Reservoir-induced topological order & quantized transport in open systems

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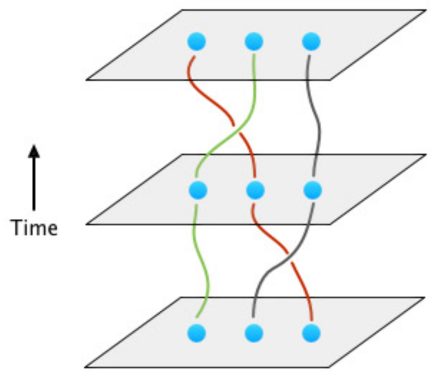
Bad Honnef 09.05.2016

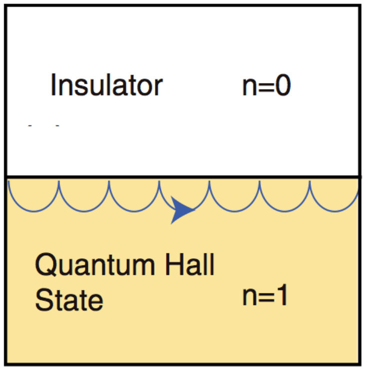
picture: wikipedia

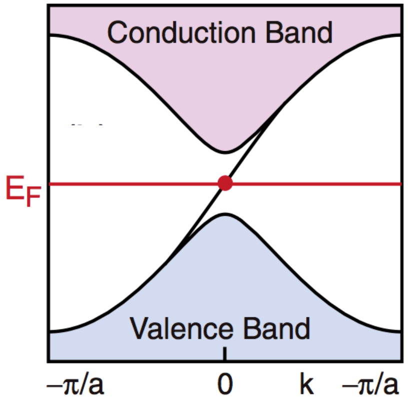
topological states

exotic quantum states

topological protection







Abelian & non-Abelian anyons

protected edge states & edge transport

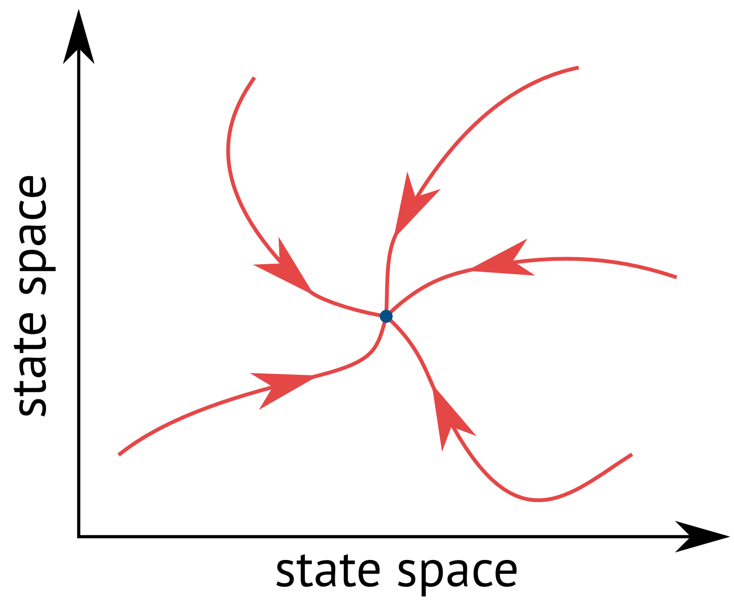
but: in general no protection against losses topological order in the steady state

of an open system ??

open-system dynamics

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i\left[\hat{H}, \rho\right] + \frac{1}{2}\sum_{\mu}\left(2L_\mu \rho L_\mu^\dagger - \{L_\mu^\dagger L_\mu, \rho\}\right) = 0$$

- open dynamics drives the system to a steady state

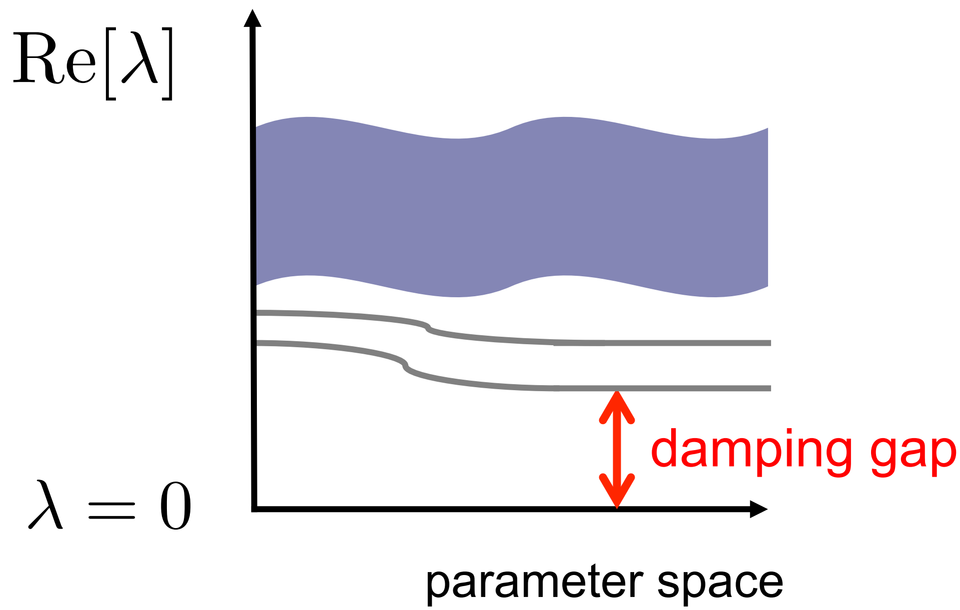


- steady state: **attractor**

robustness of the steady state

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = \mathcal{L}\,\rho \qquad\qquad \mathcal{L}\rho_\lambda = -\lambda\,\rho_\lambda$$

- gapped open systems



outline

- topological invariants & open systems
- Su-Schrieffer-Heeger model & Thouless pump
- quantized topological transport in open spin chain with interactions
- detection of topological invariant

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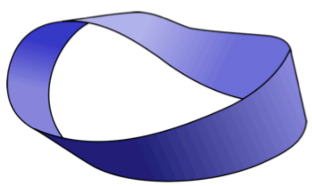
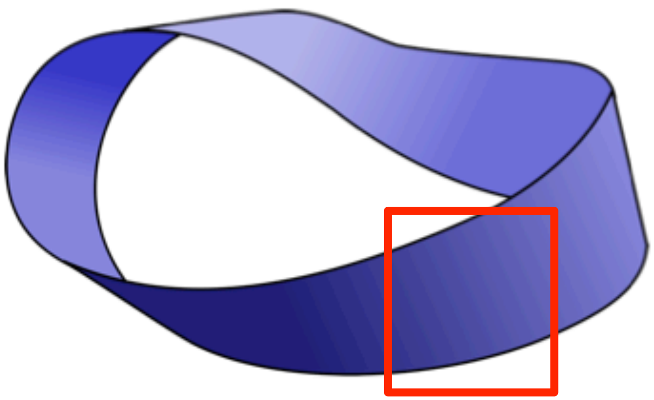
- topological invariants & open systems
- Su-Schrieffer-Heeger model & Thouless pump
- quantized topological transport in open spin chain with interactions
- **detection of topological invariant**

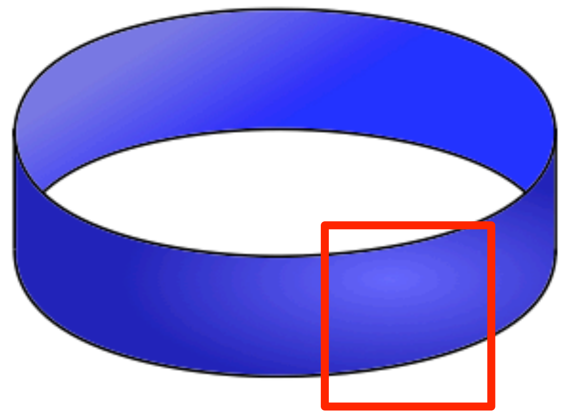
topological invariants & open systems



topology

- **Möbius strip:**

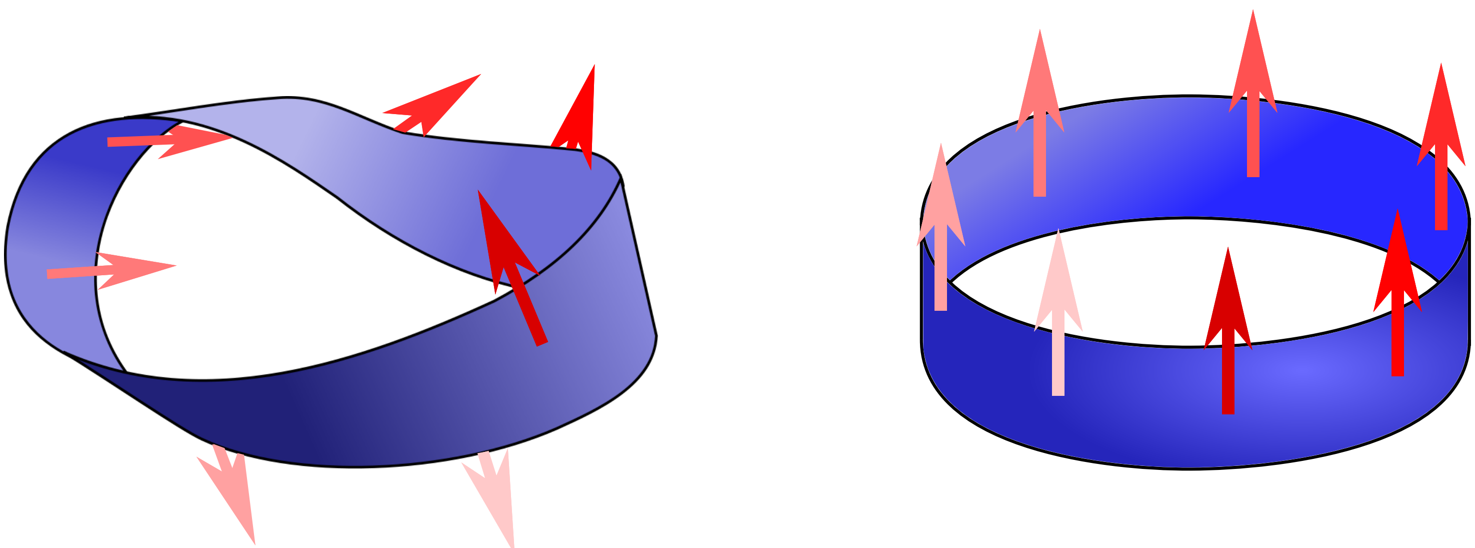




locally indistinguishable

topology

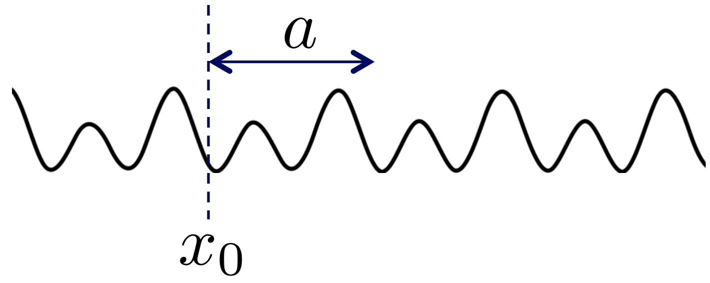
- **Möbius strip:**



differ by global properties !

topological invariants: geometric phases

- **Zak (Berry) phase**



$$\boxed{\phi_{\mathrm{Zak}} = \int_{-\pi/a}^{\pi/a} dk \, \langle u_k | i\partial_k | u_k \rangle}$$

Zak PRL (1989)

choice of origin matters $\quad |u_k'\rangle = e^{ikx_0}|u_k\rangle \quad \phi_{\mathrm{Zak}}' = \phi_{\mathrm{Zak}} + \frac{2\pi}{a}x_0$

- **Chern number**

$$\boxed{C = \frac{i}{2\pi}\int\!\!\int_{\mathrm{BZ}} \mathrm{d}^2k \left\{ \langle \partial_{k_y} u_k | \partial_{k_x} u_k \rangle - \langle \partial_{k_x} u_k | \partial_{k_y} u_k \rangle \right\} \quad \in \mathcal{Z}}$$

$C \neq 0$ $\longrightarrow$ no global gauge

geometric phases for density matrices

- **Uhlmann connection**

$$\rho = w\, w^\dagger$$

gauge degree of freedom: *U(N)*

$$w \to w\, U \qquad\qquad w^\dagger \to U^\dagger\, w^\dagger$$

O. Viyuela, et al. Phys. Rev. Lett. (2014)
Z. Huang, D. P. Arovas, Phys. Rev. Lett. (2014)

U(1) Uhlmann phase

$$e^{i\phi} = \oint \mathrm{d}\lambda \; \mathrm{Tr}\left[w \partial_\lambda w^\dagger \right]$$

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Berry phases for density matrices

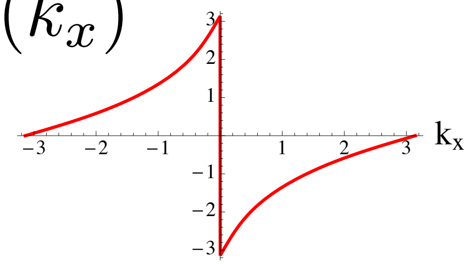
J. C. Budich, S. Diehl 1501.04135:

$$H(k) = \sum_j d^j(k)\,\hat{\sigma}_j$$

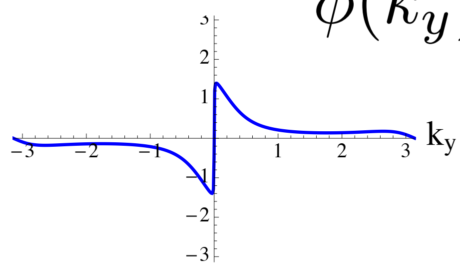
finite-T state of a Chern insulator

$$d^1 = \sin(k_x) \quad d^2 = 3\sin(k_y) \quad d^3 = 1 - \cos(k_x) - \cos(k_y)$$

$\phi(k_x)$



$\phi(k_y)$



$$C = \frac{1}{2\pi}\int dk_y \left(\frac{\partial\phi(k_y)}{\partial k_y}\right) \neq C' = \frac{1}{2\pi}\int dk_x \left(\frac{\partial\phi(k_x)}{\partial k_x}\right)$$

Furthermore without constraints: trivial global gauge

$$w = \sqrt{\rho}$$

non-interacting fermions

- **Gaussian systems** C.E. Bardyn, et al. New J. Phys (2013)

$$H = \sum_{ij} h_{ij}\, \hat{c}_i^{(\dagger)}\, \hat{c}_j \qquad\qquad L_j \sim \alpha\, \hat{c}_j^\dagger + \beta\, \hat{c}_j$$

covariance matrix

$$\Gamma_{jk} \sim \mathrm{Im}\,\mathrm{Tr}\{\rho w_i w_j\} \qquad\qquad w_i \sim \hat{c}_i \pm \hat{c}_i^\dagger$$

density matrix

$$\rho \sim \exp\left\{-\frac{i}{2} w_j\, \Gamma_{jk} w_k\right\}$$

non-interacting fermions

topological classification in terms of Γ_{ij}

$$\gamma(k) = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} \sim (1 + \vec{n}(k) \cdot \vec{\sigma})$$

topological phase transition

(I) closing of the damping gap (criticality)

(II) closing of the purity gap = gap of effective Hamiltonian

$$\rho \sim \exp\left\{ -\frac{i}{2} w_j \, \Gamma_{jk} w_k \right\} \qquad H_{\text{eff}} = i \sum_{jk} \Gamma_{jk} w_j w_k$$

→ **beyond Gaussian systems ??**

polarization

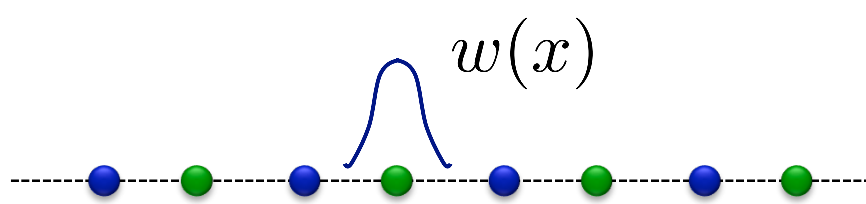
Thouless, Kohmoto, Nightingale, den Nijs (TKNN) PRL (1982)

topology ←→ quantized bulk transport

- **Zak phase & Polarization**

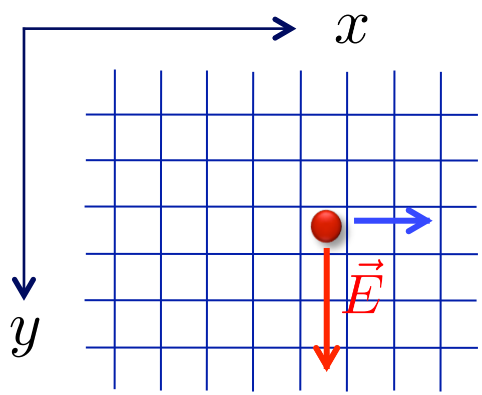
King-Smith, Vanderbilt PRB (1983)

$$P = \int dx\, w^*(x)\, x\, w(x)$$

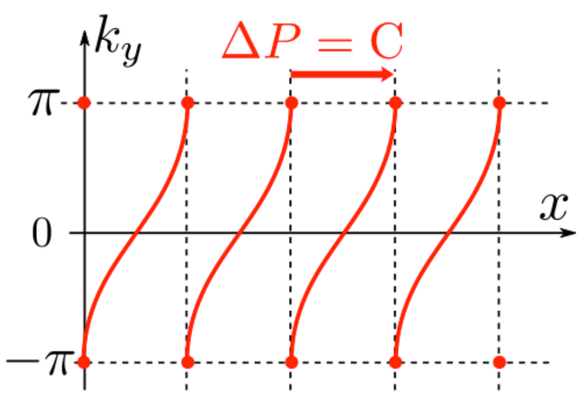


$$\boxed{\Delta P = \frac{a}{2\pi} \Delta \phi_{\mathrm{Zak}}}$$

quantization of Hall conductance



$$C = \frac{1}{2\pi} \int_0^{2\pi/a} dk_y \, \partial_{k_y} \phi_{\mathrm{Zak}}(k_y)$$



$$dk_y = E_y dt$$

$$\sigma_{xy} = \frac{j_x}{E_y} = \frac{dP}{dt}\frac{1}{E_y} = \frac{dP}{dk_y} = C$$

Su-Schrieffer-Heeger model & Thouless pump

Su, Schrieffer, Heeger, PRL (1979)

D.J. Thouless, PRB (1983)

Attala et al. (I. Bloch), Nature Physics (2013)

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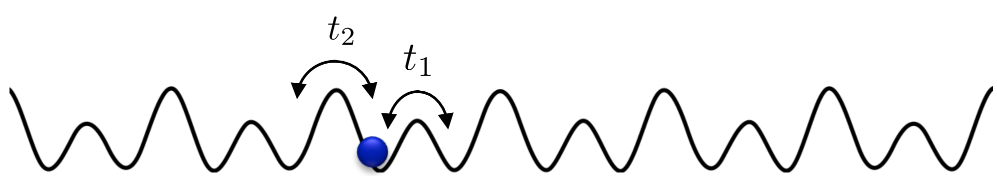
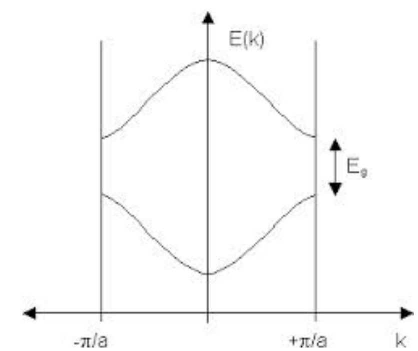
SSH

- **Model: free fermions on a superlattice with inversion symmetry**

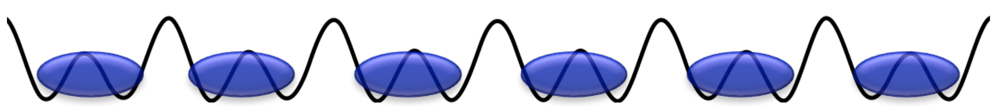


$$H = -t_1 \sum_{\text{odd}} \hat{c}_i \hat{c}^\dagger_{i+1} - t_2 \sum_{\text{even}} \hat{c}_i \hat{c}^\dagger_{i+1} + h.a.$$



$$E_g \sim t_1 - t_2$$

→ half filling = band insulator of lower sub-band

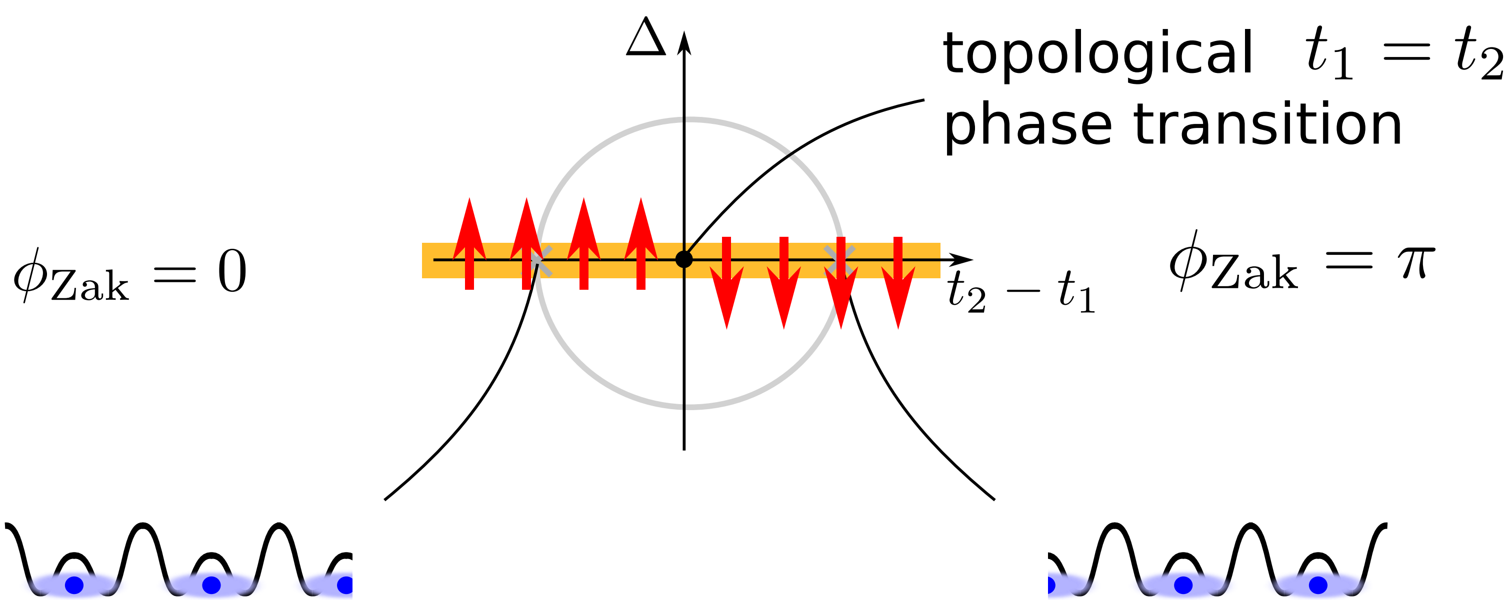


SSH → Rice Mele Hamiltonian

$$H = -t_1 \sum_{\text{odd}} \hat{c}_i^\dagger \hat{c}_{i+1} - t_2 \sum_{\text{even}} \hat{c}_i^\dagger \hat{c}_{i+1}$$

Inversion symmeric SSH symmetry breaking term

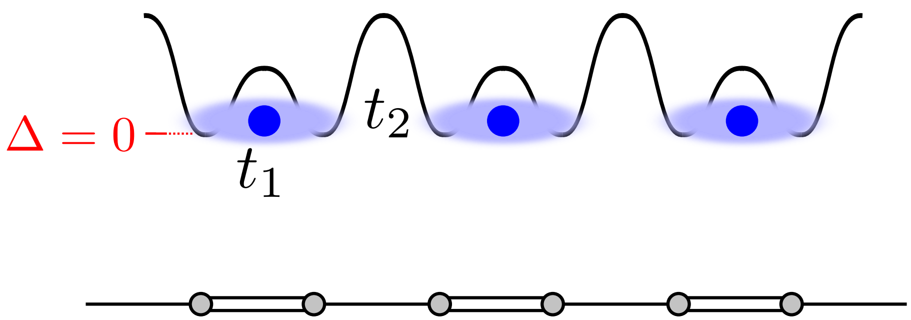
M.J. Rice & E.J. Mele, PRL(1982)

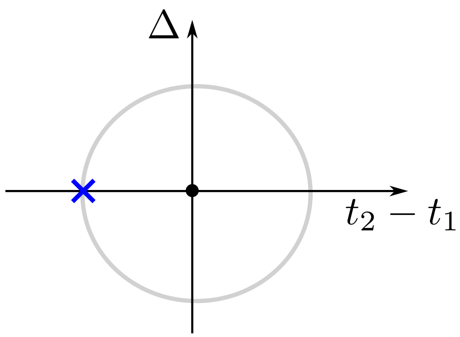


Rice-Mele Hamiltonian

- **breaking inversion symmetry & Thouless pump**

$$H = -t_1 \sum_{\text{odd}} \hat{c}_i^\dagger \hat{c}_{i+1} - t_2 \sum_{\text{even}} \hat{c}_i^\dagger \hat{c}_{i+1} + \Delta \left(\sum_{\text{odd}} \hat{c}_i^\dagger \hat{c}_i - \sum_{\text{even}} \hat{c}_i^\dagger \hat{c}_i \right)$$





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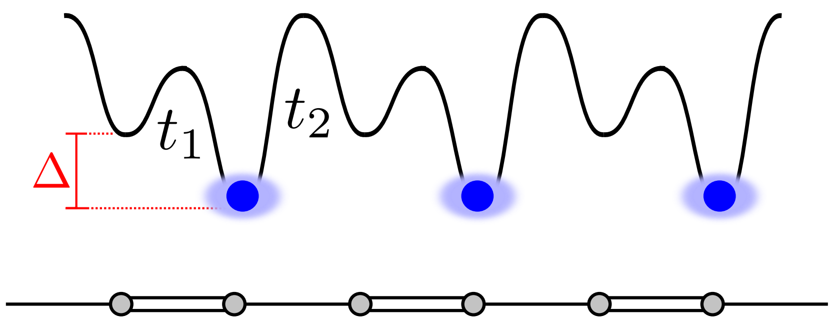

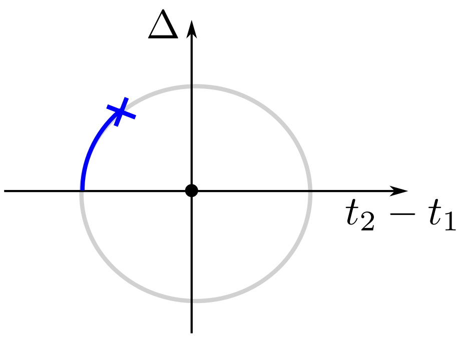

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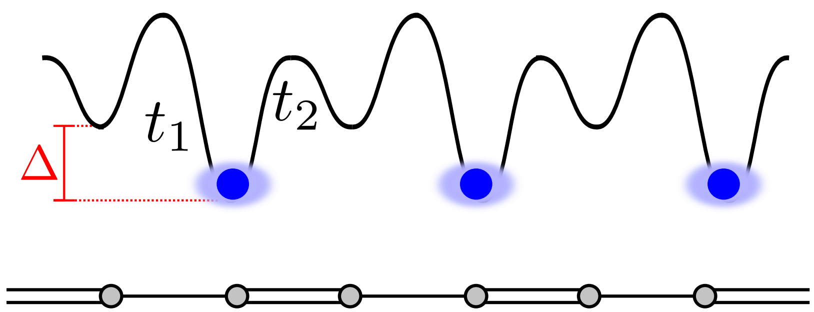

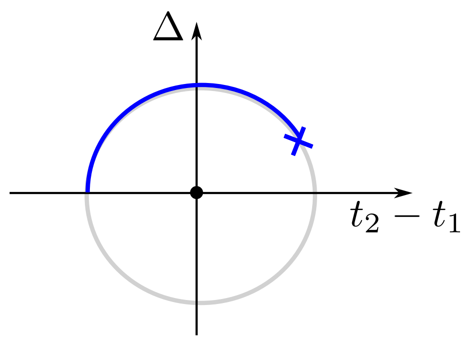

Rice-Mele Hamiltonian

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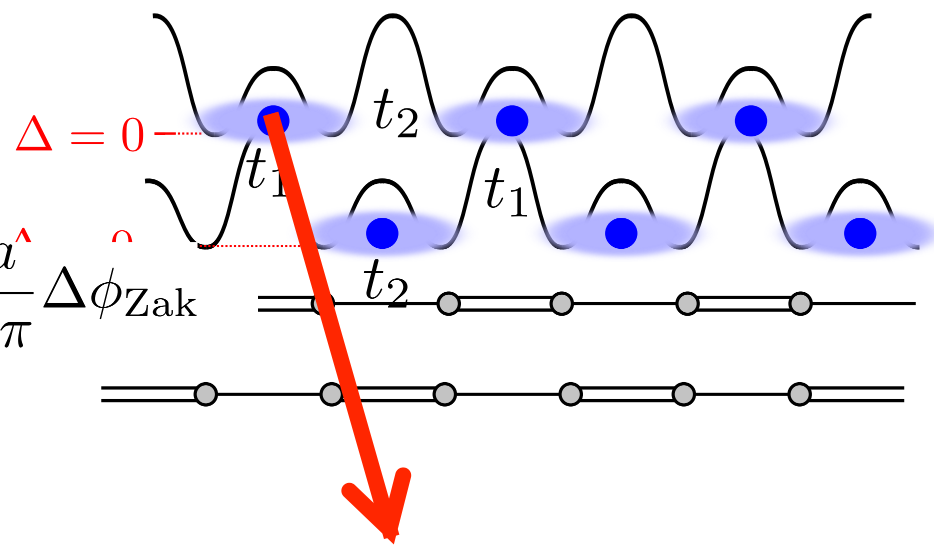
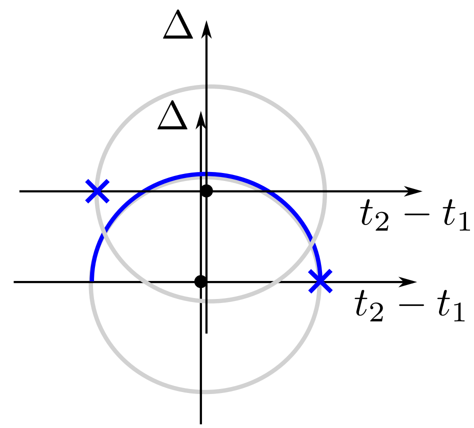


$$\Delta P = \frac{a}{2\pi} \Delta\phi_{\text{Zak}}$$



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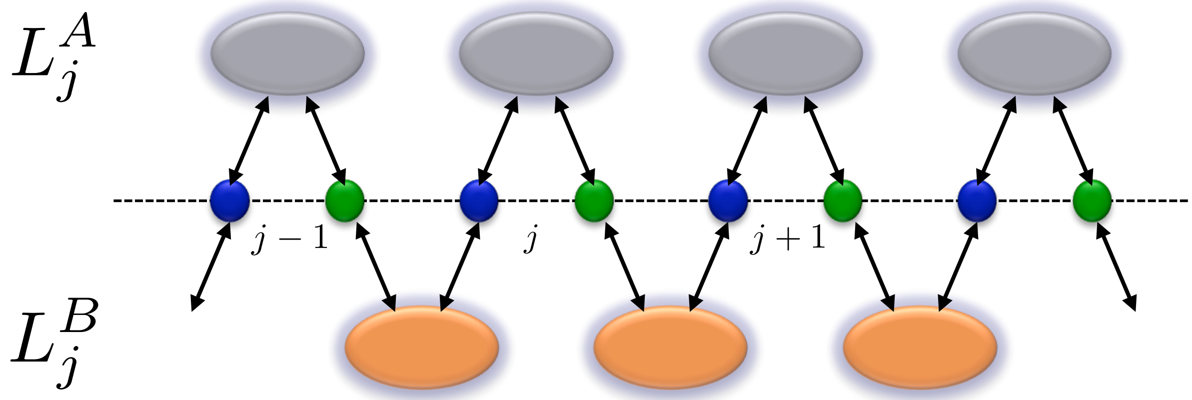
quantized topological transport in an open spin chain

D. Linzner, F. Grusdt, M. Fleischhauer, arxiv:1605.00756

model



$$\dot{\rho} = \mathcal{L}\rho = \sum_{j,\mu} \left(2L_j^\mu \rho L_j^{\mu\dagger} - L_j^{\mu\dagger} L_j^\mu \rho - \rho L_j^{\mu\dagger} L_j^\mu \right)$$

Lindblad generators

$$L_j^A = \sqrt{1+\varepsilon}\Big[(1-\lambda)\left(\hat{\sigma}_{L,j} + \hat{\sigma}^+_{R,j}\right) + (1+\lambda)\left(\hat{\sigma}^+_{L,j} + \hat{\sigma}_{R,j}\right)\Big]$$
$$L_j^B = \sqrt{1-\varepsilon}\Big[(1-\lambda)\left(\hat{\sigma}_{L,j+1} + \hat{\sigma}^+_{R,j}\right) + (1+\lambda)\left(\hat{\sigma}^+_{L,j+1} + \hat{\sigma}_{R,j}\right)\Big]$$

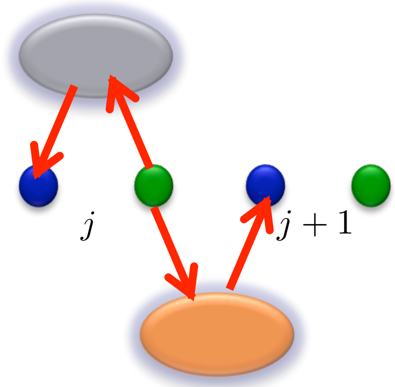
model

- **action of Lindblad generators**

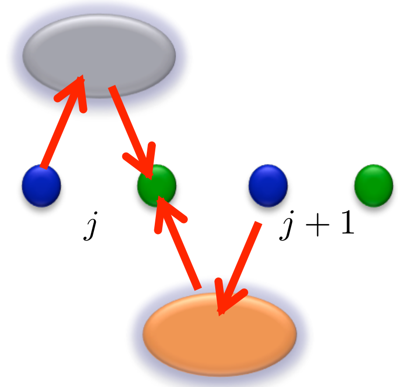
$$L_j^A = \sqrt{1+\varepsilon}\left[(1-\lambda)\left(\hat{\sigma}_{L,j} + \hat{\sigma}_{R,j}^{+}\right) + (1+\lambda)\left(\hat{\sigma}_{L,j}^{+} + \hat{\sigma}_{R,j}\right)\right]$$

$$L_j^B = \sqrt{1-\varepsilon}\left[(1-\lambda)\left(\hat{\sigma}_{L,j+1} + \hat{\sigma}_{R,j}^{+}\right) + (1+\lambda)\left(\hat{\sigma}_{L,j+1}^{+} + \hat{\sigma}_{R,j}\right)\right]$$

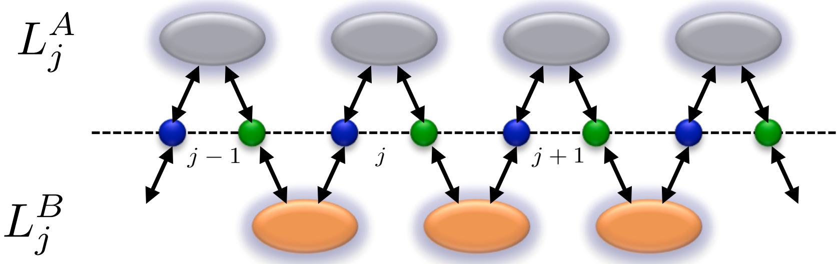
$$\boxed{\lambda = +1}$$



$$\boxed{\lambda = -1}$$



symmetries



$$L_j^A = \sqrt{1+\varepsilon}\left[(1-\lambda)\left(\hat{\sigma}_{L,j} + \hat{\sigma}^+_{R,j}\right) + (1+\lambda)\left(\hat{\sigma}^+_{L,j} + \hat{\sigma}_{R,j}\right)\right]$$

$$L_j^B = \sqrt{1-\varepsilon}\left[(1-\lambda)\left(\hat{\sigma}_{L,j+1} + \hat{\sigma}^+_{R,j}\right) + (1+\lambda)\left(\hat{\sigma}^+_{L,j+1} + \hat{\sigma}_{R,j}\right)\right]$$

- **particle-hole symmetry**

$$\sigma^z_R \to -\sigma^z_L \qquad \langle\sigma^z_R\rangle + \langle\sigma^z_L\rangle = 0$$

$$\lambda = 0 \qquad \sigma^z_j \to -\sigma^z_j \qquad \langle\sigma^z_j\rangle = 0$$

- **inversion symmetry**

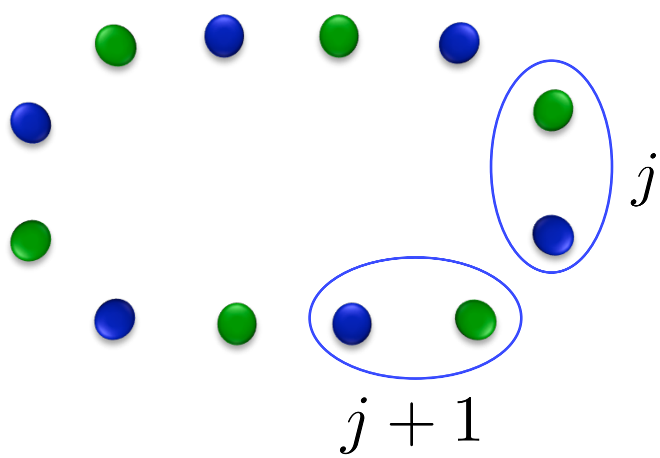
$\lambda = 0$ or $\varepsilon = 0$

steady-state Thouless pump

- **polarization in finite system with PBC**

R. Resta PRL **80**, 1800 (1998)

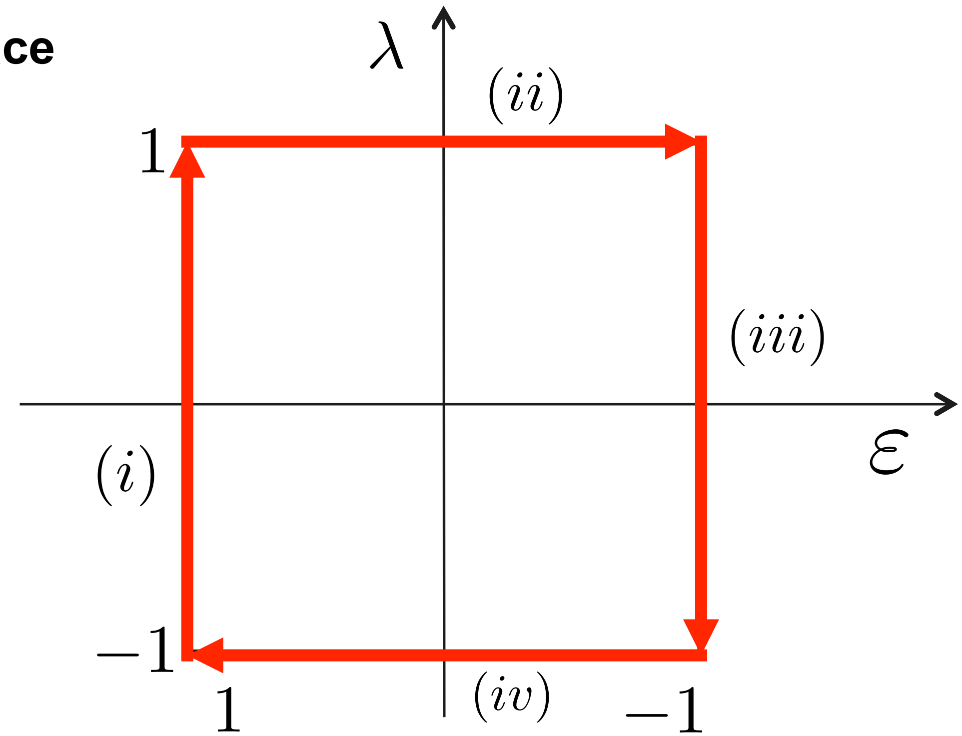
$$P = \frac{1}{2\pi} \mathrm{Im} \ln \left\langle \exp\left\{ i \frac{2\pi}{L} \sum_j j \hat{n}_j \right\} \right\rangle$$



steady-state Thouless pump

- **periodic cycle in parameter space**

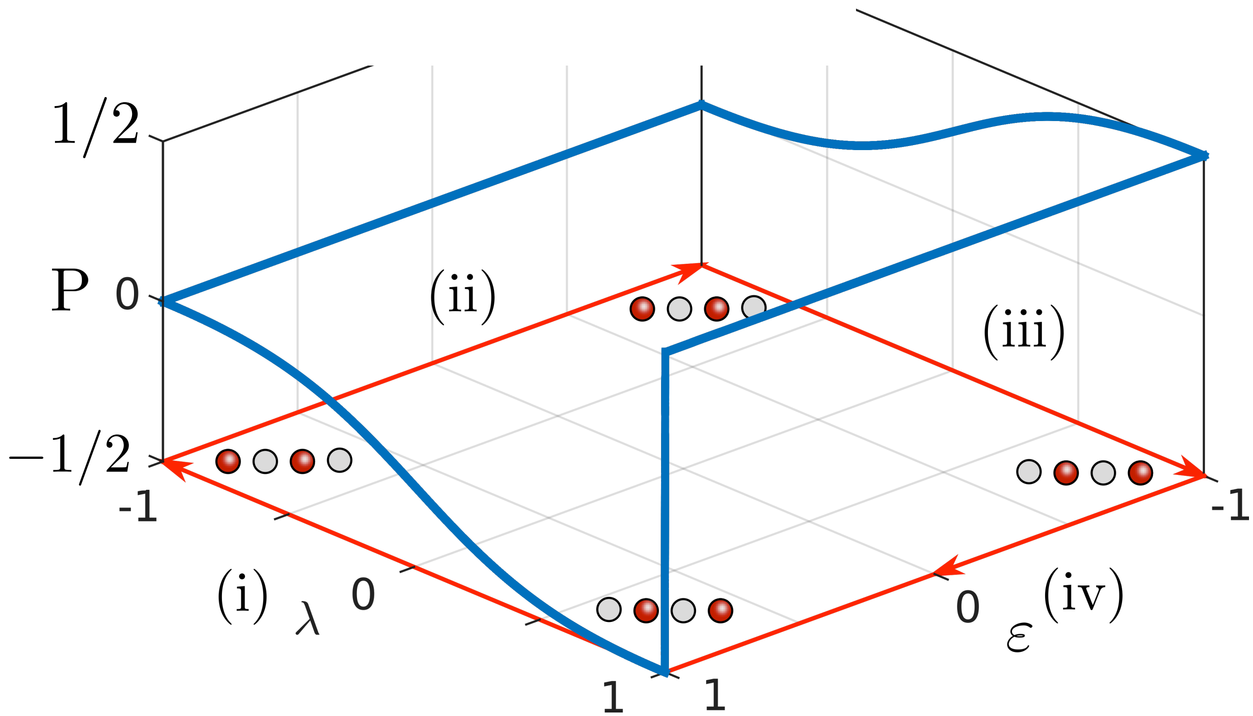
  steady state is a pure state
  (dark state)



$$P = \mp \frac{1}{2}\left(\frac{1}{2} + \frac{\lambda}{1+\lambda^2}\right)$$

steady-state Thouless pump

- **periodic cycle in parameter space**



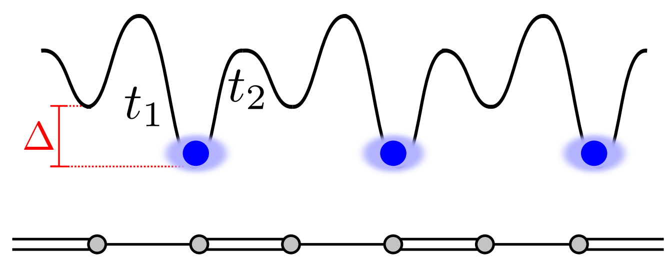
winding !!

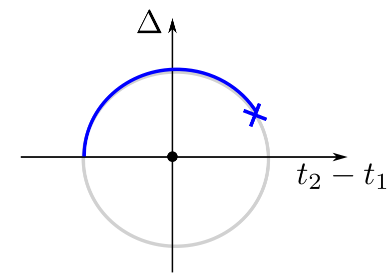
steady-state Thouless pump

- **parent Hamiltonian** $$H = \sum_\mu L_\mu^\dagger L_\mu$$

$$H = -\sum_j \left(t_1\, \hat\sigma^-_{L,j} \hat\sigma^+_{R,j} + t_2\, \hat\sigma^-_{L,j+1} \hat\sigma^+_{R,j} + h.a. \right) + \Delta \sum_j \left(\hat\sigma^+_{L,j} \hat\sigma^-_{L,j} - \hat\sigma^+_{R,j} \hat\sigma^-_{R,j} \right)$$

$$t_1 = 2\Gamma(1+\varepsilon)(1-\lambda^2)$$
$$t_2 = 2\Gamma(1-\varepsilon)(1-\lambda^2)$$
$$\Delta = 8\Gamma\lambda$$





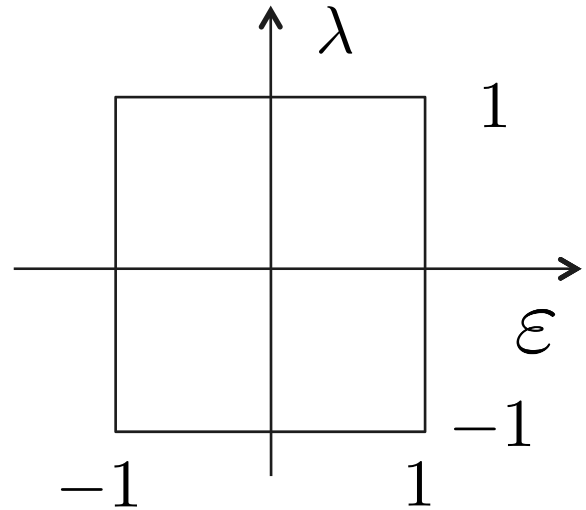
= Rice-Mele Hamiltonian: winding → quantized bulk transport

- **topological invariant = Zak phase / Chern number**

$$\boxed{\Delta P = \frac{a}{2\pi} \Delta\phi_{\mathrm{Zak}}}$$

steady-state Thouless pump

- **inner part of parameter space**



$$\lambda = 0 \quad \longrightarrow \quad L_\mu = L_\mu^\dagger$$

$$\dot{\rho} = \mathcal{L}\rho = \sum_{j,\mu} \left(2L_j^\mu \rho L_j^{\mu\dagger} - L_j^{\mu\dagger} L_j^\mu \rho - \rho L_j^{\mu\dagger} L_j^\mu \right)$$

totally mixed state is also steady state !

→ lift degeneracy by (generic) nonlinear term

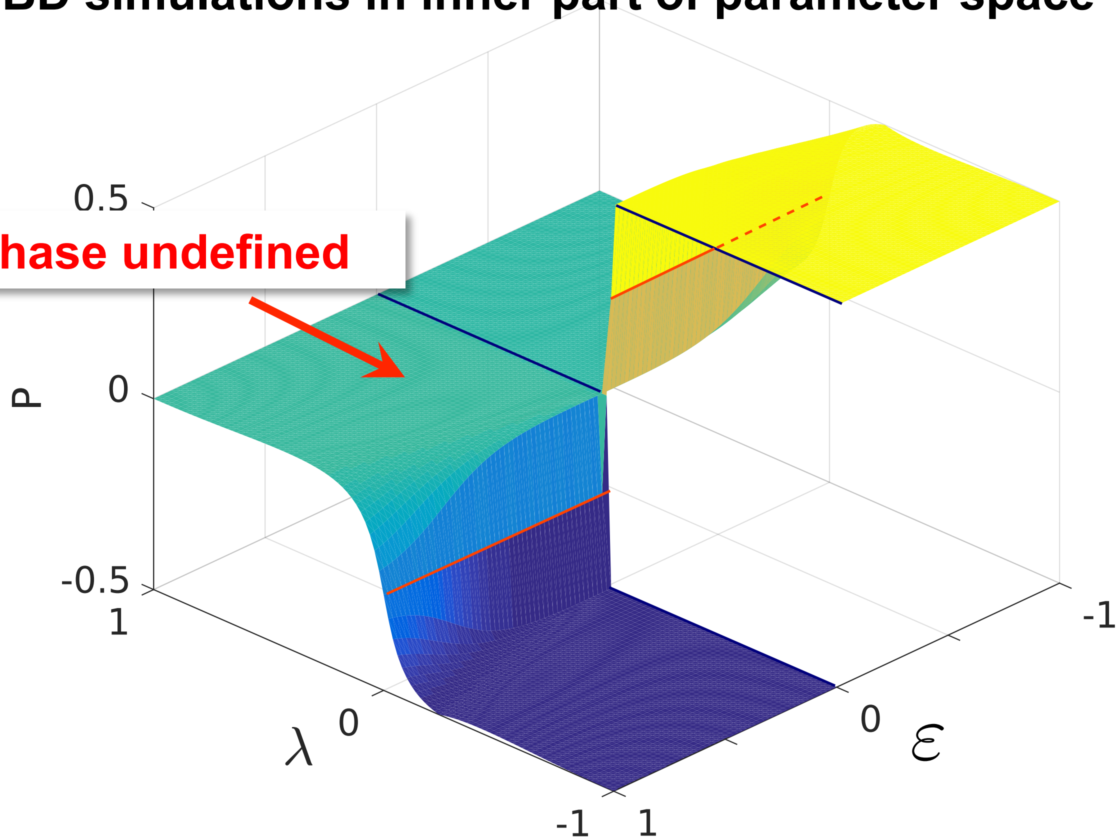
$$L_j^A \;\rightarrow\; L_j^A + \sqrt{\Gamma(1+\varepsilon)} \left(\hat{\sigma}_{L,j}^+ \hat{\sigma}_{R,j}^+ - \hat{\sigma}_{L,j}^- \hat{\sigma}_{R,j}^- \right)$$

$$L_j^B \;\rightarrow\; L_j^B + \sqrt{\Gamma(1-\varepsilon)} \left(\hat{\sigma}_{L,j+1}^+ \hat{\sigma}_{R,j}^+ - \hat{\sigma}_{L,j+1}^- \hat{\sigma}_{R,j}^- \right)$$

steady-state Thouless pump

- **TEBD simulations in inner part of parameter space**

Zak phase undefined

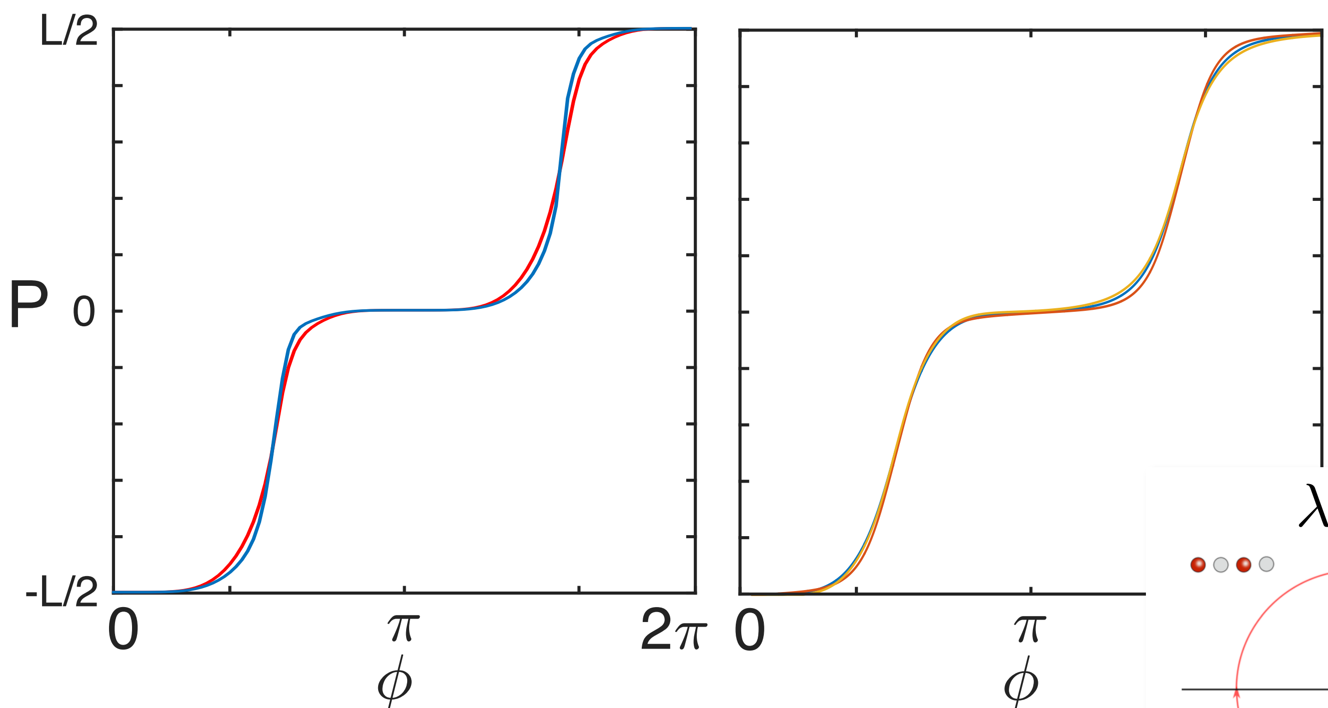


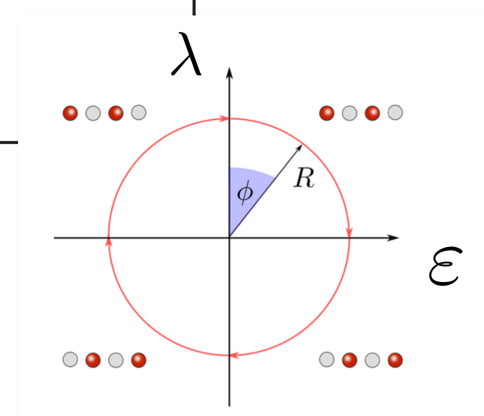
winding defines topological invariant

robustness

robustness

Hamiltonian disorder homogeneous local losses



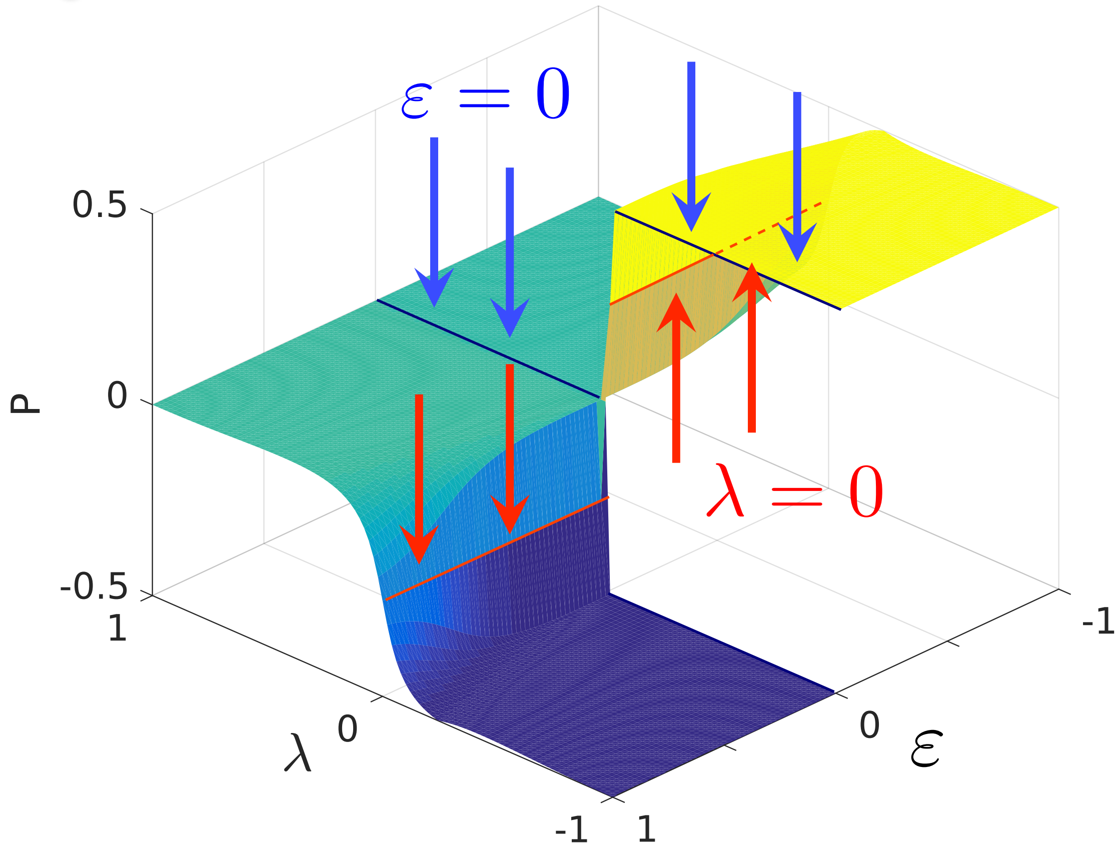


robust to disorder and losses

symmetry protected topological order

symmetry-protected topology

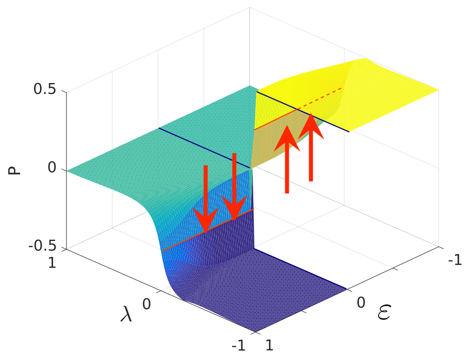
- inversion symmetric axes



polarization constant & jumps at sigularity

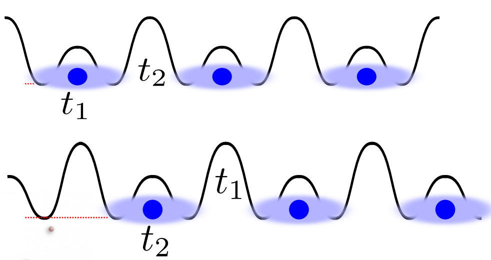
symmetry-protected topology

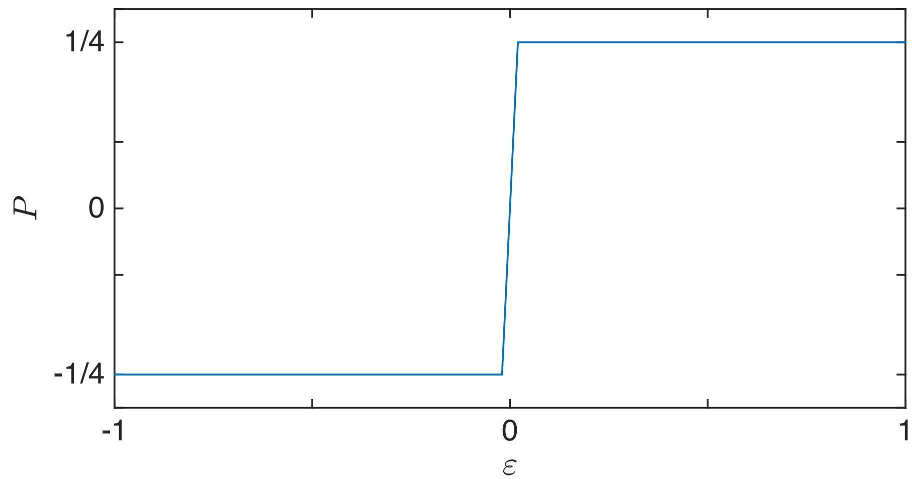
- **Inversion symmetry** $\lambda = 0$

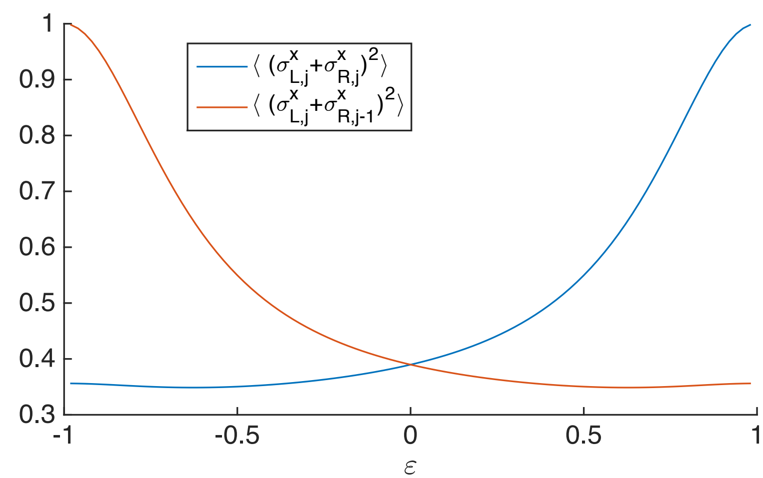


$$P_{\lambda=0}(\varepsilon) = P_{\lambda=0}(-\varepsilon) + \frac{1}{2}$$

$$\langle \sigma_j^z \rangle = 0$$

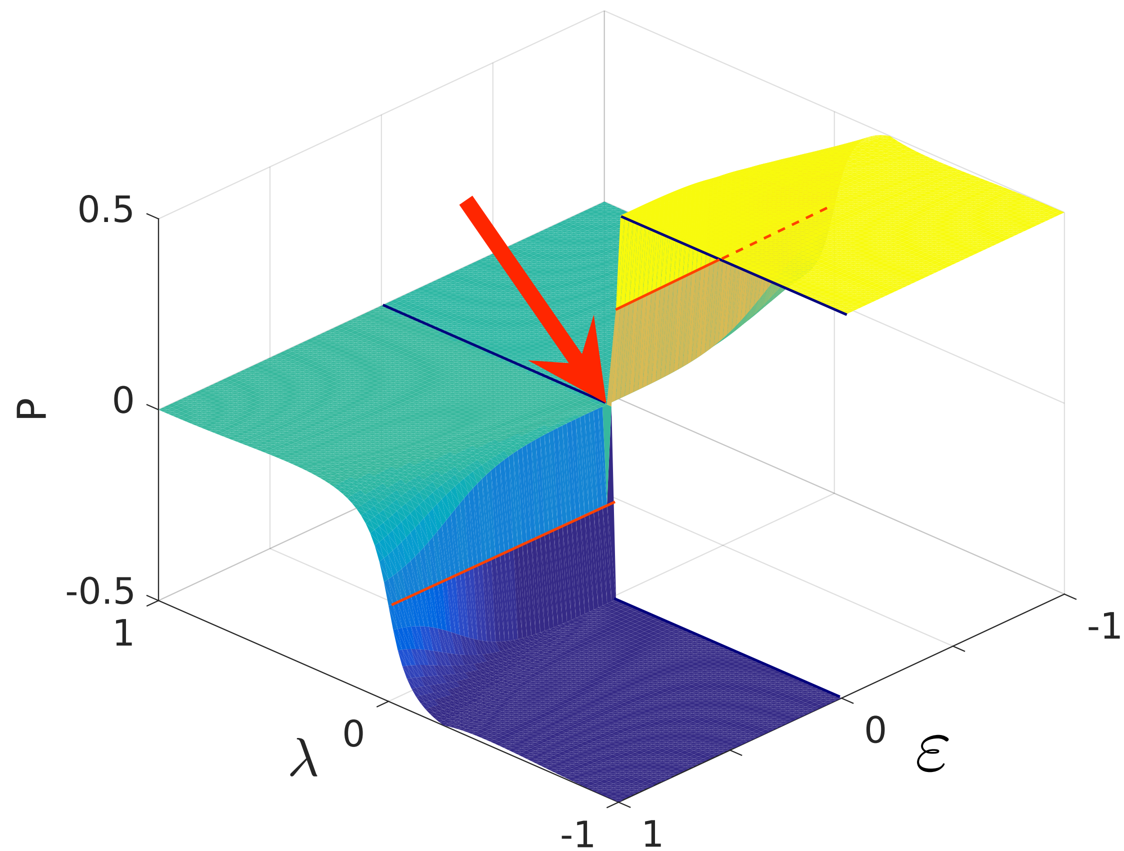






topological singularity

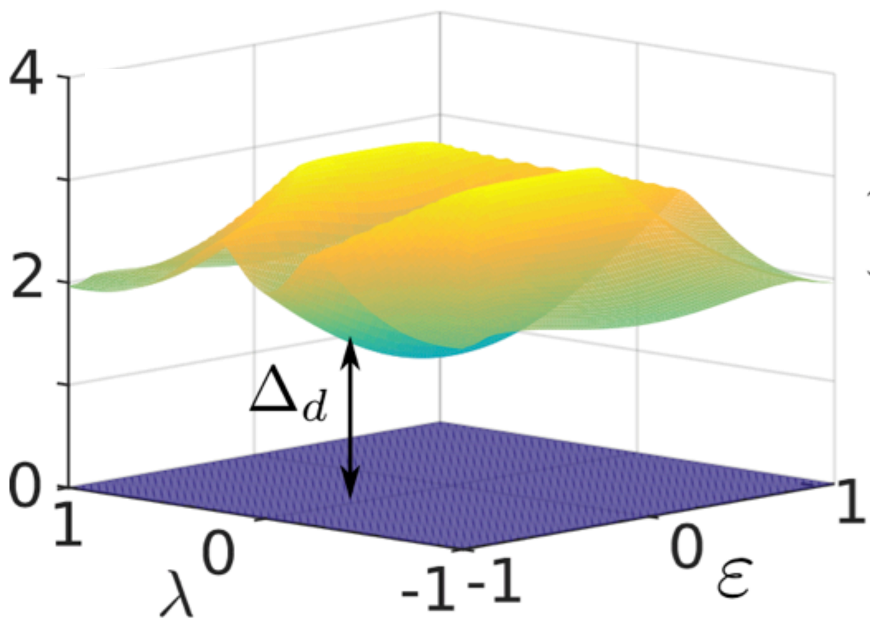
topological singularity



damping spectrum (4 sites)

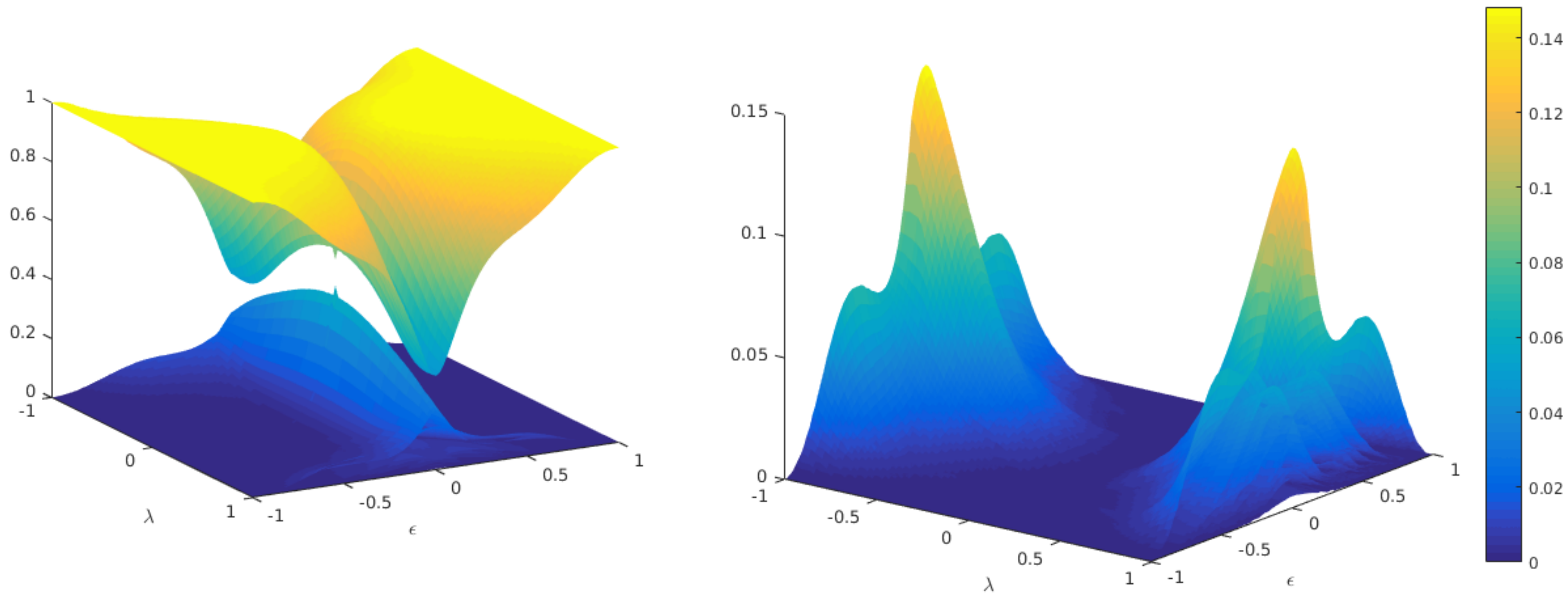
$$\mathcal{L}\rho_\nu = -\nu\rho_\nu \qquad \rho_\nu(t) = \rho_\nu(0)\, e^{-\nu t}$$

$\mathrm{Re}[\nu]$



***no* closing of damping gap**

eigenvalues of the density matrix (4 sites)



→ no degeneracies in the density matrix !

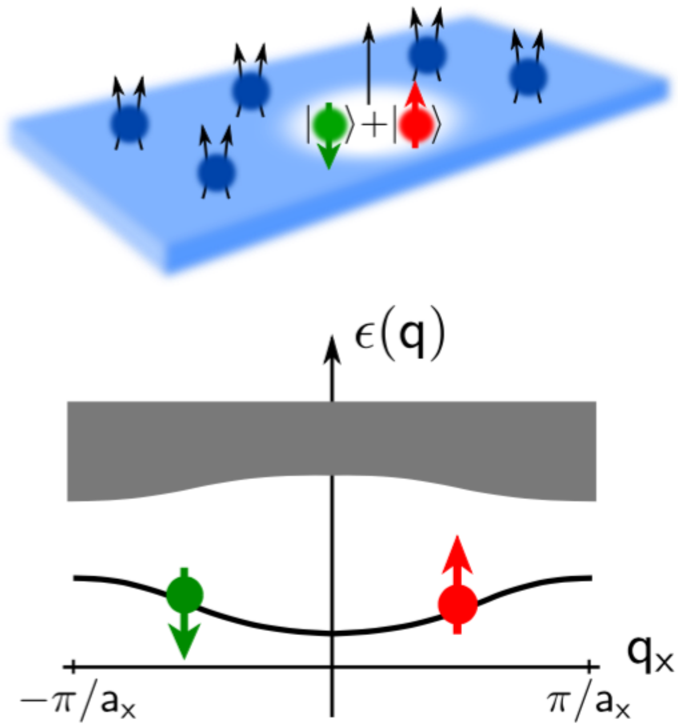
no closing of generalized "purity" gap

detection of topological invariant

detection of top. invariants in interacting systems

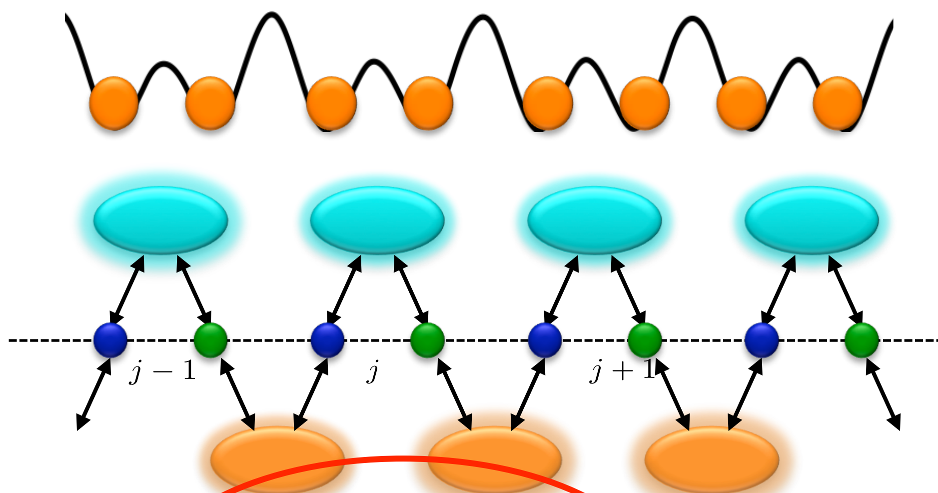
- topological polarons

F. Grusdt, N. Yao, D. Abanin, M.F., E. Demler, arxiv:1512.03407



auxiliary system

- coupling of spin chain to closed fermion system



$$H = -t \sum_j \hat{c}_j^\dagger \left[1 - \frac{1}{2}\epsilon \left(\hat{\sigma}_{Lj}^x + \hat{\sigma}_{Rj}^x \right)^2 \right] \hat{c}_{j+1} + h.a. - \eta \sum_j \hat{c}_j^\dagger \hat{c}_j \, \hat{\sigma}_j^z$$

effective Hamiltonian of auxiliary system

- perturbative limit → no effect on open spin system

  mean-field dynamics in auxiliary system

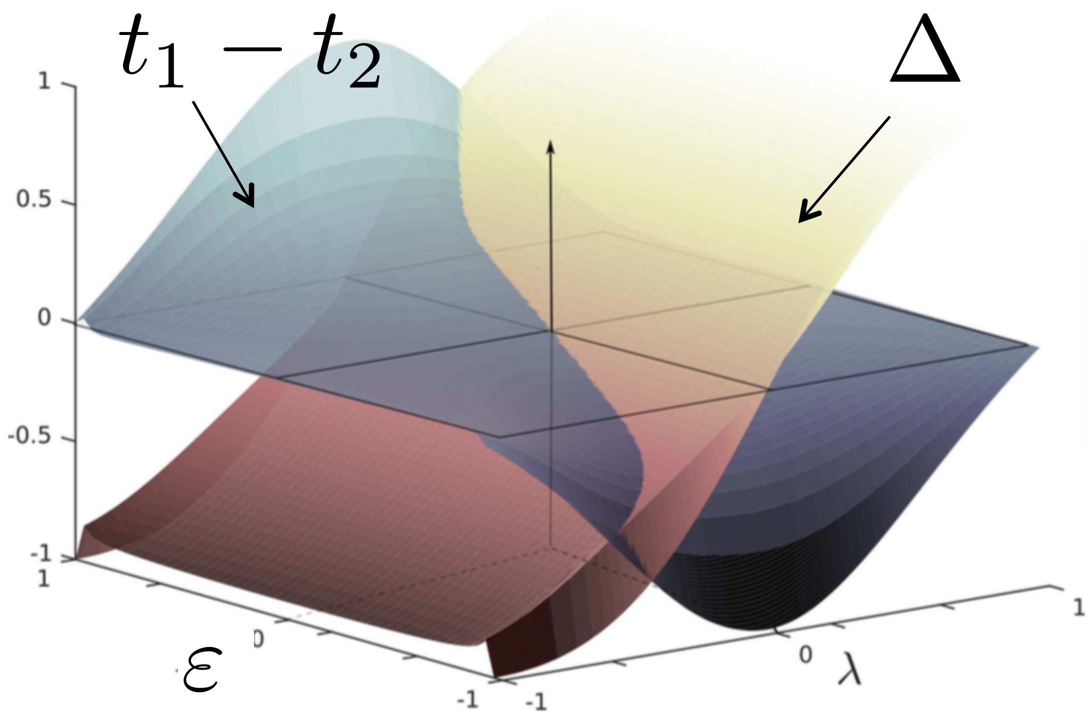
$$H = -t\sum_j \hat{c}_j^\dagger \left[1 - \frac{1}{2}\epsilon\left(\hat{\sigma}_{Lj}^x + \hat{\sigma}_{Rj}^x\right)^2\right]\hat{c}_{j+1} + h.a. - \eta\sum_j \hat{c}_j^\dagger \hat{c}_j\, \hat{\sigma}_j^z$$

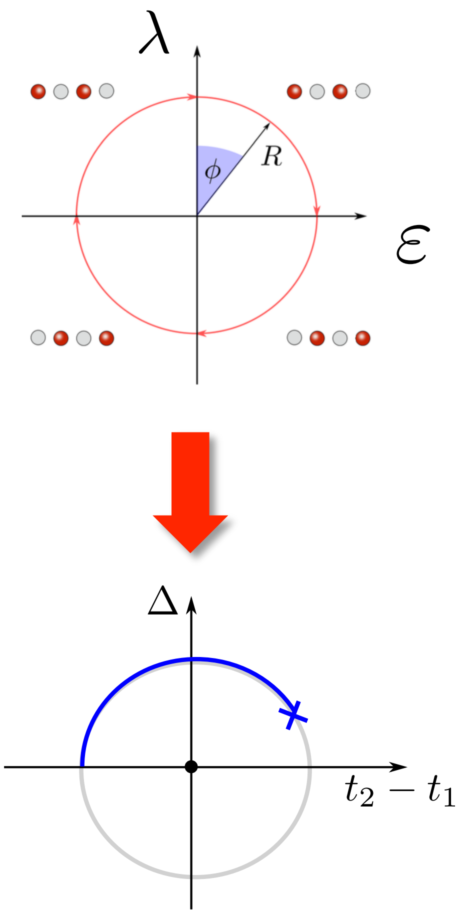
$$H = -t_1\sum_{\mathrm{odd}} \hat{c}_i^\dagger \hat{c}_{i+1} - t_2\sum_{\mathrm{even}} \hat{c}_i^\dagger \hat{c}_{i+1} - \sum_j \Delta_j \hat{c}_j^\dagger \hat{c}_j + h.a.$$

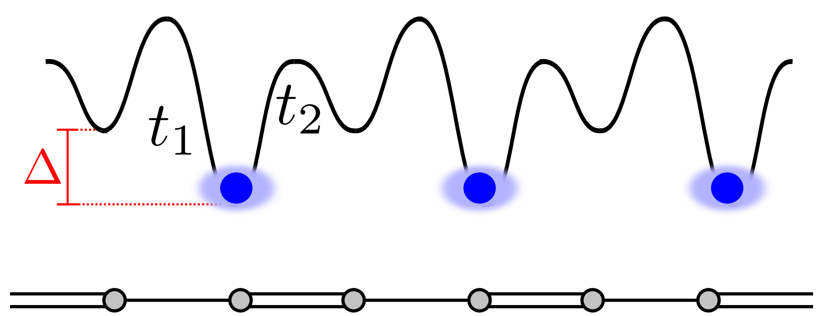
tunneling rates $\quad t_{1/2} = t\left[1 - \frac{1}{2}\varepsilon\langle(\hat{\sigma}_{L_l/R_l}^x + \hat{\sigma}_{R_l/L_{l+1}}^x)^2\rangle\right]$

staggered potential $\quad \Delta_j = \eta\langle\hat{\sigma}_j^z\rangle$

induced Thouless pump (Rice-Mele)







summary

- notion of topological order in open systems beyond Gaussian systems open problem
- quantized polarization winding to classify topology also in non-Gaussian systems
- reservoir induced topological polarization winding in open interacting spin chain
  - robust to Hamiltonian perturbation, dephasing & losses
  - symmetry protected topological order
  - nature of topological singularity unlcear (no damping gap nor purity gap closing)
  - detection scheme

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OPTIMAS light-spin-matter

Thanks !