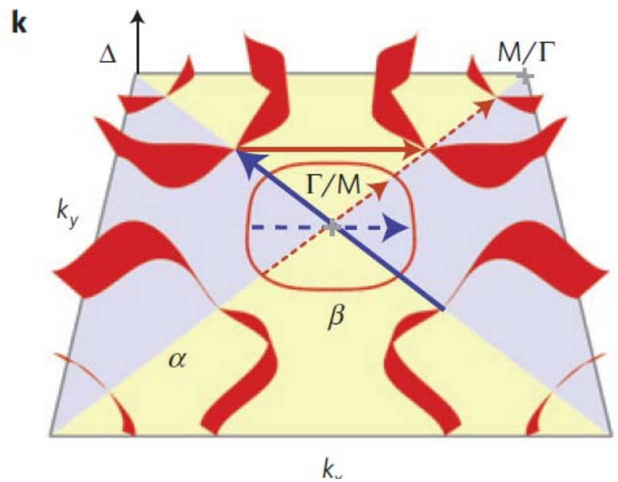
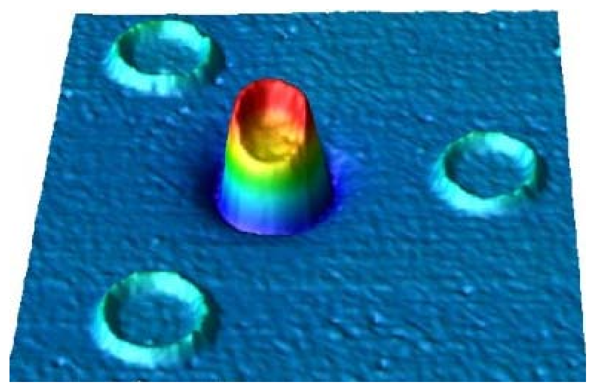
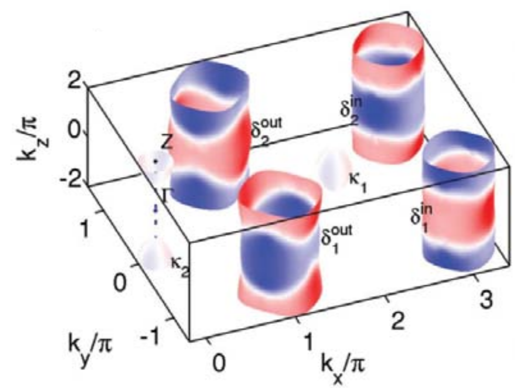
Introduction to unconventional superconductivity

P.J. Hirschfeld, U. Florida







SPP 1458

Outline

- ~~Review of conventional SC~~ (Blundel lecture!)
- What symmetries for Δ are allowed beyond BCS?
- What is unconventional superconductivity?
- What are pairing mechanisms besides phonons?
- Materials: cuprates, Fe-based, heavy fermions...

  similarities & differences? Higher T_c?

Conventional superconductors

- BCS theory (1957)

Quantum mechanical behavior at the macroscopic scale

Leon Cooper

John Bardeen

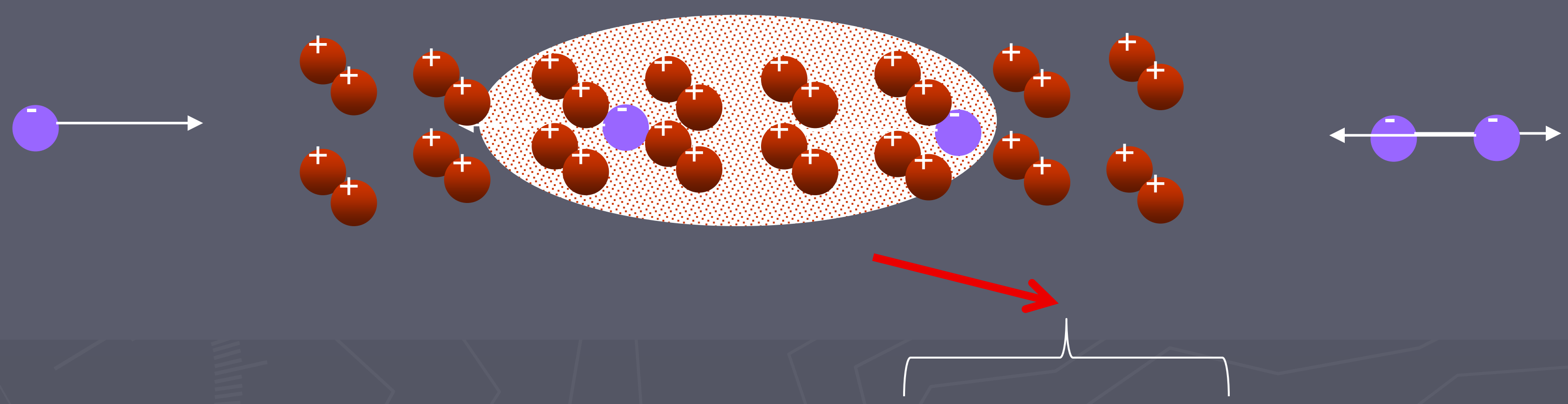
Robert Schrieffer

Nobel prize : 1972

Macro. Quantum State $\Psi_{BCS} = \prod_k \left(u_k + v_k c^*_{k\uparrow} c^*_{-k\downarrow}\right) | 0 >$

s-wave symmetry $\Delta \equiv V\langle c_{-k\downarrow}\ c_{k\uparrow}\rangle \sim \Delta_0 e^{i\phi}$

How Cooper pairs form in conventional superconductors: the "glue": electron-phonon interaction



Effective e-e interaction

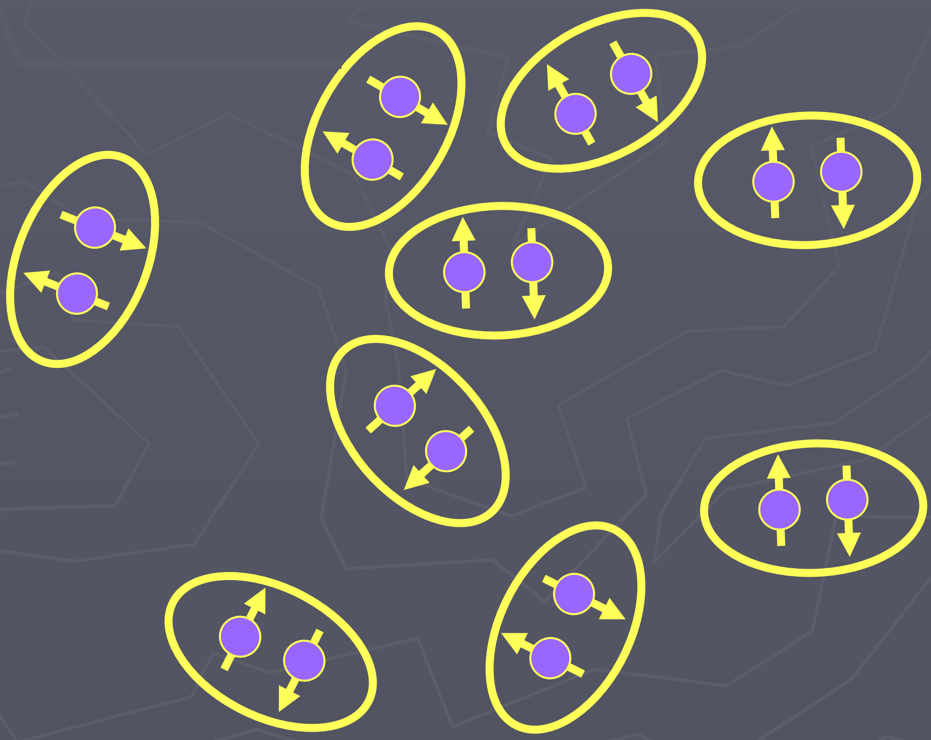
$$V(\mathbf{q},\omega) = \frac{4\pi e^2}{q^2 + k_s^2} + \frac{4\pi e^2}{q^2 + k_s^2}\,\frac{\omega_q^2}{\omega^2 - \omega_q^2}$$

Screened Coulomb Electron-phonon attraction

Note: electrons avoid Coulomb repulsion in *time* (interaction is retarded)

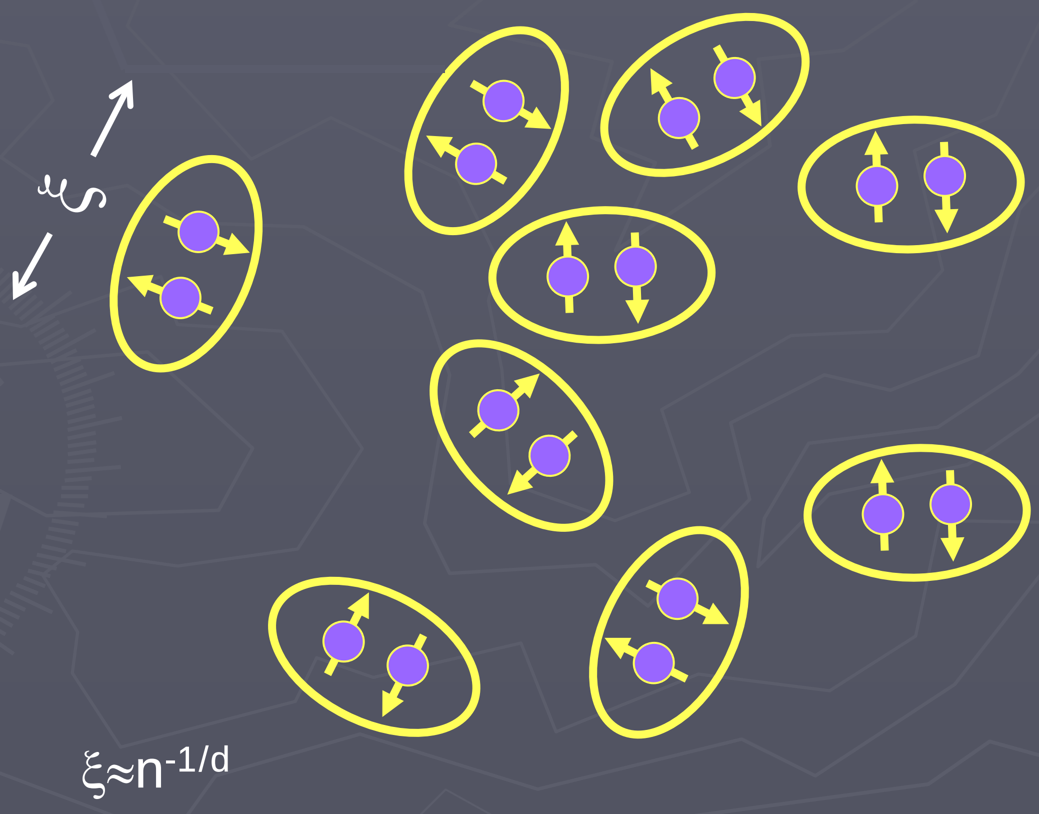
Superconductivity: Ground state

Puzzle 1: is this a good picture of Cooper pairs?



Superconductivity: Ground state

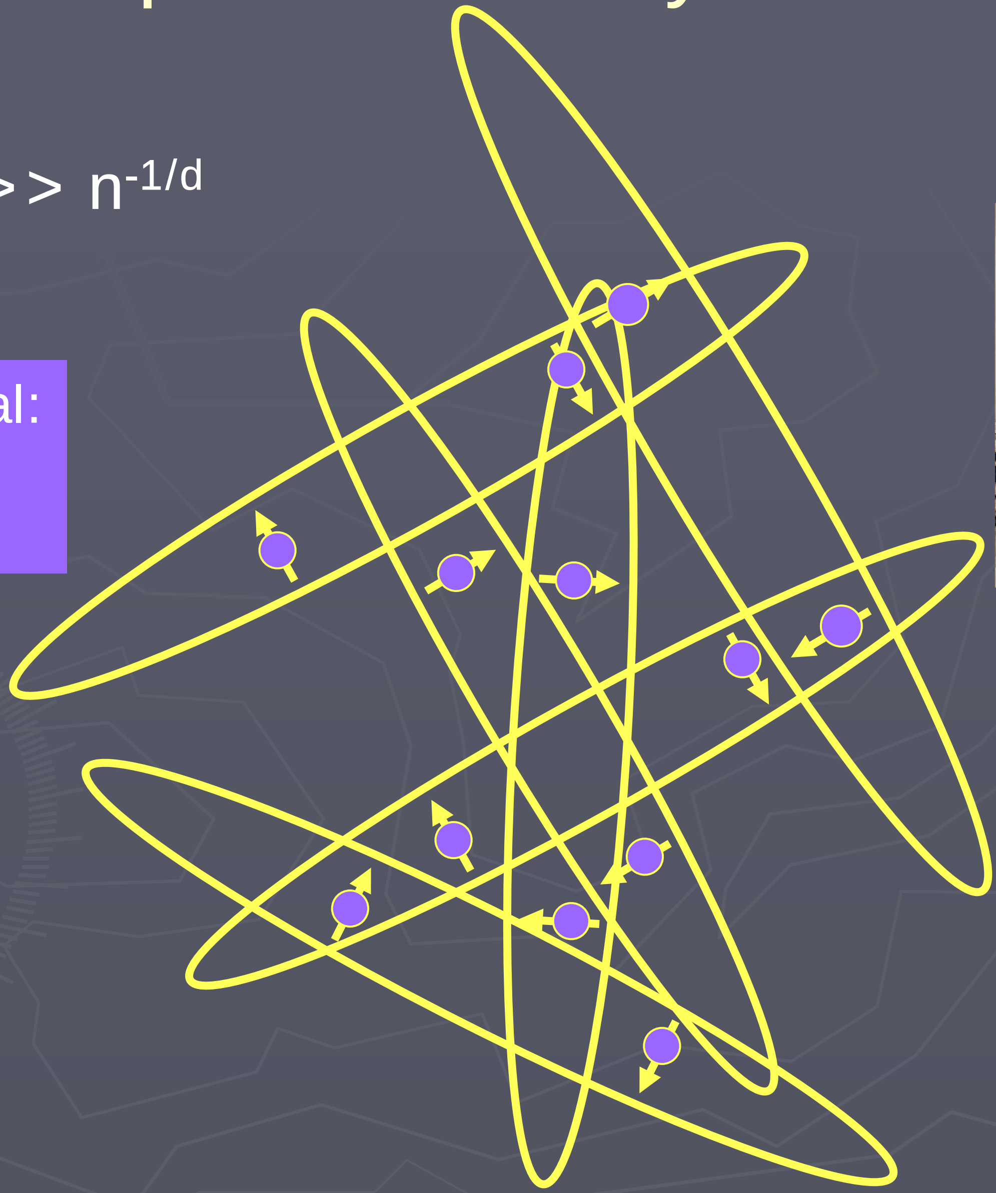
A: No! For most SC, pair size $\xi >> n^{-1/d}$



Superconductivity: Ground state

$\xi = v_F/\Delta >> n^{-1/d}$

Simple metal:
$\xi \sim 10^3$ A
$n^{-1/d} \sim 1$A

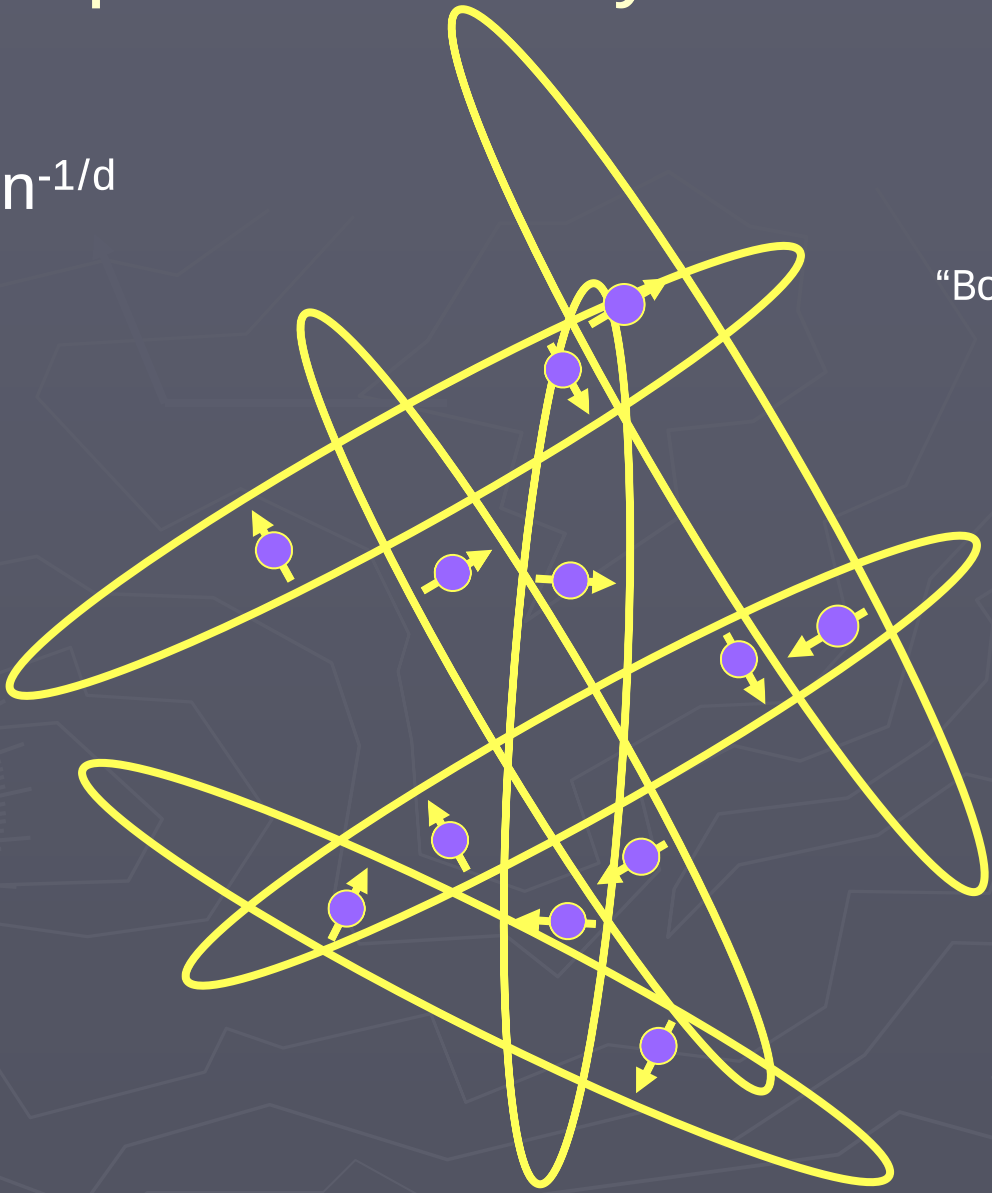


Remember that all pairs are phase coherent!

St. Matthew's Passion
Oxford, UK

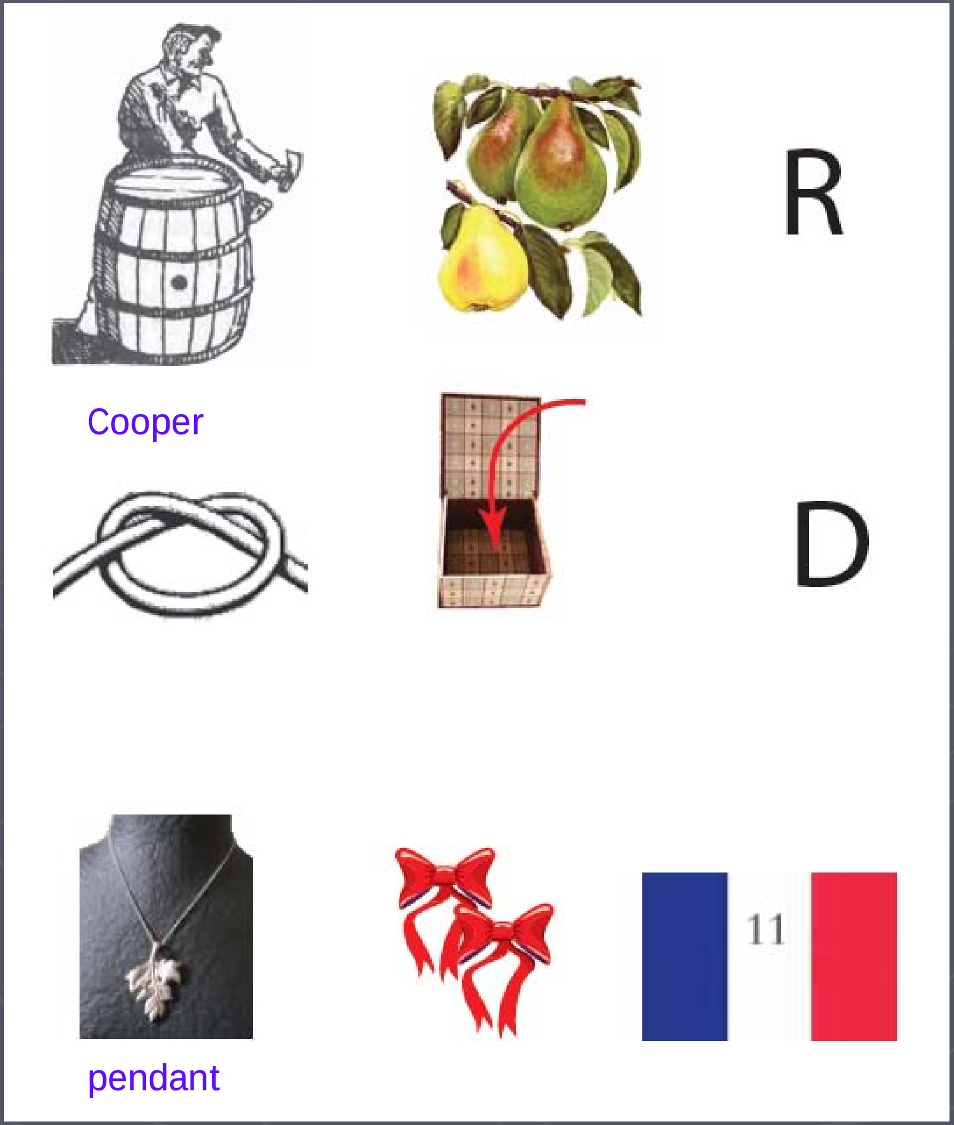
Superconductivity: Excited states

$\xi = v_F/\Delta >> n^{-1/d}$



"Bogoliubov quasiparticle"

Puzzle #2:



Cooper pairs are not independent bosons!

Grace à Henri Alloul

Is that all there is? Brian Pippard and “The Cat and the Cream” speech IBM 1961

Is that all there is? Brian Pippard and "The Cat and the Cream" speech IBM 1961

"I think I might remark that in low-temperature physics the disappearance of liquid helium, superconductivity, and magneto-resistance from the list of major unsolved problems has left this branch of research looking pretty sick from the point of view of any young innocent who thinks he's going to break new ground."

Discovery of heavy fermion superconductivity in $CeCu_2Si_2$ 1979

F. Steglich

Superconductivity in the Presence of Strong Pauli Paramagnetism: $CeCu_2Si_2$

F. Steglich
Institut für Festkörperphysik, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany

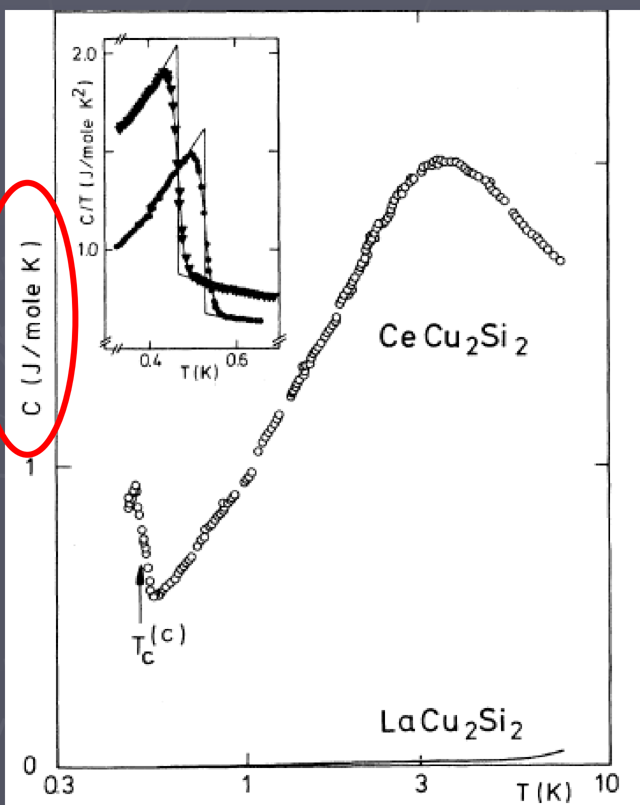
and

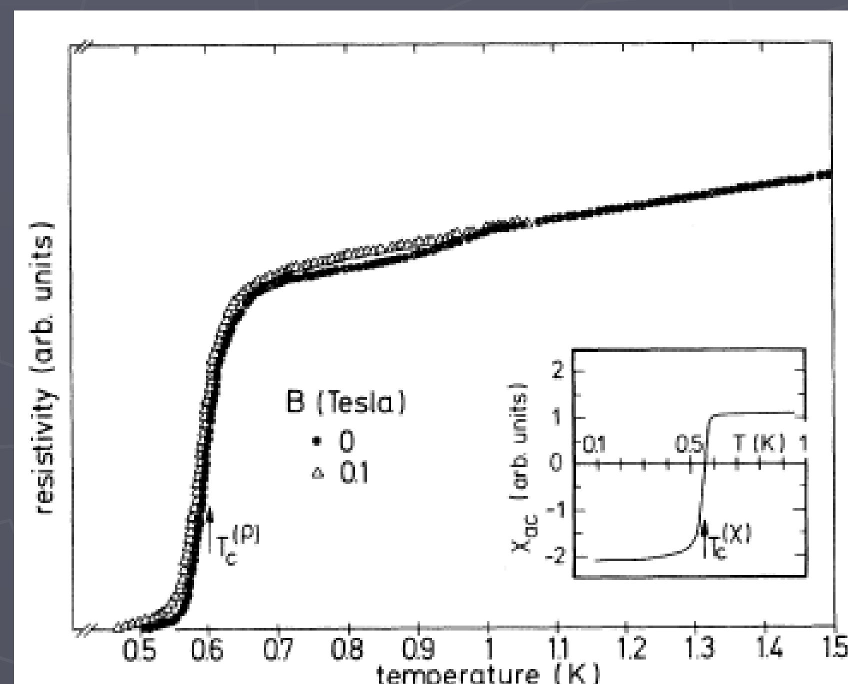
J. Aarts, C. D. Bredl, W. Lieke, D. Meschede, and W. Franz
II. Physikalisches Institut, Universität zu Köln, D-5000 Köln 41, West Germany

and

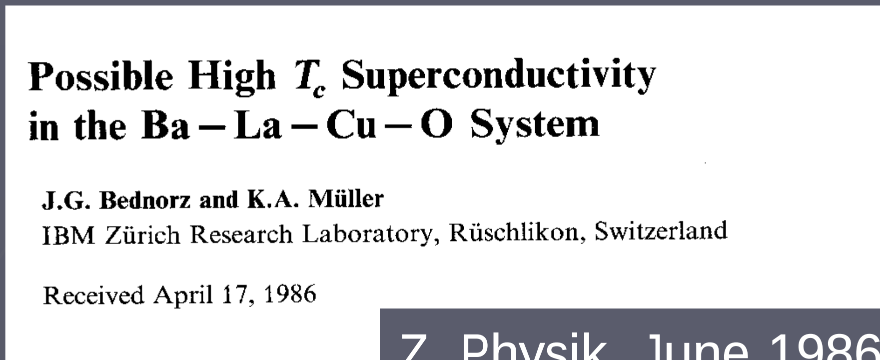
H. Schäfer
Eduard-Zintl-Institut, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany
(Received 10 August 1979; revised manuscript received 7 November 1979)

A comparison was made between four low-temperature properties of $LaCu_2Si_2$ and $CeCu_2Si_2$. Whereas $LaCu_2Si_2$ behaves like a normal metal, $CeCu_2Si_2$ shows (i) low-temperature anomalies typical of "unstable 4*f* shell" behavior and (ii) a transition into a superconducting state at $T_c \simeq 0.5$ K. Our experiments demonstrate for the first time that superconductivity can exist in a metal in which many-body interactions, probably magnetic in origin, have strongly renormalized the properties of the conduction-electron gas.





High temperature superconductivity



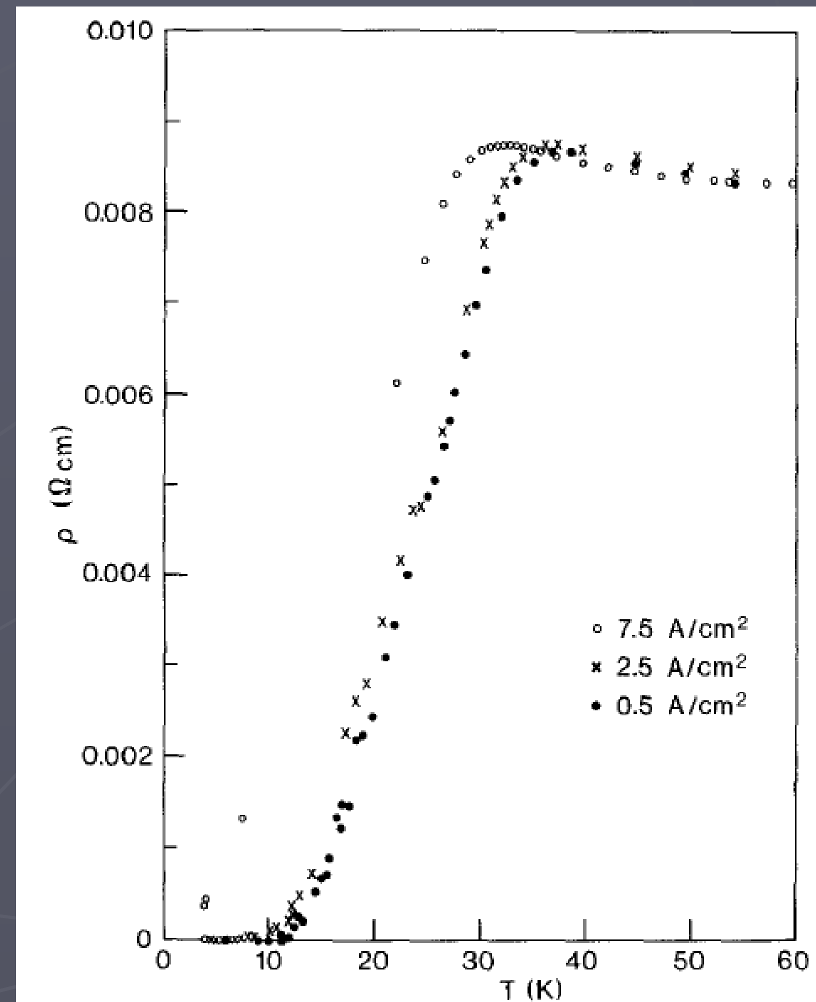
Possible High T_c Superconductivity in the Ba – La – Cu – O System

J.G. Bednorz and K.A. Müller
IBM Zürich Research Laboratory, Rüschlikon, Switzerland

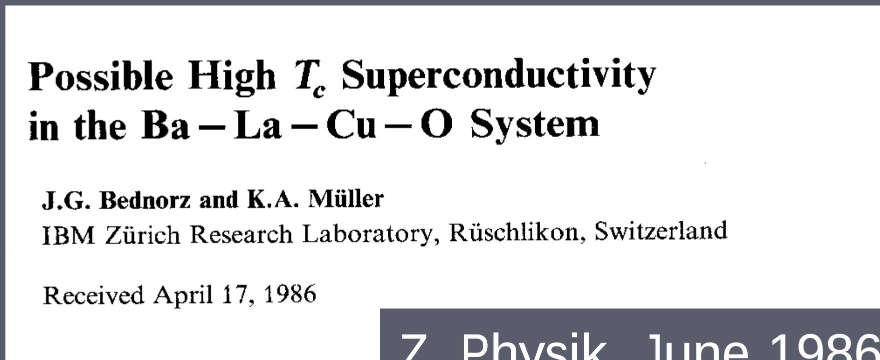
Received April 17, 1986

Z. Physik, June 1986

Alex Müller and Georg Bednorz



High temperature superconductivity



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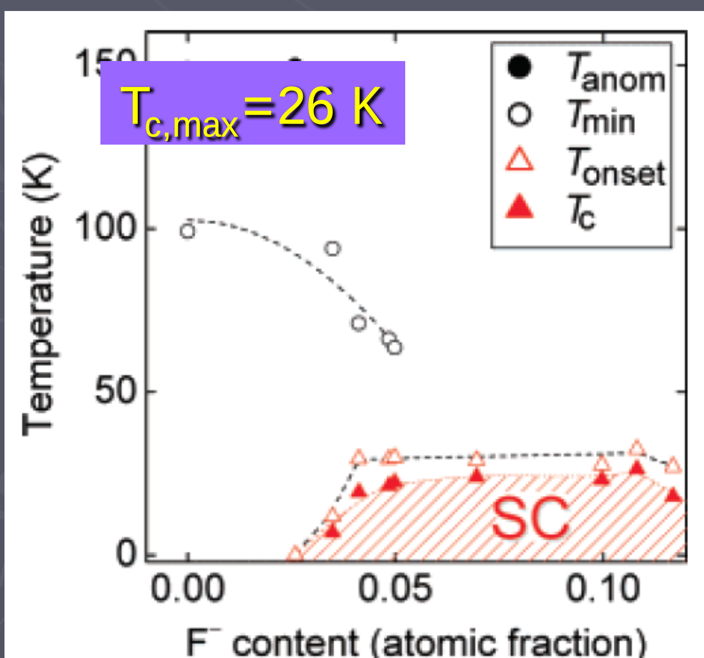
Grüneisen parameter coupling in heavy fermion systems		169-174
DOI	10.1007/BF01303699	
Authors	M. Yoshizawa, B. Lüthi and K. D. Schotte	
Text	PDF (494 kb)	
Anomalous temperature dependence of the magnetic field penetration depth in superconducting UBe_{13}		175-188
DOI	10.1007/BF01303700	
Authors	F. Gross, B. S. Chandrasekhar, D. Einzel, K. Andres, P. J. Hirschfeld, H. R. Ott, J. Beuers, Z. Fisk and J. L. Smith	
Text	PDF (1,206 kb)	
Possible high T_c superconductivity in the Ba–La–Cu–O system		189-193
DOI	10.1007/BF01303701	
Authors	J. G. Bednorz and K. A. Müller	
Text	PDF (396 kb)	

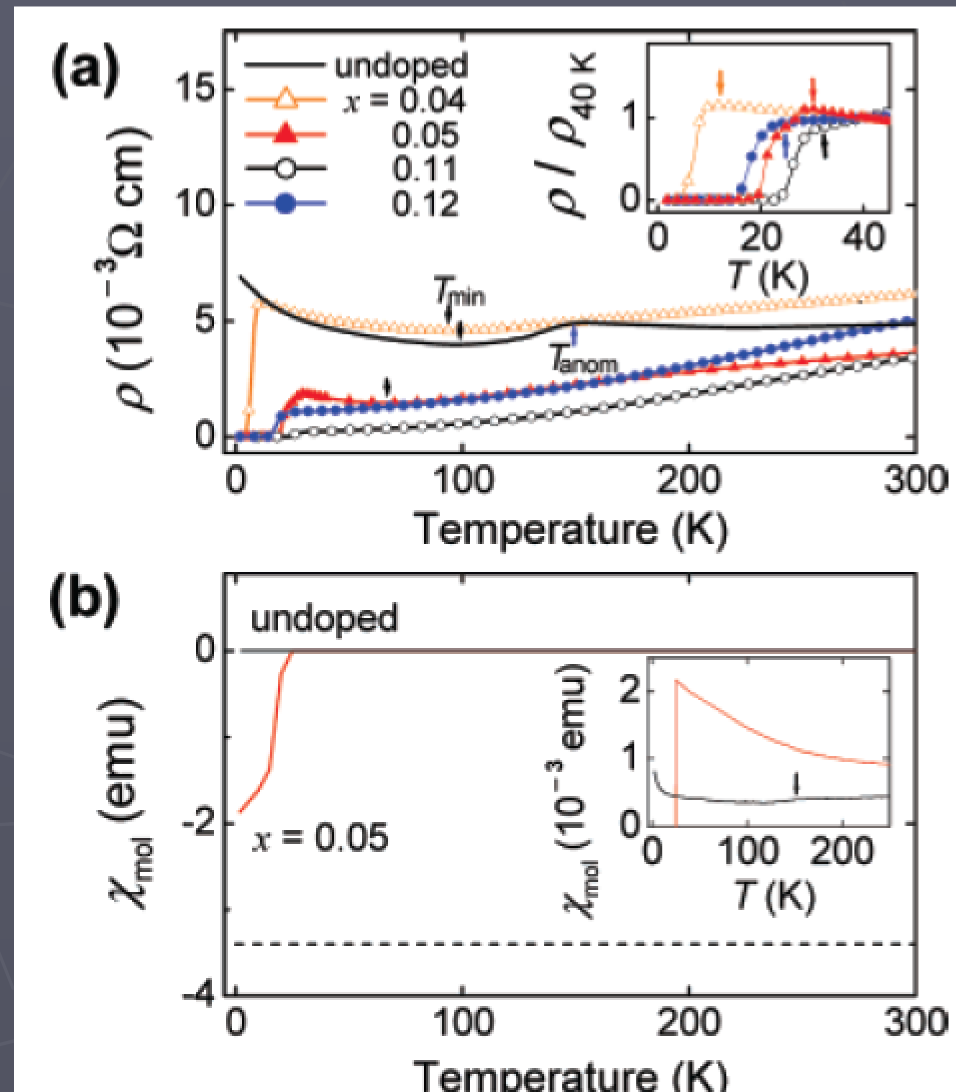
20 Articles First | 1-10 | 11-20 | Next

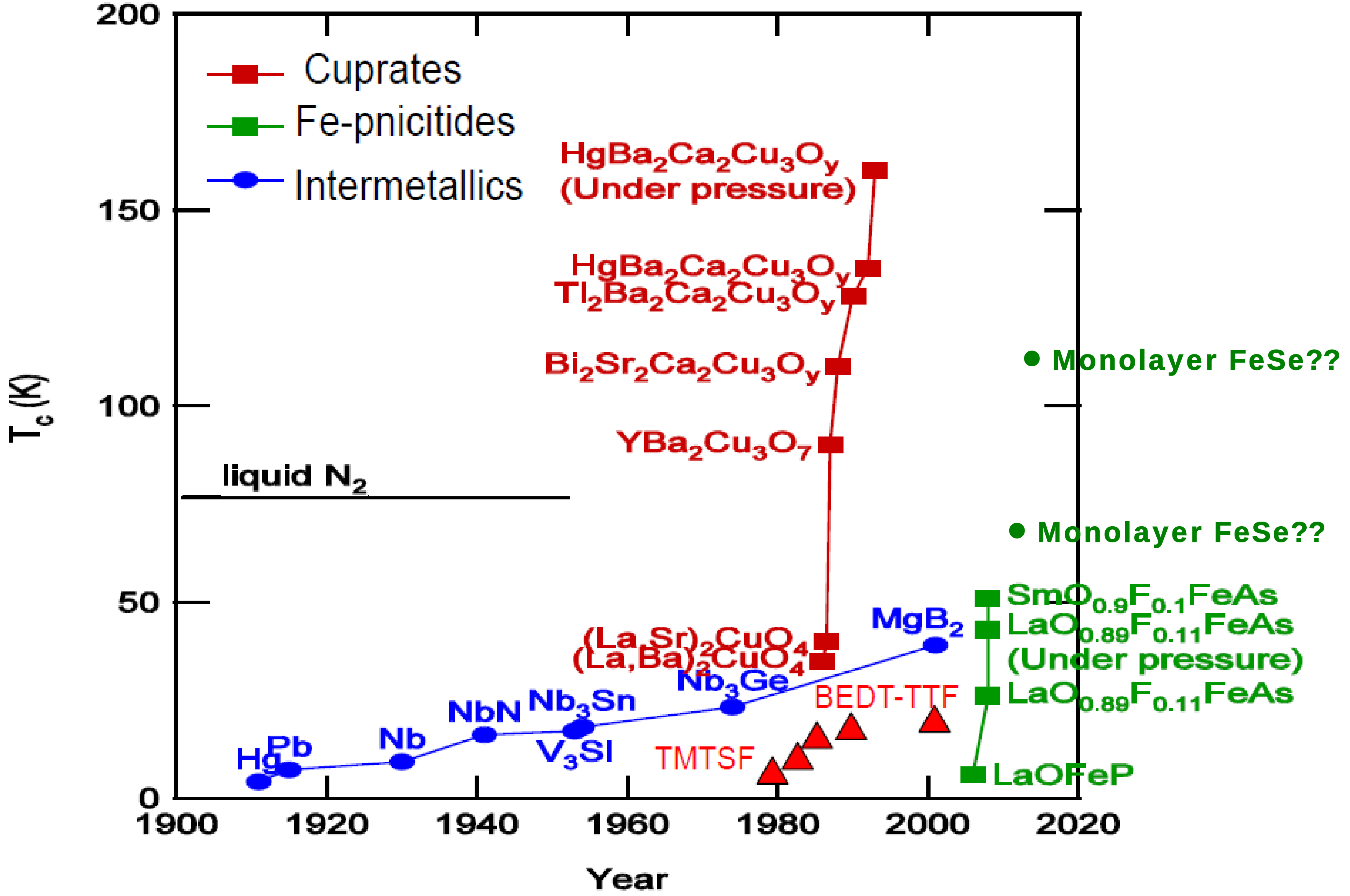
Discovery of $LaO_{1-x}F_xFeAs$

Kamihara et al JACS 2008

H. Hosono



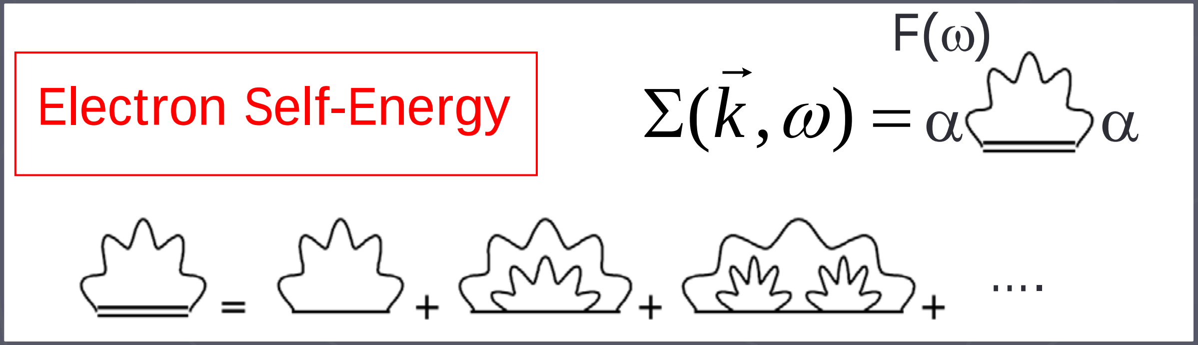


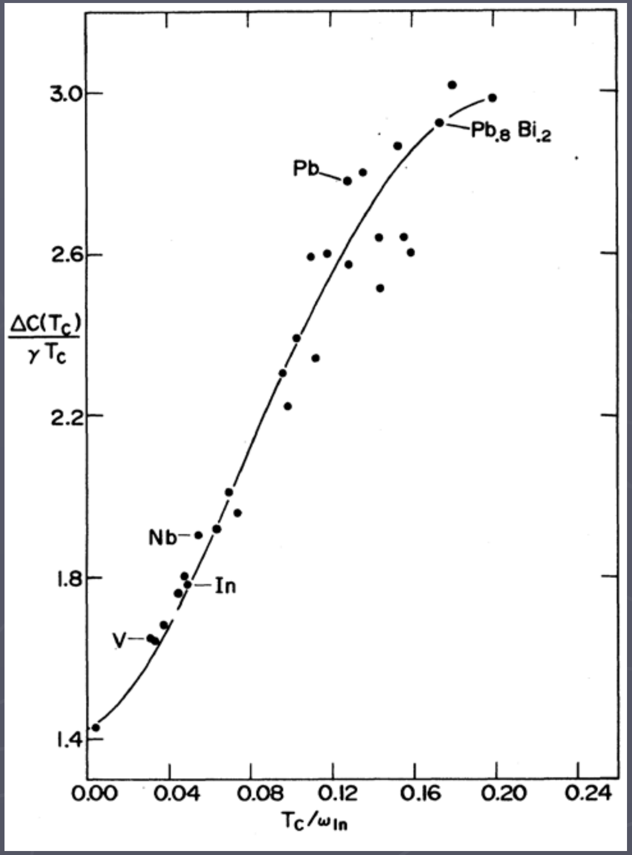


Cuprates
Fe-pnicitides
Intermetallics
$HgBa_2Ca_2Cu_3O_y$ (Under pressure)
$HgBa_2Ca_2Cu_3O_y$
$Tl_2Ba_2Ca_2Cu_3O_y$
$Bi_2Sr_2Ca_2Cu_3O_y$
$YBa_2Cu_3O_7$
liquid N_2
Monolayer FeSe??
Monolayer FeSe??
$SmO_{0.9}F_{0.1}FeAs$
$LaO_{0.89}F_{0.11}FeAs$ (Under pressure)
$LaO_{0.89}F_{0.11}FeAs$
LaOFeP
MgB_2
$(La,Sr)_2CuO_4$
$(La,Ba)_2CuO_4$
Nb_3Ge
BEDT-TTF
TMTSF
Nb_3Sn
V_3Si
NbN
Nb
Pb
Hg
T_c (K)
Year
0
50
100
150
200
1900
1920
1940
1960
1980
2000
2020

Can we get high T_c from conventional superconductivity?

First: Eliashberg strong coupling theory for electron-phonon systems





Strong coupling Eliashberg theory provides quantitively accurate predictions for all conventional superconductors based on knowing the electron-phonon interaction, summarized in the phonon spectral density $\alpha^2F(\omega)$, which can be calculated or measured by experiment.

There are deviations from BCS for most materials, even elements.

Can we get high T_c from conventional superconductivity?

Electron-phonon
T_c^{max} ~ 40K

PRL '87

Electronic Band Properties and Superconductivity in $La_{2-y}X_yCuO_4$

L. F. Mattheiss
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 7 January 1987)

The results of electronic-structure calculations for tetragonal La_2CuO_4 provide insight concerning the origin of high-temperature superconductivity in the $La_{2-y}X_yCuO_4$ alloys. A half-filled Cu(3*d*)-O(2*p*) band with two-dimensional character and a nearly square Fermi surface produces a Peierls instability for $y=0$ that opens a semiconductor gap over the Fermi surface. Alloying with divalent or tetravalent atoms should spoil the nesting features while maintaining the strong coupling of O phonons to the conduction electrons.

PACS numbers: 72.15.Nj, 71.25.Pi, 74.20.−z, 74.60.Mj

Electron-phonon
T_c^{max} ~ 20-30K

PRB '88

Electron-phonon interaction in $Ba_2YCu_3O_7$

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AT&T Bell Laboratories, Murray Hill, New Jersey 07974
and Kernforschungszentrum Karlsruhe, Institut für Nukleare Festkörperphysik, Postfach 3640, D-7500 Karlsruhe 1, Federal Republic of Germany

L. F. Mattheiss
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 18 August 1987)

A realistic tight-binding theory, based on the energy-band results of Mattheiss and Hamann, is applied to study the electron-phonon interaction in $Ba_2YCu_3O_7$. In contrast to previous results for the 40-K superconductor $La_{2-x}(Ba,Sr)_xCuO_4$, the theoretical values for the electron-phonon coupling are much too small to yield superconducting transition temperatures in the 90-K range.

Pairing and the Pauli principle

I. 1-band systems with inversion and time-reversal symmetry

Single-particle states |k↑⟩ and |-k↓⟩ = $\mathcal{T}$|k↑⟩ are degenerate if $\mathcal{T}$-symmetry is preserved (Kramers). Superconducting interaction is maximized by pairing degenerate states.

BCS chose “pair wave function” $$b_{\mathbf{k}} = \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle$$

Centrosymmetric crystal ⇒ |k↑⟩ and |-k↑⟩ = $\mathcal{P}$|k↑⟩ degenerate also!

Then 4 states are degenerate: $|\mathbf{k},\uparrow\rangle,\ |\mathbf{k},\downarrow\rangle,\ |-\mathbf{k},\uparrow\rangle,\ |-\mathbf{k},\downarrow\rangle$

General pair wave fctn. $b_{\mathbf{k}\sigma\sigma'} = \langle c_{-\mathbf{k}\sigma} c_{\mathbf{k}\sigma'} \rangle$ must obey Pauli principle: $b_{-\mathbf{k}\sigma'\sigma} = -b_{\mathbf{k}\sigma\sigma'}$

two possibilities:

1) $b_{\mathbf{k}}$ is *even* under $\mathbf{k} \to -\mathbf{k}$ $\Rightarrow b_{\mathbf{k}\sigma\sigma'} = b_{-\mathbf{k}\sigma\sigma'} = -b_{\mathbf{k}\sigma'\sigma}$,
i.e. *odd* under spin exchange (singlet, S=0).

2) $b_{\mathbf{k}}$ is *odd* under $\mathbf{k} \to -\mathbf{k}$ $\Rightarrow b_{\mathbf{k}\sigma\sigma'} = -b_{-\mathbf{k}\sigma\sigma'} = b_{\mathbf{k}\sigma'\sigma}$.
i.e. *even* under spin exchange. (triplet, $S = 1$).

Pairing and the Pauli principle

II. Generalized BCS theory

$$H \simeq H_0 - \left(\Delta \sum_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + h.c.\right) + \Delta \langle c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow}\rangle^*,$$

Conventional BCS gap eqn

$$\Delta = V {\sum}' \overbrace{\langle c_{-k\downarrow} c_{k\uparrow}\rangle}^{b_k} = V \sum_{\mathbf{k}'} \frac{\Delta^*_{\mathbf{k}'}}{2E_{\mathbf{k}'}} \tanh \frac{E_{\mathbf{k}'}}{2T}$$

Generalized BCS gap equation

$$\Delta_{\mathbf{k}\sigma_1\sigma_2} = \sum_{\mathbf{k}\sigma_3\sigma_4} V^{\sigma_2\sigma_1\sigma_3\sigma_4}_{\mathbf{kk}'} b_{\mathbf{k}\sigma_4\sigma_3}$$

$\Delta_{\mathbf{k}\sigma_1\sigma_2}$: “the gap fctn” or “the order parameter”

$V^{\sigma_2\sigma_1\sigma_3\sigma_4}_{\mathbf{kk}'}$: “the pair potential” or “the glue”

$b_{\mathbf{k}\sigma_4\sigma_3}$: “the condensate” or “the pair wave function”

Pairing and the Pauli principle

III. Singlet vs. triplet pairing

Gap functions for different spin pairs

$$\underline{\Delta} = \begin{bmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{bmatrix}$$

- **Singlet pairing** $(S=0)$

$$\underline{\Delta}_{\mathbf{k}} = i\sigma_y \Delta_{\mathbf{k}}; \qquad \Delta_{-\mathbf{k}} = \Delta_{\mathbf{k}}.$$

Why is this a singlet state? Because since $i\sigma_y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, we have $\Delta_{\mathbf{k}\uparrow\downarrow} = -\Delta_{\mathbf{k}\downarrow\uparrow}$. The orbital part of the order parameter, $\Delta_{\mathbf{k}}$, is even under parity as it must be according to Pauli.

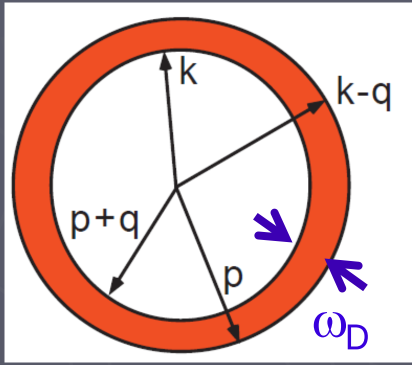
- **Triplet pairing** $(S=1)$ (Balian & Werthamer Phys. Rev. 131, 1553 (1963))

$$\underline{\Delta}_{\mathbf{k}} = i\sigma_y \mathbf{d}(\mathbf{k}) \cdot \vec{\sigma} = \begin{pmatrix} -d_x + i d_y & d_z \\ d_z & d_x + i d_y \end{pmatrix},$$

e.g. d ǁ z $\Rightarrow$ $\Delta_{\uparrow\downarrow} = \Delta_{\downarrow\uparrow}$, i.e. the Sz=0 component of the triplet $|\uparrow\downarrow + \downarrow\uparrow\rangle/\sqrt{2}$

Pairing and the Pauli principle

IV. Orbital symmetry of Cooper pairs



BCS: pairing is confined to a thin shell of energies near the Fermi surface:

"weak coupling": pair wave function "lives on the Fermi surface", i.e.

$$b_{\mathbf{k}} \simeq b_{\hat{k}} \delta(\epsilon_{\mathbf{k}} - \epsilon_F)$$

So expand:

$$b_{\hat{k}} = b_0 + \sum_{m=-1}^{1} b_{1m} Y_{1m} + \sum_{m=-2}^{2} b_{2m} Y_{2m} + \ldots$$

&

$$V_{\mathbf{k},\mathbf{k}'} = V_0 + V_1 \sum_{m=-1}^{1} Y_{1m}(\hat{k}) Y_{1m}(\hat{k}')^* + \ldots$$

& insert into BCS gap eqn.:

$$\Delta_{\mathbf{k}\sigma_1\sigma_2} = \sum_{\mathbf{k}\sigma_3\sigma_4} V_{\mathbf{k}\mathbf{k}'}^{\sigma_2\sigma_1\sigma_3\sigma_4} b_{\mathbf{k}\sigma_4\sigma_3}$$

Project out each ℓ-channel. Usually only single ℓ channel important since $T_c^\ell \simeq \omega_D e^{-1/N_0 V_\ell}$

Pairing and the Pauli principle

V. Consequences of Pauli principle for multiple bands

Pauli tells us that $b_{\mathbf{k}\sigma\nu\sigma'\nu'} = b_{-\mathbf{k}\sigma'\nu'\sigma\nu}$. We can then have, for even and odd parity respectively,

1) $b_{\mathbf{k}}$ is *even* under $\mathbf{k} \to -\mathbf{k}$ $\Rightarrow b_{\mathbf{k}\sigma\nu\sigma'\nu'} = b_{-\mathbf{k}\sigma\nu\sigma'\nu'} = -b_{\mathbf{k}\sigma'\nu'\sigma\nu}$., yielding now *two* possibilities, either

i.e. a) *odd* under spin exchange (singlet, S=0), *even* under band exchange

or

b) *even* under spin exchange (triplet, S=1), *odd* under band exchange

Similarly if

2) $b_{\mathbf{k}}$ is *odd* under $\mathbf{k} \to -\mathbf{k}$ $\Rightarrow b_{\mathbf{k}\sigma\nu\sigma'\nu'} = -b_{-\mathbf{k}\sigma\nu\sigma'\nu'} = b_{\mathbf{k}\sigma'\nu'\sigma\nu}$.

i.e. a) *even* under spin exchange (triplet, $S = 1$), *even* under band exchange.

or

b) *odd* under spin exchange (singlet, $S = 0$), *odd* under band exchange.

Note "exotic" possibilities a) even parity S=1 and b) odd parity S=0 involve intraband pairing of k and –k, hence are *energetically disfavored.*

Terminology

- Conventional/unconventional:

"unconventional pairing" occurs when electrons are bound by exchange of electronic excitations rather than phonons.

- Trivial/nontrivial:

"nontrivial pairing" refers to "non-s-wave" pairing, i.e the Cooper pair wave function has a symmetry less than that of the lattice.

Warning: "unconventional" is used in many early papers to mean "nontrivial"

Two paradigms for superconductivity

- Conventional pairing:

USUALLY occurs in ℓ=0 pairing channel to take advantage of the attractive electron-phonon interaction at r=0 – avoid Coulomb repulsion in time

- Unconventional pairing:

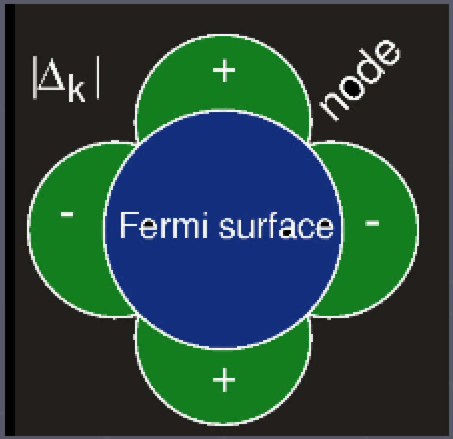
USUALLY occurs in higher-ℓ pairing channel to avoid the Coulomb interaction in space – Ψ has node at r=0

Warning: weird counterexamples: theories of d-wave pairing from phonons, extended s-wave pairing from electronic excitations

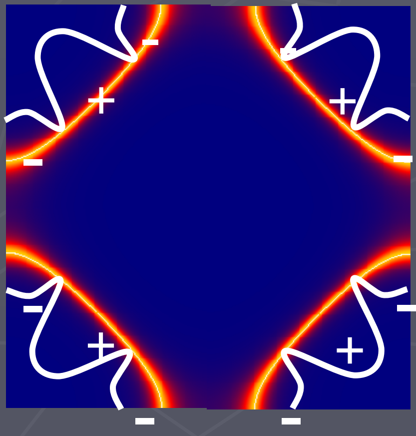
Consequences of nontrivial pairing

I. Low energy quasiparticle excitations (nodes)

- can be required by symmetry
  e.g. d-wave $\Delta_k \sim k_x^2 - k_y^2$

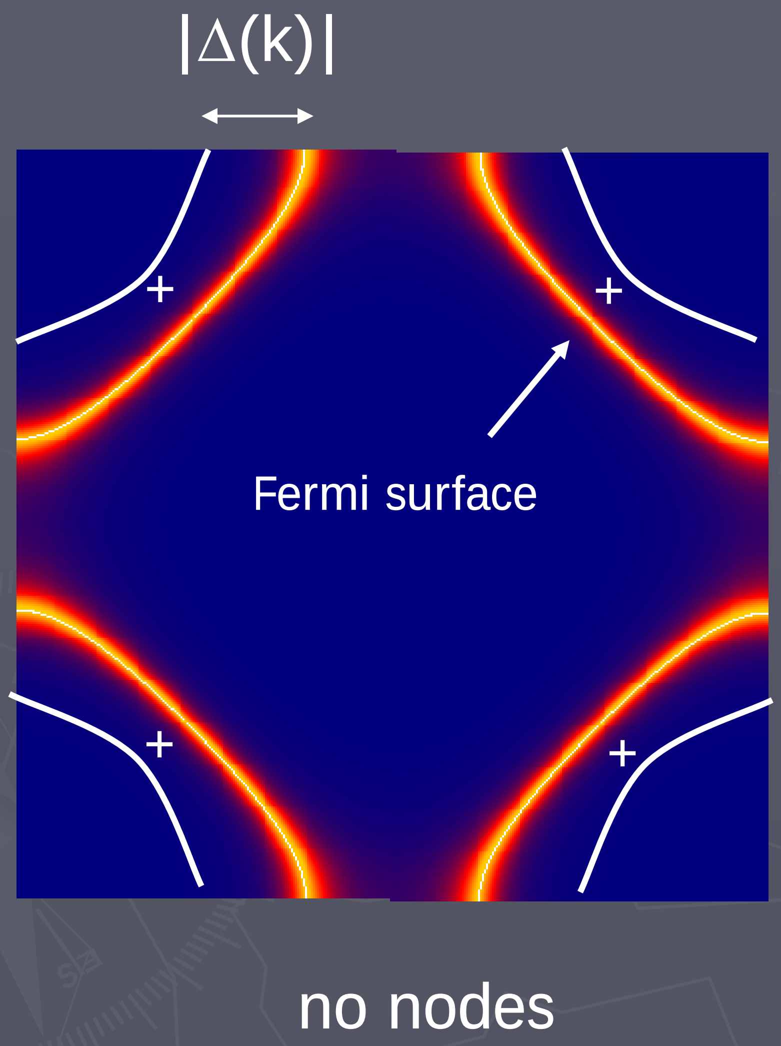


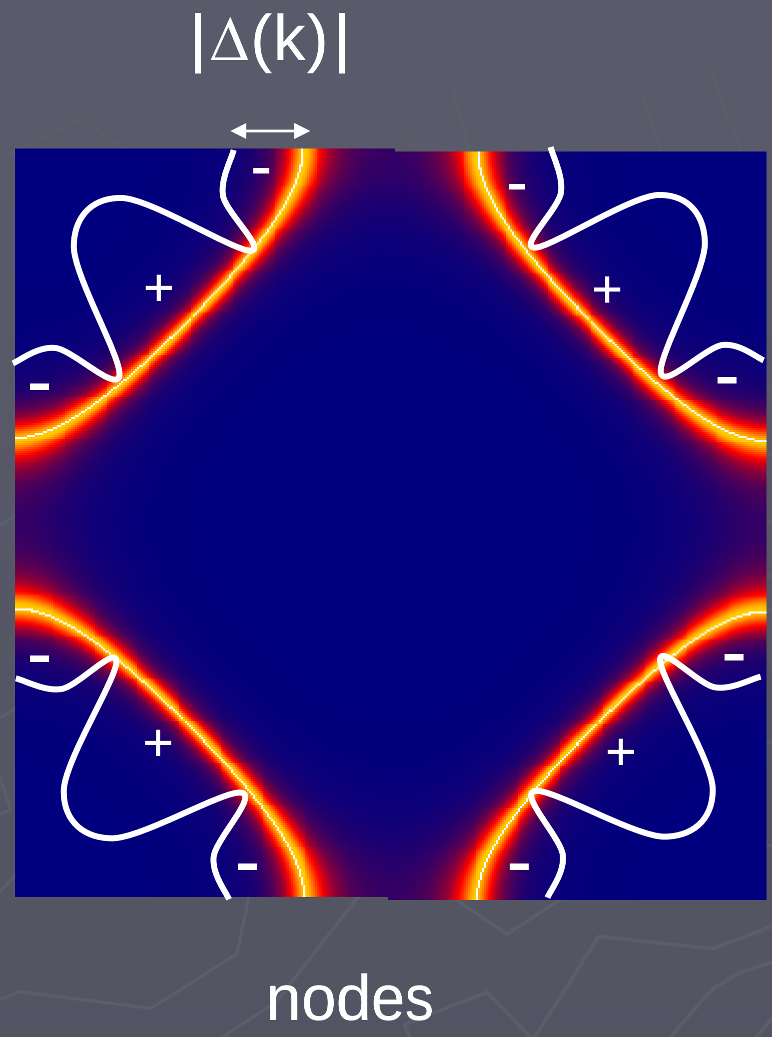
- can be "accidental", due to details of pair potential $V_{kk'}$



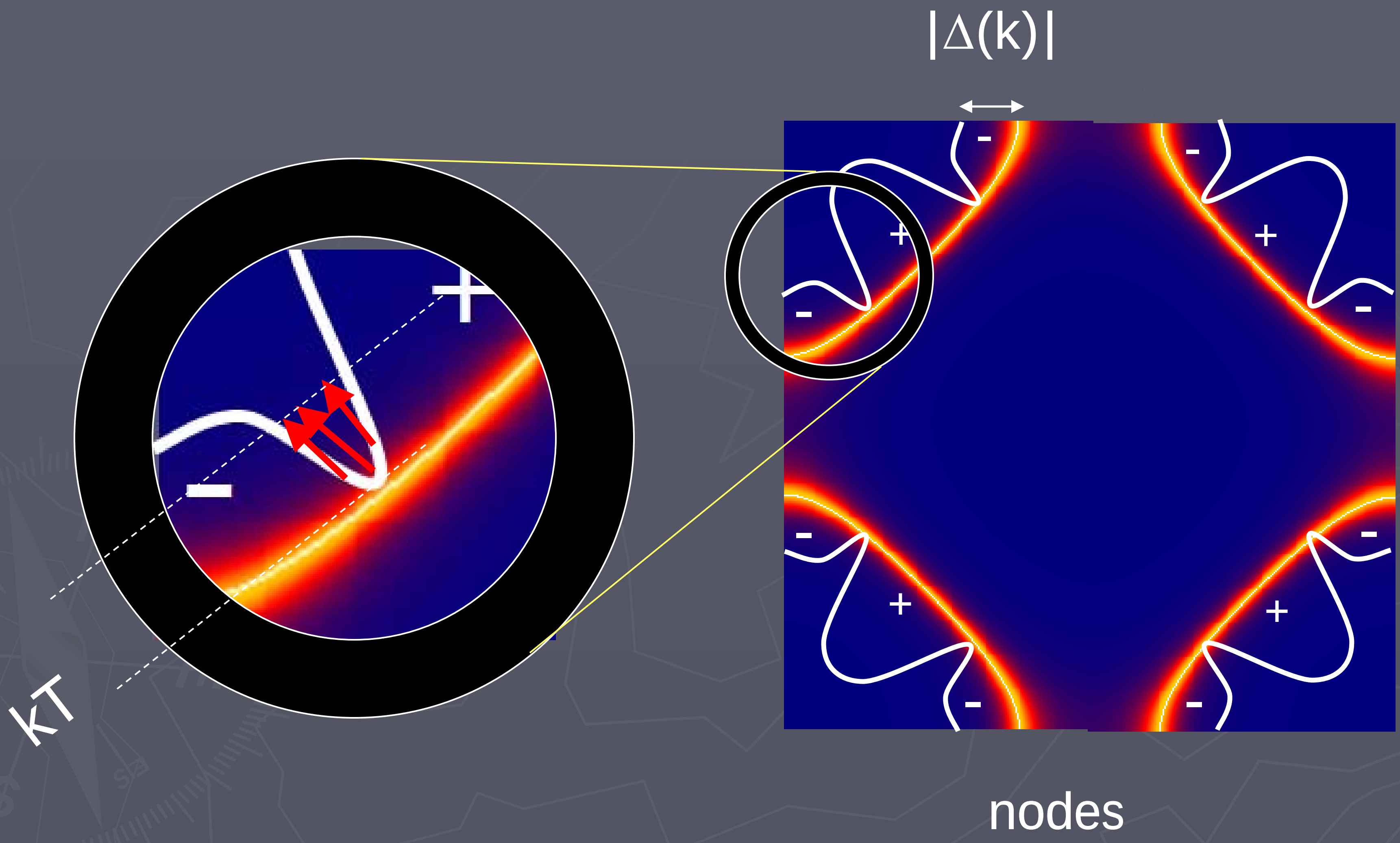
N.B. Pt. group G has finite # irreps ⇒ sum over many functions with same symmetry
e.g. A_{1g}: 1, cos 4θ,... or B_{1g}: cos 2θ, cos 6θ, ...

Order parameter Δ(k) shape in A_{1g} representations—1 band

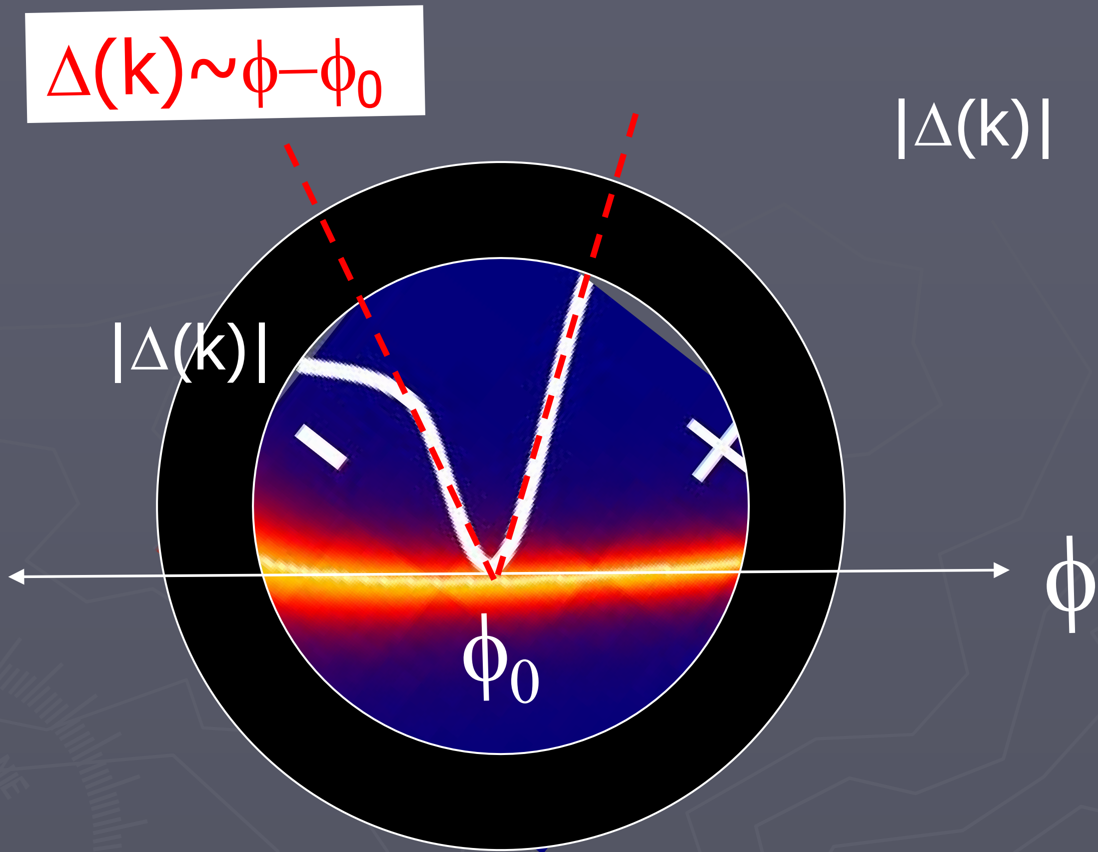


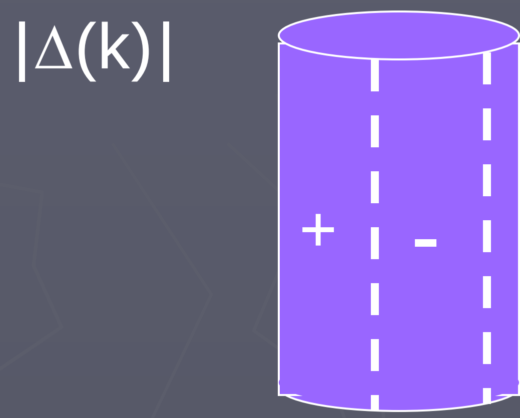


Nodal excitations dominate low T properties

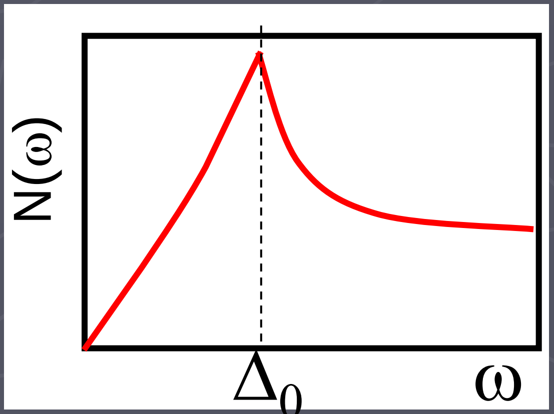


Linear DOS from *line* nodes

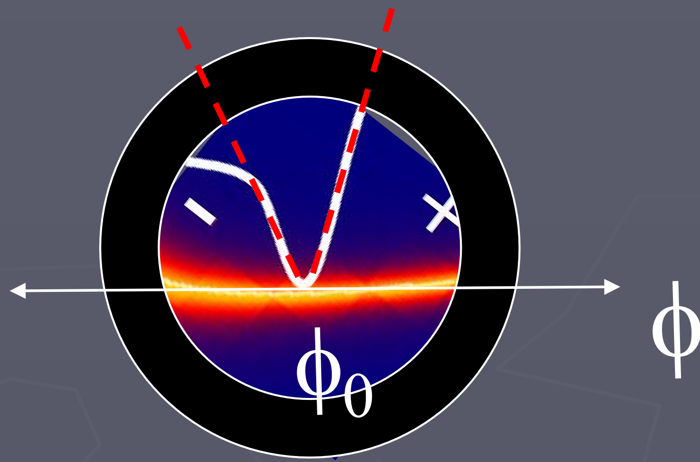




$$N(\omega)=\int\frac{d\phi}{2\pi}\,\mathrm{Re}\frac{\omega}{\sqrt{\omega^2-\Delta_0{}^2(\phi-\phi_0)^2}}\approx\frac{\omega}{\Delta_0}$$



Example: T^2 specific heat from line nodes



$$N(\omega) \approx \frac{\omega}{\Delta_0}$$

\# excitations

energy/ excitation

Estimate for energy of free Fermi gas:

$$E = \int d\omega\, \omega N(\omega) f(\omega) \simeq N_0 \int d\omega\, \omega f(\omega) \sim \left(\frac{T}{E_F}\right) \bullet T \sim \frac{T^2}{E_F}$$

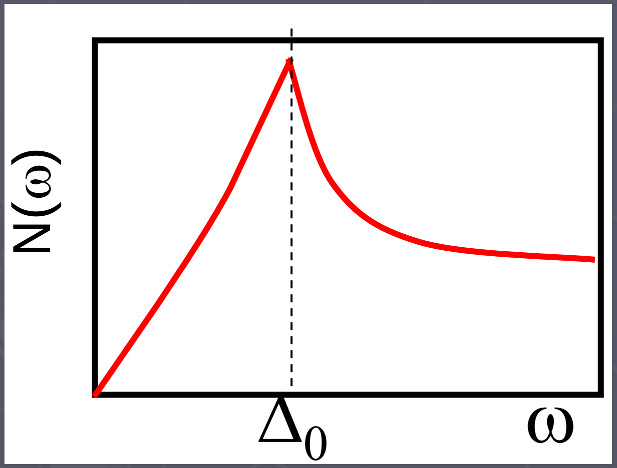
$$C = \frac{dE}{dT} \sim \frac{T}{E_F}$$

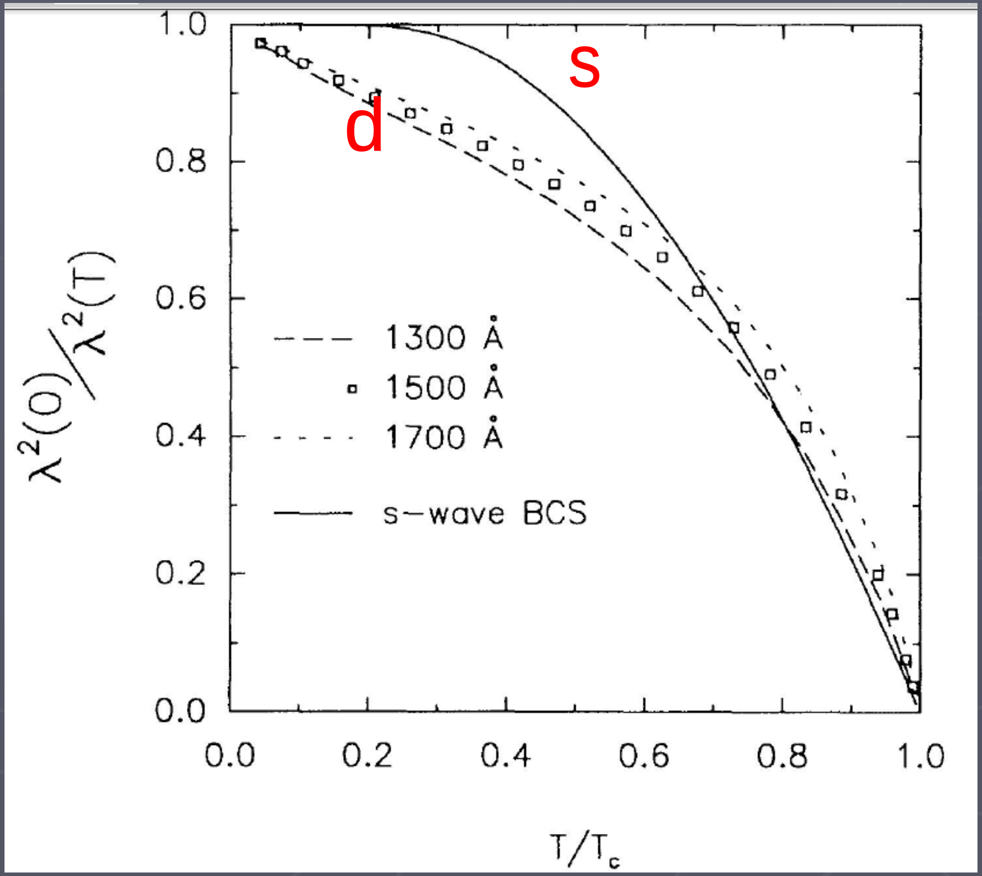
Estimate for energy of nodal SC:

$$E = \int d\omega\, \omega N(\omega) f(\omega) \simeq N_0 \int d\omega \left(\frac{\omega}{\Delta_0}\right) \omega f(\omega) \sim \left(\frac{T^2}{\Delta_0 E_F}\right) \bullet T \sim \frac{T^3}{E_F}$$

$$C = \frac{dE}{dT} \sim \frac{T^2}{\Delta_0 E_F}$$

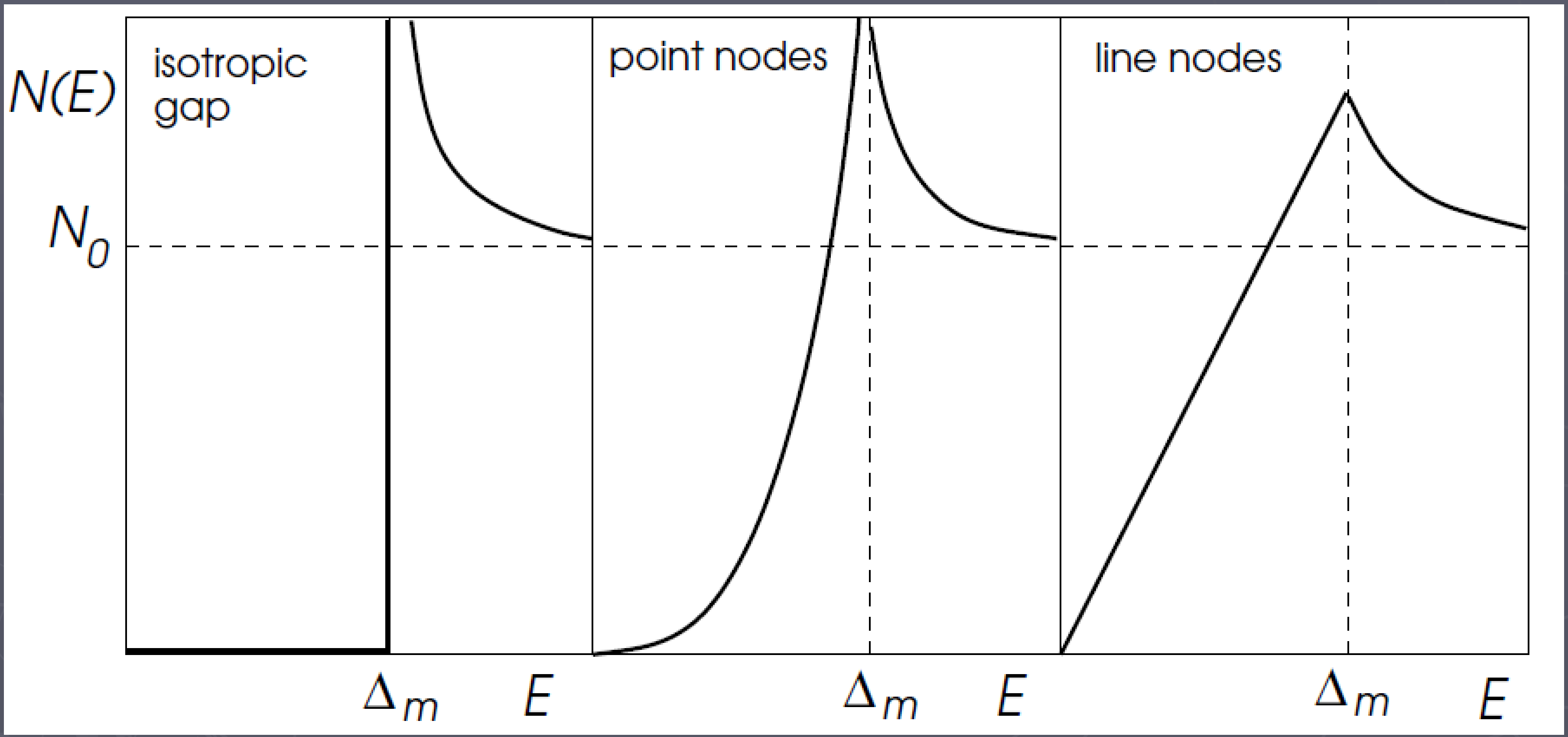
Detecting low-energy quasiparticle states





$$\Delta\lambda \simeq \int d\omega \left(-\frac{\partial f}{\partial \omega}\right) N(\omega) \sim T/\Delta_0$$

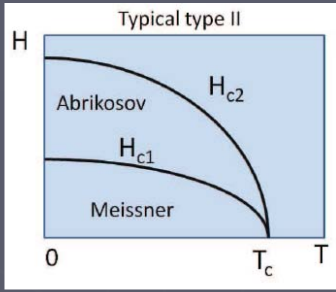
Dimension of nodal surface



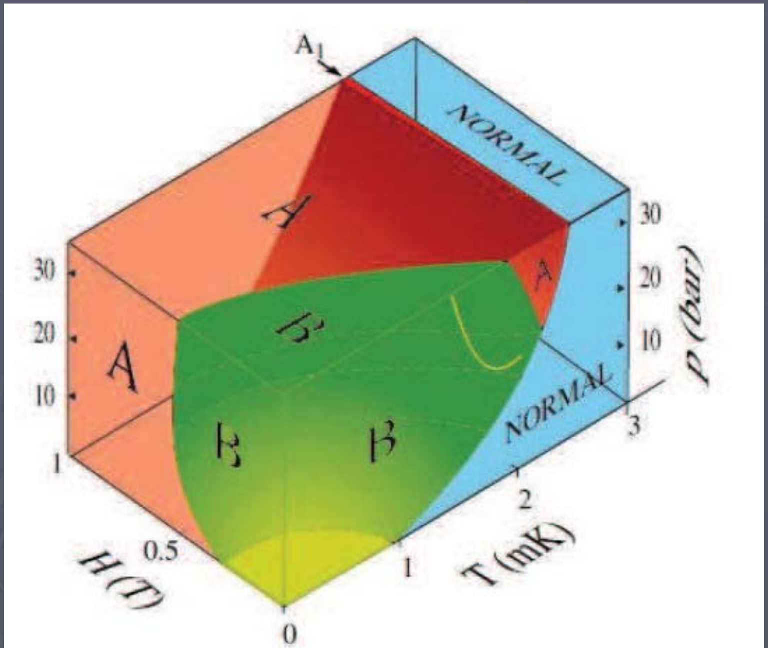
$C(T) \sim \exp(-\Delta/T)$ $C(T) \sim T^3$ $C(T) \sim T^2$

Consequences of nontrivial pairing

II. Possible nontrivial phase diagrams



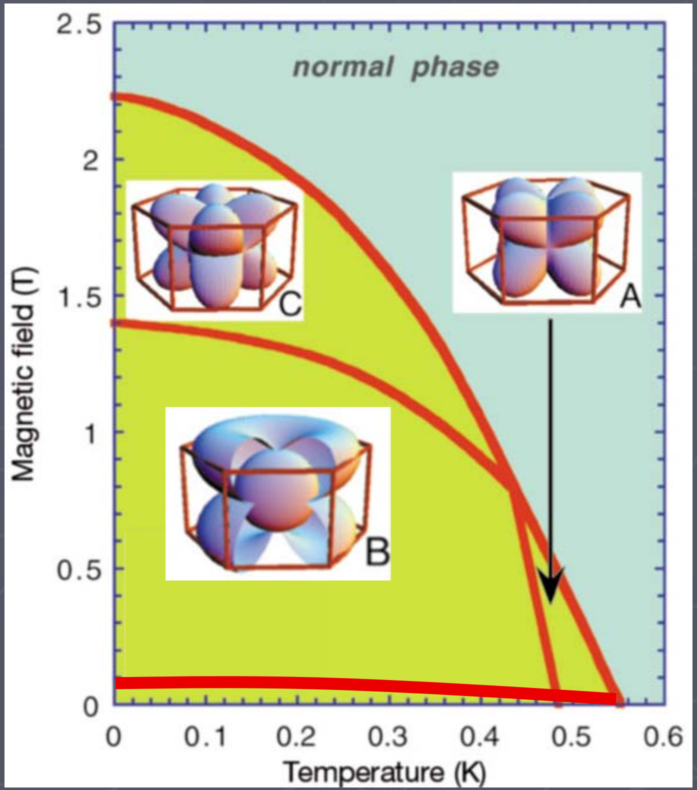
Superfluid ^{3}He



$$\Delta_k = i\sigma_y \mathbf{d}(\mathbf{k}) \cdot \vec{\sigma} = \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix}$$

9 complex components ($d_\mu = A_{\mu i} k_i$)

UPt_3

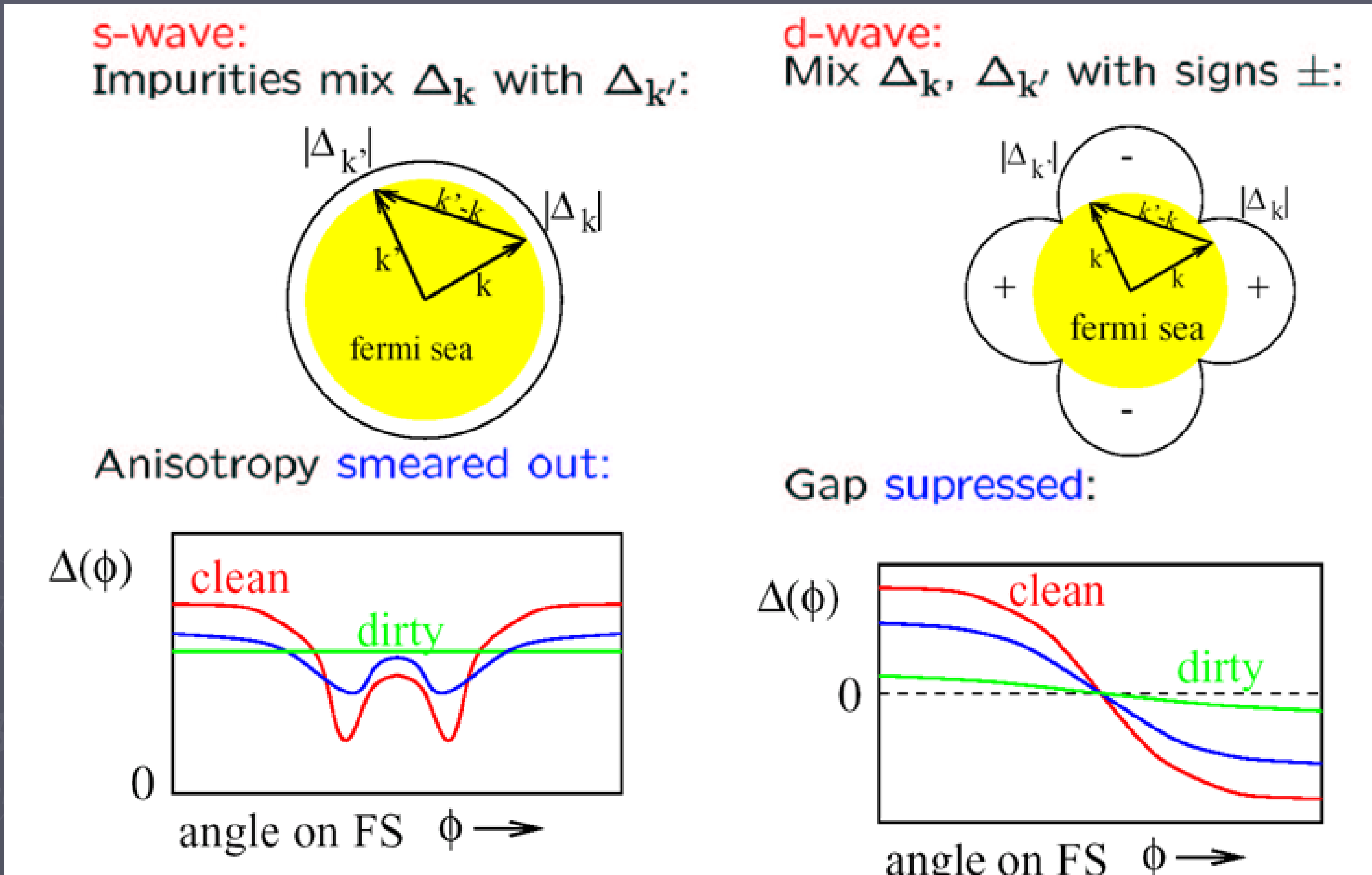


$$\Delta(\mathbf{k}) = \Delta_1 \psi_1(\mathbf{k}) + \Delta_2 \psi_2(\mathbf{k})$$

2 complex components

Consequences of nontrivial pairing

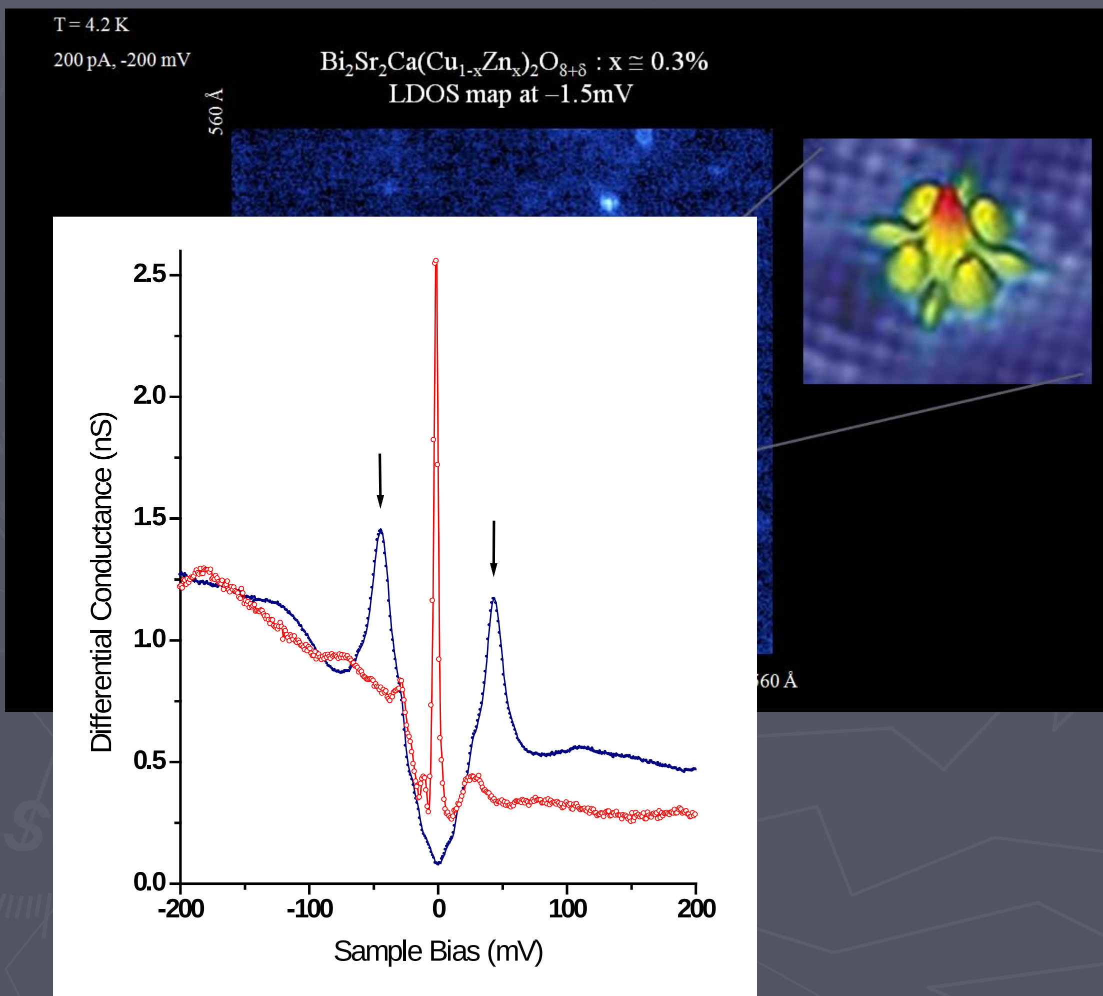
III. Nonmagnetic impurities and surfaces break pairs
(anisotropic and/or sign-changing gap)



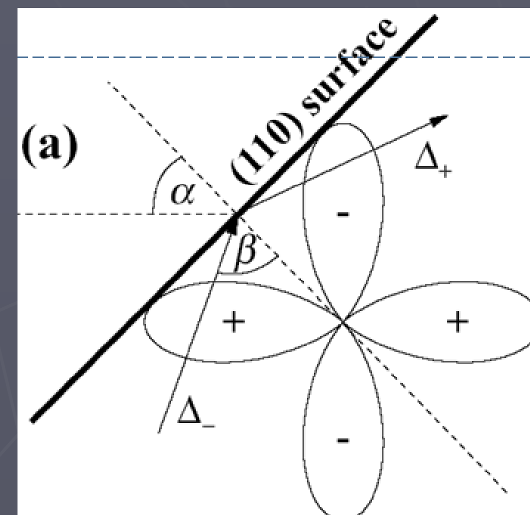
Consequences of nontrivial pairing

III. Nonmagnetic impurities and surfaces break pairs (sign-changing gap)

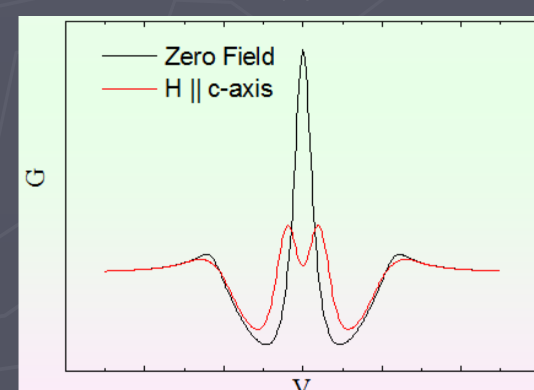
Zn impurity at surface of d-wave SC



Andreev bound state at 110 of d-wave SC



YBCO



Consequences of nontrivial pairing

IV. Order parameter collective modes (multicomponent order param)

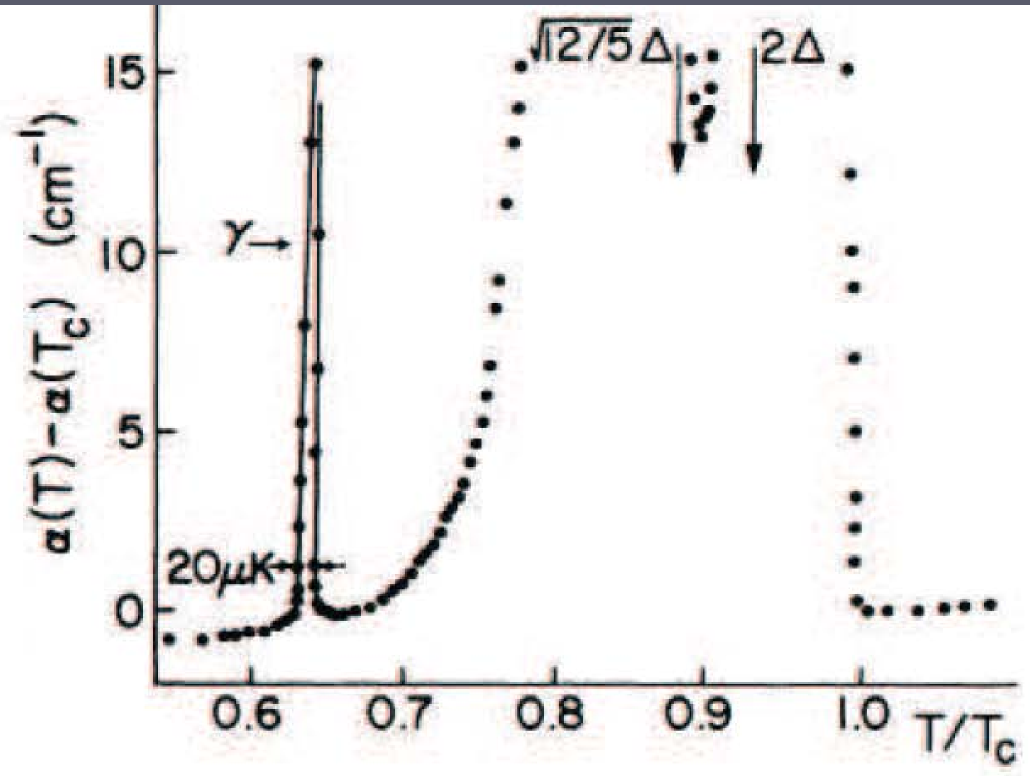


Figure 5: Sound attenuation in ^{3}He-B (Giannetta et al., Phys. Rev. Lett. 45, 262 (1980).

Not yet observed convincingly in superconductors!

Consequences of nontrivial pairing

V. Novel types of vortex structures

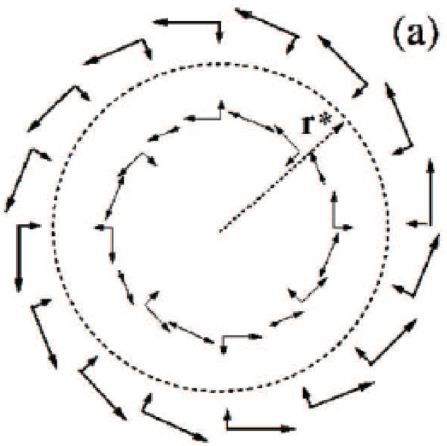


Figure 6: Vortex in a d-wave superconductor with subdominant s-wave interaction at low T (Li et al., Phys. Rev. 63, 054504 (2001)). Long arrows: phase of d-wave component; short arrows: phase of induced s-wave component.

Consequences of nontrivial pairing

VI. Novel Josephson effects

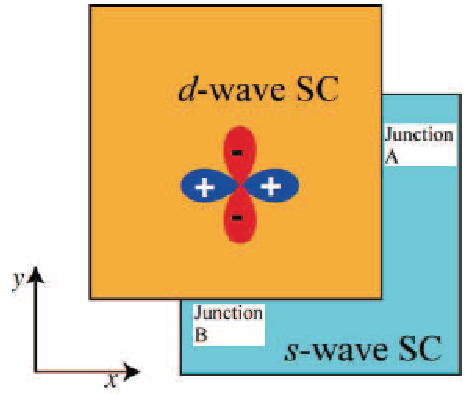


Figure 7: Corner junction geometry to detect d-wave symmetry (Wollman et al., Phys. Rev. Lett. 71, 2134 (1993).

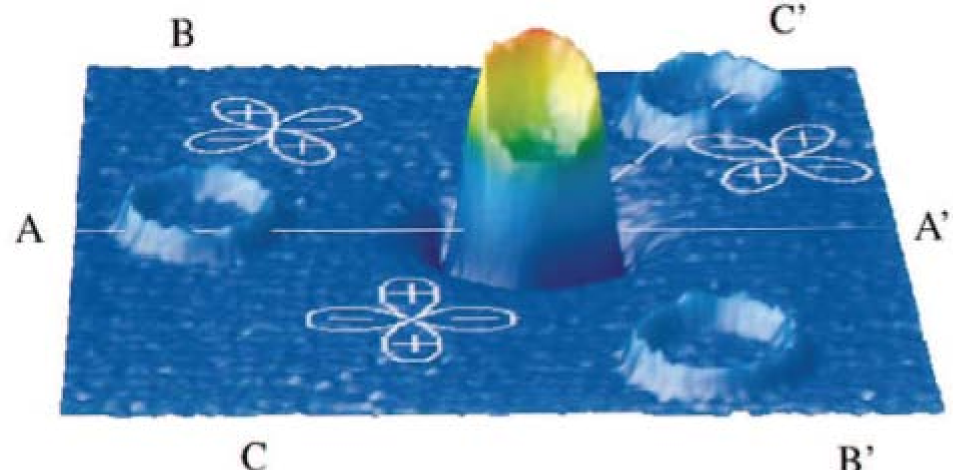


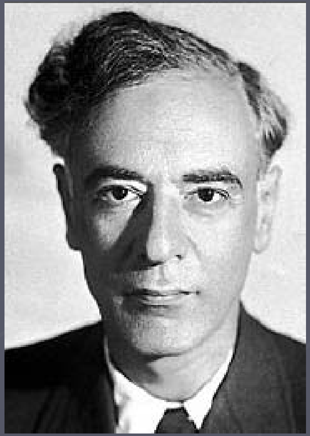
Figure 45: tricrystal sample with YBCO rings fabricated both with single crystal orientations, and across all three tricrystals (center). False color is magnitude of flux detected by scanning SQUID probe (Tsuei/Kirtley RMP 2000)

Unconventional pairing

Prehistory: Kohn-Luttinger 1965


Walter Kohn


Quinn Luttinger



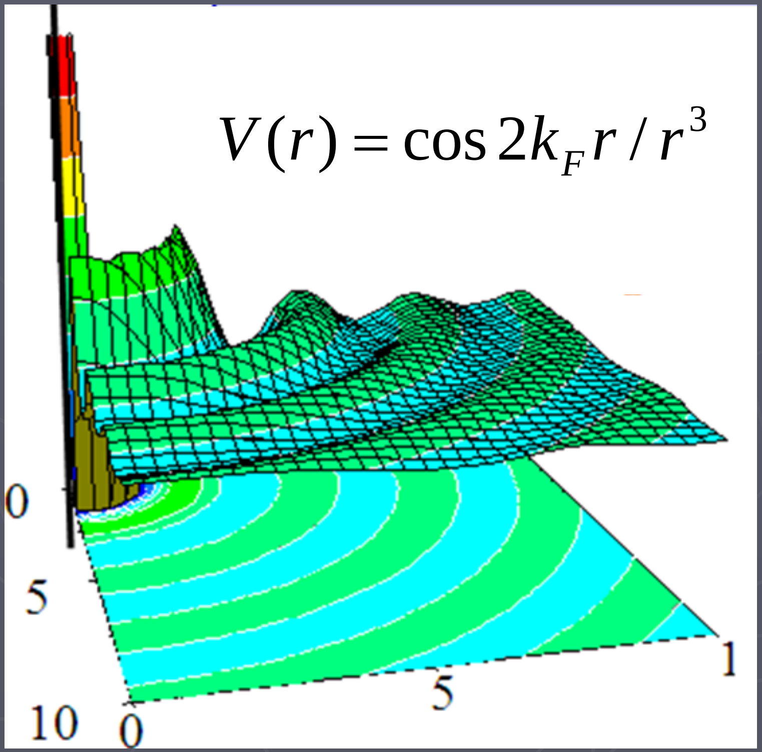
Also: Landau and Pitaevskii

KL (1962): an electron gas with no phonons and only repulsive Coulomb interactions can be a superconductor!

A new paradigm: electrons avoid repulsive part of Coulomb interaction in space rather than time!

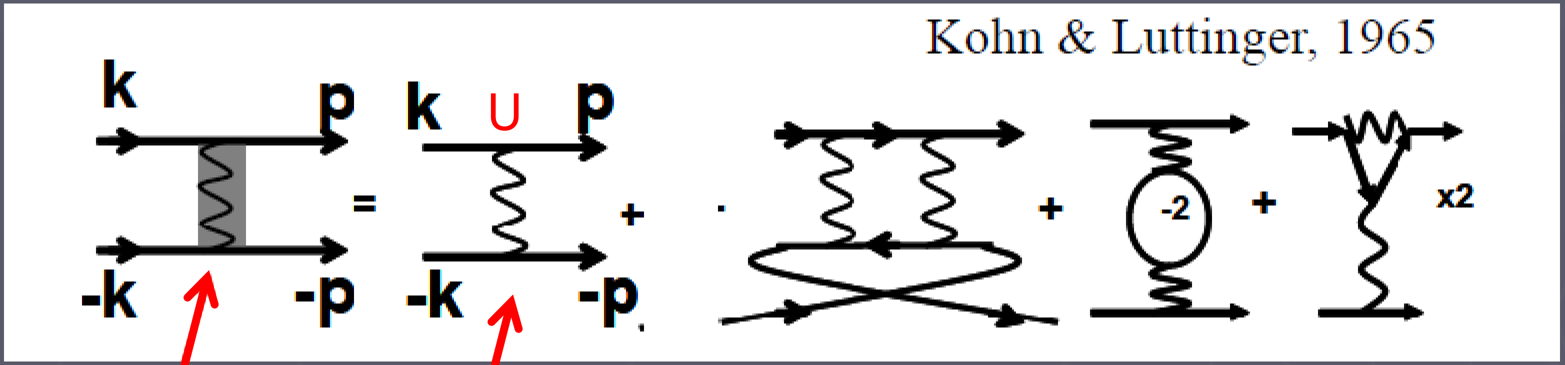
Prehistory: Kohn-Luttinger 1965

Friedel: screened Coulomb interaction



At finite distances, screened Coulomb interaction becomes attractive: finite-L pairing

Prehistory: Kohn-Luttinger 1965



effective pairing interaction

bare interaction (repulsive)

screening terms (attractive in some L-channels)

Example: short range U>0 for rotationally invariant system (≈ ^{3}He)

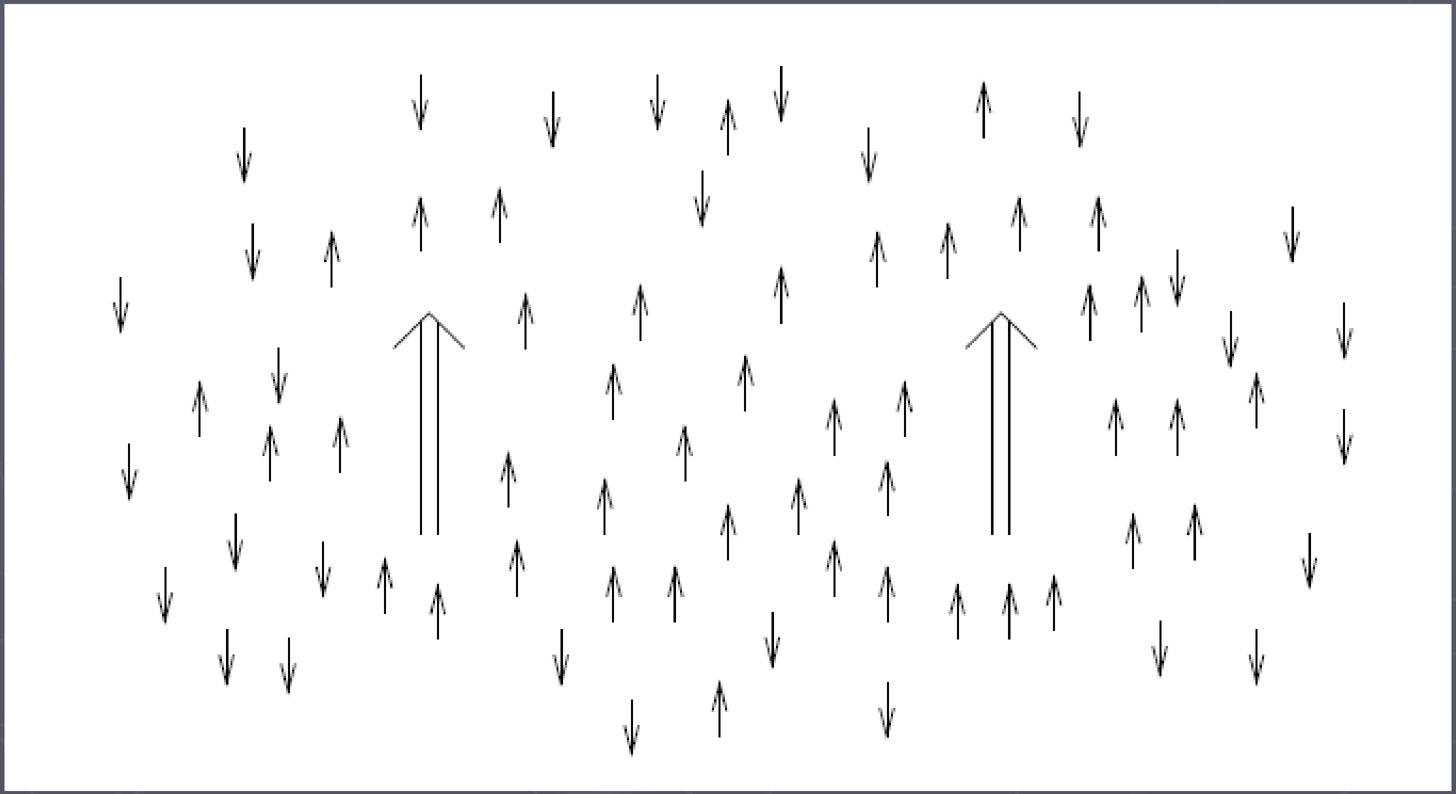
$$T_c \approx E_F \exp(-2.5L^4)$$

Best calculation in 1965: Brueckner Soda Anderson Morel PR 1960 :
predicted *L*=2 for ^{3}He $\Rightarrow$ $T_c \sim 10^{-17}$K

But had they taken *L*=1 they would have gotten $T_c \sim 1$ mK!

Spin fluctuations

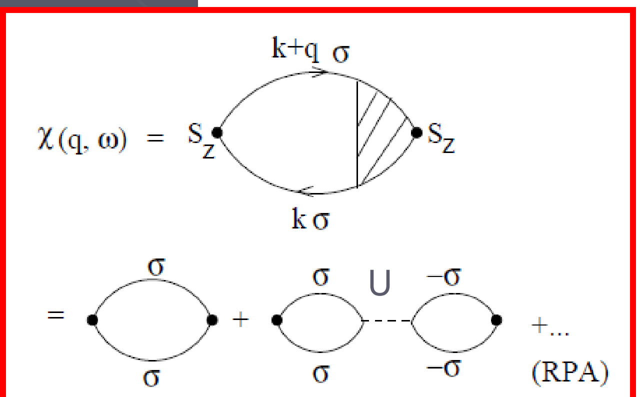
(ferromagnetic)

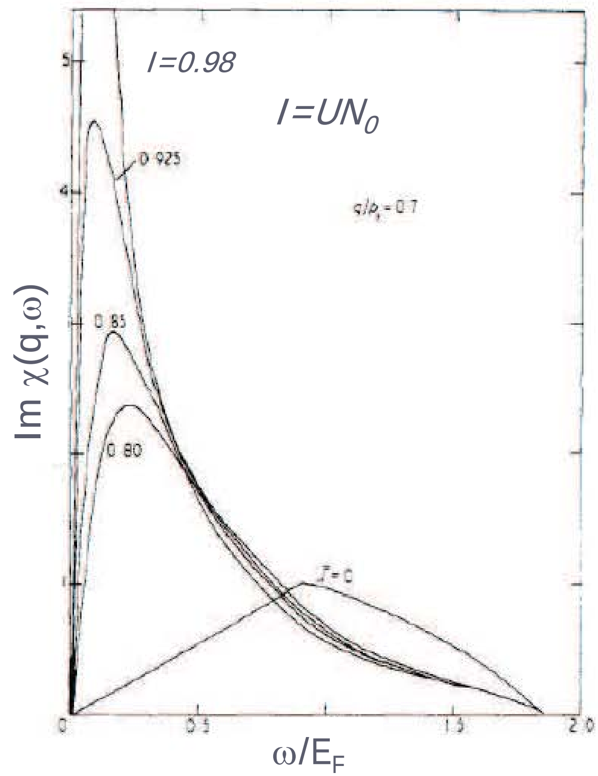


1st electron polarizes medium ferromagnetically, 2nd lowers its energy by aligning
$\Rightarrow$ attraction

Stoner theory: enhanced polarization from interactions

$$\chi(\mathbf{q},\omega) = \frac{\chi_0(\mathbf{q},\omega)}{1 - U\chi_0(\mathbf{q},\omega)}$$





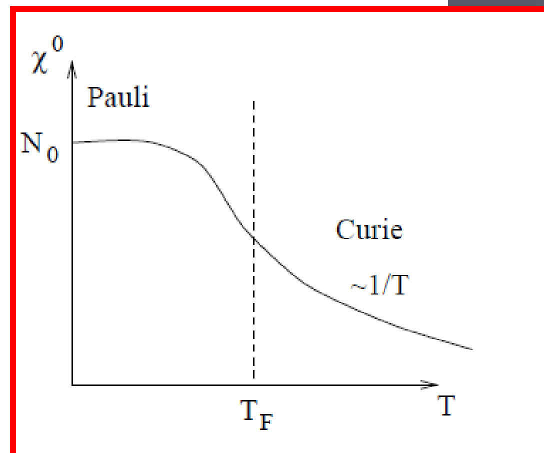


Figure 31: Spin fluctuation spectrum Im $\chi(\mathbf{q},\omega)$ for $q/p_F = 0.7$ in Stoner model of electron gas.

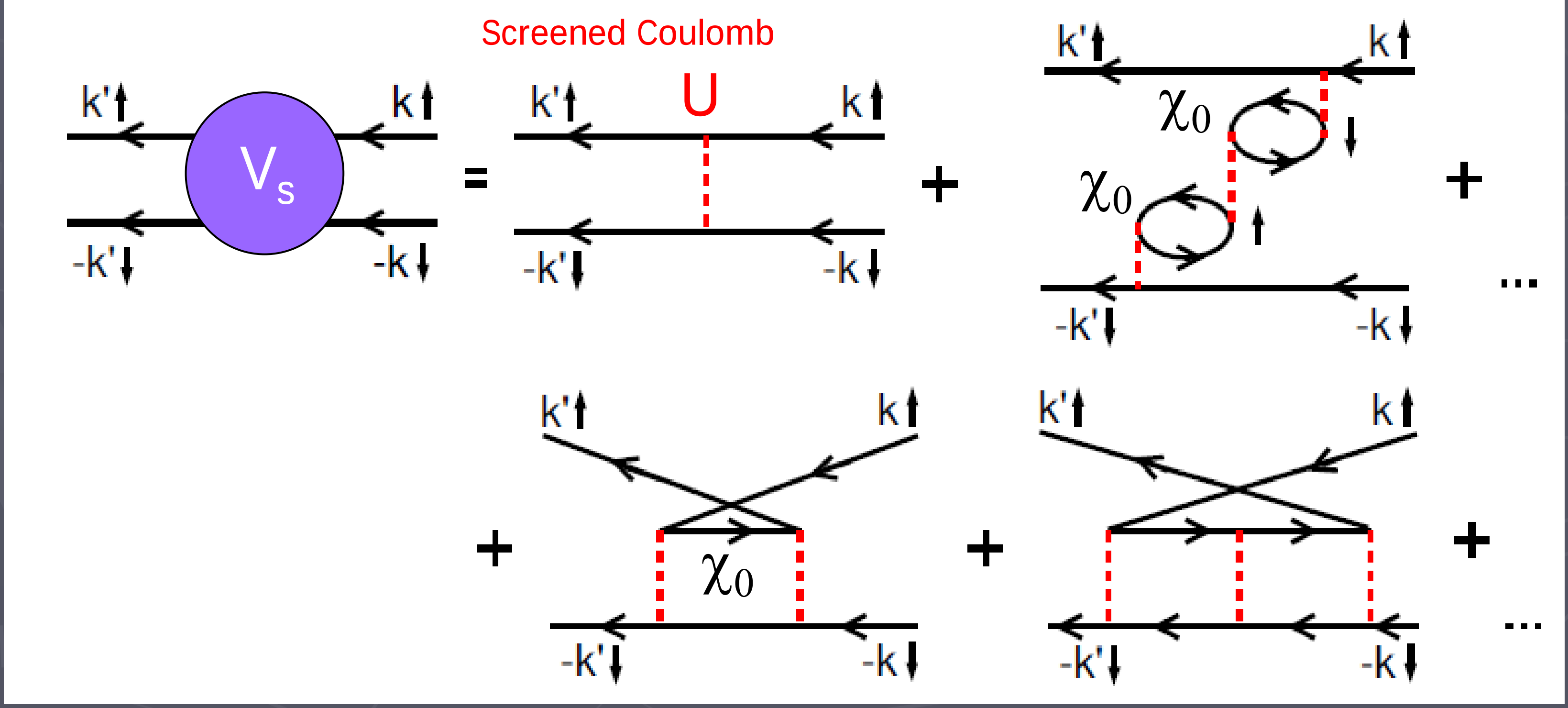
In limit $UN_0 \rightarrow 1$, excitations become very sharp (``paramagnons")

Spin fluctuation theories of pairing

Effective singlet interaction from spin fluctuations (Berk-Schrieffer 1966)

$$V_s(q,\omega) \cong \frac{3}{2}\,\frac{\bar{U}^2\chi_0(q,\omega)}{1-\bar{U}\chi_0(q,\omega)}$$

$$\chi_0(q,\omega) = \int \frac{d^3p}{(2\pi)^3}\,\frac{f(\varepsilon_{p+q})-f(\varepsilon_p)}{\omega-(\varepsilon_{p+q}-\varepsilon_p)+i\delta}$$



Results for pairing interactions

$$\Gamma_{\uparrow\uparrow} = \frac{-U^2\chi_0(\mathbf{k}'-\mathbf{k})}{1-U^2{\chi_0}^2(k'-k)},$$

$$\Gamma_{\uparrow\downarrow} = \frac{U}{1-U^2{\chi_0}^2(\mathbf{k}'-\mathbf{k})} + \frac{U^2\chi_0(\mathbf{k}'+\mathbf{k})}{1-U^2{\chi_0}^2(\mathbf{k}'+\mathbf{k})}$$

attractive

repulsive

Total pairing singlet channel:

$$V_s(k,k') = \tfrac{1}{2}(2\Gamma_{\uparrow\downarrow} - \Gamma_{\uparrow\uparrow}) = U^2\left(\frac{3}{2}\chi^s - \frac{1}{2}\chi^c\right) + U$$

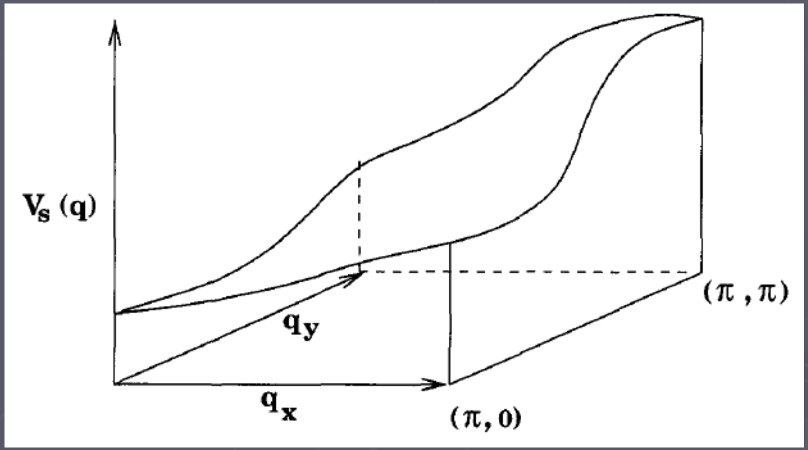
Spin fluctuation theories of pairing

Effective interaction from spin fluctuations (Berk-Schrieffer 1961)

paradigm: *d*-wave in cuprates from antiferromagnetic spin fluctuations

$$V_s(q,\omega) \cong \frac{3}{2}\,\frac{\bar{U}^2\chi_0(q,\omega)}{1-\bar{U}\chi_0(q,\omega)}$$

$$\chi_0(q,\omega) = \int \frac{d^3p}{(2\pi)^3}\,\frac{f(\varepsilon_{p+q})-f(\varepsilon_p)}{\omega-(\varepsilon_{p+q}-\varepsilon_p)+i\delta}$$



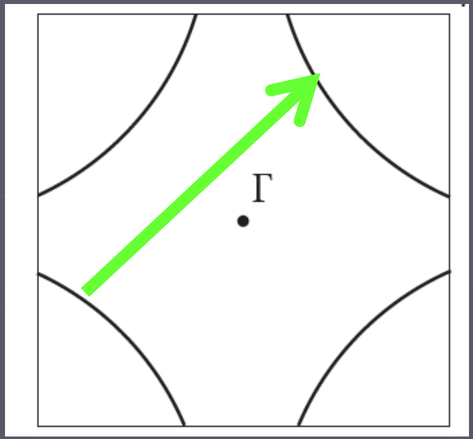
repulsive interactions!!!

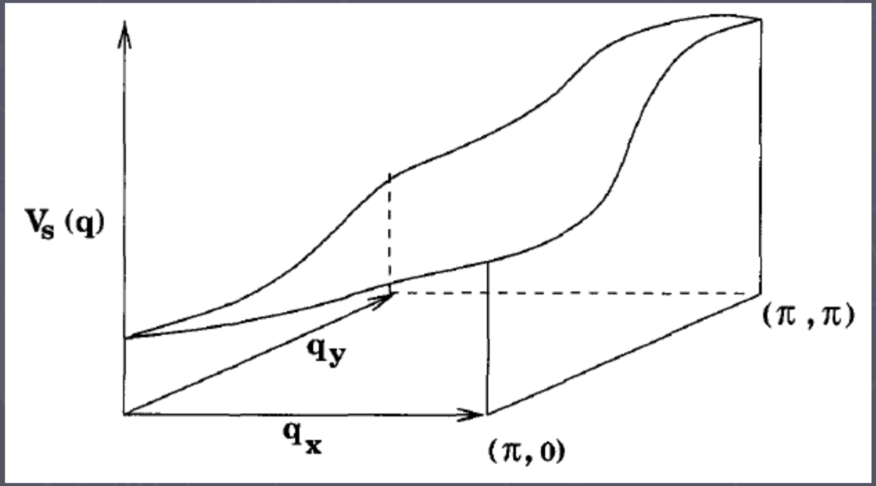
$$\Delta_p = -\sum_{p'} \frac{V(p-p')\Delta_{p'}}{2E_{p'}}$$

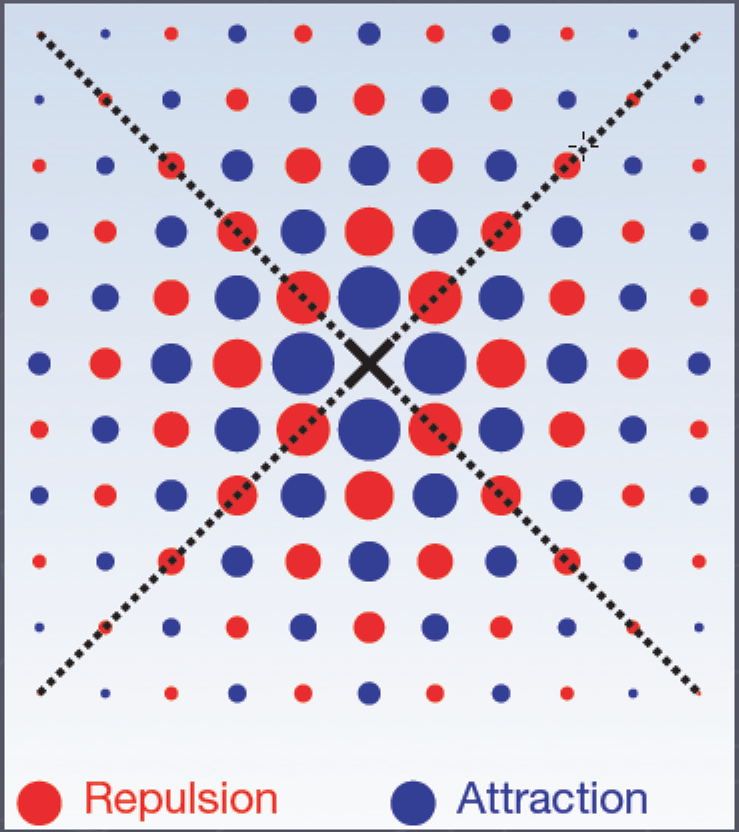
d-wave takes advantage of peak in spin fluct. interaction at π,π!

$$\Delta_{p+(\pi,\pi)} = -\Delta_p$$

remember at least some channels must be attractive in order to form Cooper bound state







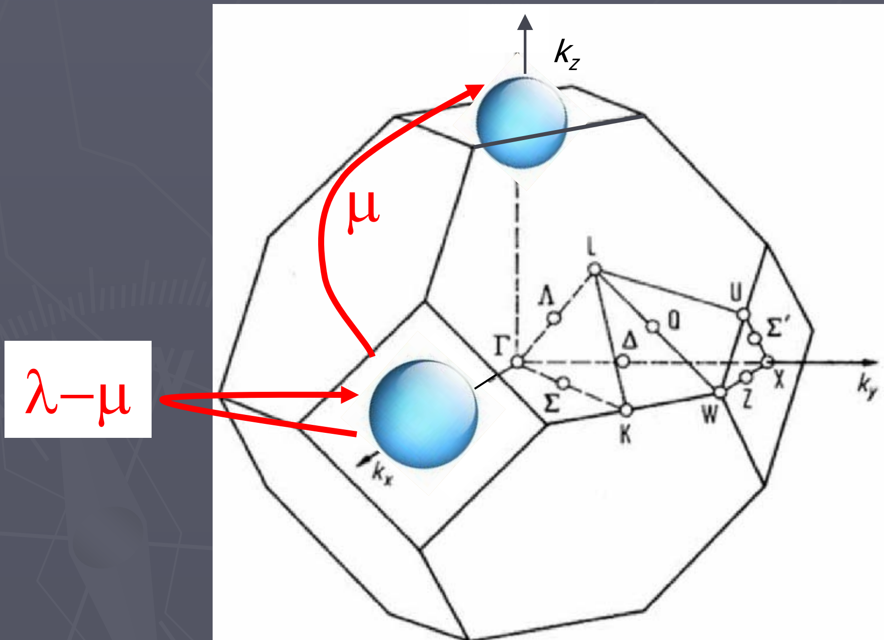
k-space:

$V_s(k-k') \sim V_0 + V_2 \phi_d(k)\ \phi_d(k') + \ldots$

r-space

Unconventional pairing from multiple Fermi pockets around high symmetry points

D. F. Agterberg , V. Barzykin, L.P. Gor’kov PRB 80, 14868 (1999)



$$\lambda_{\alpha\beta} = \lambda\,\delta_{\alpha\beta} + \mu(1 - \delta_{\alpha\beta})$$

possible singlet BCS solutions:

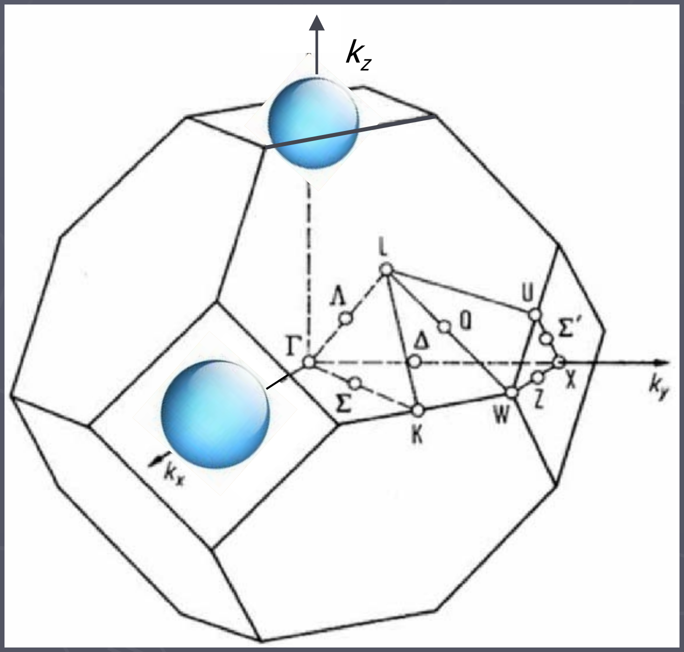
1D:	A_{1g}	s-wave
3D:	E_{1g}	d-wave

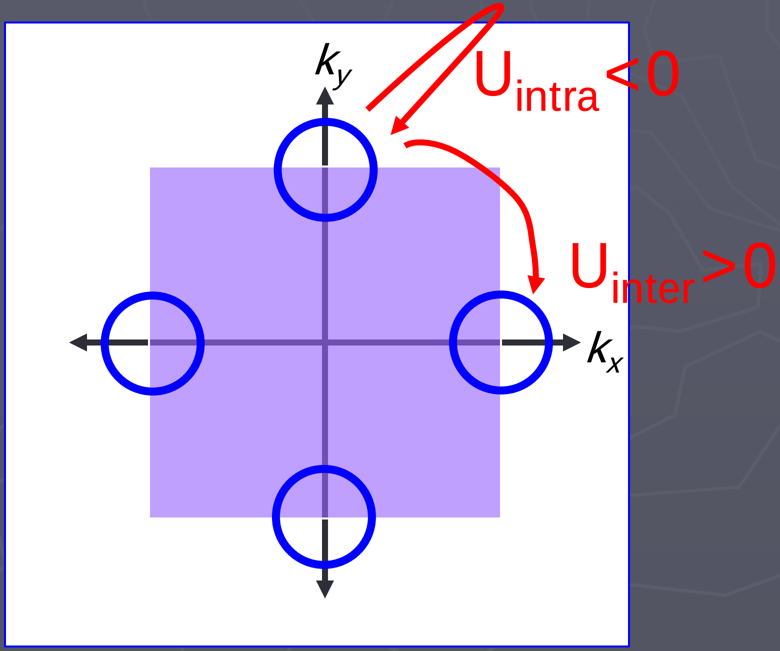
“The nontrivial 3D representation is stable if $\lambda - \mu < 0$ and $\mu > 0$, i.e., if the interaction is *attractive* for each pocket alone, while it is *repulsive* between two different pockets.”

Unconventional pairing from multiple Fermi pockets around high symmetry points

D. F. Agterberg , V. Barzykin, L.P. Gor'kov PRB 80, 14868 (1999)

Same idea, only easier, in 2D





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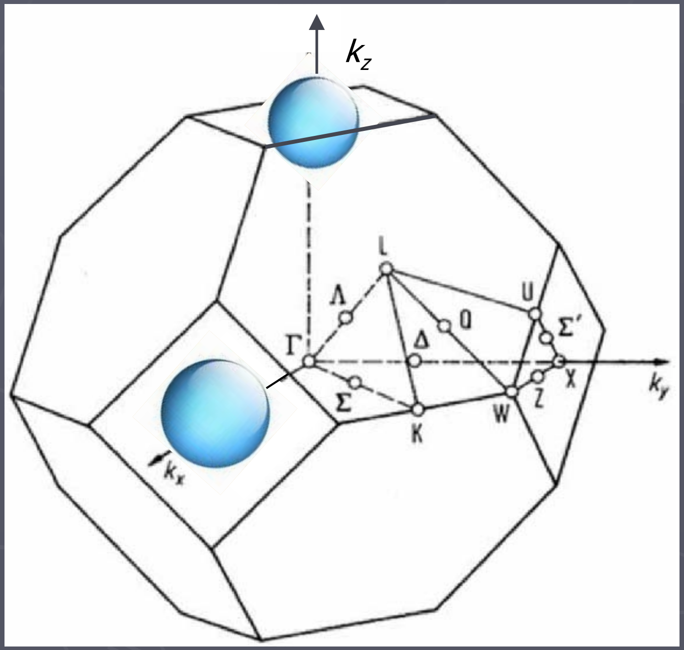
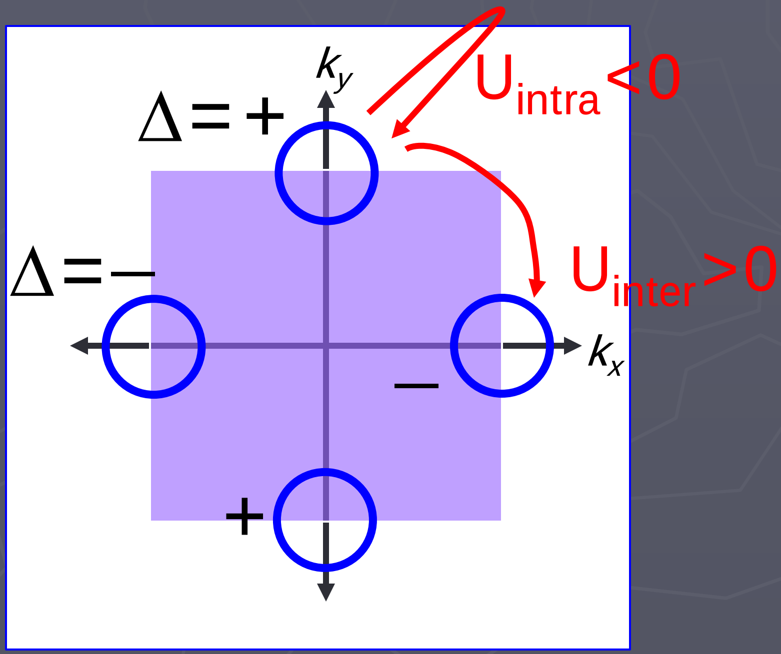
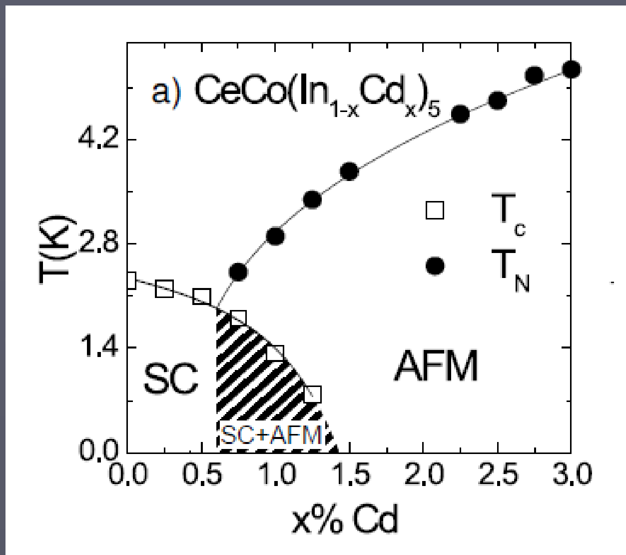




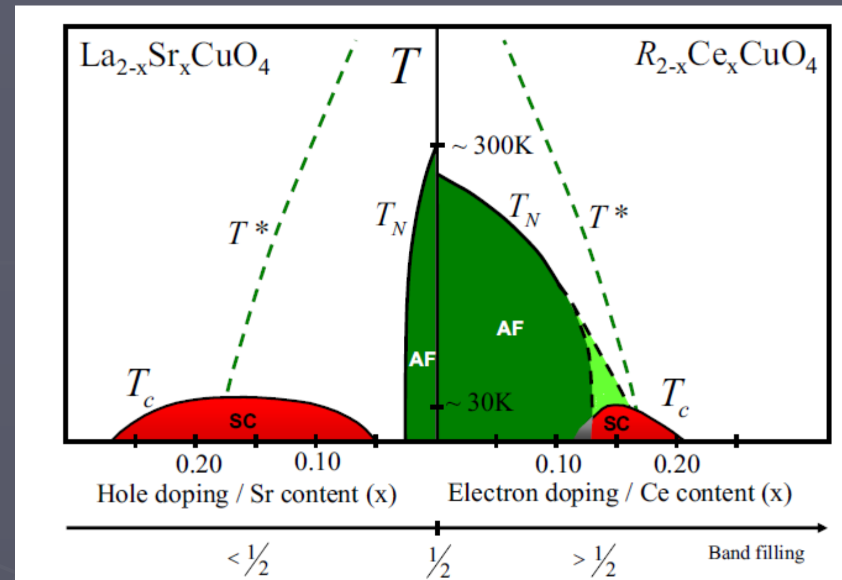
(nodeless) d-wave

Materials: phase diagrams

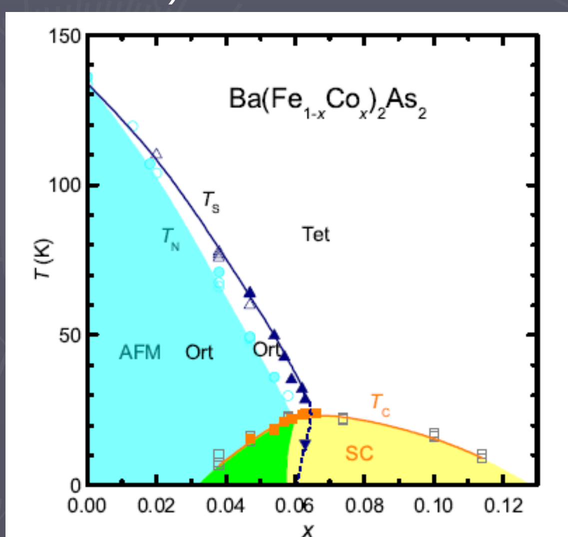
a) heavy fermions



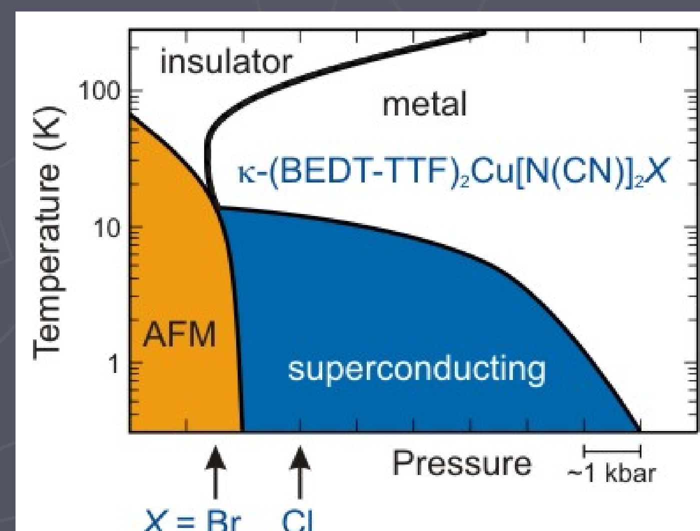
b) cuprates



c) Fe-based

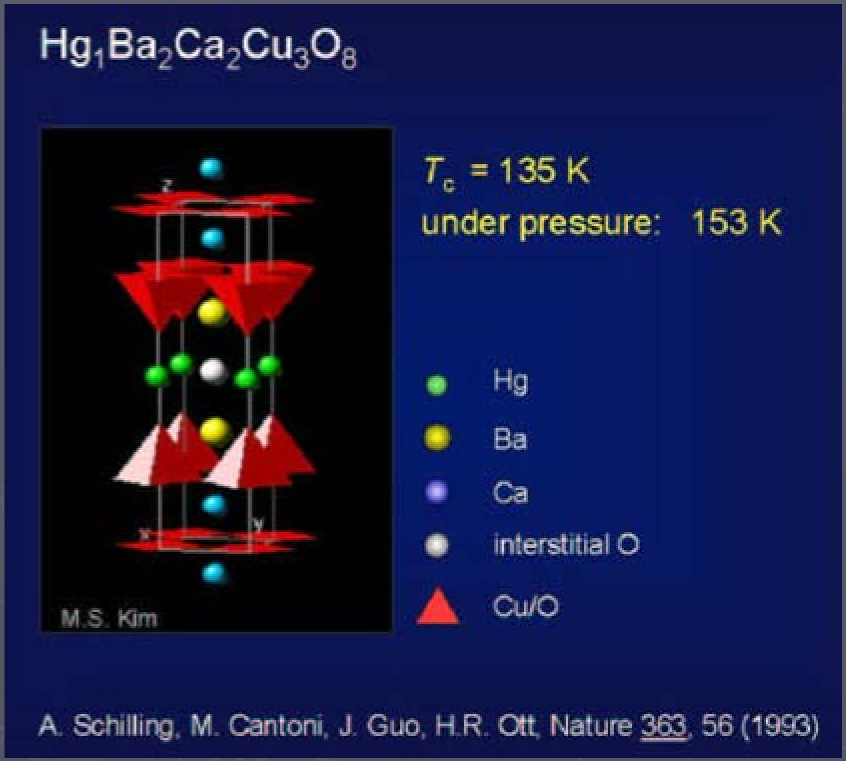


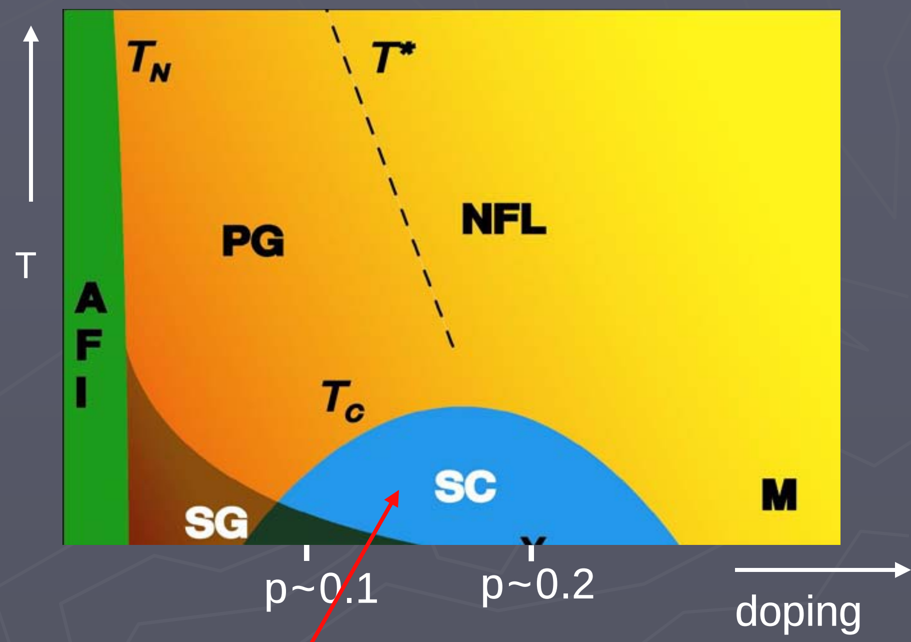
d) Organic charge-transfer salts



Similar phase diagrams: "A common thread?" D.J. Scalapino, RMP 2013

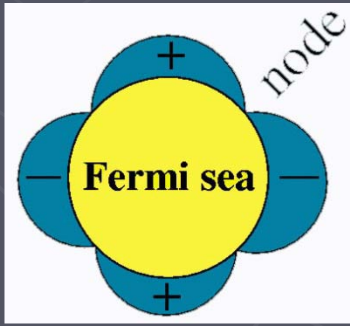
Cuprates: status report





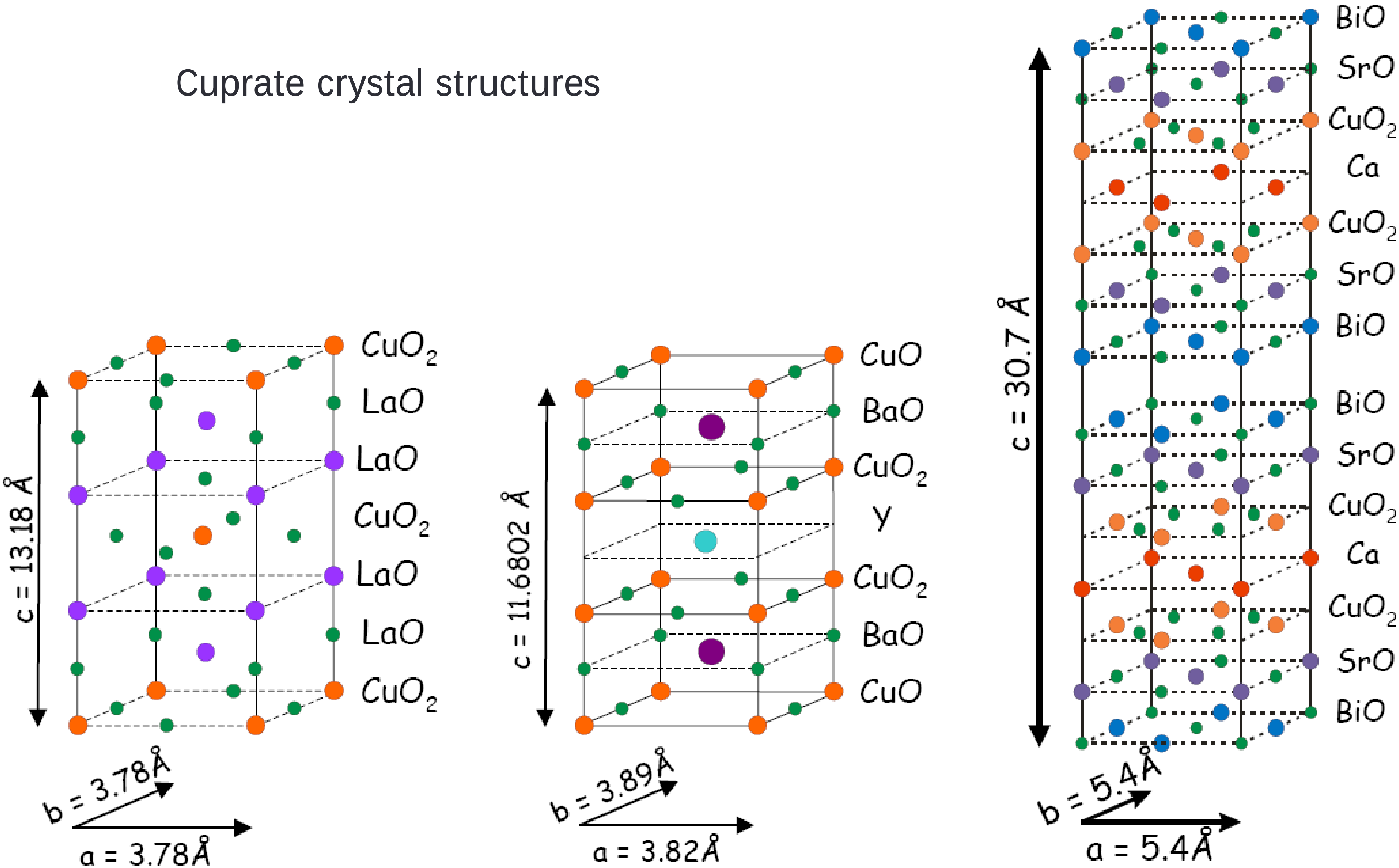
d-wave SC:

$$\Delta_k = \frac{\Delta_0}{2}\left(\cos k_x - \cos k_y\right)$$



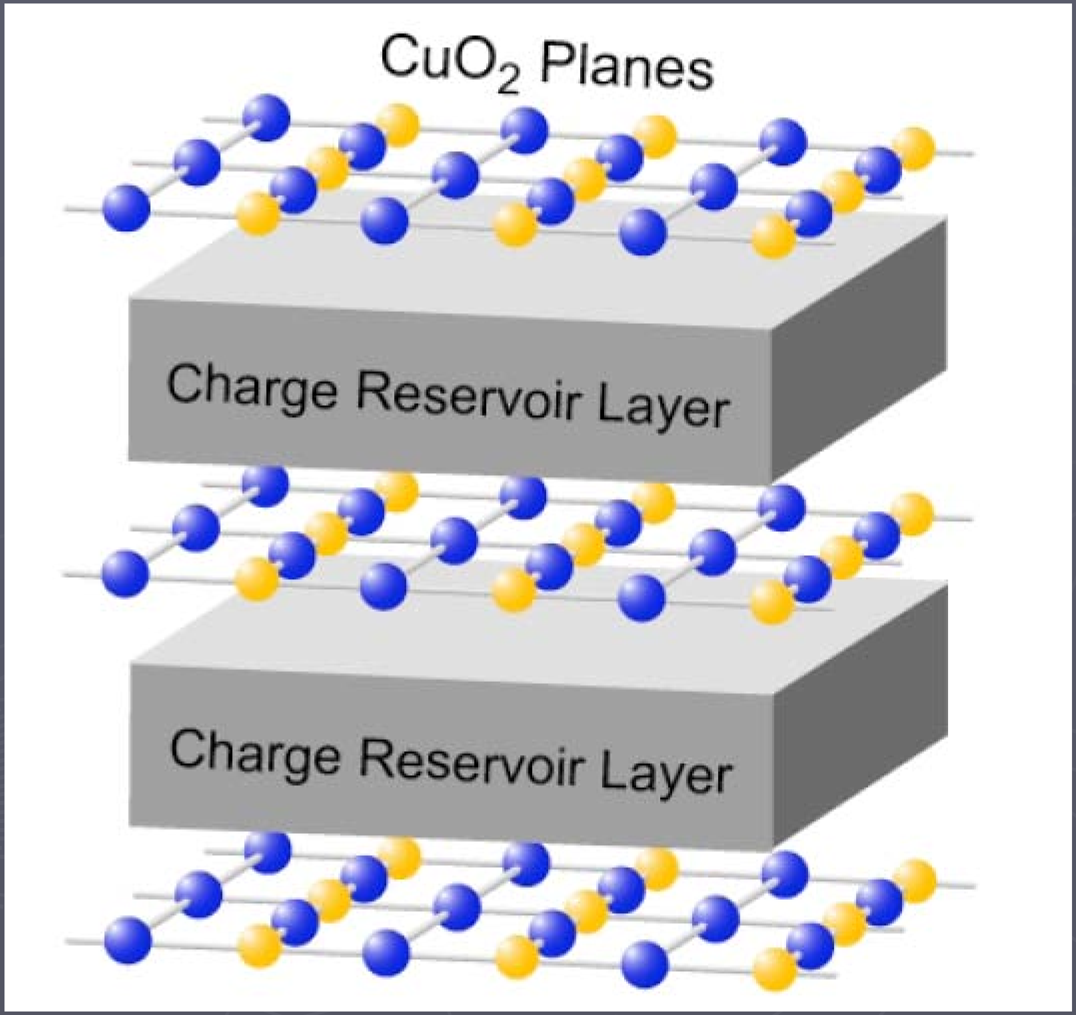
T_c is too high for electron-phonon “glue” to work! What holds pairs together?

Cuprate crystal structures



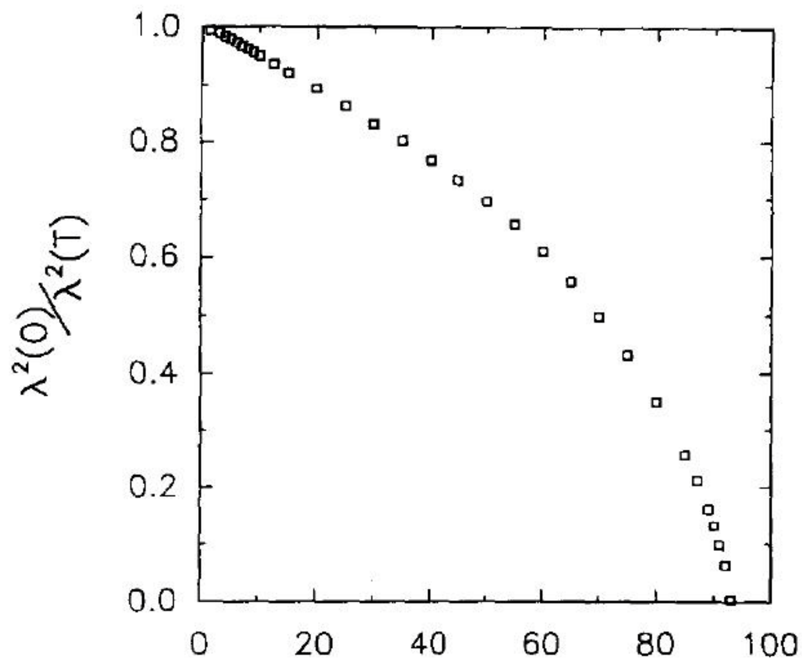
(a) LSCO (b) YBCO (c) BSCCO

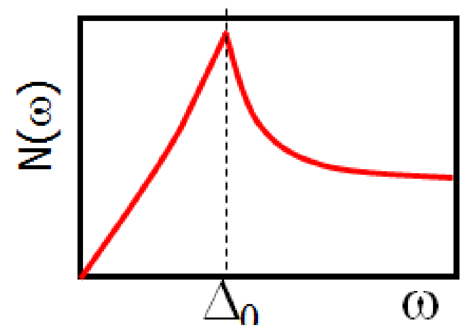
Action takes place in CuO2 planes doped by charge reservoirs



d-wave pairing in cuprates: 3 crucial experiments

1. London penetration depth. W. Hardy et al. PRL 1993





superfluid density

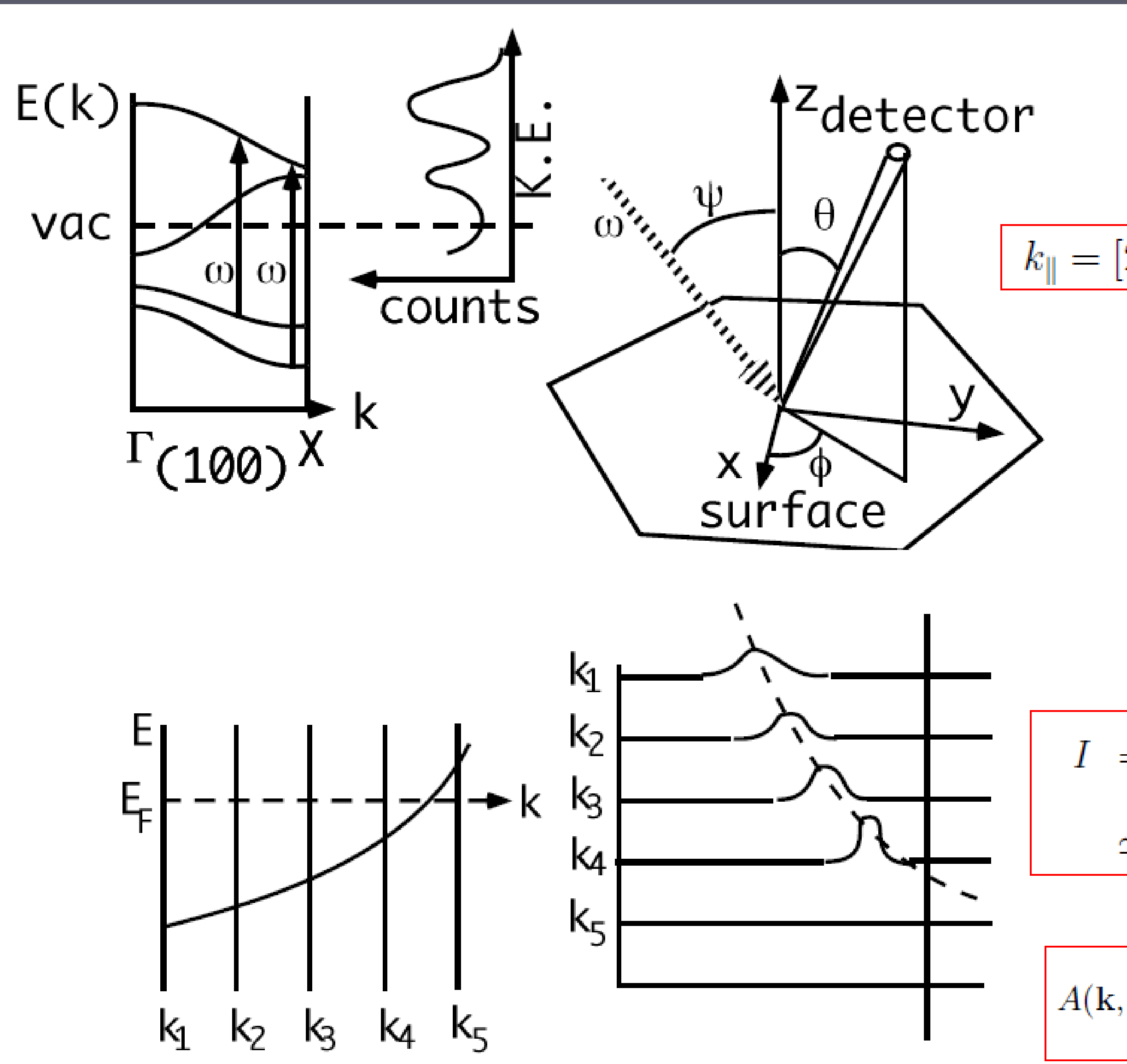
$$n_s = n\left[1 - \int d\xi_k \left(\frac{-\partial f}{\partial E_k}\right)\right] \quad (234)$$

$$= n\left[1 - \int dE N(E)\left(\frac{-\partial f}{\partial E}\right)\right], \quad (235)$$

which, if one substitutes the low-energy d-wave DOS obtained above, $N(E) \sim N_0 E/\Delta_0$, yields immediately for $T \ll \Delta_0$,

$$n_s \simeq n\left[1 - \frac{T}{\Delta_0}\right], \quad (236)$$

ARPES=Angle Resolved Photoemission Spectroscopy



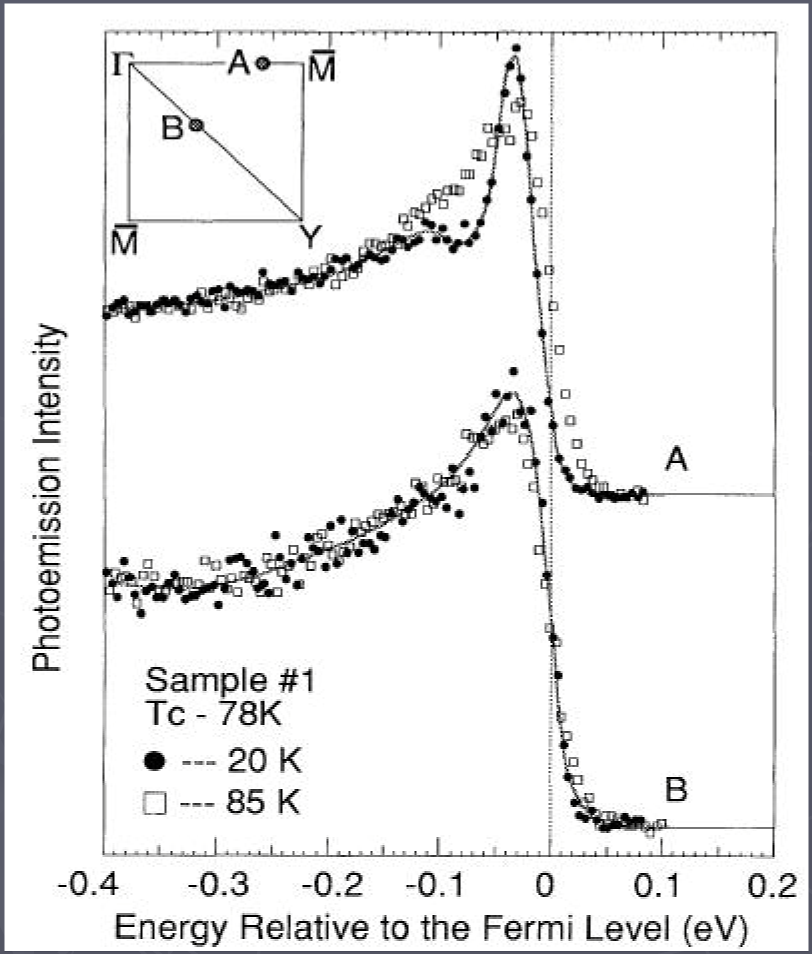
$$k_\parallel = [2m(\omega - \varepsilon_{n,\mathbf{k}} + \mu - W)]^{1/2} \sin\theta$$

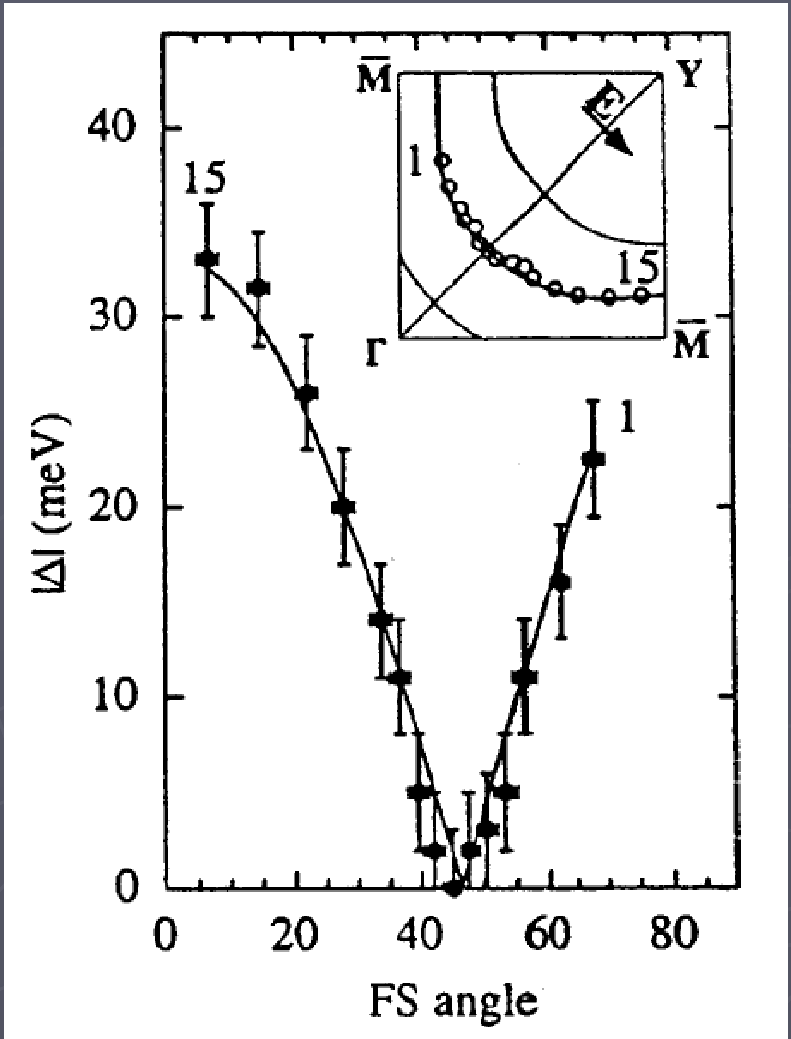
$$\begin{aligned} I &= 2\pi \sum_n |\langle n; -\mathbf{k}|\hat{\psi}|0\rangle|^2 \delta(\omega + \varepsilon_n - \mu), \\ &\simeq I_0 |M(\mathbf{k},\omega)|^2 A(\mathbf{k},\omega) f(\omega), \end{aligned}$$

$$A(\mathbf{k},\omega) = -\frac{1}{\pi} \frac{\Sigma''(\mathbf{k},\omega)}{(\omega - \epsilon_{\mathbf{k}} - \Sigma'(\mathbf{k},\omega))^2 + \Sigma''(\mathbf{k},\omega)^2}$$

d-wave pairing in cuprates: 3 crucial experiments

2. ARPES ZX Shen et al. PRL 1993



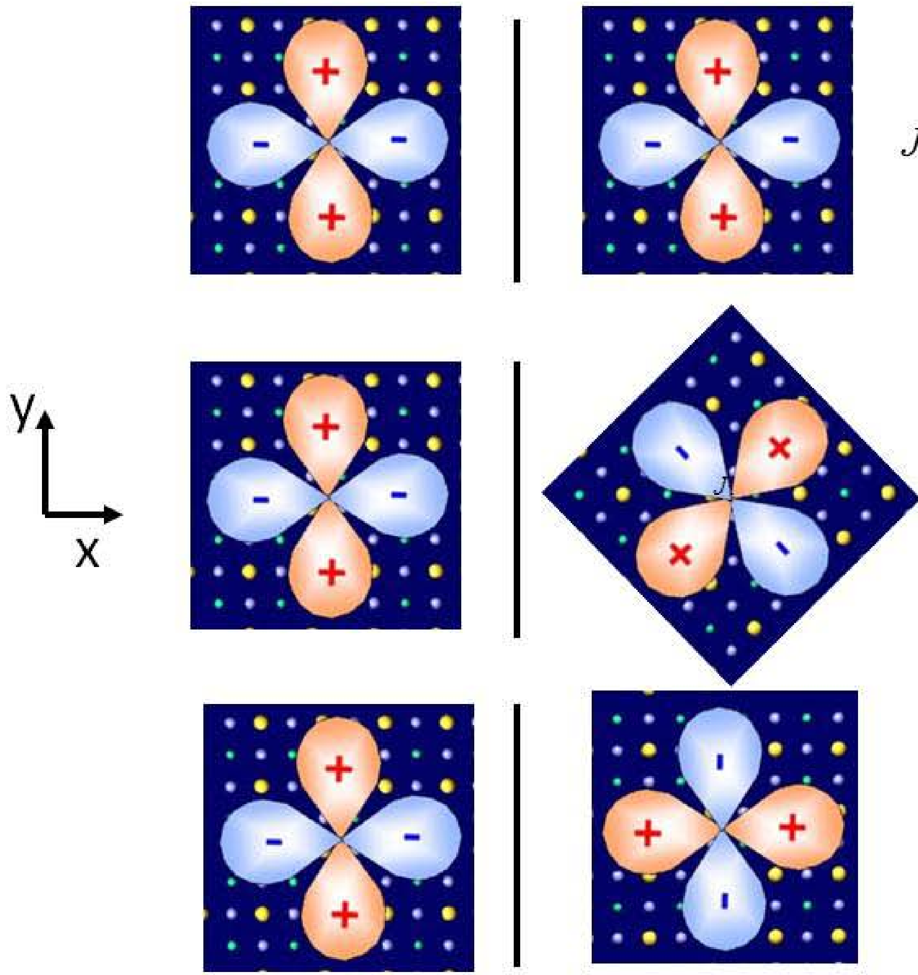


Fits $\Delta_k = \Delta_0 (\cos k_x - \cos k_y)$ well!

d-wave pairing in cuprates: 3 crucial experiments

3. Phase sensitive experiments—Josephson tunneling

$$J_s = A_s(n_x^2 - n_y^2)_L(n_x^2 - n_y^2)_R \sin\gamma$$



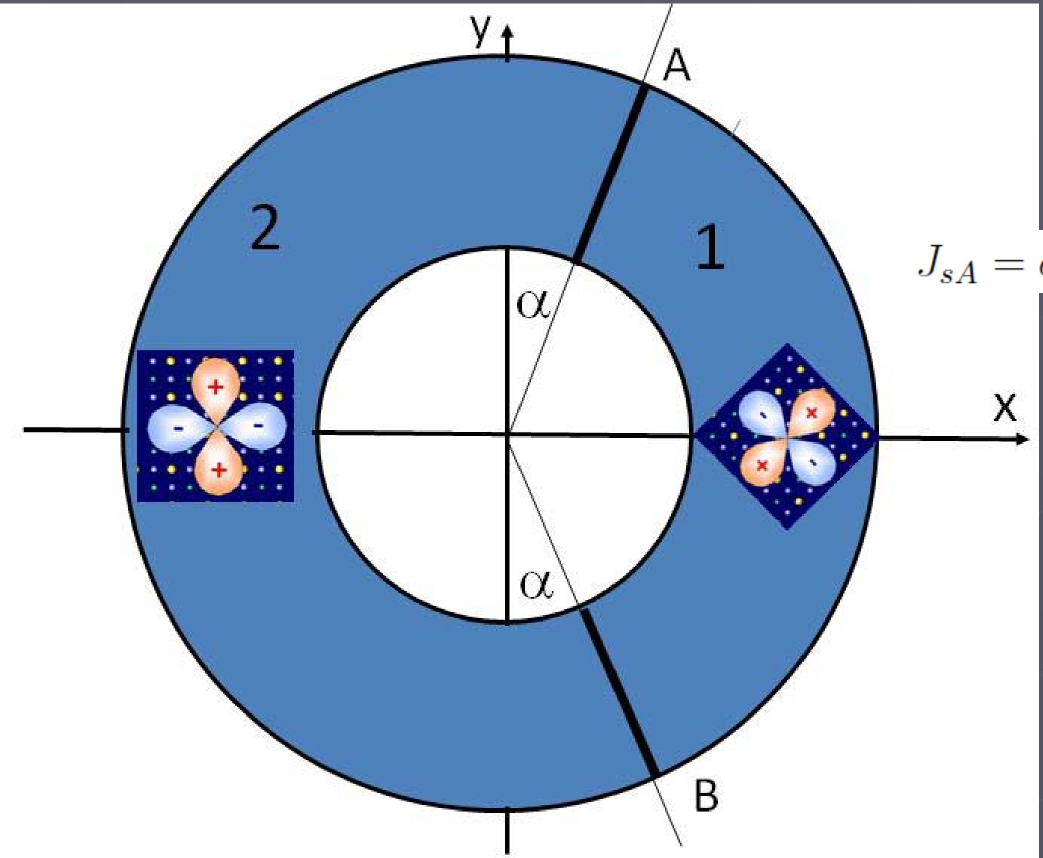
$J_s \propto n_x^2 n_x^2 \approx 1$ (like s – wave)

$J_s \propto n_x^2 \cdot 0 \approx 0$

$J_s = -|J_s|$ (π – junction)

Bicrystal ring

$$\delta f \sim c_A(\psi_{1A}\psi_{2A}^* + c.c.) + c_B(\psi_{1B}\psi_{2B}^* + c.c.)$$



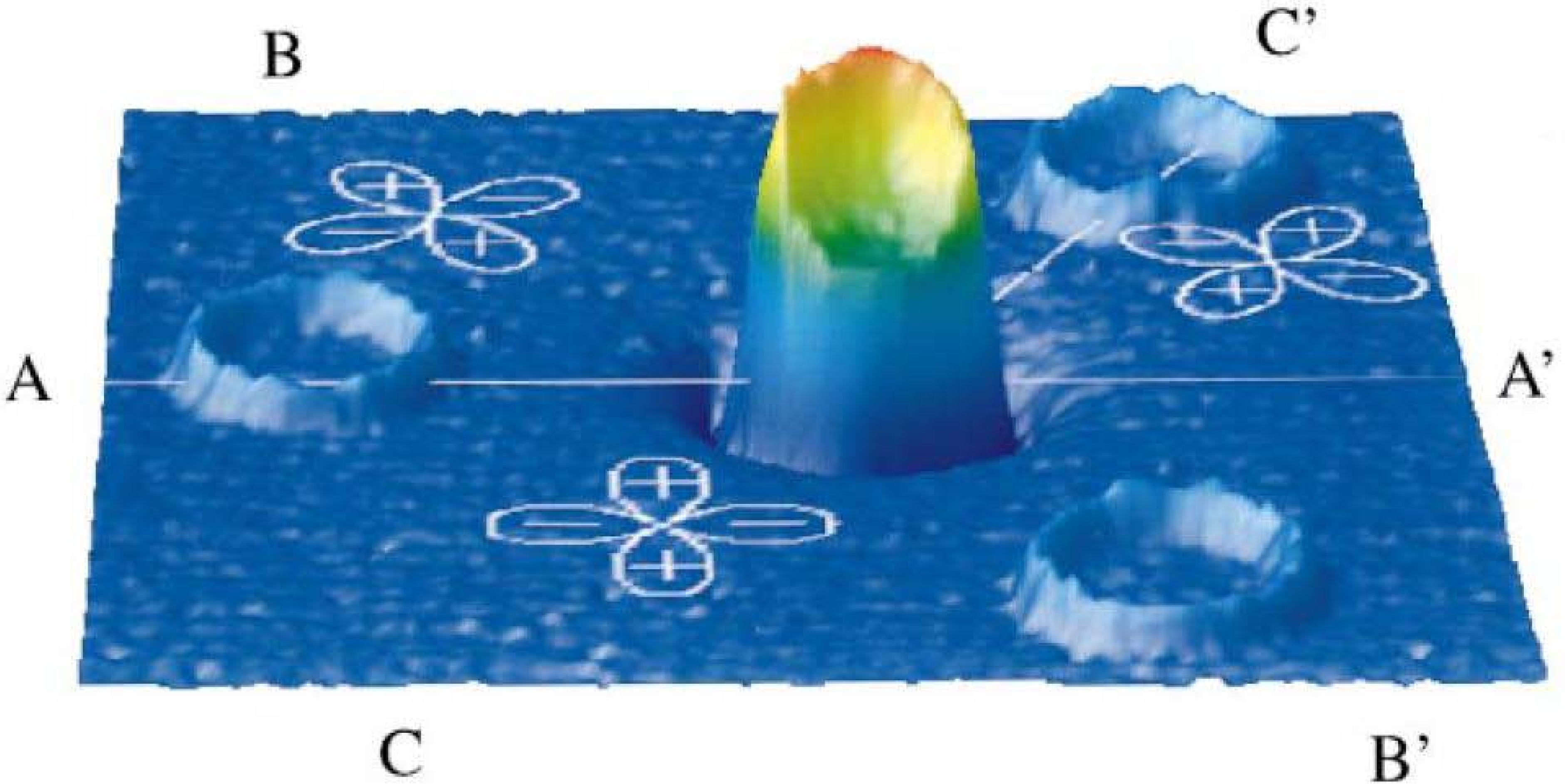
s-wave $c_A = c_B$

d-wave $c_A = -c_B$

$J_{sA} = \delta f/\delta\phi_A$ is minus the current $J_{sB} = \delta f/\delta\phi_B$

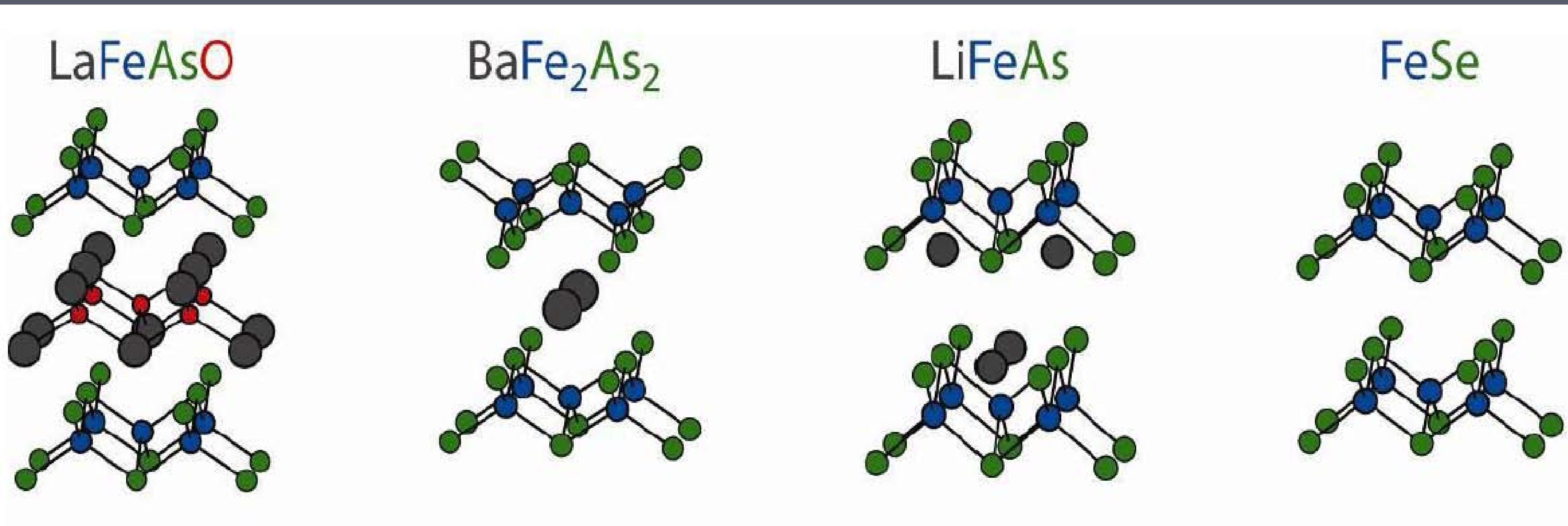
"pi-junction" $\Rightarrow$ flux quantized in $1/2\Phi_0$

Tsuei/Kirtley tricrystal expt.: YBCO on STO, etc.



Iron-based superconductors

Recent reviews: G.R. Stewart RMP 2012 Paglione & Greene Nat Phys 2010; Johnston Adv. Phys. 2010



T_c=28K
(55K for Sm)

- Kamihara et al JACS (2008)
- Ren et al Chin. Phys. Lett. (2008)

T_c=38K

- Rotter et al. arXiv: PRL (2008)
- Ni et al Phys. Rev. B 2008 (single xtals)

T_c=18K

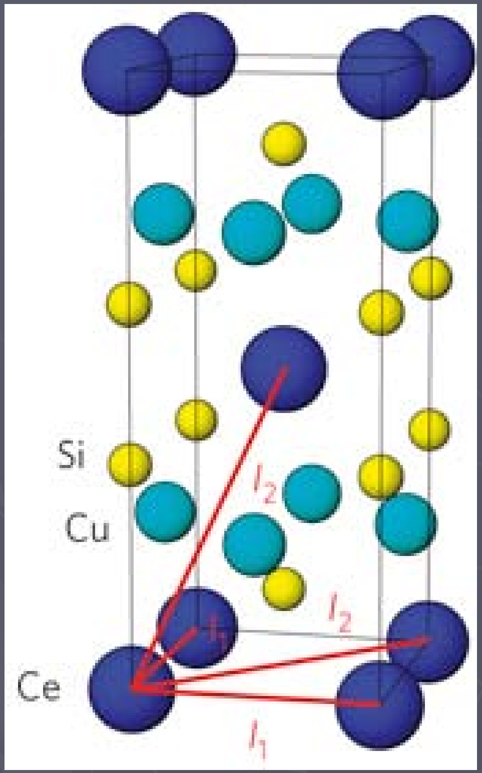
Wang et al
Sol. St. Comm. 2008

T_c=8K

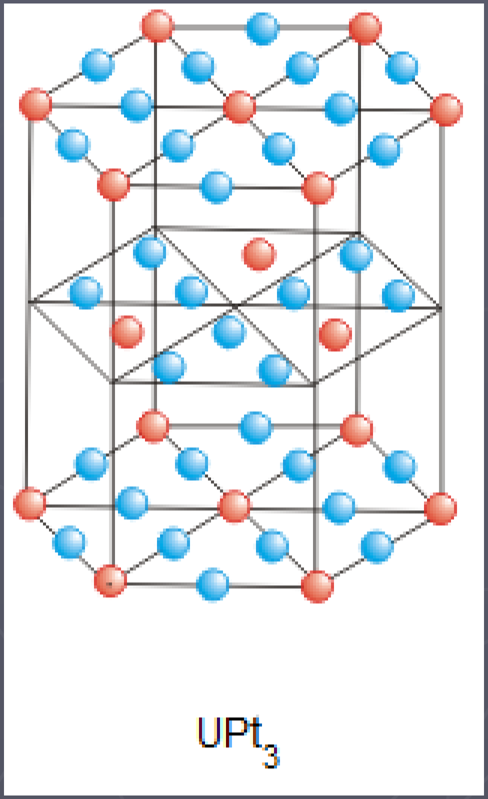
Hsu et al
PNAS 2008

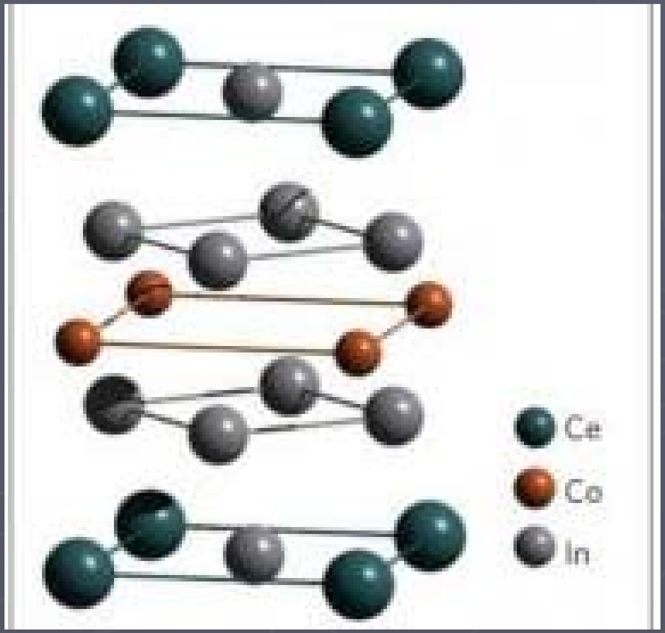
No arsenic ☺!

Heavy fermion materials



$CeCu_2Si_2$

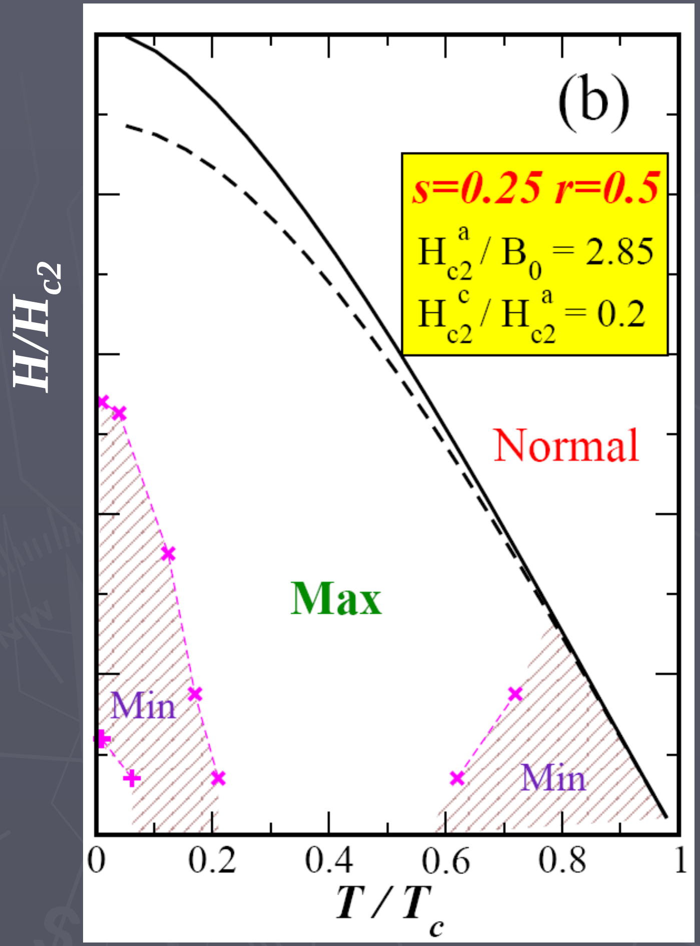




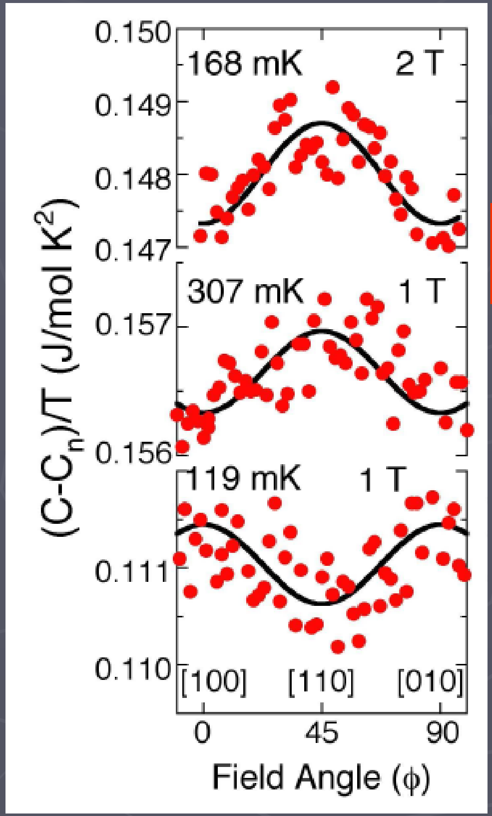
$CeCoIn_5$

d-wave pairing in $CeCoIn_5$: specific heat anisotropy

A. Vorontsov and IV, '06-07



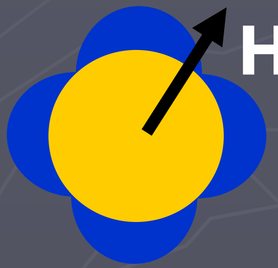
- Shaded area: *C/T* minimum for **H**||node
- Unshaded: *C/T* maximum for **H**||node
- suggestive of $d_{x^2-y^2}$ pairing in $CeCoIn_5$
- prediction: anisotropy inversion at lower *T, H*



K. An et al. '10

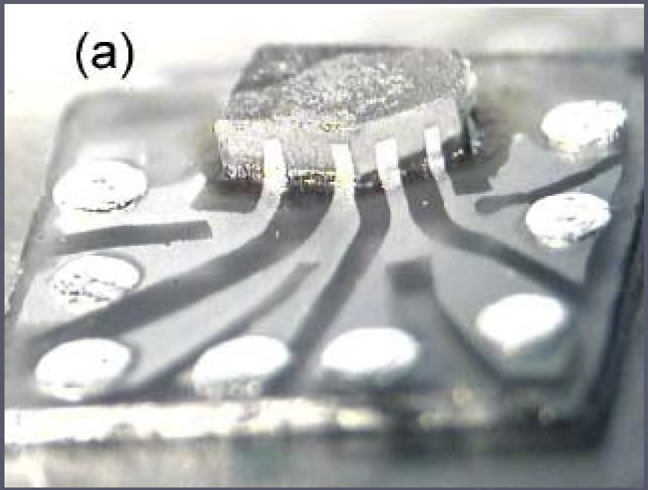
$CeCoIn_5$

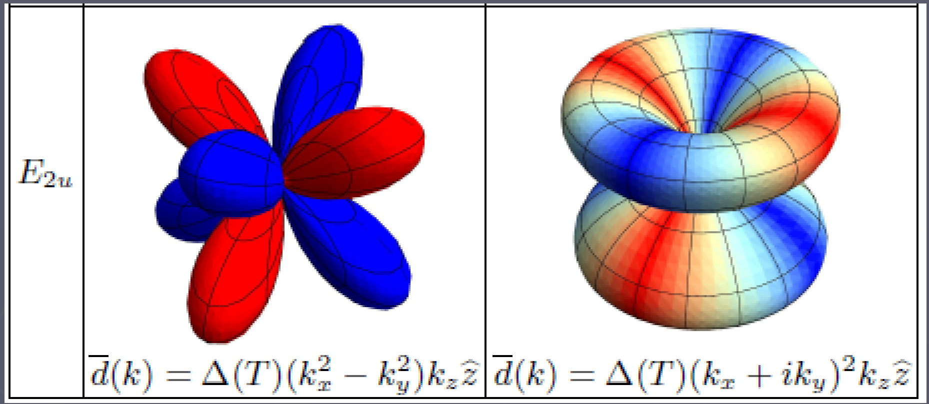
$d_{x^2-y^2}$

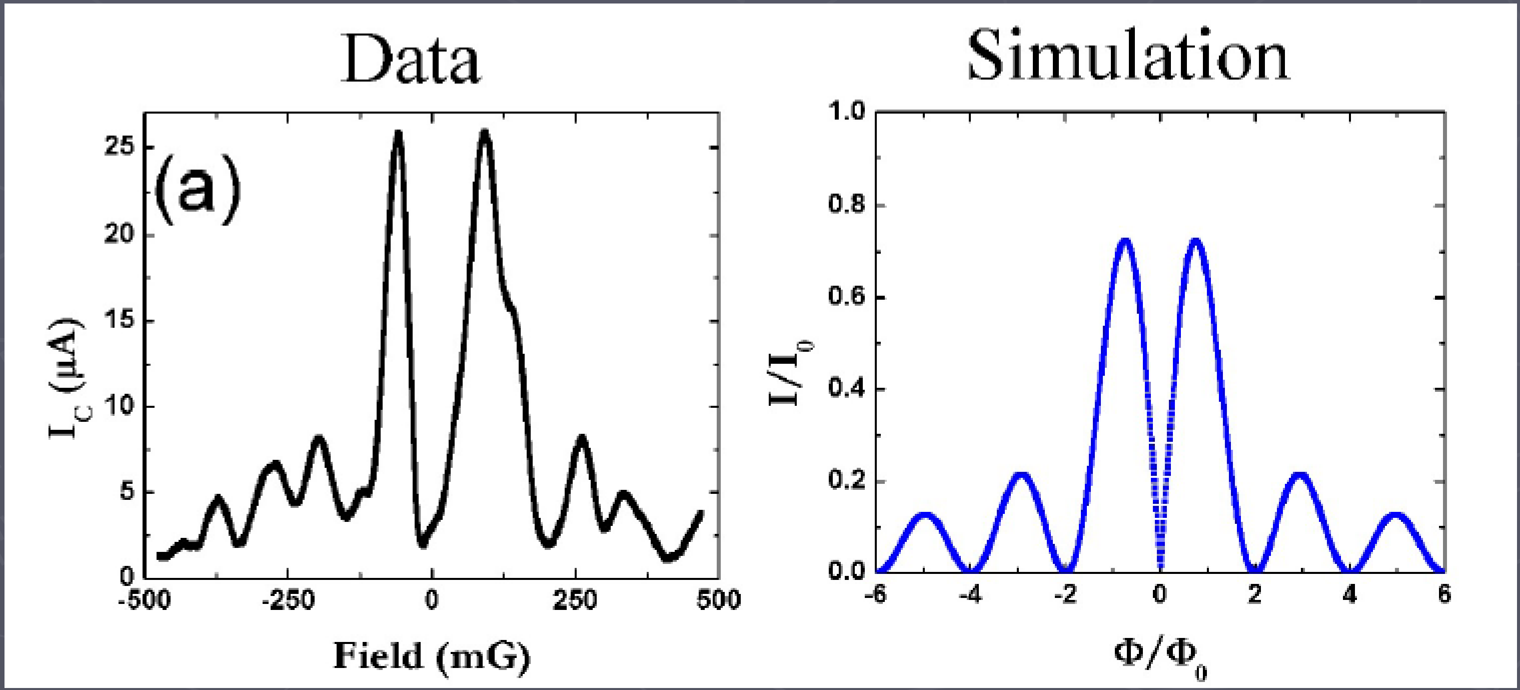


f-wave pairing in UPt_3: Josephson-Frauenhofer spectroscopy

Strand et al PRL 2009







Conclusions

- **Conventional pairing:**
  USUALLY occurs in $\ell=0$ pairing channel to take advantage of the attractive electron-phonon interaction at r=0 – avoid Coulomb repulsion in time

- **Unconventional pairing:**
  USUALLY occurs in higher-ℓ pairing channel to avoid the Coulomb interaction in space – Ψ has node at r=0

- Exotic effects in SC state due to non $\ell=0$ *symmetry*

Reading:

"Phenomenological theory of unconventional superconductivity**", M. Sigrist and K. Ueda, Rev. Mod. Phys. 63, 239** (1991);
"Introduction to Unconventional Superconductivity", by V. P. Mineev and K.V. Samokhin (Gordon and Breach, Amsterdam), 1999;
"Pairing symmetry in cuprate superconductors**", C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. 72, 974 (2000);**
"Introduction to Unconventional Superconductivity, Manfred Sigrist, Lecture Notes **AIP Conference Proceedings 789, 165 (2005)** [Available online]