

# Superfluidity and spin superfluidity (in spinor Bose gases and magnetic insulators)

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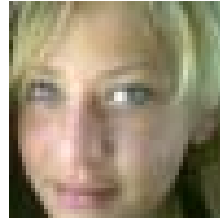
*Nature Physics* 11, 1022–1026 (2015), *Phys. Rev. Lett.* 116, 117201 (2016),  
[arXiv:1603.01996](https://arxiv.org/abs/1603.01996) [cond-mat.quant-gas], [arXiv:1604.03706](https://arxiv.org/abs/1604.03706) [cond-mat.mes-hall]

# Collaborators

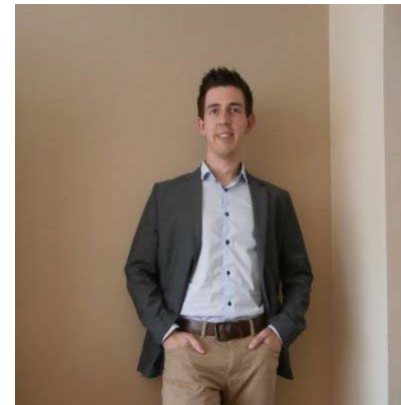


## Utrecht:

*Benedetta Flebus*



*Kevin Peters*



Gerrit Bauer (also Delft/Sendai)

## Other:

*Jogundas Armaitis (Vilnius)*



Ludo Cornelissen, J. Liu,

Bart van Wees (Groningen)

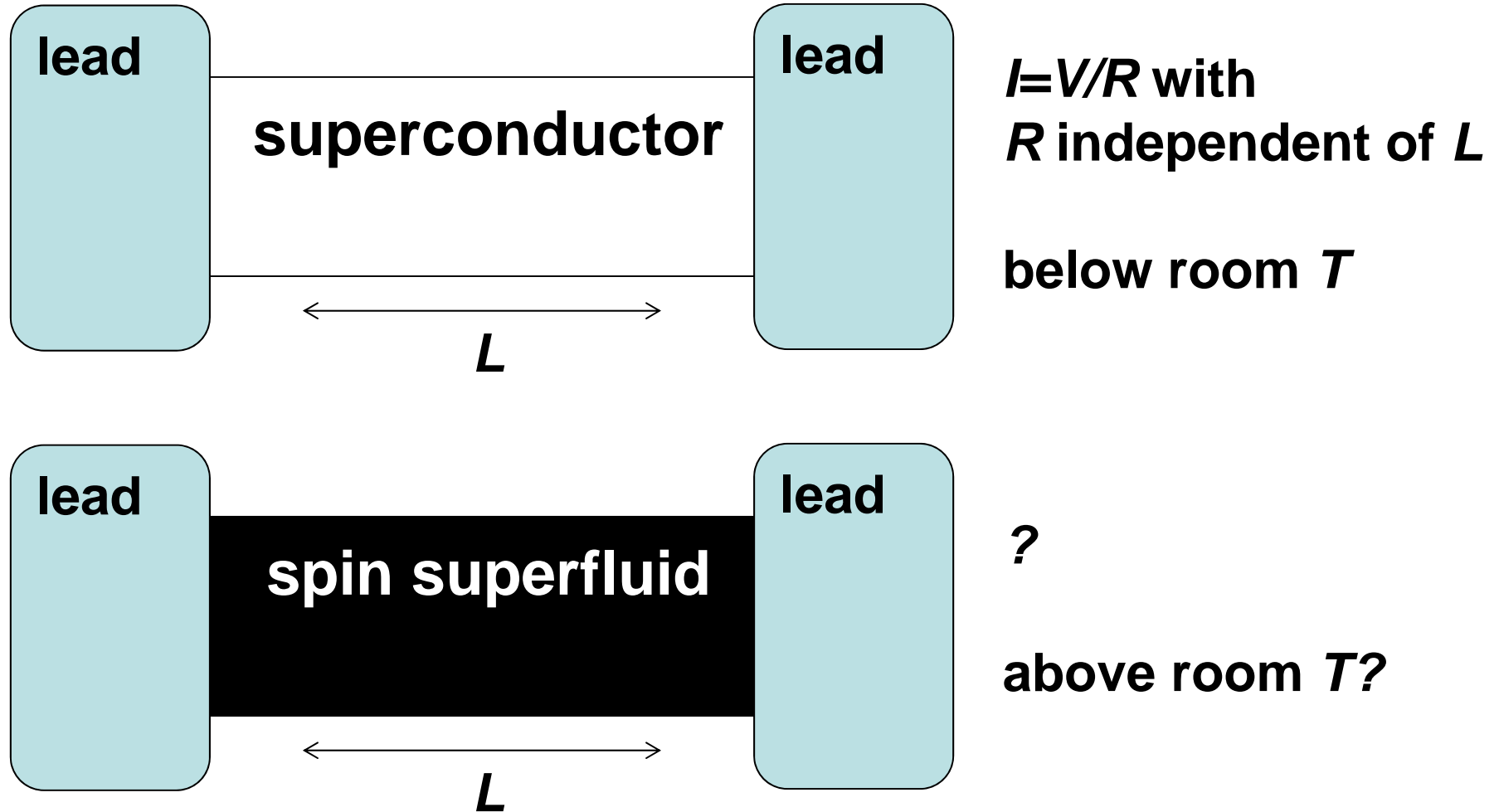
Yaroslav Tserkovnyak, *Scott Bender (UCLA->UU)*

J. Ben Youssef (Brest), Arne Brataas (NTNU)



European Research Council

# Long-term motivation

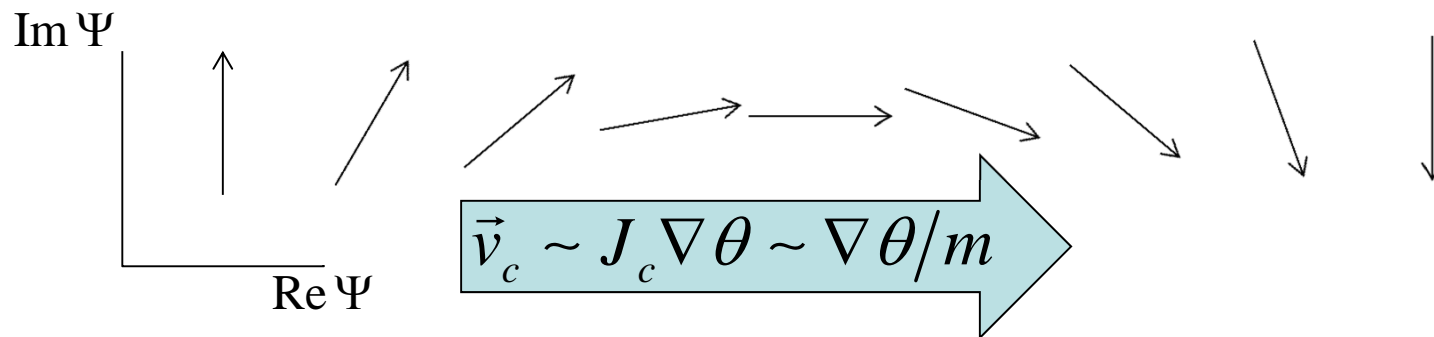


# Outline

- **Introduction superfluidity and spin superfluidity**
- **Spinor gases: combining superfluidity and spin superfluidity (theory)**
- **Spin transport through magnetic insulators (theory and experiment)**

# Superfluid (mass) transport

Superfluid  $U(1)$  order parameter:  $\Psi = \sqrt{\rho_c} e^{i\theta}$



$$\frac{\partial \rho_c}{\partial t} = -\nabla \cdot \vec{j}_c$$

supercurrent:

+

$$\vec{j}_c = \rho_c \vec{v}_c$$

$$\frac{\partial \theta}{\partial t} = -\frac{\mu}{\hbar}$$

carried by condensate

chemical potential  $\mu$

Wave-like excitations:

$$\omega \sim \sqrt{J_c \left( \frac{\partial \mu}{\partial \rho} \right) k}$$

stiffness

interactions  $\propto g\rho_c$

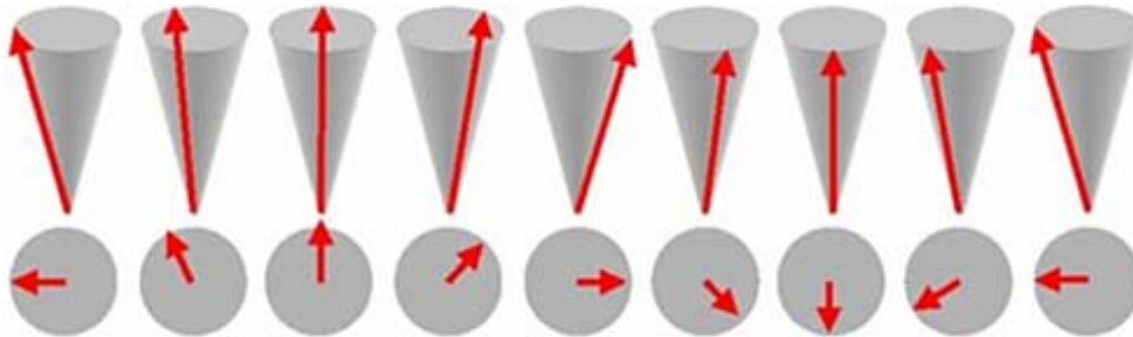
If  $\rho_c$  zero then  $\omega \sim J_s k^2$

(ballistic transport)

# (ballistic) spin transport

Magnetic insulator  
(exchange only)

$$H = -J_{xc} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$



Landau-Lifschitz  
equation:

$$\hbar \frac{d\vec{S}_i}{dt} = J_{xc} \vec{S}_i \times \left( \sum_{j \in \text{neighbours } i} \vec{S}_j \right) \sim J_{xc} \vec{S} \times \nabla^2 \vec{S} = -\nabla \cdot \vec{j}$$

spin current:

$$\vec{j}_\alpha \sim J_{xc} \vec{S} \times \partial_\alpha \vec{S}; \quad \alpha \in \{x, y, z\}$$

particle-like excitations:  $\omega \sim J_{xc} k^2$  spin waves/magnons

# Spin superfluidity (I)

Easy-plane magnetic insulator [SO(3) → U(1)]

$$H = -J_{xc} \sum_i \vec{S}_i \cdot (\vec{S}_{i-1} + \vec{S}_{i+1}) + K \sum_i (S_i^z)^2 + B \sum_i S_i^z$$

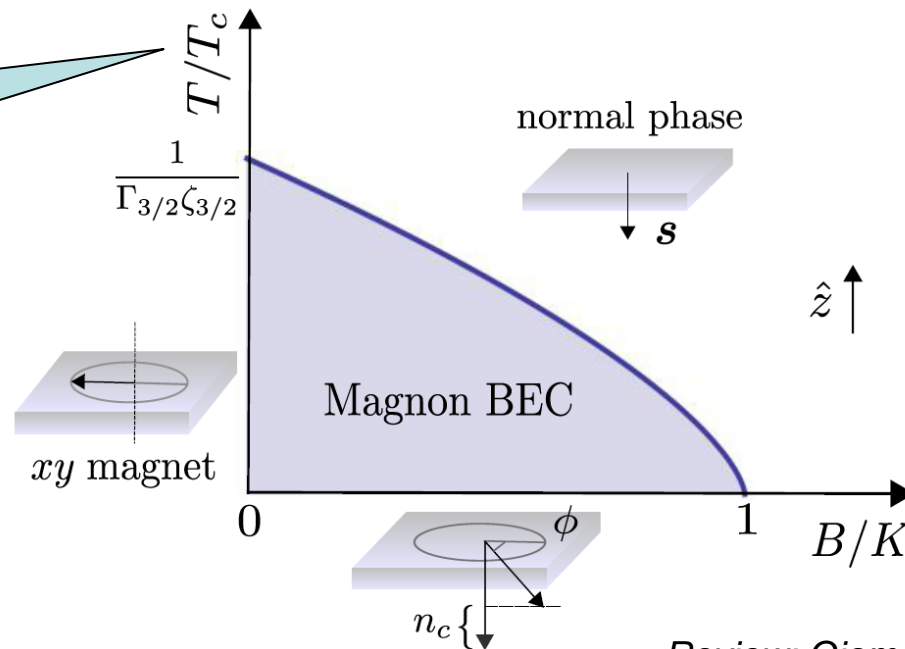
exchange

anisotropy

field

$$T_{BEC} \sim k_B J_{xc} (1 - B/K)^{2/3}$$

Holstein-Primakoff  
trafo:  $\vec{S} \sim \begin{pmatrix} \sqrt{2} (\text{Re} \langle \hat{a} \rangle) \\ \sqrt{2} (\text{Im} \langle \hat{a} \rangle) \\ \langle \hat{a}^\dagger \hat{a} \rangle - 1 \end{pmatrix}$



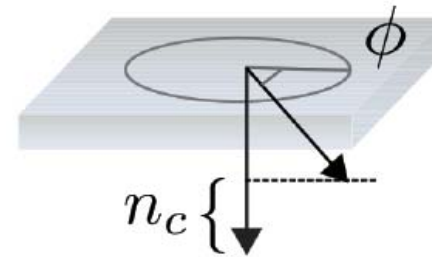
# Spin superfluidity (II)

Zero  $T$  dynamics governed by Landau-Lifshitz equation:

$$\hbar \frac{d\vec{S}}{dt} = J_{xc} \vec{S} \times \left[ \nabla^2 \vec{S} + K S^z \hat{z} + B \hat{z} \right]$$

Write:

$$\vec{S} \sim \begin{pmatrix} \sqrt{2n_c} \cos \phi \\ \sqrt{2n_c} \sin \phi \\ n_c - 1 \end{pmatrix}$$



Same as equations  
for mass superfluid!

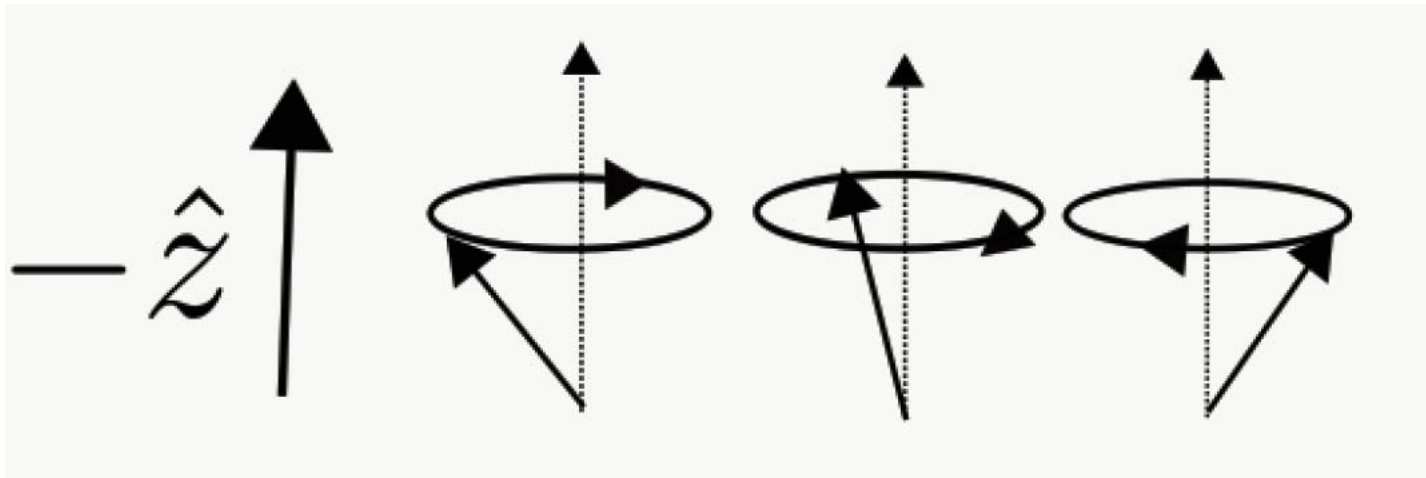
$$\omega \sim \sqrt{J_{xc} K n_c} k$$

$$\frac{\partial n_c}{\partial t} \sim J_{xc} n_c \nabla^2 \phi \equiv -\nabla \cdot \vec{j}_s$$

$$\frac{\partial \phi}{\partial t} \sim B - K + K n_c \equiv \mu / \hbar$$



# Spin superfluidity (III)

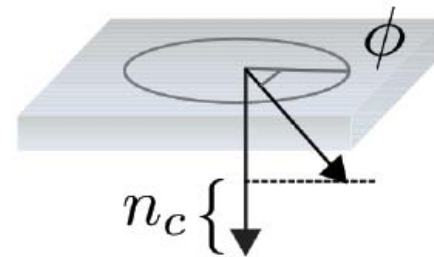


$$j_{s,\alpha} \sim J_{xc} n_c \partial_\alpha \phi; \text{ polarization in } z\text{-direction}$$

# Mass vs. spin superfluidity

- mass current  $j_c$
- phase of superfluid order parameter  $\theta$
- superfluid stiffness  $J_s$
- interparticle interactions  $g$
- spin current  $j_s$
- in-plane angle of magnetization  $\phi$
- exchange interactions  $J_{xc}$
- easy-plane anisotropy  $K$

$$\Psi = \sqrt{\rho_c} e^{i\theta}$$



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# Related work

- **Our work: hydrodynamic description treating spin and mass superfluidity on equal footing**
- **Spin currents in spinor gases:**
  - K. Kudo and Y. Kawaguchi, Physical Review A **84**, 043607 (2011).
  - H. Flayac, *et al.*, Physical Review B **88**, 184503 (2013).
  - Q. Zhu, Q.-f. Sun, and B. Wu, Phys. Rev. A **91**, 023633 (2015).
- **Stability of spirals:**
  - R. W. Cherng, *et al.*, Phys. Rev. Lett. **100**, 180404 (2008).
- **Magnon condensation:**
  - Fang, *et al.*, Phys. Rev. Lett. **116**, 095301 (2016) – tonight's evening talk (?).
- **Reviews:**
  - Y. Kawaguchi and M. Ueda, Physics Reports **520**, 253 (2012).
  - D. M. Stamper-Kurn and M. Ueda, Rev. Mod. Phys. **85**, 1191 (2013).

# Questions

- Which system is both a mass and spin superfluid?
- What is the interaction between mass and spin superfluidity in such a system?

**NB: a ferromagnetic mass superfluid**

**$\neq$**

**spin superfluid**

# Ferromagnetic spinor Bose with quadratic Zeeman effect

We have  
hamiltonian:

$$\hat{H} = \int d\vec{x} \hat{\psi}^\dagger \left[ -\frac{\hbar^2 \nabla^2}{2m} - BF^z - K (F^z)^2 \right] \hat{\psi} + g_0 (\hat{\psi}^\dagger \hat{\psi})^2 + g_1 (\hat{\psi}^\dagger \vec{F} \hat{\psi})^2$$

field

easy-plane  
anisotropy

leads to Bose  
condensation

$g_1 < 0$ , leads to  
ferromagnetism

**Deep in ferromagnetic regime, zero  $T$  linearized mean-field equations reduce to equations for uncoupled mass and spin superfluid with:**

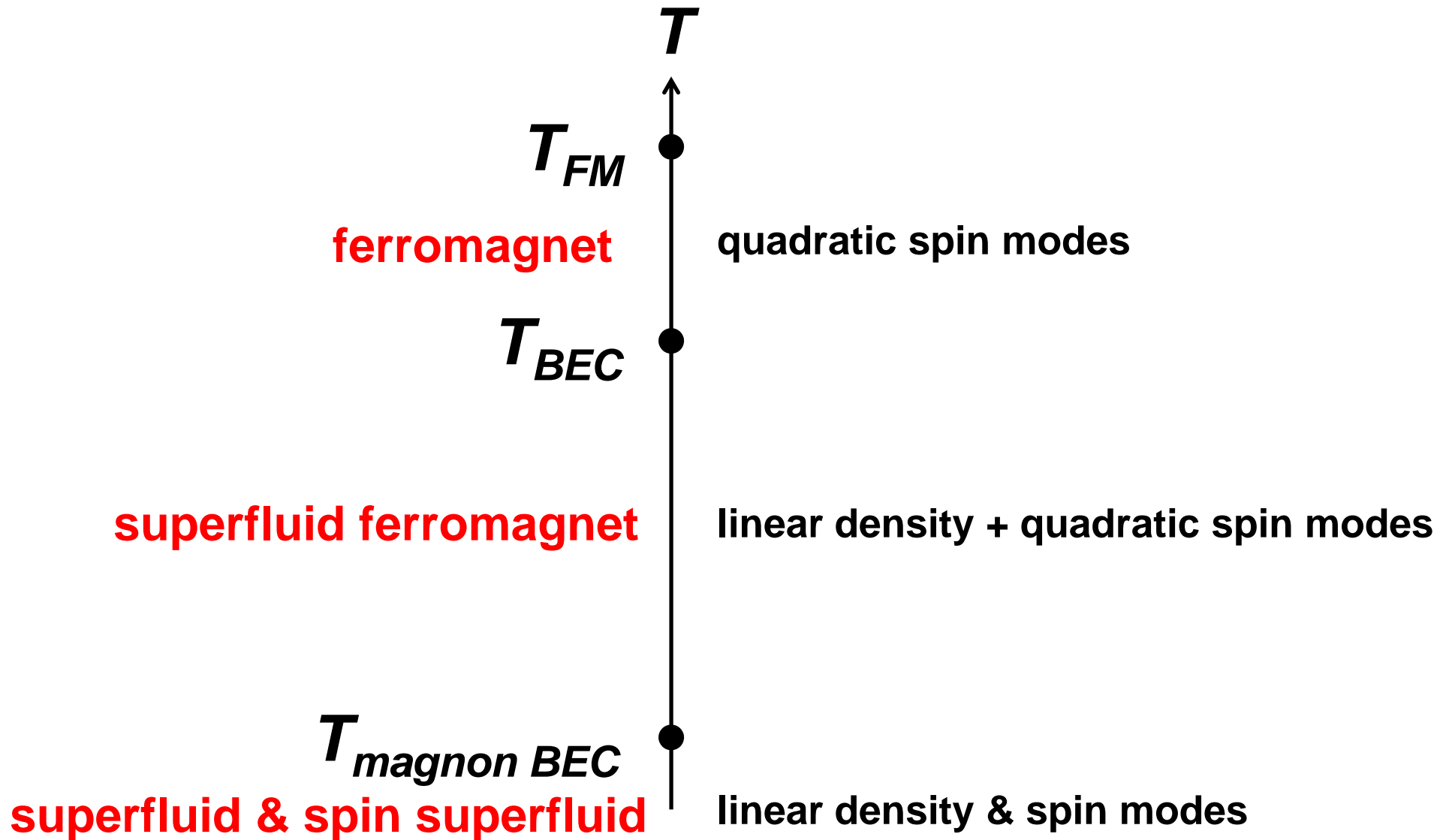
**Superfluid and spin stiffness:  $J_s \sim J_{xc} \sim \hbar/m$**

**Collective modes:**

$$\omega_{mass} \sim \sqrt{\hbar \rho_c (g_0 + g_1) / mk}$$

$$\omega_{spin} \sim k \sqrt{\frac{\hbar K (K - B)}{m}}$$

# Collective (Goldstone) modes



# Coupling between spin and mass superfluid

Occurs via hydrodynamic derivative:

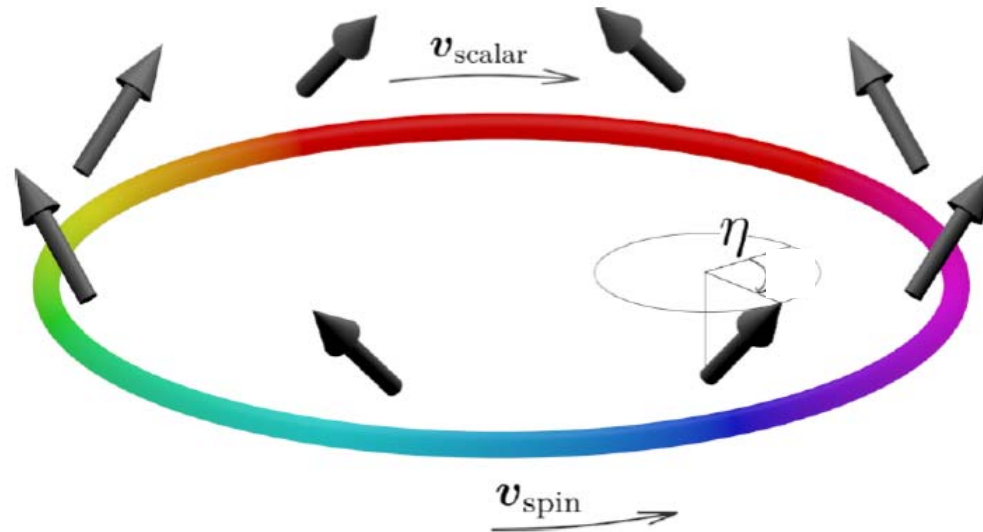
$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \quad \text{with} \quad \vec{v} = \frac{\hbar}{m} (\nabla \theta - n_z \nabla \phi)$$

Influence of spin polarized mass current on magnetization dynamics: “spin transfer”

Current driven by magnetic texture



# Stationary solutions



**Stationary solutions possible if:**

$$\theta' = \frac{n_z}{2} \left( \phi' + \frac{K}{J_s \phi'} \right)$$

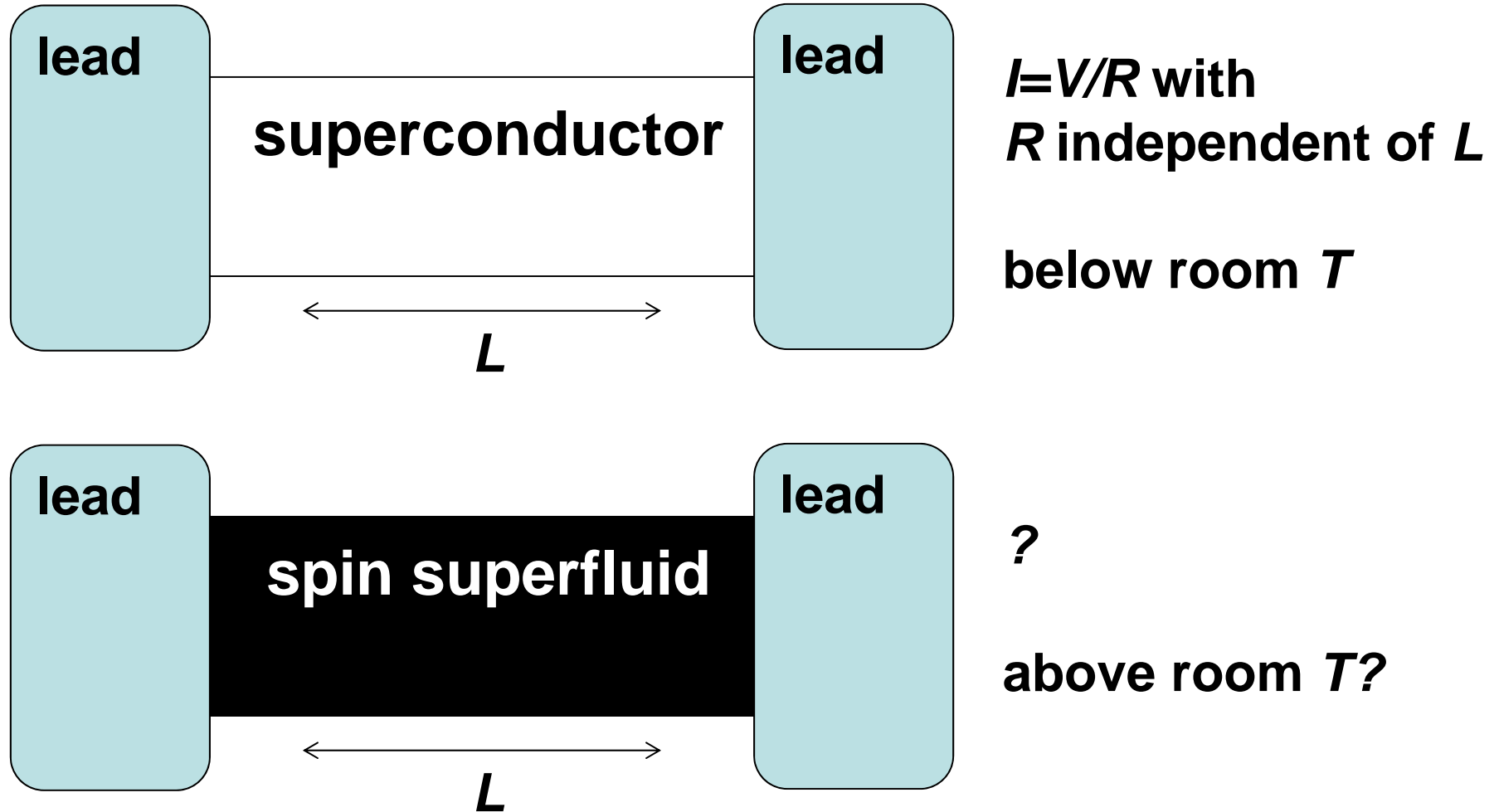
# Discussion

- Dipolar interactions (lower critical current)
- Upper critical current
- Quasi-equilibrium magnon condensation
- Magnon kinetics
- Coupling to nematic/antiferromagnetic order

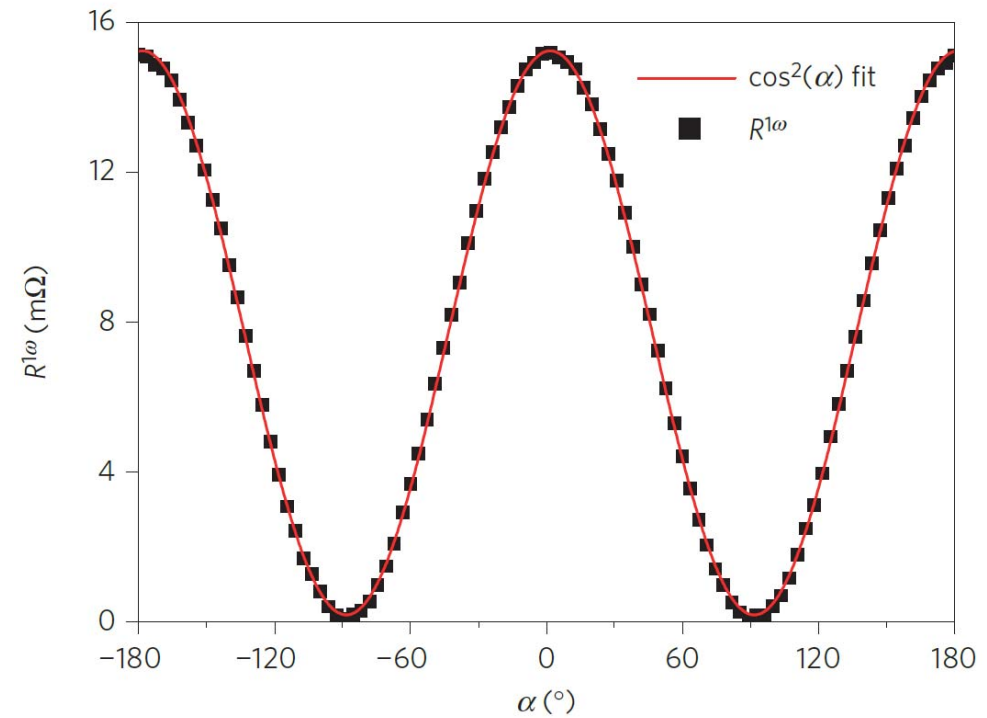
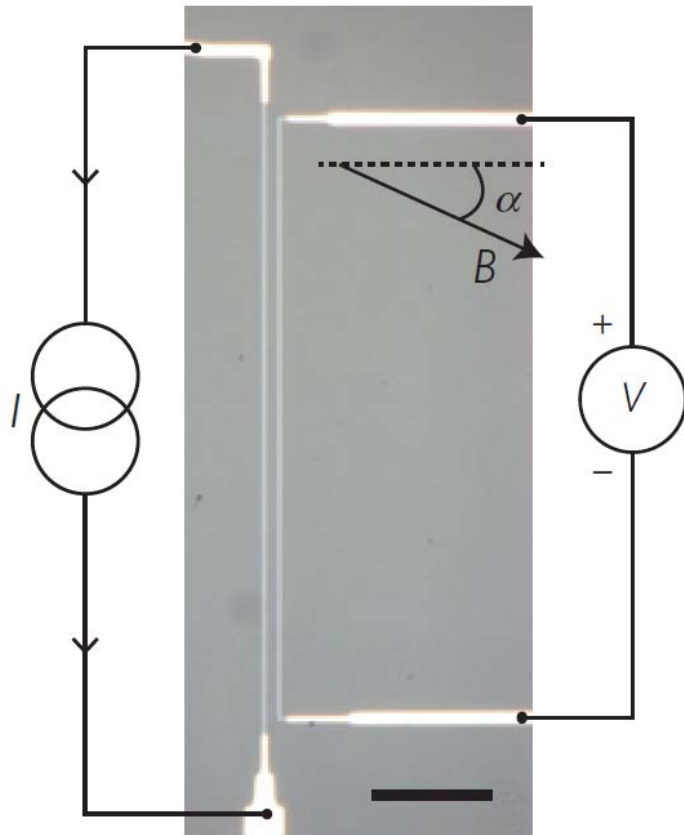
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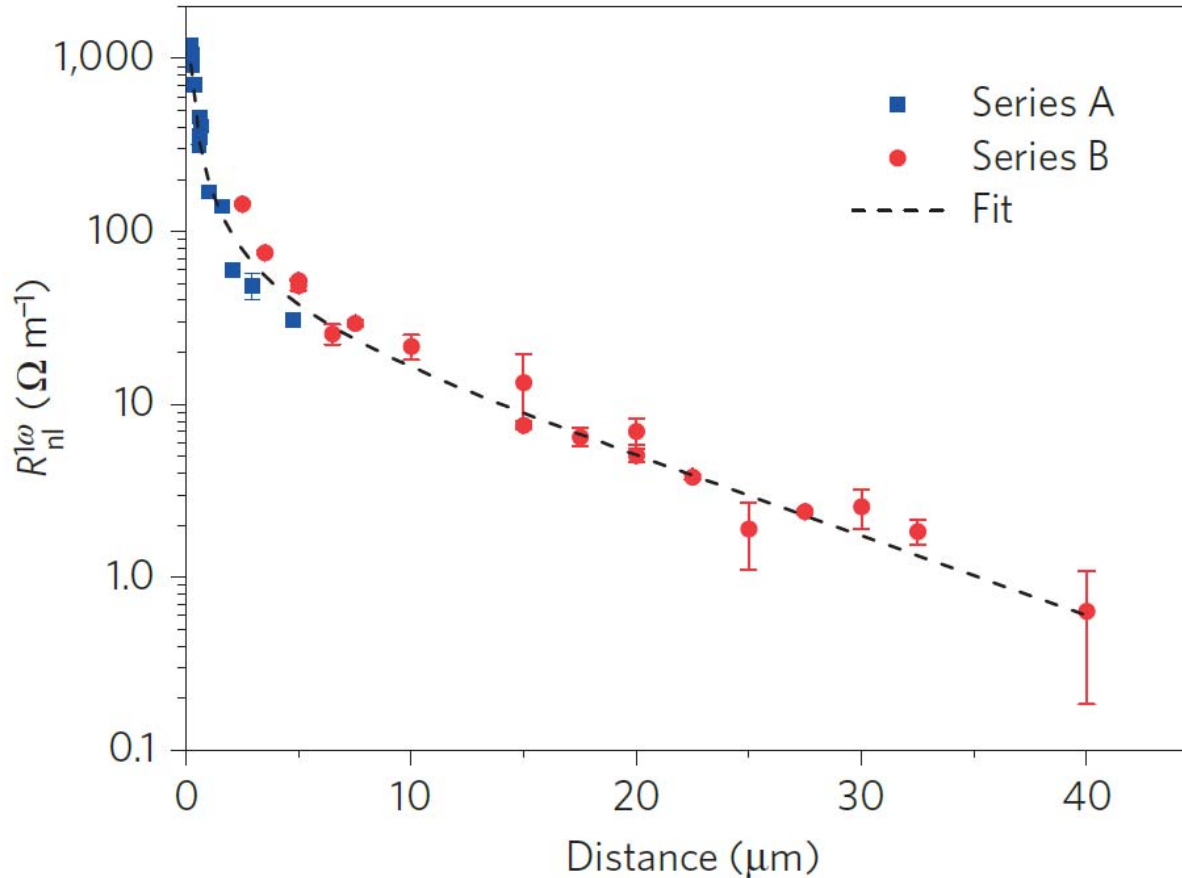


# Experiment (I)



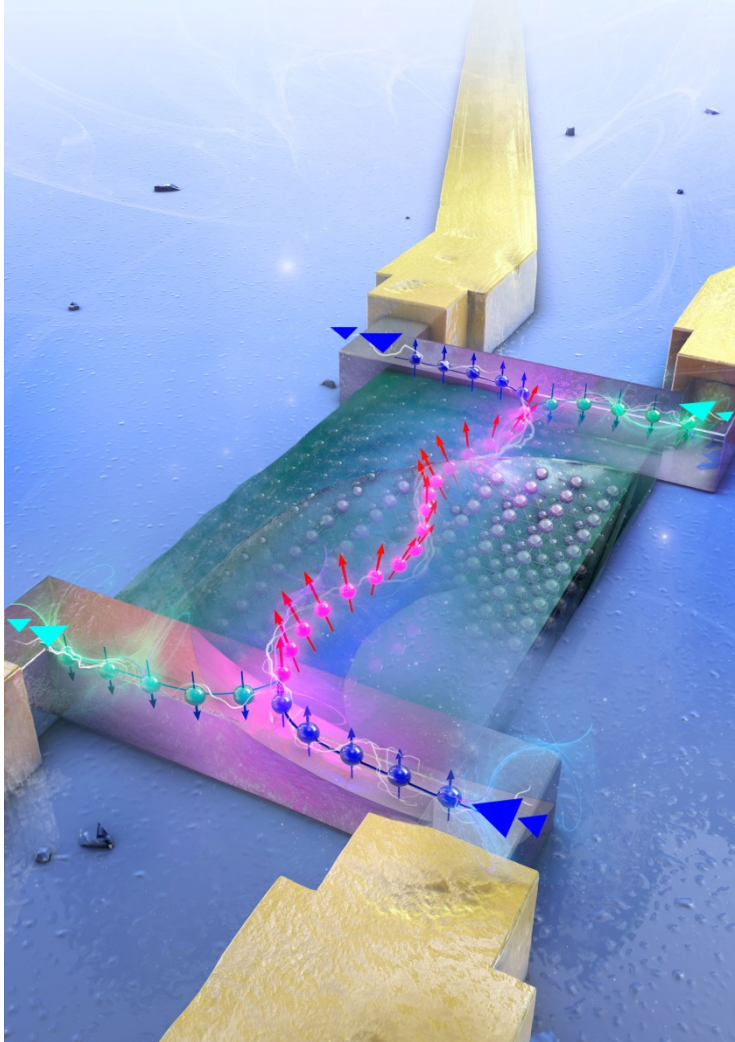
- Pt strips on magnetic insulator Yttrium-Iron-Garnett (YIG)
- Non-local (“drag”) resistance  $R_D = V/I$
- Room  $T$

# Experiment (II)



- Short distance:  $1/\text{distance}$
- Long distance: exponential decay (length scale  $10 \mu\text{m}$ )

# Idea



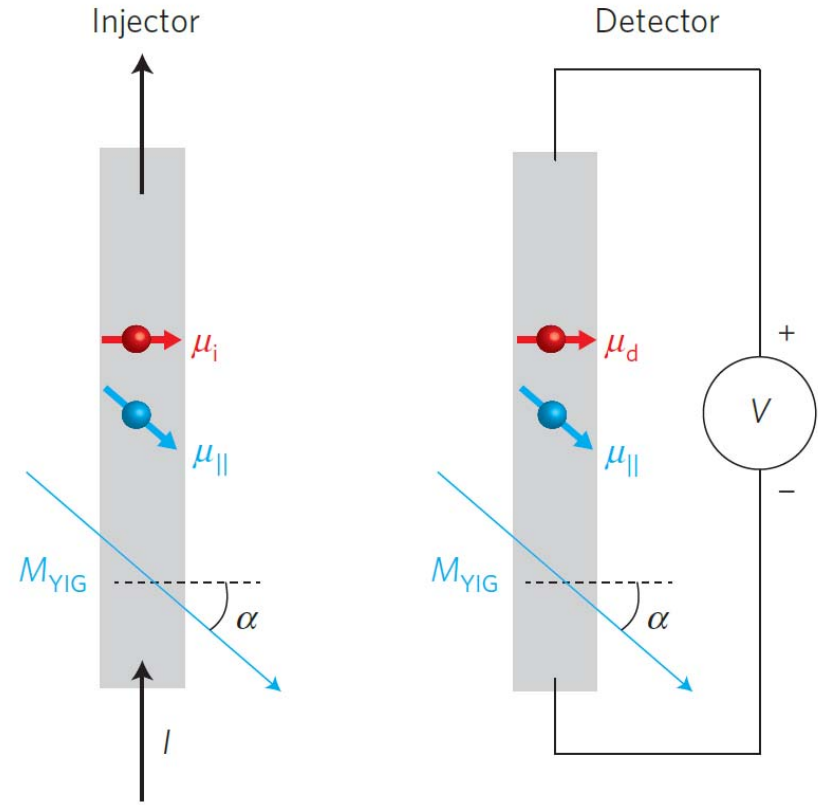
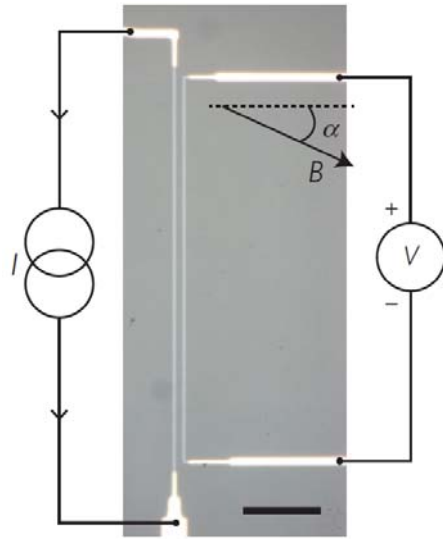
*Image courtesy Ludo Cornelissen  
and Bart van Wees*

**Magnon spintronics:  
electrical control over  
magnon spin currents**

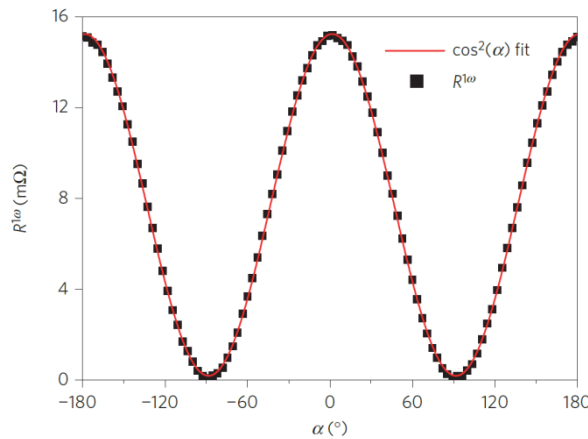
**Magnonic many-body  
physics**

**Route to spin currents  
with low dissipation**

# Interfacial spin current



$$R = V/I$$



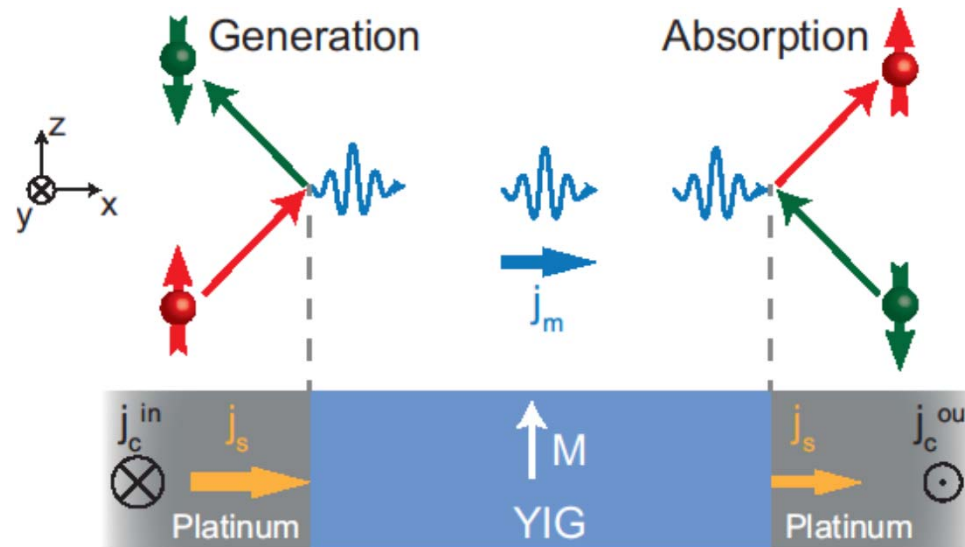
$$\cos^2 \alpha = \cos \alpha$$

\*

$$\cos \alpha$$



# Model



magnon diffusion  
& relaxation:

$$\vec{j}_s = -\sigma_m \nabla \mu$$

$$\nabla^2 \mu = \frac{\mu}{\ell_m^2}$$

$$\text{signal} \sim e^{-L/\ell_m}$$

For spin superfluid this would be  $\sim \frac{1}{1 + CL}$

# QUANTUM SPINTRONICS

## SPIN TRANSPORT THROUGH QUANTUM MAGNETIC MATERIALS

Workshop September 21<sup>st</sup> - 23<sup>rd</sup> 2016  
JGU MITP/SPICE, Mainz, Germany

ORGANIZERS:  
Rembert Duine  
Yaroslav Tserkovnyak

SPICE CO-ORGANISER:  
Jairo Sinova

SP/CE



### INVITED SPEAKERS:

Leon Balents (KITP)  
Arne Brataas (NTNU)  
Hans-Benjamin Braun (Dublin)  
David G. Cahill (UIUC)  
Ludo J. Cornelissen (Groningen)  
Benedetta Flebus (Utrecht)  
Thierry Giamarchi (Geneve)  
Sebastian Goennenwein (Munich)

Fabian Heidrich-Meissner (Munich)  
Peter Kopietz (Frankfurt)  
Ilya Krivorotov (UC Irvine)  
Daniel Loss (Basel)  
Allan MacDonald (UT Austin)  
Tamara Nunner (Berlin)  
Teruo Ono (Kyoto)  
Christian Rüegg (PSI)

Subir Sachdev (Harvard)  
Eiji Saitoh (Sendai)  
Dirk Schuricht (Utrecht)  
Alexander Serga (Kaiserslautern)  
So Takei (New York)  
Wolfgang Wernsdorfer (Néel)  
Amir Yacoby (Harvard)

The Spin Phenomena Interdisciplinary Center aims to bring together scientists from diverse disciplines and of varying seniority in the broad area of spin-related research. Its goal is to break down scientific barriers and foster emergent areas of research that combine the strengths of different fields.

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# Take home messages

- **Spin superfluidity is not ferromagnetic superfluidity**
- **Spinor Bose gas can be spin and/or mass superfluid**
- **Recent developments of spintronics pave the way for integration electronic and magnonic (eventually superfluid) low-dissipation spin currents & bosonic many-body physics**