Generation & dynamics of solitonic defects: Kibble-Zurek in reduced dimensionality

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G. Lamporesi, G. Ferrari, S.-C. Gou, I. Carusotto , F. Dalfovo and N. P. Proukakis

616th WE-Heraeus Seminar, 12 May 2016
Bad Honnef - Germany

UNIVERSITÀ DEGLI STUDI DI TRENTO

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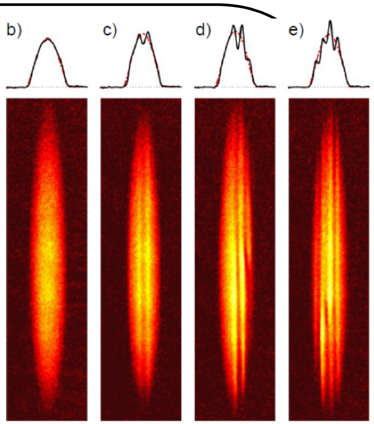
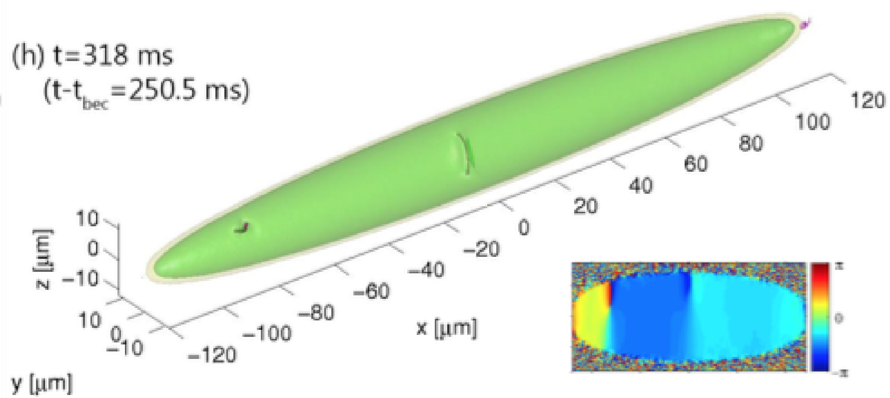
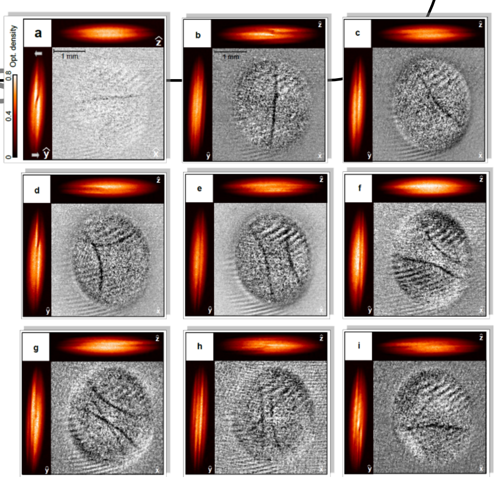
Outline

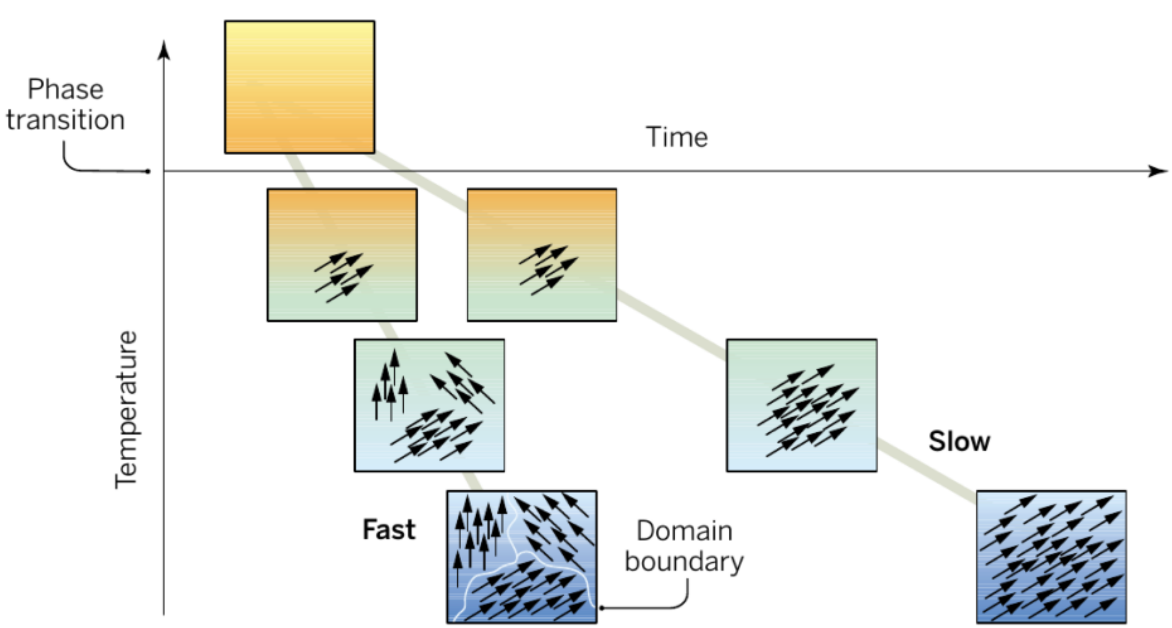
- Kibble-Zurek Mechanism;
- a relevant experiment (Nat. Phys. 9, 2013);
- solitons *vs* “solitonic” vortices (PRL 113, 201
- stochastic defect generation;
- solitonic vortex dynamics and interactions (PRL 115, 2015);
- q … orrections

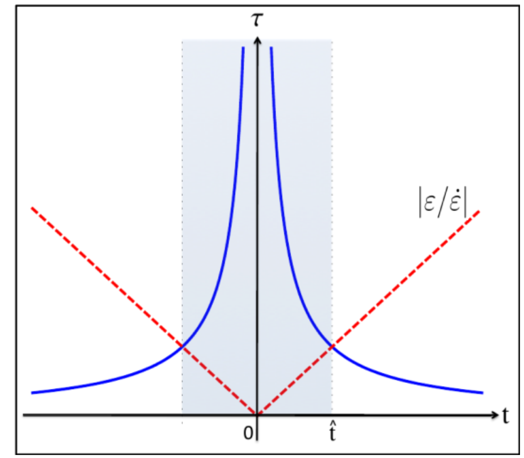






Kibble-Zurek Mechanism





$$\epsilon(t) = \frac{T - T_C}{T_C}$$

The system evolution slows down at the phase transition. Fast enough quenches could result in defect creation

Defect number $N_S \propto \tau_q^{-\alpha}$

ν, z equilibrium critical exponents

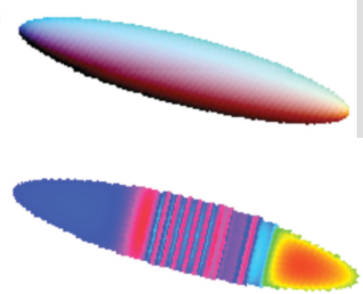
exponents

$$\xi(t) = \frac{\xi_0}{|\epsilon(t)|^{\nu}}$$

$$\tau(t) = \frac{\tau_0}{|\epsilon(t)|^{z\nu}}$$

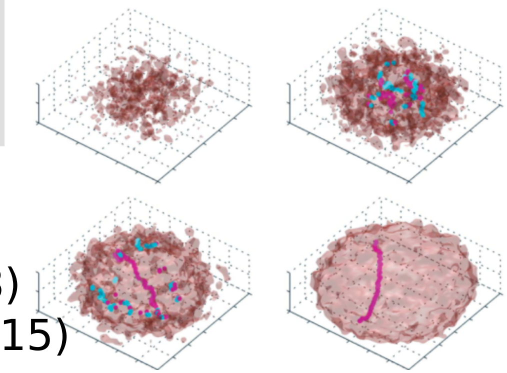
Kibble-Zurek in cold atoms

1D:
Soliton Formation



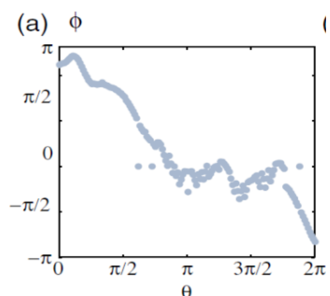
Zurek et al.
PRL 102, 105702 (2009)
PRL 104, 160404 (2010)
ngels / Schmiedmayer et al.

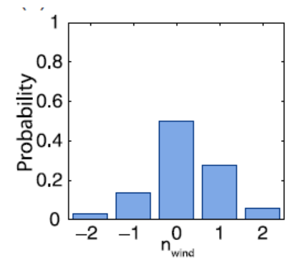
Quasi-2D / 3D:
Vortex Formation



Weiler-Davis et al.
Nature 455, 948 (2008)
Dalibard Nat. Comm. (2015)

Ring Trap:
Persistent Current





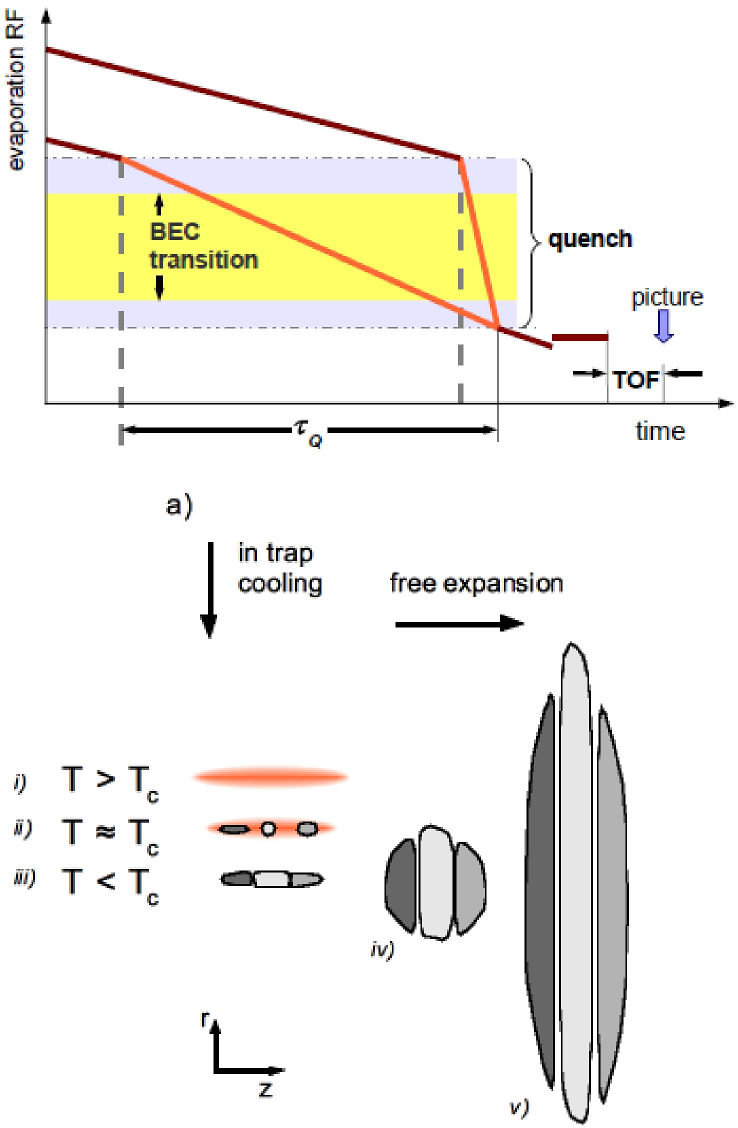
Experiment: Dalibard/Beugnon
PRL 113, 135302 (2014)

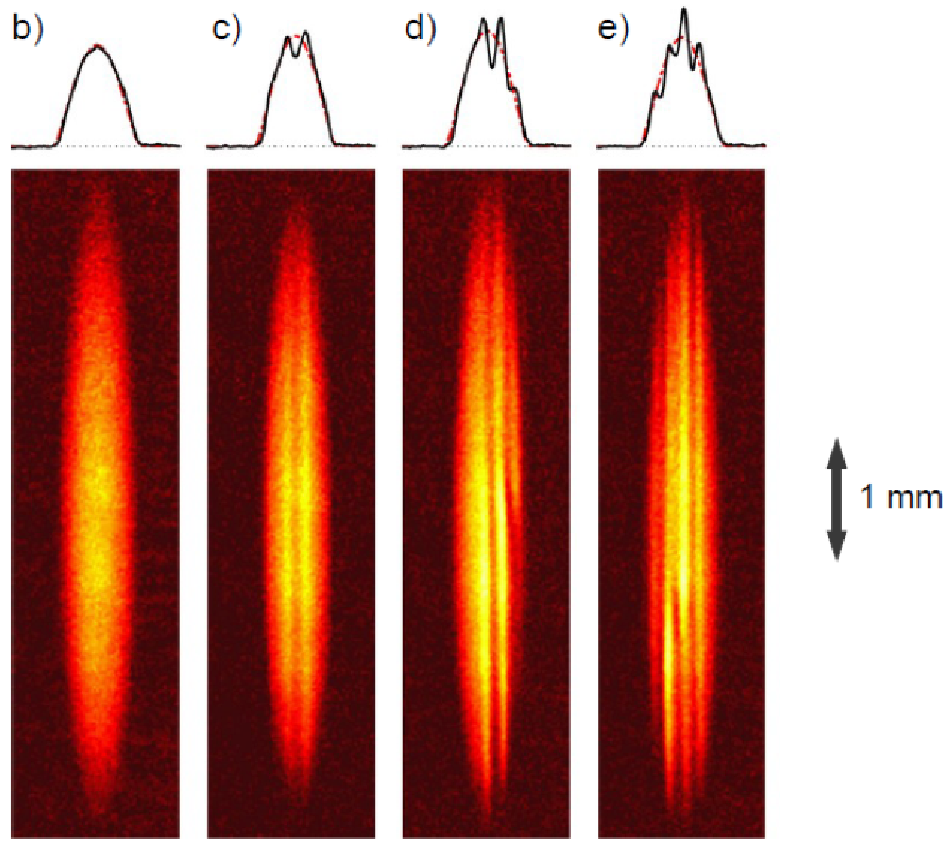
Brand et al.
PRL 110, 215302 (2013)
Davis et al
PRL 107, 230402 (2011)
Zurek et al. Sci. Rep (2012)

Critical Exponents Experimentally
Characterised in a Box-like Trap

Hadzibabic et al, Science 347
(2015)

Quench with different rates

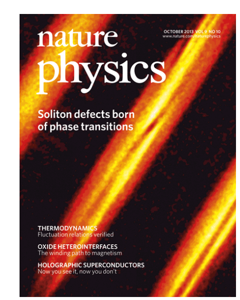




Slow → Fast

Solitons?

G.Lamporesi *et al.*, Nat. Phys. 9, 656 (2013)



The lifetime puzzle

Solitons in 3D are expected to undergo two kinds of instability:

- Thermal (unless at $T = 0$)
- Dynamical (snaking instability)

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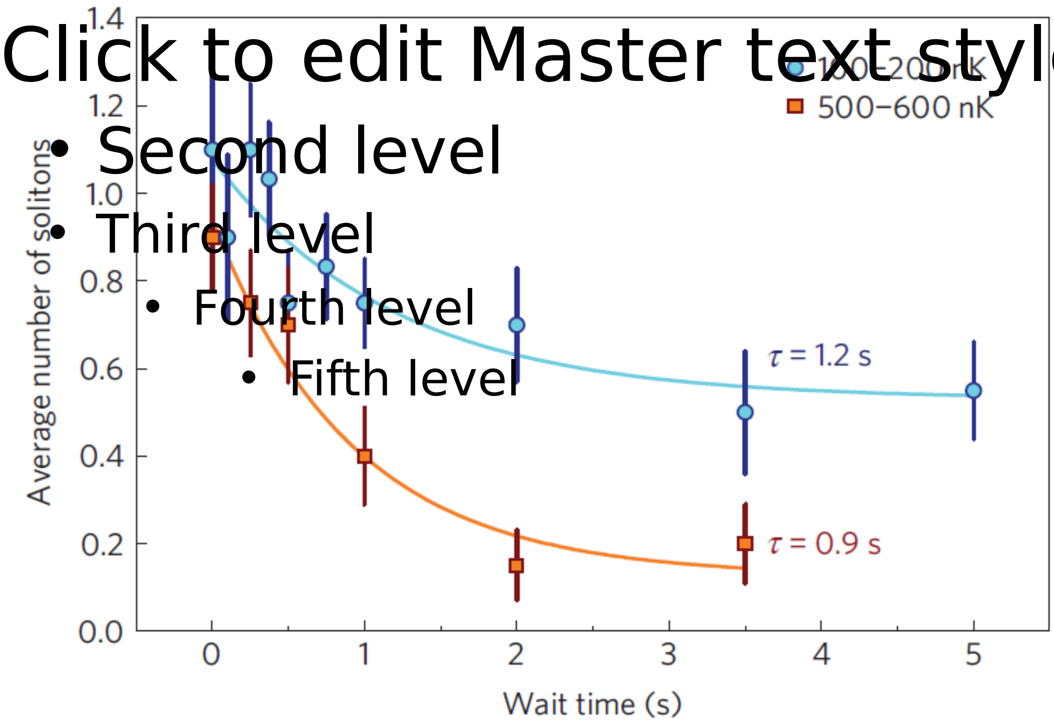
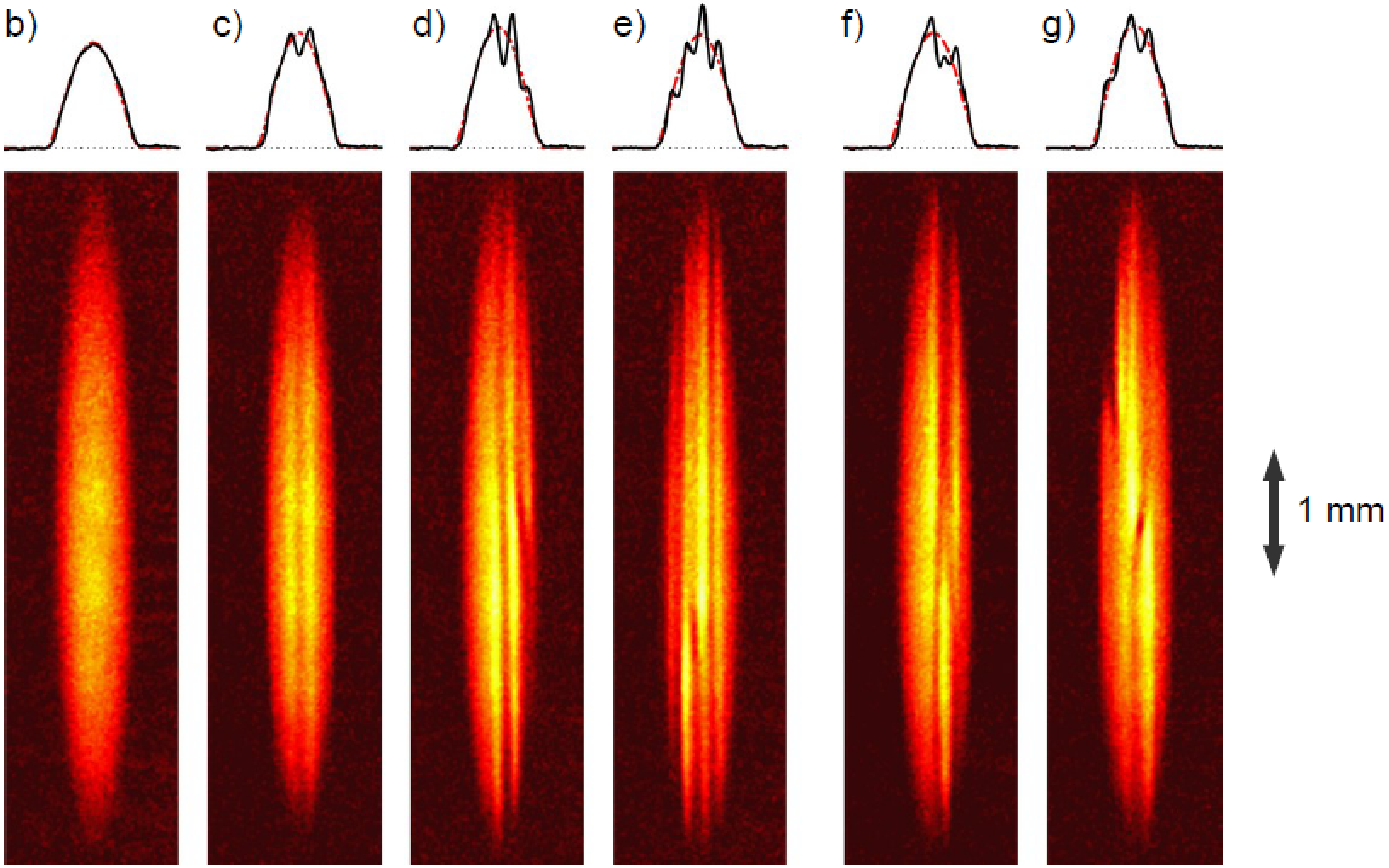


Why do they live for such a long time?

G.Lamporesi *et al.*, Nat. Phys. 9, 656 (2013)

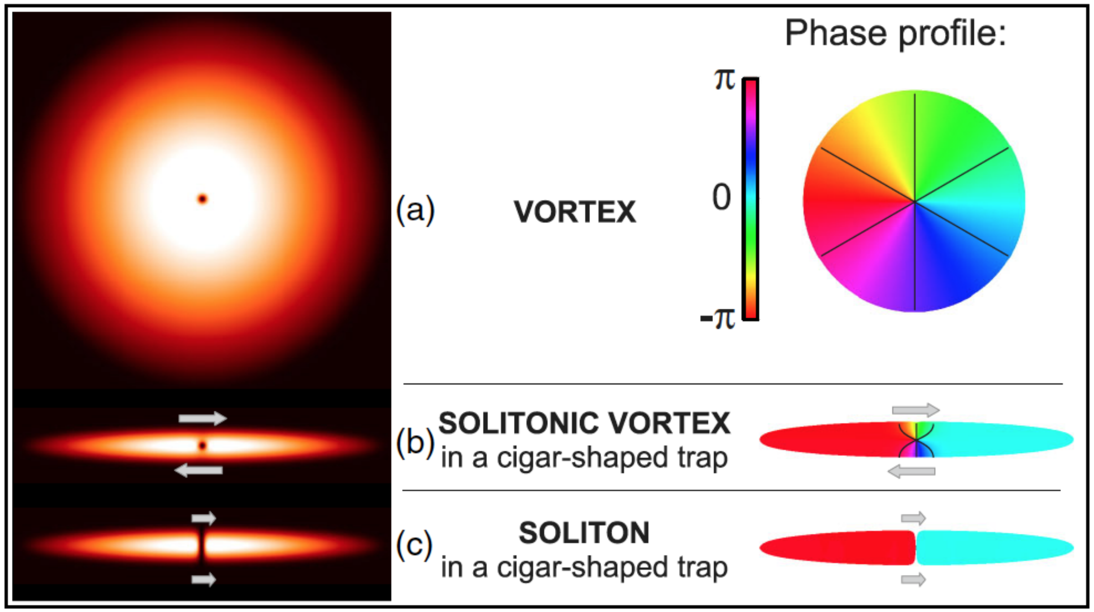
Are they really solitons?

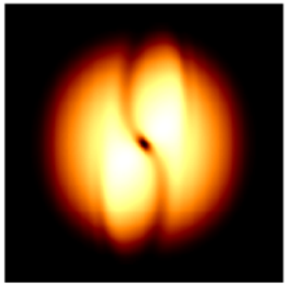


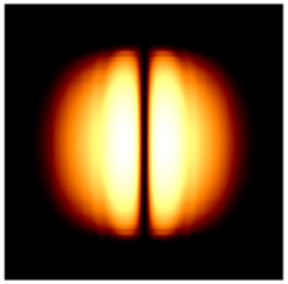
Solitonic vortices

- Vortex oriented perpendicularly to the axis of an axisymmetric elongated trap.
- Quantized vorticity
- Anisotropic phase pattern - Planar

(

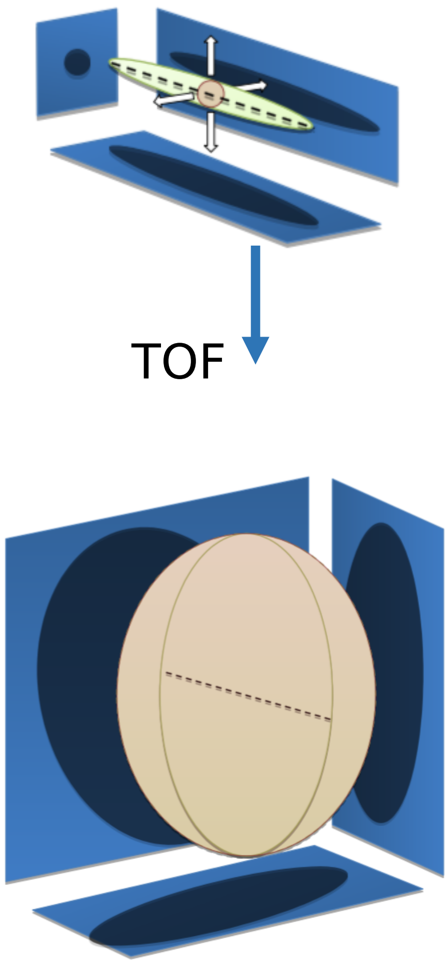






M. Tylutki *et al.*, EPJ-ST **224**, 577 (2015

Random orientation



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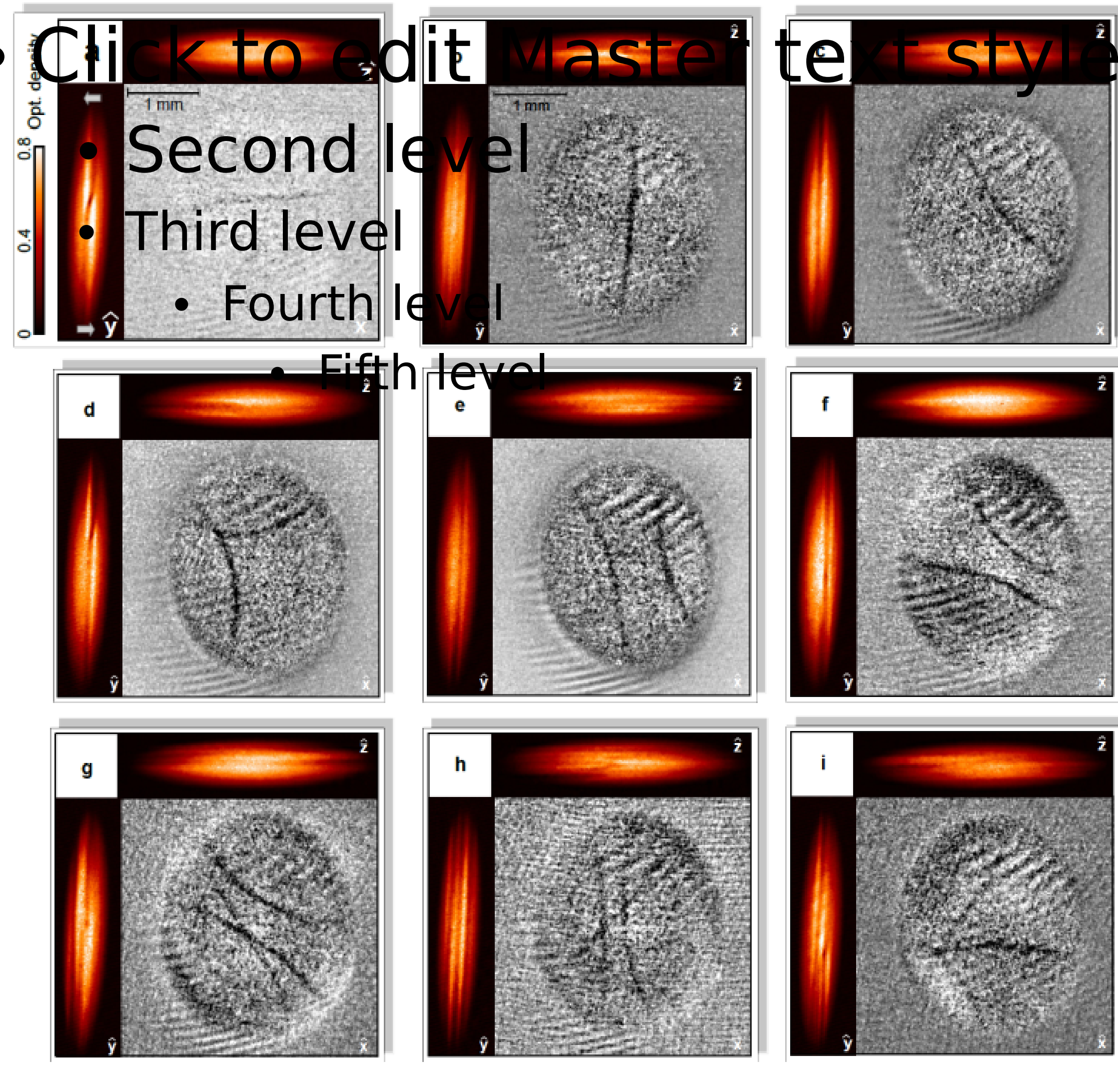
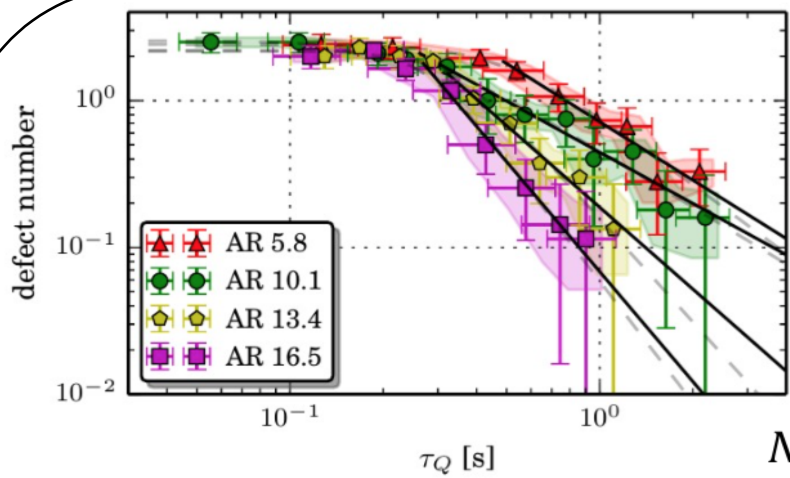
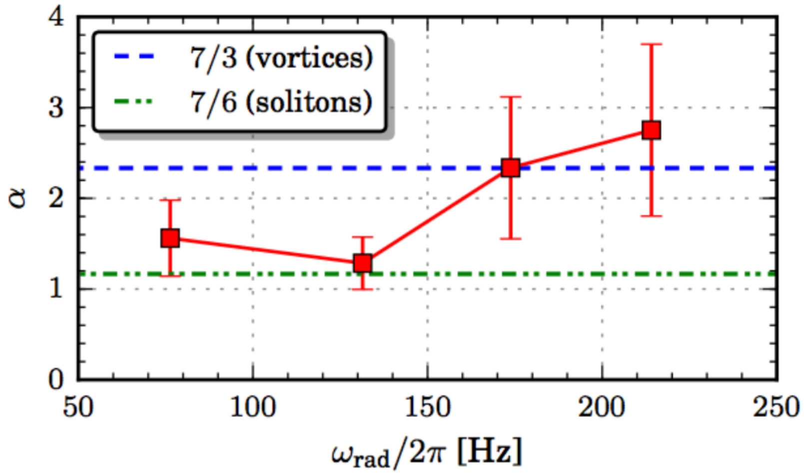


S. Donadello et al., Phys. Rev. Lett. **113**, 065302 (2

Scaling and aspect ratio

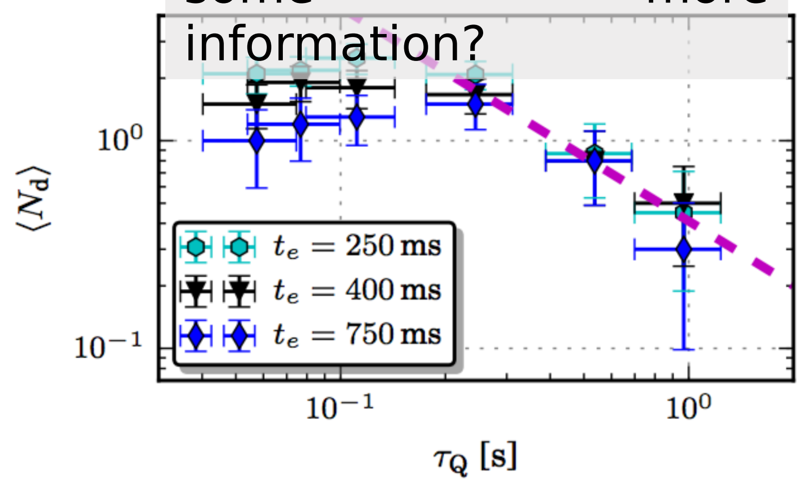




- power-law scaling for slow ramps
- aspect ratio dependent exponent
- at plateau for fast ramps
- plateau independent on aspect ratio

Experiments show some intriguing behaviour, how to get some more information?

$$N_S \propto \tau_Q^{-\alpha}$$



New on arXiv::1605.02982

Del Campo *et al.*, NJP **13**, 083022

Zurek, PRL **102**, 105702 (2009)

Simulations:

Stochastic Projected Gross-Pitaevskii equation

$$d\psi = \hat{P}\left\{-\frac{i}{\hbar}\hat{L}_0\psi + \frac{\gamma}{K_B T(t)}\left[\mu(t) - \hat{L}_0\right]\psi\right\}dt + dW$$

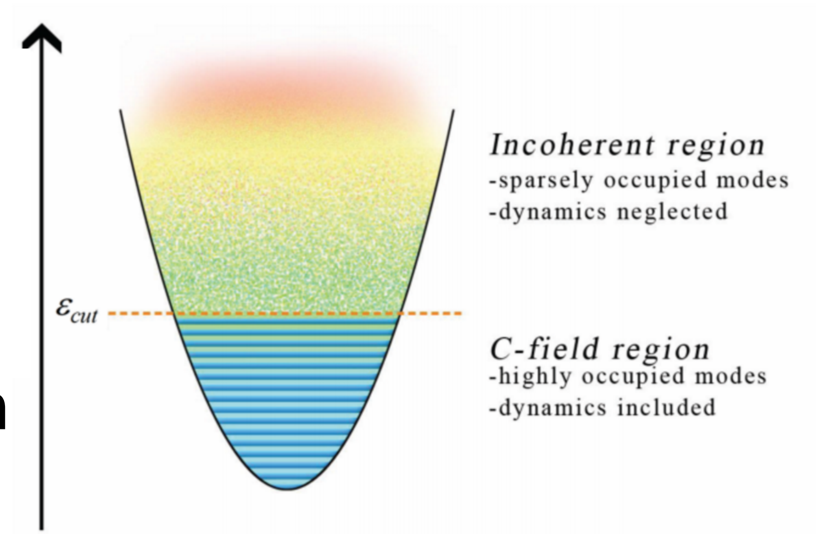
$$\hat{L}_0 = -\frac{\hbar^2\nabla^2}{2\mathrm{m}} + V(\boldsymbol{r}) + g|\psi|^2$$

$$\langle dW^*(\boldsymbol{r})dW(\boldsymbol{r}')\rangle = 2\gamma\delta(\boldsymbol{r}-\boldsymbol{r}')dt$$

Experimental data matched through:

$$N_{TOT}^{exp} = \int d^3r\left[|\psi|^2 + \int d^3p\, n_{E>E_R}(\boldsymbol{r},\boldsymbol{p})\right]$$

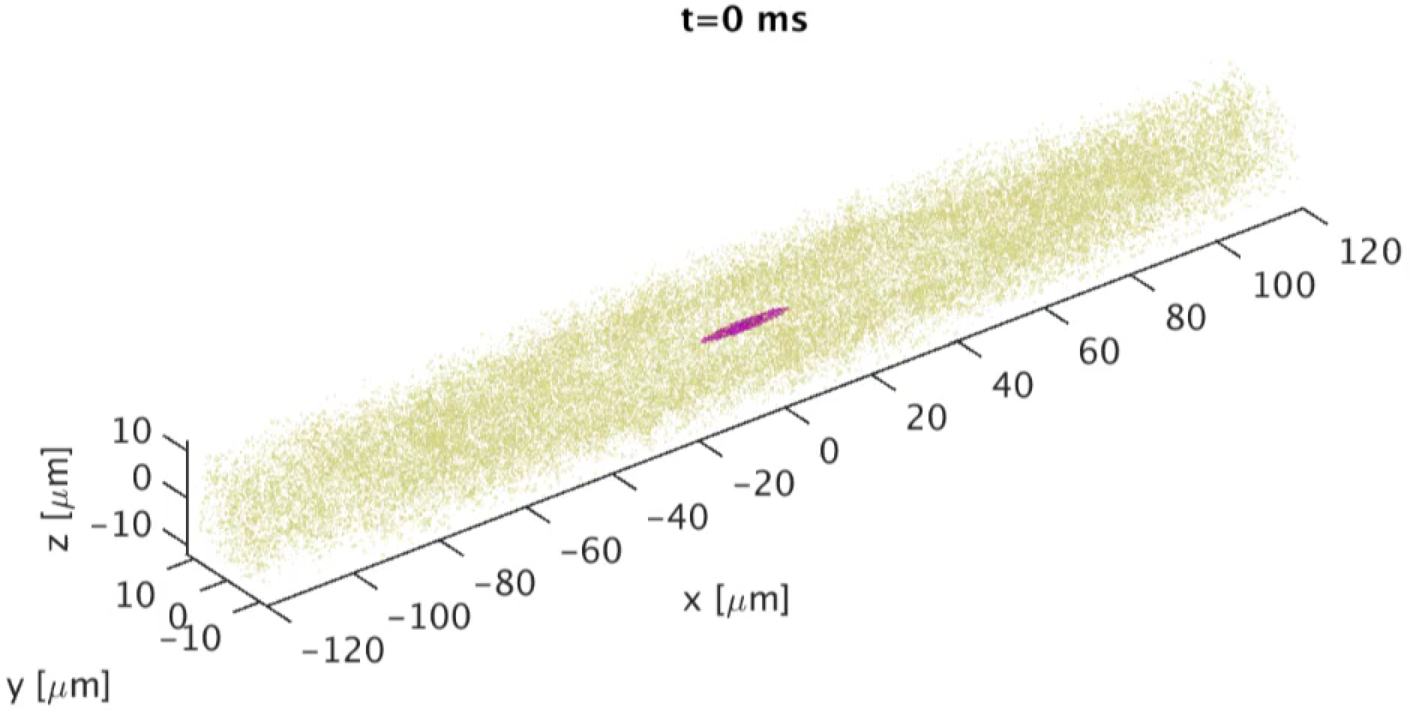
- ψ represents the condensate and a number of thermal modes.
- Condensate extracted by numerical diagonalisation [Penrose-Onsager].



numerical diagonalisation [Penrose-Onsager].

Blakie et al., Adv. in Phys., 57 (2008)

Quench:



-K. Liu et al, in preparation.

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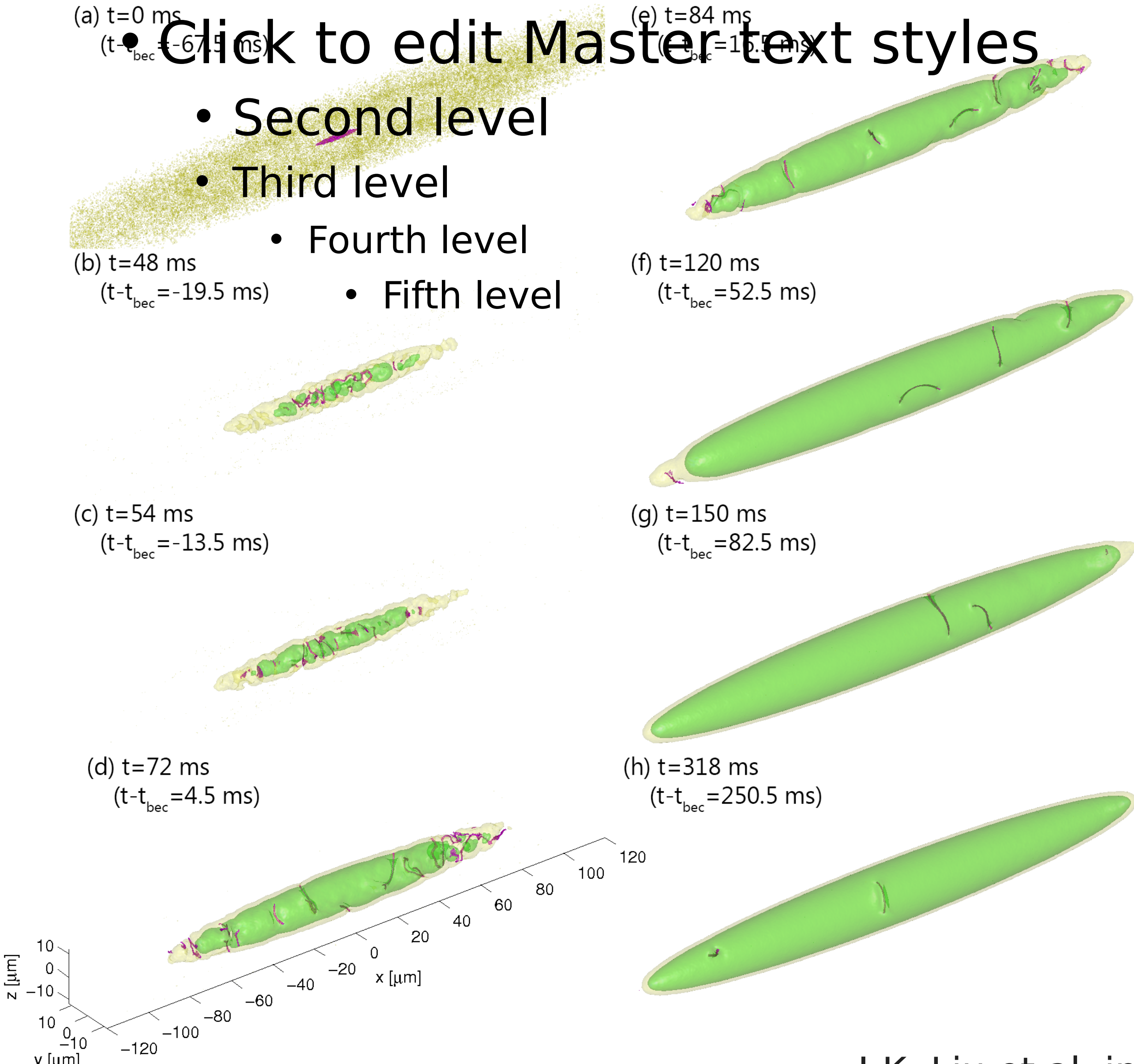
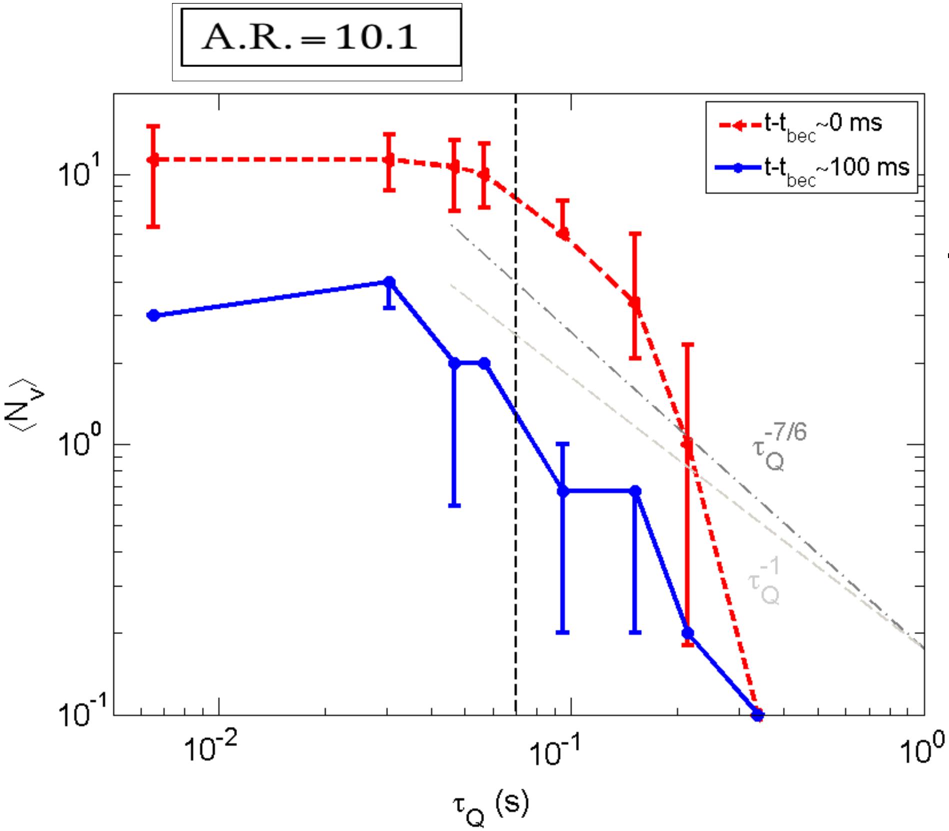


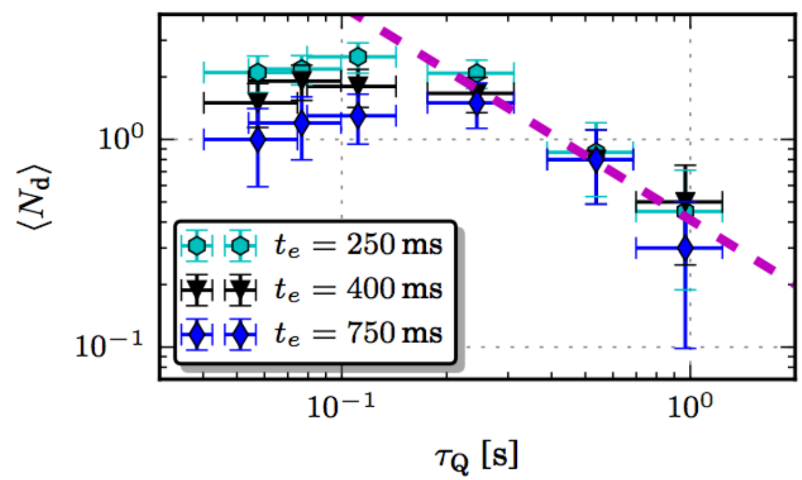
I-K. Liu et al, in preparatio

Scaling exponent for the defect density



Qualitative agreement with the experiments:

- Plateau for fast quenches
- Power decay for slower quenches
- Lowering with waiting time



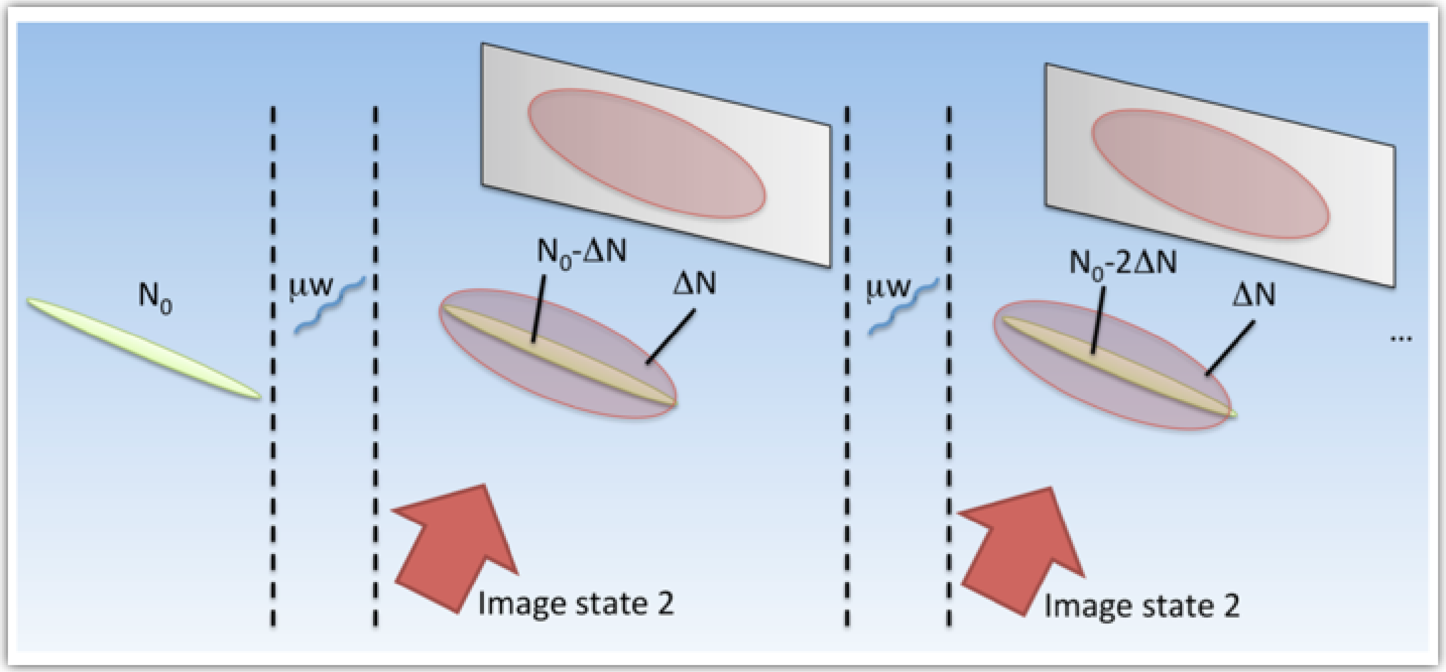
The number of defects goes down as the waiting time increases.

$$(t - t_{bec})^{exp} = 250\text{ms}$$

-K. Liu et al, in preparation.

Quasi-non destructive stroboscopic imaging:

$$\frac{\Delta N}{N_0} \approx 4\%$$
$$N_0 \approx 10^7$$



- Magnetic harmonic trap in $|1,-1\rangle$ with $\{\omega_\perp, \omega_z\}/2\pi = \{131{,}13\}$ Hz;
- 13 ms of expansion in $|2,-2\rangle$, with RF refocusing dressing;
- Up to 20 consecutive extractions.

Serafini et al., Phys. Rev. Lett. **115**, 170402

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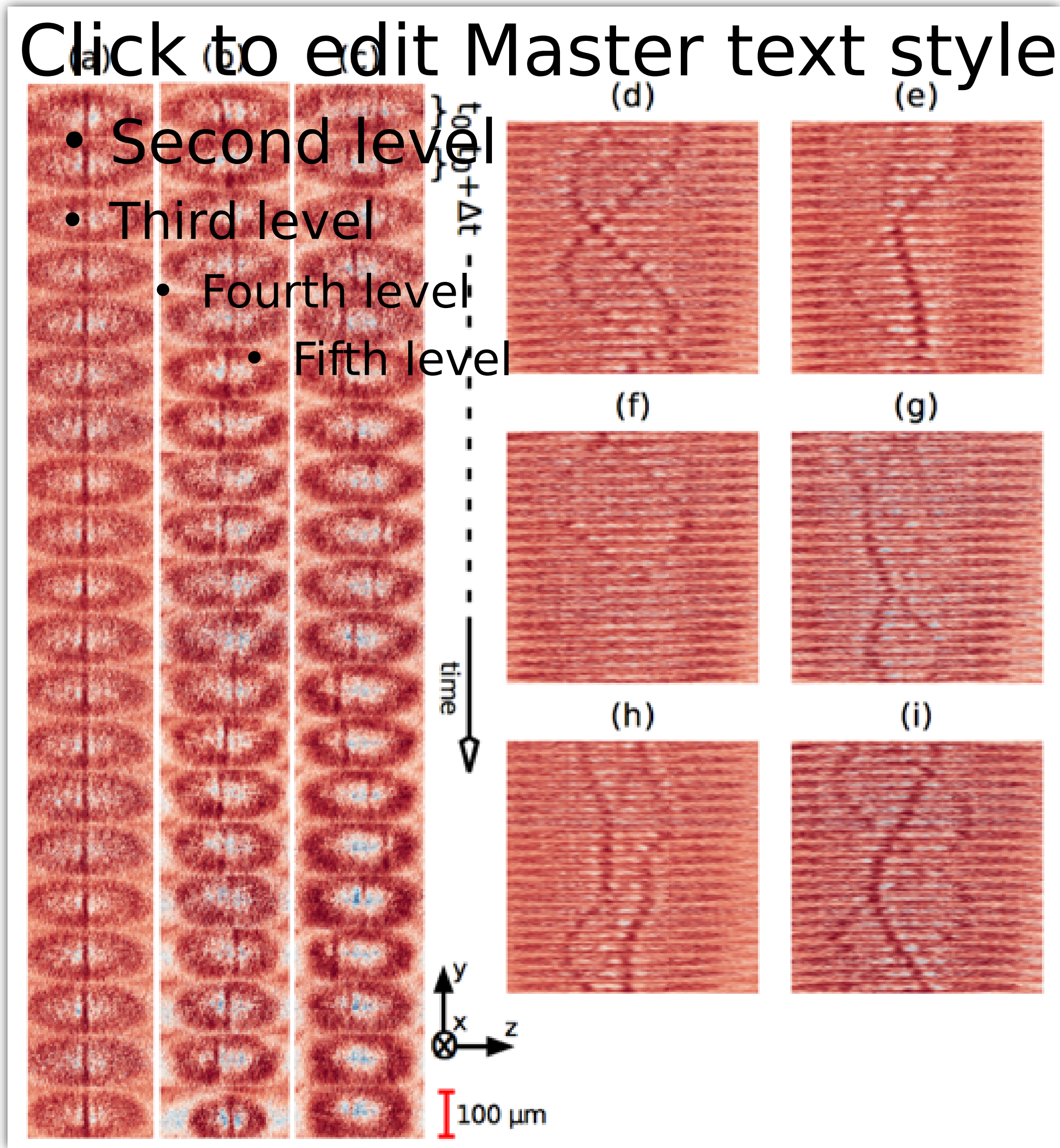


- Expansion in the anti-trapped state
- Optical levitation
- Imaging of the outcoupled part only

Serafini et al., Phys. Rev. Lett. **115**, 170402 (2015)

Vortex dynamics

- A straight vortex line should precess in an inhomogeneous non-rotating condensate
- It follows equipotential elliptical orbits around

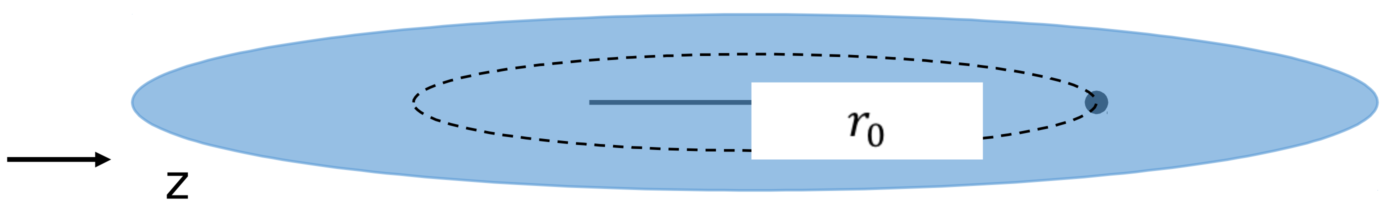
Orbital period:

$$T_{SV} = \frac{4(1 - r_0^2)\mu}{3\hbar\omega_\perp \ln(R_\perp/\xi)} T_z$$

being: $T_z = \frac{2\pi}{\omega_z}$ the axial trapping frequency;

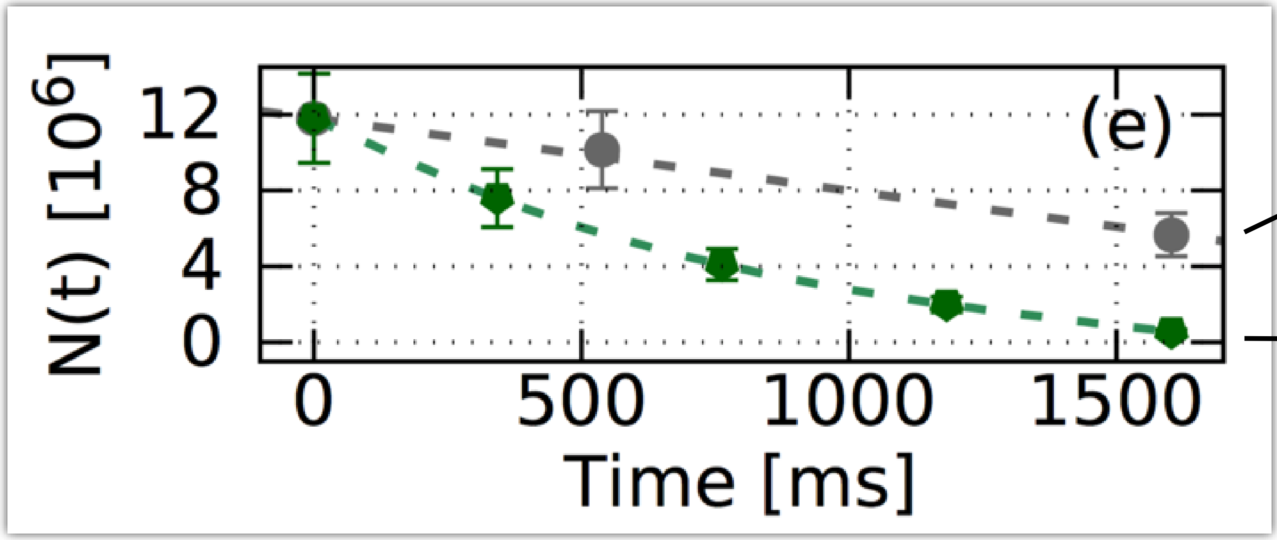
$r_0 = \frac{z_{max}}{R_z} = \frac{y_{max}}{R_y}$ the maximum amplitude;

ξ the condensate healing length.



Vortex dynamics

The extraction procedure changes the number of particles in time:



Decay without extraction

Decay with extractio

Hence, the period itself should depend on time:

$$T_{SV} \propto \mu \propto N(t)^{2/5}$$

Serafini et al., Phys. Rev. Lett. **115**, 1704

Period decay

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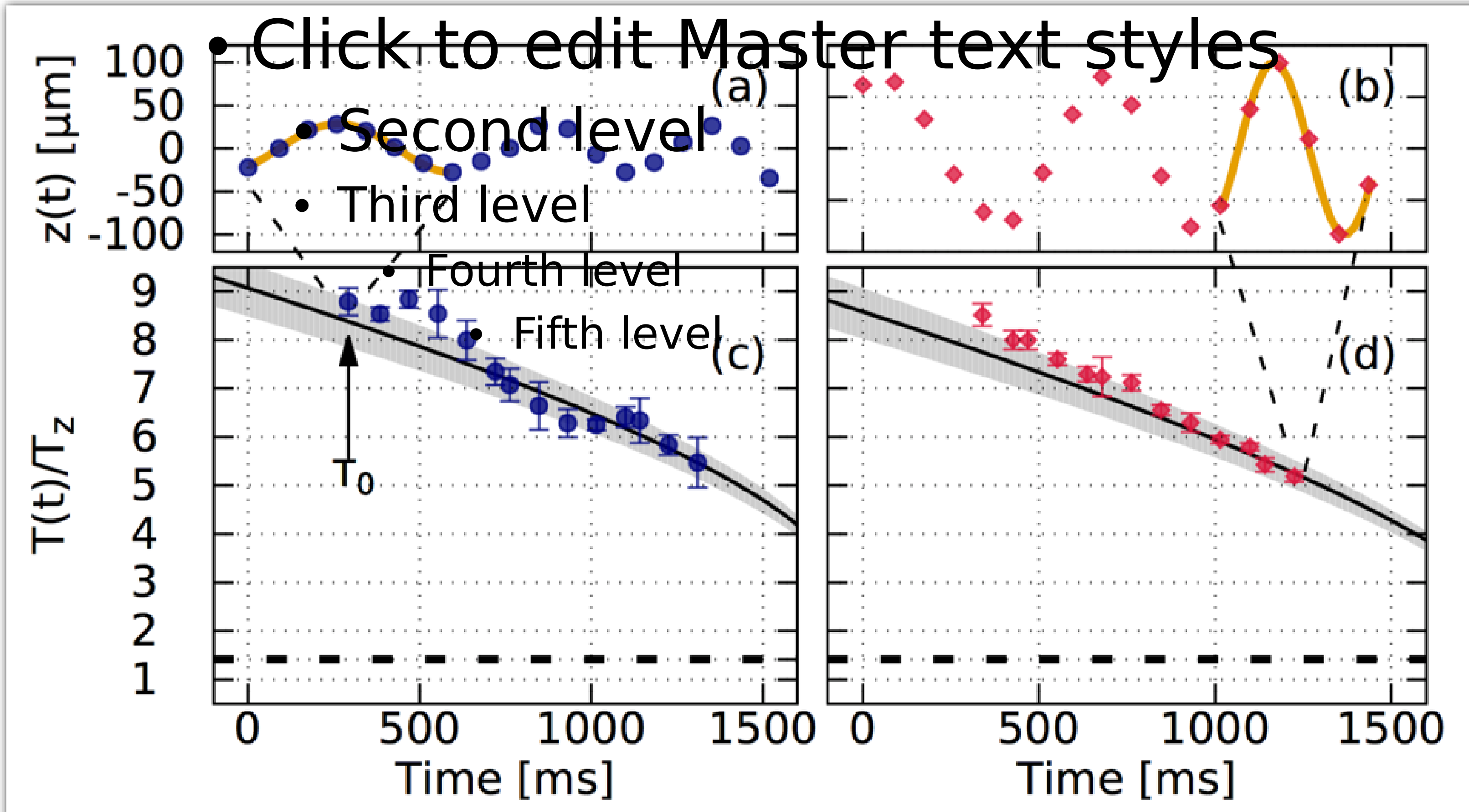

Serafini et al., Phys. Rev. Lett. **115**, 1704

Vortex interactions?

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Destructive absorption images show random orientation of vortex lines

The experimental system seems a good benchmark for studying in real time vortex decay processes and reconnections, if only an axial non-destructive observation method is developed.

Vortex crossings

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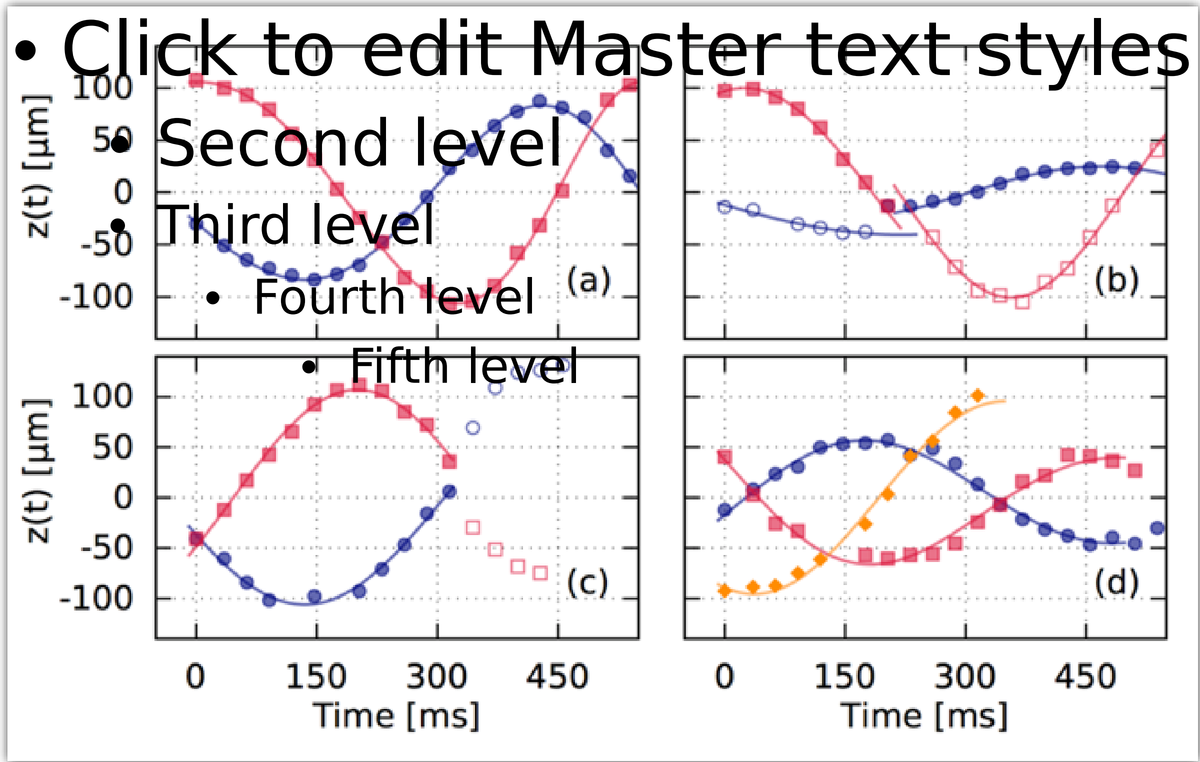
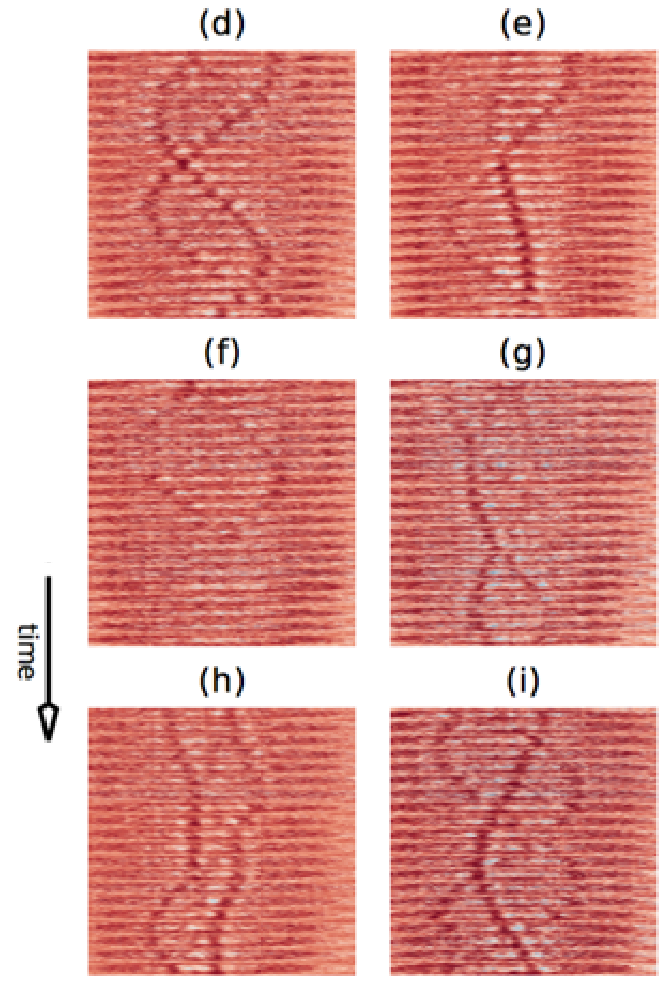
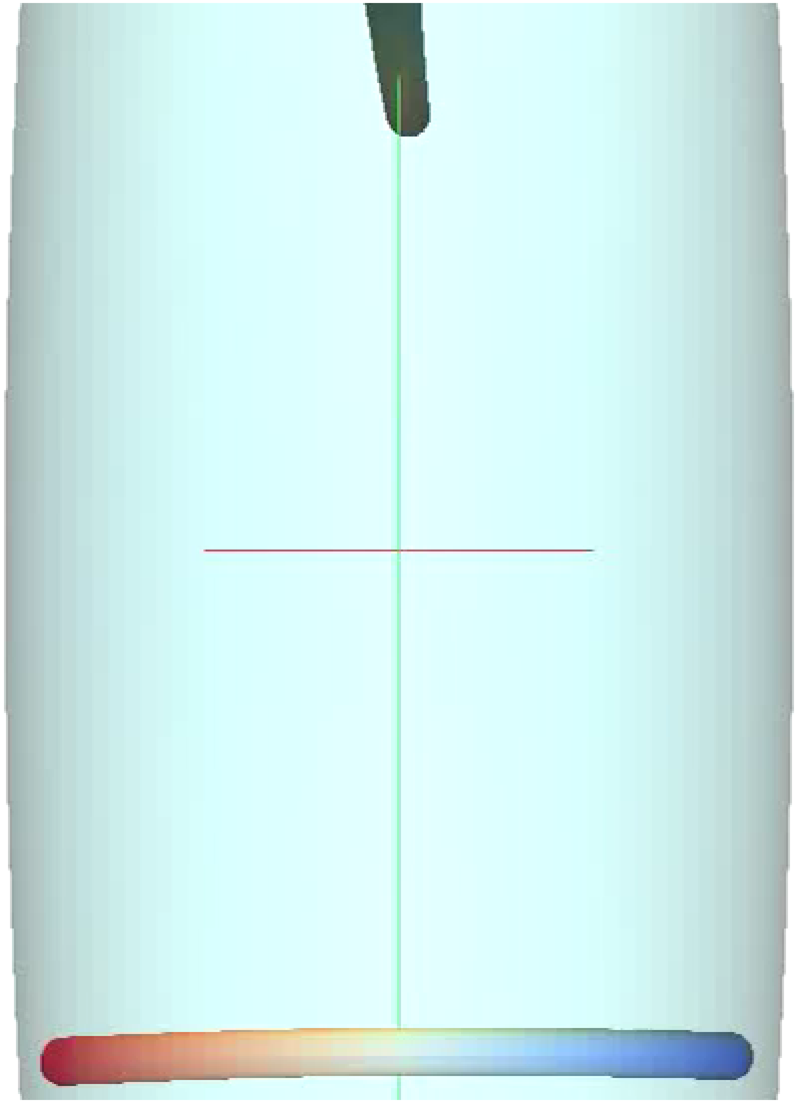


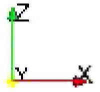


Observations:

- Unperturbed trajectory *or*
- Change in visibility *and/or*
- Trajectory phase shift.

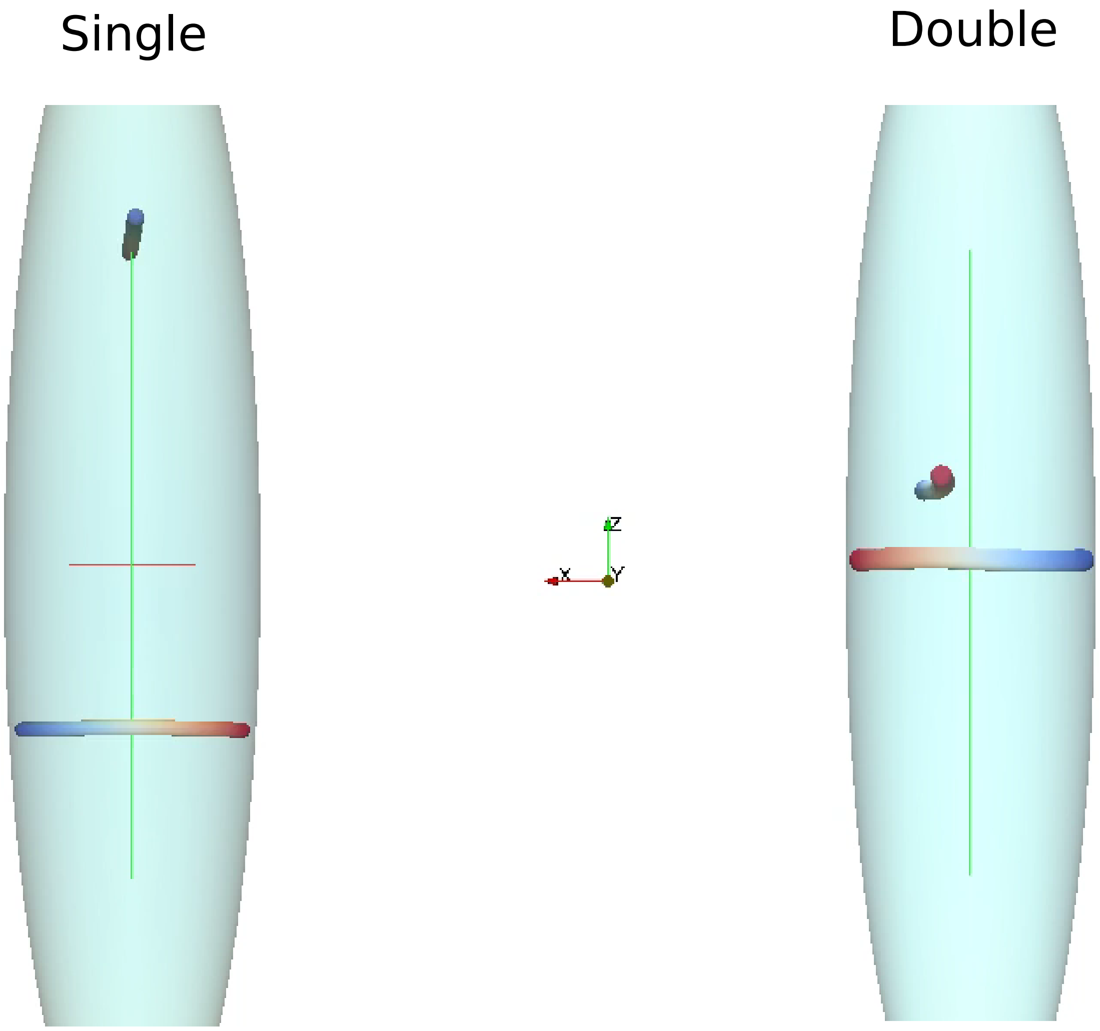
Bouncing





L. Galantucci and C. Barenghi, in

Reconnections



L. Galantucci and C. Barenghi, in

BEC Two-dimensional systems

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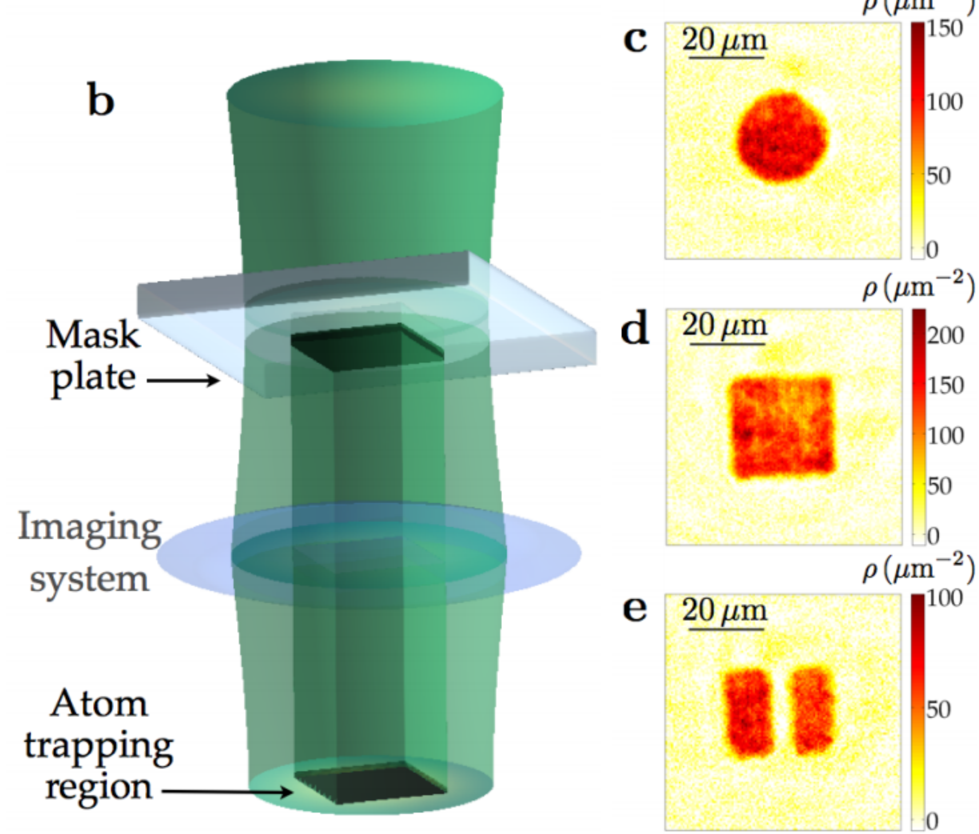
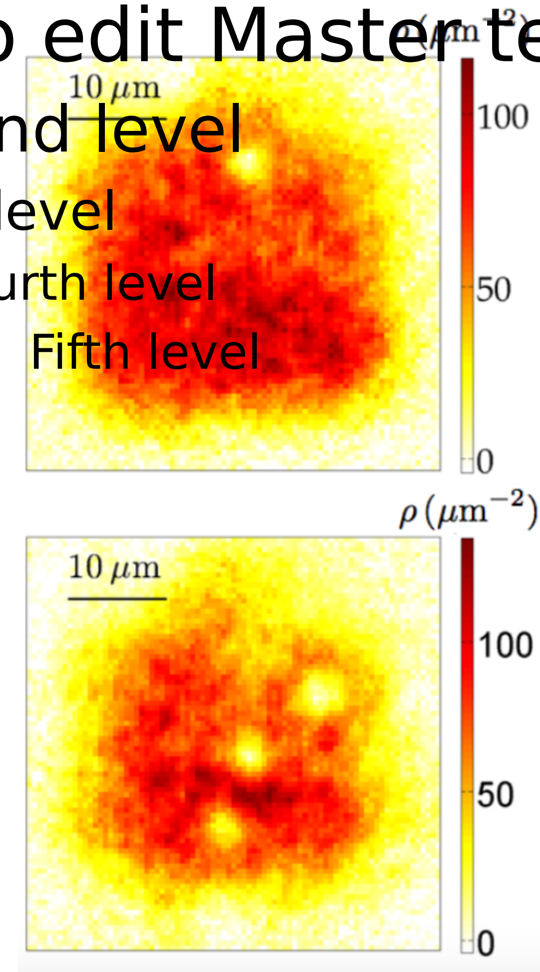
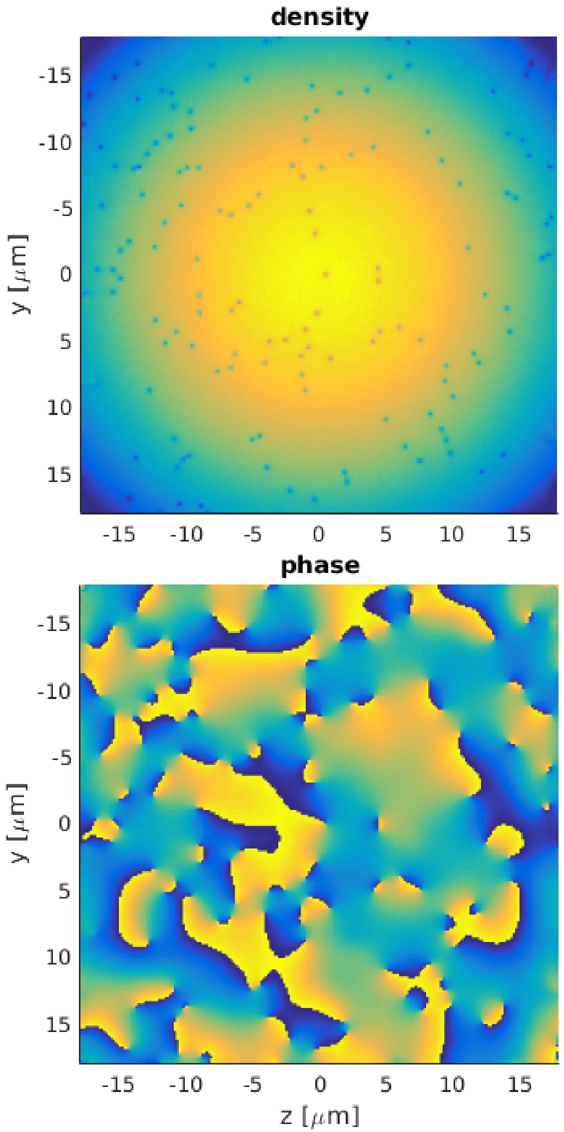




L. Chomaz et al., Nat. Comm. 6, 2015

Two-dimensional systems



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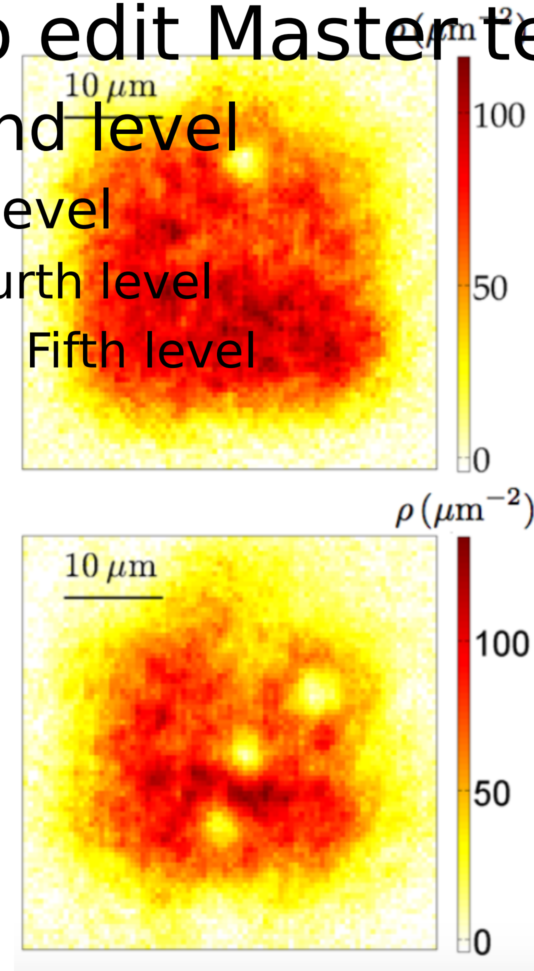
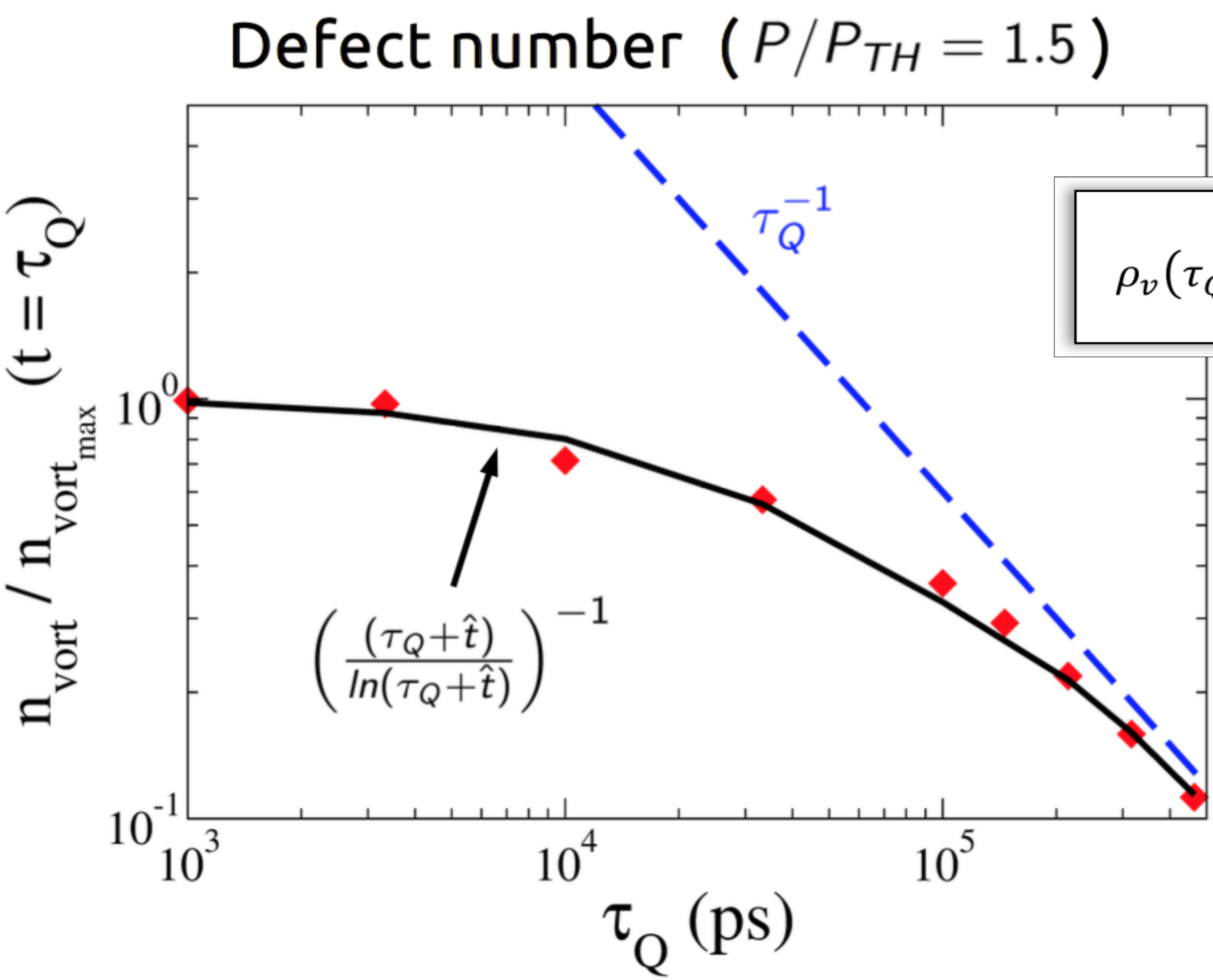


L. Chomaz et al., Nat. Comm. 6, 2015

Vortex decay vs τ_Q in polaritons



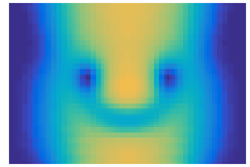
$$\rho_v(\tau_Q) \simeq \left(\lambda \, \frac{\tau_Q + \hat{t}}{\ln((\tau_Q+\hat{t})/t_0)}\right)^{-1}$$

Fit according to

P. Comaron et al., in

A. Jelić et al., J. Stat. Mech.

Thank you!



Happy soliton

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<u>Registration deadline: 30th June, 2016</u>

ultiple contributed talk slots available to applicants.

Backslides

Kibble-Zurek Mechanism

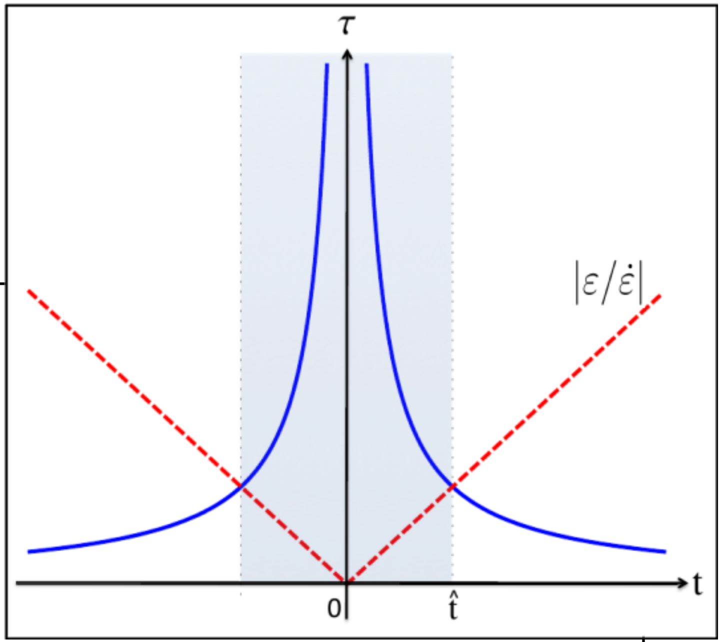
Power law scaling:

coherence length

$$\boxed{\xi(t) = \frac{\xi_0}{|\varepsilon(t)|^{\nu}}}$$

relaxation time

$$\boxed{\tau(t) = \frac{\tau_0}{|\varepsilon(t)|^{z\nu}}}$$



If the quench is linear

$$\varepsilon(t) = t/\tau_Q$$

"Freezing" time:

$$\hat{t} \sim \left(\tau_0 \tau_Q^{z\nu}\right)^{\frac{1}{1+z\nu}}$$

domain size:

$$\hat{\xi} = \xi(\hat{t}) = \xi_0 \left(\frac{\tau_Q}{\tau_0}\right)^{\frac{\nu}{1+z\nu}}$$

defect density:

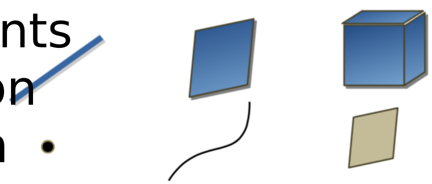
$$d \sim \hat{\xi}^{-D}$$

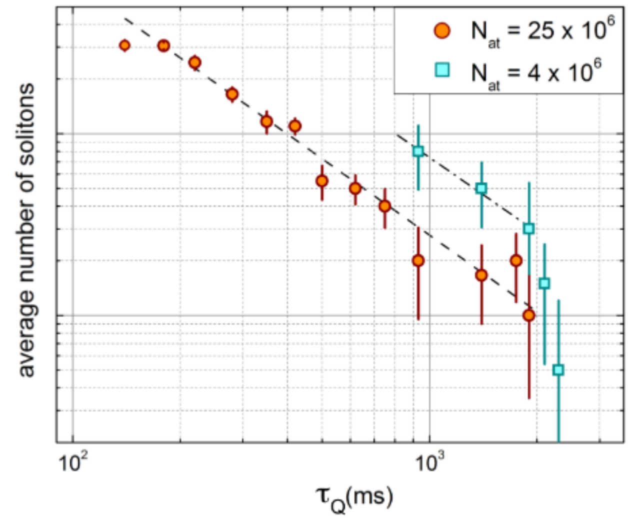
A. del Campo, W. H. Zurek, Int. J. Mod. Phys. A **29**, 1430018 (2014)

Scaling exponents

$$n \sim \frac{\hat{\xi}^d}{\hat{\xi}^D} = \frac{1}{{\xi_0}^{D-d}} \left(\frac{\tau_0}{\tau_Q}\right)^{(D-d)\frac{\nu}{1+z\nu}}$$

ν, z: critical exponents
D: system dimension
d: defect dimension





(D-d)=2				(D-d)=1	
Critical exponents \ Trap	Homogeneous	Harmonic	Toroidal	Homog.	Harm.
Arbitrary (ν, z)	$\frac{2\nu}{1+\nu z}$	$\frac{2(1+2\nu)}{1+\nu z}$	$\frac{1+3\nu}{1+\nu z}$	$\frac{\nu}{1+\nu z}$	$\frac{1+2\nu}{1+\nu z}$
Mean-field theory ($\nu = \frac{1}{2}$, $z = 2$)	$\frac{1}{2}$	2	$\frac{5}{4}$	1/4	1
Experiments/F model ($\nu = \frac{2}{3}$, $z = \frac{3}{2}$)	$\frac{2}{3}$	$\frac{7}{3}$	$\frac{3}{2}$	1/3	7/6

→ Dimensionality has a role in scaling!

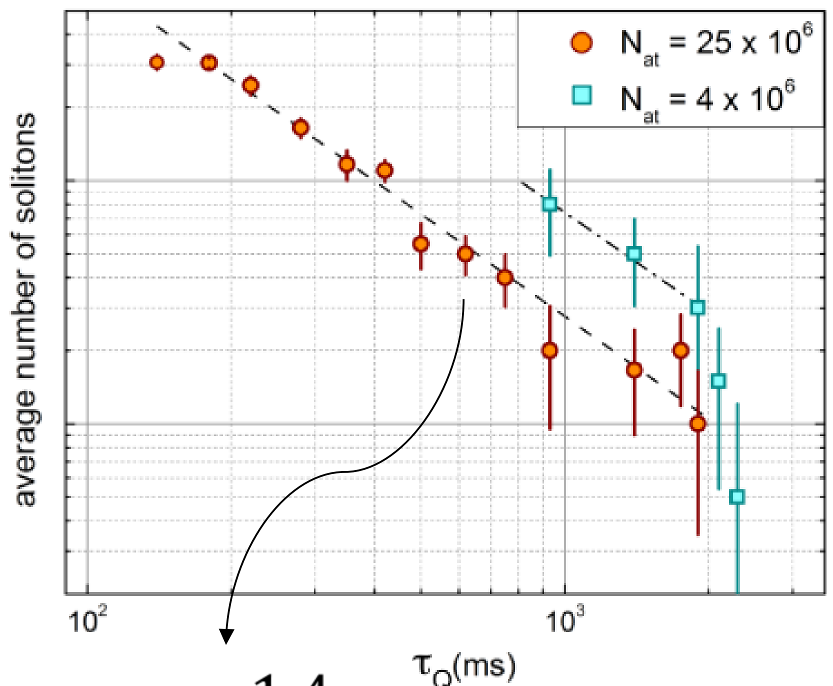
Del Campo *et al.*, NJP **13**, 083022 (2011)
Zurek, PRL **102**, 105702 (2009)

Scaling exponent

- The number of defects is expected to follow a power-law as a function of the quench time (fixed size of the system)

$$N_S \propto \tau_Q^{-\alpha}$$

where α is determined by the critical exponents of the phase transition.



$\alpha = 1.4$

F-model prediction
for solitons in 3D:

$\alpha = 7/6 = 1.17.$

G.Lamporesi *et al.*, Nat. Phys. 9, 656 (2013)

Zurek, W. H. Phys. Rev. Lett. 102, 105702 (2009)

Other period characterisations

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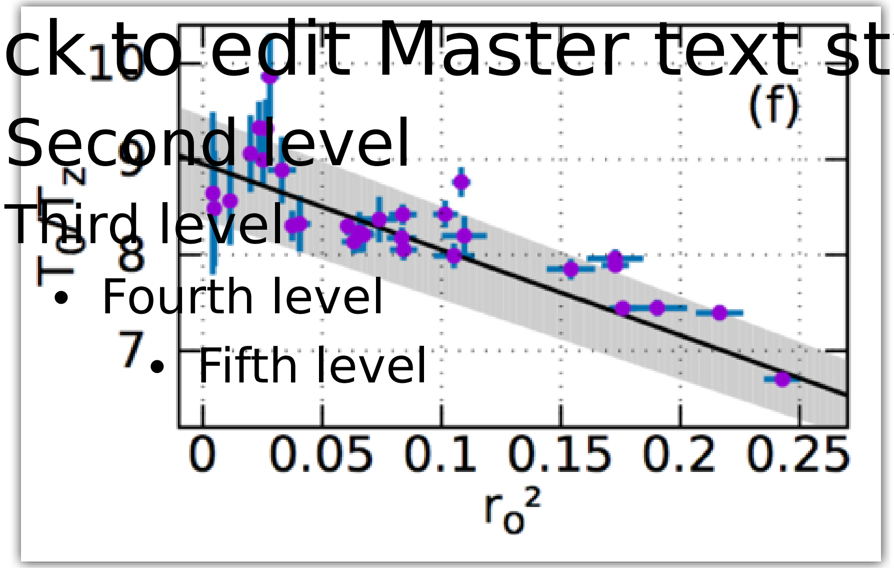
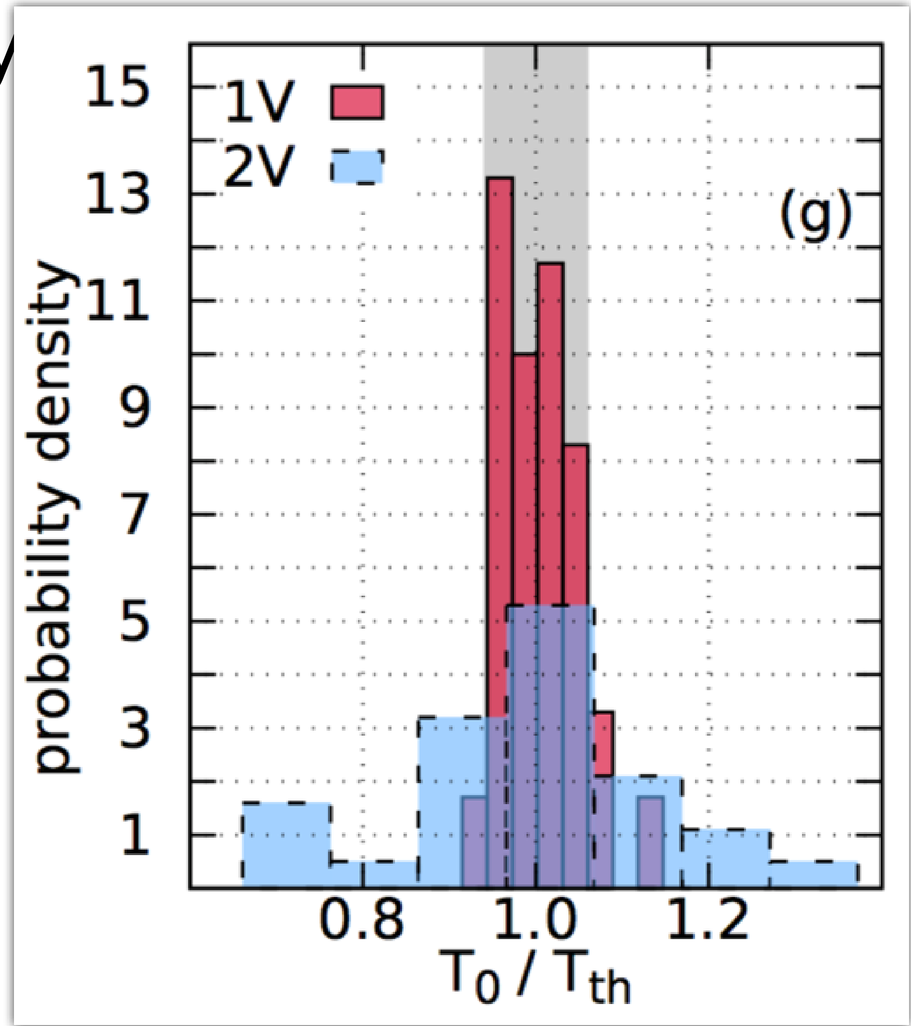
- Second level
- Third level
  - Fourth level
    - Fifth level



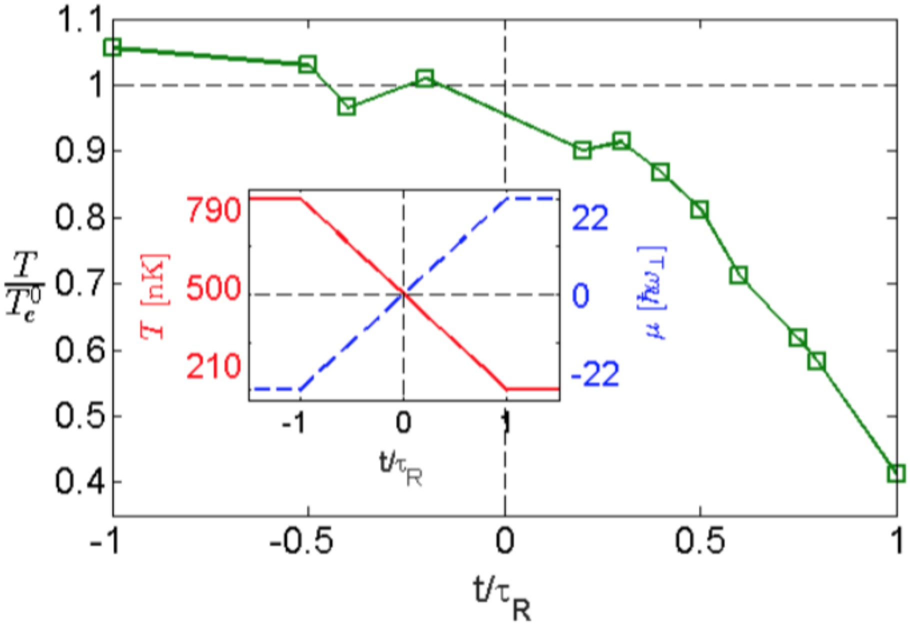
$$\frac{T_{SV}}{T_z} = \frac{4(1-r_0^2)\mu}{3\hbar\omega_\perp \ln(R_\perp/\xi)}$$

$$N_V = 1: \quad \frac{T_0}{T_{th}} = 0.97 \pm 0.04$$

$$N_V = 2: \quad \frac{T_0}{T_{th}} = 0.96 \pm 0.14$$

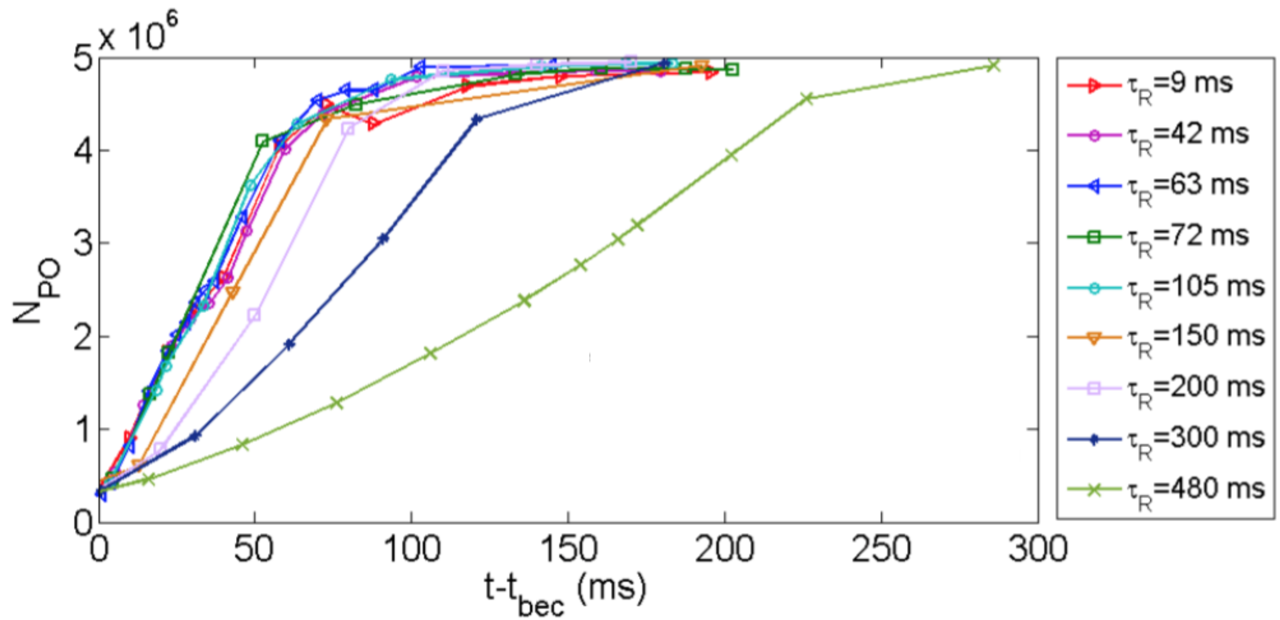


Serafini et al., Phys. Rev. Lett. **115**, 1704



A quench is performed simultaneously in temperature and in chemical potential (inset).

$$n_{PO}(t_{BEC}) = 5\%\, n_{PO}(t_{final})$$

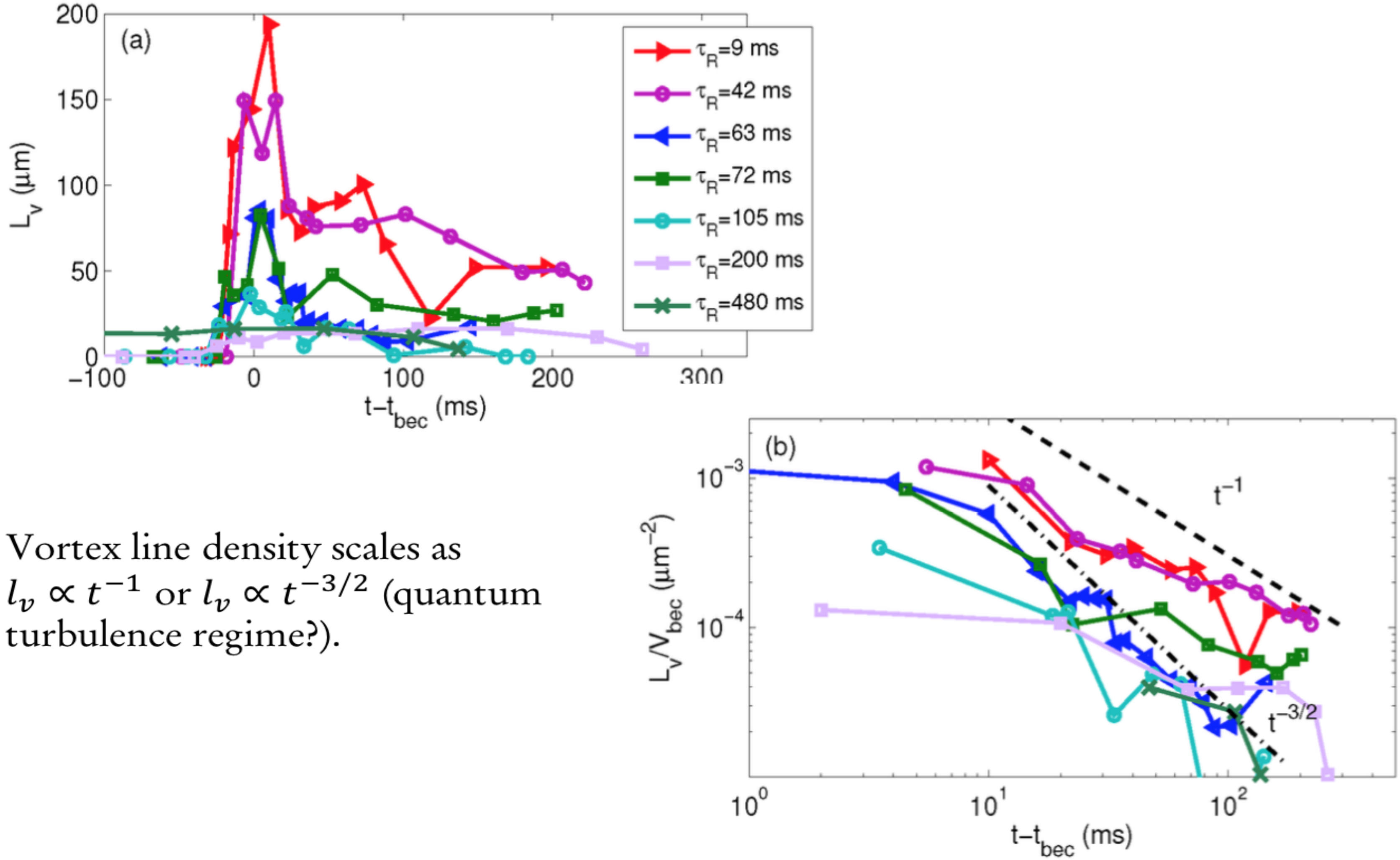


P-O mode growth. τ_R is the ramp time.

is the ramp time.

-K. Liu et al, in preparation.

Vortex length

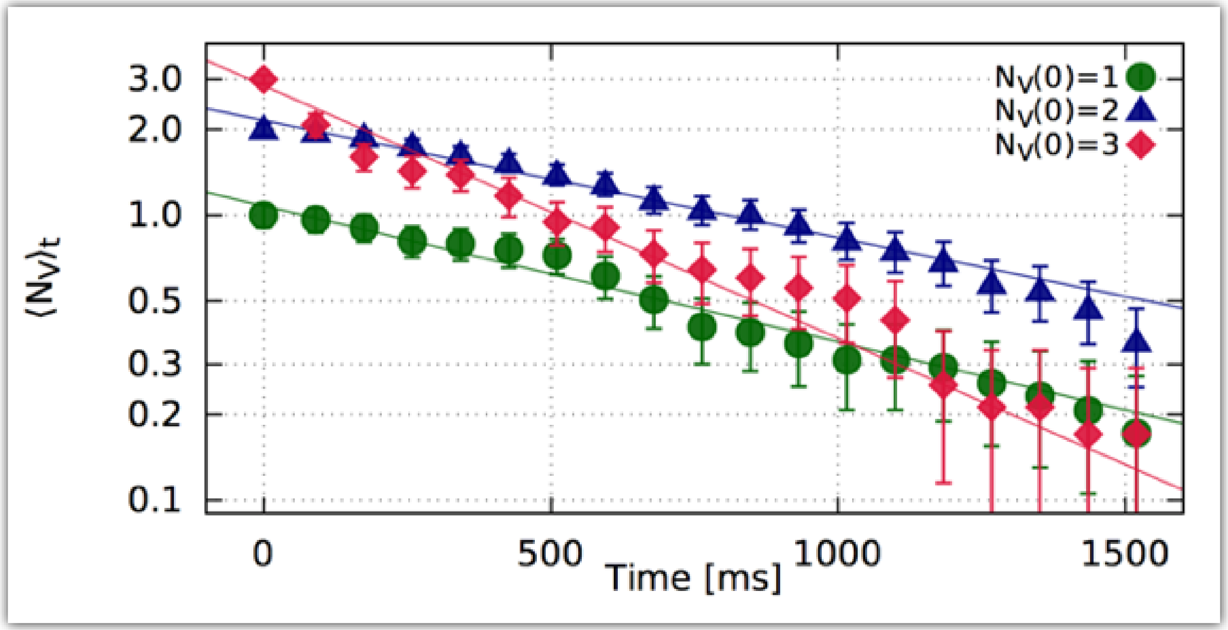


Vortex line density scales as $l_v \propto t^{-1}$ or $l_v \propto t^{-3/2}$ (quantum turbulence regime?).

P. M. Walmsley and A. I. Golov, Phys. Rev. **100**, 245301

Vortex decay

Single vortex lifetime is limited by scattering with thermal excitations.



$\tau_1 = (910 \pm 100)\mathrm{ms}$

$\tau_2 = (1050 \pm 100)\mathrm{ms}$

$\tau_3 = (490 \pm 100)\mathrm{ms}$

τ is compatible for $N_V(0) = 1, 2$, but not for $N_V(0) = 3$.
Does this mean that two-vortex interactions are suppressed?

suppressed?

Two-dimensional KZM

- BKT free vortices give a correction in the predicted vortex density for KZM:

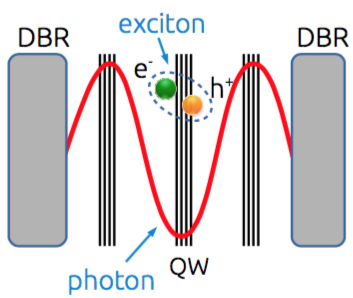
$$\rho_v(\tau_Q) \simeq \left(\lambda \frac{\tau_Q + \hat{t}}{\ln((\tau_Q+\hat{t})/t_0)}\right)^{-1}$$

A. Jelić et al., J. Stat. Mech. 2011(02), 2011

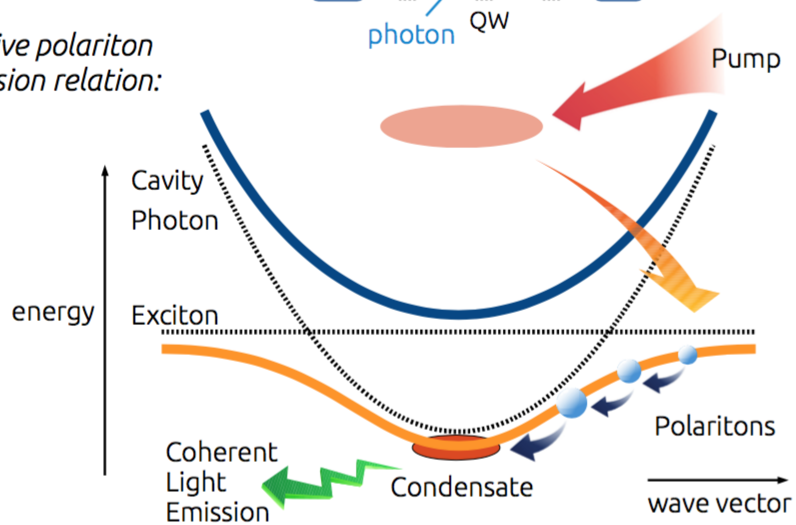
Polaritons

$$i\hbar\frac{\partial\psi}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m}+i\frac{\hbar}{2}\left(\frac{P_0(\mathbf{r},t)}{1+\frac{|\psi|^2}{n_s}}-\gamma\right)+\frac{\hbar}{2}\frac{P_0(\mathbf{r},t)}{\Omega}\frac{\partial}{\partial t}+g\left(|\psi|^2-\frac{1}{dV}\right)\right]\psi+\eta(\mathbf{r},t)$$

Schematic of exciton-polaritons in quantum-well:



Effective polariton dispersion relation:

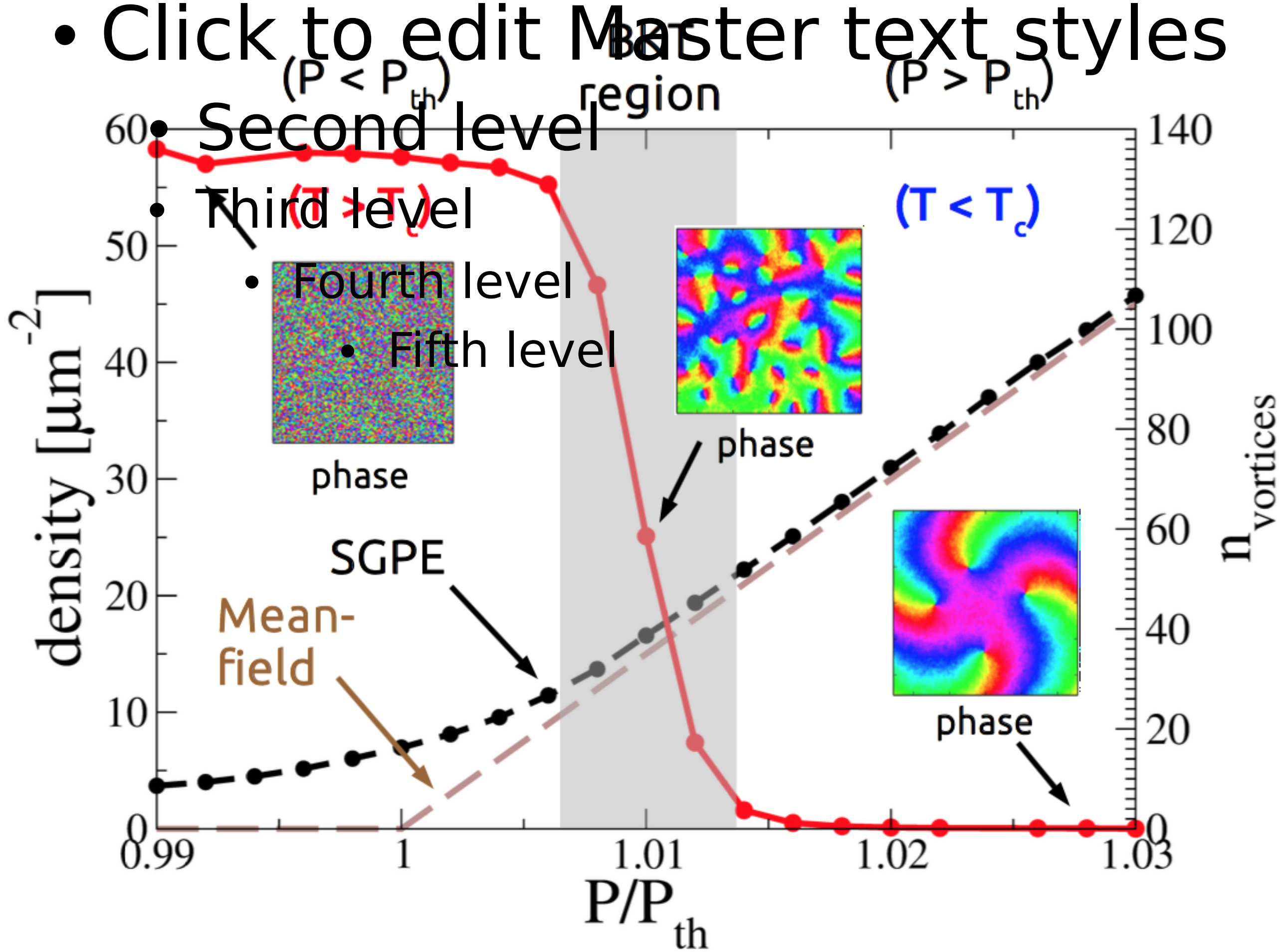


γ loss rate;
P_0 pumping strength;
n_s saturation density;

A steady state is reached when the system equilibrates between driving and dissipation.

Vortex number decay

- Click to edit Master text styles
  - Second level
    - Third level
      - Fourth level
        - Fifth level



P. Comaron et al., in

Two-dimensional KZM

- BKT phase transition:
  - Due to the Mermin-Wagner theorem, no condensation in an infinite 2D system for any $T > 0$.
  - However, a superfluid transition occurs at finite T.
  - Berezinskii-Kosterlitz-Thouless (BKT) at T_{BKT}:
    - For $T < T_{BKT}$ vortices of opposite circulation are coupled in pairs.
    - For $T > T_{BKT}$ they gradually become free.