Introduction to unconventional superconductivity

P.J. Hirschfeld, U. Florida
Outline

- Review of conventional SC (Blundel lecture!)
- What symmetries for $\Delta$ are allowed beyond BCS?
- What is unconventional superconductivity?
- What are pairing mechanisms besides phonons?
- Materials: cuprates, Fe-based, heavy fermions… similarities & differences? Higher $T_c$?
Conventional superconductors

• BCS theory (1957)

Quantum mechanical behavior at the macroscopic scale

Leon Cooper

Nobel prize : 1972

John Bardeen

Robert Schrieffer

Macro. Quantum State

\[ \Psi_{BCS} = \prod_k \left( u_k + v_k c_{k\uparrow}^* c_{-k\downarrow}^* \right) |0\rangle \]

s-wave symmetry

\[ \Delta \equiv \mathcal{V} \langle c_{-k\downarrow} c_{k\uparrow} \rangle \sim \Delta_0 e^{i\phi} \]
How Cooper pairs form in conventional superconductors:

the “glue”: electron-phonon interaction

Effective e-e interaction

Screened Coulomb

Electron-phonon attraction

Note: electrons avoid Coulomb repulsion in time (interaction is retarded)
Superconductivity: Ground state

Puzzle 1: is this a good picture of Cooper pairs?
A: No! For most SC, pair size $\xi >> n^{-1/d}$
Superconductivity: Ground state

\[ \xi = \frac{v_F}{\Delta} \gg n^{-1/d} \]

Simple metal:
\[ \xi \sim 10^3 \text{ A} \]
\[ n^{-1/d} \sim 1 \text{ A} \]

Remember that all pairs are phase coherent!

St. Matthew’s Passion
Oxford, UK
Superconductivity: Excited states

\[ \xi = \frac{v_F}{\Delta} \gg n^{-1/d} \]

"Bogoliubov quasiparticle"
Puzzle #2:

Cooper pairs are not independent bosons!
Is that all there is?  Brian Pippard and “The Cat and the Cream” speech IBM 1961
“I think I might remark that in low-temperature physics the disappearance of liquid helium, superconductivity, and magneto-resistance from the list of major unsolved problems has left this branch of research looking pretty sick from the point of view of any young innocent who thinks he's going to break new ground.”
Superconductivity in the Presence of Strong Pauli Paramagnetism: CeCu$_2$Si$_2$

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(Received 10 August 1979; revised manuscript received 7 November 1979)

A comparison was made between four low-temperature properties of LaCu$_2$Si$_2$ and CeCu$_2$Si$_2$. Whereas LaCu$_2$Si$_2$ behaves like a normal metal, CeCu$_2$Si$_2$ shows (i) low-temperature anomalies typical of "unstable $4f$ shell" behavior and (ii) a transition into a superconducting state at $T_c = 0.5$ K. Our experiments demonstrate for the first time that superconductivity can exist in a metal in which many-body interactions, probably magnetic in origin, have strongly renormalized the properties of the conduction-electron gas.
Possible High $T_c$ Superconductivity in the Ba–La–Cu–O System

J.G. Bednorz and K.A. Müller
IBM Zürich Research Laboratory, Rüslikon, Switzerland

Received April 17, 1986

Z. Physik, June 1986

Alex Müller and Georg Bednorz
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Grüneisen parameter coupling in heavy fermion systems

DOI: 10.1007/BF01303699
Authors: M. Yoshizawa, B. Lüthi, and K. D. Schotte
Text: PDF (494 kb)

Anomalous temperature dependence of the magnetic field penetration depth in superconducting $\text{UBe}_{13}$

DOI: 10.1007/BF01303700
Text: PDF (1,206 kb)

Possible high $T_c$ superconductivity in the Ba–La–Cu–O system

DOI: 10.1007/BF01303701
Authors: J. G. Bednorz and K. A. Müller
Text: PDF (396 kb)
Discovery of $\text{LaO}_{1-x}\text{F}_x\text{FeAs}$

Kamihara et al. JACS 2008

$T_{c,\text{max}} = 26$ K
Can we get high $T_c$ from conventional superconductivity?

First: Eliashberg strong coupling theory for electron-phonon systems

Electron Self-Energy

\[
\Sigma(\vec{k}, \omega) = \alpha \alpha \quad F(\omega)
\]

Strong coupling Eliashberg theory provides quantitatively accurate predictions for all conventional superconductors based on knowing the electron-phonon interaction, summarized in the phonon spectral density $\alpha^2 F(\omega)$, which can be calculated or measured by experiment.

There are deviations from BCS for most materials, even elements.
Can we get high $T_c$ from conventional superconductivity?

**Electronic Band Properties and Superconductivity in $\text{La}_{2-\gamma}\text{Y}_{\gamma}\text{CuO}_4$**

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(Received 7 January 1987)

The results of electronic-structure calculations for tetragonal $\text{La}_2\text{CuO}_4$ provide insight concerning the origin of high-temperature superconductivity in the $\text{La}_{2-\gamma}\text{Y}_{\gamma}\text{CuO}_4$ alloys. A half-filled $\text{Cu}(3d)\text{-O}(2p)$ band with two-dimensional character and a nearly square Fermi surface produces a Peierls instability for $\gamma=0$ that opens a semiconductor gap over the Fermi surface. Alloying with divalent or tetravalent atoms should spoil the nesting features while maintaining the strong coupling of O phonons to the conduction electrons.

PACS numbers: 72.15.Nj, 71.25.Pi, 74.20.-z, 74.60.Mj

**Electron-phonon interaction in $\text{Ba}_2\text{YC}_3\text{O}_7$**

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(Received 18 August 1987)

A realistic tight-binding theory, based on the energy-band results of Mattheiss and Hamann, is applied to study the electron-phonon interaction in $\text{Ba}_2\text{YC}_3\text{O}_7$. In contrast to previous results for the 40-K superconductor $\text{La}_{2-\gamma}(\text{Ba},\text{Sr})\text{CuO}_4$, the theoretical values for the electron-phonon coupling are much too small to yield superconducting transition temperatures in the 90-K range.
Pairing and the Pauli principle

I. 1-band systems with inversion and time-reversal symmetry

Single-particle states $|k\uparrow\rangle$ and $|-k\downarrow\rangle = \mathcal{T}|k\uparrow\rangle$ are degenerate if $\mathcal{T}$-symmetry is preserved (Kramers). Superconducting interaction is maximized by pairing degenerate states.

BCS chose “pair wave function”

$\begin{align*}
 b_k &= \langle c_{-k\downarrow} c_{k\uparrow} \rangle
\end{align*}$

Centrosymmetric crystal $\Rightarrow |k\uparrow\rangle$ and $|-k\uparrow\rangle = \mathcal{P}|k\uparrow\rangle$ degenerate also!

Then 4 states are degenerate:

$|k, \uparrow\rangle, |k, \downarrow\rangle, |-k, \uparrow\rangle, |-k, \downarrow\rangle$

General pair wave fctn.

$\begin{align*}
 b_{k\sigma\sigma'} &= \langle c_{-k\sigma} c_{k\sigma'} \rangle
\end{align*}$

must obey Pauli principle:

$\begin{align*}
 b_{-k\sigma'\sigma} &= -b_{k\sigma\sigma'}
\end{align*}$

1) $b_k$ is even under $k \rightarrow -k \Rightarrow b_{k\sigma\sigma'} = b_{-k\sigma\sigma'} = -b_{k\sigma'\sigma}$, i.e. odd under spin exchange (singlet, $S=0$).

2) $b_k$ is odd under $k \rightarrow -k \Rightarrow b_{k\sigma\sigma'} = -b_{-k\sigma\sigma'} = b_{k\sigma'\sigma}$, i.e. even under spin exchange. (triplet, $S=1$).
Pairing and the Pauli principle

II. Generalized BCS theory

Conventional BCS gap eqn

\[ H \simeq H_0 - (\Delta \sum_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger + h.c.) + \Delta \langle c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \rangle^*, \]

\[ \Delta = V \sum_{k'} \langle c_{-k\downarrow} c_{k\uparrow}^\dagger \rangle = V \sum_{k'} \frac{\Delta_{k'}^*}{2E_{k'}} \tanh \frac{E_{k'}}{2T} \]

Generalized BCS gap equation

\[ \Delta_{k\sigma_1 \sigma_2} = \sum_{k\sigma_3 \sigma_4} V_{kk'}^{\sigma_2 \sigma_1 \sigma_3 \sigma_4} b_{k\sigma_4 \sigma_3} \]

“the gap func” or “the order parameter”

“the pair potential” or “the glue”

“the condensate” or “the pair wave function”
Pairing and the Pauli principle

III. Singlet vs. triplet pairing

Gap functions for different spin pairs

\[ \Delta = \begin{bmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{bmatrix} \]

- **Singlet pairing** \((S = 0)\)

  \[ \Delta_k = i \sigma_y \Delta_k; \quad \Delta_{-k} = \Delta_k. \]

  Why is this a singlet state? Because since \(i \sigma_y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\), we have \(\Delta_{k\uparrow\downarrow} = -\Delta_{k\downarrow\uparrow}\). The orbital part of the order parameter, \(\Delta_k\), is even under parity as it must be according to Pauli.

- **Triplet pairing** \((S = 1)\) (Balian & Werthamer Phys. Rev. 131, 1553 (1963))

  \[ \Delta_k = i \sigma_y \mathbf{d}(k) \cdot \vec{\sigma} = \begin{pmatrix} -d_x + id_y \\ d_y \\ d_x + id_y \end{pmatrix}, \]

  e.g. \(d \parallel z \Rightarrow \Delta_{\uparrow\downarrow} = \Delta_{\downarrow\uparrow}\), i.e. the Sz=0 component of the triplet \(|\uparrow\downarrow + \downarrow\uparrow\rangle/\sqrt{2}|\)
Pairing and the Pauli principle

IV. Orbital symmetry of Cooper pairs

BCS: pairing is confined to a thin shell of energies near the Fermi surface:

“weak coupling”: pair wave function “lives on the Fermi surface”, i.e.

\[ b_k \simeq b_k \delta(\epsilon_k - \epsilon_F) \]

So expand:

\[ b_k = b_0 + \sum_{m=-1}^{1} b_{1m} Y_{1m} + \sum_{m=-2}^{2} b_{2m} Y_{2m} + \ldots \]

\[ V_{k,k'} = V_0 + V_1 \sum_{m=-1}^{1} Y_{1m}(\hat{k}) Y_{1m}(\hat{k}')^* + \ldots \]

& insert into BCS gap eqn.:

\[ \Delta_{k\sigma_1\sigma_2} = \sum_{k\sigma_3\sigma_4} V_{kk'}^{\sigma_2\sigma_1\sigma_3\sigma_4} b_{k\sigma_4\sigma_3} \]

Project out each \( \ell \)-channel. Usually only single \( \ell \) channel important since

\[ T_c^\ell \simeq \omega_D e^{-1/N_0 V_\ell} \]
Pairing and the Pauli principle

V. Consequences of Pauli principle for multiple bands

Pauli tells us that \( b_{\mathbf{k}\sigma\nu\sigma'\nu'} = b_{-\mathbf{k}\sigma'\nu'\sigma\nu} \). We can then have, for even and odd parity respectively,

1) \( b_{\mathbf{k}} \) is even under \( \mathbf{k} \rightarrow -\mathbf{k} \) \( \Rightarrow \) \( b_{\mathbf{k}\sigma\nu\sigma'\nu'} = b_{-\mathbf{k}\sigma'\nu'\sigma\nu} = -b_{\mathbf{k}\sigma'\nu'\sigma\nu} \), yielding now two possibilities, either
   i.e. a) odd under spin exchange (singlet, \( S=0 \), even under band exchange or

   b) even under spin exchange (triplet, \( S=1 \), odd under band exchange

Similarly if

2) \( b_{\mathbf{k}} \) is odd under \( \mathbf{k} \rightarrow -\mathbf{k} \) \( \Rightarrow \) \( b_{\mathbf{k}\sigma\nu\sigma'\nu'} = -b_{-\mathbf{k}\sigma'\nu'\sigma\nu} = b_{\mathbf{k}\sigma'\nu'\sigma\nu} \).
   i.e. a) even under spin exchange (triplet, \( S=1 \), even under band exchange.
   or

b) odd under spin exchange (singlet, \( S=0 \), odd under band exchange.

Note “exotic” possibilities a) even parity \( S=1 \) and b) odd parity \( S=0 \) involve intraband pairing of \( \mathbf{k} \) and \(-\mathbf{k} \), hence are energetically disfavored.
Terminology

• Conventional/unconventional:
  “unconventional pairing” occurs when electrons are bound by exchange of electronic excitations rather than phonons.

• Trivial/nontrivial:
  “nontrivial pairing” refers to “non-s-wave” pairing, i.e. the Cooper pair wave function has a symmetry less than that of the lattice.

Warning: “unconventional” is used in many early papers to mean “nontrivial”
Two paradigms for superconductivity

- **Conventional pairing:**
  USUALLY occurs in $\ell=0$ pairing channel to take advantage of the attractive electron-phonon interaction at $r=0$ – avoid Coulomb repulsion in time

- **Unconventional pairing:**
  USUALLY occurs in higher-$\ell$ pairing channel to avoid the Coulomb interaction in space – $\Psi$ has node at $r=0$

Warning: weird counterexamples: theories of d-wave pairing from phonons, extended s-wave pairing from electronic excitations
Consequences of nontrivial pairing

1. Low energy quasiparticle excitations (nodes)

- can be required by symmetry e.g. d-wave $\Delta_k \sim k_x^2 - k_y^2$

- can be “accidental”, due to details of pair potential $V_{kk'}$

N.B. Pt. group G has finite # irreps $\Rightarrow$ sum over many functions with same symmetry e.g. $A_{1g}$: 1, cos 4$\theta$,... or $B_{1g}$: cos 2$\theta$, cos 6$\theta$, ...
Order parameter $\Delta(k)$ shape in $A_{1g}$ representations—1 band

$|\Delta(k)|$

Fermi surface

no nodes

nodes
Nodal excitations dominate low T properties

|Δ(k)|

nodes

kT
Linear DOS from *line* nodes

\[ \Delta(k) \sim \phi - \phi_0 \]

\[ N(\omega) = \int \frac{d\phi}{2\pi} \Re \frac{\omega}{\sqrt{\omega^2 - \Delta_0^2 (\phi - \phi_0)^2}} \approx \frac{\omega}{\Delta_0} \]
Example: $T^2$ specific heat from line nodes

Estimate for energy of free Fermi gas:

$$E = \int d\omega \omega N(\omega) f(\omega) \approx N_0 \int d\omega \omega f(\omega) \sim \frac{T}{E_F}$$

$$C = \frac{dE}{dT} \sim \frac{T}{E_F}$$

Estimate for energy of nodal SC:

$$E = \int d\omega \omega N(\omega) f(\omega) \approx N_0 \int d\omega \left( \frac{\omega}{\Delta_0} \right) \omega f(\omega) \sim \frac{T^2}{\Delta_0 E_F}$$

$$C = \frac{dE}{dT} \sim \frac{T^2}{\Delta_0 E_F}$$
Detecting low-energy quasiparticle states

\[ \Delta \lambda \approx \int d\omega \left( -\frac{\partial f}{\partial \omega} \right) N(\omega) \sim T/\Delta_0 \]
Dimension of nodal surface

\[ C(T) \sim \exp\left(-\frac{\Delta}{T}\right) \]
\[ C(T) \sim T^3 \]
\[ C(T) \sim T^2 \]
Consequences of nontrivial pairing

II. Possible nontrivial phase diagrams

Superfluid $^3$He

$\Delta_k = i\sigma_y d(k) \cdot \vec{\sigma} = \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix}$

9 complex components ($d_{ij} = A_{ij} |k_j$)

$\Delta(k) = \Delta_1 \psi_1(k) + \Delta_2 \psi_2(k)$

2 complex components

UPt$_3$
Consequences of nontrivial pairing

III. Nonmagnetic impurities and surfaces break pairs
(anisotropic and/or sign-changing gap)
Consequences of nontrivial pairing

III. Nonmagnetic impurities and surfaces break pairs (sign-changing gap)

Zn impurity at surface of d-wave SC

Andreev bound state at 110 of d-wave SC

Bi$_2$Sr$_2$Ca(Cu$_{1-x}$Zn$_x$)$_2$O$_{8+y}$ : $x \approx 0.3$

LDOS map at -1.5mV

Differential Conductance (nS)

Sample Bias (mV)

YBCO
Consequences of nontrivial pairing

IV. Order parameter collective modes (multicomponent order param)

Figure 5: Sound attenuation in $^3$He-B (Giannetta et al., Phys. Rev. Lett. 45, 262 (1980)).

Not yet observed convincingly in superconductors!
Consequences of nontrivial pairing

V. Novel types of vortex structures

Figure 6: Vortex in a $d$-wave superconductor with subdominant $s$-wave interaction at low $T$ (Li et al., Phys. Rev. 63, 054504 (2001)). Long arrows: phase of $d$-wave component; short arrows: phase of induced $s$-wave component.
Consequences of nontrivial pairing

VI. Novel Josephson effects

Figure 7: Corner junction geometry to detect $d$-wave symmetry (Wollman et al., Phys. Rev. Lett. 71, 2134 (1993)).

Figure 45: tricrystal sample with YBCO rings fabricated both with single crystal orientations, and across all three tricrystals (center). False color is magnitude of flux detected by scanning SQUID probe (Tsuei/Kirtley RMP 2000)
Unconventional pairing

Prehistory: Kohn-Luttinger 1965

KL (1962): an electron gas with no phonons and only repulsive Coulomb interactions can be a superconductor!

A new paradigm: electrons avoid repulsive part of Coulomb interaction in space rather than time!

Walter Kohn  
Quinn Luttinger  
Also: Landau and Pitaevskii
Prehistory: Kohn-Luttinger 1965

Friedel: screened Coulomb interaction

\[ V(r) = \cos 2k_F r / r^3 \]

At finite distances, screened Coulomb interaction becomes attractive: finite-L pairing.
Prehistory: Kohn-Luttinger 1965

Example: short range $U>0$ for rotationally invariant system ($\approx ^3$He)

$$T_c \approx E_F \exp(-2.5L^4)$$

Best calculation in 1965: Brueckner Soda Anderson Morel PR 1960:
predicted $L=2$ for $^3$He $\Rightarrow T_c \sim 10^{-17}$K

*But* had they taken $L=1$ they would have gotten $T_c \sim 1$ mK!
Spin fluctuations

(ferromagnetic)

1st electron polarizes medium ferromagnetically, 2nd lowers its energy by aligning ⇒ attraction
Stoner theory: enhanced polarization from interactions

\[ \chi(q, \omega) = \frac{\chi_0(q, \omega)}{1 - U\chi_0(q, \omega)} \]

In limit \( \lim_{U\to 1} N_0 \to 1 \), excitations become very sharp (``paramagnons'')

Figure 31: Spin fluctuation spectrum \( \text{Im} \chi(q, \omega) \) for \( q/p_F = 0.7 \) in Stoner model of electron gas.
Spin fluctuation theories of pairing

Effective singlet interaction from spin fluctuations (Berk-Schrieffer 1966)

\[
V_s(q, \omega) \approx \frac{3}{2} \frac{U^2 \chi_0(q, \omega)}{1 - U \chi_0(q, \omega)}
\]

\[
\chi_0(q, \omega) = \int \frac{d^3p}{(2\pi)^3} \frac{f(\varepsilon_{p+q}) - f(\varepsilon_p)}{\omega - (\varepsilon_{p+q} - \varepsilon_p) + i\delta}
\]
Results for pairing interactions

\[ \Gamma_{\uparrow\uparrow} = \frac{-U^2 \chi_0 (k' - k)}{1 - U^2 \chi_0^2 (k' - k)} \]

\[ \Gamma_{\uparrow\downarrow} = \frac{U}{1 - U^2 \chi_0^2 (k' - k)} + \frac{U^2 \chi_0 (k' + k)}{1 - U^2 \chi_0^2 (k' + k)} \]

attractive

repulsive

Total pairing singlet channel:

\[ V_s(k,k') = \frac{1}{2} (2\Gamma_{\uparrow\downarrow} - \Gamma_{\uparrow\uparrow}) = U^2 \left( \frac{3}{2} \chi^s - \frac{1}{2} \chi^c \right) + U \]
Spin fluctuation theories of pairing

Effective interaction from spin fluctuations (Berk-Schrieffer 1961)

*paradigm:* *d*-wave in cuprates from antiferromagnetic spin fluctuations

\[ V_s(q, \omega) \approx \frac{3}{2} \frac{U^2 \chi_0(q, \omega)}{1 - U \chi_0(q, \omega)} \]

\[ \chi_0(q, \omega) = \int \frac{d^3p}{(2\pi)^3} \frac{f(\epsilon_{p+q}) - f(\epsilon_p)}{\omega - (\epsilon_{p+q} - \epsilon_p) + i\delta} \]

\[ \Delta_p = -\sum_{p'} \frac{V(p - p') \Delta_{p'}}{2E_{p'}} \]

\[ \Delta_{p+(\pi,\pi)} = -\Delta_p \]

d-wave takes advantage of peak in spin fluct. interaction at \( \pi,\pi \)!
remember at least some channels must be attractive in order to form Cooper bound state

$k$-space:
\( V_s(k-k') \sim V_0 + V_2 \phi_d(k) \phi_d(k') + \ldots \)

r-space
Unconventional pairing from multiple Fermi pockets around high symmetry points

D. F. Agterberg, V. Barzykin, L.P. Gor’kov PRB 80, 14868 (1999)

\[
\lambda_{\alpha\beta} = \lambda \delta_{\alpha\beta} + \mu (1 - \delta_{\alpha\beta})
\]

possible singlet BCS solutions:

1D: \( A_{1g} \) s-wave
3D: \( E_{1g} \) d-wave

"The nontrivial 3D representation is stable if \( \lambda - \mu < 0 \) and \( \mu > 0 \), i.e., if the interaction is attractive for each pocket alone, while it is repulsive between two different pockets."
Unconventional pairing from multiple Fermi pockets around high symmetry points

D. F. Agterberg, V. Barzykin, L.P. Gor’kov PRB 80, 14868 (1999)

Same idea, only easier, in 2D
Unconventional pairing from multiple Fermi pockets around high symmetry points

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Same idea, only easier, in 2D

(nodeless) d-wave
Materials: phase diagrams

a) heavy fermions

\[ T(\text{K}) \]

\[ x\% \text{Cd} \]

b) cuprates

La\(_{2-x}\)Sr\(_x\)CuO\(_4\)

\[ T \]

\[ R_{2-x}\text{Ce}_x\text{CuO}_4 \]

\[ T^* \]

\[ T_N \]

\[ \sim 300K \]

\[ \sim 30K \]

Hole doping / Sr content (x)

Electron doping / Ce content (x)

Band filling

< \(1/2\)  \quad  \(1/2\)  \quad > \(1/2\)

c) Fe-based

\[ \text{Ba(Fe}_{1-x}\text{Co}_x\text{)}\text{As}_2 \]

\[ T_g \]

\[ T_N \]

\[ T_{\text{el}} \]

d) Organic charge-transfer salts

\[ \kappa-(\text{BEDT-TTF})\text{Cu[N(CN)]}_2\text{X} \]

\[ T_c \]

\[ T_{sc} \]

\[ x = \text{Br, Cl} \]

Similar phase diagrams: “A common thread?” D.J. Scalapino, RMP 2013
Cuprates: status report

$T_c$ is too high for electron-phonon “glue” to work! What holds pairs together?

$T_c = 135 \text{ K}$ under pressure: $153 \text{ K}$

$d$-wave SC:

$$\Delta_k = \frac{\Delta_0}{2} (\cos k_x - \cos k_y)$$
Cuprate crystal structures

(a) LSCO
- $b = 3.78\, \text{Å}$
- $a = 3.78\, \text{Å}$
  
(b) YBCO
- $b = 3.89\, \text{Å}$
- $a = 3.82\, \text{Å}$
  
(c) BSCCO
- $b = 5.4\, \text{Å}$
- $a = 5.4\, \text{Å}$
Action takes place in CuO2 planes doped by charge reservoirs
d-wave pairing in cuprates: 3 crucial experiments

1. London penetration depth. W. Hardy et al. PRL 1993

\[
\frac{\lambda^2(0)}{\lambda^2(T)} = n \left[ 1 - \int d\xi_k \left( \frac{-\partial f}{\partial E_k} \right) \right] = n \left[ 1 - \int dE N(E) \left( \frac{-\partial f}{\partial E} \right) \right],
\]

which, if one substitutes the low-energy d-wave DOS obtained above, \(N(E) \sim N_0 E / \Delta_0\), yields immediately for \(T \ll \Delta_0\),

\[
n_s \simeq n \left[ 1 - \frac{T}{\Delta_0} \right],
\]
ARPES = Angle Resolved Photoemission Spectroscopy

\[ k_\parallel = \left[ 2m(\omega - \varepsilon_{n,k} + \mu - W) \right]^{1/2} \sin \theta \]

\[ I = 2\pi \sum_n \left| \langle n; -k|\hat{\psi}|0 \rangle \right|^2 \delta(\omega + \varepsilon_n - \mu), \]
\[ \approx I_0 |M(k,\omega)|^2 A(k,\omega) f(\omega), \]

\[ A(k,\omega) = -\frac{1}{\pi} \frac{\Sigma''(k,\omega)}{(\omega - \varepsilon_k - \Sigma'(k,\omega))^2 + \Sigma''(k,\omega)^2} \]
d-wave pairing in cuprates: 3 crucial experiments

2. ARPES ZX Shen et al. PRL 1993

Fits $\Delta_k = \Delta_0 (\cos k_x - \cos k_y)$ well!
d-wave pairing in cuprates: 3 crucial experiments

3. Phase sensitive experiments—Josephson tunneling

\[ J_s = A_s (n_x^2 - n_y^2)_L (n_x^2 - n_y^2)_R \sin \gamma \]

\[ J_s \propto n_x^2 n_x^2 \approx 1 \quad (\text{like } s\text{-wave}) \]

\[ J_s \propto n_x^2 \cdot 0 \approx 0 \]

\[ J_s = -|J_s| \quad (\pi\text{-junction}) \]
Bicrystal ring

\[ \delta f \sim c_A (\psi_{1A} \psi_{2A}^* + c.c.) + c_B (\psi_{1B} \psi_{2B}^* + c.c.) \]

s-wave \quad c_A = c_B

d-wave \quad c_A = -c_B

\[ J_{sA} = \frac{\delta f}{\delta \phi_A} \text{ is minus the current} \quad J_{sB} = \frac{\delta f}{\delta \phi_B} \]

“pi-junction” \Rightarrow \text{flux quantized in } 1/2\Phi_0
Tsuei/Kirtley tricrystal expt.: YBCO on STO, etc.
Iron-based superconductors


LaFeAsO


$T_c = 28K$
(55K for Sm)

BaFe$_2$As$_2$


$T_c = 38K$

LiFeAs

- Wang et al

$T_c = 18K$

FeSe

- Hsu et al
PNAS 2008

No arsenic 😊!
Heavy fermion materials

CeCu$_2$Si$_2$

CeCoIn$_5$
d-wave pairing in CeCoIn$_5$: specific heat anisotropy

- Shaded area: $C/T$ minimum for $H||$node
- Unshaded: $C/T$ maximum for $H||$node
- suggestive of $d_{x^2-y^2}$ pairing in CeCoIn$_5$
- prediction: anisotropy inversion at lower $T, H$

$H/H_{c2}$

$T/T_c$

$H_{c2}^a / B_0 = 2.85$
$H_{c2}^c / H_{c2}^a = 0.2$

Shaded area: $C/T$ minimum for $H||$node
Unshaded: $C/T$ maximum for $H||$node
suggestive of $d_{x^2-y^2}$ pairing in CeCoIn$_5$
prediction: anisotropy inversion at lower $T, H$

CeCoIn$_5$

$d_{x^2-y^2}$

K. An et al. '10
f-wave pairing in UPt$_3$: Josephson-Frauenhofer spectroscopy

Strand et al PRL 2009

$\bar{d}(k) = \Delta(T)(k_x^2 - k_y^2)k_z \hat{z}$

$\bar{d}(k) = \Delta(T)(k_x + ik_y)^2k_z \hat{z}$

Data

Simulation

$I_C (\mu A)$

$I/I_0$

Field (mG)

$\Phi/\Phi_0$
Conclusions

• Conventional pairing:
  USUALLY occurs in ℓ=0 pairing channel to take advantage of the attractive electron-phonon interaction at r=0 – avoid Coulomb repulsion in time

• Unconventional pairing:
  USUALLY occurs in higher-ℓ pairing channel to avoid the Coulomb interaction in space – Ψ has node at r=0

• Exotic effects in SC state due to non ℓ=0 symmetry

Reading:

“Phenomenological theory of unconventional superconductivity”, M. Sigrist and K. Ueda, Rev. Mod. Phys. 63, 239 (1991);
“Introduction to Unconventional Superconductivity”, by V. P. Mineev and K.V. Samokhin (Gordon and Breach, Amsterdam), 1999;
“Pairing symmetry in cuprate superconductors”, C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. 72, 974 (2000);