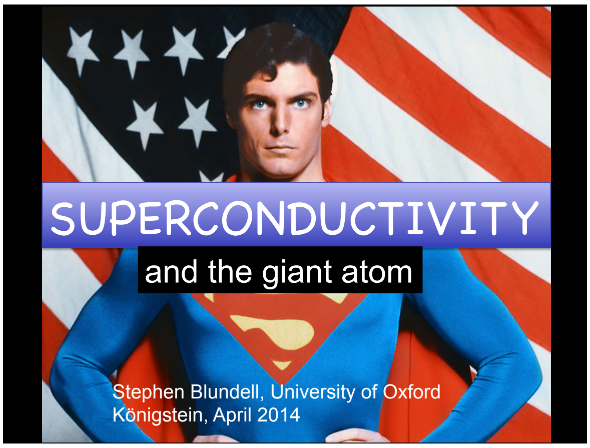

Outline

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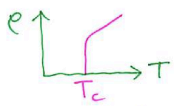
Superconductivity: basic phenomenonlogy

- $\boxed{\vec{E}=0}$ $\because$ $\rho = 0$ for $T<T_c$



- persistent currents

$\Phi = \int \vec{B}\cdot d\vec{S}$

$\oint \vec{E}\cdot d\vec{l} = 0$

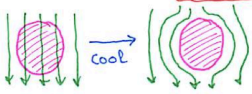
$\Rightarrow \int \nabla \times \vec{E}\cdot d\vec{S} = 0$

$\Rightarrow \dot{\Phi} = 0$

- no thermopower or Peltier effect
  poor conductors of heat

The Meissner effect and perfect diamagnetism

- Perfect diamagnetism (Meissner effect)
  $\vec{M} = -\vec{H} \Rightarrow \boxed{\vec{B}=0}$

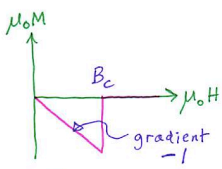


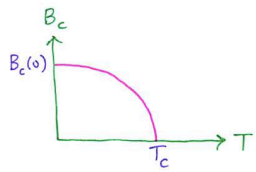
FLUX EXCLUSION & EXPULSION

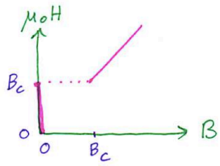
(A perfect conductor only has $\dot{\vec{B}} = 0$
$\therefore$ can trap flux $\therefore$ no obvious ground state)

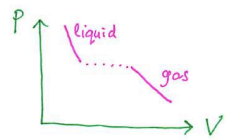
$\vec{E}=0$ & $\vec{B}=0$ $\Rightarrow$ can do reversible thermodynamics

M, B and H

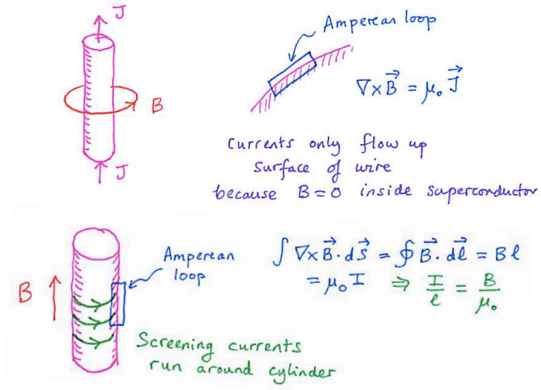








Where are the currents?



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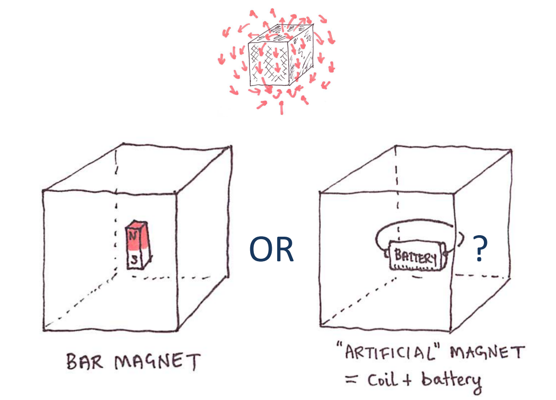
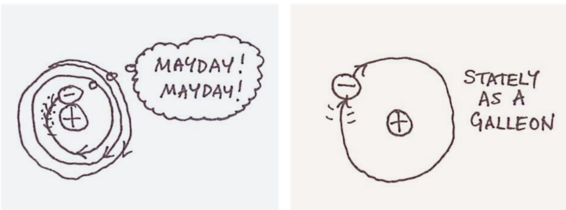
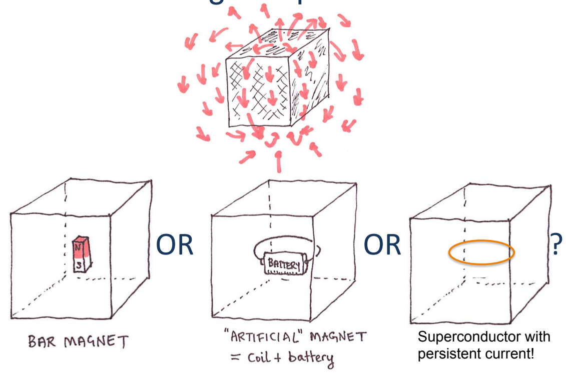


The atom



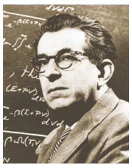
Thought experiment



Our house was a two storey house. I was in the kitchen cooking and suddenly the upstairs door was opened by Fritz. `Edith, Edith come, we have it. Come up, we have it.' And maybe the wind closed the door. I do not know what had happened upstairs. I left everything, ran up and, then, the door was opened in my face. On my forehead I had a bruise for a week. Fritz said `The equations are established. We have the solution. We can explain it.'

Edith London

London equations



Fritz London: superconductivity due to macroscopic quantum phenomenon

LRO of momentum vector

$\Rightarrow$ condensation in p-space

Rigidity of superconducting wave function ψ responsible for diamagnetism.

London equation: Condense into $\vec{p}=0$ state

$$\vec{p} = -i\hbar\vec{\nabla} = m\vec{v} + q\vec{A} = 0$$

$$\Rightarrow \vec{v} = -\frac{q\vec{A}}{m} \qquad (\vec{J} = nq\vec{v})$$

$$\Rightarrow \boxed{\vec{J} = -\frac{nq^2}{m}\vec{A}} \quad \text{London equation}$$

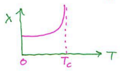
$$\nabla\times\vec{B} = \mu_0\vec{J}$$

$$\underbrace{\nabla\times(\nabla\times\vec{B})}_{\nabla(\underbrace{\nabla\cdot\vec{B}}_{=0})-\nabla^2\vec{B}} = \mu_0\nabla\times\vec{J} = -\underbrace{\frac{\mu_0 nq^2}{m}}_{\equiv 1/\lambda^2}\underbrace{\nabla\times\vec{A}}_{\vec{B}}$$

$$\Rightarrow \boxed{\nabla^2\vec{B} = \frac{1}{\lambda^2}\vec{B}}$$

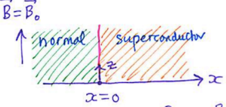
"diffusion equation"

λ is called the penetration depth



$$\lambda = \sqrt{\frac{m}{\mu_0 nq^2}}$$

$\vec{B}=\vec{B}_0$



$$\frac{d^2B_z}{dx} = \frac{B_z}{\lambda^2} \qquad x>0$$

$$B_z = \underbrace{B_1 e^{x/\lambda}}_{\text{blows up at } x\to\infty} + B_2 e^{-x/\lambda}$$

$$B_z(x) = B_0 e^{-x/\lambda} \qquad x>0$$

Gauge

$$\hat{\vec{p}} = -i\hbar\nabla$$

MOMENTUM OPERATOR

MAGNETIC VECTOR POTENTIAL

$$\hat{\vec{p}} = m\vec{v} + q\vec{A}$$

NB $\vec{A}\to\vec{A}+\nabla\chi$ has no effect on PHYSICS!

$\Rightarrow$ no unique description of $\hat{\vec{p}}$.

Connection with phase of wave function

$$\hat{\vec{p}}\, e^{i\theta(\vec{r})} = (\hbar\nabla\theta)\, e^{i\theta(\vec{r})}$$

If $\psi = |\psi|e^{i\theta(\vec{r})}$ then $\vec{p} = m\vec{v} + q\vec{A} = \hbar\nabla\theta$

Fritz London (1936)

$$\Psi(\underline{r}) = |\Psi(\underline{r})|\, e^{i\theta(\underline{r})}$$

ORDER PARAMETER

AMPLITUDE

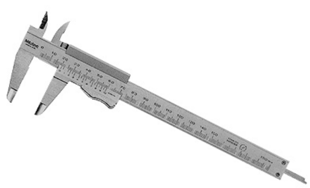
PHASE

RIGID IN THE SUPERCONDUCTING STATE

$$m\underline{v} = \hbar\nabla\theta - q\underline{A}$$

GAUGE INVARIANT

REGAUGING



$\underline{A}\to\underline{A}+\nabla\chi$

$\phi\to\phi-\dot{\chi}$

$\underline{p}\to\underline{p}+q\nabla\chi$

$\theta\to\theta+\frac{q\chi}{\hbar}$

$$m\underline{v} = \hbar\nabla\theta - q\underline{A}$$

GAUGE INVARIANT

Flux quantization

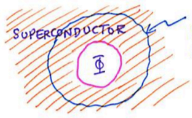
Twist a rigid wavefunction

$\Psi = |\Psi| e^{i\theta(\vec r)}$ ← phase of wave function can twist

$\hat{\vec p}\,\Psi = \hbar\vec\nabla\theta\,\Psi$

$\vec p = m\vec v + q\vec A = \hbar\nabla\theta$

$\vec v = \frac{\hbar}{m}\nabla\theta - \frac{q}{m}\vec A$ $\qquad \vec J = \frac{nq}{m}(\hbar\nabla\theta - q\vec A)$



LOOP WHICH LIES IN SUPERCONDUCTOR $\vec J = 0$

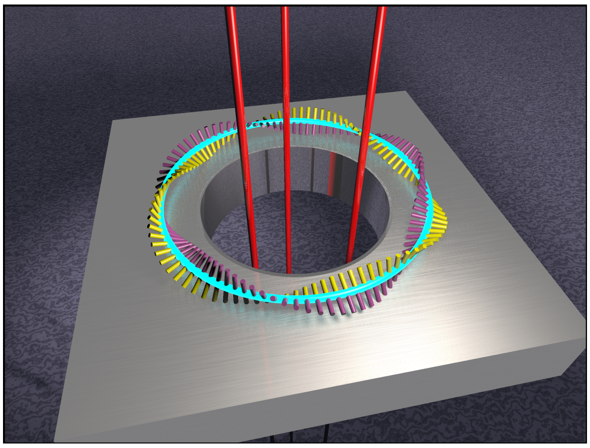
$\oint \vec J\cdot d\vec\ell = 0$

$\Rightarrow \hbar\underbrace{\oint\nabla\theta\cdot d\vec\ell}_{\Delta\theta = 2\pi N} = q\underbrace{\oint\vec A\cdot d\vec\ell}_{\iint\vec B\cdot d\vec S}$

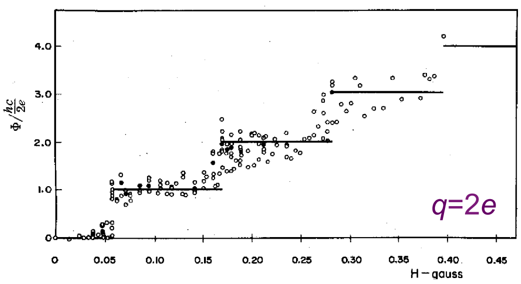
integer

$\Rightarrow \Phi = N\frac{h}{q}$ ← experiment ⇒ 2e

FLUX QUANTIZATION

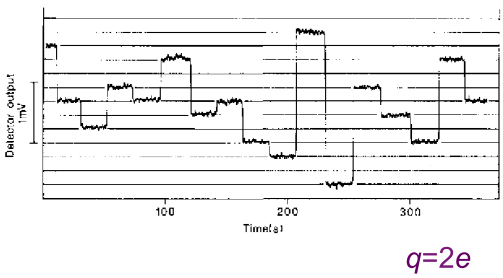


Measurements on a Sn cylinder: flux quantization!



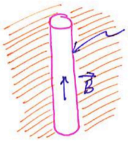
1961: B. S. Deaver and W. M. Fairbank, Phys. Rev. Lett. 7, 43 (1961)

Measurements on a YBCO ring: flux quantization!



1987: C. E. Gough et al., Nature 326, 855 (1987). Flux jumps (0.97±0.04) h/(2e)

Single superconducting vortex



normal core with field in it

Solve $\nabla^2\vec B = \frac{1}{\lambda^2}\vec B$ in cylindrical polars in region outside

Result: $B_z(r) = \frac{\Phi_0}{2\pi\lambda^2}K_0\left(\frac{r}{\lambda}\right)$ modified Bessel function

$K_0\left(\frac{r}{\lambda}\right) \sim -\ln\frac{r}{\lambda}$ at small r

∴ blows up. (In practice cut off by ξ).

$K_0\left(\frac{r}{\lambda}\right) \sim \sqrt{\frac{\pi\lambda}{2r}}\, e^{-r/\lambda}$ at large r

$\int B_z(r)\, d^2r = \Phi_0$ $\qquad \vec J \sim \frac{1}{r}\vec e_\phi$

vortex ⇒ 1 flux quantum $\qquad$ circulating current irrotational

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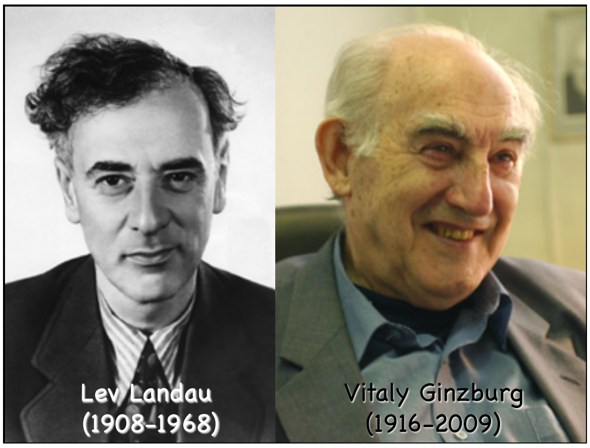
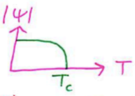


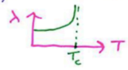
Ginzburg-Landau theory

$$f_s(T) = f_n(T) + \underbrace{a(T)}_{a_0(T-T_c)}|\psi|^2 + \frac{1}{2}\underbrace{b(T)}_{b}|\psi|^4 + \ldots$$

$$\frac{\partial f_s}{\partial \psi} = 0 \quad \Rightarrow \quad |\psi| = \begin{cases} \left(\frac{a_0}{b}\right)^{1/2}(T_c-T)^{1/2} & T<T_c \\ 0 & T>T_c \end{cases}$$



$$n = |\psi|^2 = -\frac{a(T)}{b(T)} \quad \Rightarrow \quad \lambda = \left(\frac{m}{\mu_0 n q^2}\right)^{1/2} \propto (T_c-T)^{-1/2}$$



$$f_s(T) - f_n(T) = -\frac{a_0^2(T_c-T)^2}{2b} = -\frac{B_c^2}{2\mu_0}$$

$$\Rightarrow B_c = a_0\sqrt{\frac{\mu_0}{b}}(T_c-T)$$

$$S_s(T) - S_n(T) = -\frac{\partial}{\partial T}(f_s - f_n) \quad \therefore \quad C_s - C_n = \begin{cases} T\frac{a_0^2}{b} & T<T_c \\ 0 & T>T_c \end{cases}$$

Ginzburg-Landau theory

ψ varies in space, add in gauge-invariant K.E. term

$$\frac{1}{2}mv^2 = \frac{1}{2m}|-i\hbar\nabla\psi + 2e\vec{A}\psi|^2 \qquad q = -2e$$

- $\vec{A} = 0$

$$f_s - f_n = \int d\vec{r}\ \underbrace{\left(a|\psi|^2 + \frac{1}{2}b|\psi|^4 + \frac{\hbar^2}{2m}|\nabla\psi|^2\right)}_{f}$$

vary ψ $\Rightarrow$ $(a + b\psi^2)\psi - \frac{\hbar^2}{2m}\nabla^2\psi = 0$

nonlinear Schrödinger equation

near T_c, neglect $b\psi^2 \ll a$

$$\nabla^2\psi = \frac{\psi}{\xi^2} \qquad \xi = \sqrt{\frac{\hbar^2}{2m|a|}}$$

$\psi \propto e^{-x/\xi}$ solutions

COHERENCE LENGTH

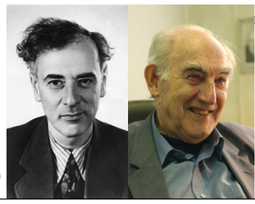
$$\xi \propto (T_c-T)^{-1/2}$$

GINZBURG–LANDAU THEORY

$$F_s = F_n + \int d^3r\left[\underbrace{a|\psi|^2 + \frac{b}{2}|\psi|^4}_{\text{uniform terms}} + \underbrace{\frac{1}{2m}|-i\hbar\nabla\psi + 2e\underline{A}\psi|^2}_{\text{gradient term}} + \underbrace{\frac{(B-B_0)^2}{2\mu_0}}_{\text{field energy}}\right]$$

$$\underbrace{\psi(\underline{r})}_{\text{ORDER PARAMETER}} = \underbrace{|\psi(\underline{r})|}_{\text{AMPLITUDE}}\, e^{i\overbrace{\theta(\underline{r})}^{\text{PHASE}}}$$

Lev Landau
(1908-1968)



Vitaly Ginzburg
(1916-2009)

Ginzburg-Landau theory

- $\vec{A} \neq 0$ vary ψ $\therefore$ $-\frac{\hbar^2}{2m}\left(\nabla + \frac{2e}{\hbar}i\vec{A}\right)^2\psi + (a + b\psi^2)\psi = 0$

vary $\vec{A}$ $\therefore$ $\vec{J} = \underbrace{-\frac{i\hbar}{2m^*}(2e)[\psi^*\nabla\psi - \psi\nabla\psi^*]}_{\text{usual probability current}} - \frac{(2e)^2}{m^*}|\psi|^2\vec{A}$

Writing $\psi = |\psi|e^{i\theta(\vec{r})}$ $\Rightarrow$ $\vec{J} = \frac{nq}{m}(\hbar\nabla\theta - q\vec{A})$

(with $q = -2e$) i.e. recover the London equation.

Type I and Type II

For $B < B_c$, condensation energy $\left|\frac{a^2}{2b}\right|$ saved beats the $\frac{B^2}{2\mu_0}$ cost.

- Type I

$\xi > \sqrt{2}\lambda$ surface energy > 0

lose condensation energy saving over $\sim\xi$
gain field energy cost only over $\sim\lambda$

Surface stable (like surface tension)

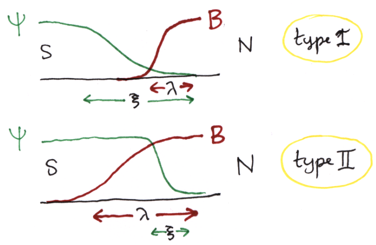
- Type II

$\xi < \sqrt{2}\lambda$ surface energy < 0

reverse argument!

Surface unstable $\therefore$ make lots of surfaces

Type I and Type II



$\kappa < \frac{1}{\sqrt{2}}$ type I

$\kappa > \frac{1}{\sqrt{2}}$ type II

$\kappa = \frac{\lambda}{\xi} = \left(\frac{m^2 b}{2\mu_0 e^2 \hbar^2}\right)^{1/2}$

independent of T

GINZBURG-LANDAU PARAMETER

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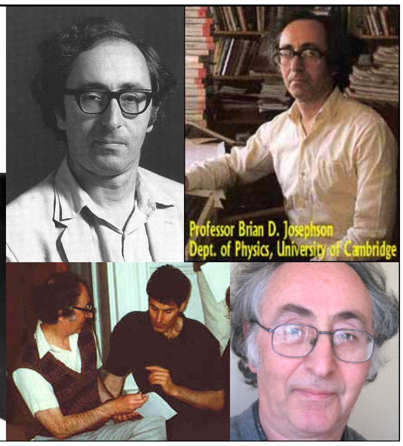
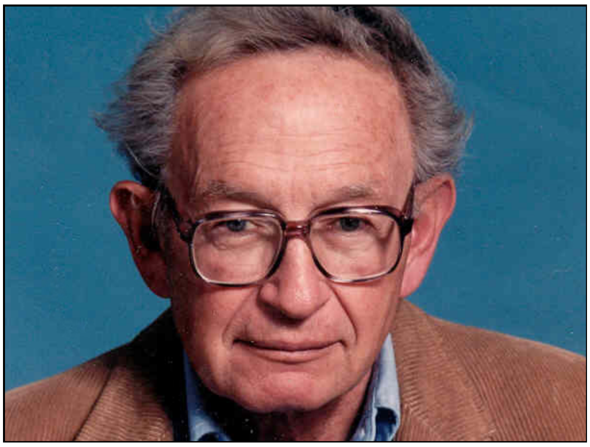
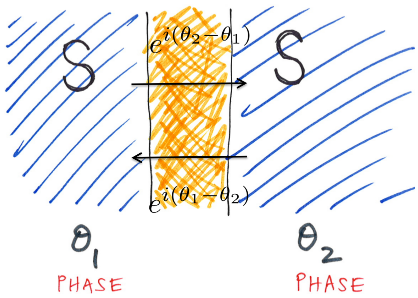
Brian Pippard
(1920 – 2008)

Philip Anderson
(1923 –)

Brian
Josephson
(1940–)







$$I = I_J \sin\phi \qquad \phi = \theta_1 - \theta_2$$

$$\hbar\dot{\phi} = 2eV \qquad I = I_J \sin\left[2eVt/\hbar + \phi(0)\right]$$

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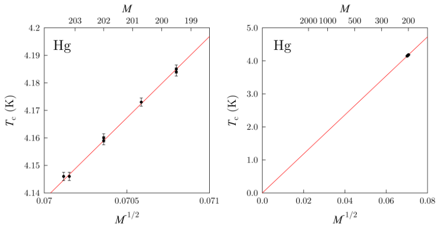
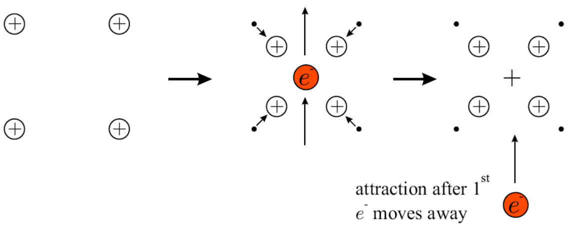
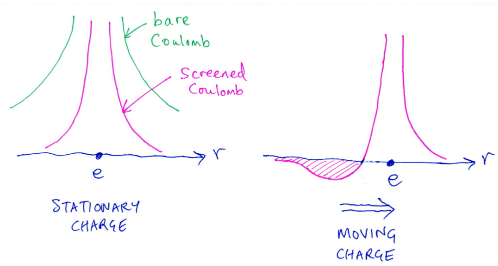
The isotope effect

The transition temperature $T_c \propto M^{-1/2}$ where M is the mass of the isotope.

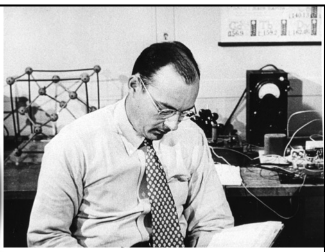


This is very good evidence for the role of *phonons* in superconductivity.





John Bardeen
(1908-1991)

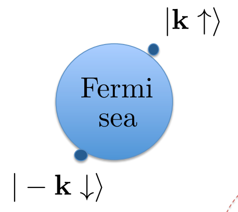


Nobel prizes in physics

1956 The transistor (with Brattain and Shockley)

1972 BCS theory (with Cooper and Schrieffer)

Formation of Cooper pairs due to attractive interaction



Cooper pair:

$|\mathbf{k}\uparrow\rangle$ $|-\mathbf{k}\downarrow\rangle$

$P_{\text{tot}} = 0$

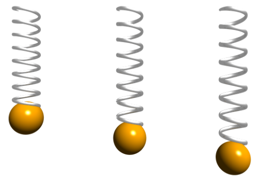
$L_{\text{tot}} = 0$

$S_{\text{tot}} = 0$

Many body state: *coherent state* of Cooper pairs

Mass on a spring

$$\mathcal{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{q}^2$$



Write in reduced units

$$\mathcal{H} = \hbar\omega(\hat{P}^2 + \hat{Q}^2)$$

$$(\hat{Q} - i\hat{P})(\hat{Q} + i\hat{P}) = \hat{Q}^2 + \hat{P}^2 + i(\hat{Q}\hat{P} - \hat{P}\hat{Q})$$

$$\mathcal{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{q}^2$$

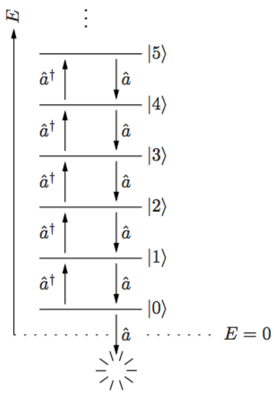
$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$$

$$\hat{a} = \frac{\hat{Q} + i\hat{P}}{\sqrt{2}}$$

Annihilation operator

$$\hat{a}^\dagger = \frac{\hat{Q} - i\hat{P}}{\sqrt{2}}$$

Creation operator



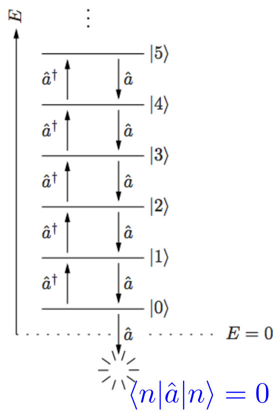
$$\mathcal{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{q}^2$$

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$$

$$\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$$
$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$\hat{a}^\dagger\hat{a}|n\rangle = n\,|n\rangle$$

Number operator



$$\langle n|\hat{a}|n\rangle = 0$$

Coherent states

Annihilation operator

$$\hat{a} = (\hat{Q} + i\hat{P})/\sqrt{2}$$

A **coherent state** |α> is an eigenstate of the annihilation operator

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

Explicit form:

$$|\alpha\rangle = \textstyle\sum_{n=0}^{\infty} c_n|n\rangle$$

$$|\alpha\rangle = c_0\left(|0\rangle + \frac{\alpha}{\sqrt{1!}}|1\rangle + \frac{\alpha^2}{\sqrt{2!}}|2\rangle + \frac{\alpha^3}{\sqrt{3!}}|3\rangle + ...\right)$$
$$= c_0\left(1 + \frac{\alpha}{\sqrt{1!}}\hat{a}^\dagger + \frac{\alpha^2}{\sqrt{2!}}(\hat{a}^\dagger)^2 + \frac{\alpha^3}{\sqrt{3!}}(\hat{a}^\dagger)^3 + ...\right)|0\rangle$$

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}}e^{\alpha\hat{a}^\dagger}|0\rangle$$

Properties of coherent states

Mean number of particles in a coherent state |α>

$$\langle\hat{n}\rangle = \langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle = |\alpha|^2$$

Uncertainty in number of particles in a coherent state |α>

$$\Delta n = \sqrt{\langle\hat{n}^2\rangle - \langle n\rangle^2}$$

$$\Delta n = |\alpha|$$

Number-phase uncertainty

$$\Delta n \Delta\phi \geq \frac{1}{2}$$

$$\langle n|\hat{a}|n\rangle = 0 \qquad \langle\alpha|\hat{a}|\alpha\rangle = \alpha \neq 0$$

Properties of coherent states

Coherent state in a superfluid:

$$|\psi\rangle = |\alpha_{\boldsymbol{p}_0}\alpha_{\boldsymbol{p}_1}\ldots\rangle$$

This is an eigenstate of the field annihilation operator:

$$\hat{\Psi}(\boldsymbol{x}) = \frac{1}{\sqrt{\mathcal{V}}}\sum_{\boldsymbol{p}} \hat{a}_{\boldsymbol{p}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \qquad \hat{\Psi}(\boldsymbol{x})|\psi\rangle = \psi(\boldsymbol{x})|\psi\rangle$$

Eigenvalue is a **macroscopic wave function**

$$\langle\psi|\hat{\psi}(\boldsymbol{x})|\psi\rangle = \psi(\boldsymbol{x}) = \frac{1}{\sqrt{\mathcal{V}}}\sum_{\boldsymbol{p}} \alpha_{\boldsymbol{p}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}}$$

$\hat{c}^\dagger_{\mathbf{k}\sigma}$ = creation operator, electron with momentum ***k***, spin σ

$\hat{c}_{\mathbf{k}\sigma}$ = annihilation operator, electron with momentum ***k***, spin σ

$\hat{P}^\dagger_{\mathbf{k}} = \hat{c}^\dagger_{\mathbf{k}\uparrow}\hat{c}^\dagger_{-\mathbf{k}\downarrow}$ = pair creation operator

$$|\text{Fermi sea}\rangle = \prod_{k<k_F} \hat{P}^\dagger_{\mathbf{k}}|0\rangle$$

$$|\Psi_{\text{BCS}}\rangle = \text{constant} \times \prod_k \exp(\alpha_k \hat{P}^\dagger_{\mathbf{k}})|0\rangle$$

$\hat{P}^\dagger_{\mathbf{k}}\hat{P}^\dagger_{\mathbf{k}} = 0$ because electrons identical fermions

Therefore

$$|\Psi_{\text{BCS}}\rangle = \text{constant} \times \prod_k \exp(\alpha_k \hat{P}^\dagger_{\mathbf{k}})|0\rangle$$

can be simplified

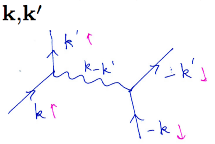
$$\exp(\alpha_{\mathbf{k}} P^\dagger_{\mathbf{k}}) = 1 + \alpha_{\mathbf{k}}\hat{P}^\dagger_{\mathbf{k}}$$

$$\Rightarrow \quad |\Psi_{\text{BCS}}\rangle = \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} P^\dagger_{\mathbf{k}})|0\rangle$$

We don't know what u_k and v_k are: treat as variational parameters

BCS Hamiltonian

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma} - |g_{\text{eff}}|^2 \sum_{\mathbf{k},\mathbf{k}'} c^\dagger_{\mathbf{k}\uparrow}c^\dagger_{-\mathbf{k}\downarrow}c_{-\mathbf{k}'\downarrow}c_{\mathbf{k}'\uparrow}$$

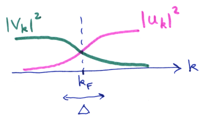


$$\boxed{|\Psi_{BCS}\rangle = \prod_k (u_k + v_k P_k^+)|0\rangle}$$ BCS wave function

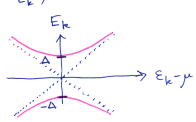
Treat u_k and v_k as variational parameters, and put in BCS Hamiltonian: $\hat{H} = \sum_{k,\sigma} \varepsilon_k c^+_{k\sigma}c_{k\sigma} - |g_{eff}|^2 \sum_{k,k'} c^+_{k\uparrow}c^+_{-k\downarrow}c_{-k'\downarrow}c_{k'\uparrow}$

The results of this are

$$|u_k|^2 = \frac{1}{2}\left(1 + \frac{\varepsilon_k - \mu}{E_k}\right); \quad |v_k|^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_k - \mu}{E_k}\right); \quad E_k = \sqrt{(\varepsilon_k - \mu)^2 + |\Delta|^2}$$



and



here, $\Delta = |g_{eff}|^2 \sum_k u_k v_k^*$ is the gap parameter, and $u_k v_k^* = \frac{\Delta}{2E_k}$

$$\Rightarrow \quad \Delta = |g_{eff}|^2 \sum_k \frac{\Delta}{2E_k} \quad \text{(BCS gap equation)}$$

BCS gap equation:

$$\Delta = |g_{eff}^2| \sum_k \frac{\Delta}{2E_k} \qquad (T=0)$$

$$= \underbrace{|g_{eff}^2|\, g(E_F)}_{\lambda} \int_0^{\hbar\omega_D} \frac{\Delta\, d\varepsilon}{\sqrt{\Delta^2+\varepsilon^2}}$$

$$\therefore \quad \frac{1}{\lambda} = \int_0^{\hbar\omega_D} \frac{d\varepsilon}{\sqrt{\Delta^2+\varepsilon^2}} = \sinh^{-1}\left(\frac{\hbar\omega_D}{\Delta}\right)$$

$$\Delta \ll \hbar\omega_D \Rightarrow \frac{e^{1/\lambda}}{2} \approx \frac{\hbar\omega_D}{\Delta} \Rightarrow \boxed{\Delta \approx 2\hbar\omega_D e^{-1/\lambda}} \quad ①$$

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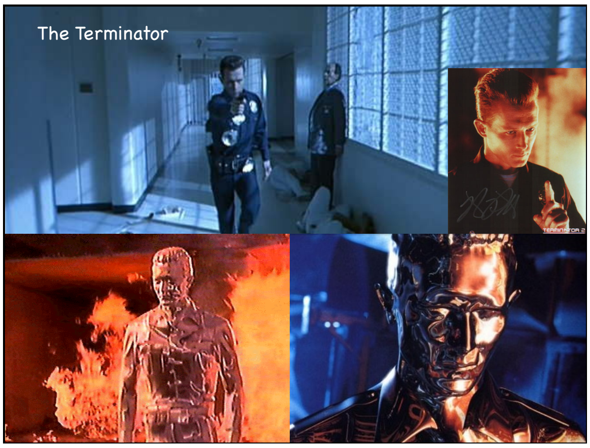
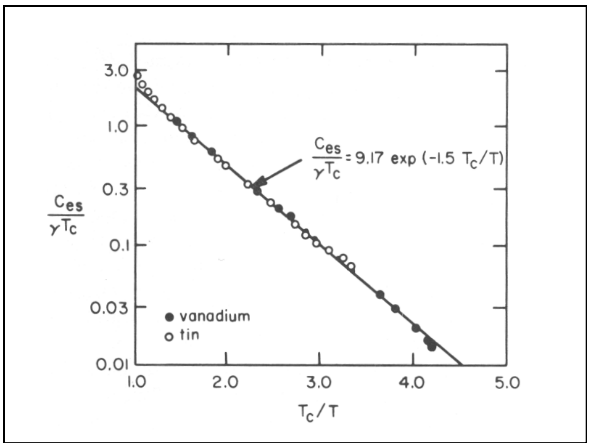
$$T \neq 0 \qquad \Delta = \lambda \int_0^{\hbar\omega_D} \frac{\Delta\, d\varepsilon}{\sqrt{\Delta^2+\varepsilon^2}}(1 - 2f(E))$$

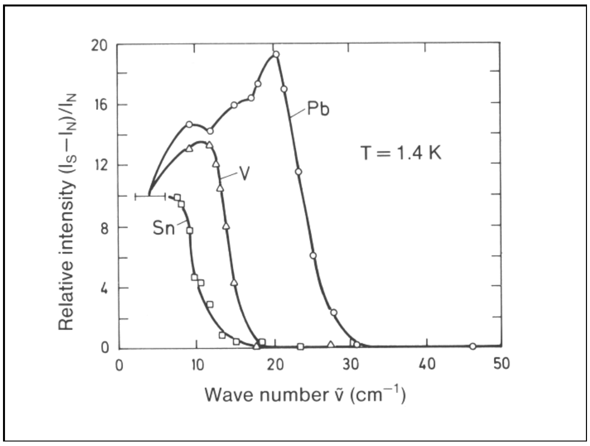
so for $T = T_c$, $\Delta = 0$ and $\frac{1}{\lambda} = \int_0^{x_D} \frac{\tanh x}{x}\, dx \quad x_D = \frac{\hbar\omega}{2k_BT_c}$

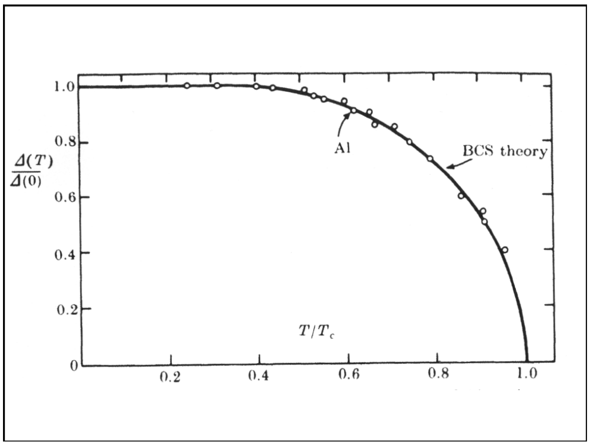
$$\Rightarrow \boxed{k_BT_c = 1.13\,\hbar\omega_D e^{-1/\lambda}} \quad ②$$

$$① \,\&\, ② \Rightarrow \boxed{2\Delta(0) = 3.52\, k_BT_c}$$









More complex pairing?

Pair wave function $\Psi_{\mathbf{k}ss'} = \langle \Psi | \hat{c}_{-\mathbf{k}s'} \hat{c}_{\mathbf{k}s} | \Psi \rangle$

Symmetry of pairs of identical electrons

$$\Psi_{\mathbf{k}ss'} = g(\mathbf{k}) \chi_{ss'}$$

Wave function totally asymmetric under particle exchange

$$\mathbf{k} \rightarrow -\mathbf{k} \qquad s \rightarrow s'$$

Even parity	$L = 0, 2, 4, \cdots$ even	$S = 0$ odd	singlet
Odd parity	$L = 1, 3, 5 \cdots$ odd	$S = 1$ even	triplet