616th WE-Heraeus Seminar Bad Honnef, May 9-13 2016

Ultracold Quantum Gases - Current Trends & Future Perspectives

UNIVERSITY OF INNSBRUCK

‘Dark State’ Optical Lattices for Cold Atoms

Peter Zoller

ÖAW

UQUAM
ERC Synergy Grant

SFB
FoQuS

siqs

RYQS

T. Esslinger: "optical lattices as the Swiss army knife for AMO"

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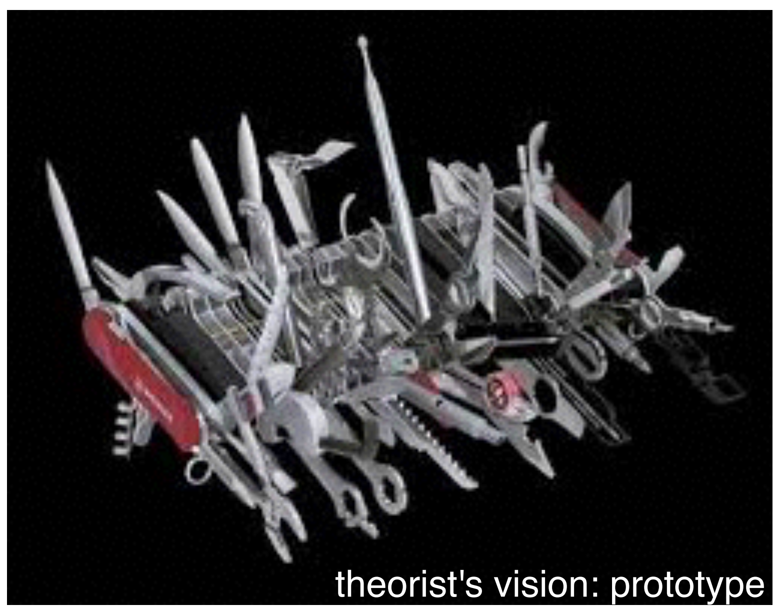

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‘Dark State’ Optical Lattices for Cold Atoms

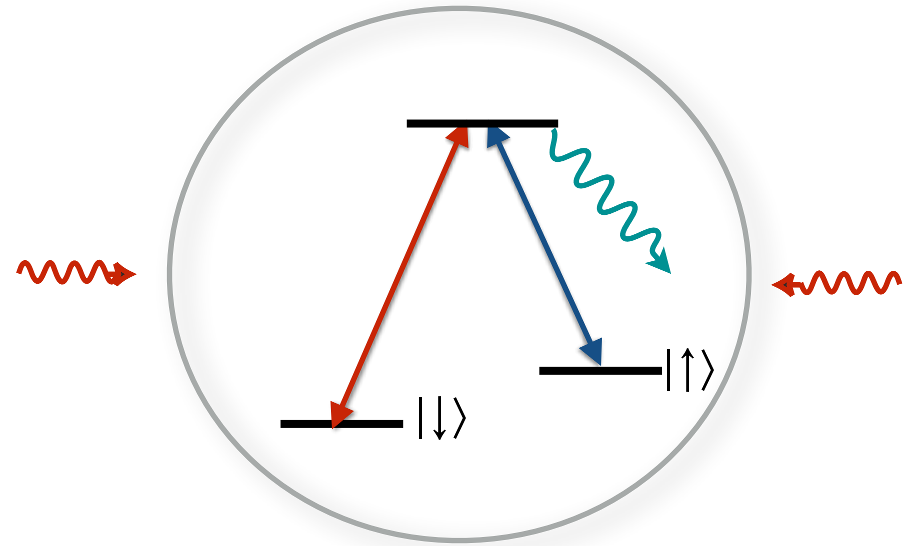
Peter Zoller

In collaboration with :


Mateusz Łącki


Misha Baranov


Hannes Pichler
→ ITAMP



dark states in Λ-systems

‘Dark State’ Optical Lattice

single particle physics

- near-resonant / dissipation-less optical lattice
- sub-wavelength structures
  - ‘Kronig-Penney’ box-like lattices
  - sub-wavelength spin structures

- AMO
  - ✓ Alkali / Alkaline Earth (magnetic)
  - ✓ polar molecules (electric dipoles)

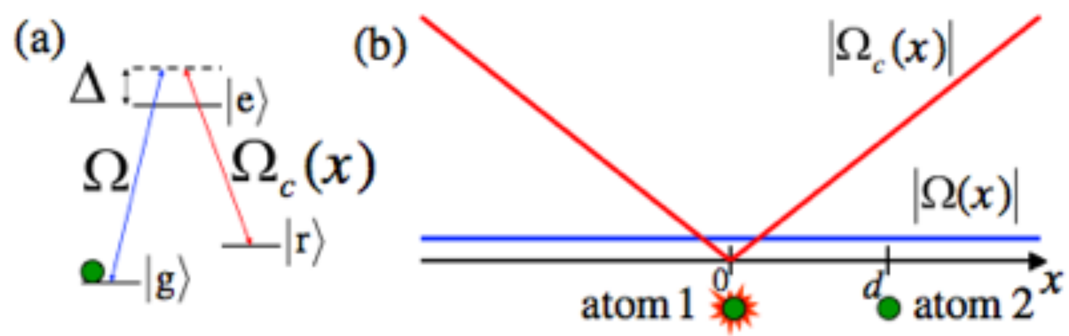
quantum many-body physics

work in progress

PRL **100,** 093005 (2008) PHYSICAL REVIEW LETTERS week ending 7 MARCH 2008

Coherent Quantum Optical Control with Subwavelength Resolution

Alexey V. Gorshkov,[1] Liang Jiang,[1] Markus Greiner,[1] Peter Zoller,[2] and Mikhail D. Lukin[1]
[1]*Physics Department, Harvard University, Cambridge, Massachusetts 02138, USA*
[2]*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria*
(Received 11 December 2007; published 7 March 2008)



vs. incoherent sub-wavelength microscopy: S. Hell

PRL **95,** 010404 (2005) PHYSICAL REVIEW LETTERS week ending 1 JULY 2005

Non-Abelian Gauge Potentials for Ultracold Atoms with Degenerate Dark States

J. Ruseckas,[1,2] G. Juzeliūnas,[1] P. Öhberg,[3] and M.
[1]*Institute of Theoretical Physics and Astronomy of Vilnius University, A. G*
[2]*Fachbereich Physik, Technische Universität Kaiserslautern, D-676*
[3]*Department of Physics, University of Strathclyde, Glasgow G*
(Received 8 March 2005; published 28 June

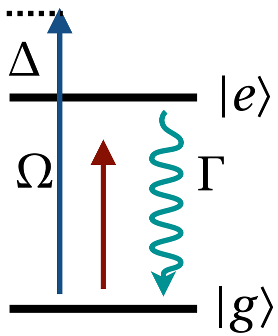
We show that the adiabatic motion of ultracold, multilevel atoms in
give rise to effective non-Abelian gauge fields if degenerate adiabat

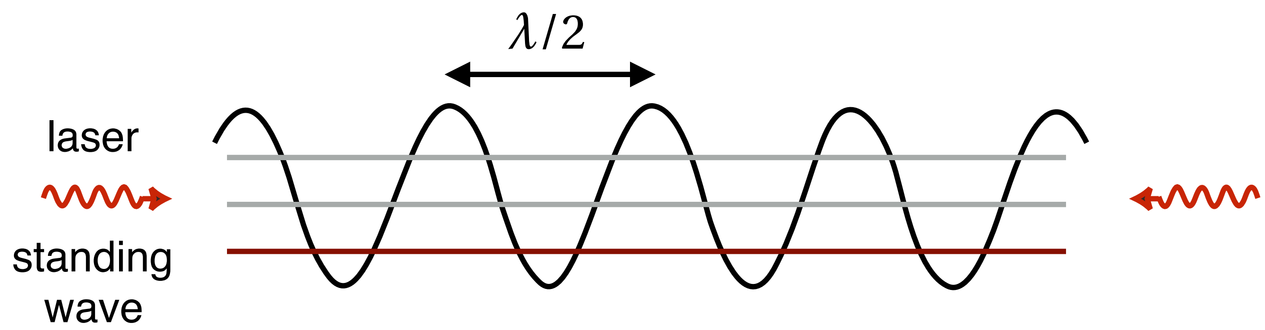
Geometric Manipulation of Trapped Ions for Quantum Computation

www.sciencemag.org SCIENCE VOL 292 1 JUNE 2001

'Off-Resonant' Optical Lattices [vs. 'Dark State']

- **far off-resonant optical lattice**

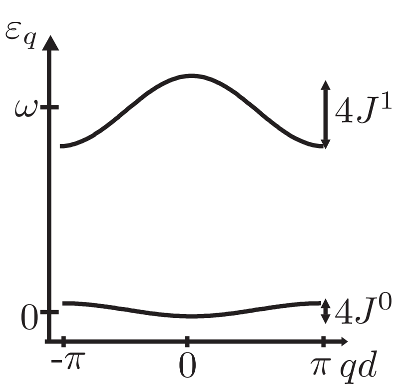




$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_0 \sin^2(kx)$$

optical potential (1D)

- **Bloch bands**



band structure (1D)

✓ AC-Stark shift as optical potential

$$\Delta E_g \sim I(x) \sim V_0 \sin^2(kx)$$

$$\frac{\Omega^2}{4}\frac{1}{\Delta - i\frac{1}{2}\Gamma}$$

lattice spacing / energy scale

$\lambda/2$

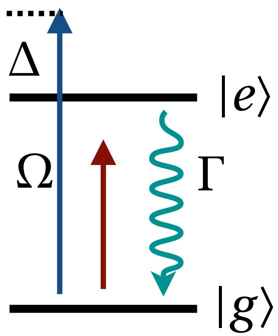
✓ off-resonant laser

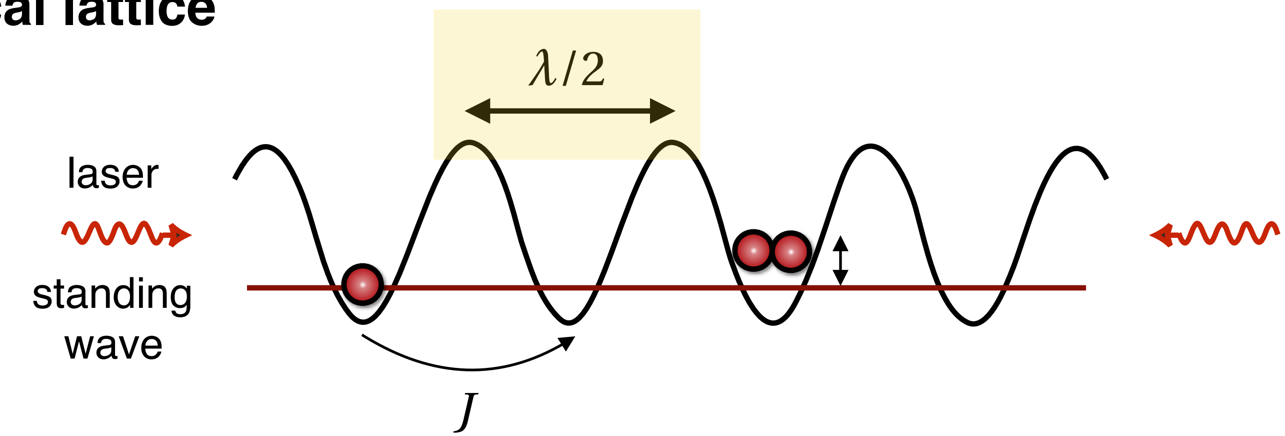
$$|\Delta| \gg \Gamma$$

small dissipation

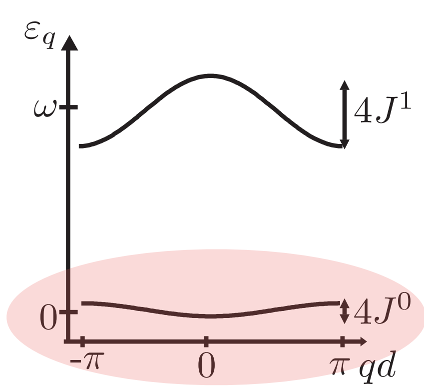
'Off-Resonant' Optical Lattices: Hubbard Models

- **far off-resonant optical lattice**





- **Bloch bands**



band structure (1D)

- **many particle physics: Bose / Fermi Hubbard**

$$H = -J \sum_{<i,j>} b_i^\dagger b_j + U \sum_i b_i^{\dagger 2} b_i^2$$

✓ Hubbard toolbox, …

✓ energy scales $\quad J \ll E_R = \frac{\hbar^2 k^2}{2m} \leftrightarrow U \leftrightarrow T \quad \sim \lambda$

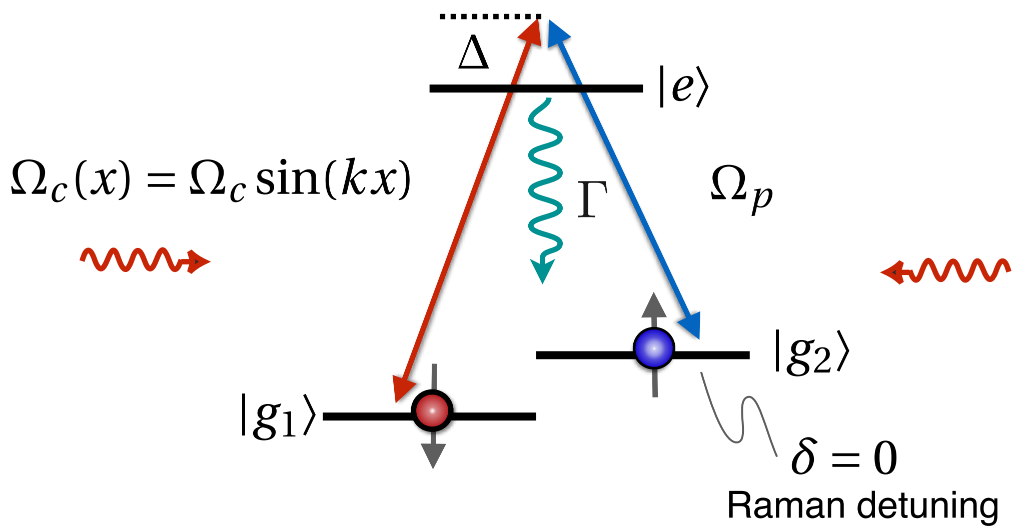
sub-wavelength lattices:

W Yi, AJ Daley, G Pupillo, P Zoller - NJP 2008

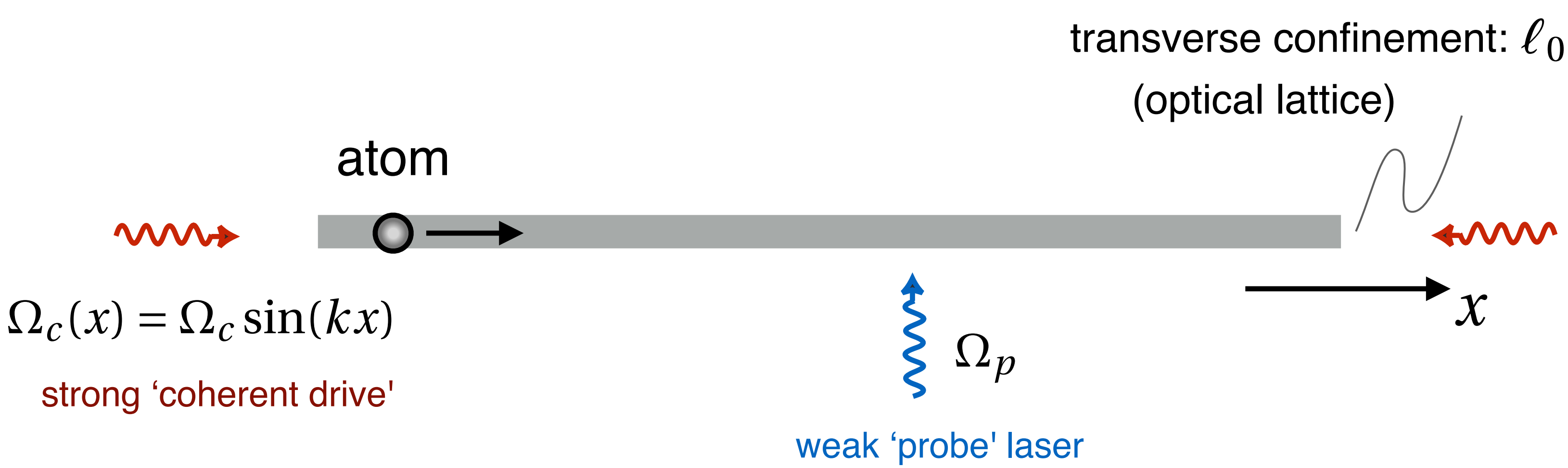
S Nascimbene, N Goldman, NR Cooper, J Dalibard - PRL 2015

Atom in Λ-Configuration: 1D Quantum Motion

- **atomic configuration**

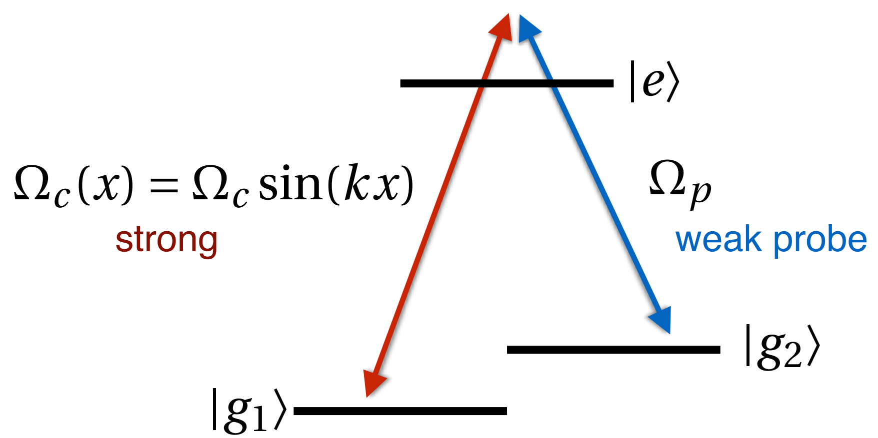


- **quantum motion of atom in 1D**

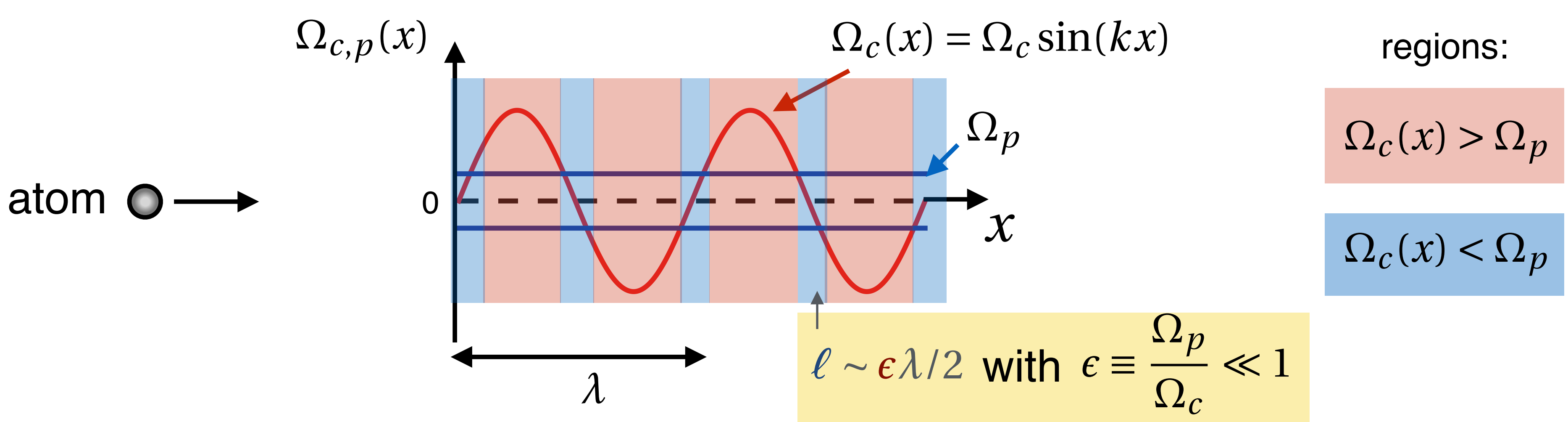


Atom in Λ-Configuration: 1D Quantum Motion

- **atomic configuration**

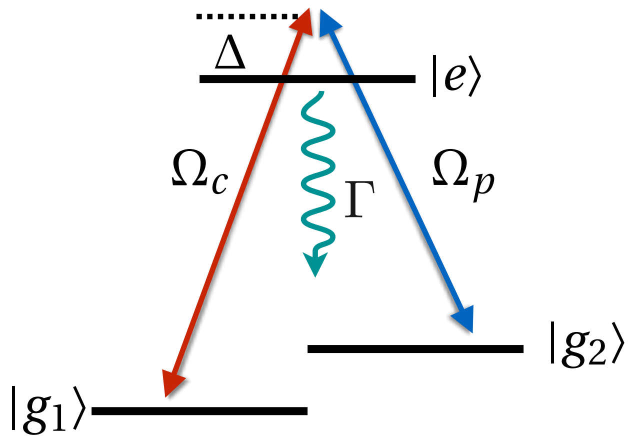


- **Rabi frequencies in space**



Atom in Λ-Configuration: 1D Quantum Motion

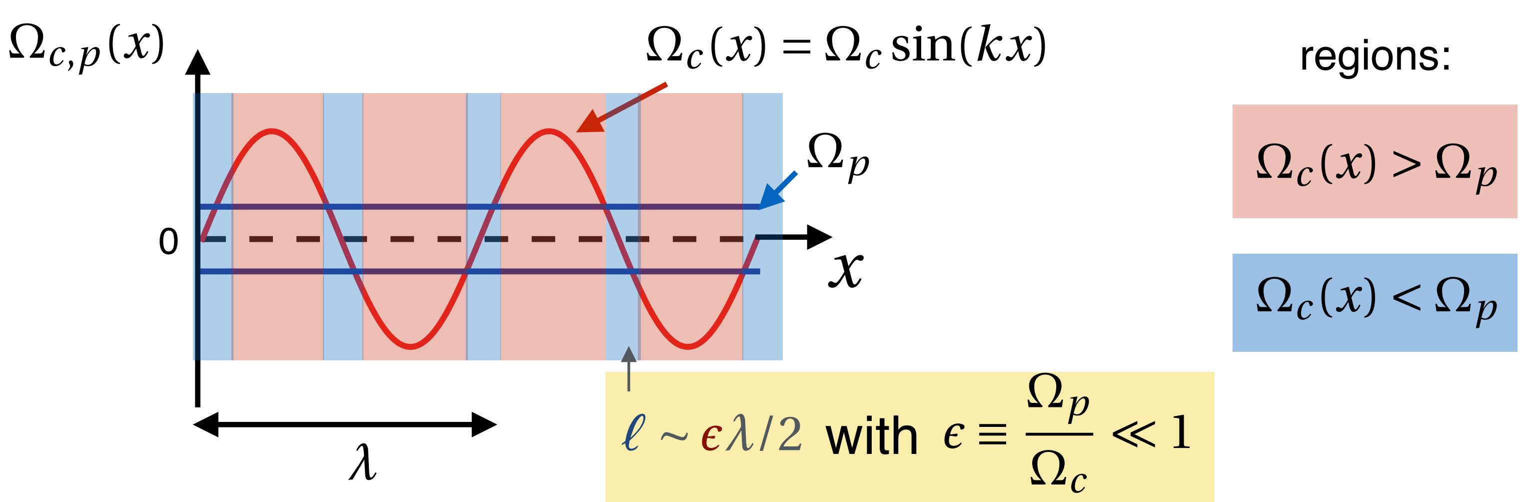
- **Hamiltonian**



$$
\begin{array}{cccc}
 & |g_1\rangle & |e\rangle & |g_2\rangle \\
H = -\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2} + & \multicolumn{3}{c}{\begin{pmatrix} 0 & \frac{1}{2}\Omega_c(x) & 0 \\ \frac{1}{2}\Omega_c(x) & -\Delta - i\frac{1}{2}\Gamma & \frac{1}{2}\Omega_p \\ 0 & \frac{1}{2}\Omega_p & 0 \end{pmatrix}}
\end{array}
$$

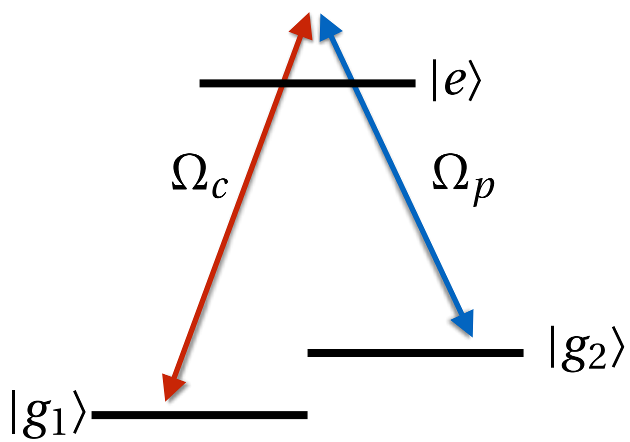
kinetic energy internal

- **Rabi frequencies in space**



Born-Oppenheimer (Adiabatic) Approximation

- **Hamiltonian**



$$H = \cancel{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}} + \begin{pmatrix} 0 & \frac{1}{2}\Omega_c(x) & 0 \\ \frac{1}{2}\Omega_c(x) & -\Delta - i\frac{1}{2}\Gamma & \frac{1}{2}\Omega_p \\ 0 & \frac{1}{2}\Omega_p & 0 \end{pmatrix}$$

- **Born-Oppenheimer (adiabatic) approximation**

dark state

$$E_0 = 0$$

$$|0\rangle \quad \sim \quad \Omega_p|g_1\rangle - \Omega_c(x)|g_2\rangle$$

no excited state admixed:

bright states

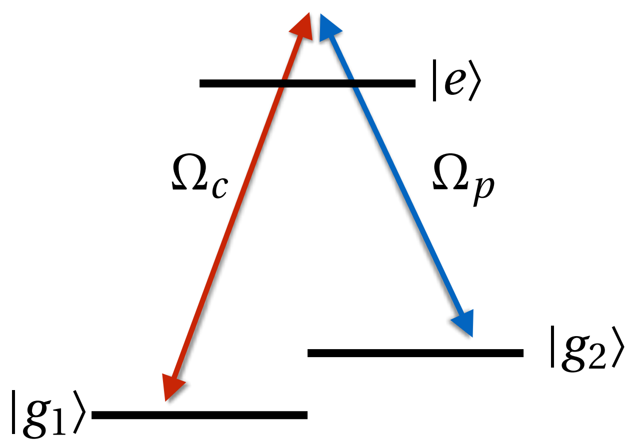
$$E_\pm = \pm E(x) \equiv \pm\frac{1}{2}\sqrt{\Omega_p^2 + \Omega_c(x)^2} - i\frac{1}{4}\Gamma$$

$$|\pm\rangle \quad \sim \quad |e\rangle \pm \frac{1}{E(x)}\left[\Omega_c(x)|g_1\rangle + \Omega_p|g_2\rangle\right]$$

here: $\Omega_{p.c} \gg \Gamma$ and $\Delta = 0$

Born-Oppenheimer (Adiabatic) Approximation

- **Hamiltonian**



$$H = \cancel{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}} + \begin{pmatrix} 0 & \frac{1}{2}\Omega_c(x) & 0 \\ \frac{1}{2}\Omega_c(x) & -\Delta - i\frac{1}{2}\Gamma & \frac{1}{2}\Omega_p \\ 0 & \frac{1}{2}\Omega_p & 0 \end{pmatrix}$$

- **Born-Oppenheimer (adiabatic) approximation**

dark state

$$E_0 = 0$$

$$|0\rangle = \cos\alpha(x)\,\big|g_1\big\rangle - \sin\alpha(x)\,\big|g_2\big\rangle$$

$$\tan\alpha(x) = \frac{\Omega_c(x)}{\Omega_p}$$

bright states

$$E_\pm = \pm E(x) \equiv \pm\frac{1}{2}\sqrt{\Omega_p^2 + \Omega_c(x)^2} - i\frac{1}{4}\Gamma$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left\{|e\rangle \pm \left[\sin\alpha(x)\,\big|g_1\big\rangle + \cos\alpha(x)\,\big|g_2\big\rangle\right]\right\}$$

we will expand the atomic wave function in these BO states

Expanding in Adiabatic Channels: Version 1

- **We expand in (adiabatic) Born-Oppenheimer channels**

$$\big|\psi(x,t)\big\rangle = \psi_0(x,t)\,|0\rangle_x + \psi_+(x,t)\,|+\rangle_x + \psi_-(x,t)\,|-\rangle_x$$

'dark' BO channel — 'bright' BO channels

spin/x-dependent dressed states

'approximate decoupling'

Rem.: compare Sisyphus laser cooling

- **Expansion in bare atomic states**

$$|\psi(x,t)\rangle = f_{g_1}(x,t)|g_1\rangle + f_e(x,t)|e\rangle + f_{g_2}(x,t)|g_2\rangle$$

bare atomic states

- see below -

Expanding in Adiabatic Channels: Version 1

- **… to obtain the Hamiltonian** for wave functions (ψ_0, ψ_+, ψ_-)

$$H = -\frac{\hbar^2}{2m}\left[\frac{\partial}{\partial x} + \frac{\alpha'}{\sqrt{2}}\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\right]^2 + E(x)\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

vector potential (non-adiabatic coupling)

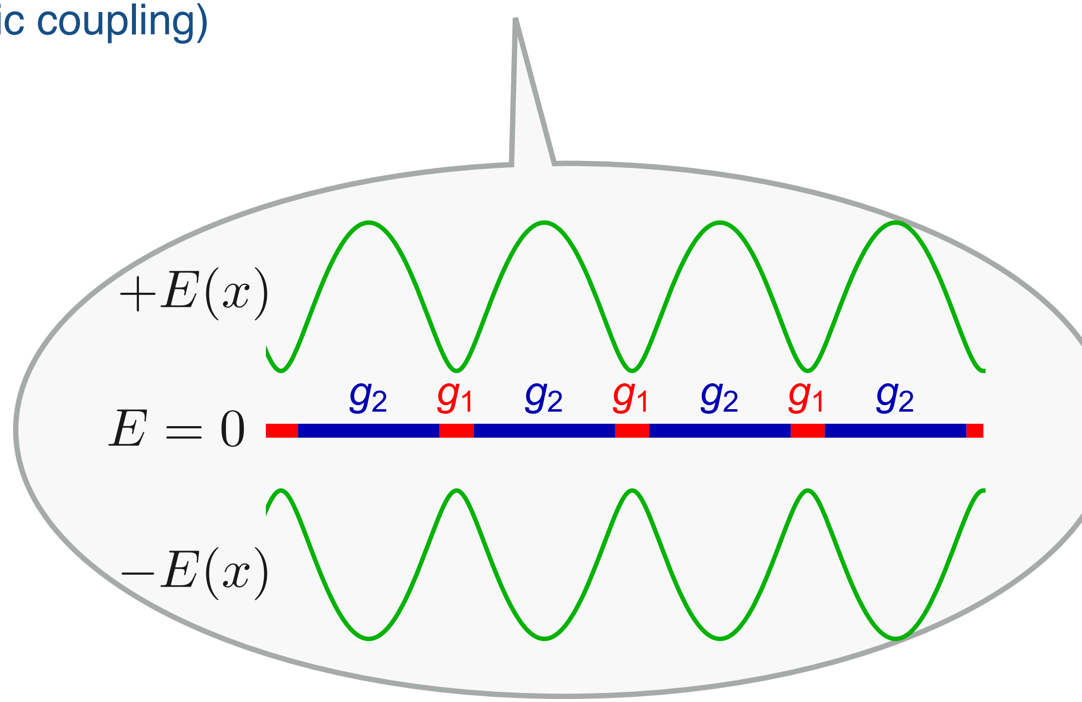
potential

$$\alpha' \equiv \frac{d\alpha}{dx} = k\varepsilon\frac{\cos(kx)}{\varepsilon^2 + \sin^2(kx)}$$

validity of adiabatic approximation:

$$\frac{1}{\epsilon}E_R \equiv \frac{\hbar^2 k^2}{2m} \ll \Omega_c, \Omega_p$$

$$\epsilon \sim \frac{\ell}{\lambda} \ll 1$$



Expanding in Adiabatic Channels: Version 2

- **… to obtain the Hamiltonian** for wave functions (ψ_0, ψ_+, ψ_-)

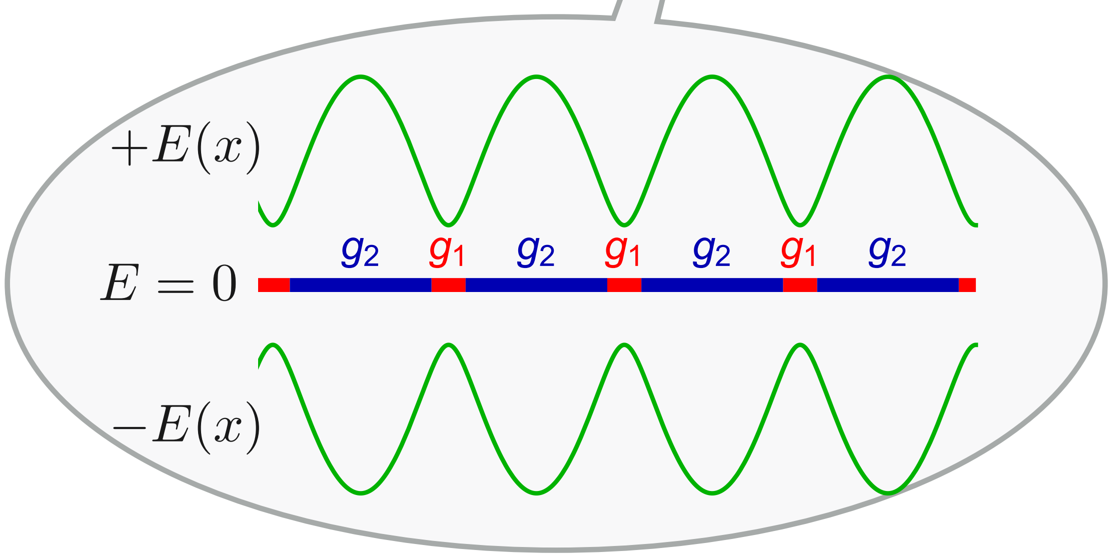
$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + E(x)\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

← **adiabatic**

$$\cancel{-\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial x}\frac{\alpha'}{\sqrt{2}} + \frac{\alpha'}{\sqrt{2}}\frac{\partial}{\partial x}\right)\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}} + \frac{\hbar^2}{2m}\frac{(\alpha')^2}{2}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

$V_{\text{na}}(x)$

first order correction



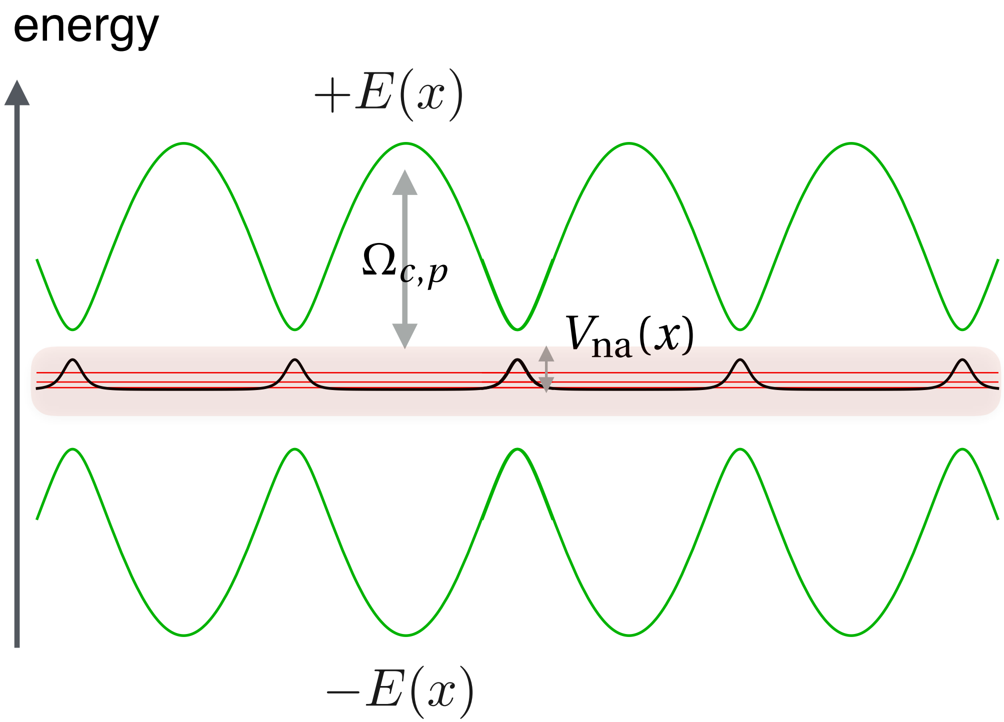
‘Dark State’ Optical Lattice

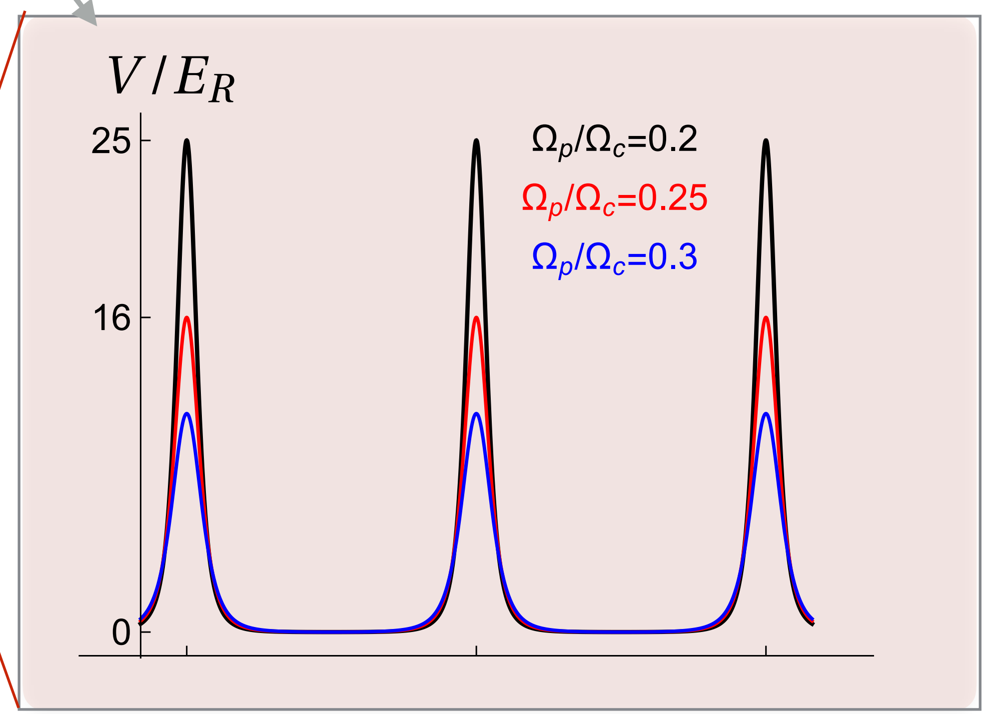
- **… including the first order non-adiabatic correction**

$$i\hbar \frac{\partial}{\partial t}\psi_0(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_{\text{opt}}(x)\right]\psi_0(x,t)$$

‘dark state’ channel

$$V_{\text{opt}}(x) \equiv V_{\text{na}}(x) = E_R \frac{\varepsilon^2 \cos^2(kx)}{[\varepsilon^2 + \sin^2(kx)]^2}$$

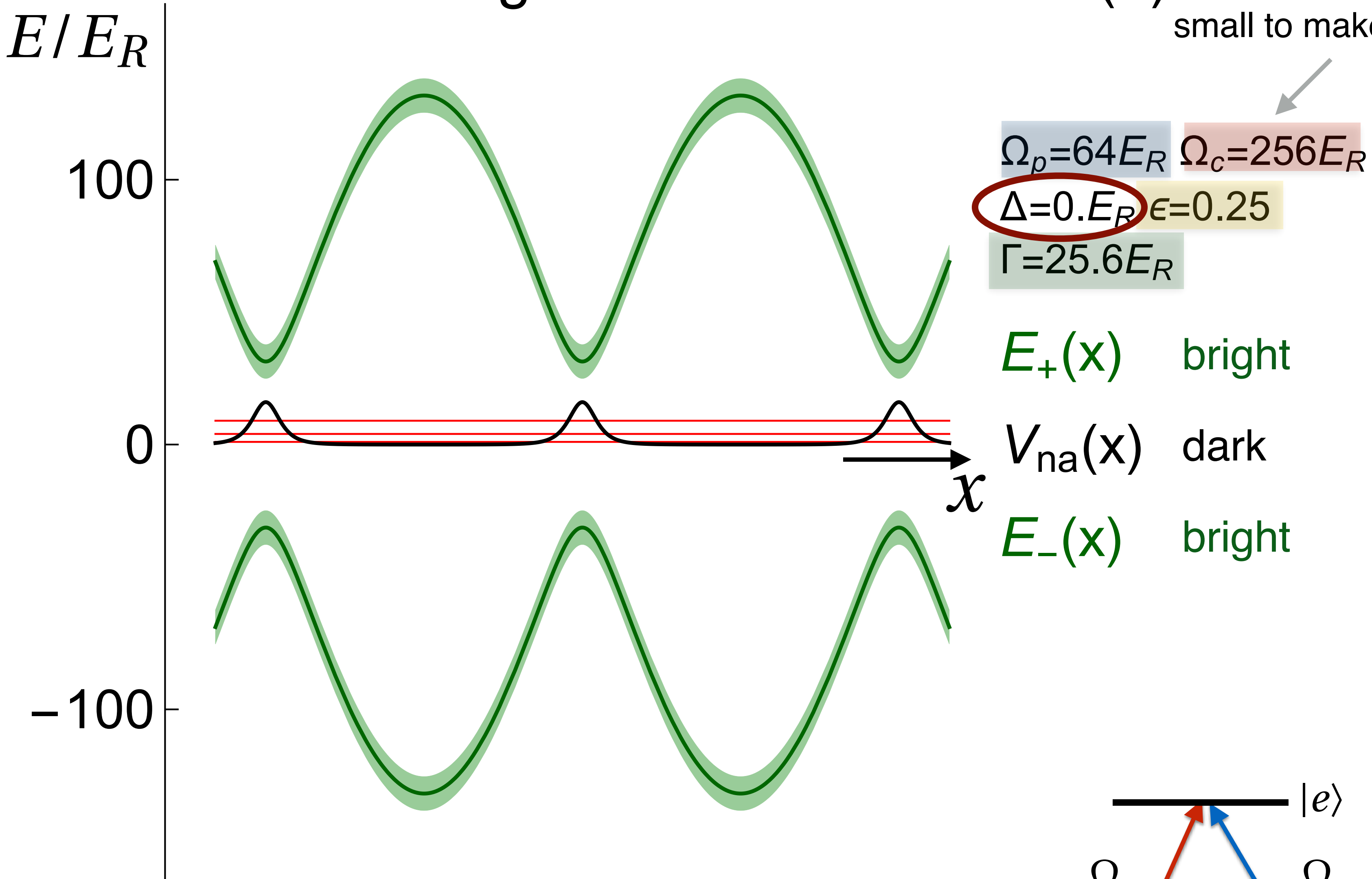




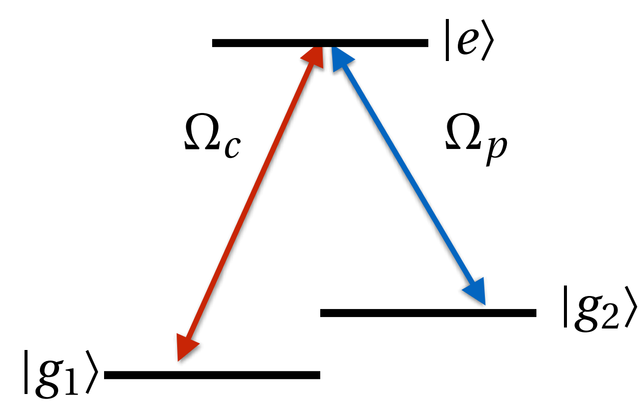
✓ conservative

✓ sub-wavelength structures

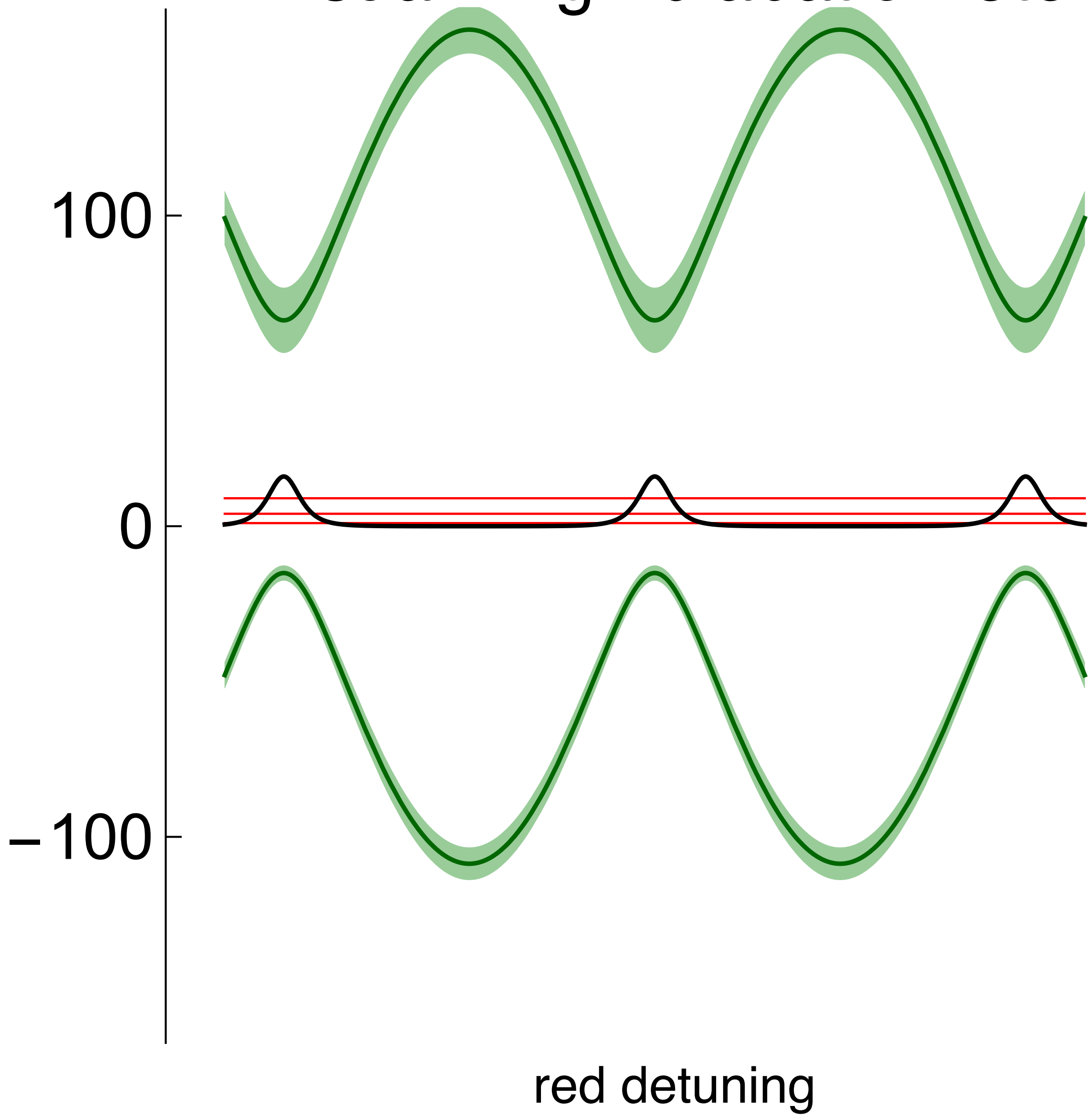
Visualizing Adiabatic Potentials (1)

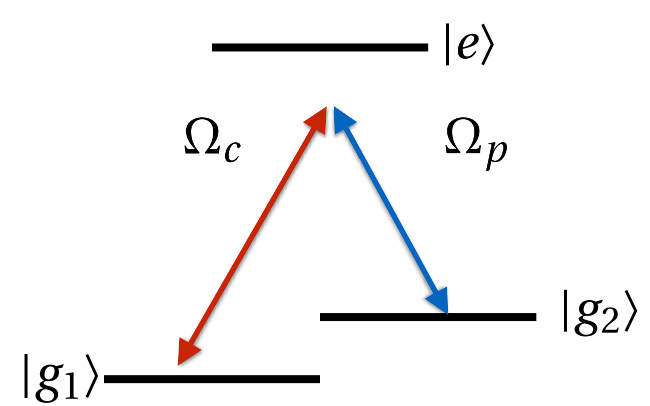


on resonance

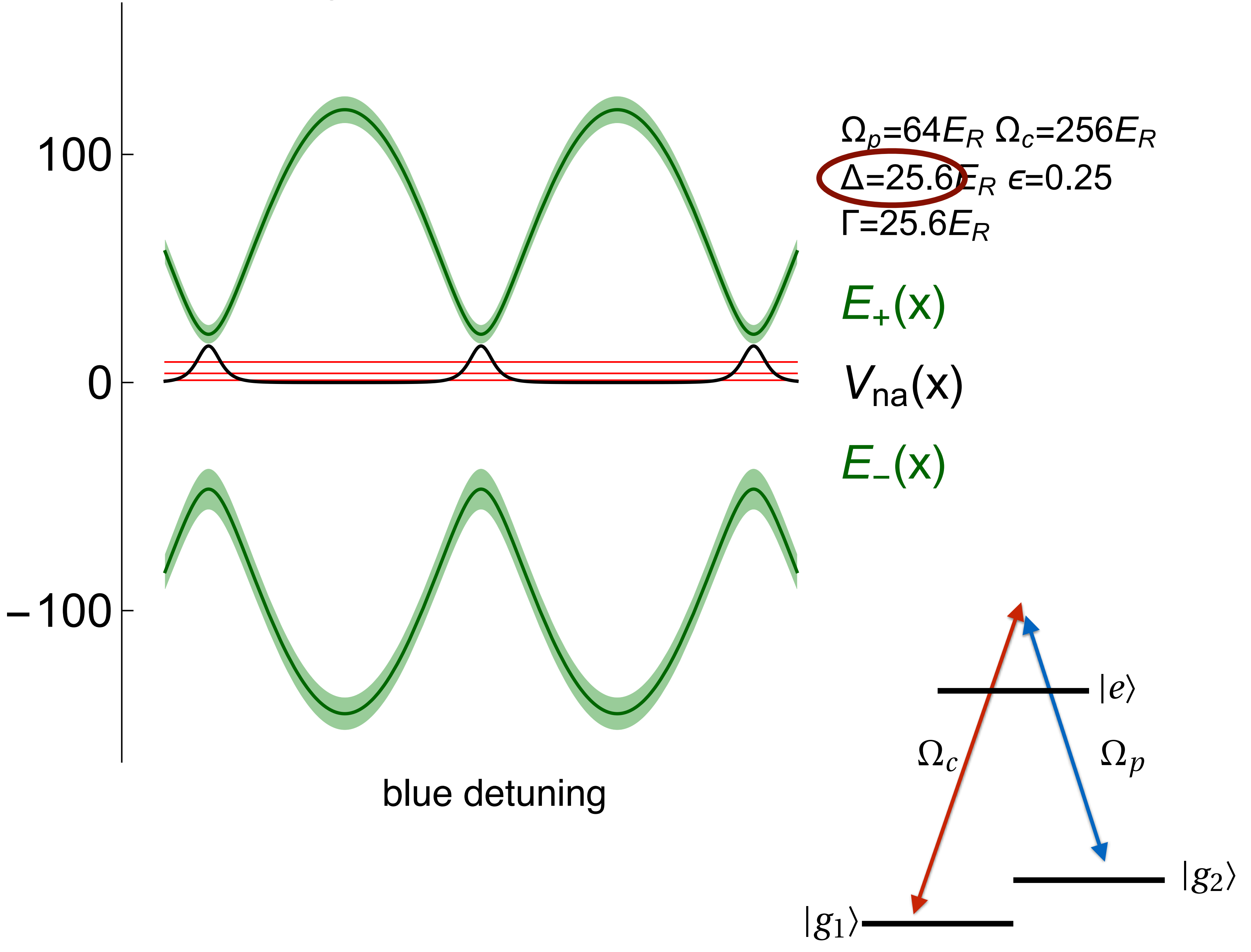


Visualizing Adiabatic Potentials (1)

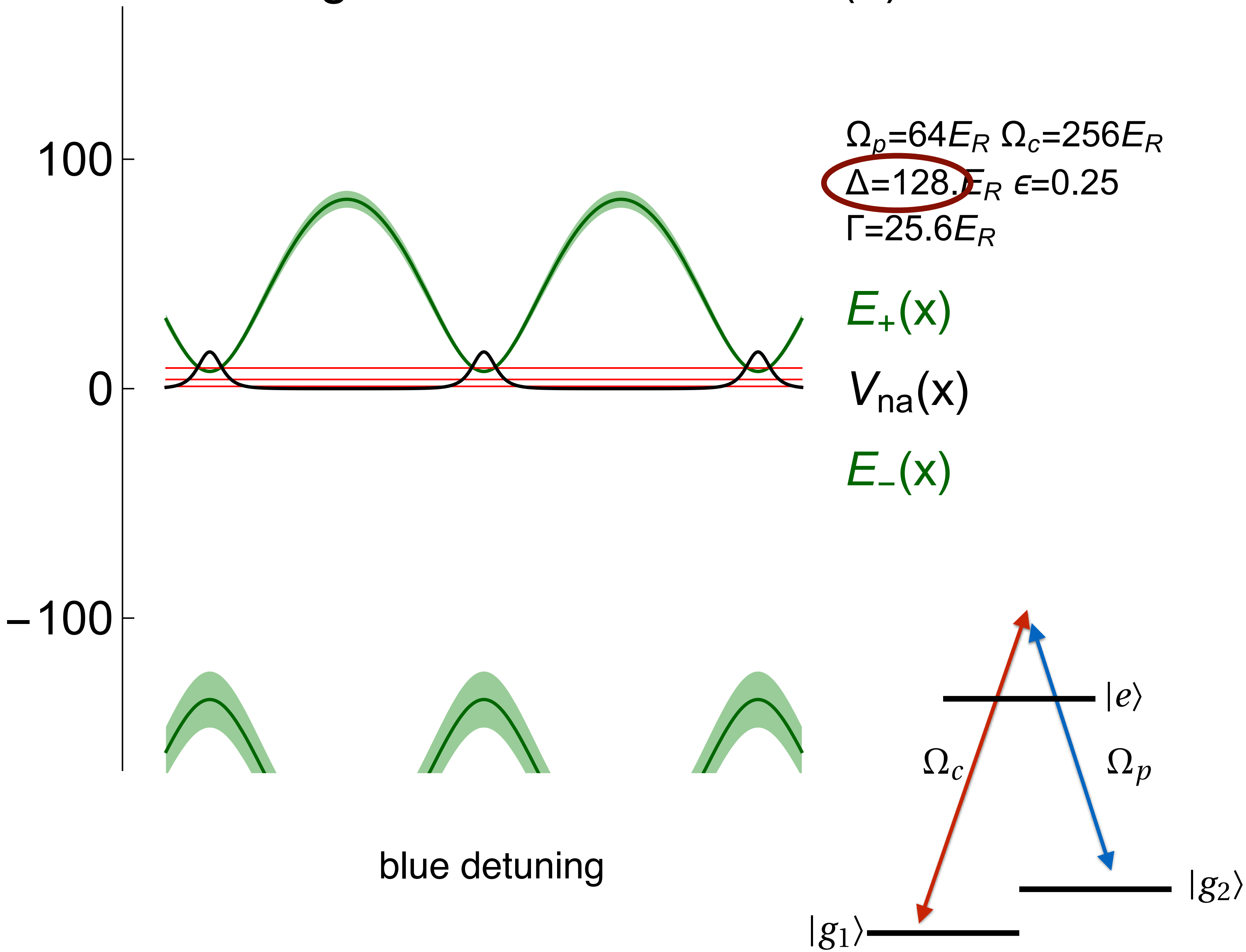




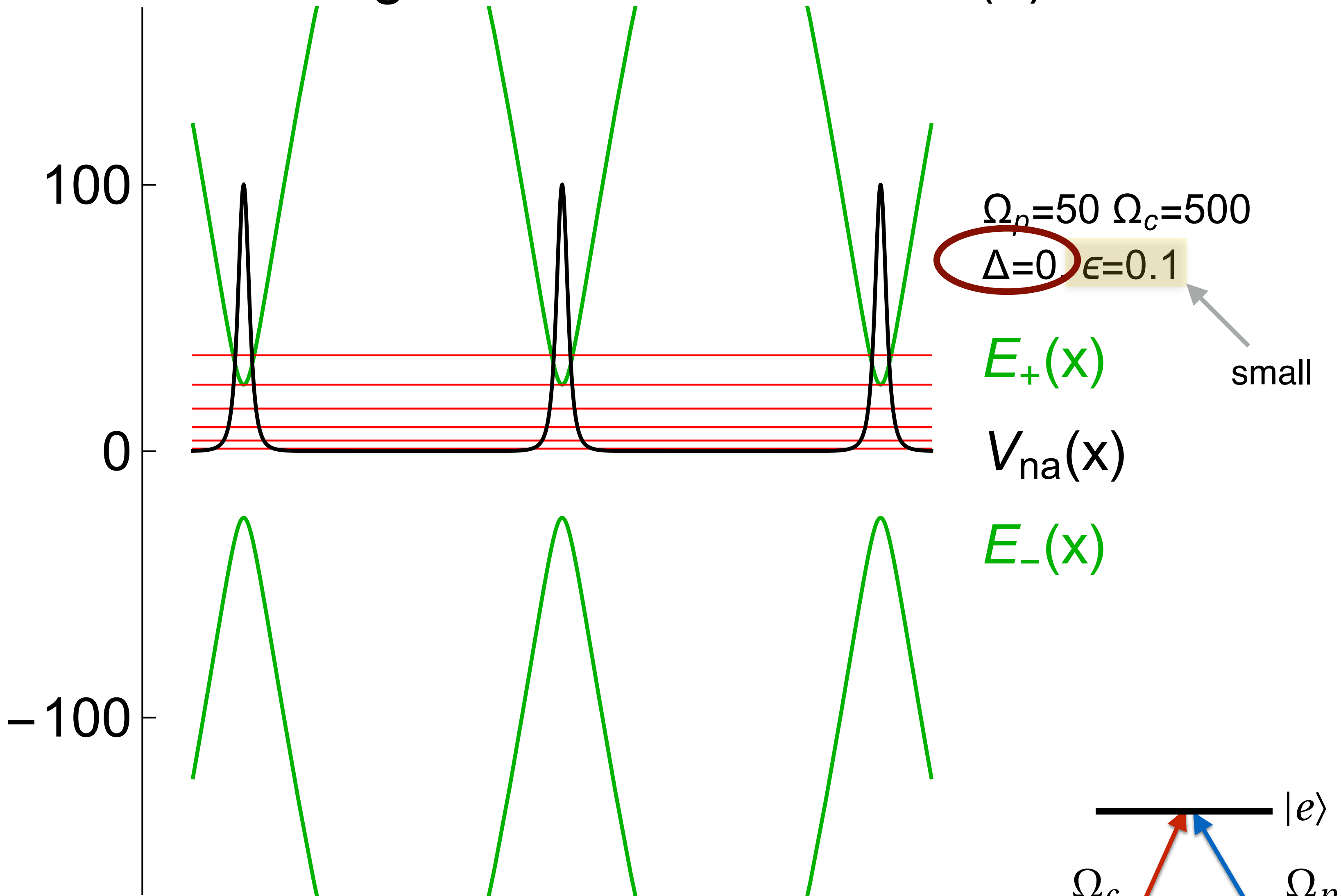
Visualizing Adiabatic Potentials (2)

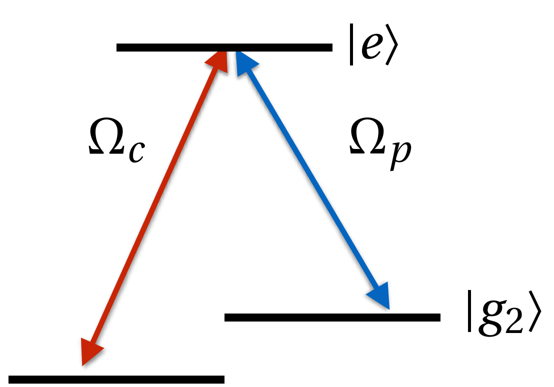


Visualizing Adiabatic Potentials (2)

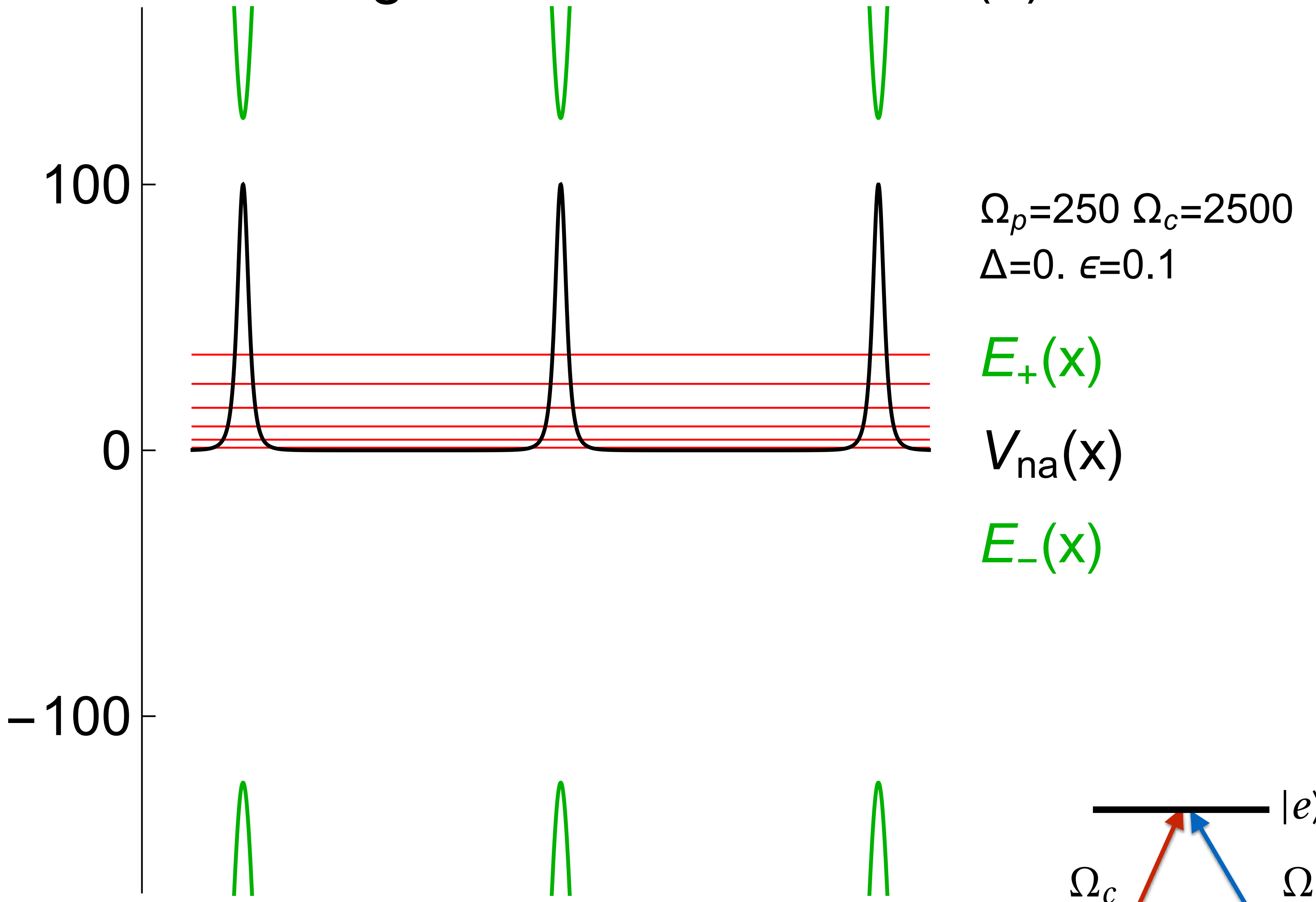


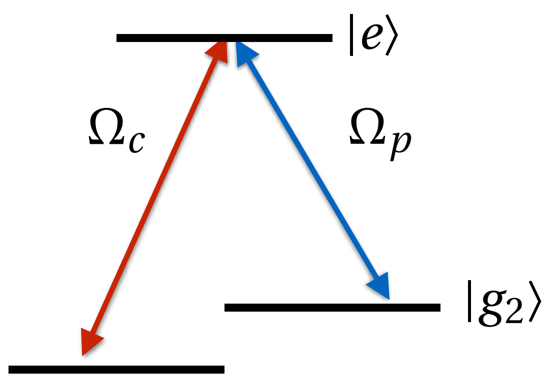
Visualizing Adiabatic Potentials (3)



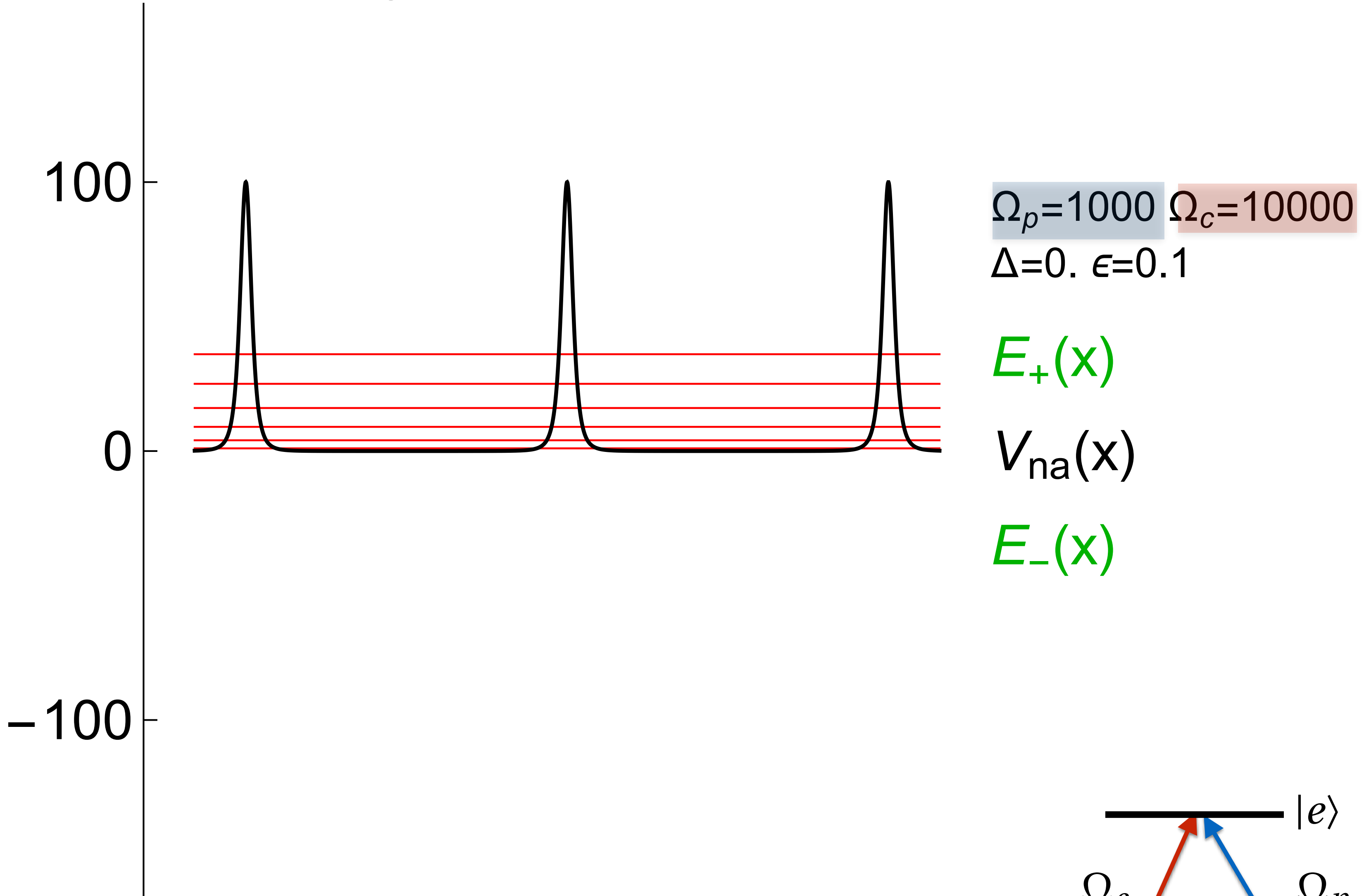


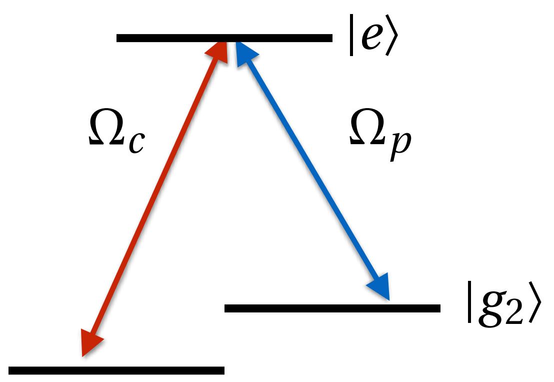
Visualizing Adiabatic Potentials (3)





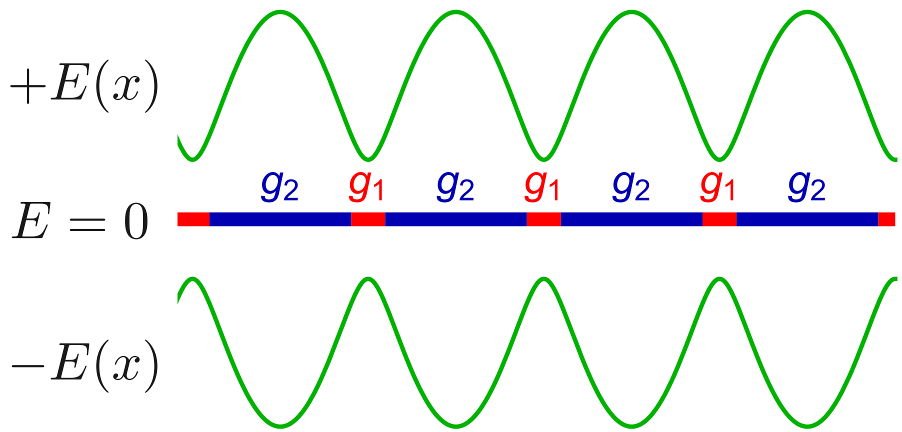
Visualizing Adiabatic Potentials (3)





Discussion

1. Zero order adiabatic approximation

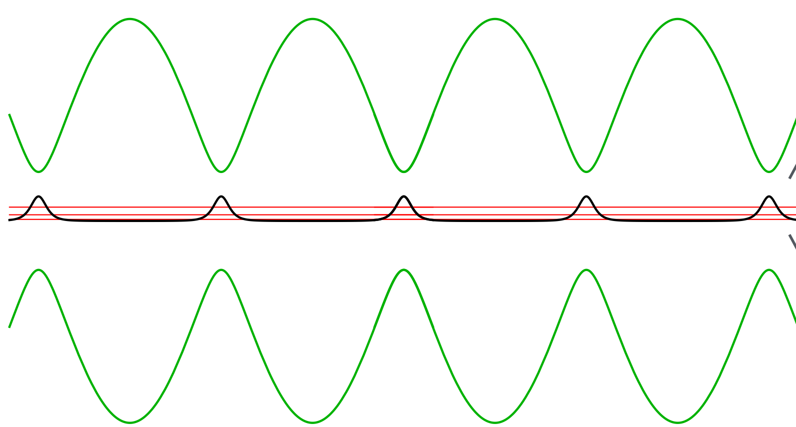


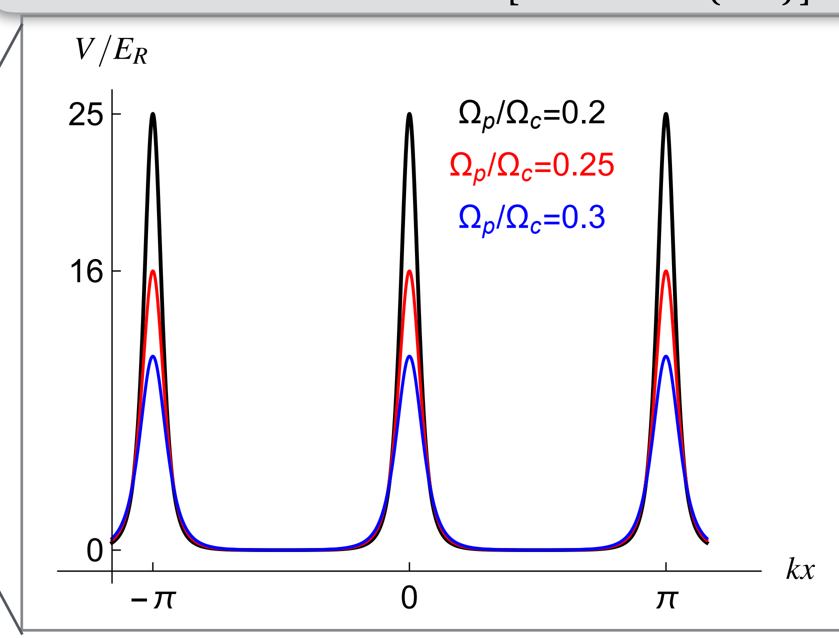
✓ no optical potential

✓ sub-wavelength structure in internal state / interaction

$$V_{\text{opt}}(x) \equiv V_{na}(x) = E_R \frac{\varepsilon^2 \cos^2(kx)}{[\varepsilon^2 + \sin^2(kx)]^2}$$

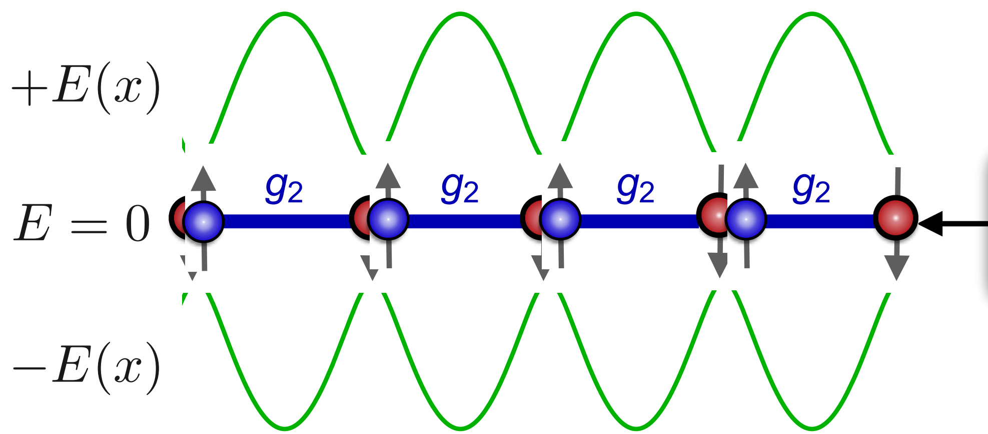
2. First order adiabatic approximation





3. Exact bandstructure: lifetime due to channel couplings

1. Zero order adiabatic approximation



free particle

$$i\hbar\frac{\partial}{\partial t}\psi_0(x,t)=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+0\right]\psi_0(x,t)$$

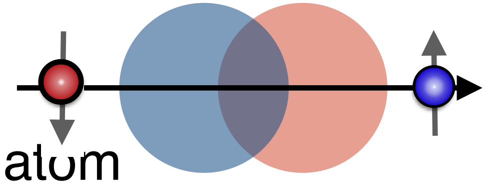
while internal state changes

$$|0\rangle=\cos\alpha(x)\,|g_1\rangle-\sin\alpha(x)\,|g_2\rangle$$

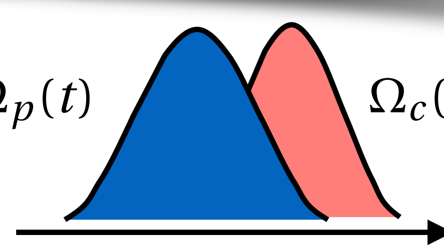
THE JOURNAL OF CHEMICAL PHYSICS **142**, 170901 (2015)

Perspective: Stimulated Raman adiabatic passage: The status after 25 years

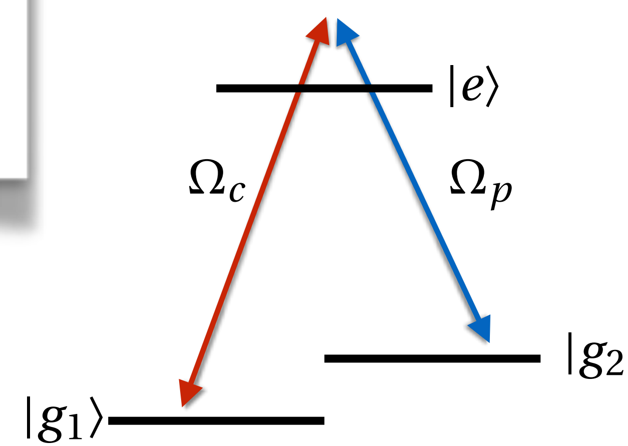
Klaas Bergmann,[1] Nikolay V. Vitanov,[2] and Bruce W. Shore[3]



classical trajectory

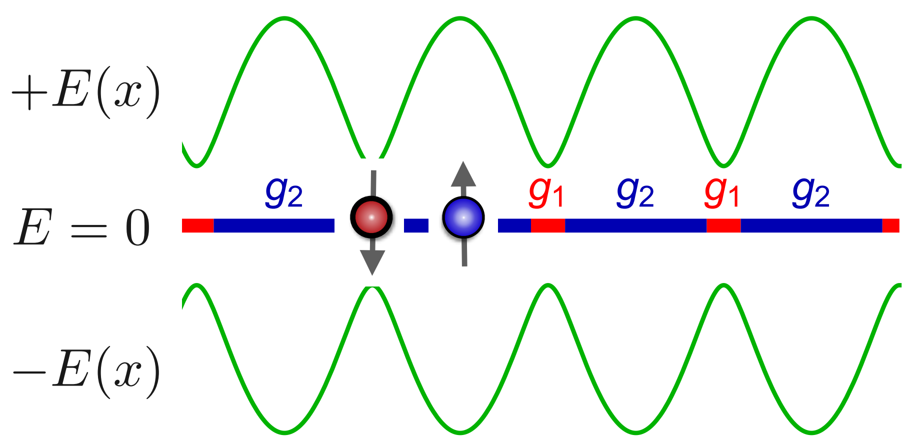


counter intuitive
pulse sequence

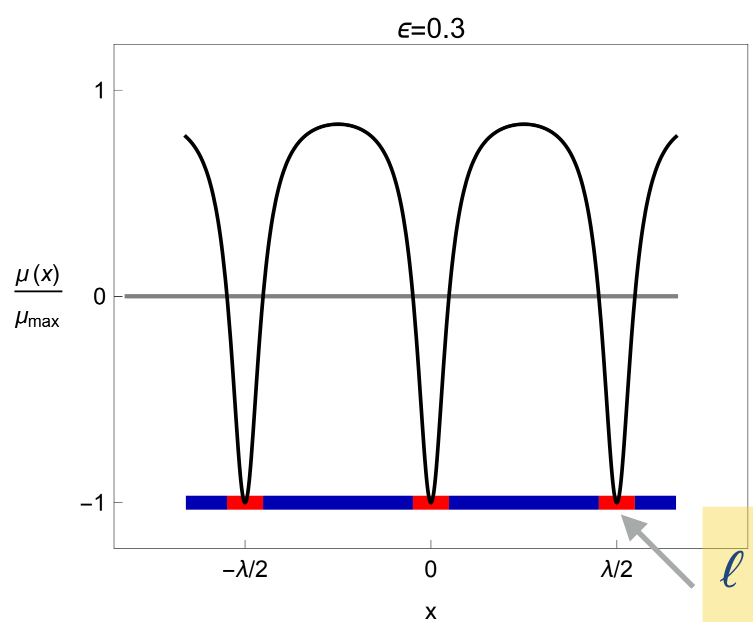


$$|g_1\rangle \xrightarrow{\Omega_p\gg\Omega_c} |E_0\rangle \xrightarrow{\Omega_p\ll\Omega_c} |g_2\rangle$$

1. Zero order adiabatic approximation



spatial variation of dipole moment



✓ dipole moment can be any angle

✓ limit to spatial structure:

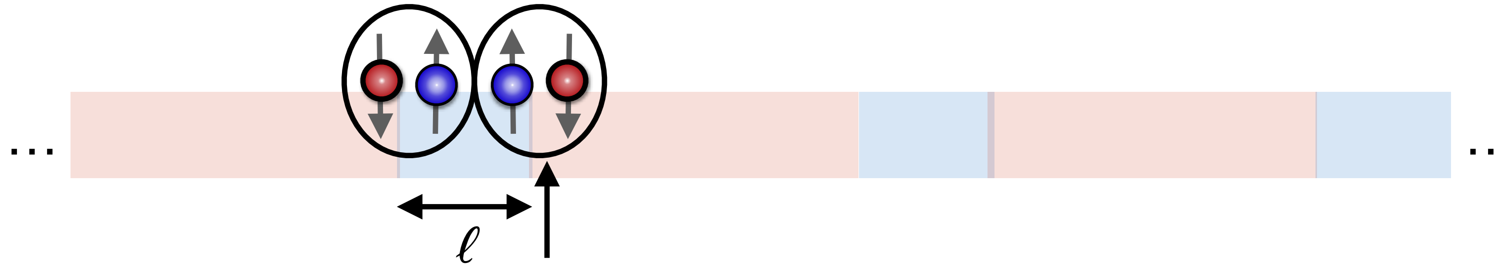
$$\ell > \ell_0$$

(at least in our 1D model)

$\ell \sim \epsilon\lambda/2$ with $\epsilon \equiv \dfrac{\Omega_p}{\Omega_c} \ll 1$

Quantum Many-Body Physics

- **Two-particles**



'domain wall molecule' as bound state

✓ molecule sees a lattice

Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2} + \frac{\mu(x_1)\mu(x_2)}{|x_1 - x_2|^3}$$

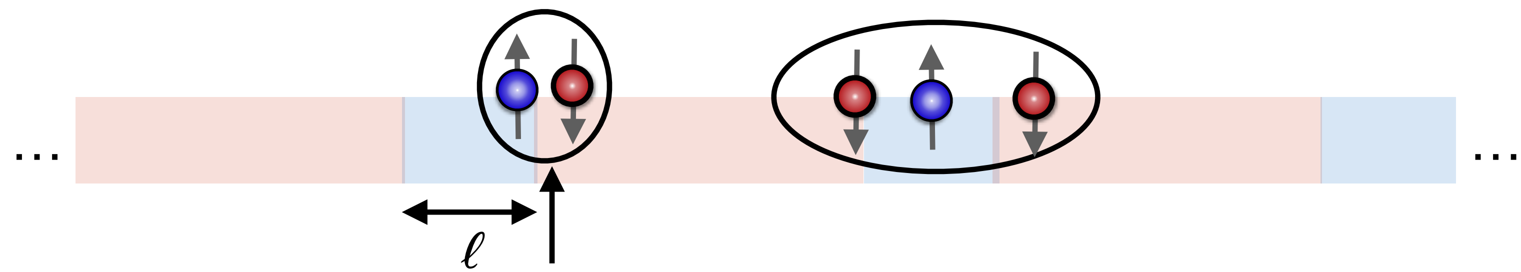
spatial variation of dipole moment

✓ sub-wavelength

✓ + cutoff for $|x_1 - x_2| < \ell_0$

Quantum Many-Body Physics

- **Two-particles**



'domain wall molecule' as bound state

✓ molecule sees a lattice

Hamiltonian

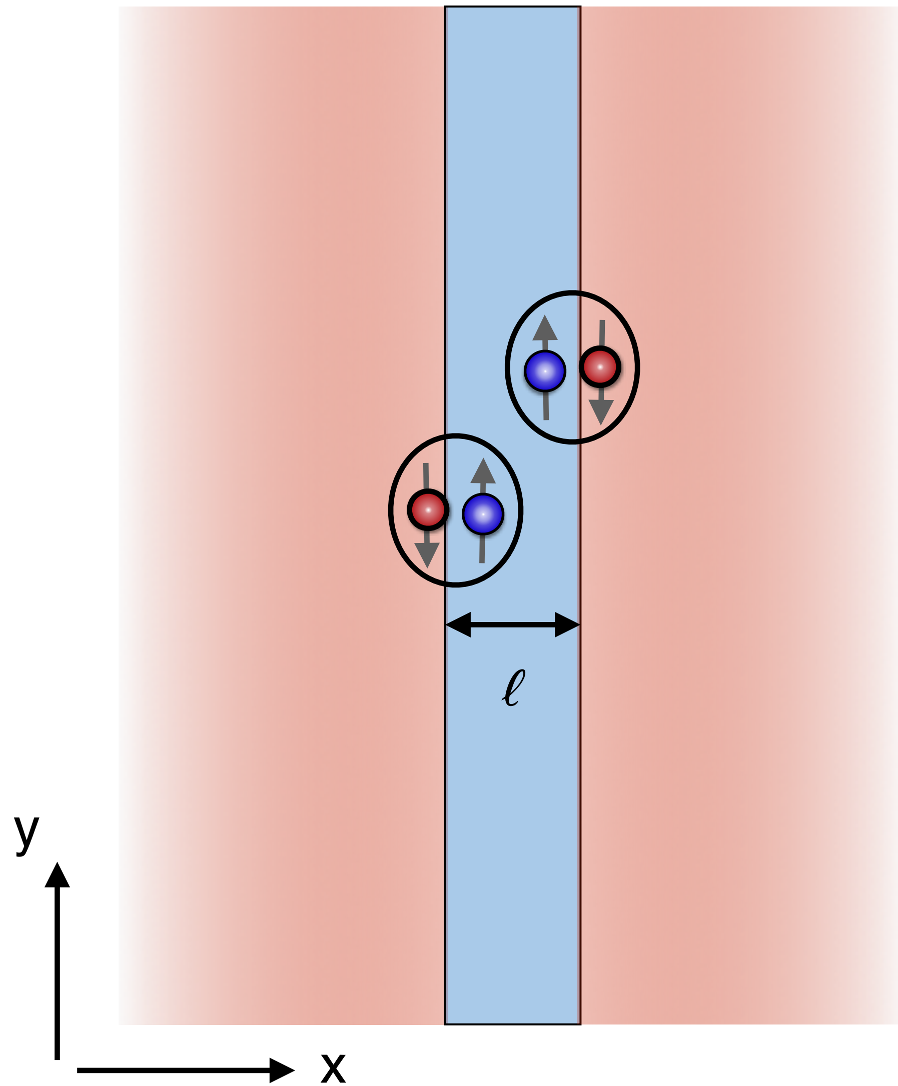
$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2} + V_{\rm na}(x_1) + V_{\rm na}(x_2) + \frac{\mu(x_1)\mu(x_2)}{|x_1 - x_2|^3}$$

see below

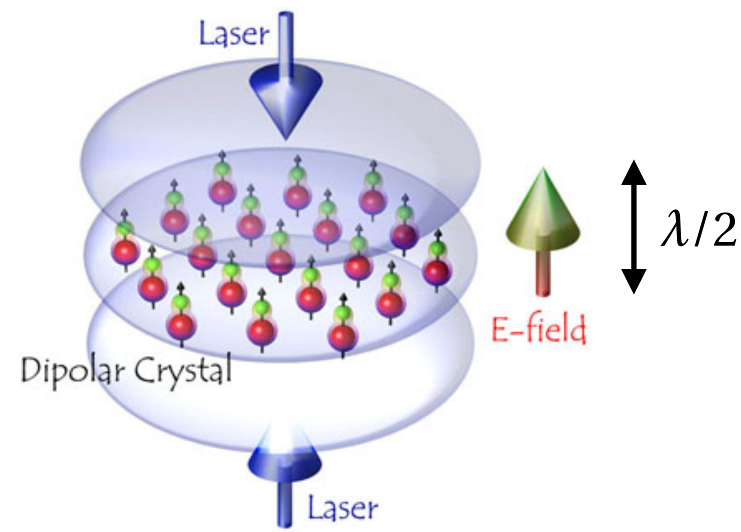
- **Three-particles**

Quantum Many-Body Physics

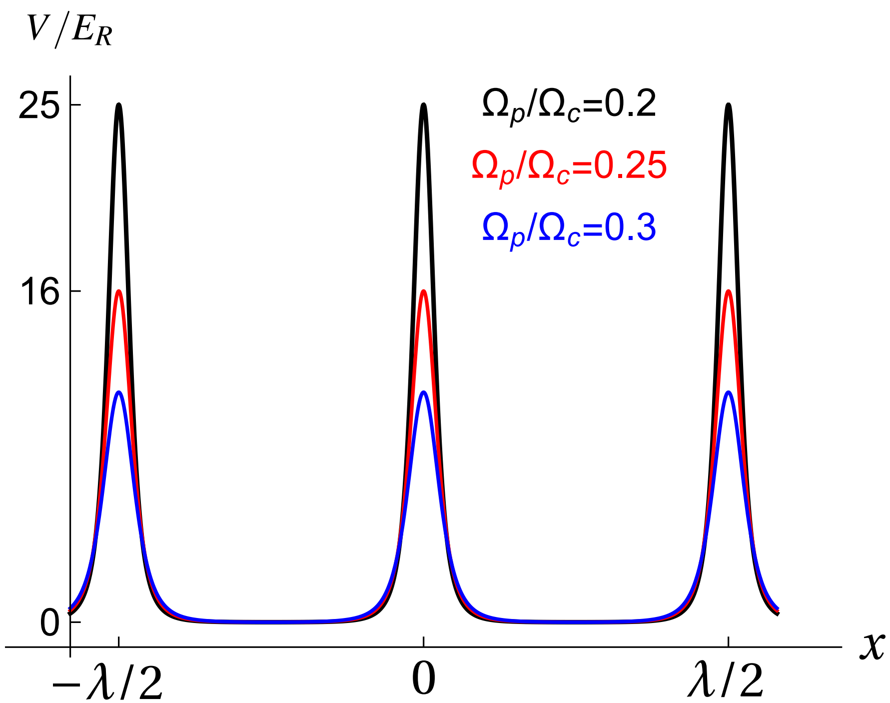
- ***Sub-wavelength*** **bilayer**



- **polar molecules in bilayer from standing light wave**



2. First order adiabatic approximation



$$V_{\mathrm{opt}}(x) \equiv V_{na}(x) = E_R \frac{\varepsilon^2 \cos^2(kx)}{[\varepsilon^2 + \sin^2(kx)]^2}$$

Mapping to a Kronig-Penney potential:

$$H_\delta = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \boxed{\frac{\pi \lambda E_R}{4\epsilon}} \sum_n \delta\left(x - \frac{\lambda}{2} n\right)$$

$$\epsilon \ll 1$$

properties:

✓ immune against laser noise

- intensity noise
- laser bandwidth ~ dephasing

  for Ω_c and Ω_p derived from same laser

Band Structure

- **Bloch ansatz**

$$\psi_q(x) = e^{iqx} u_q(x)$$

$$u_q(x) = u_q(x + a)$$

$q \in [-\frac{\pi}{a}, +\frac{\pi}{a})$

Brillouin zone

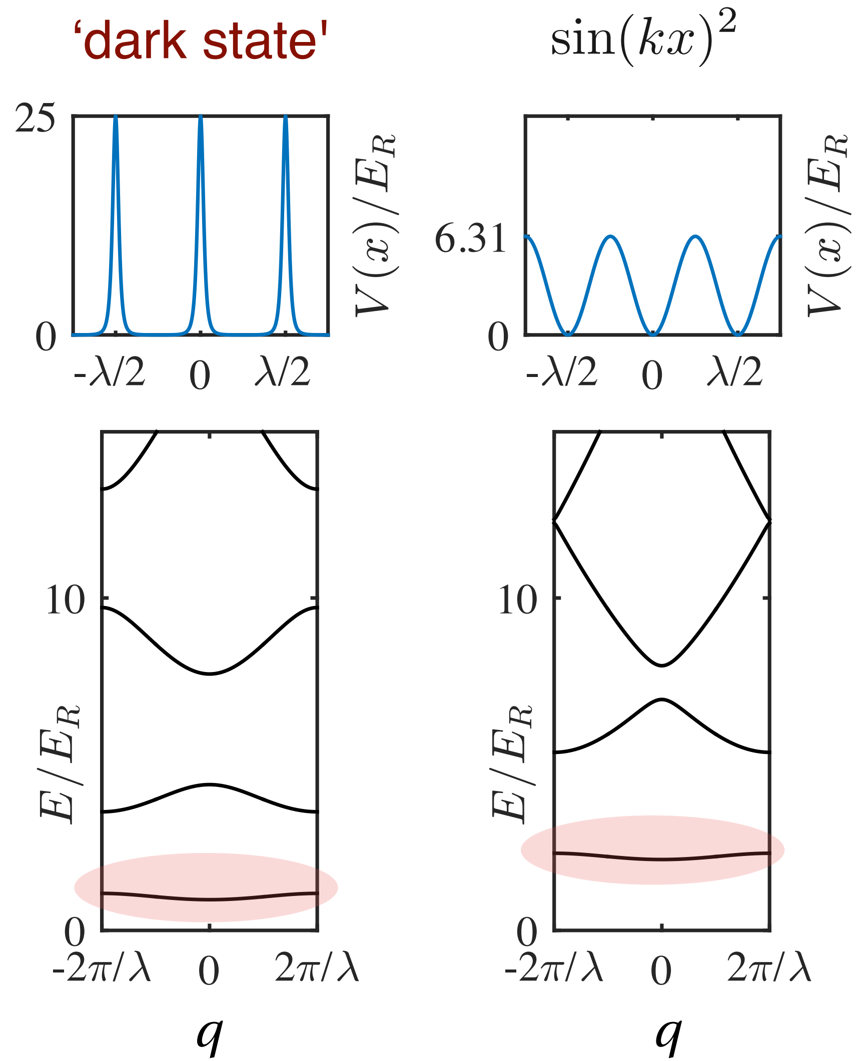
$a = \lambda/2$

lattice spacing

band spacing:

n^2 vs. n

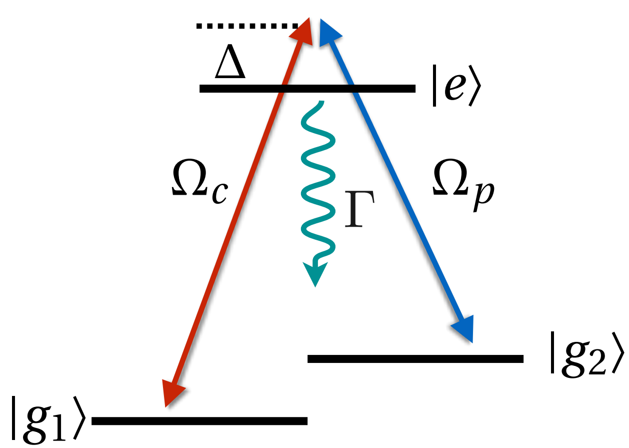
- **Band structure**



same Bloch bandwidth J

3. Band Structure for Coupled Channels

- **Hamiltonian**



- **Multichannel Bloch ansatz**

bare channel functions

$$\psi_q(x) = e^{iqx} \begin{pmatrix} u_{g_1}(x) \\ u_e(x) \\ u_{g_2}(x) \end{pmatrix}, \quad u_\lambda(x+a) = u_\lambda(x)$$

λ

lattice spacing

$q \in [-\pi/\lambda, \pi/\lambda)$

quasi-momentum

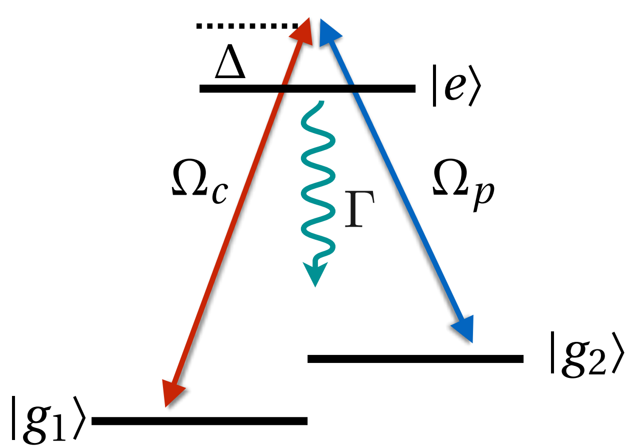
- **Band structure**

$$\left[\frac{(\frac{\hbar}{i}\frac{\partial}{\partial x} + q)^2}{2m} + \begin{pmatrix} 0 & \frac{1}{2}\Omega_c(x) & 0 \\ \frac{1}{2}\Omega_c(x) & -\Delta - i\frac{1}{2}\Gamma & \frac{1}{2}\Omega_p \\ 0 & \frac{1}{2}\Omega_p & 0 \end{pmatrix} \right] \begin{pmatrix} u_{g_1} \\ u_e \\ u_{g_2} \end{pmatrix} = E(q) \begin{pmatrix} u_{g_1} \\ u_e \\ u_{g_2} \end{pmatrix}$$

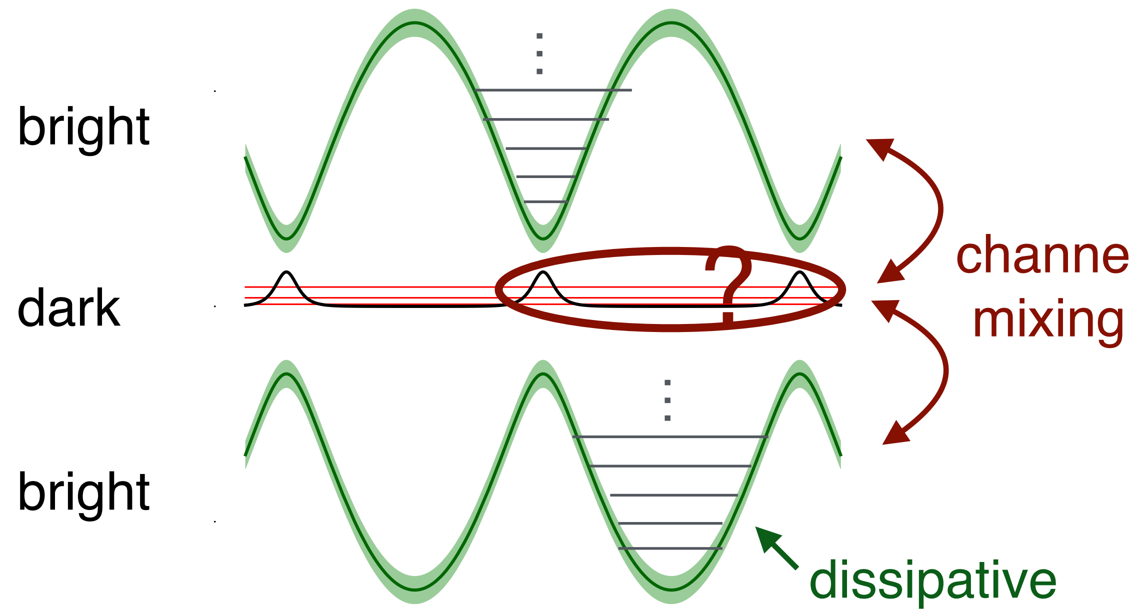
complex / lossy band structure

3. Band Structure for Coupled Channels

- **Hamiltonian**



- **Multichannel:** adiabatic potential



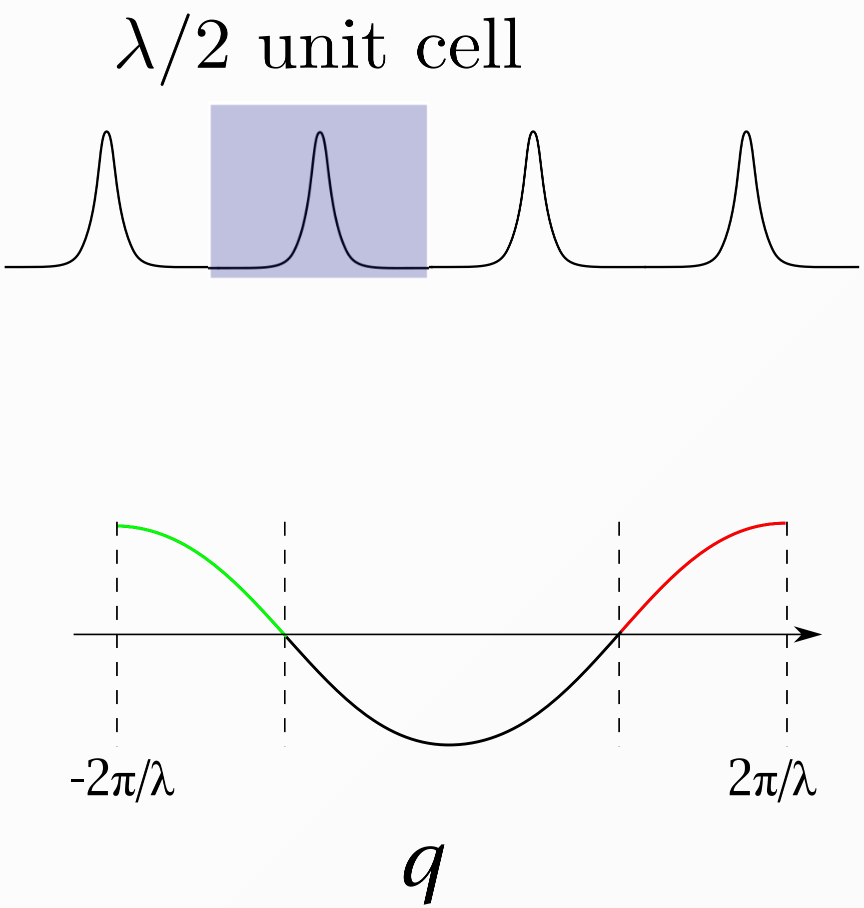
- **Band structure**

$$\left[\frac{(\frac{\hbar}{i}\frac{\partial}{\partial x}+q)^2}{2m} + \begin{pmatrix} 0 & \frac{1}{2}\Omega_c(x) & 0\\ \frac{1}{2}\Omega_c(x) & -\Delta - i\frac{1}{2}\Gamma & \frac{1}{2}\Omega_p \\ 0 & \frac{1}{2}\Omega_p & 0\end{pmatrix}\right]\begin{pmatrix}u_{g_1}\\ u_e\\ u_{g_2}\end{pmatrix} = E(q)\begin{pmatrix}u_{g_1}\\ u_e\\ u_{g_2}\end{pmatrix}$$

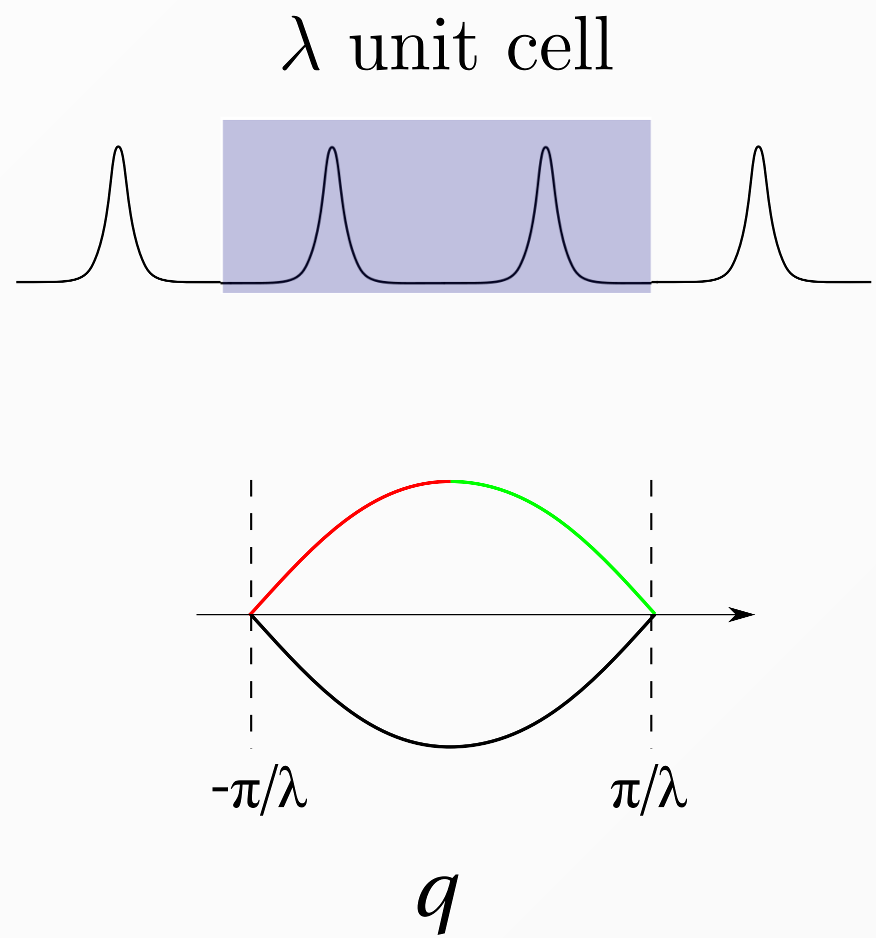
complex / lossy band structure

Remark: Brillouin zones

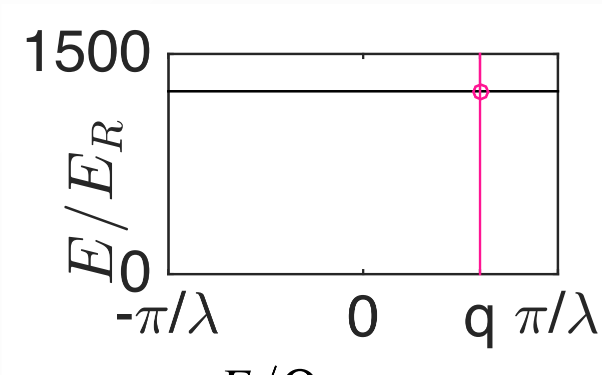
so far

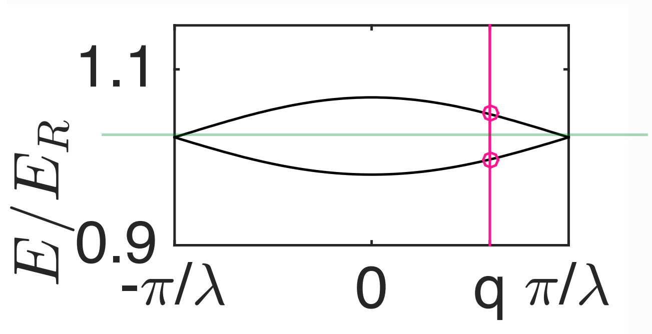


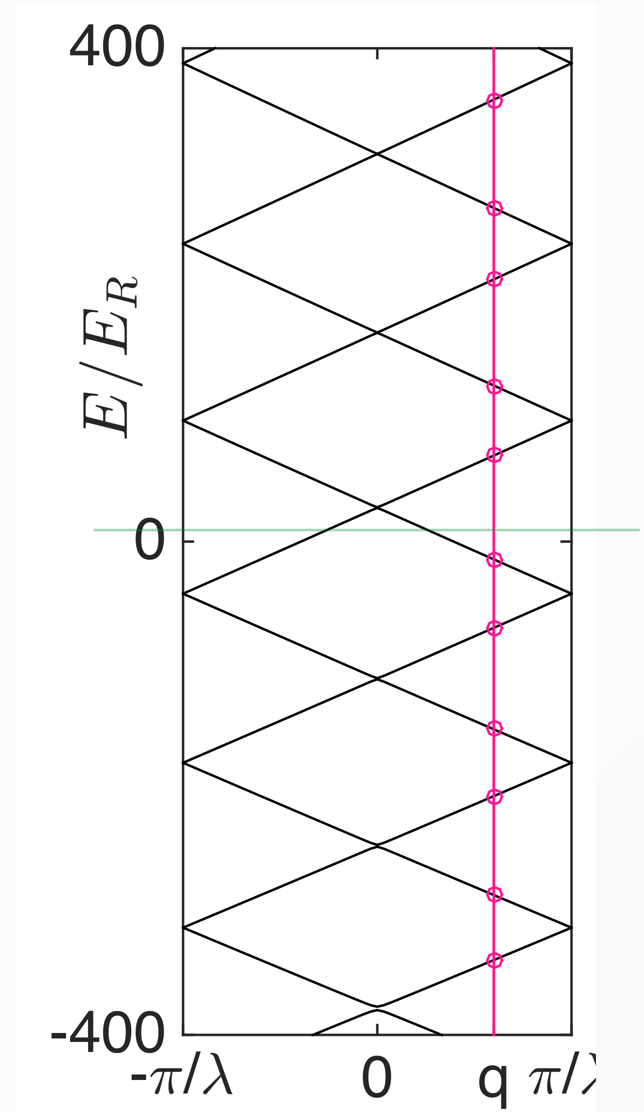
now

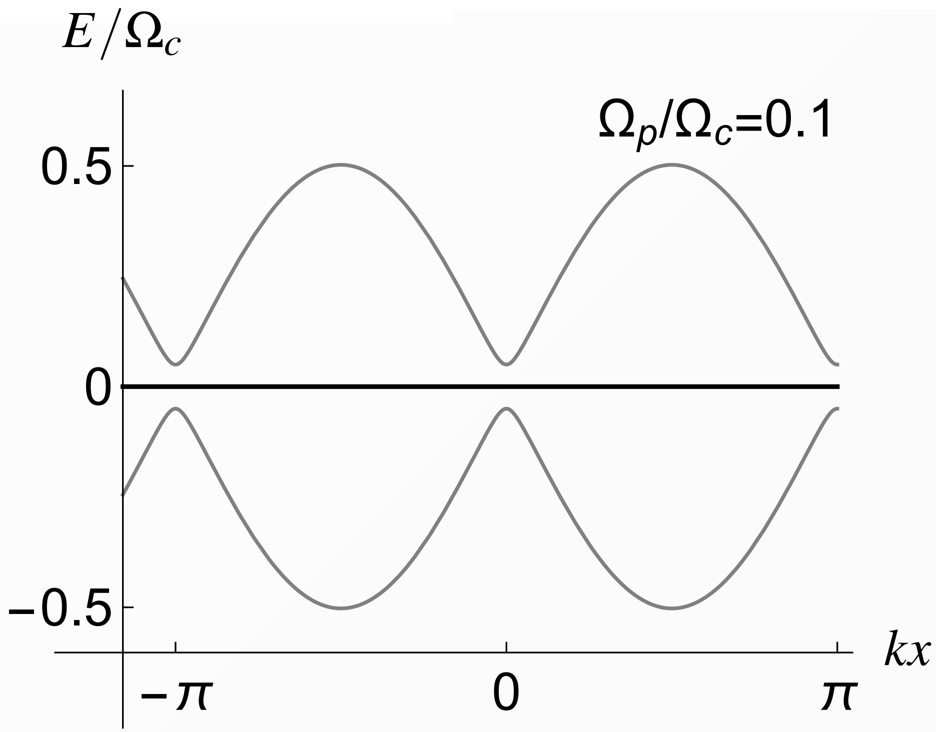


adiabatic channels (realistic energy ranges)









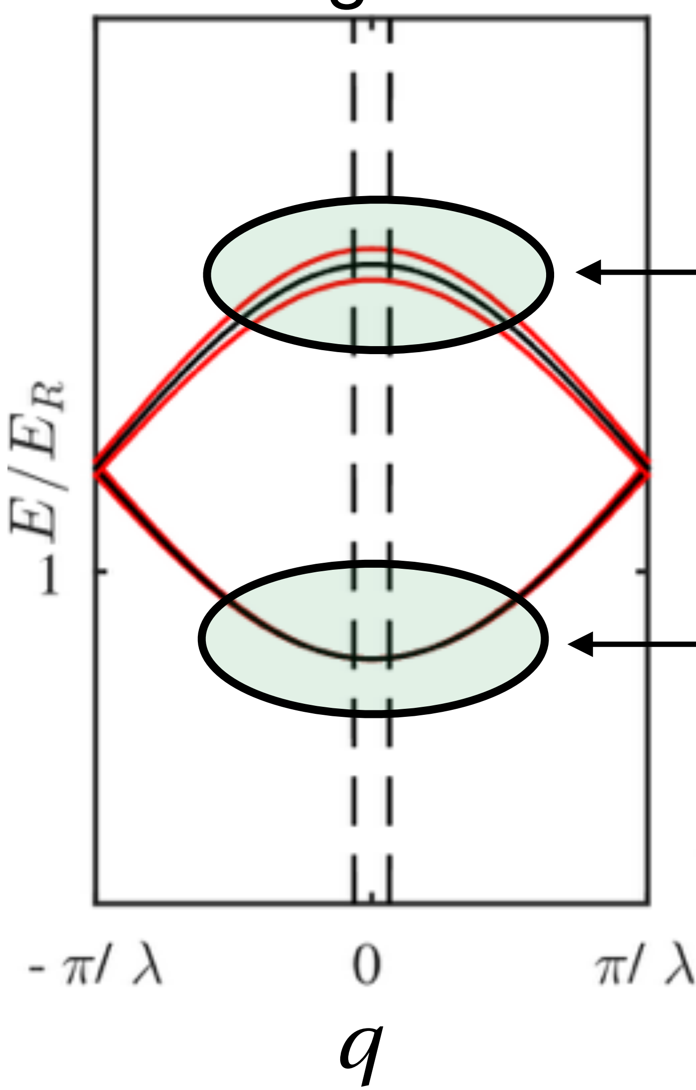
$$\Omega_c = 20000 E_r$$

Lowest Blochband: Non-adiabatic Losses

$$E(q) = \epsilon(q) - i\frac{1}{2}\gamma(q)$$

dissipation (engineering)

large Γ

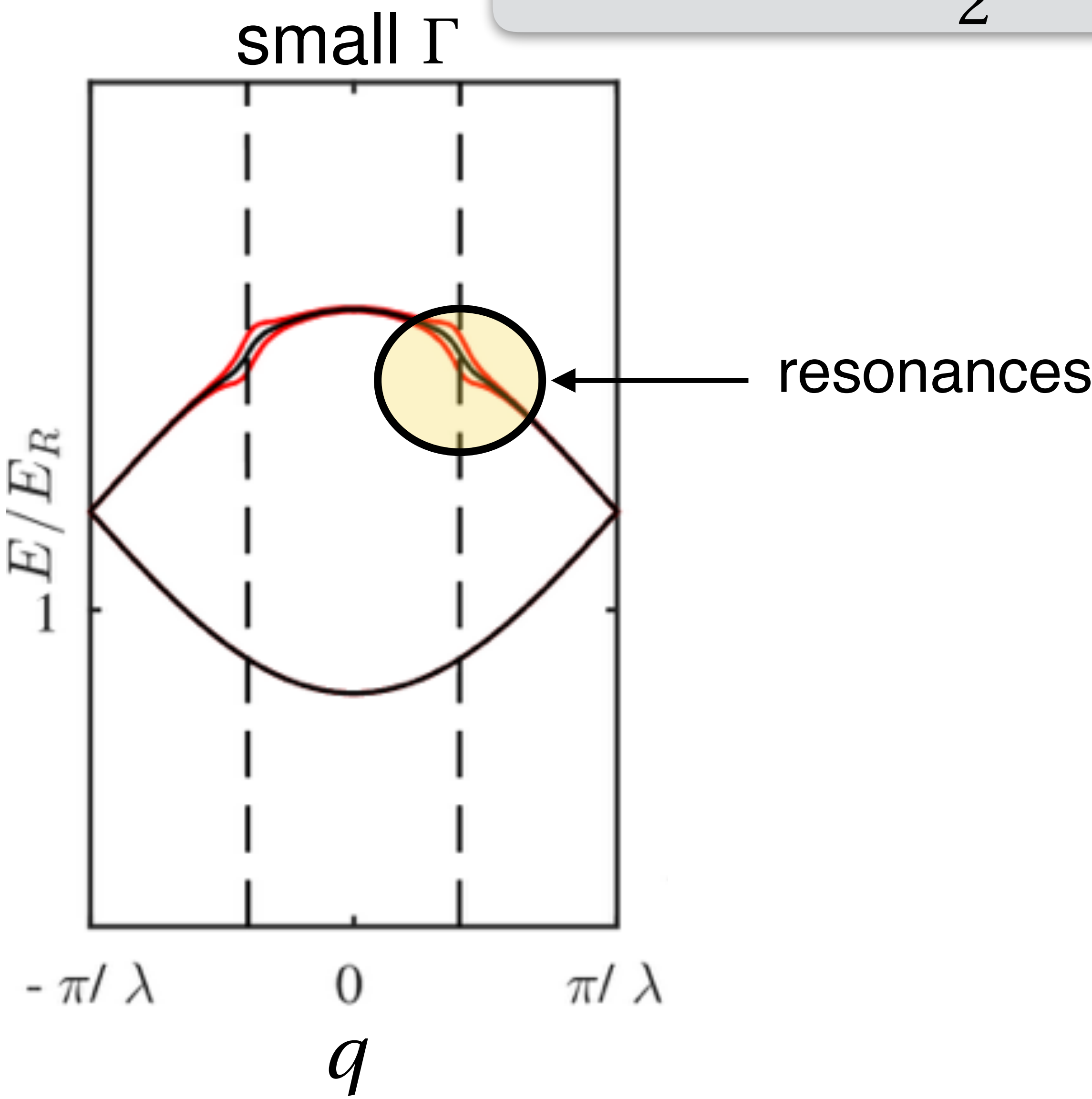


The faster the atom, the higher the losses

stable

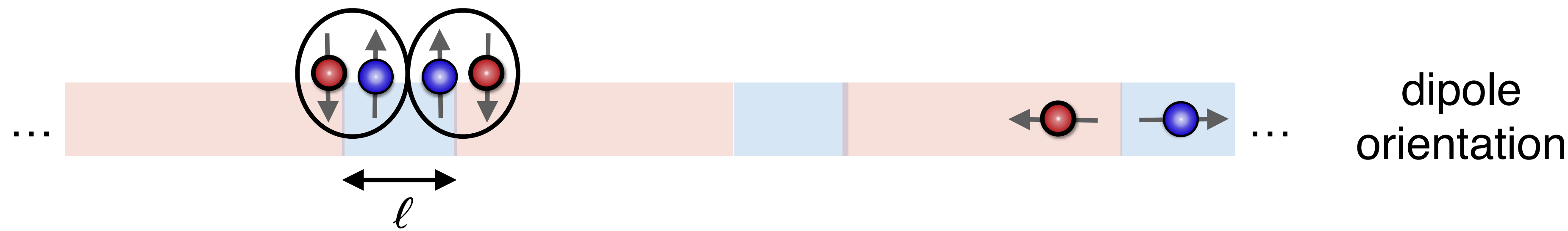
Lowest Blochband: Non-adiabatic Losses

$$E(q) = \epsilon(q) - i\frac{1}{2}\gamma(q)$$

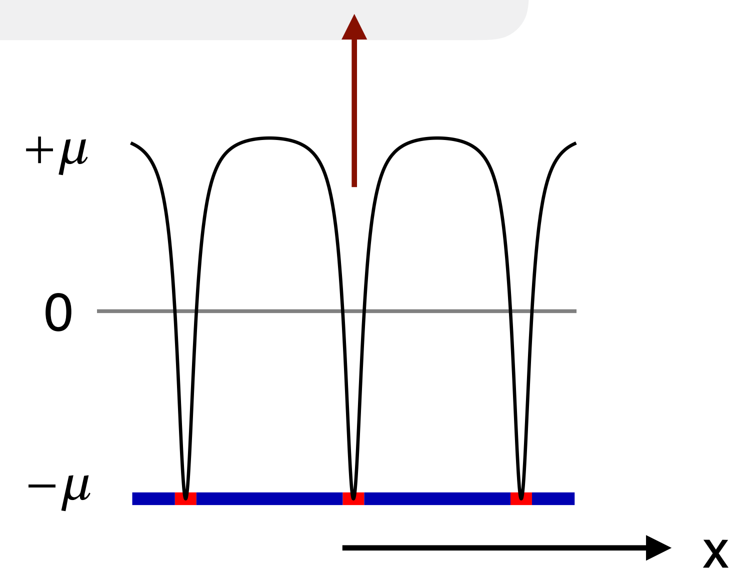


A few more topics & Outlook

1. Bound States - Domain Wall Molecules

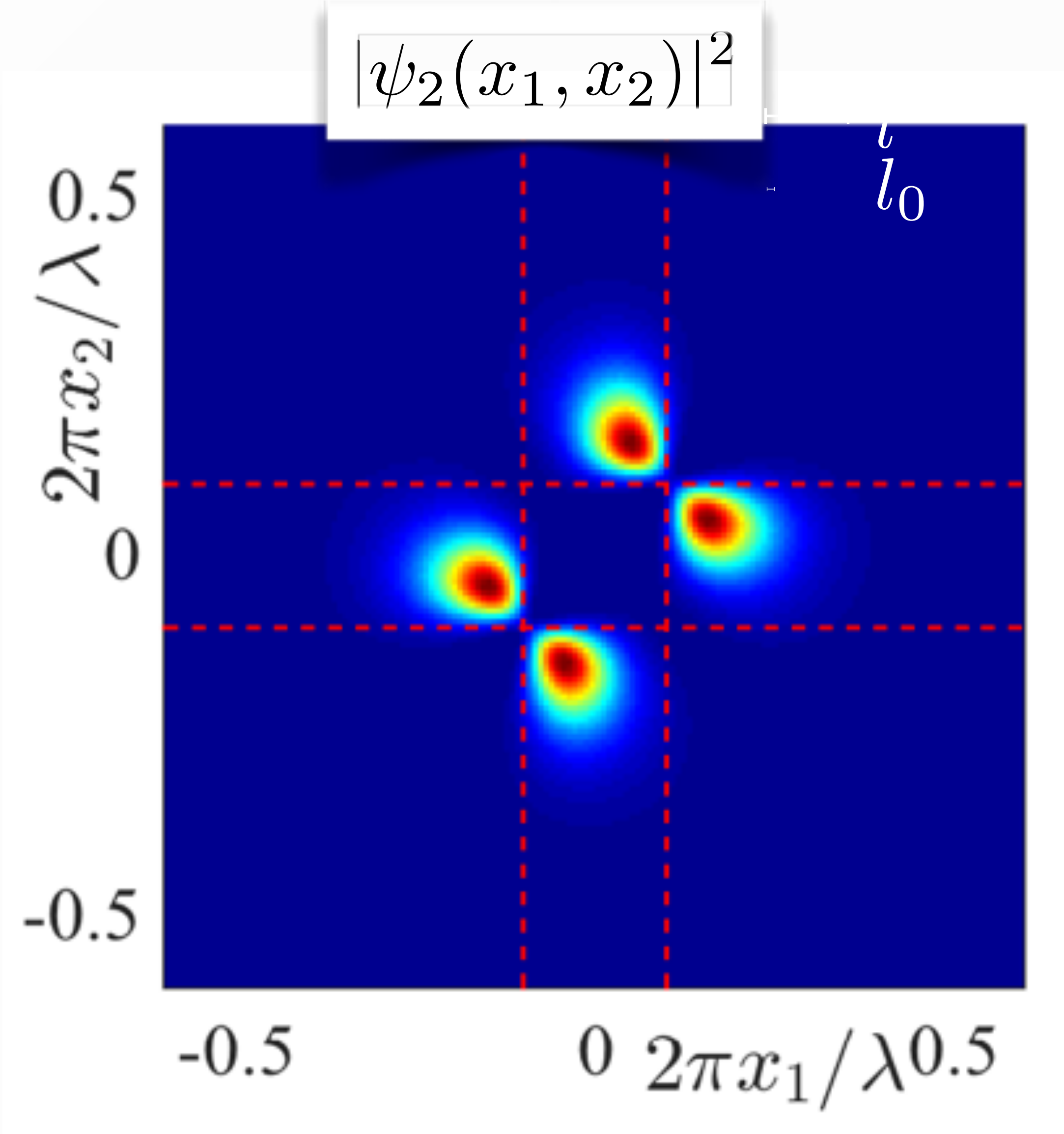


$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2} + V_{\mathrm{na}}(x_1) + V_{\mathrm{na}}(x_2) + \frac{\mu(x_1)\mu(x_2)}{|x_1 - x_2|^3}$$



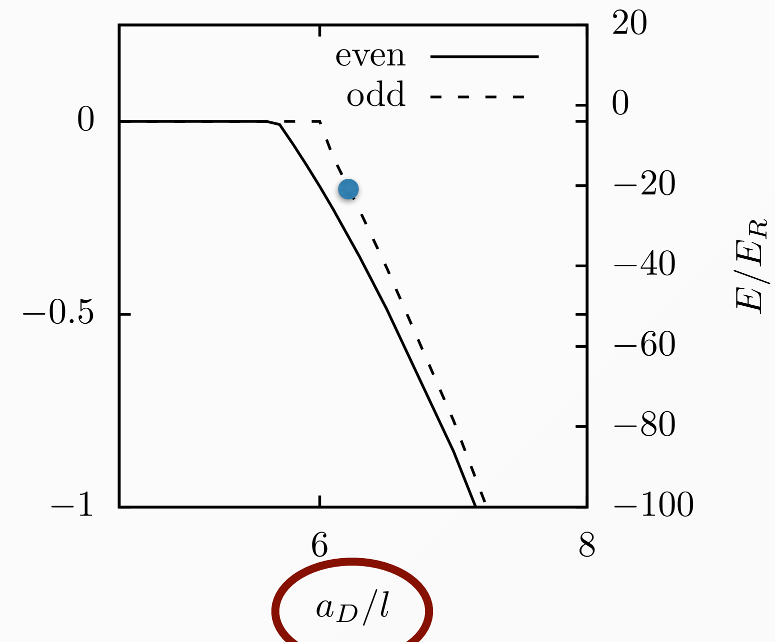
The bound states are symmetric and antisymmetric combination of pairs of particles bound at each interface

$$\left[-\frac{\partial^2}{\partial z_1^2}-\frac{\partial^2}{\partial z_2^2}+\frac{a_D\ell^2}{l_0^3\sqrt{2}}f(z_1)f(z_2)F\left(\frac{\ell(z_1-z_2)}{l_0\sqrt{2}}\right)+(k\ell)^2\frac{(\Omega_p\tilde{\Omega}_c')^2}{(\Omega_p^2+\tilde{\Omega}_c(z)^2)^2}\right]\psi(z_1,z_2)=(E/\frac{\hbar^2}{2m\ell^2})\psi(z_1,z_2)$$



$$l = \frac{\lambda}{2\pi}\epsilon$$

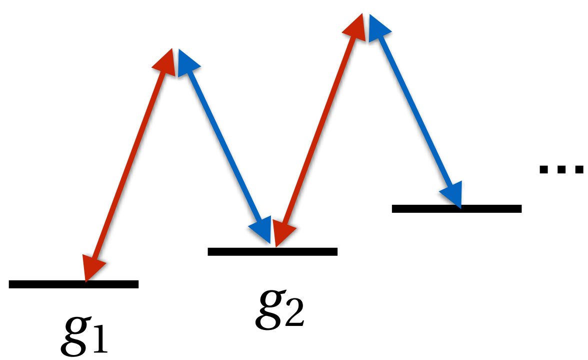
binding energy



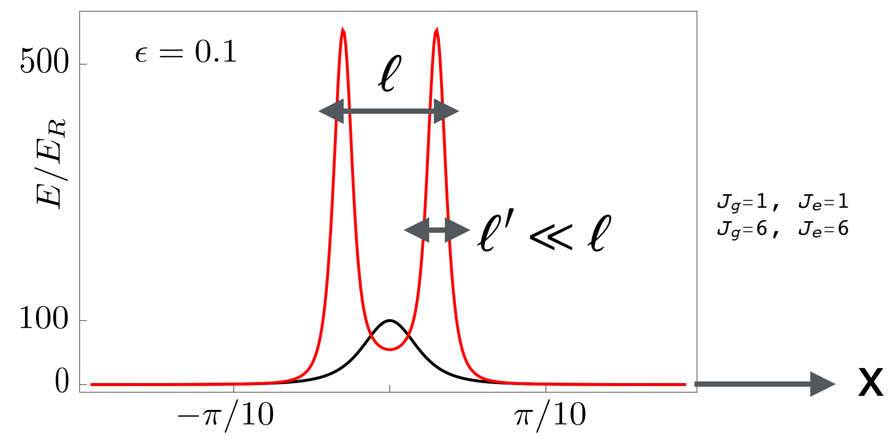
dipole length

2. Other atomic configurations

- **Zig-Zag atomic configuration**

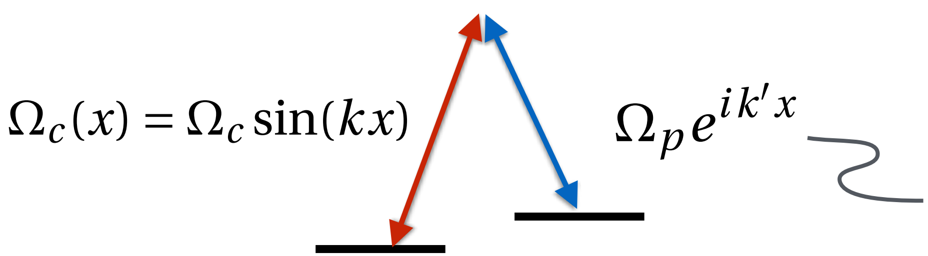


'atomic quantum dots'

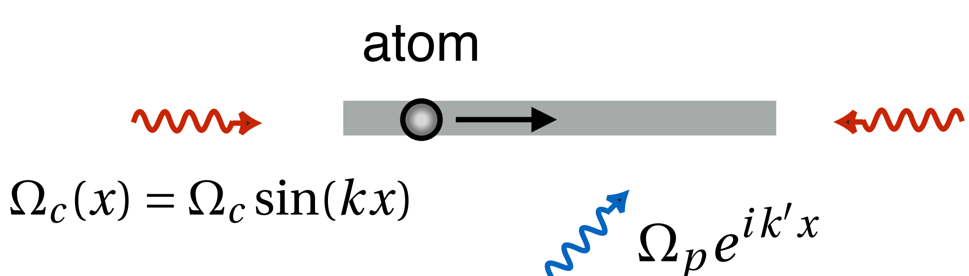


patch-work lattice

- **spin-orbit coupling**



compare: Spielman et al.



We get sub-wavelength spin-orbit structures