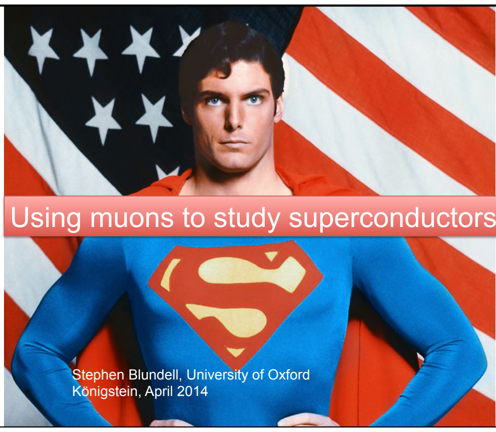


Using muons to study superconductors

Stephen Blundell, University of Oxford
Königstein, April 2014

Superconductivity: A Very Short Introduction
S. J. Blundell, OUP (2008)

The forgotten brothers
S. J. Blundell Physics World **24(April)**, 26 (2011)

Outline

1. Introduction to muons
2. Low energy muons and the penetration depth
3. Vortex lattices
4. Magnetism and superconductivity
5. Superfluid stiffness and T_c

What the technique is not!
(mistakes to avoid)

- “~~Mu ng~~ – no, the muons stop in the sample and they do not scatter!
- “~~Mu ce~~” - no, unlike NMR and ESR you do not need to use resonant techniques (though you can).

Electron

mass m_e, charge –e, spin 1/2

moment –1.001μ_B, γ_e=28024.2 MHz/T

lifetime > 4x10^{23} years

Muon

mass 206.8m_e, charge +e, spin 1/2

moment 0.00484μ_B, γ_μ=135.53 MHz/T

lifetime = 2.19903x10^{-6} s

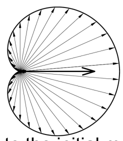
Muon spin rotation (μSR)

- Muons produced via pion decay:

$$\underset{\text{PION}}{\pi^+} \to \underset{\text{MUON}}{\mu^+} + \underset{\text{NEUTRINO}}{\nu_\mu}$$

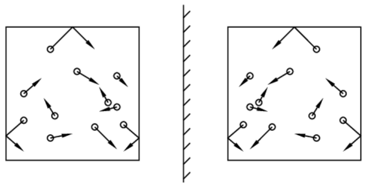
- Muons 100% spin polarized, speed~c/4, K.E.~4 MeV.
- Muon decays into a positron:

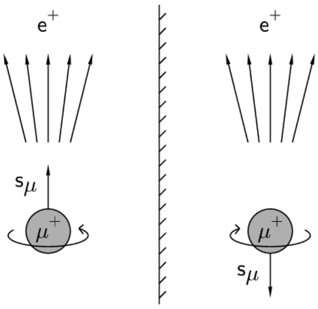
$$\underset{\text{MUON}}{\mu^+} \to \underset{\text{POSITRON}}{e^+} + \underset{\text{NEUTRINOS}}{\nu_e + \bar{\nu}_\mu}$$



- Positron decay is asymmetric with respect to the initial muon-spin polarization because of parity violation (weak interaction)

(see S.J. Blundell, Contemp. Phys. **40**, 175 (1999))

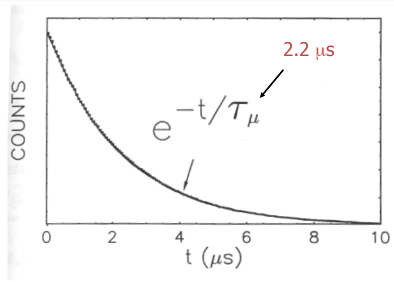




"I cannot believe God is a weak left-hander"

Wolfgang Pauli
(1900-1958)

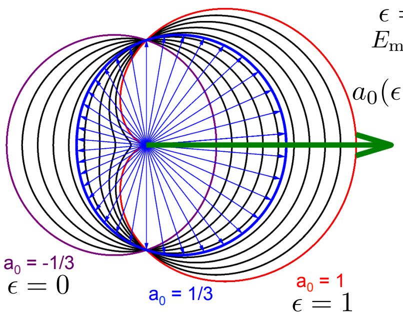
Every single μSR experiment….
…looks like this



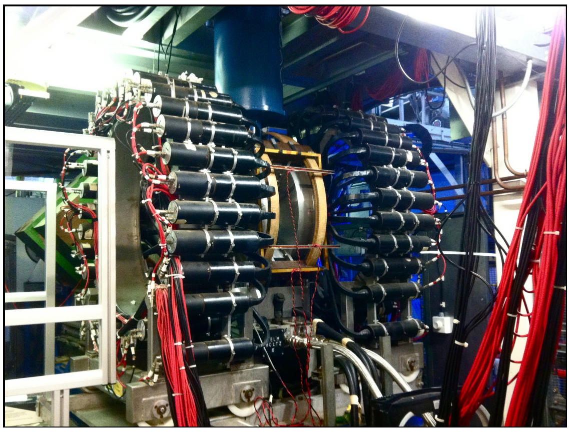
$\epsilon = E/E_{\rm max}$

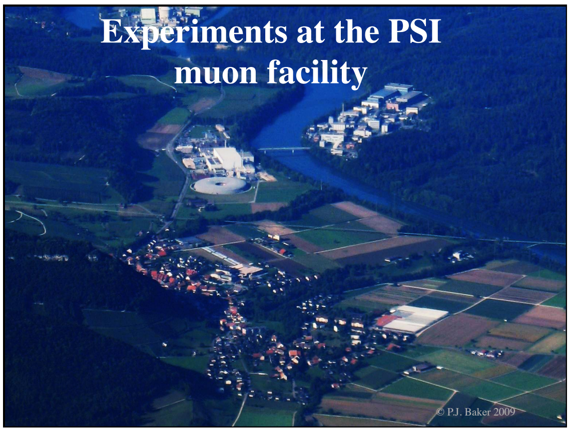
$E_{\rm max} = 52\,{\rm MeV}$

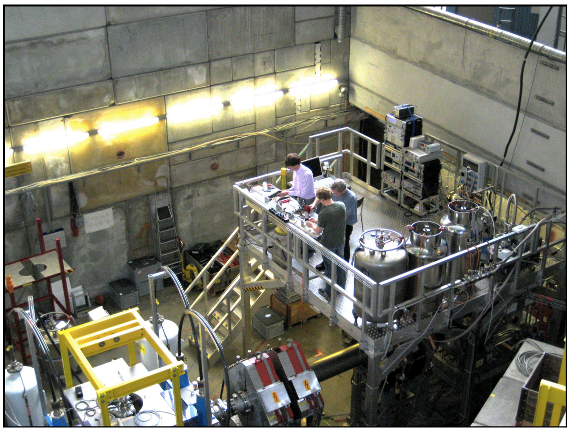
$$a_0(\epsilon) = \frac{2\epsilon - 1}{3 - 2\epsilon}$$



$$p(\theta) \propto 1 + a_0(\epsilon)\cos\theta$$







μSR worldwide

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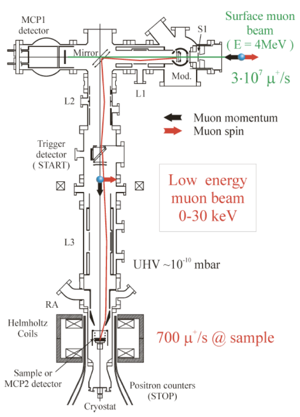
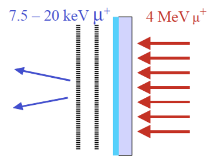
LE-μSR Apparatus

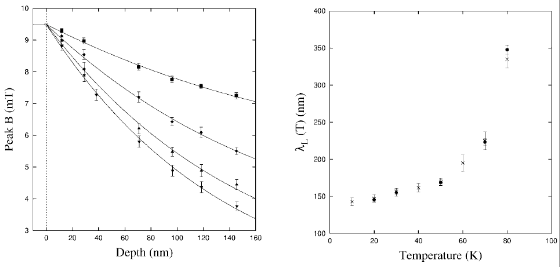


- Moderator



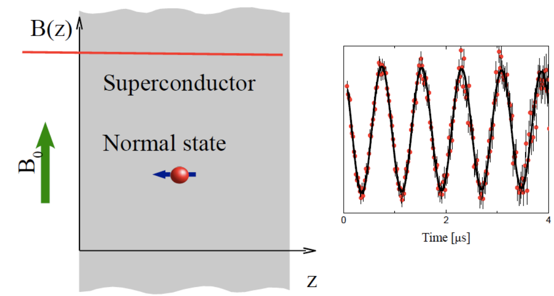
Depth-Resolved Profile of the Magnetic Field beneath the Surface of a Superconductor with a Few nm Resolution

Experiment: YBCO

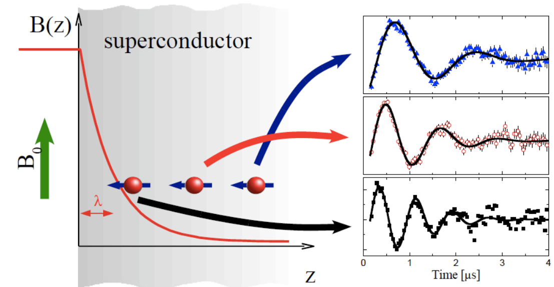


T. J. Jackson et al. Phys. Rev. Lett. **84**, 4958 (2000)

Meissner state in a Superconductor

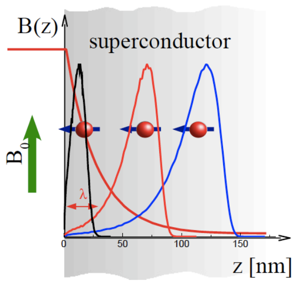


Meissner state in a Superconductor



R. F. Kiefl et al. Phys. Rev. B **81**, 180502 (2010)

What about muon stopping profile?

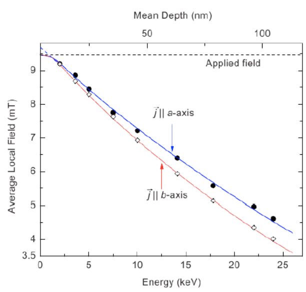


$$B(z)=B_0 e^{-\frac{z}{\lambda_{ab}(T)}}$$

$$A(t)=\int n(E,z)\cos(\gamma B(z)t+\phi)\,dz$$

R. F. Kiefl et al. Phys. Rev. B **81**, 180502 (2010)

Anisotropy of London penetration depth in the ab plane



λ_a = 128.9(1.2) nm

λ_b = 108.4(1.0) nm

A "dead-layer" near the surface

d = 10.3(5) nm

R. F. Kiefl et al. Phys. Rev. B **81**, 180502 (2010)

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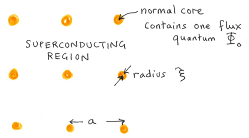
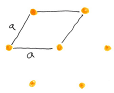
Type II superconductor ⇒ "vortex lattice"

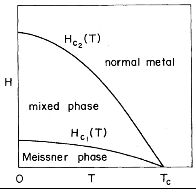


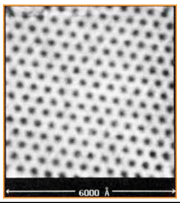
Usually have a TRIANGULAR lattice

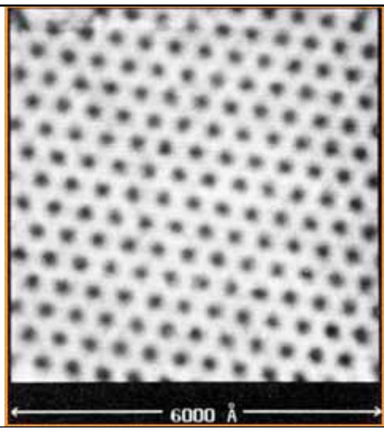


$$a = \left(\frac{2\,\Phi_0}{\sqrt{3}\,B}\right)^{1/2}$$

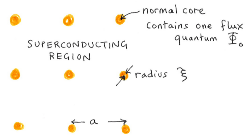
$$\Phi_0 = B a^2 \Rightarrow a = \left(\frac{\Phi_0}{B}\right)^{1/2}$$



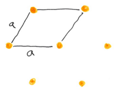




Type II superconductor ⇒ "vortex lattice"



Usually have a TRIANGULAR lattice



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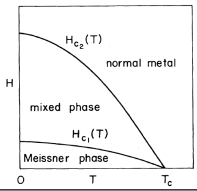
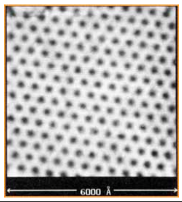


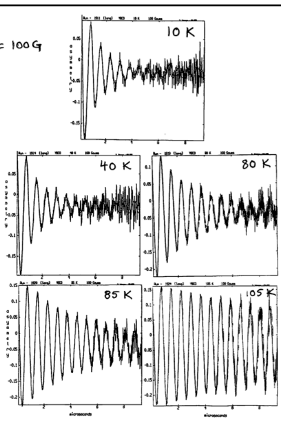


Experiment: YBCO

B = 100 G

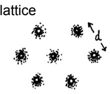
relaxation function

$$G_x(t) = e^{-\frac{\sigma^2 t^2}{2}}$$



Muons and the flux lattice

Flux lattice



Transverse field measurement.

relaxation function

$$G_x(t) = e^{-\frac{\sigma^2 t^2}{2}}$$

number of muons decaying (forwards) at time t

2.2 μs

initial decay asymmetry

$$N(t) = N(0)\, e^{-t/\tau_\mu}\left[1 + A\,G_x(t)\cos(\omega_\mu t + \phi)\right]$$

$$\Phi_0 = \frac{h}{2e}$$

$$\sigma^2 = \frac{\gamma_\mu^2}{2}\langle(\Delta B)^2\rangle = \frac{\gamma_\mu^2}{2}\left(0.00371\,\frac{\Phi_0^2}{\lambda^4}\right)$$

field variation

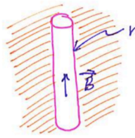
penetration depth

decay of oscillations

$$\sigma \propto \frac{1}{\lambda^2} \propto \frac{n_s}{m^*}$$

superconducting carrier density

Single superconducting vortex



Solve $\nabla^2 \vec{B} = \frac{1}{\lambda^2}\vec{B}$ in cylindrical polars in region outside

Result: $B_z(r) = \frac{\Phi_0}{2\pi\lambda^2} K_0\left(\frac{r}{\lambda}\right)$ Modified Bessel function

$K_0\left(\frac{r}{\lambda}\right) \sim -\ln\frac{r}{\lambda}$ at small r

∴ blows up. (In practice cut off by ξ).

$K_0\left(\frac{r}{\lambda}\right) \sim \sqrt{\frac{\pi\lambda}{2r}}\, e^{-r/\lambda}$ at large r

$\int B_z(r)\, d^2r = \Phi_0$ $\qquad \vec{J} \sim \frac{1}{r}\vec{e}_\phi$

vortex ⇒ 1 flux quantum $\qquad$ circulating current irrotational

Vortex lattice

$$\vec{B}(\vec{r}) + \lambda_{ab}^2\left[\nabla\times\left(\nabla\times\vec{B}(\vec{r})\right)\right] = \Phi_0\sum_n \delta(\vec{r}-\vec{r}_n)\,\hat{\vec{z}}$$

in plane penetration depth

$$\vec{B}_{\vec{k}} = n_f\int_{\text{cell}} \vec{B}(\vec{r})\, e^{-i\vec{k}\cdot\vec{r}}\, d^2\vec{r}$$

number of vortices/area

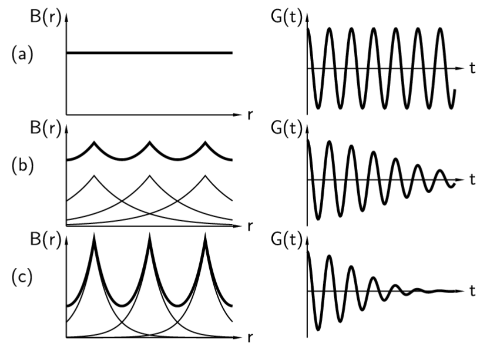
$$\vec{B}(\vec{r}) = \sum_{\vec{k}}{}' B_{\vec{k}}\, e^{-i\vec{k}\cdot\vec{r}} = B_0\sum_{\vec{k}}\frac{e^{-i\vec{k}\cdot\vec{r}}}{1+k^2\lambda_{ab}^2}\,\hat{\vec{z}}$$

$n_f\Phi_0$

Use a cut-off $e^{-k^2\xi^2/2}$ to suppress higher Fourier components [Brandt]

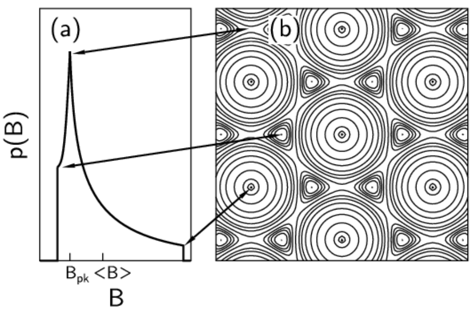
$$\vec{B}(\vec{r}) = B_0\sum_{\vec{k}}\frac{e^{-i\vec{k}\cdot\vec{r}}\, e^{-k^2\xi_{ab}^2/2(1-b)}}{1+k^2\lambda_{ab}^2/(1-b)}\,\hat{\vec{z}}$$

Muons and penetration depth



S.J. Blundell, Contemp. Phys. **40**, 175 (1999))

Muons and the flux lattice



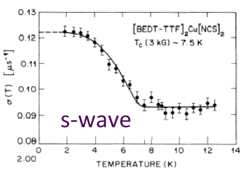
- Scale of *p(B)* gives information about penetration depth
- Shape of *p(B)* allows the study of vortex lattice melting and decoupling.

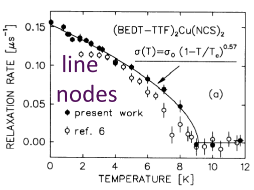
Volume 64, Number 11 PHYSICAL REVIEW LETTERS 12 March 1990

Magnetic Penetration Depth in the Organic Superconductor κ-[BEDT-TTF]$_2$Cu[NCS]$_2$

Volume 68, Number 12 PHYSICAL REVIEW LETTERS 23 March 1992

Muon-Spin-Relaxation Measurements of Magnetic Penetration Depth in Organic Superconductors (BEDT-TTF)$_2$-X: X = Cu(NCS)$_2$ and Cu[N(CN)$_2$]Br



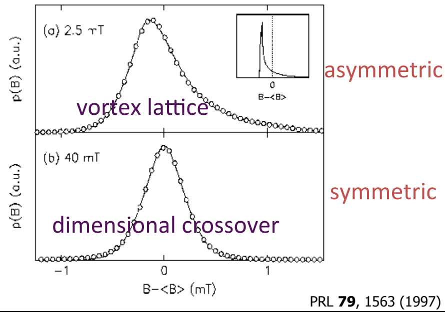


D.R. Harshman et al. 1990 $\qquad$ L.P. Le et al. 1992

Volume 79, Number 8 PHYSICAL REVIEW LETTERS 25 August 1997

Investigation of Vortex Behavior in the Organic Superconductor κ-(BEDT-TTF)$_2$Cu(SCN)$_2$ Using Muon Spin Rotation

S. L. Lee,[1] F. L. Pratt,[2,3] S. J. Blundell,[2] C. M. Aegerter,[4] P. A. Pattenden,[2] K. H. Chow,[2] E. M. Forgan,[5] T. Sasaki,[6] W. Hayes,[2] and H. Keller[4]



PRL **79**, 1563 (1997)

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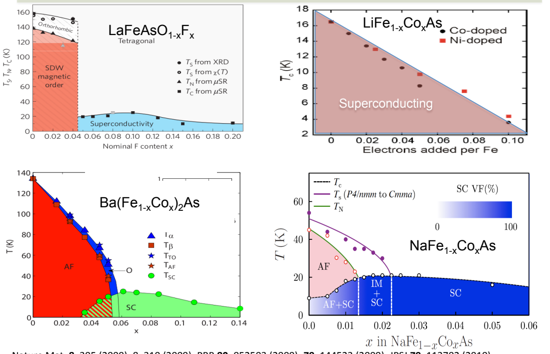
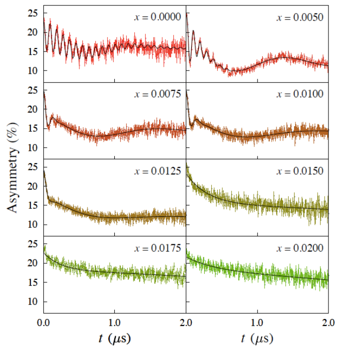
New iron-based superconductors



Nature Mat. **8**, 305 (2009); 8, 310 (2009); PRB **80**, 052503 (2009); **79**, 144523 (2009); JPSJ **79**, 113702 (2010)
J. Am. Chem. Soc. **132**, 10467 (2010); PRL **104**, 057007 (2010); PRB **85** 054503 (2012),

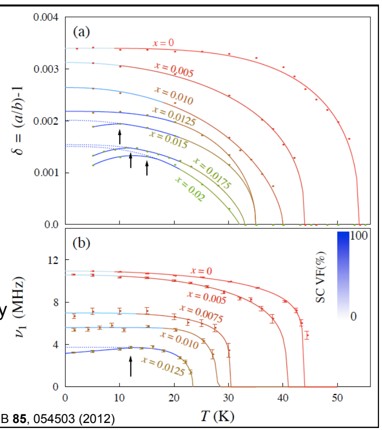
NaFe$_{1-x}$Co$_x$As



D. R. Parker et al. Phys. Rev. Lett. **104**, 057007 (2010)
J. D. Wright et al. Phys. Rev. B **85**, 054503 (2012)

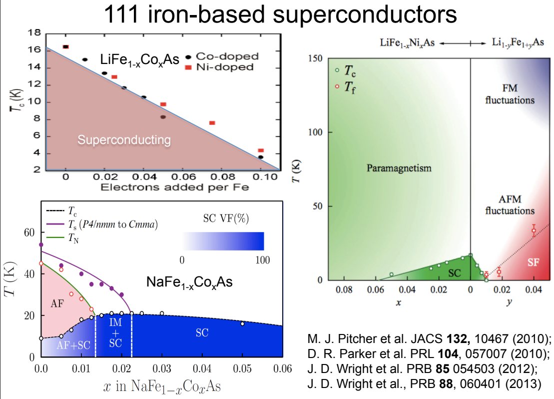
structural distortion

precession frequency



J. D. Wright et al. Phys. Rev. B **85**, 054503 (2012)

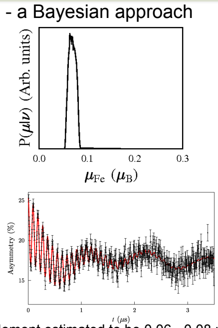
111 iron-based superconductors

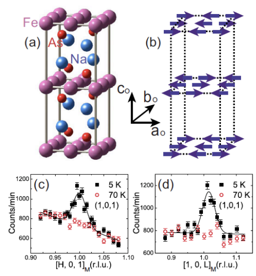


M. J. Pitcher et al. JACS **132**, 10467 (2010);
D. R. Parker et al. PRL **104**, 057007 (2010);
J. D. Wright et al. PRB **85** 054503 (2012);
J. D. Wright et al., PRB **88**, 060401 (2013)

Magnetic moment determination

- a Bayesian approach





Moment estimated to be 0.06 - 0.08 μ_B from our muon results

Moment estimated to be 0.09(4) μ_B from neutron diffraction

J. D. Wright et al. Phys. Rev. B **85**, 054503 (2012)

S. Li et al. Phys. Rev. B **80**, 020504 (2009)

Quantum states of muons in fluorides

J. S. Möller,[1,*] D. Ceresoli,[2] T. Lancaster,[3] N. Marzari,[4] and S. J. Blundell

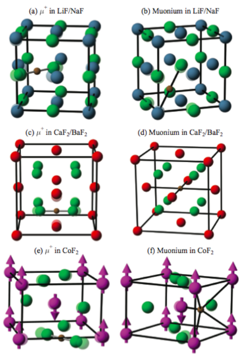


TABLE I. Calculated (DFT) and experimental (exp) properties of the diamagnetic F-μ-F states in solid and vacuum, and of the $(FHF)^-$ molecular ion in vacuum. r (Å) is the muon-fluoride bond length, ν is the frequency (cm^{-1}) of the symmetric stretch (SS), asymmetric stretch (AS), and bending (B) mode, and ZPE is the zero-point energy (eV). [a]Our calculation. [b]Experimental data (Ref. 19). [c]Ref. 20 reports 1377 cm^{-1}.

	$2r_{DFT}$	$2r_{exp}$	ν_{SS}	ν_B	ν_B	ν_{AS}	ZPE
$(FHF)^{-a}$	2.36	2.28	581	1289	1289	1611	0.30
$(FHF)^{-b}$		2.28	583	1286	1286	1331[c]	0.28
$(F\text{-}\mu\text{-}F)^-$	2.36		581	3797	3797	4748	0.80
LiF	2.34[18]	2.36(2)[12]		2825	4603	4881	0.76
NaF	2.35	2.38(1)[12]		3071	4363	4813	0.76
CaF_2	2.31	2.34(2)[12]	649	2737	4481	5446	0.83
BaF_2	2.33	2.37(2)[12]	613	3033	4130	4974	0.79
CoF_2	2.36	2.43(2)	585	3076	3473	4570	0.73

Phys. Rev. B **87**, 121108(R) (2013)

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'Uemura Plot' for cuprates and other superconductors

VOLUME 62, NUMBER 19 — PHYSICAL REVIEW LETTERS — 8 MAY 1989

Universal Correlations between T_c and n_s/m^* (Carrier Density over Effective Mass) in High-T_c Cuprate Superconductors

Y. J. Uemura,[1] G. M. Luke,[1] B. J. Sternlieb,[1] J. H. Brewer,[2] J. F. Carolan,[2] W. N. Hardy,[2] R. Kadono,[2] J. R. Kempton,[2] R. F. Kiefl,[2] S. R. Kreitzman,[2] P. Mulhern,[2] T. M. Riseman,[2] D. Ll. Williams,[2] B. X. Yang,[2] S. Uchida,[3] H. Takagi,[3] J. Gopalakrishnan,[4] A. W. Sleight,[4] M. A. Subramanian,[4] C. L. Chien,[5] M. Z. Cieplak,[5] Gang Xiao,[5] V. Y. Lee,[6] B. W. Statt,[7] C. E. Stronach,[8] W. J. Kossler,[9] and X. H. Yu[9]

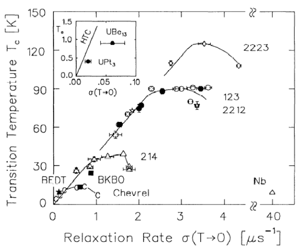
$T_c \propto \sigma$
(μSR relaxation rate)

Equivalently:

$T_c \propto n_s/m^*$

$T_c \propto \rho_s$
(superfluid strength)

$T_c \propto 1/\lambda^2$
(λ is penetration depth)



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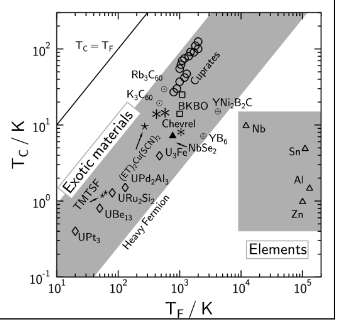
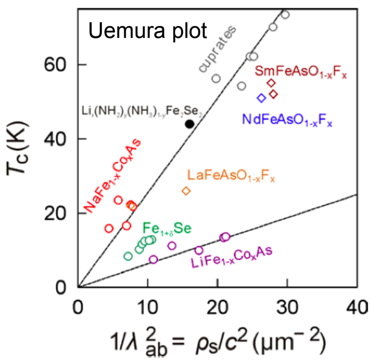
Equivalently:

$T_c \propto n_s/m^*$

$T_c \propto \rho_s$
(superfluid strength)

$T_c \propto 1/\lambda^2$
(λ is penetration depth)





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