At zero temperature, BEC is described by the time-dependent Gross-Pitaevskii equation
\[ \frac{i}{\hbar} \frac{\partial \Psi(r,t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi(r,t) + V(r) + gN(\Psi(r,t))^{2} + g_1 N^2(\Psi(r,t))^4 \]
where \( V(r) = \frac{1}{2} \mu r^2 \) is harmonic trap with anisotropy \( \mu \), and \( g, g_1 \) are parameters of two- and three-body interactions, respectively.

By using the Gaussian variational ansatz \([4]\), we obtain equations for condensate widths \( u_p \) and \( u_t \) in the dimensionless form:
\[ \frac{d}{dt} \left( \begin{array}{l} u_p(t) \\ u_t(t) \end{array} \right) = \left( \begin{array}{cc} 0 & \frac{1}{2} \lambda \tilde{u}_p(t) \\ \frac{1}{2} \lambda \tilde{u}_t(t) & 0 \end{array} \right) \left( \begin{array}{l} u_p(t) \\ u_t(t) \end{array} \right) + \frac{1}{2} \frac{k}{\mu} \tilde{u}_p(t) u^2_t(t) - \frac{1}{2} \frac{k}{\mu} \tilde{u}_t(t) u^2_p(t) = 0. \]

Dimensionless parameters are \( p = \frac{\omega}{\omega_0} \), \( \lambda = \sqrt{\frac{\mu}{\hbar}} \), \( N \) is the number of particles, \( a \) is the s-wave scattering length, and \( \lambda \) is the oscillator length.

- Initial state: \( u(0) = u_0 + e \omega_{\perp} \tilde{u}(0) / 0 \)
- Real-time dynamics for \( p = 1, k = 0.001 \), and \( \epsilon = 0.1 \)

**Equilibrium positions:**
\[ u_{eq} = \frac{1}{\omega_{\perp}} + \frac{1}{\omega_{\parallel}} + \frac{k}{\mu} \tilde{u}_{eq} = \frac{1}{\omega_{\perp}} + \frac{1}{\omega_{\parallel}} + \frac{k}{\mu} \tilde{u}_{eq} \]

**Frequencies of collective modes:**
\[ \omega_{\parallel}^2 = m_1 + m_2 \pm \sqrt{(m_1 - m_2)^2 + 8m_1} \]

where \( m_1 = 4 + \frac{2\lambda^2}{\hbar^2}, m_2 = \frac{4\lambda^2}{\hbar^2}, m_3 = 4\lambda^2 - \frac{p}{\mu} \).

**Summary and outlook**

- We have calculated frequency shifts of collective modes of an axially-symmetric BEC with two- and three-body contact interaction for varying trap aspect ratios using numerical Fourier analysis and analytical Poincaré-Lindstedt method.
- We have shown that the influence of a small repulsive three-body interaction extends the stability region of the condensate beyond the critical number of atoms in the trap.
- Due to the nonlinearity of GP equation, collective modes are coupled. Even when we excite only one mode, the others are necessarily excited in the second order of the perturbative expansion and appear in real-time dynamics of the condensate.
Geometric Resonances in Bose-Einstein Condensates with Two- and Three-Body Interactions

Hamid Al-Jibbouri¹, Ivana Vidanović², Antun Balaz², and Axel Pelster³

¹Institut für Theoretische Physik, Freie Universität Berlin, Germany
²SCL, Institute of Physics Belgrade, University of Belgrade, Serbia
³Fachbereich Physik und Forschungszentrum OPTIMAS, Technische Universität Kaiserslautern, Germany

References


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