

# A cold grip on topology: the Haldane model

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[www.quantumoptics.ethz.ch](http://www.quantumoptics.ethz.ch)











Metals

Insulators

Superconductivity

Quantum Hall Effect

$e^-$

$e^-$

Quantum Magnetism

$e^-$

Topological Insulators

Weyl Fermions

$e^-$

$e^-$

Quantum Phase Transitions

$e^-$

Dirac Fermions

Majorana Fermions

$e^-$

$e^-$

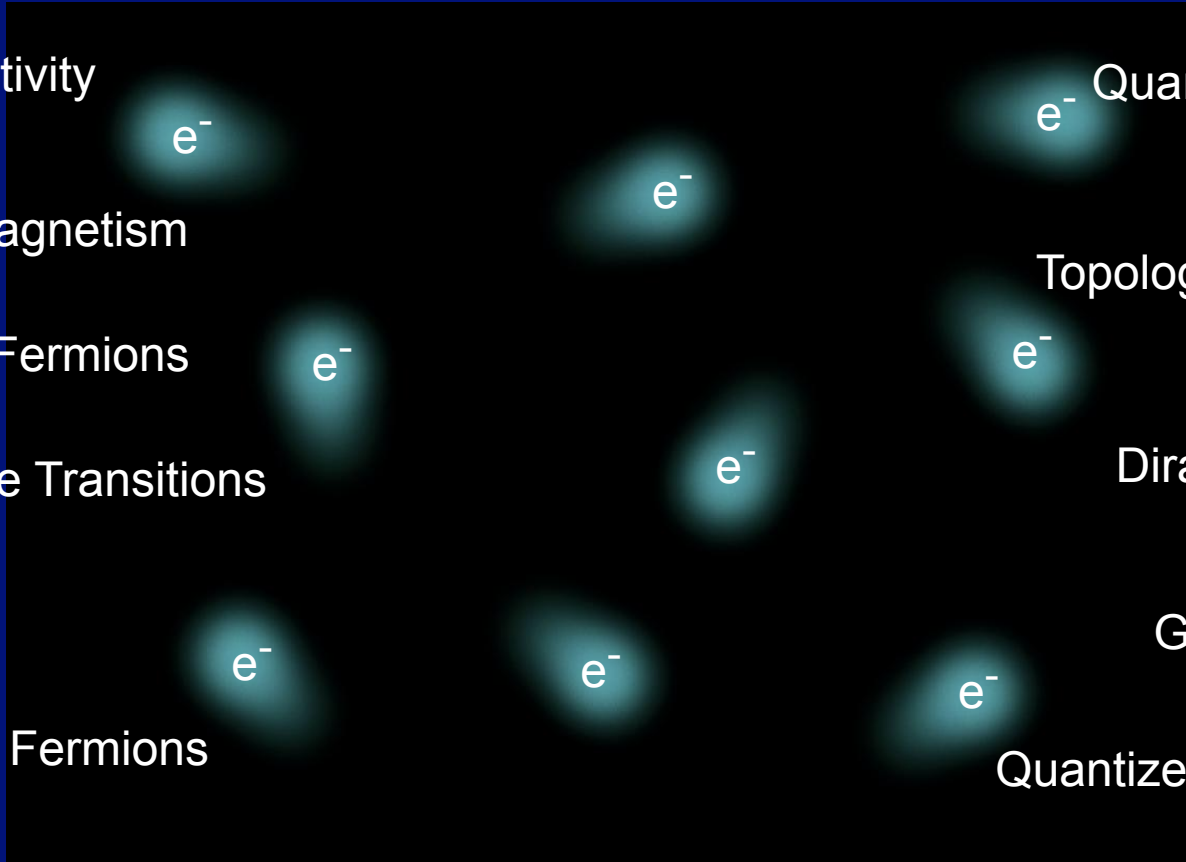
Graphene

$e^-$

Quantized Conductance

High  $T_c$

Photovoltaic Quantum Hall effect





# Why cold atoms?

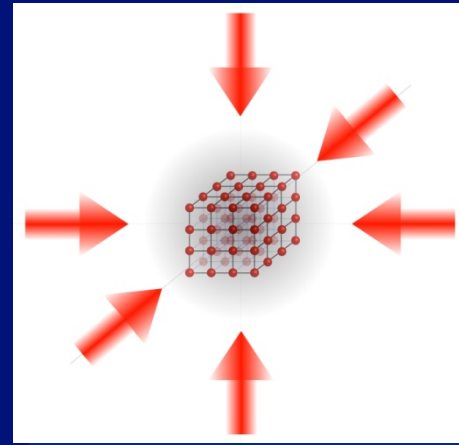
- Different approach
- New systems
- New regimes
- New questions
- Surprises



# Building the Hamiltonian



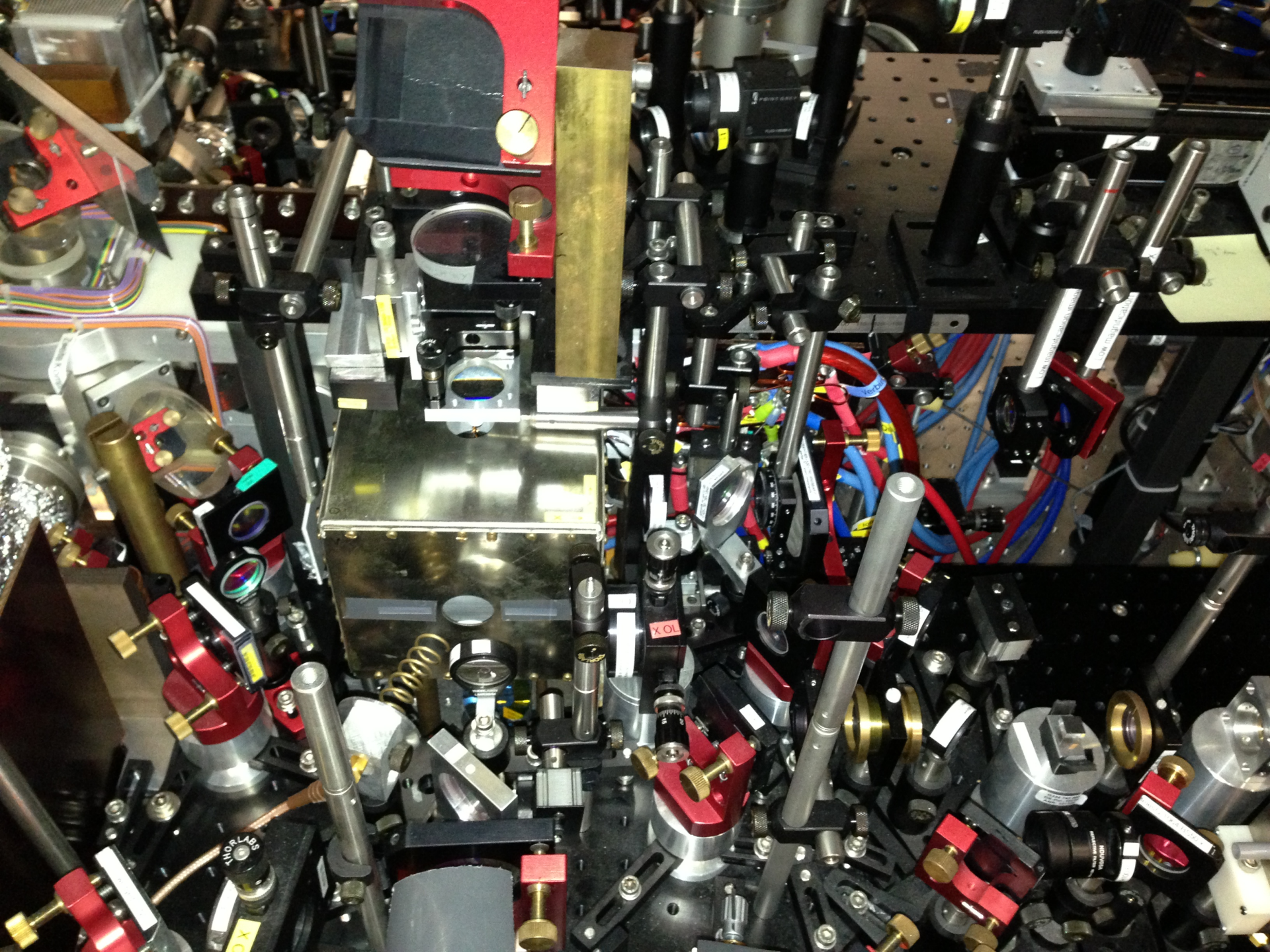
+



Quantum Gases ( $^{40}\text{K}$ )

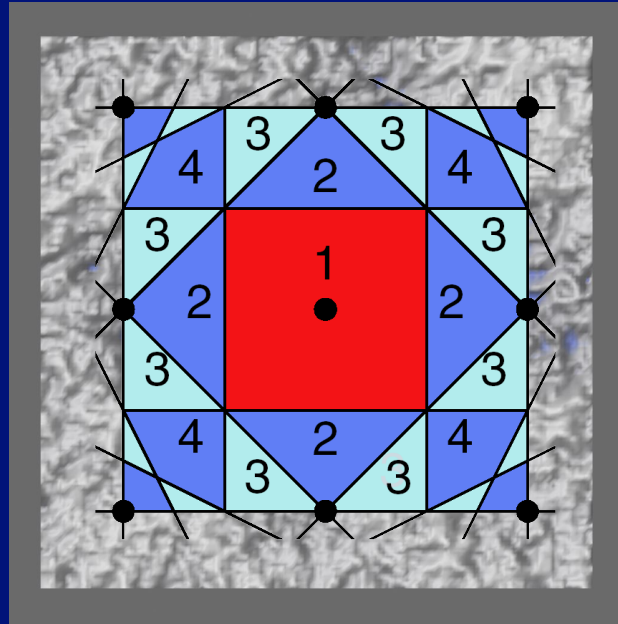
Optical Lattices

See also: Mainz/Munich, Hamburg, MIT, Illinois, Rice,...

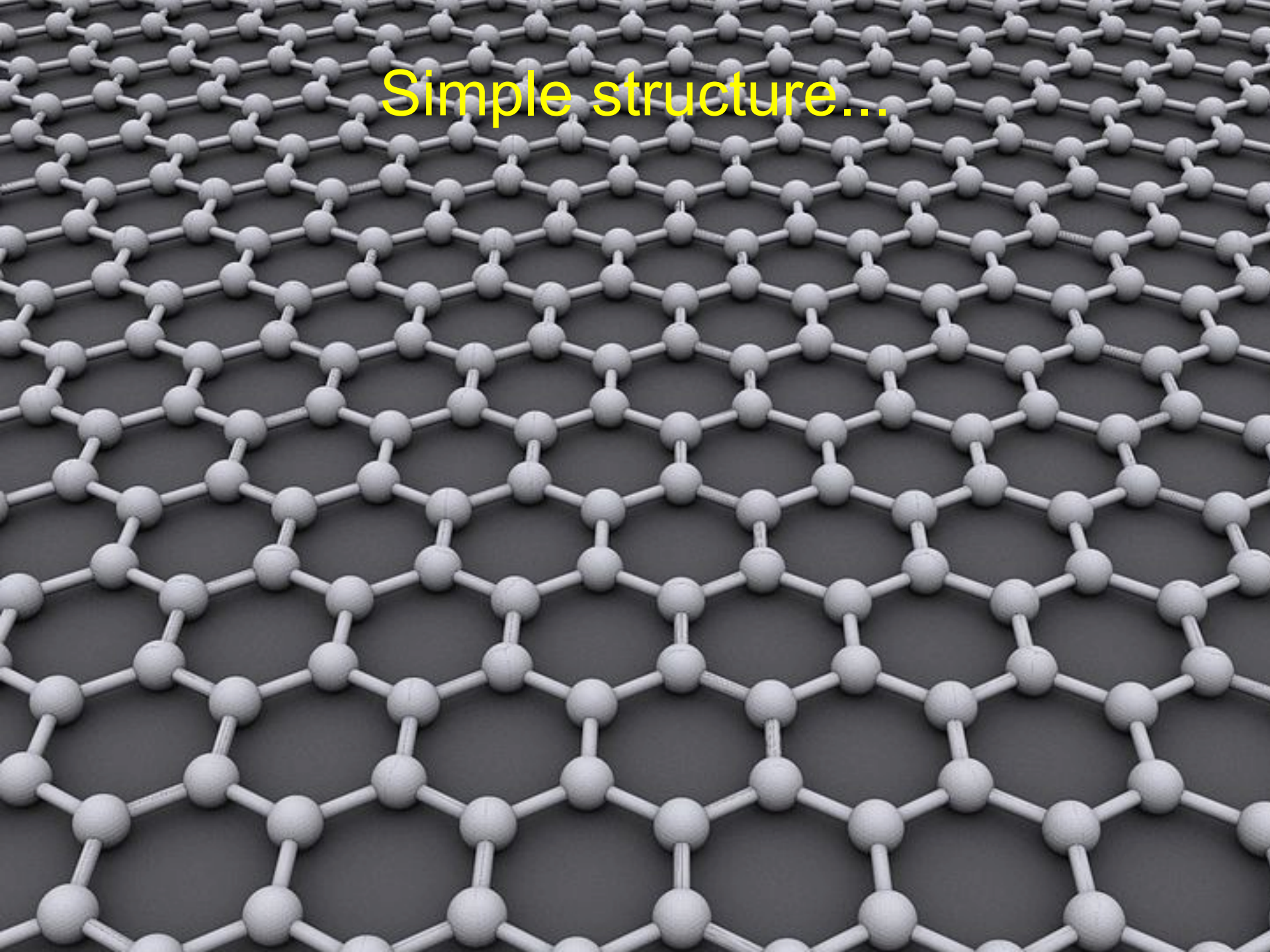




# Simple Measurement...



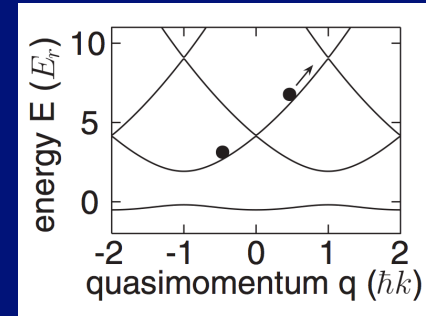
Simple structure...



# Bose gases in lattices with topological defects

## BEC in Excited bands:

- 1D « Dirac point » (Weitz group, Bonn)  
S. Kling *et al.*, Phys. Rev. Lett. 105, 215301 (2010)  
T. Salger *et al.*, Phys. Rev. Lett. 107, 240401 (2011)
- Quadratic avoided band crossing (Hemmerich group, Hamburg)  
M. Ölschläger *et al.*, Phys. Rev. Lett. 108, 075302 (2012)

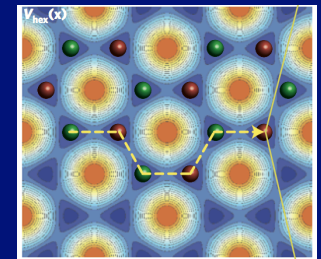


## BEC in a Honeycomb lattice:

(Sengstock group, Hamburg)

P. Soltan-Panahi *et al.*, Nature Phys. 7, 434 (2011)

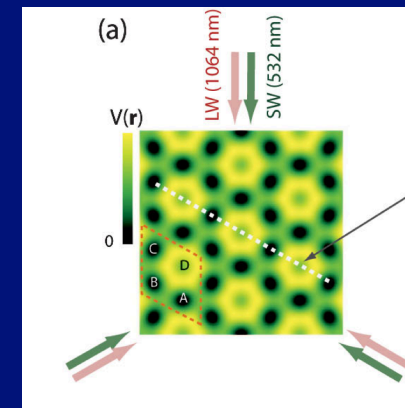
P. Soltan-Panahi *et al.*, Nature Phys. 8, 71 (2012)



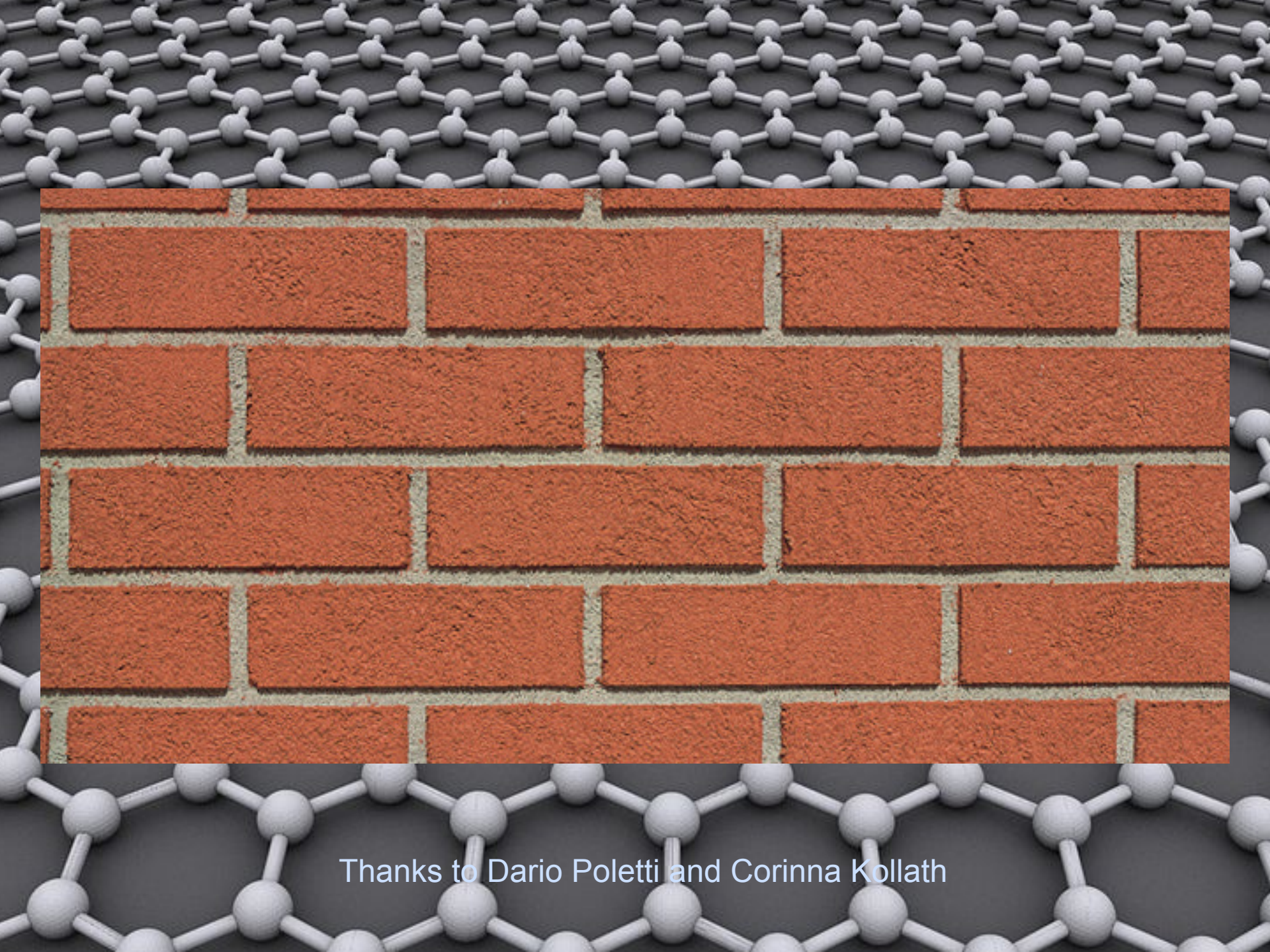
## BEC in Kagome:

(Dan Stamper-Kurn, Berkeley)

G.B. Jo *et al.* Phys. Rev. Lett. 108, 045305 (2012)

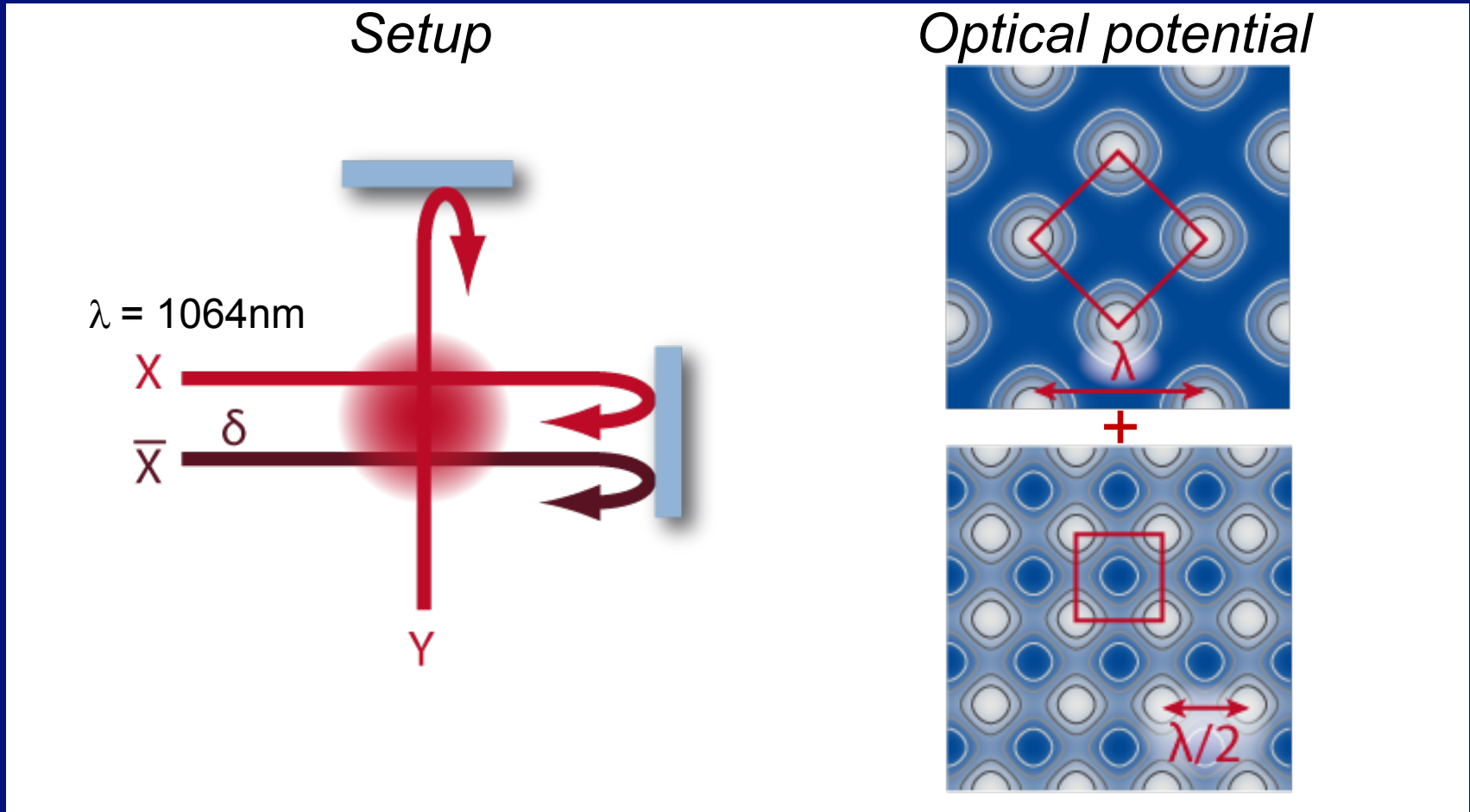






Thanks to Dario Poletti and Corinna Kollath

# Tunable Geometry Optical Lattice

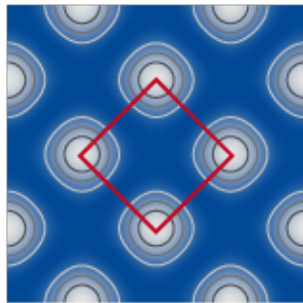


Other complex lattices: NIST, Munich, Hamburg, Berkeley

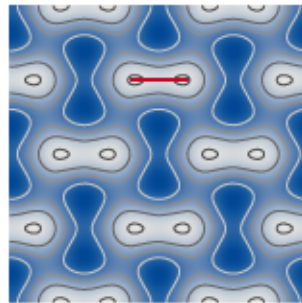


# Tunable Geometry Optical Lattice

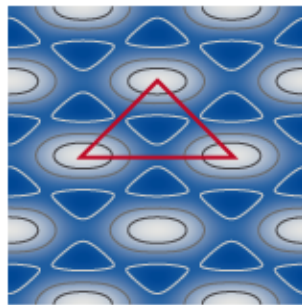
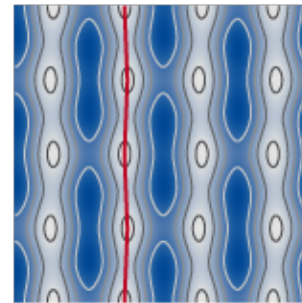
Chequerboard



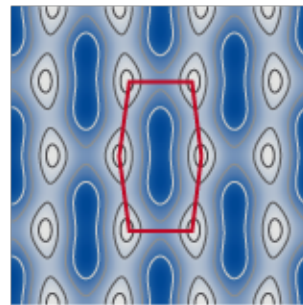
Dimer



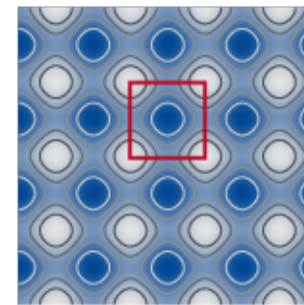
1D chains



Triangular



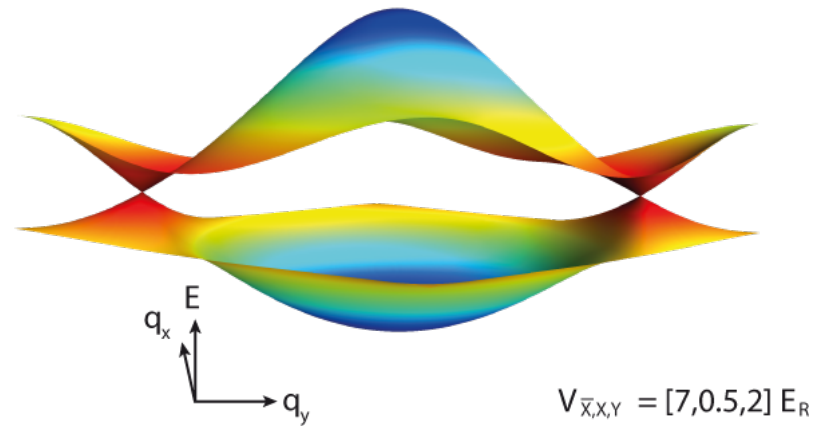
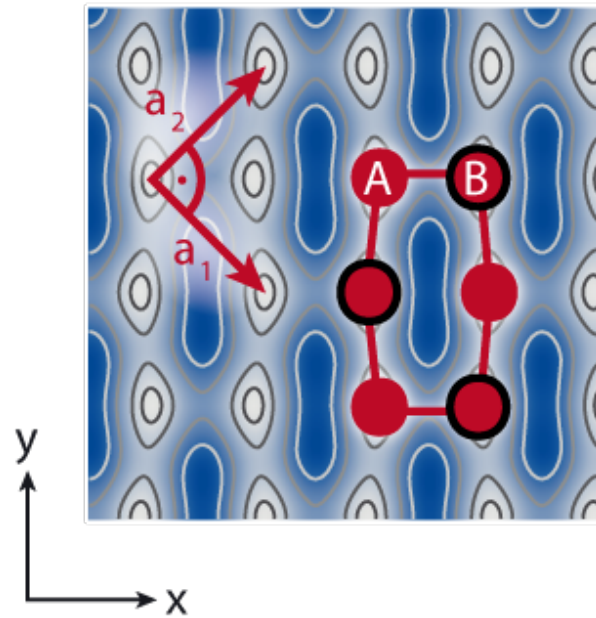
Honeycomb



Square



# Honeycomb Lattice



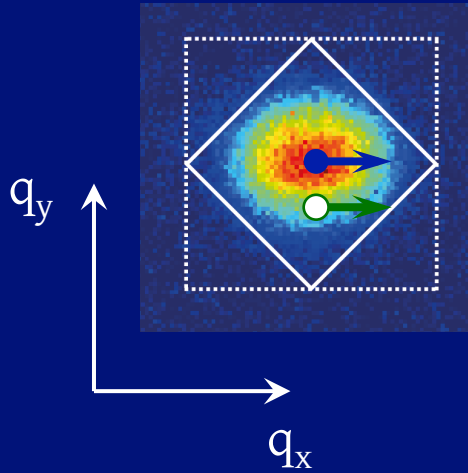
# Probing the Dirac points

vanishing density of states

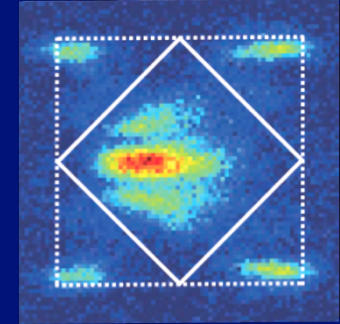
small energy scales

# Bloch oscillation and interband transitions

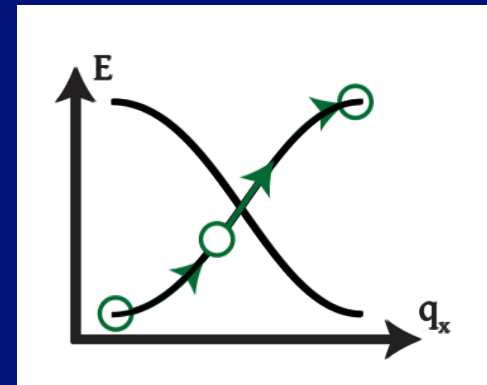
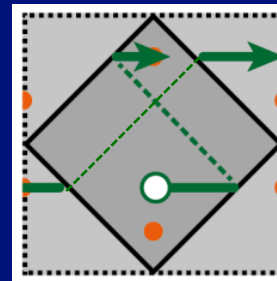
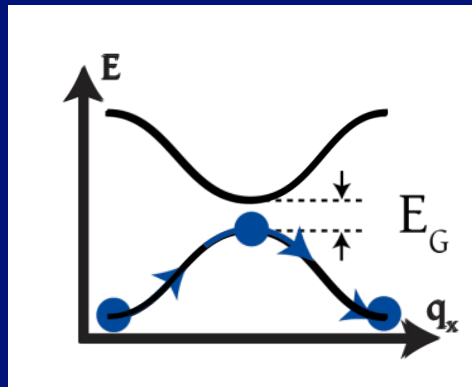
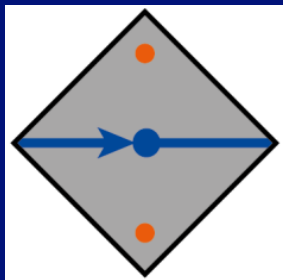
Starting point



+ magnetic gradient



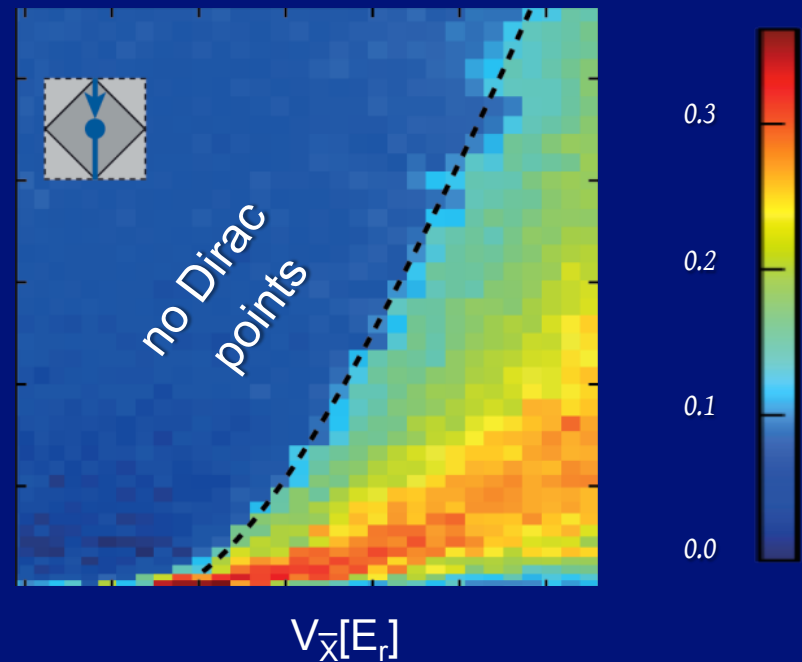
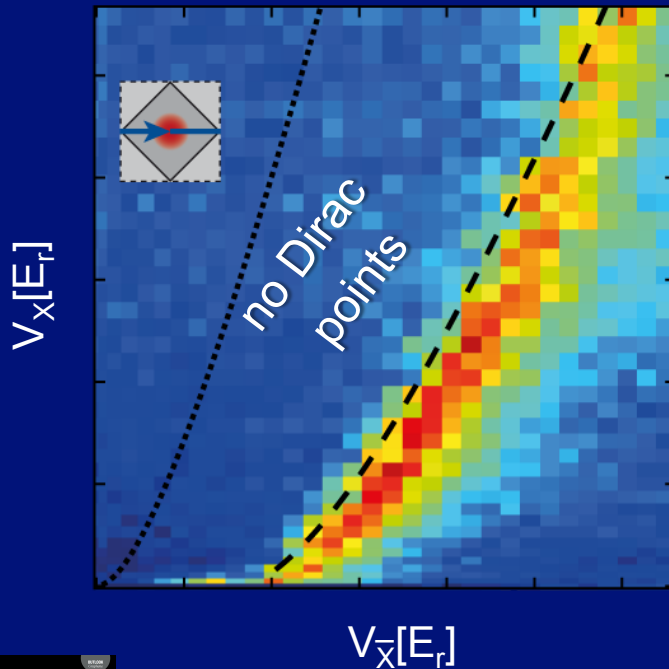
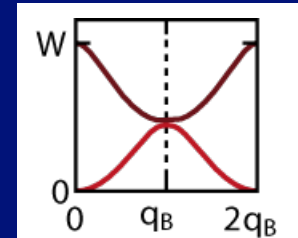
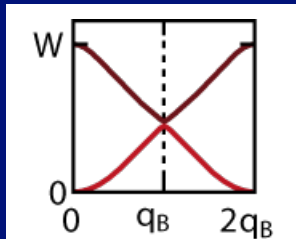
Transfer to 2nd band



Method in 1D: T. Salger et. al, Phys. Rev. Lett. 99, 190405 (2007)



# Touching Dirac points

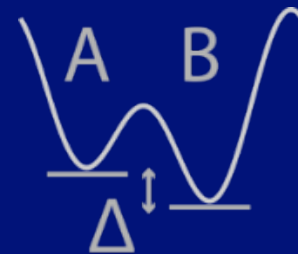
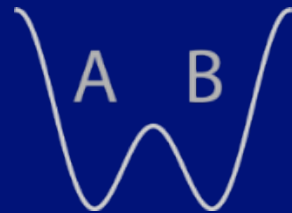
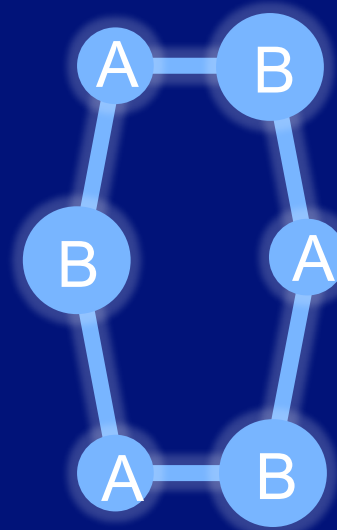
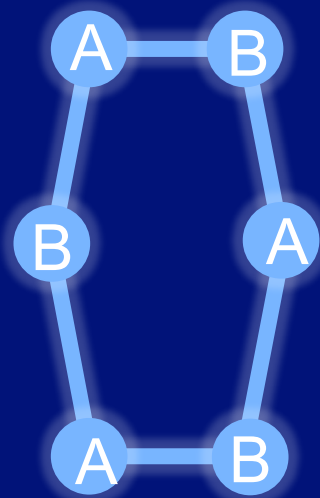


L. Tarruell, D. Greif, T. Uehlinger, G. Jotzu, and T. Esslinger, *Nature* 483, 302–305 (2012).

Theory, see also: L.-K. Lim, J.-N. Fuchs, G. Montambaux, *PRL* 108, 175303 (2012)

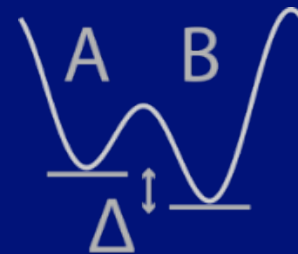
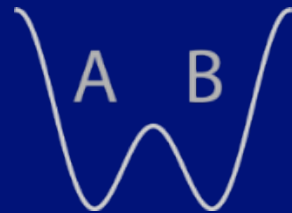
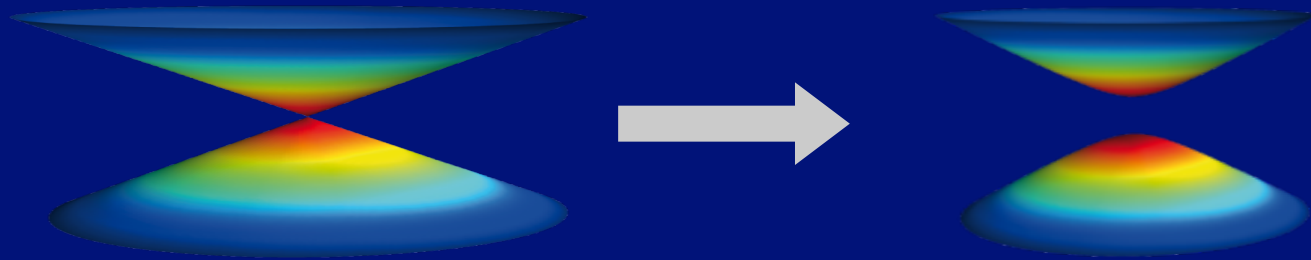
# Breaking Inversion Symmetry

# Breaking Inversion Symmetry



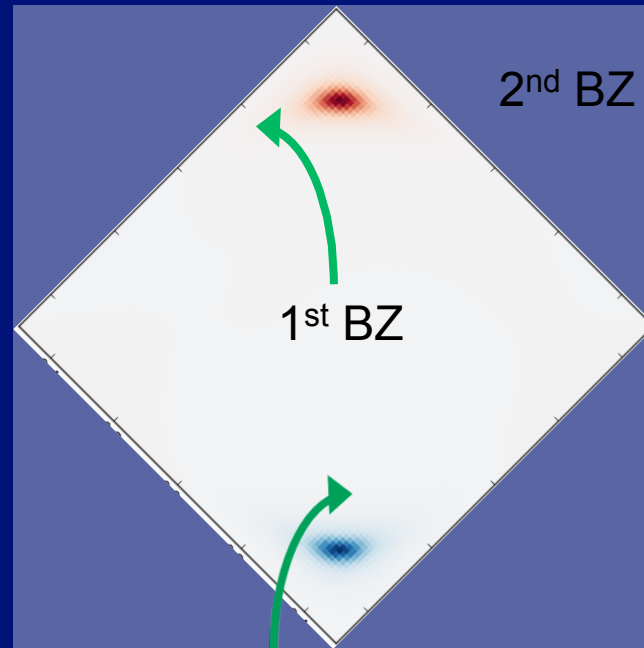


# Berry curvature



See also: L. Duca, Science 347, 288 (2015)

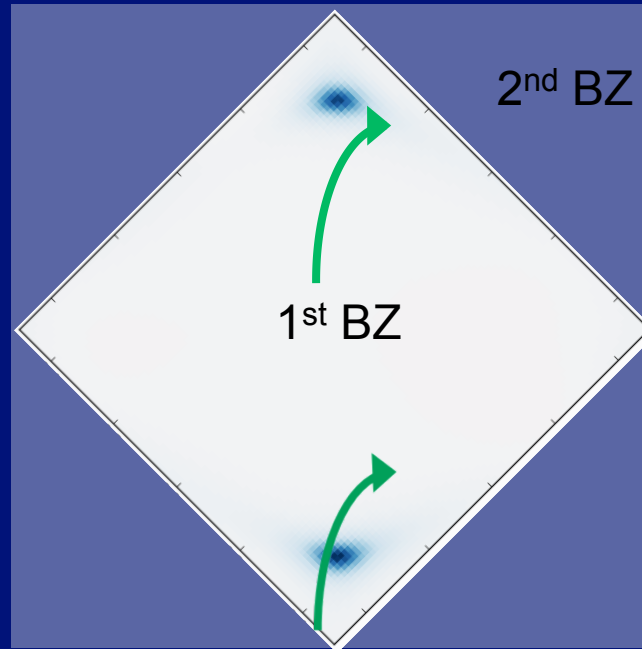
# Berry Curvature and Transverse Drift



$$\dot{\mathbf{r}} = \frac{1}{\hbar} \partial_{\mathbf{k}} \epsilon(\mathbf{k}) - \dot{\mathbf{k}} \times \boldsymbol{\Omega}(\mathbf{k})$$
$$\hbar \dot{\mathbf{k}} = \mathbf{F}(\mathbf{r})$$

Chang and Niu, PRL 75, 1348 (1995)  
Price and Cooper, PRA 85, 033620 (2012)

# Berry Curvature and Transverse Drift



Like a Hall current

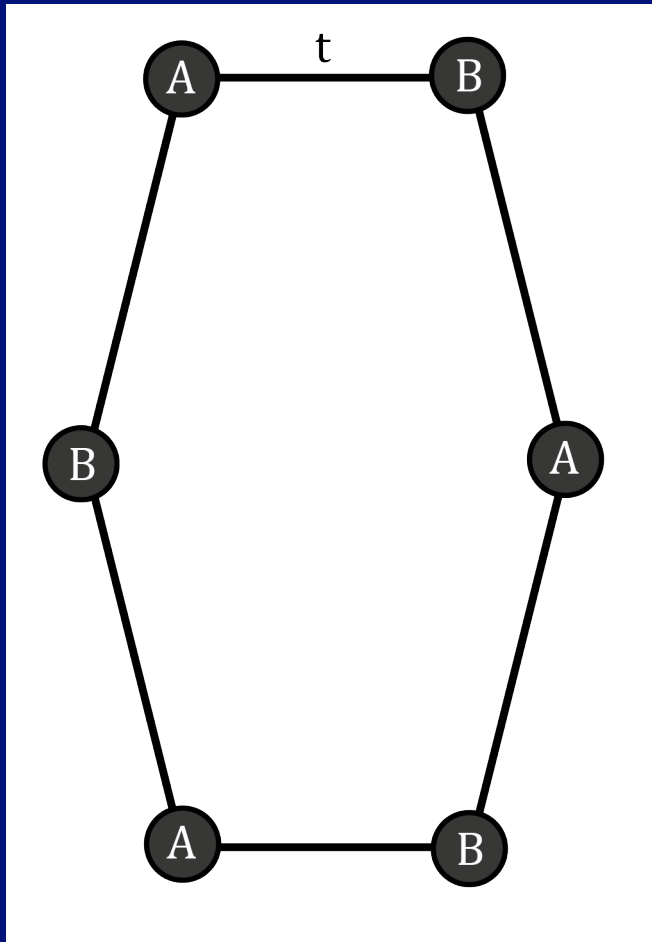


# Topological Haldane model

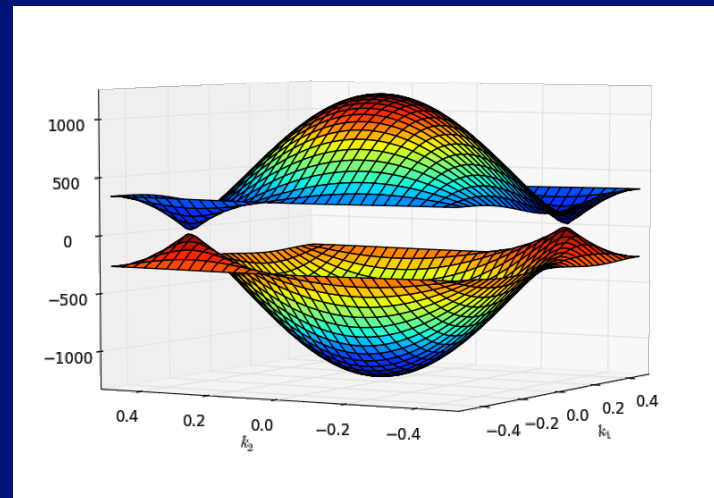
Proposal for Quantum Hall Effect *without* magnetic field!

Haldane, PRL **61**,2015-2018 (1988)

# Topological Haldane model



Start from a honeycomb lattice



inversion and time-reversal symmetry

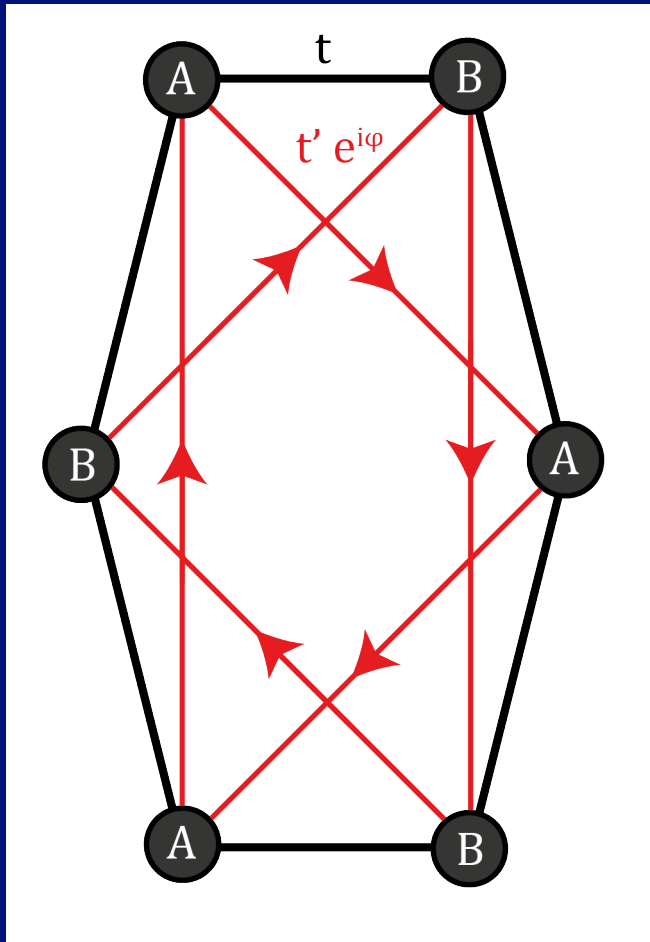


**Topological Haldane model  
break time-reversal symmetry**

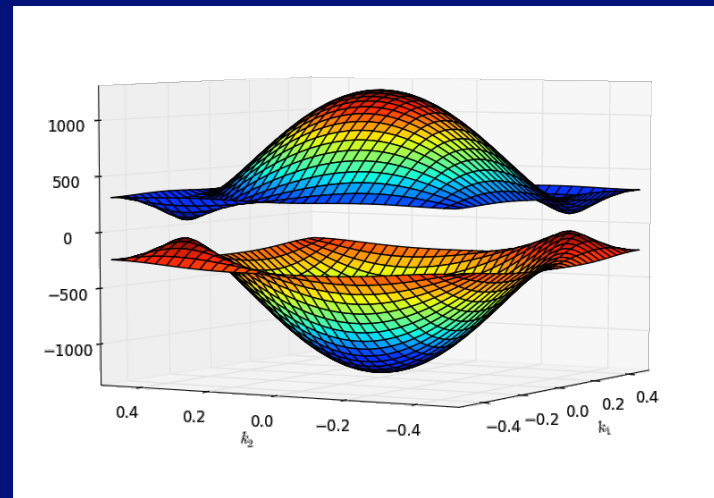




# Topological Haldane model

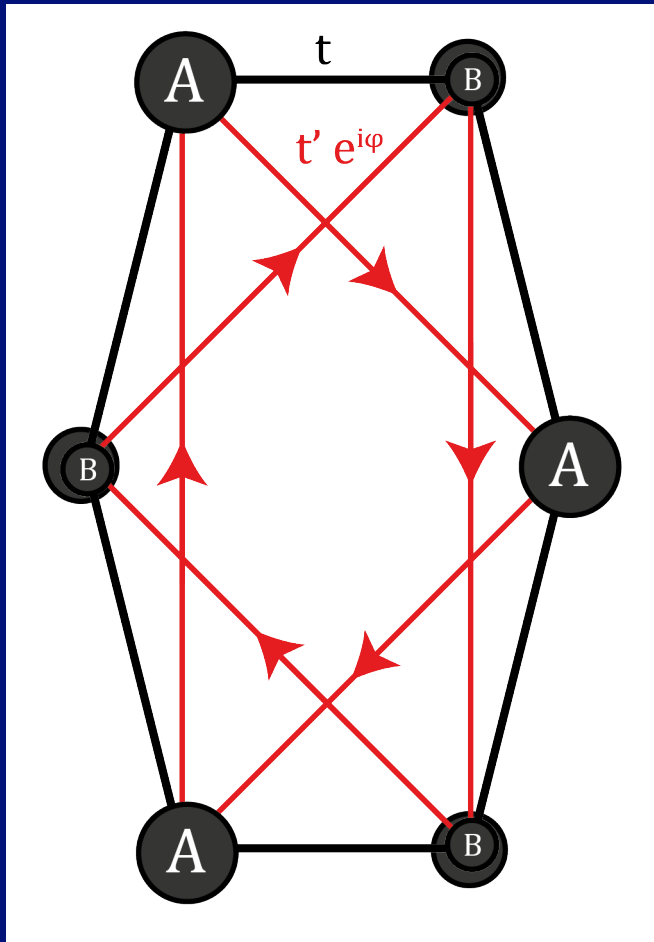


break time-reversal symmetry with complex next-nearest neighbour tunnellings

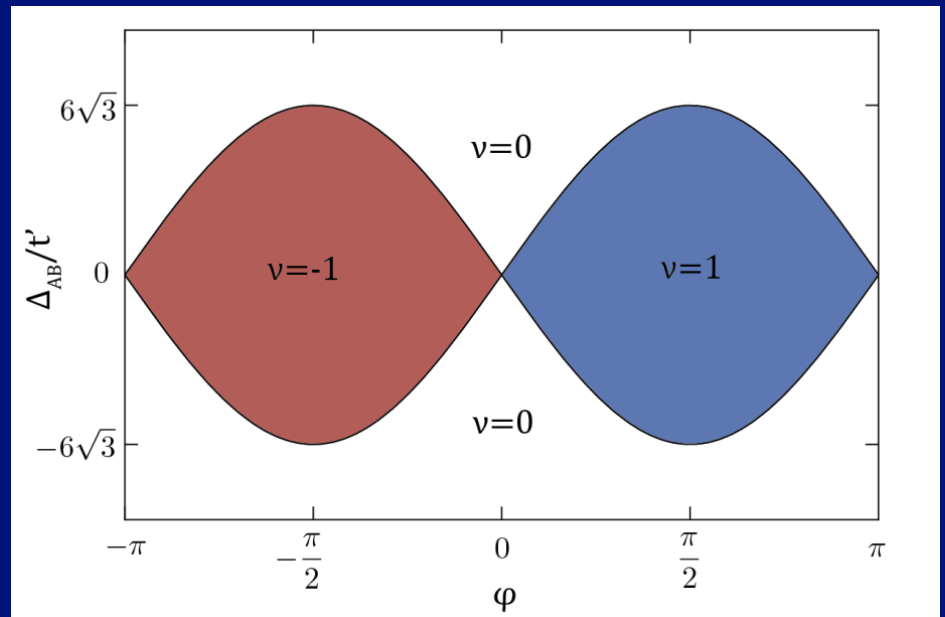


→ Topological Chern insulator, with non-zero Hall conductance

# Topological Haldane model

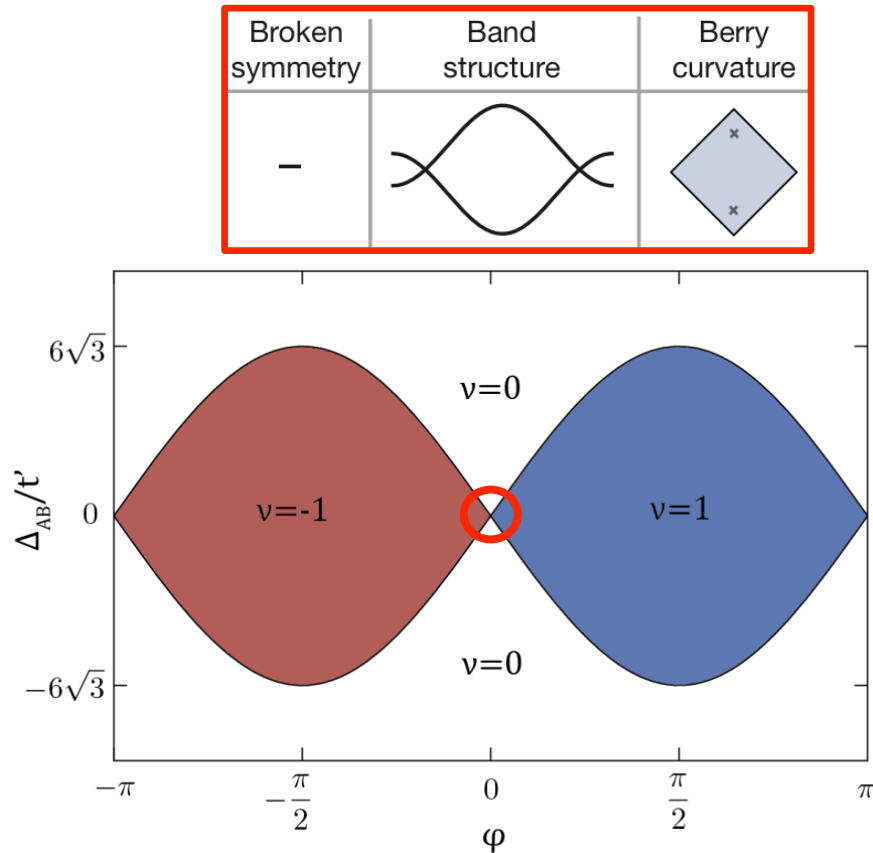


Additionally break time-reversal symmetry with energy offset



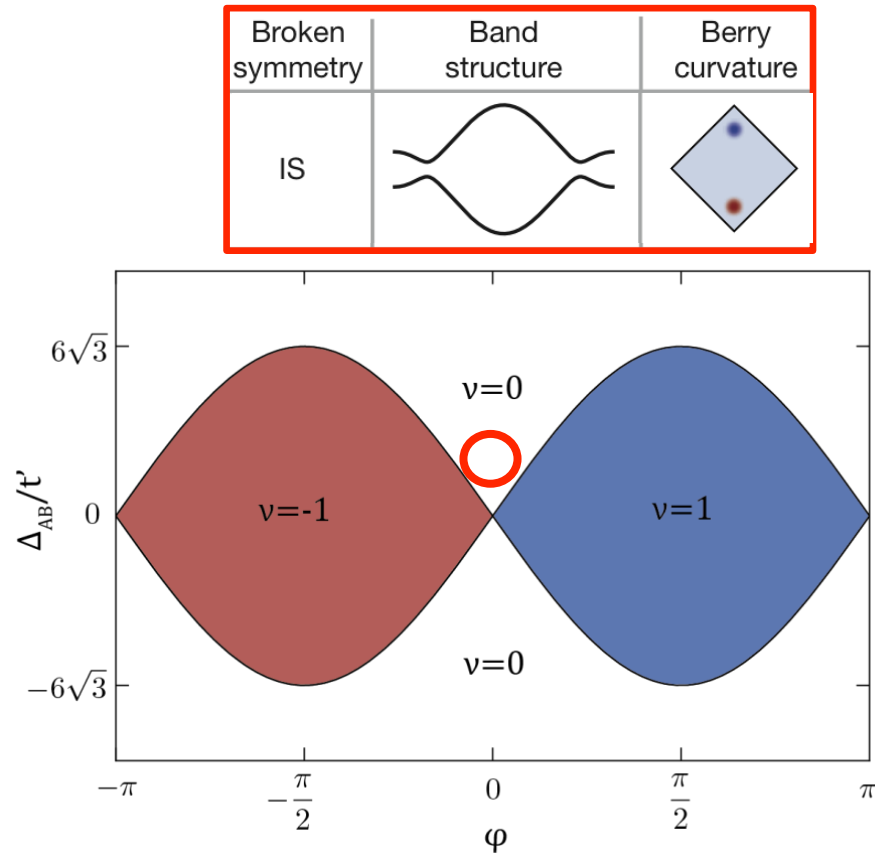
→ Distinct topological phases

# Topological Haldane model



Haldane, PRL **61**,2015-2018 (1988)

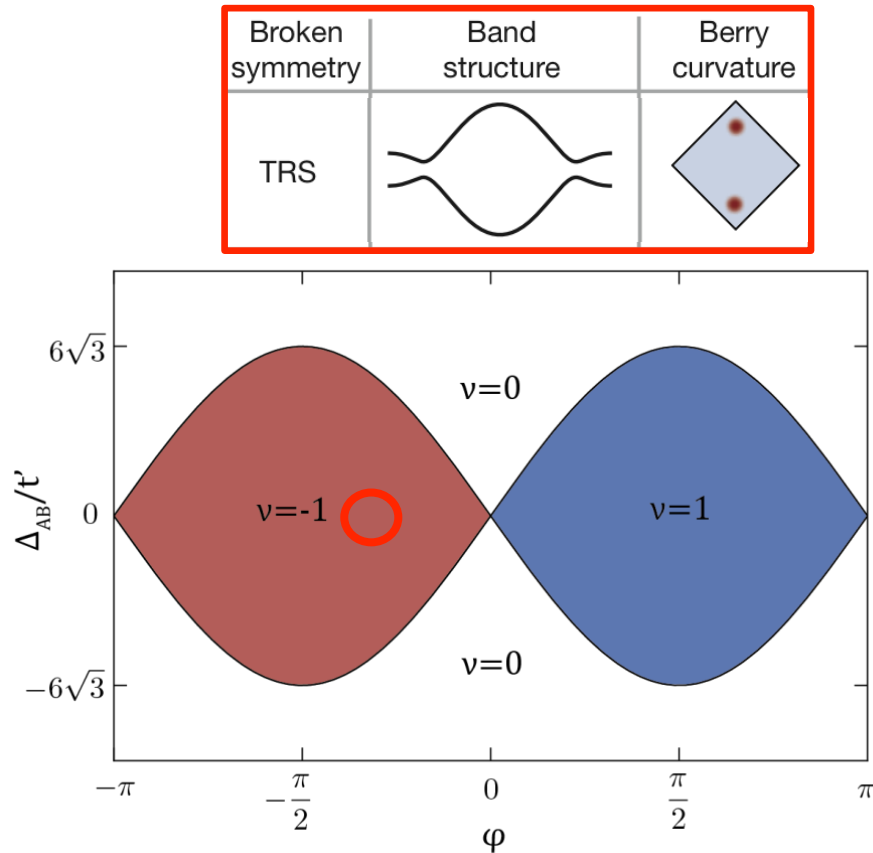
# Topological Haldane model



Haldane, PRL **61**,2015-2018 (1988)

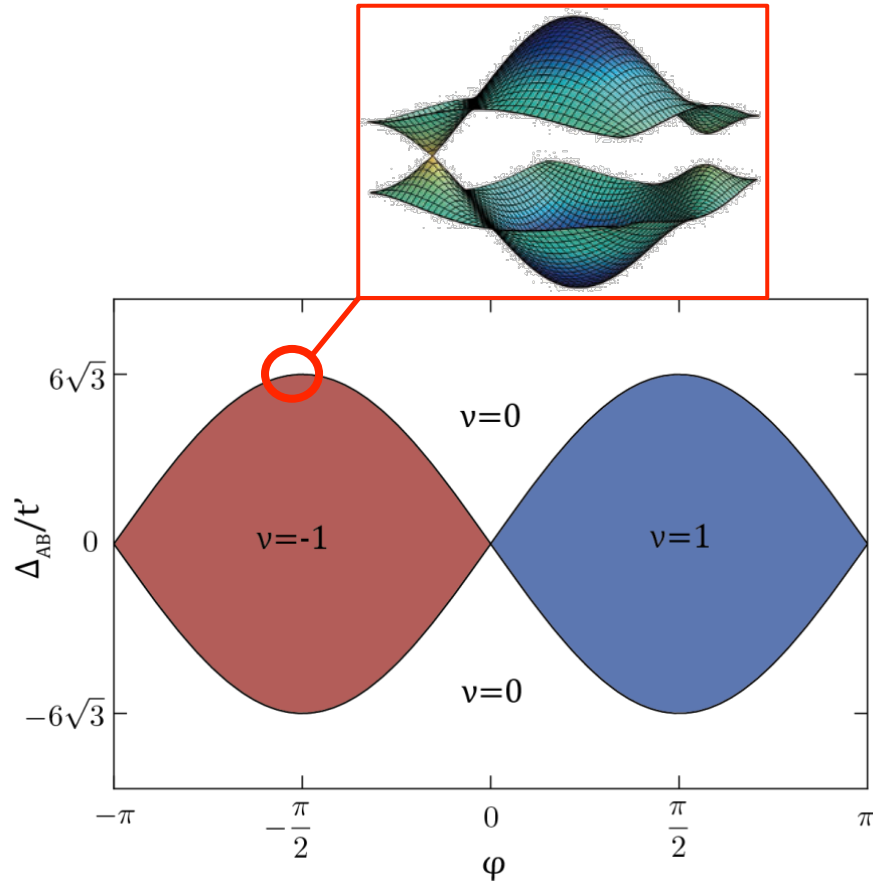


# Topological Haldane model



Haldane, PRL **61**,2015-2018 (1988)

# Topological Haldane model



Haldane, PRL **61**,2015-2018 (1988)

How?



geometrical constant of order unity, and  $g$  is the Landé  $g$  factor for the electrons.

While the particular model presented here is unlikely to be directly physically realizable, it indicates that, at least in principle, the QHE can be placed in the wider context of phenomena associated with broken time-reversal invariance, and does not necessarily require external magnetic fields, but could occur as a consequence of magnetic ordering in a quasi-two-dimensional system.



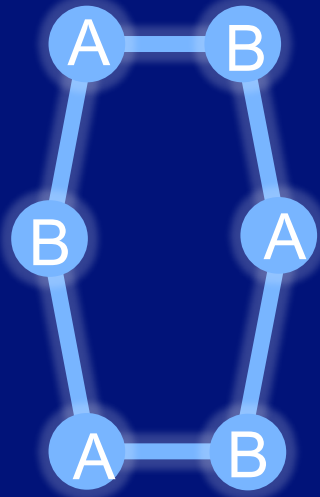
# Breaking time-reversal symmetry

Proposal for Photovoltaic Hall effect in graphene

T. Oka und H. Aoki, PRL **79**, 081406 (2009)



# Breaking time-reversal symmetry



Other proposals to realize topological Hamiltonians:

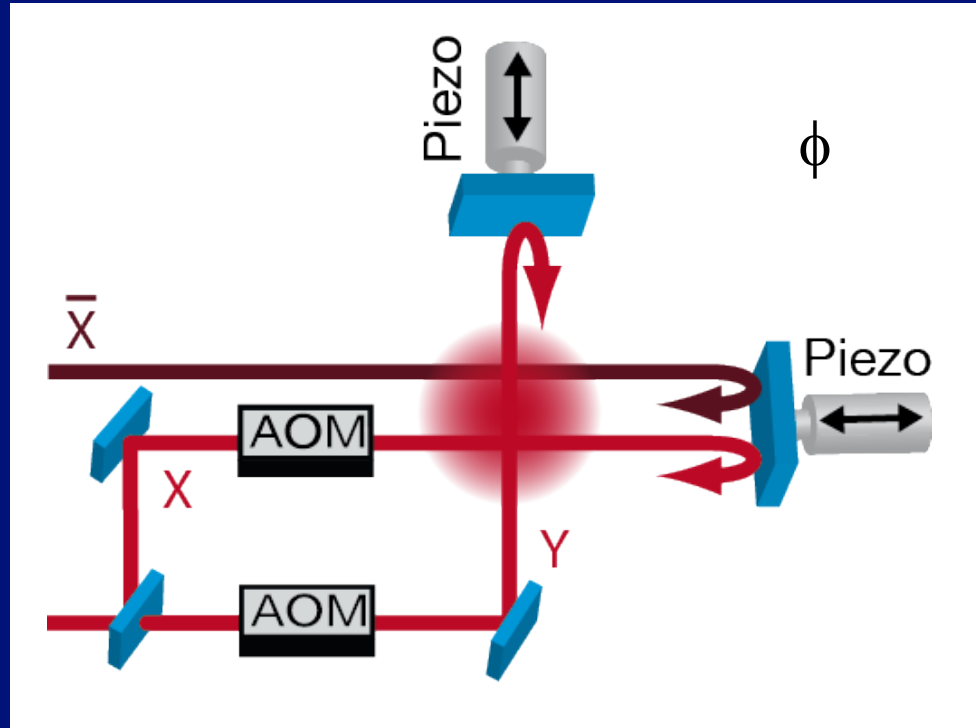
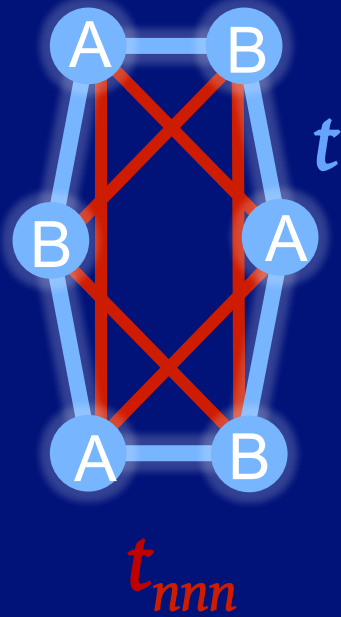
T. Kitagawa et al., Phys. Rev. B 82, 235114 (2010)

P. Hauke et al., Phys. Rev. Lett 109, 145301 (2012)

Realisation in photonic system: Rechtsman et. al Nature 496, 196–200 (2013)

# Breaking time-reversal symmetry

## Lattice Shaking



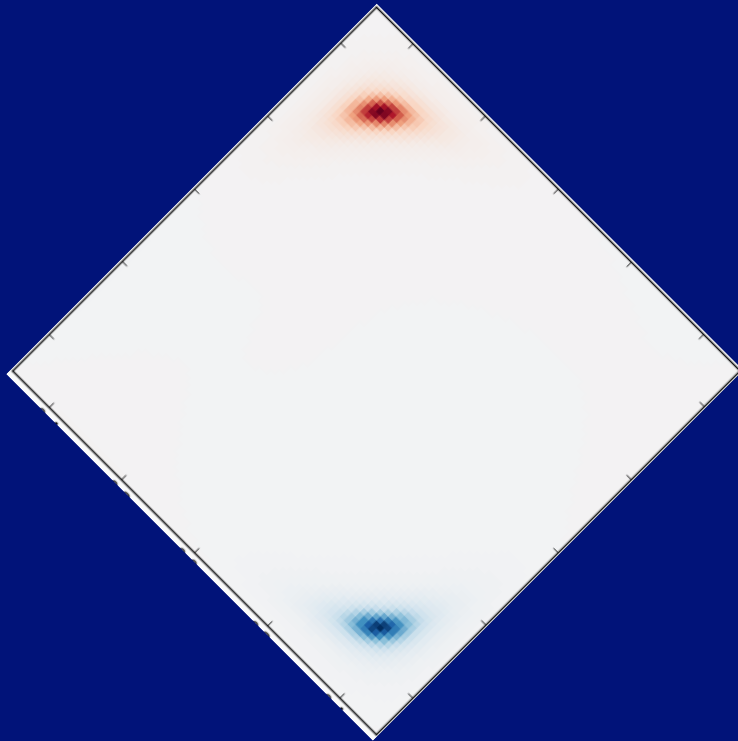
Lattice shaking: Pisa — Lignier, PRL **99**, 220403 (2007)

Hamburg/Barcelona — Struck, Science **333**, 996-9 (2011), PRL 108, 225304 (2012)

Chicago — Parker, Nat. Phys. **9**, 769-774 (2013)

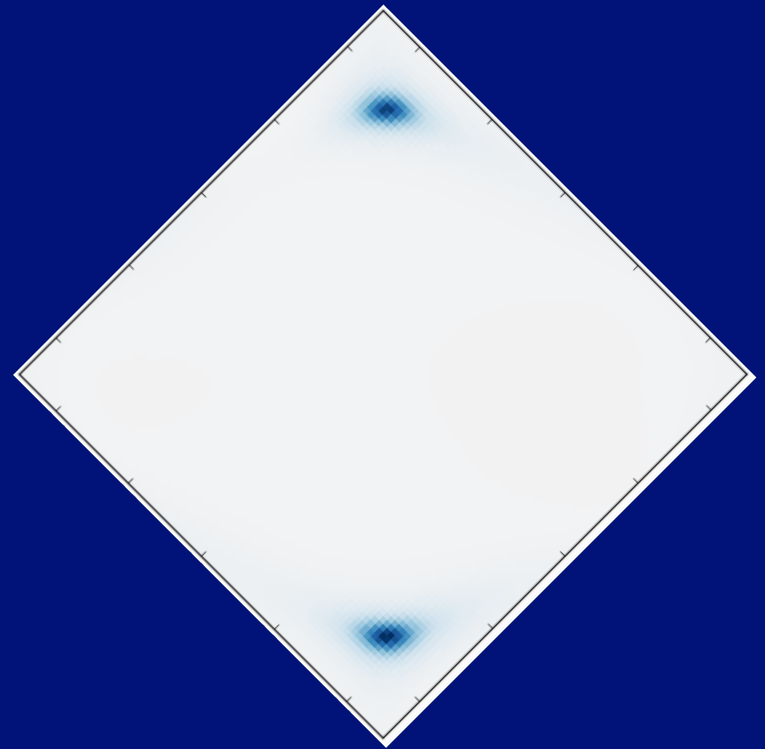
# Berry Curvature

Trivial band insulator



Chern number 0

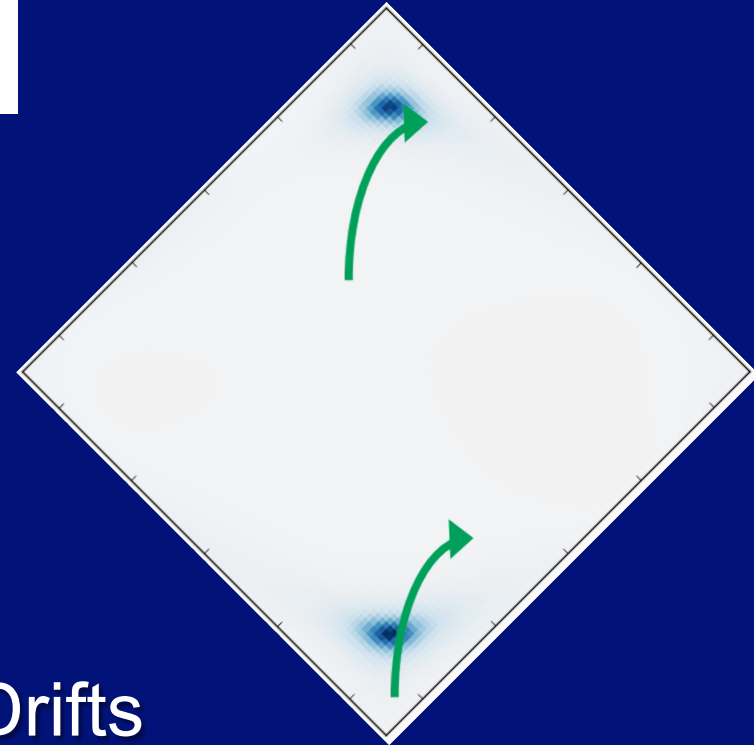
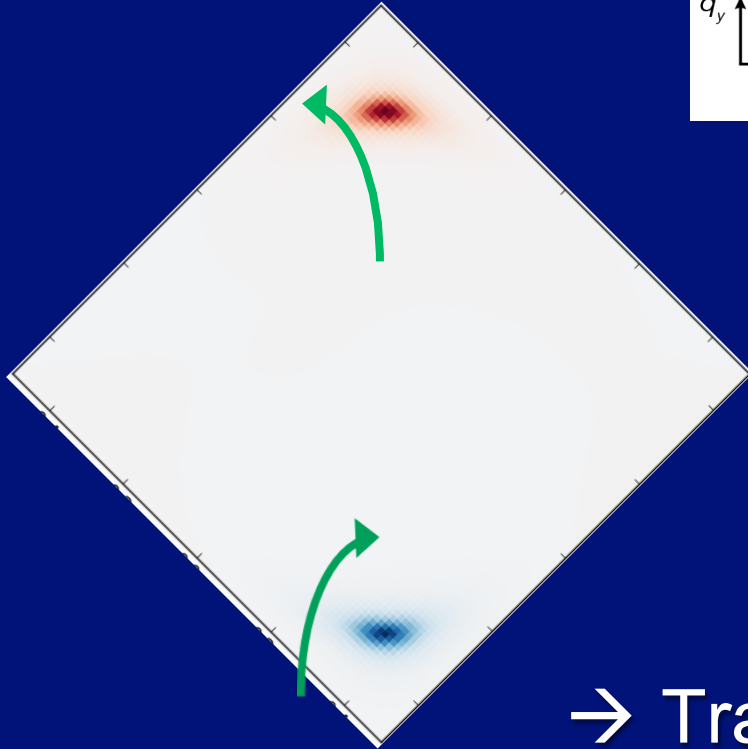
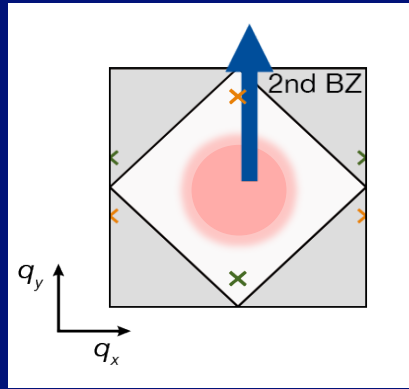
Chern insulator



Chern number -1

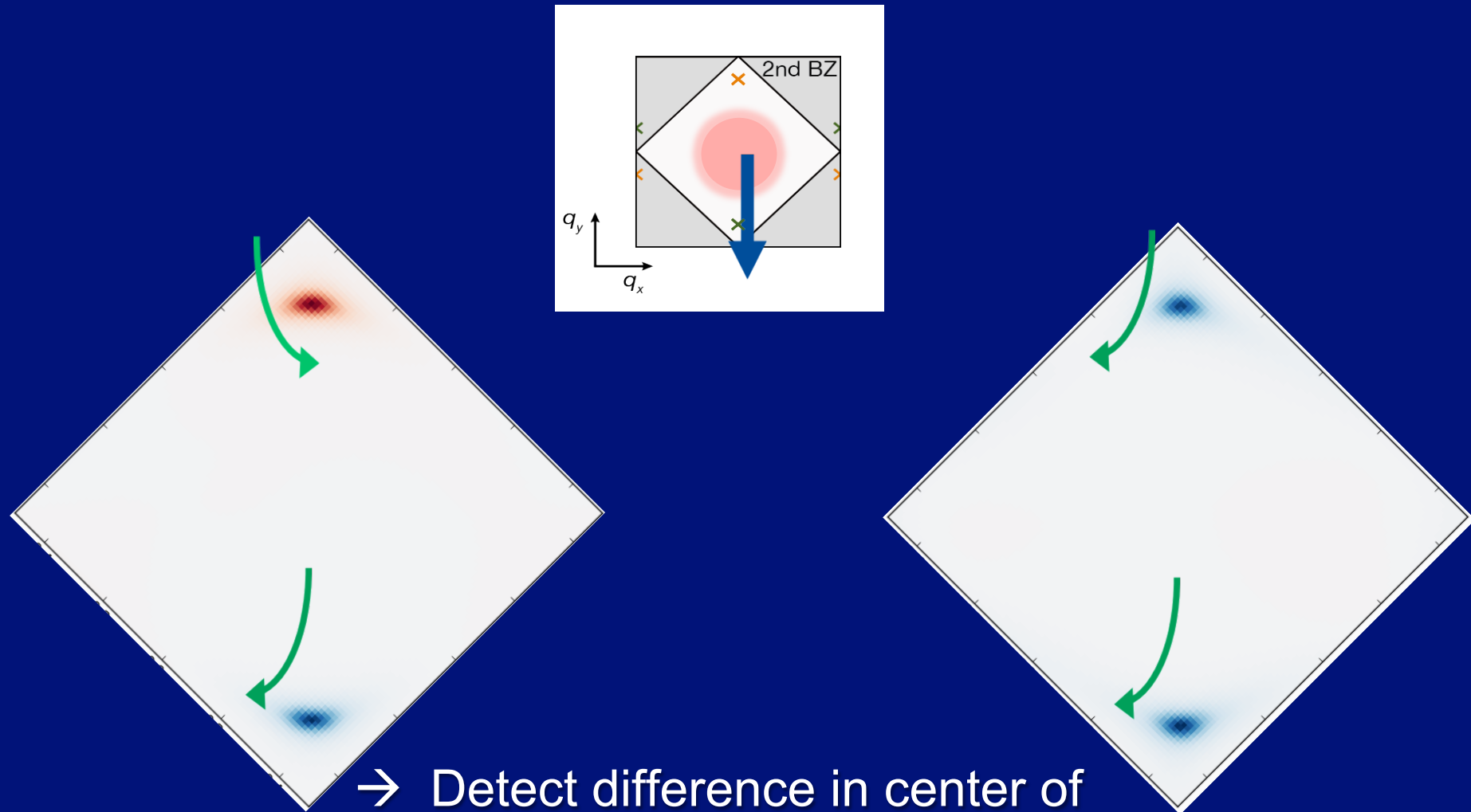


# Berry Curvature - Measurement



→ Transverse Drifts

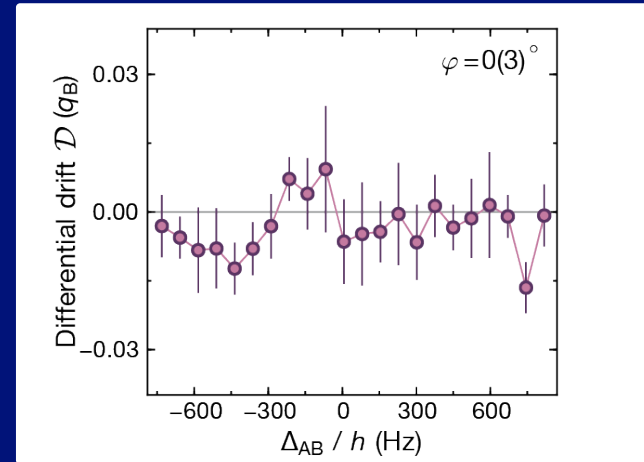
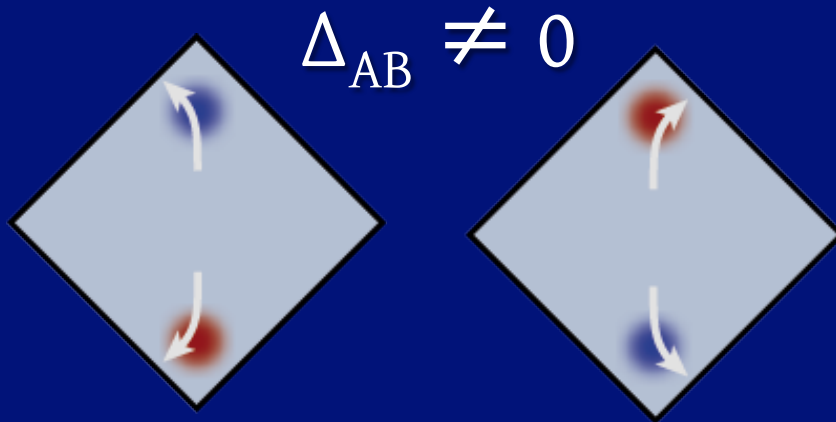
# Berry Curvature - Measurement



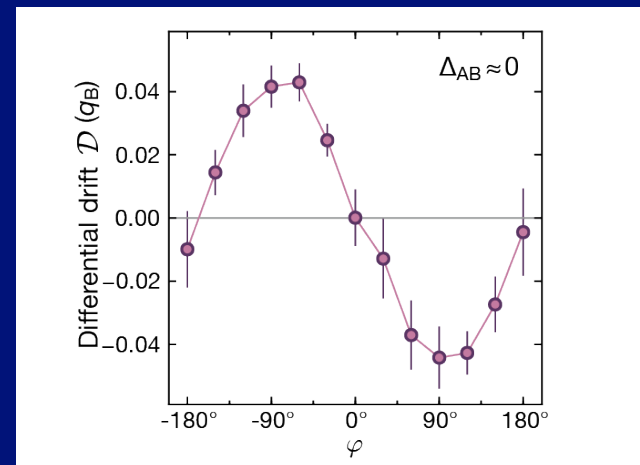
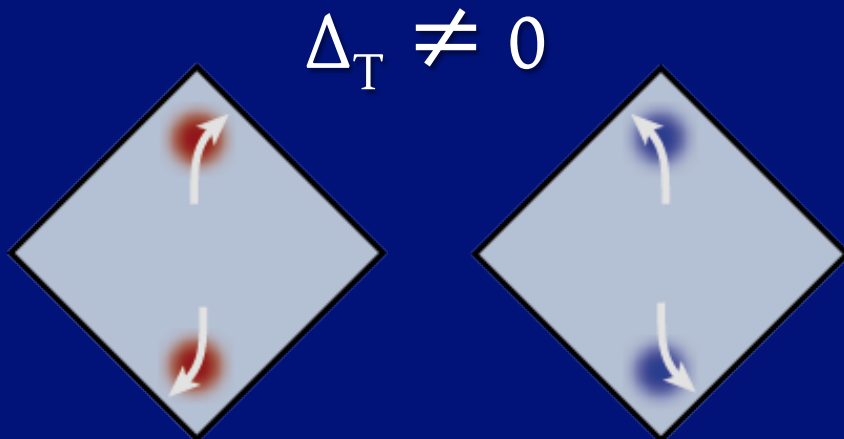
→ Detect difference in center of mass position after full Bloch cycle

# Topological features of the system

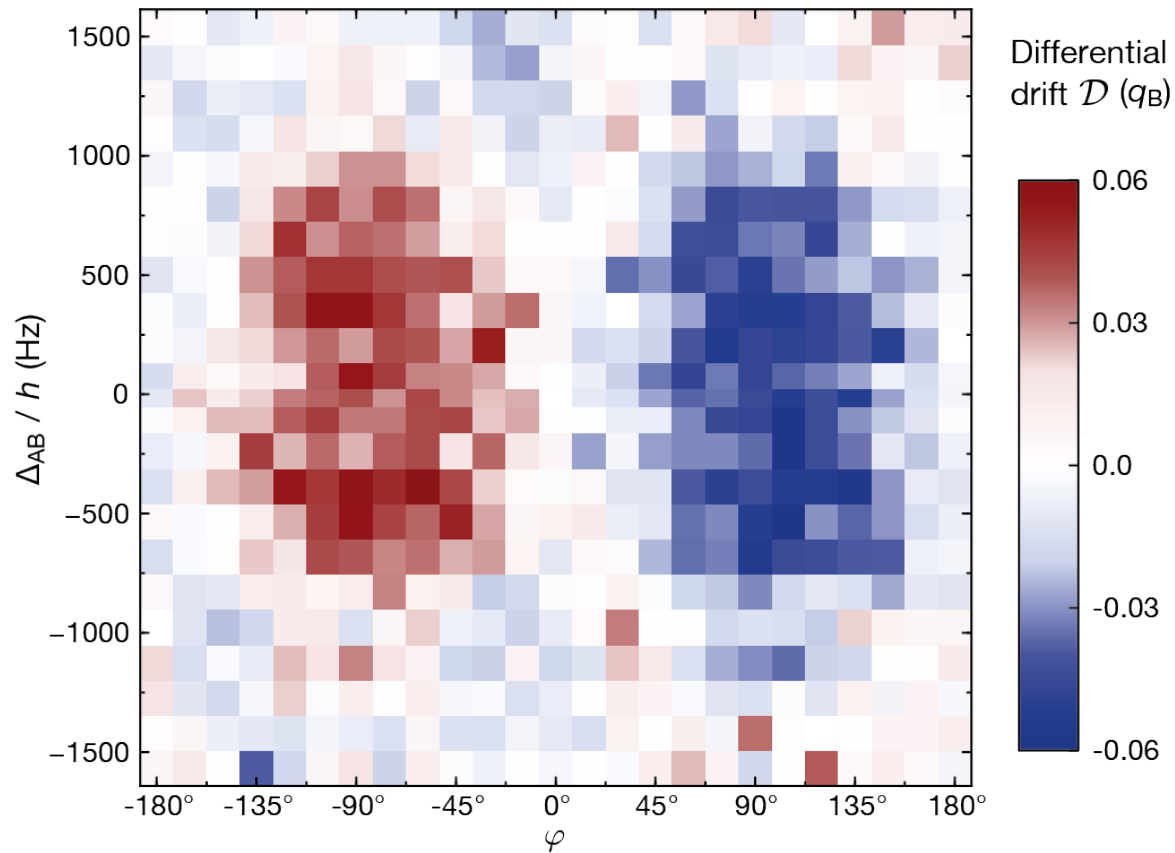
topologically trivial



nonzero Chern number

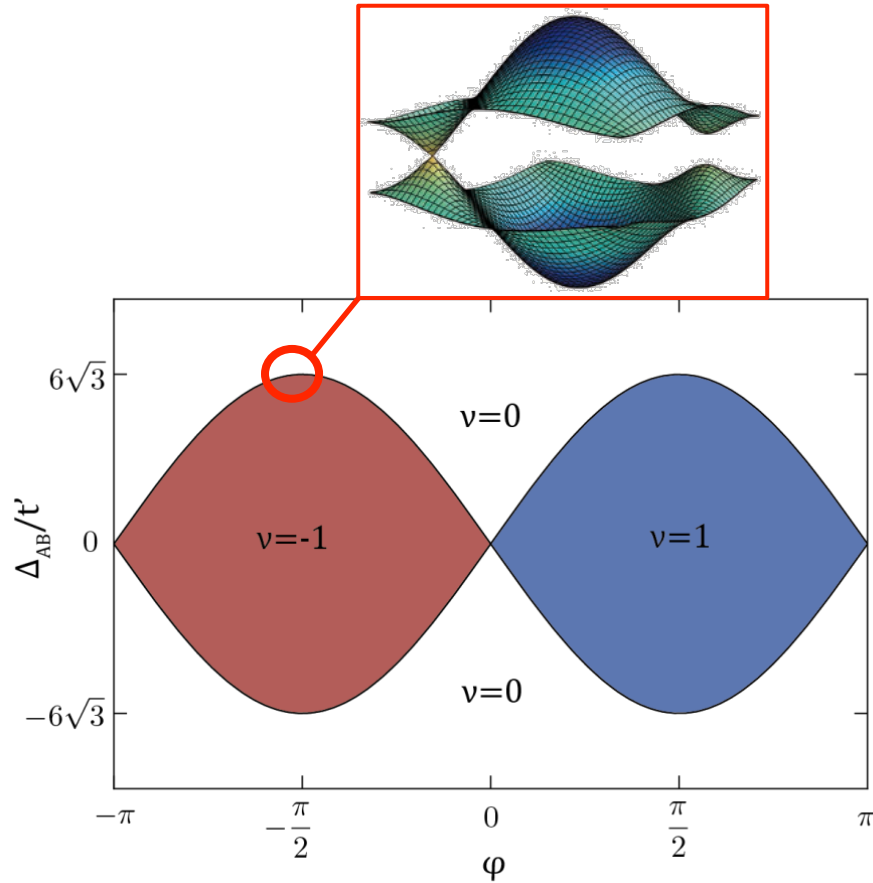


# Observing Transverse Drifts

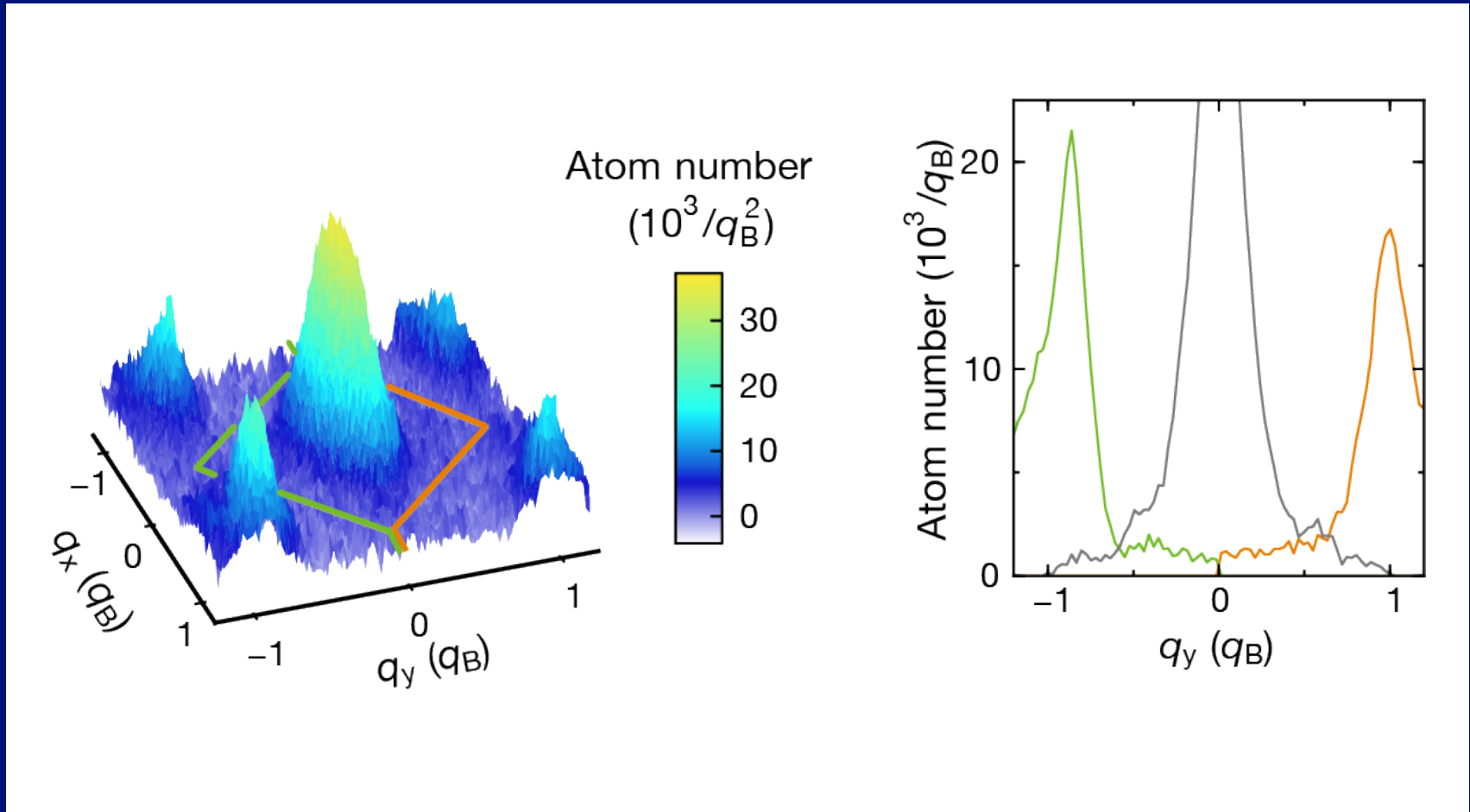




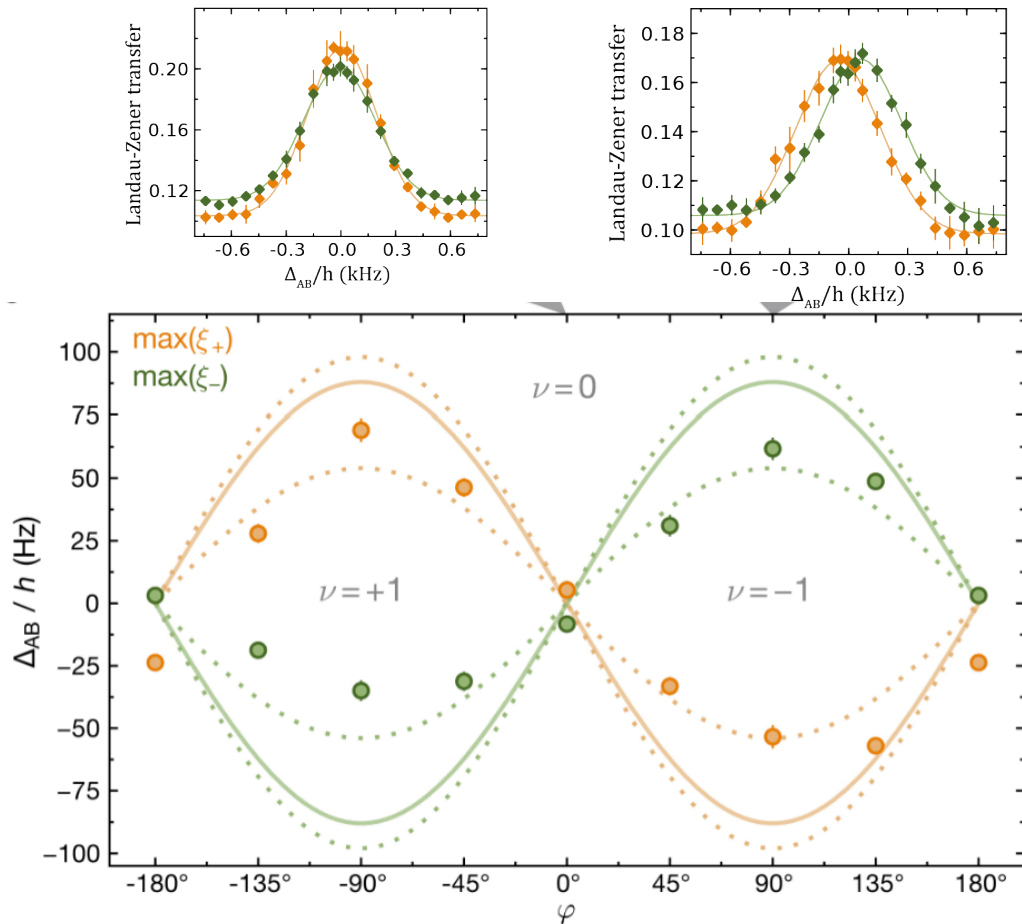
# Mapping out the transition line



# Mapping out the transition line



# Mapping out the transition line



# What about interactions?

Little is known

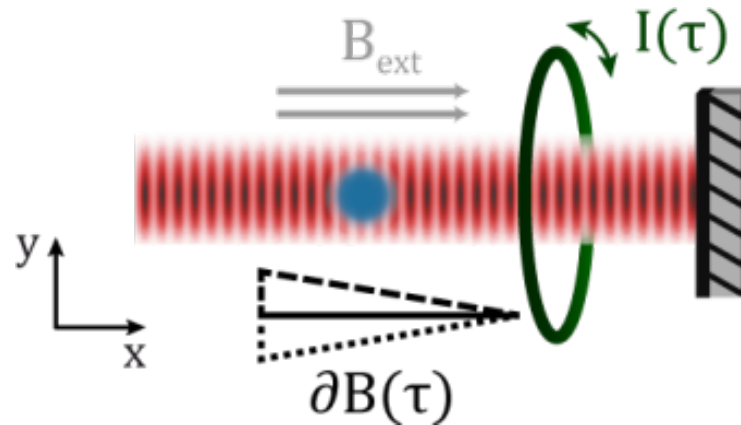


# What about interactions?

Loaded interacting gas into  
coupled layers of Haldane models

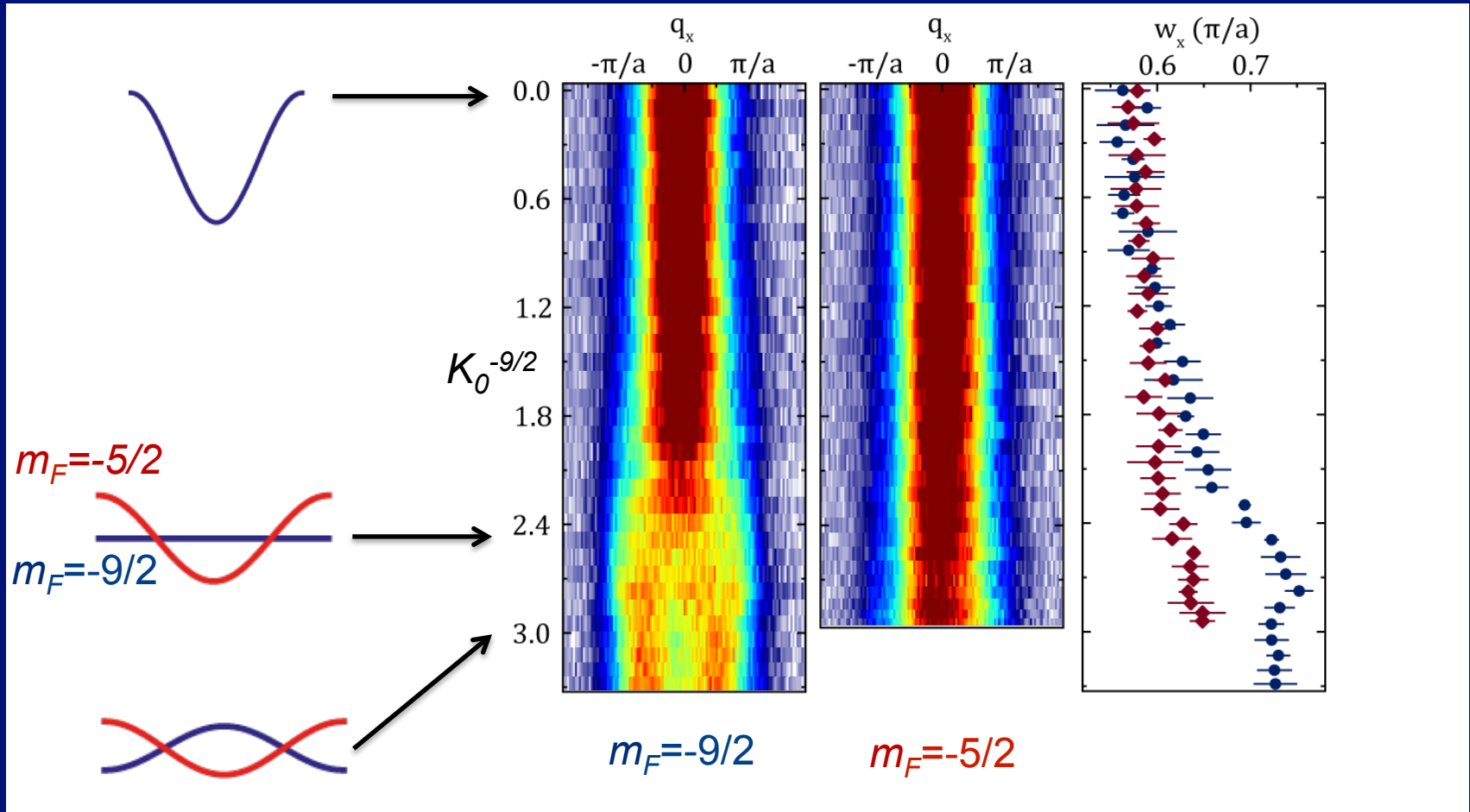
→ Observed no significant heating

# What about spin dependency?



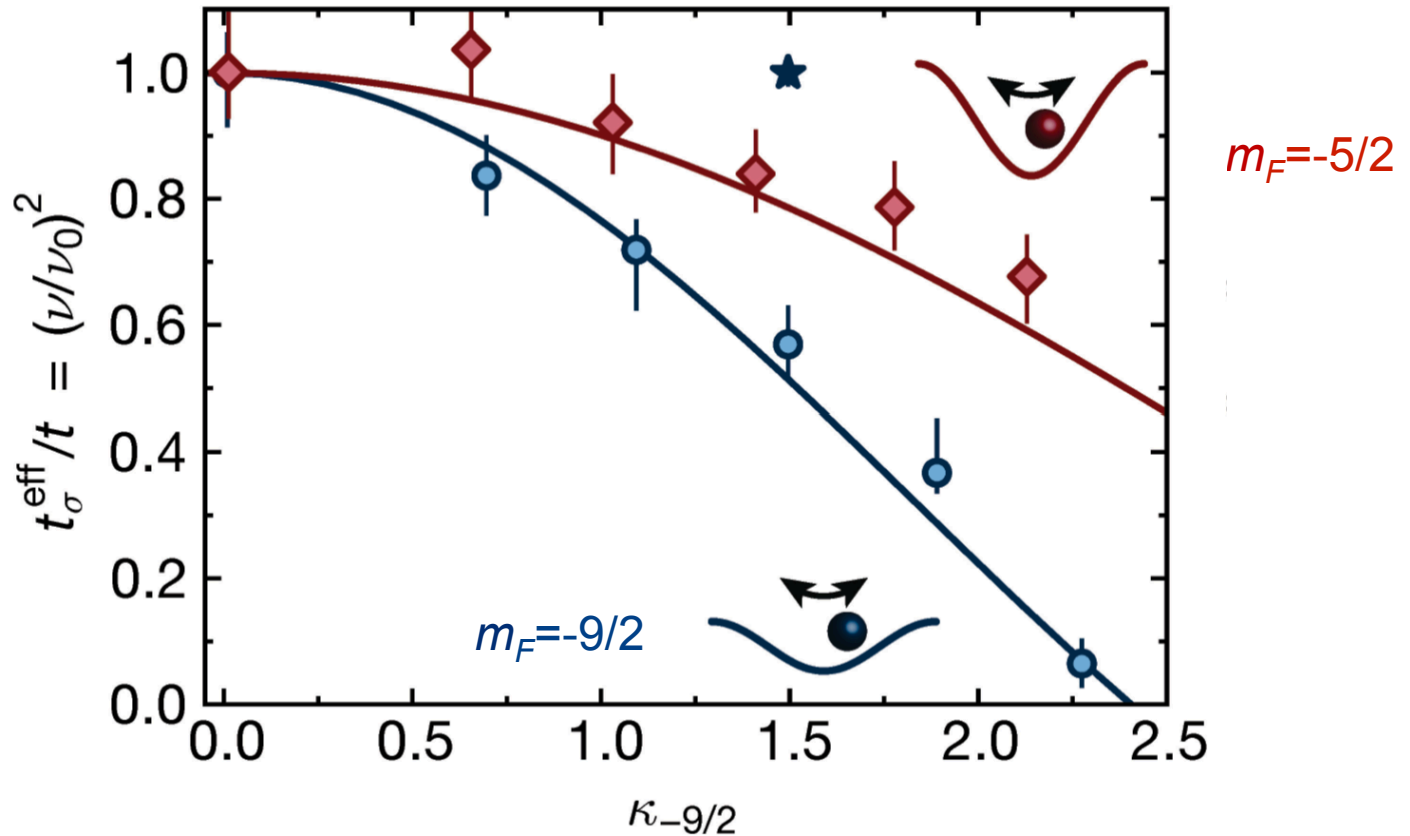
Proposed in: G. Jotzu, M. Messer, R. Desbuquois, M. Lebrat,  
T. Uehlinger, D. Greif, T. E., Nature 515, 237 (2014)

# What about spin dependence?

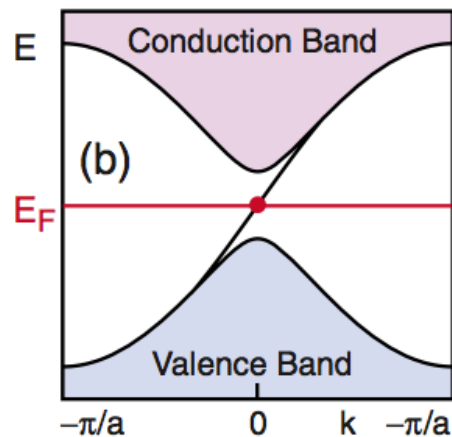


The band structure is different for each internal state

# What about spin dependence?



# What about quantized edge currents?



Hasan, Kane RMP 82, 3045 (2010)

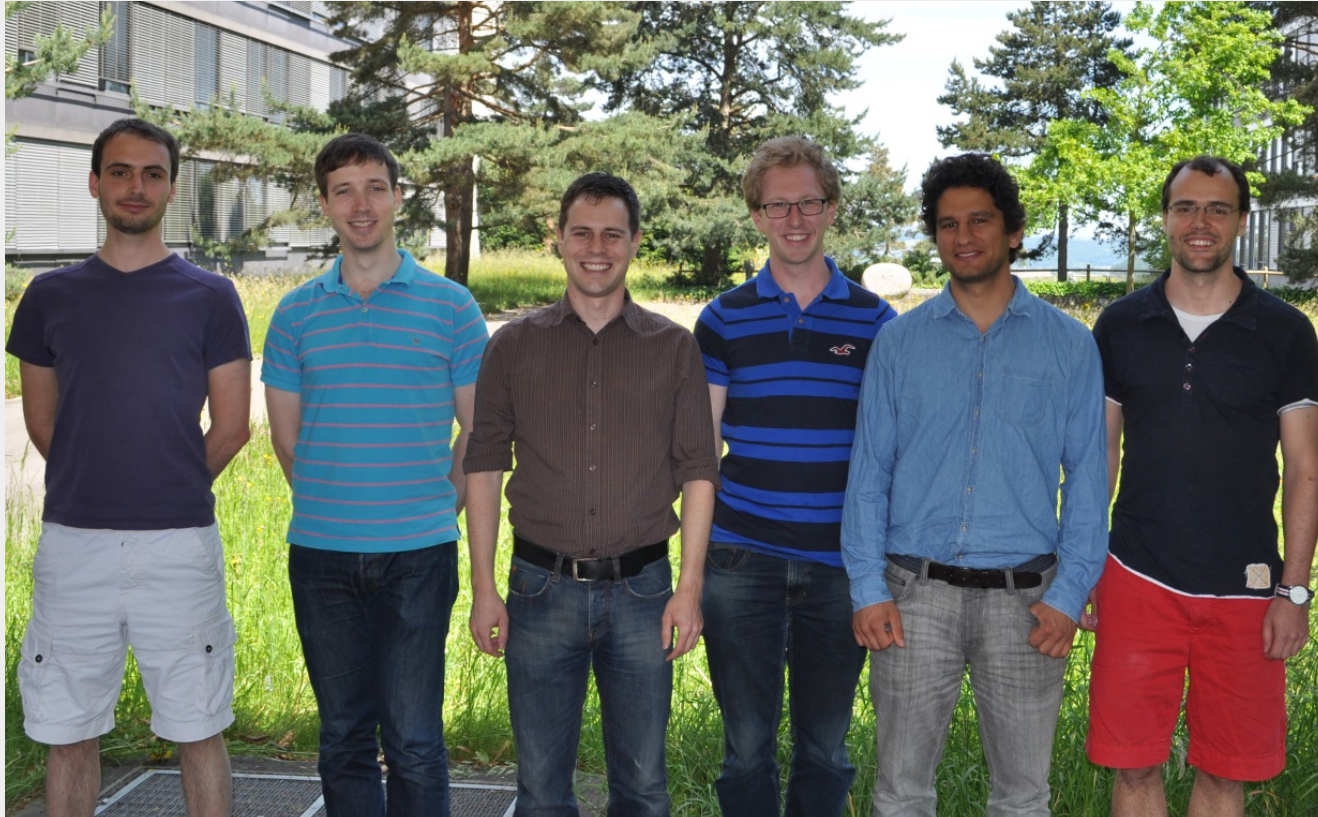
See: M. Mancini et al. Science 349, 1510 (2015)

B. K. Stuhl et al. Science 349, 1514 (2015)



# The Lattice team

+ Leticia Tarruell: now @ ICFO, Barcelona



Frederic Görg

Martin Lebrat	Rémi Desbuquouis	Thomas Uehlinger	Michael Messer	Gregor Jotzu	Daniel Greif
now@Lithium		now@Sensirion			now@Harvard



Supersolidity?

# Supersolidity?

Coexistence of:

- non-trivial diagonal long-range order
- off-diagonal long-range order

A. J. Leggett, Phys. Rev. Lett. 25, 1543 (1970)

# A possible Hamiltonian

$$H = J \sum_{\langle i,j \rangle} b_i^\dagger b_j + \frac{1}{2} U_0 \sum_i n_i (n_i - 1) \\ + \frac{1}{2} U_{\sigma_1} \sum_{\langle i,j \rangle} n_i n_j + \frac{1}{2} U_{\sigma_2} \sum_{\langle\langle i,j \rangle\rangle} n_i n_j + \dots$$

K. Goral, L. Santos, and M. Lewenstein, PRL 88, 170406 (2002)  
Scarola, V. W. & Sarma, S. D.. Phys. Rev. Lett. 95, 033003 (2005)

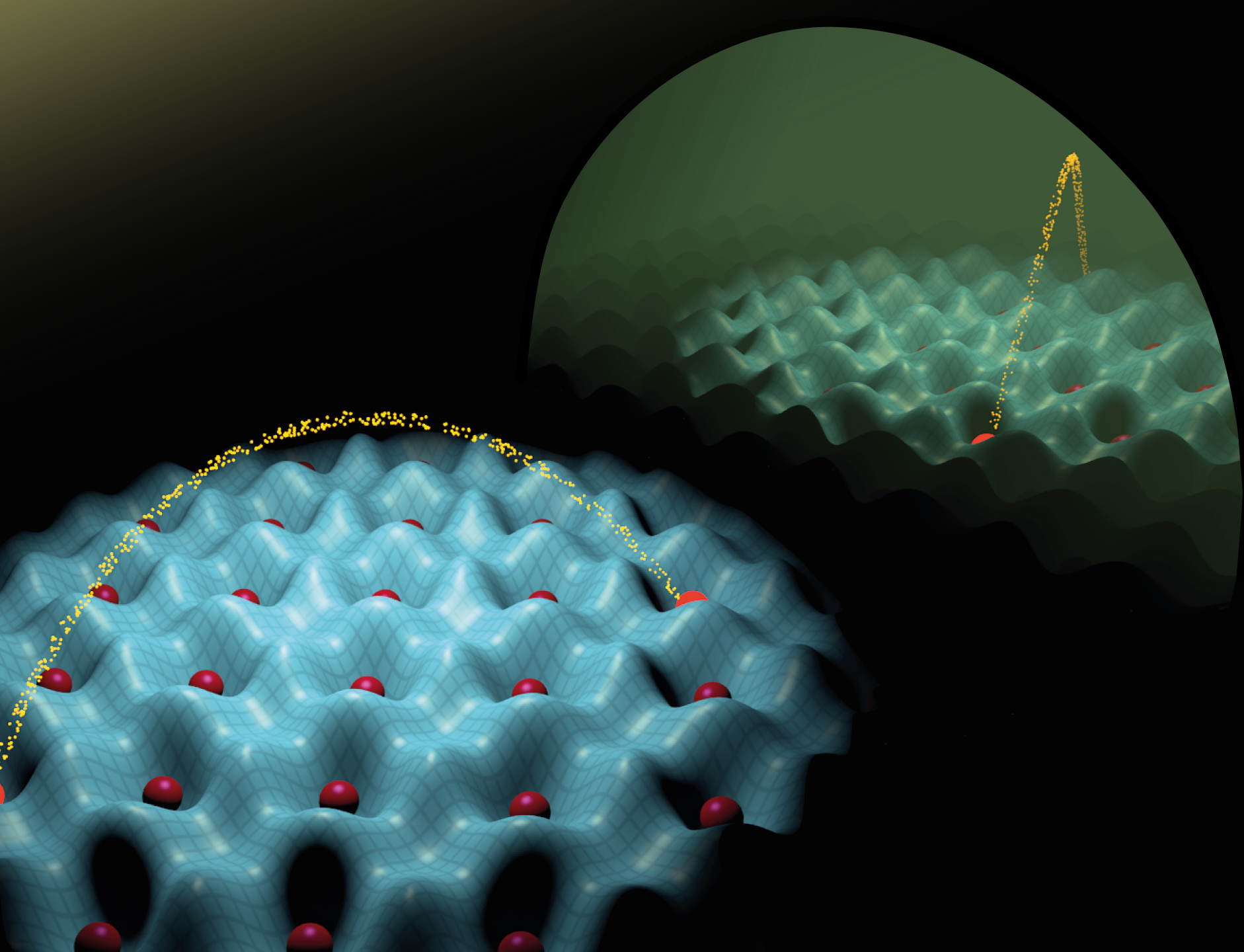


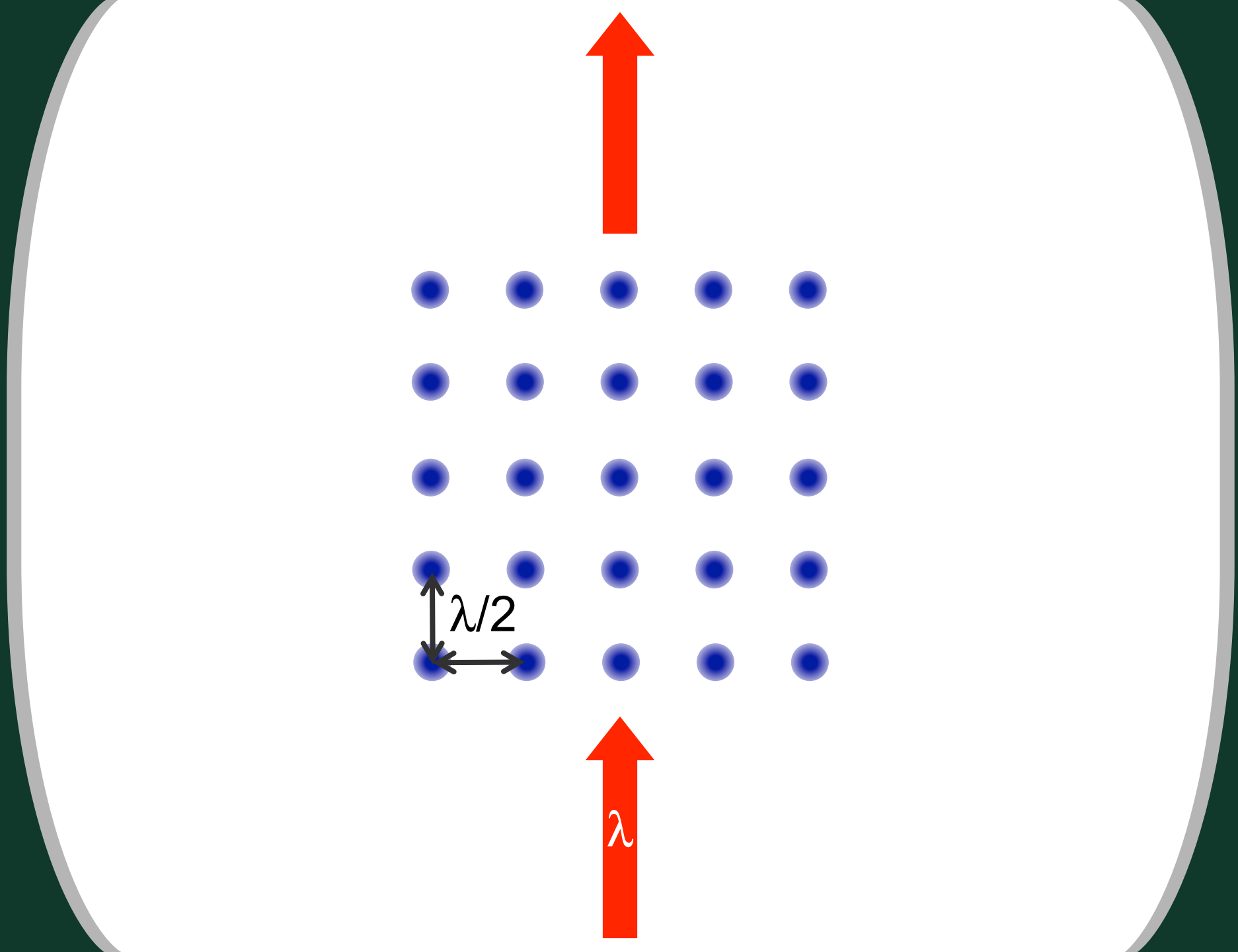
# Long-range interactions

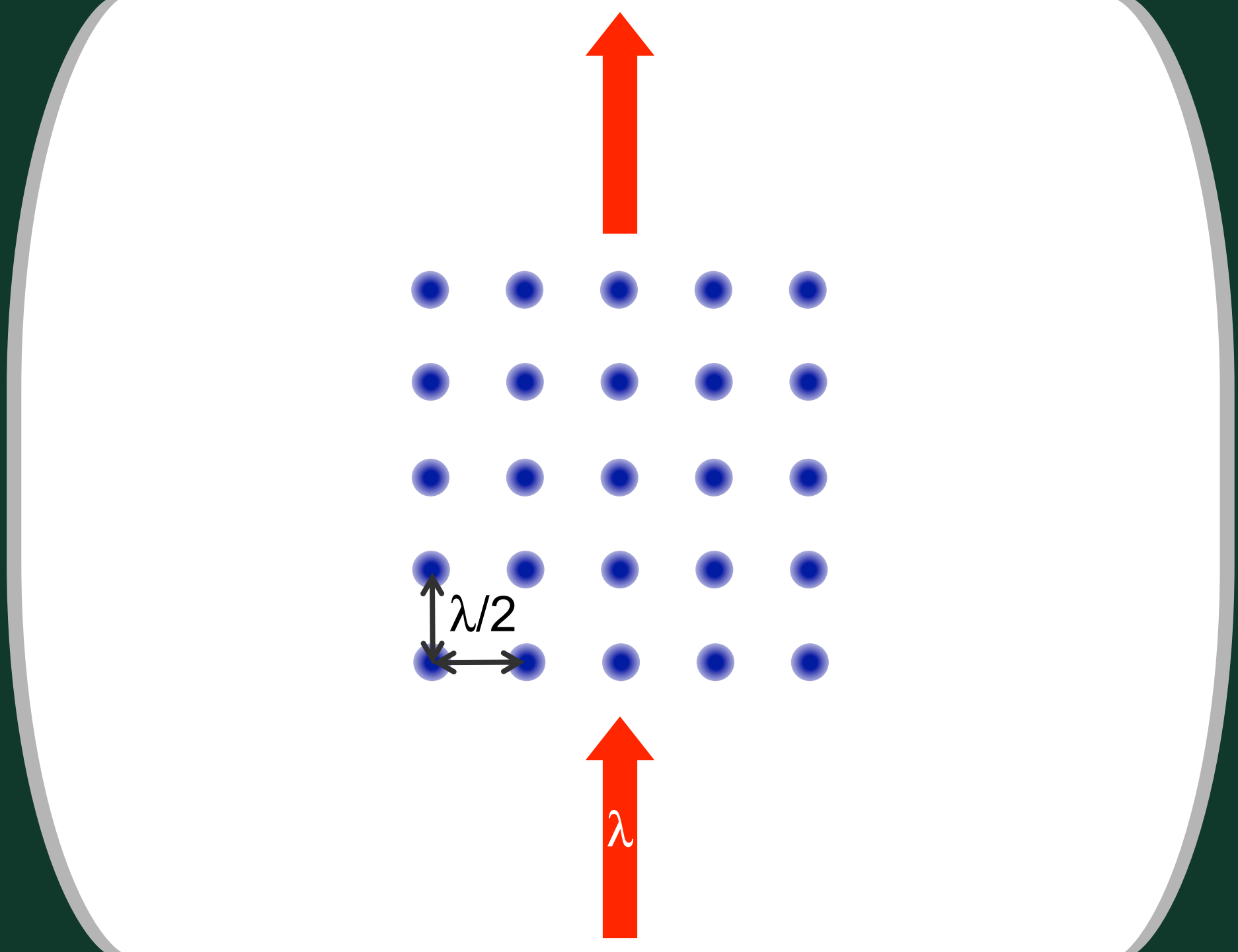
Dipolar molecules/atoms

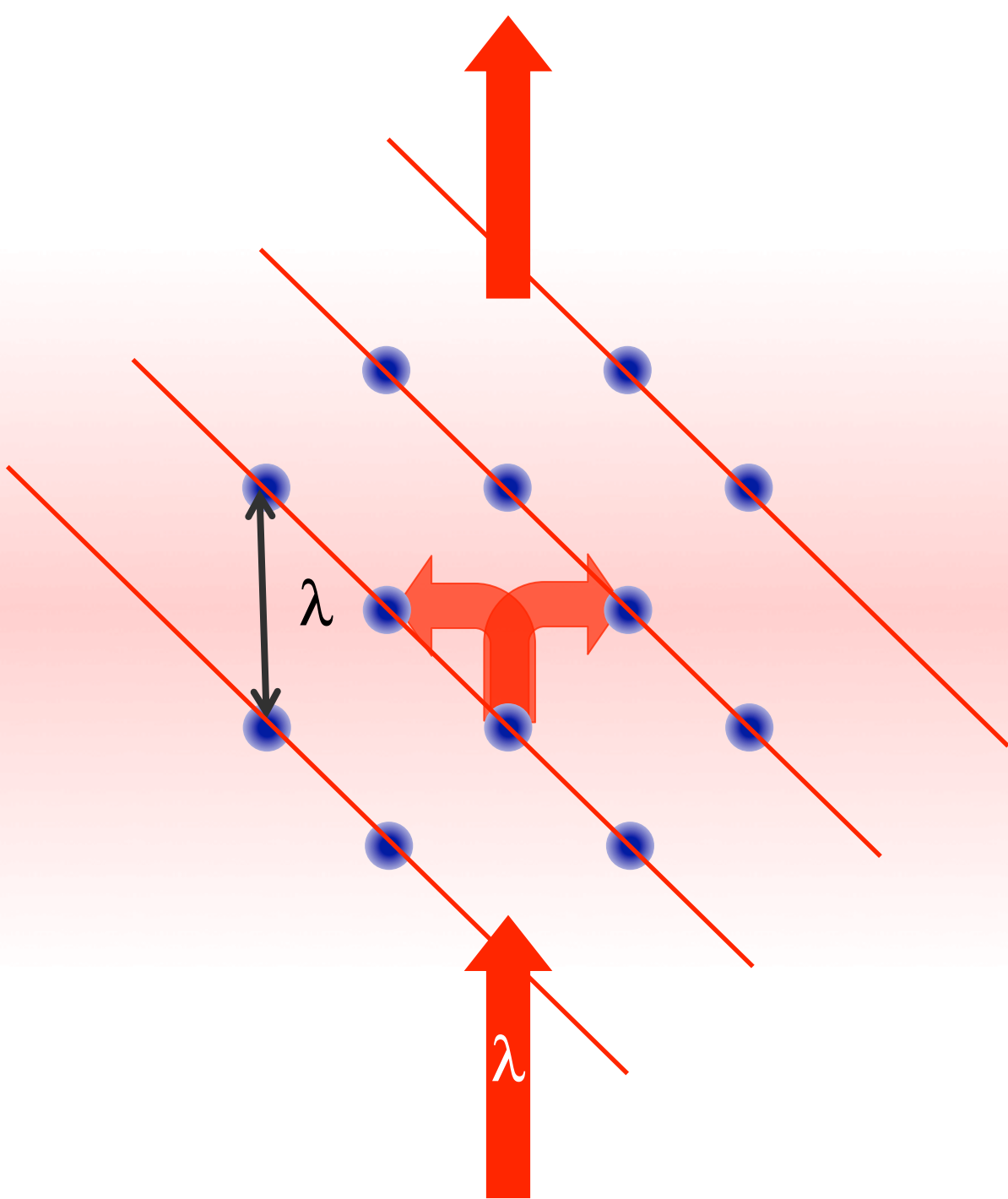
Rydberg atoms

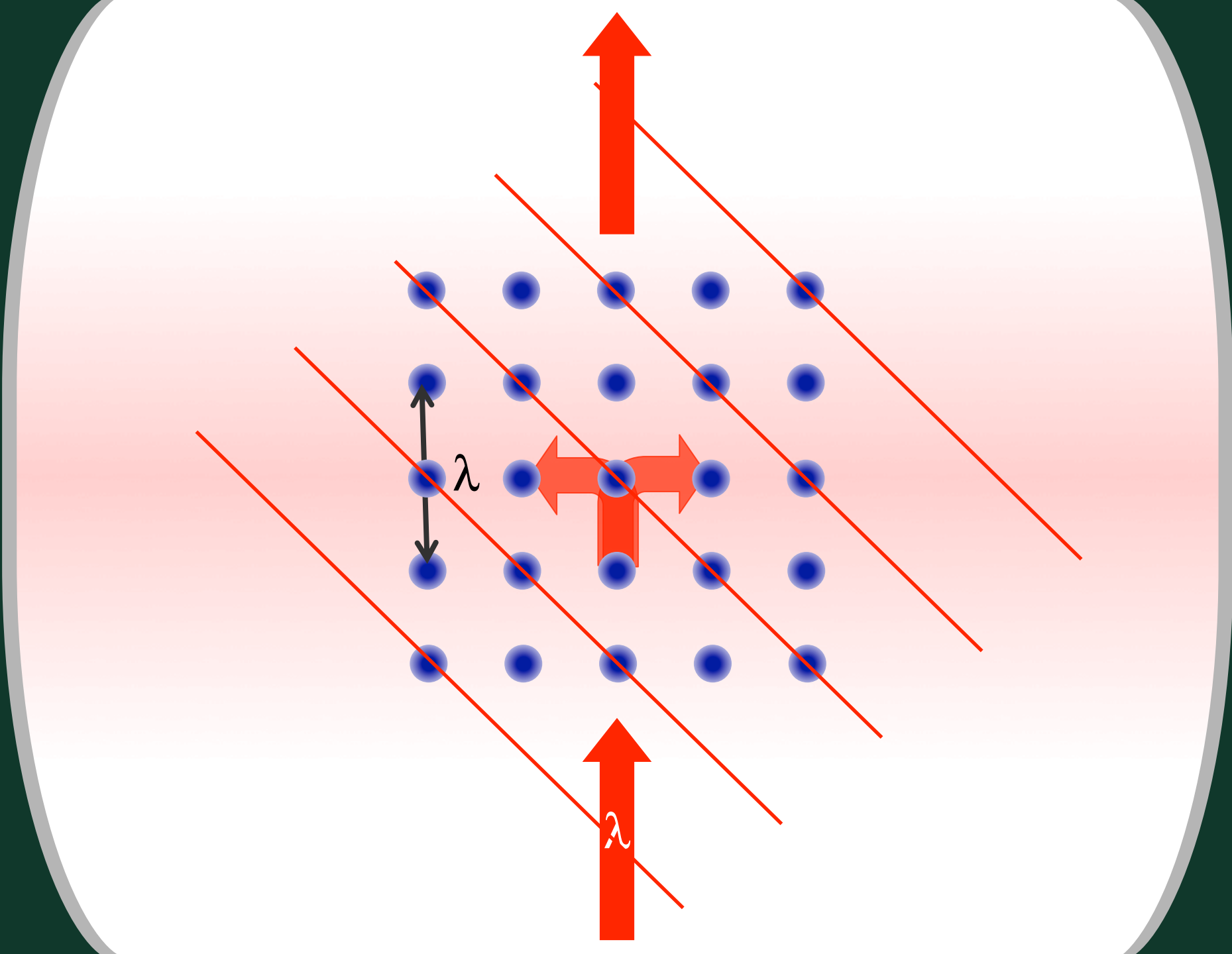
Cavity mediated interactions



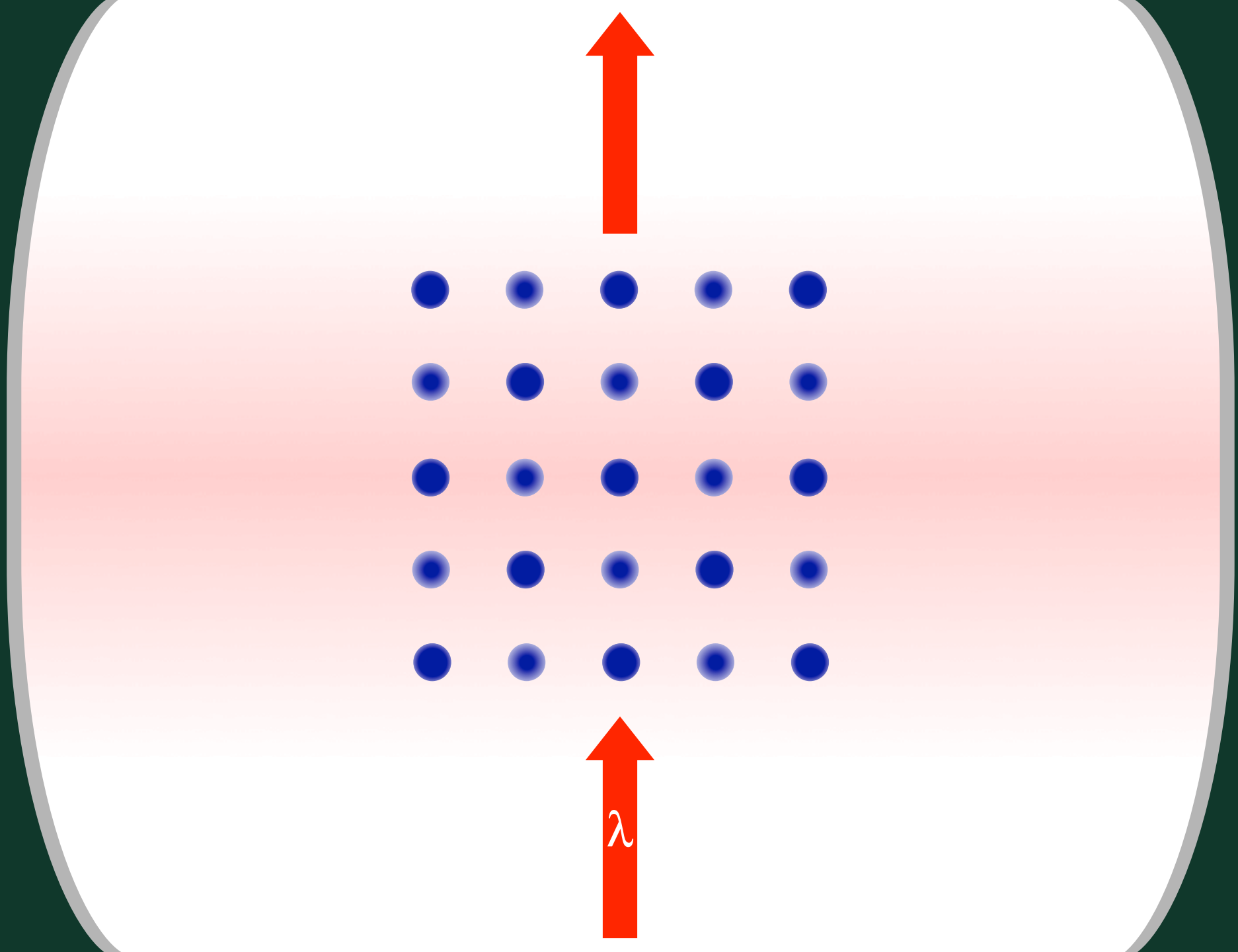


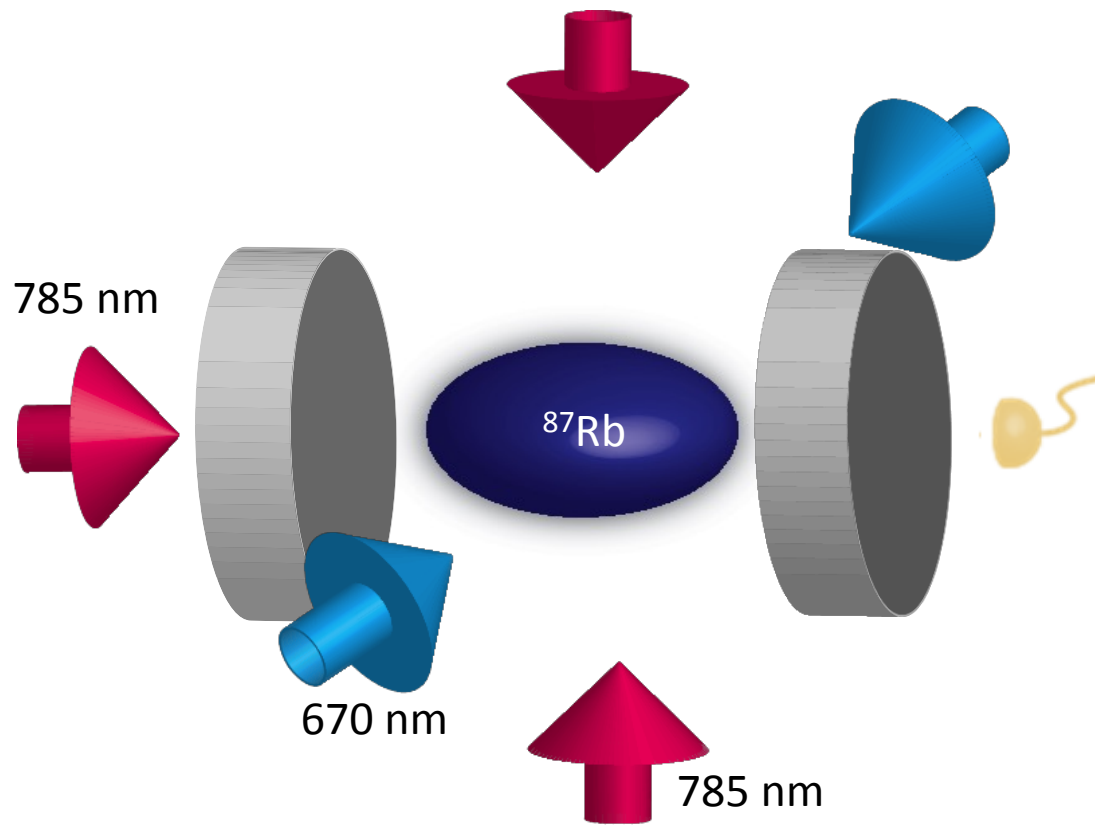








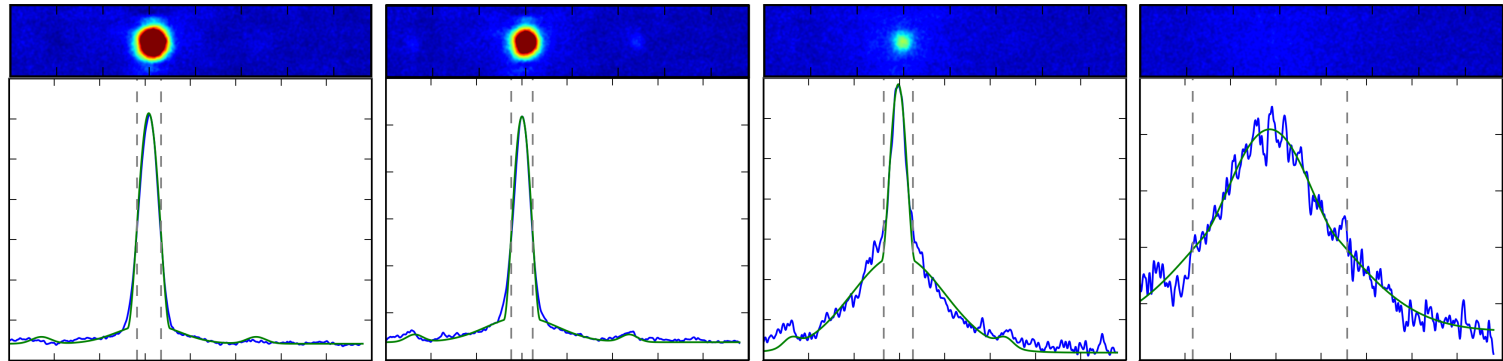




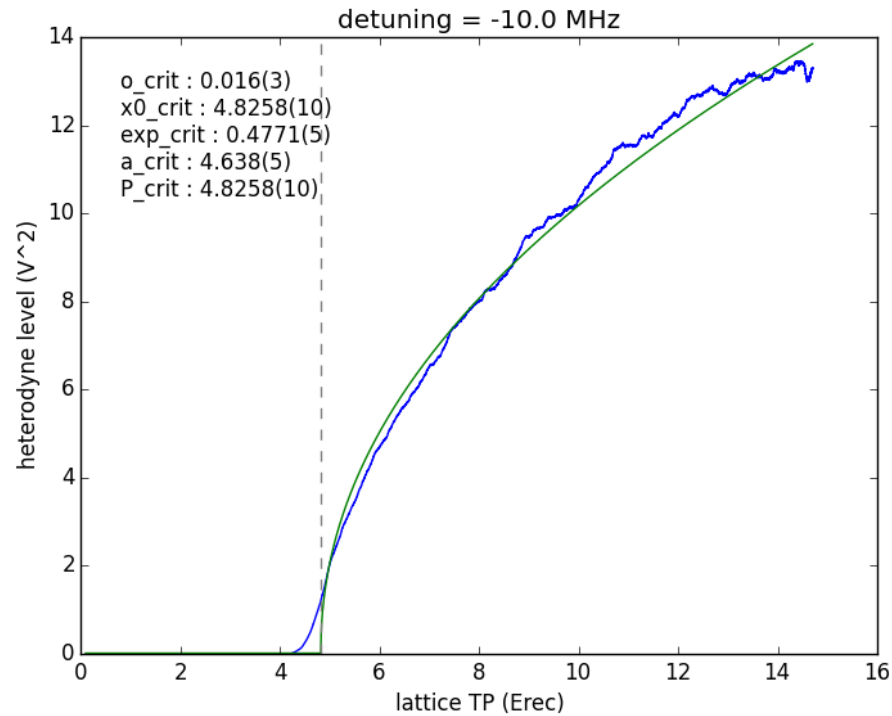
$$U_{short} \propto P_{785}$$

$$U_{long} \propto \frac{P_{785}}{\Delta_c}$$

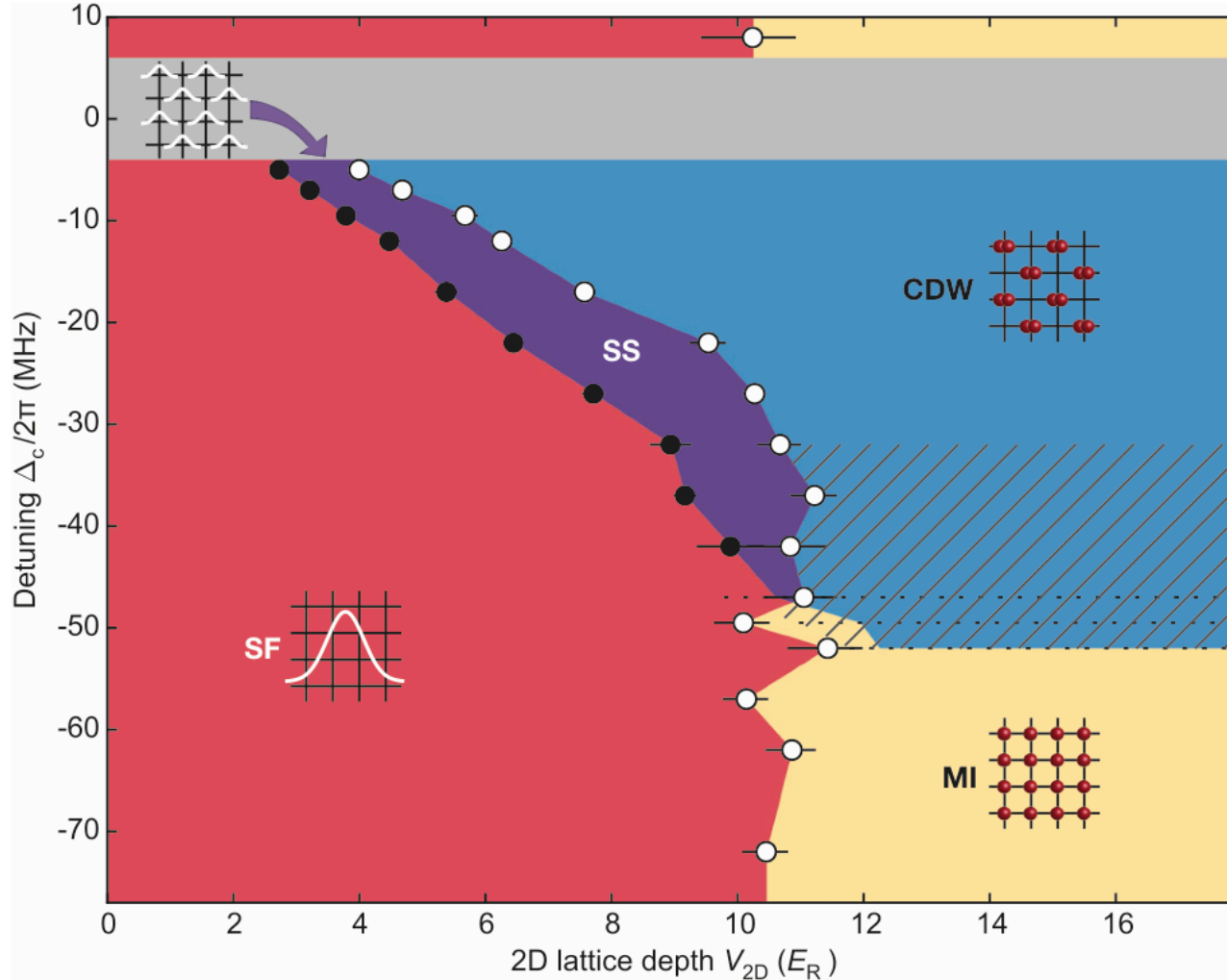
# Coherence: Measure of superfluid order parameter



# Cavity output: measure of checkerboard order parameter



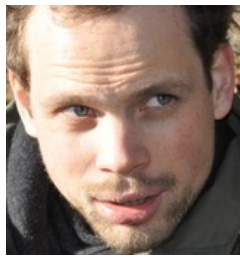
# Phase diagram



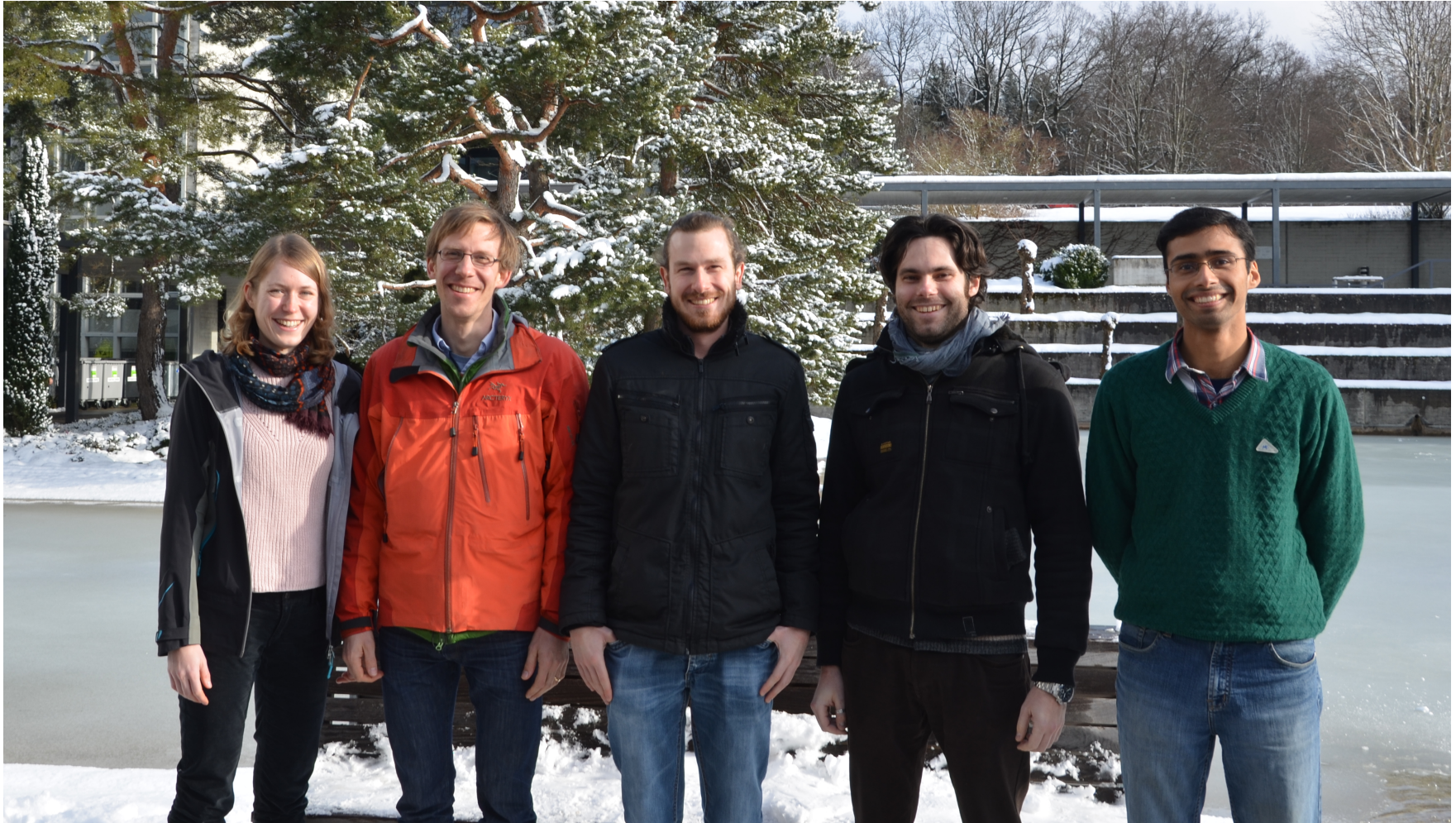
Renate Landig, Lorenz Hruby, Nishant Dogra, Manuele Landini, Rafael Mottl, Tobias Donner, TE, Nature 532, 476 (2016), arXiv:1511.00007

Related work: J. Klinder, H. Keßler, M. Reza Bakhtiari, M. Thorwart, and A. Hemmerich, Phys. Rev. Lett. 115, 230403 (2015), arXiv:1511.00850

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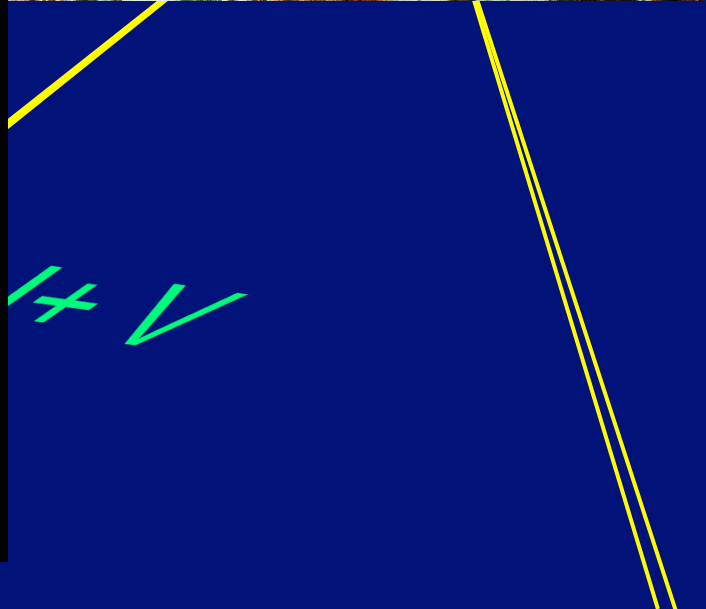
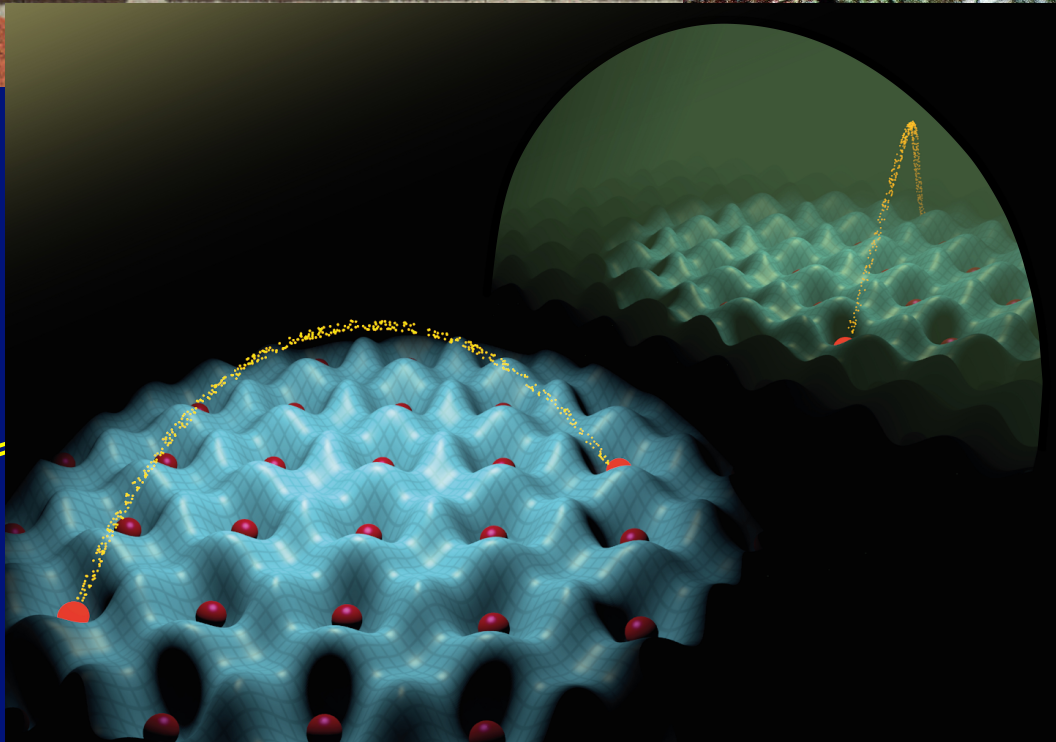
Manuele  
Landini

Nishant  
Dogra



Deeper understanding of many-body quantum physics

$$H = T + U + V$$





# Thanks !

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## Quantum Gases in Optical Lattices

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Thomas Uehlinger  
Gregor Jotzu  
Michael Messer  
Rémi Desbuquois  
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## Lithium Microscope

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## BEC and Cavity

Tobias Donner  
Renate Landig  
Lorenz Hruby  
Nishant Dogra  
Manuele Landini  
(Rafael Mottl)  
(Ferdinand Brennecke)

## Impact experiment

Tobias Donner  
Julian Leonard  
Andrea Morales  
Philip Zupancic  
(Moonjoo Lee)

## Electronics

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