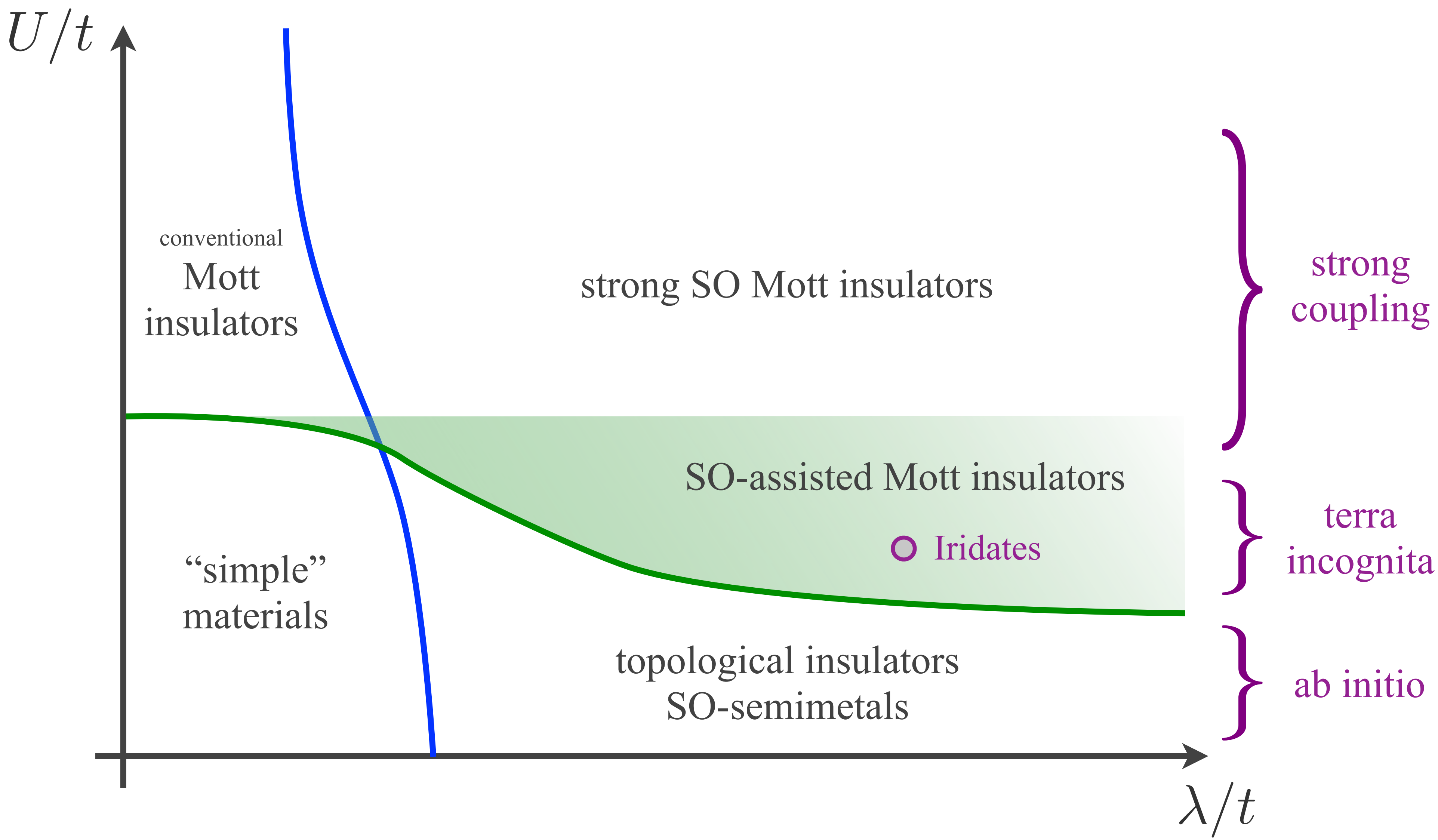
HgTe
Bi_2Se_3

SOC competes directly
with **Hubbard physics**

Strong SOC versus Hubbard physics

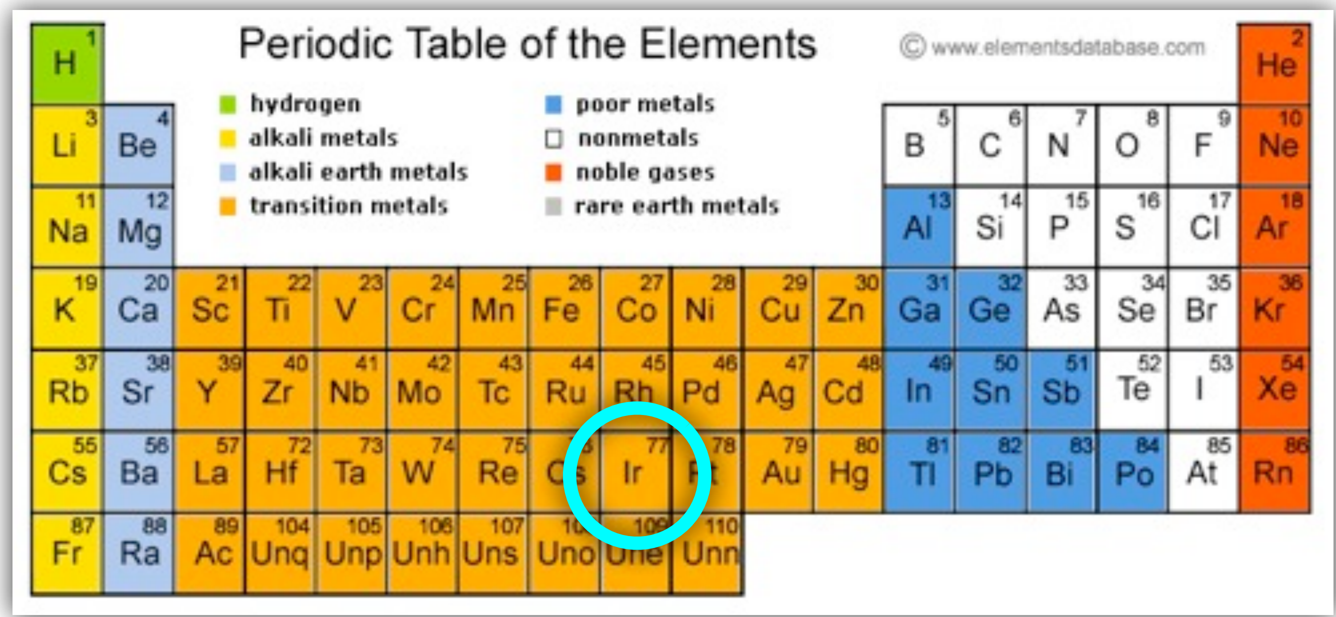


Strong SOC versus Hubbard physics



Spin-orbit assisted Mott physics

5d transition metal oxides



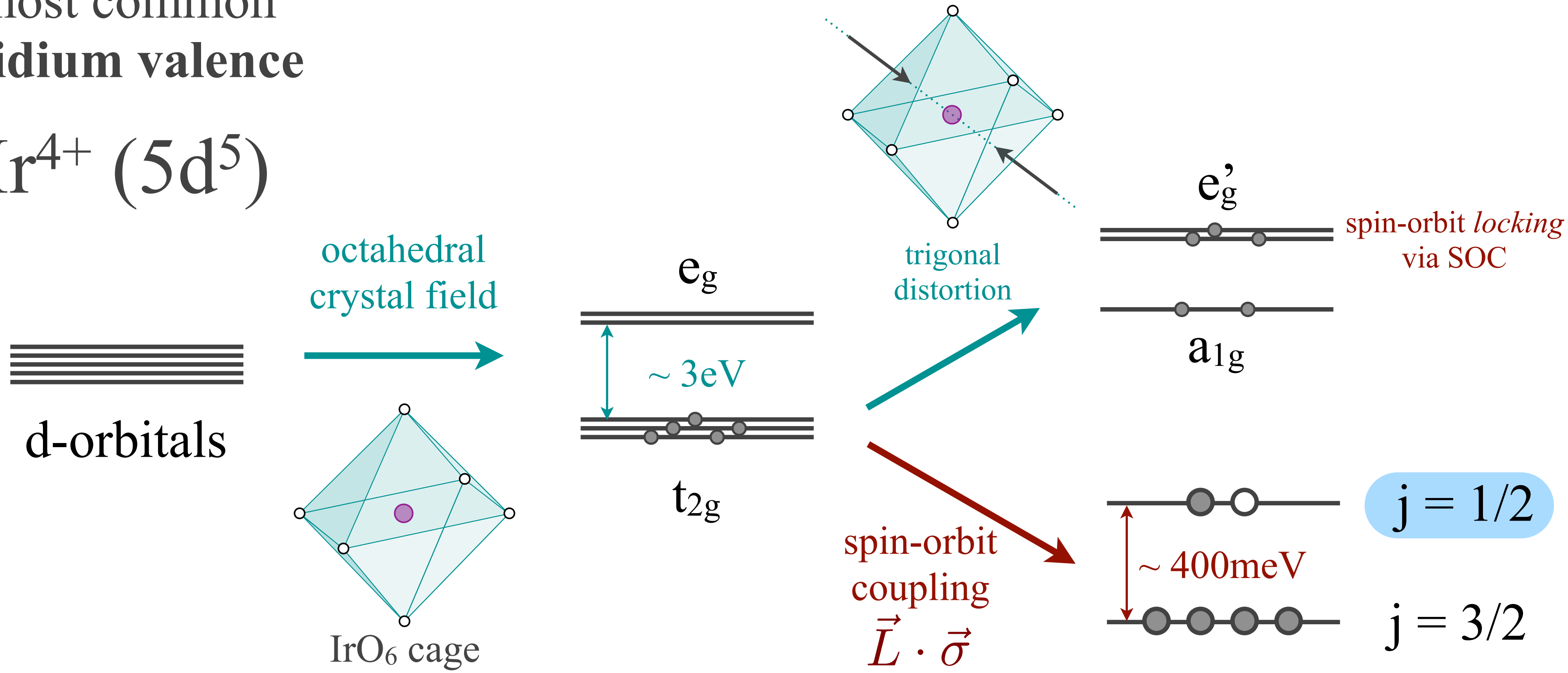
"heavy" → **strong** spin-orbit coupling

"large" → **weak** tendency to form Mott insulators
enhanced sensitivity to crystal fields

Spin-orbit entanglement in Iridates

most common
Iridium valence

Ir^{4+} $(5d^5)$



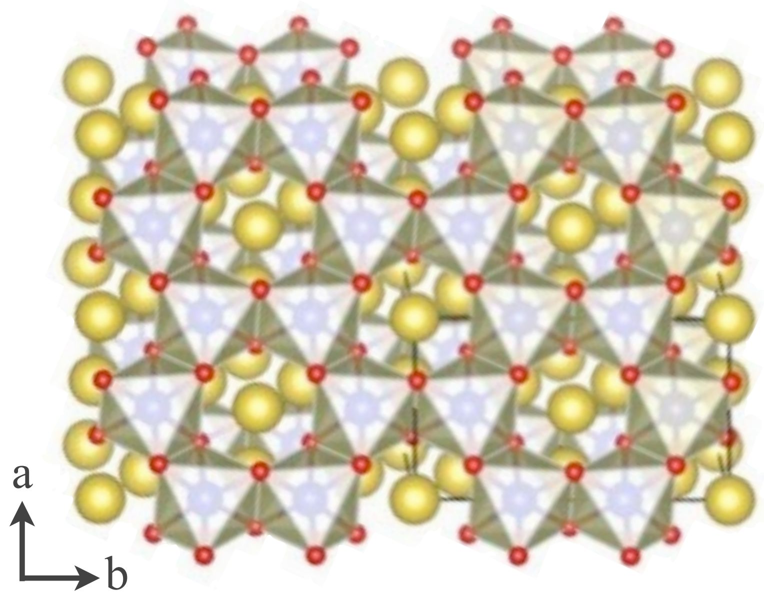
Sr_2IrO_4

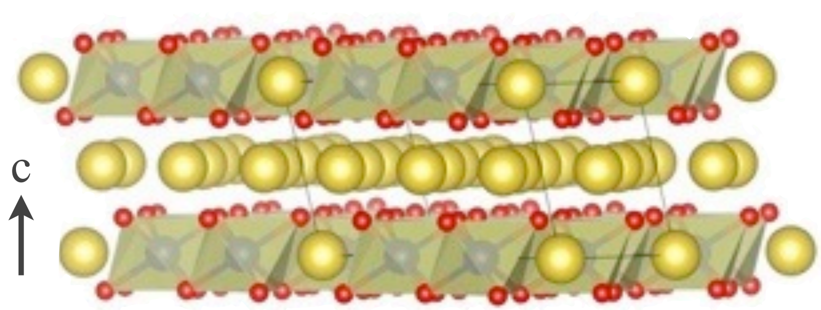
B.J. Kim *et al.* PRL **101**, 076402 (2008)
B.J. Kim *et al.* Science **323**, 1329 (2009)

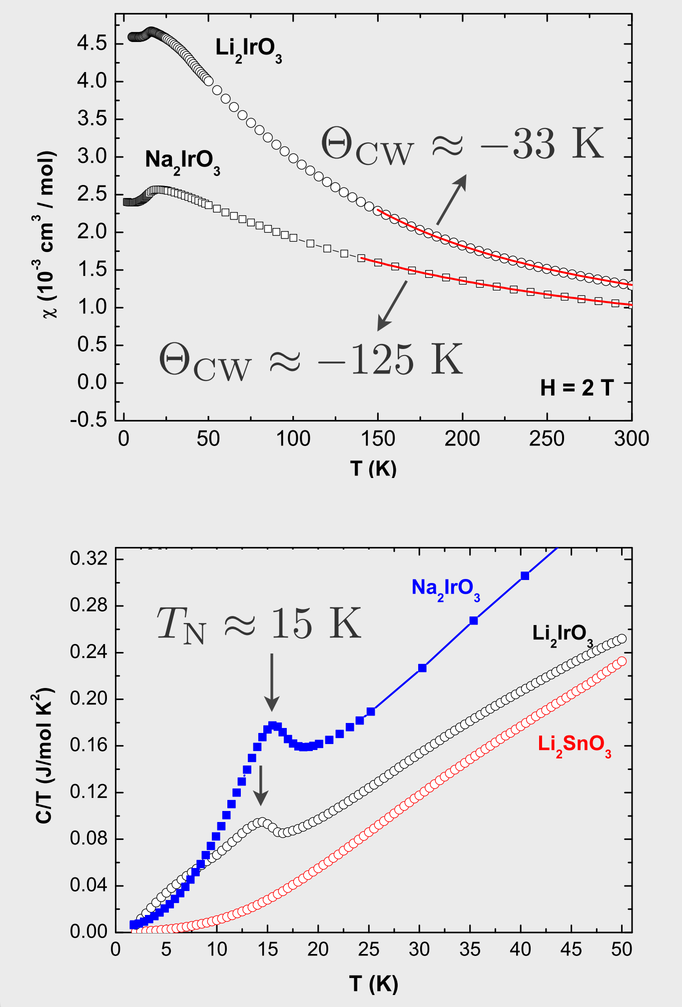
$(Na,Li)_2IrO_3$

Spin-orbit assisted Mott physics in Iridates

$(Na,Li)_2IrO_3$

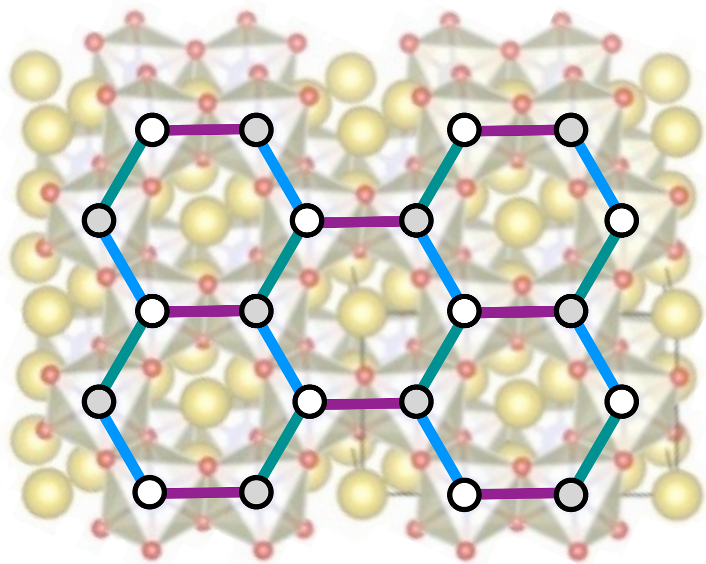


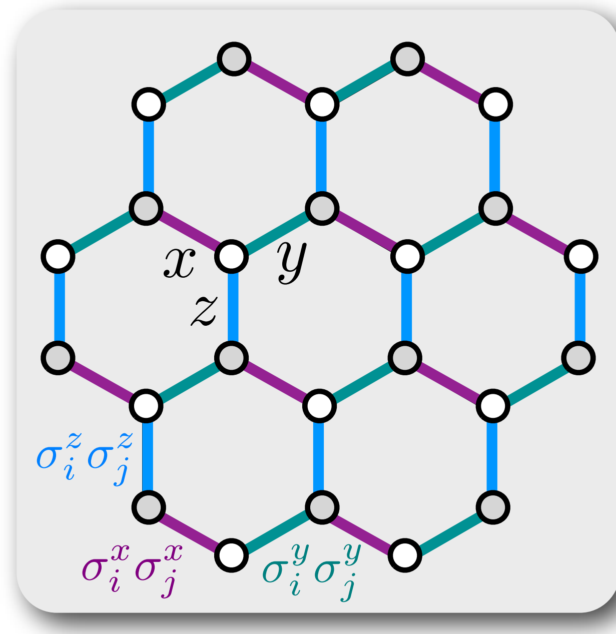




Spin-orbit assisted Mott physics in Iridates

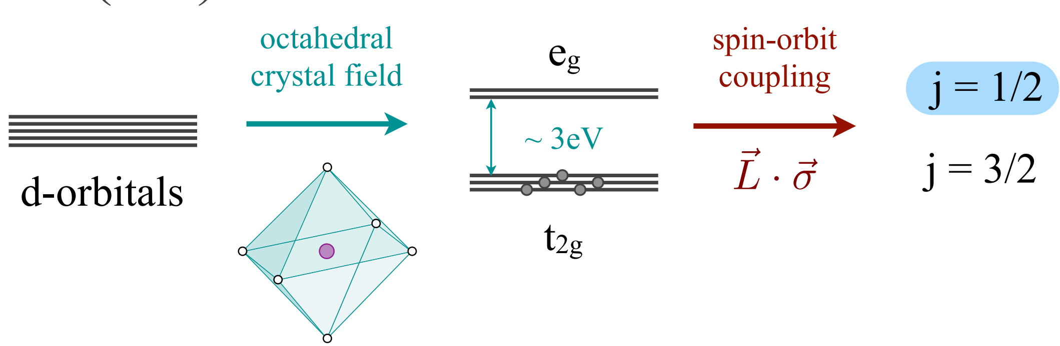
$(Na,Li)_2IrO_3$





$$H_{\text{Kitaev}} = \sum_{\gamma-\text{links}} J_\gamma \sigma_i^\gamma \sigma_j^\gamma$$

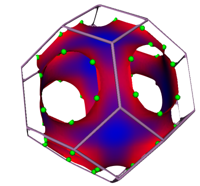
Ir^{4+} $(5d^5)$

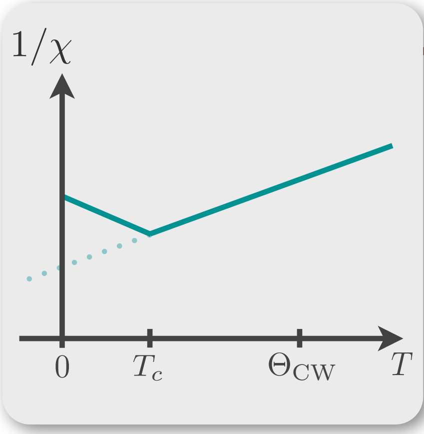


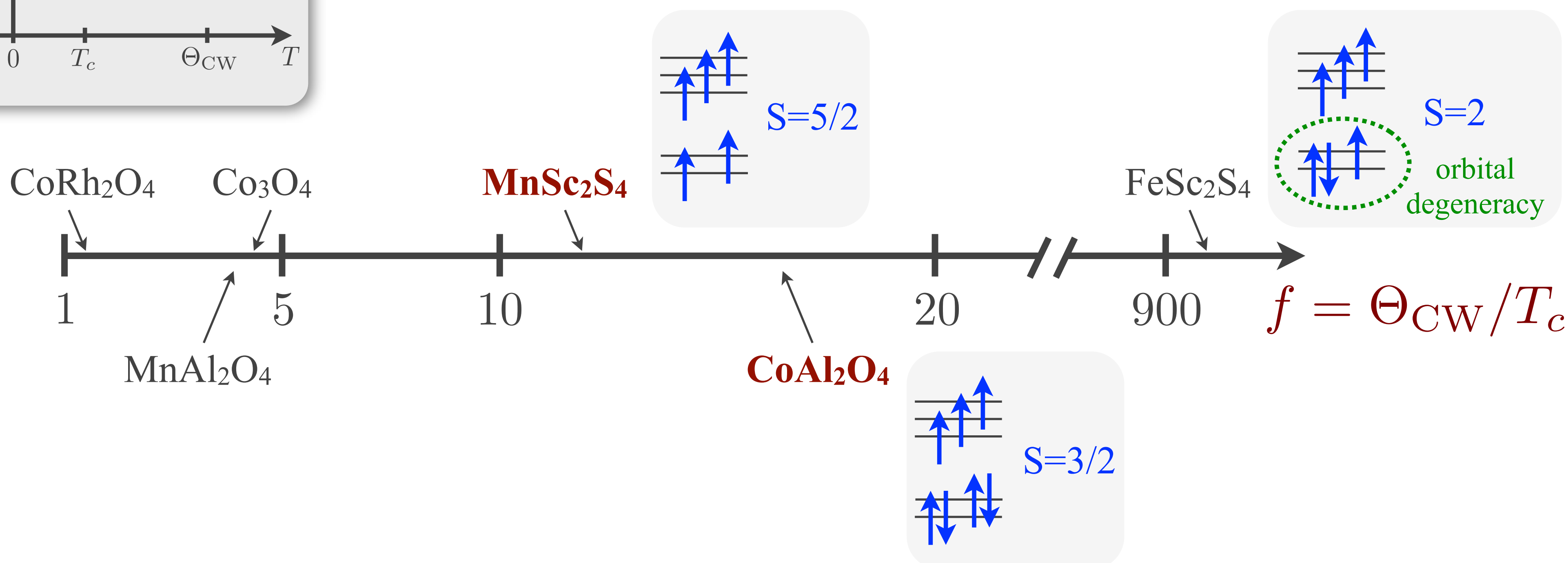
Rare combination of a model of **fundamental conceptual importance** (harboring topological phases) and an **exact analytical solution**.


spiral spin liquids

Frustrated magnets: A-site spinels





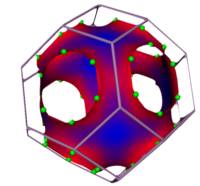


Materials forming the normal spinel structure AB_2X_4.

Focus: Spinels with magnetic A-sites (only).

Material synthesis: Loidl group (Augsburg) and Takagi group (Tokyo)

Frustration in A-site spinels

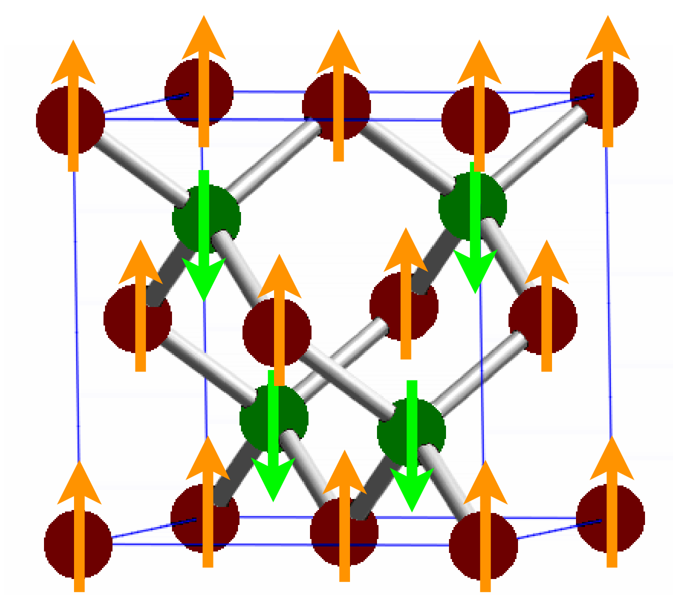


magnetic exchange

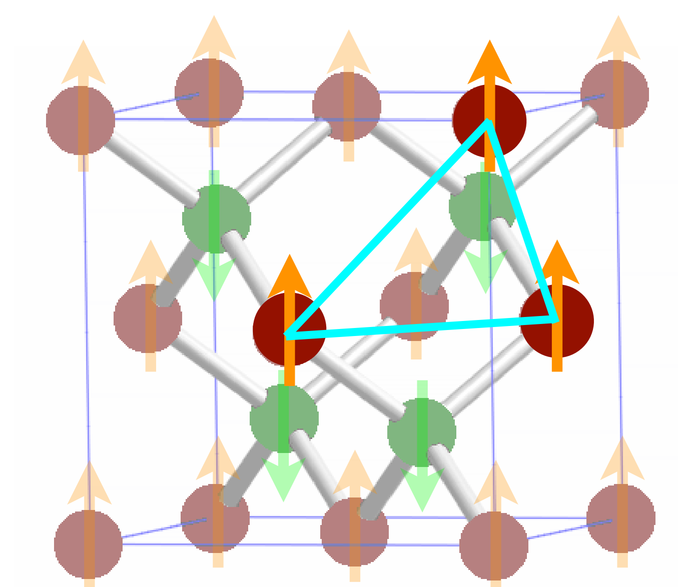
$$H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$

favors Néel state

introduces frustration

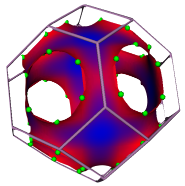


diamond lattice



FCC sublattice

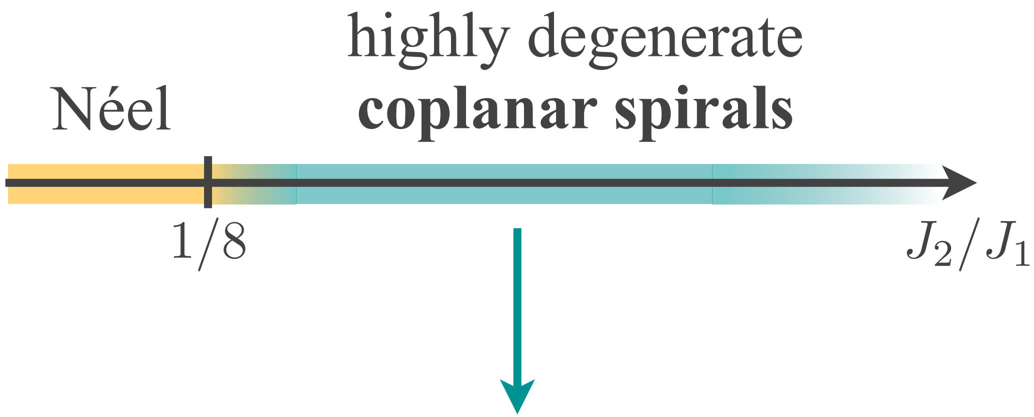
Frustration in A-site spinels

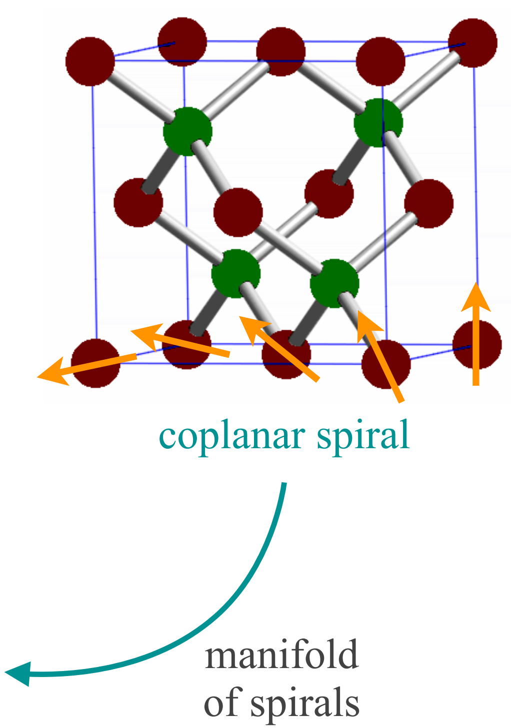


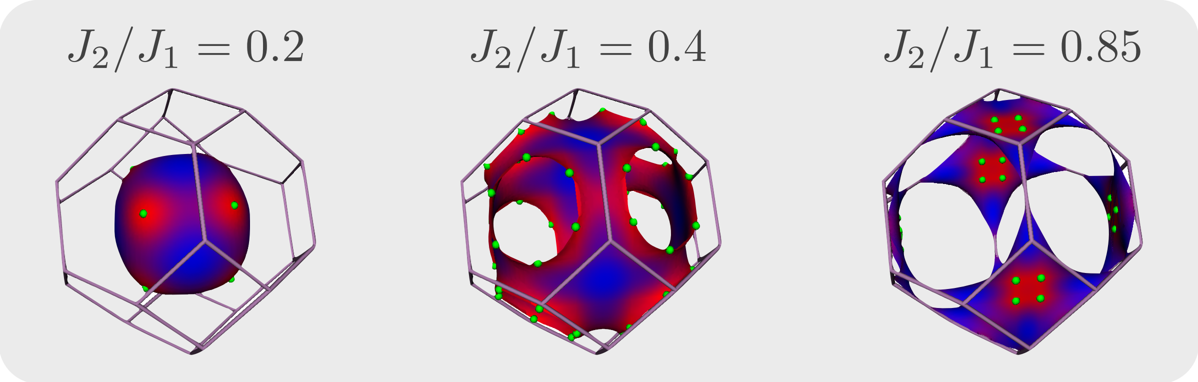
Nature Physics **3**, 487 (2007).

magnetic exchange

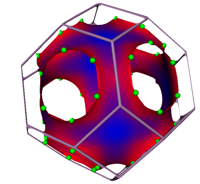
$$H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$



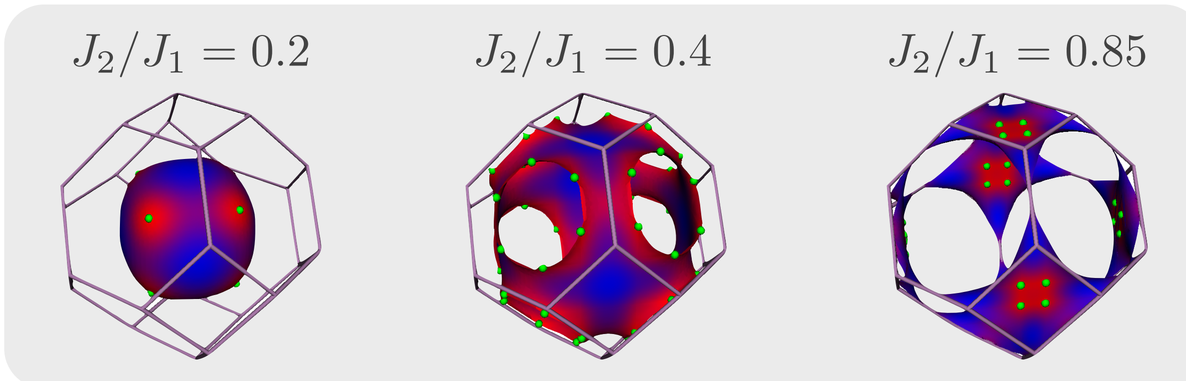


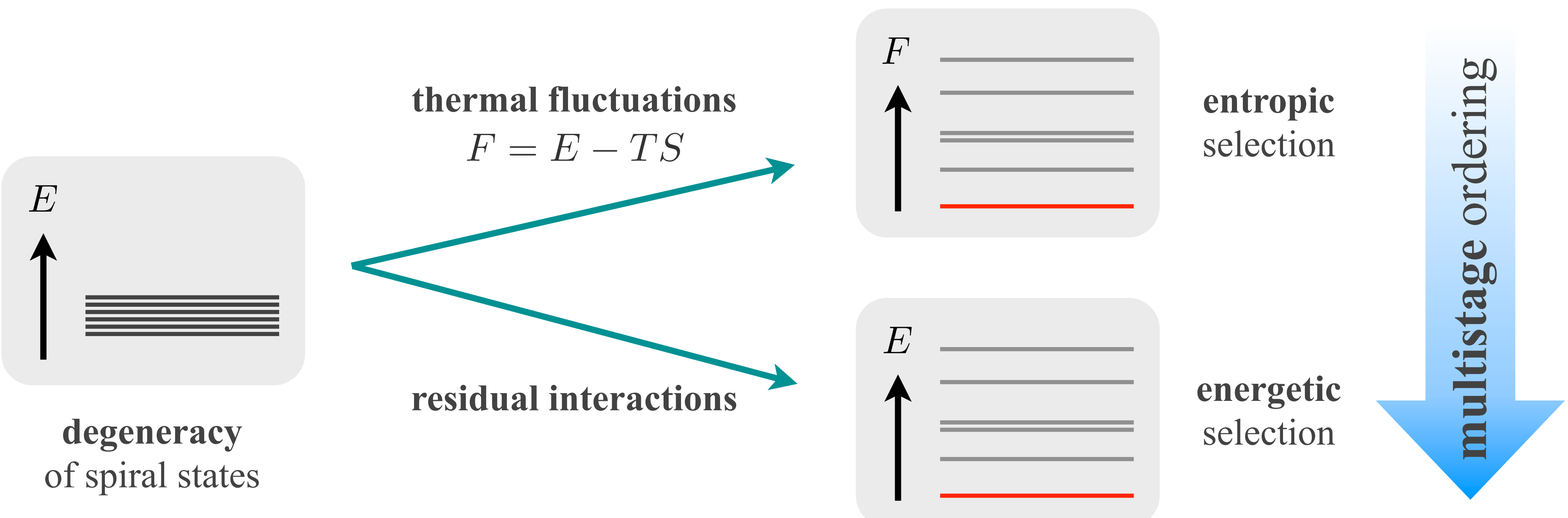


Order by disorder

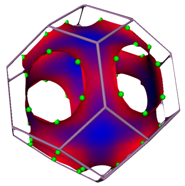


Nature Physics **3**, 487 (2007).

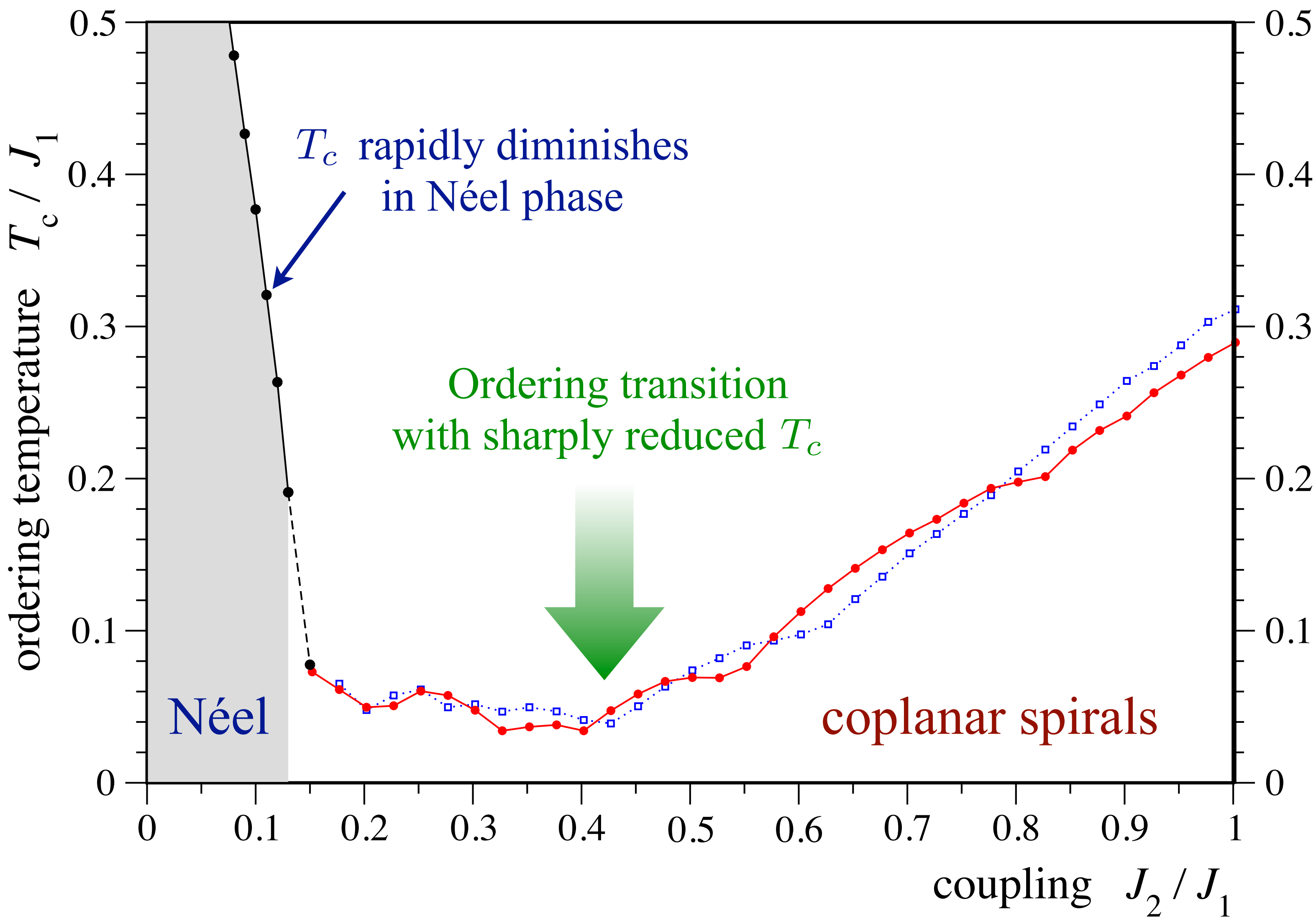




Phase diagram

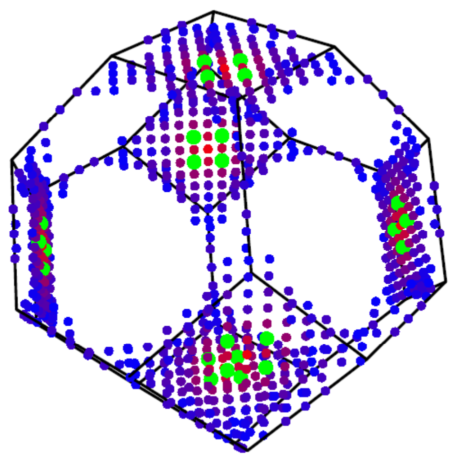


Nature Physics **3**, 487 (2007).

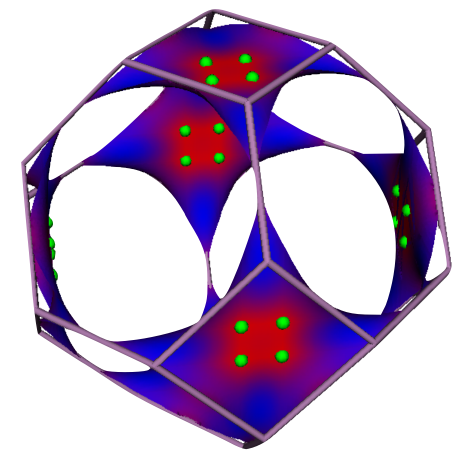


Ordering transition: magnetic correlations

numerical **structure factor**



analytic **free energy**



$J_2/J_1 = 0.85$

$MnSc_2S_4$

Spin structure factor directly images **"spiral surface"**.

Free energy corrections visible

$MnSc_2S_4$ multistage ordering

A. Krimmel *et al.*, PRB **73**, 014413 (2006); M. Müksch *et al.* (2007)

spiral order
$\vec{q} = 2\pi(3/4, 3/4, 0)$

spiral order
diffusive scattering

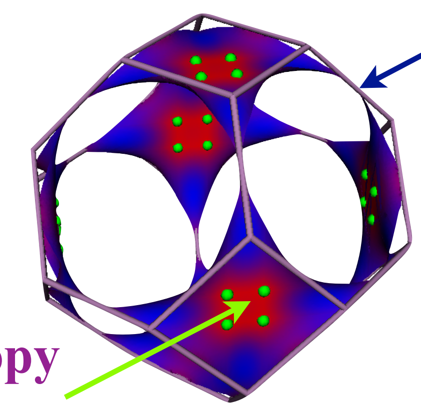
spin liquid
diffusive scattering

high temperature paramagnet

0 1.9K 2.3K Θ_{CW} T

energetic selection

entropic selection



entropy favors 100 order

experiment observes 110 order

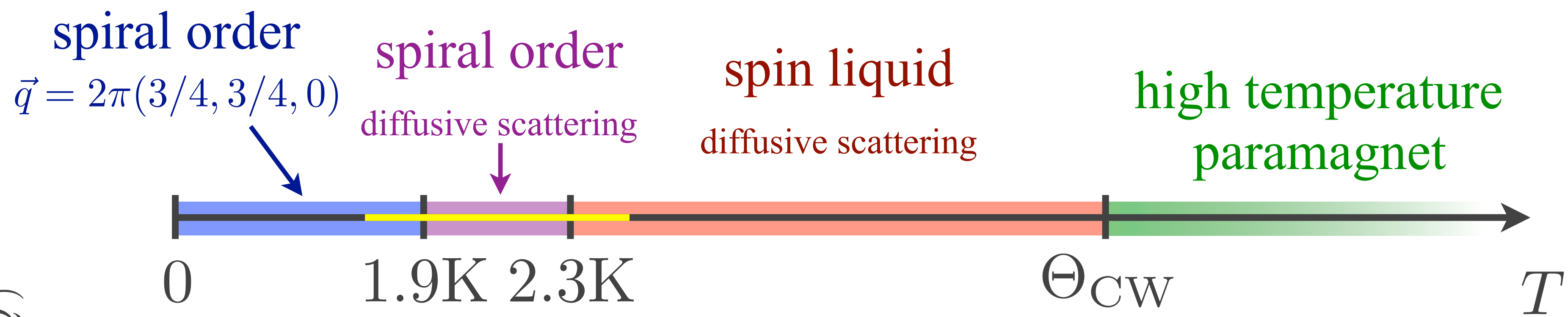
favored by AFM J_3

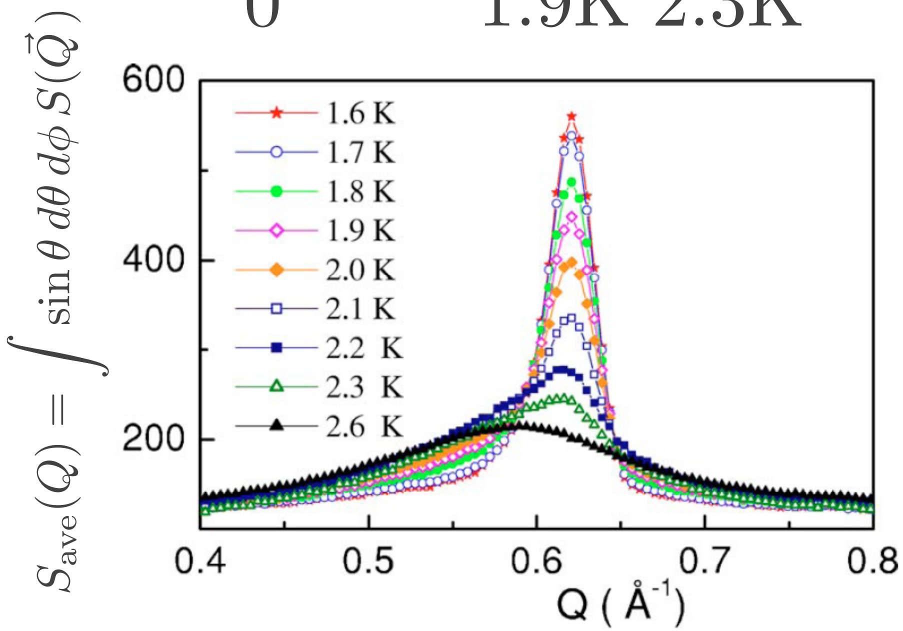
model parameters

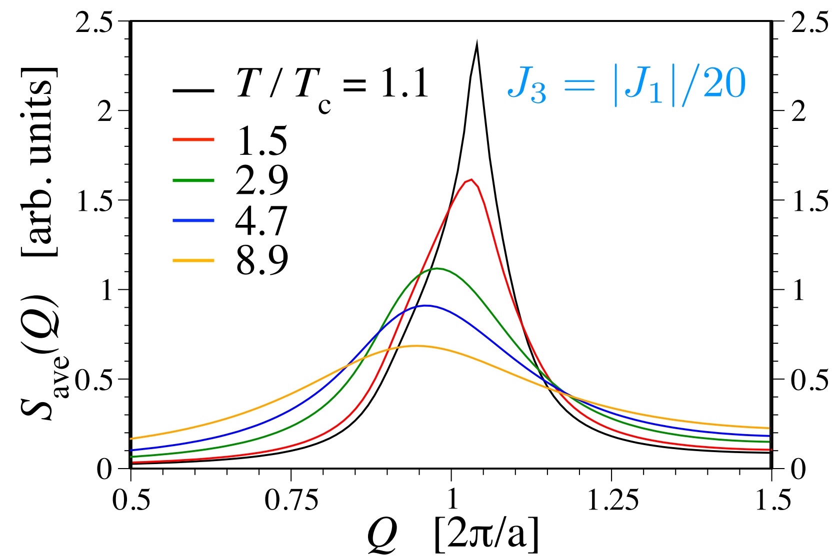
- J_1 is ferromagnetic
- $J_2/|J_1| \approx 0.85$
- $\Theta_{\mathrm{CW}} \approx -22.1\mathrm{K}$ gives
  $J_1 \approx 1.2\mathrm{K}$ $J_2 \approx 1.0\mathrm{K}$
  $T_c \approx 2.4\mathrm{K}$

$MnSc_2S_4$ diffusive scattering

A. Krimmel *et al.*, PRB **73**, 014413 (2006); M. Müksch *et al.* (2007)







Intensity shifts from $|\vec{q}|$ to "spiral surface" as T washes out J_3.

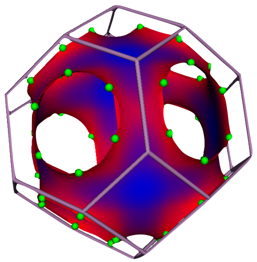
Consistent with "spiral spin liquid".

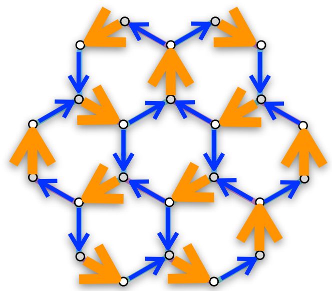
We are done with part I!

So what did we learn?

Summary

- Frustrated magnets are a source of **remarkably diverse behavior**
  - complex collective phenomena
  - exotic ordered phases
  - spin liquids





- Frustration brings along an enhanced **sensitivity to otherwise residual effects**, which will split degenerate states and reorganize the collective state of a system.

All slides of this presentation will become available on our group webpage at **www.thp.uni-koeln.de/trebst**