Self-Organization in Complex Systems: The Past, Present and Future of Synergetics
Symposium in Honour of Prof. Hermann Haken‘s 85th Birthday
Delmenhorst 15. 11. 2012

CONTROL OF SELF-ORGANIZING COMPLEX SYSTEMS AND NETWORKS WITH TIME-DELAY

Eckehard Schöll

Institut für Theoretische Physik
and
SFB 910 Control of Self-Organizing Nonlinear Systems
Technische Universität Berlin
Germany

http://www.itp.tu-berlin.de/schoell

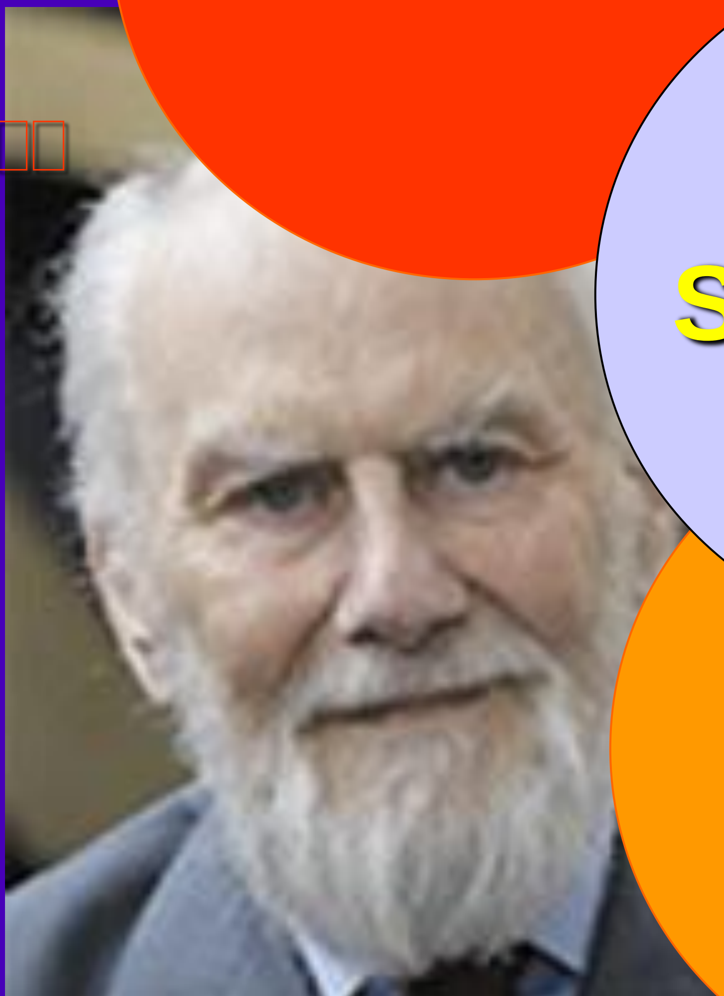
Happy Birthday to Hermann Haken!

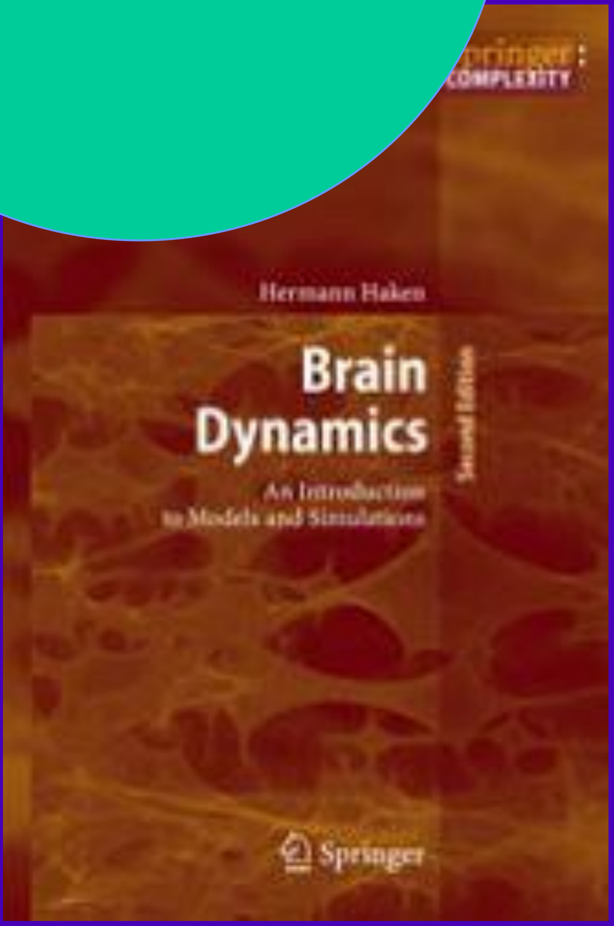
Laser

Semiconductors

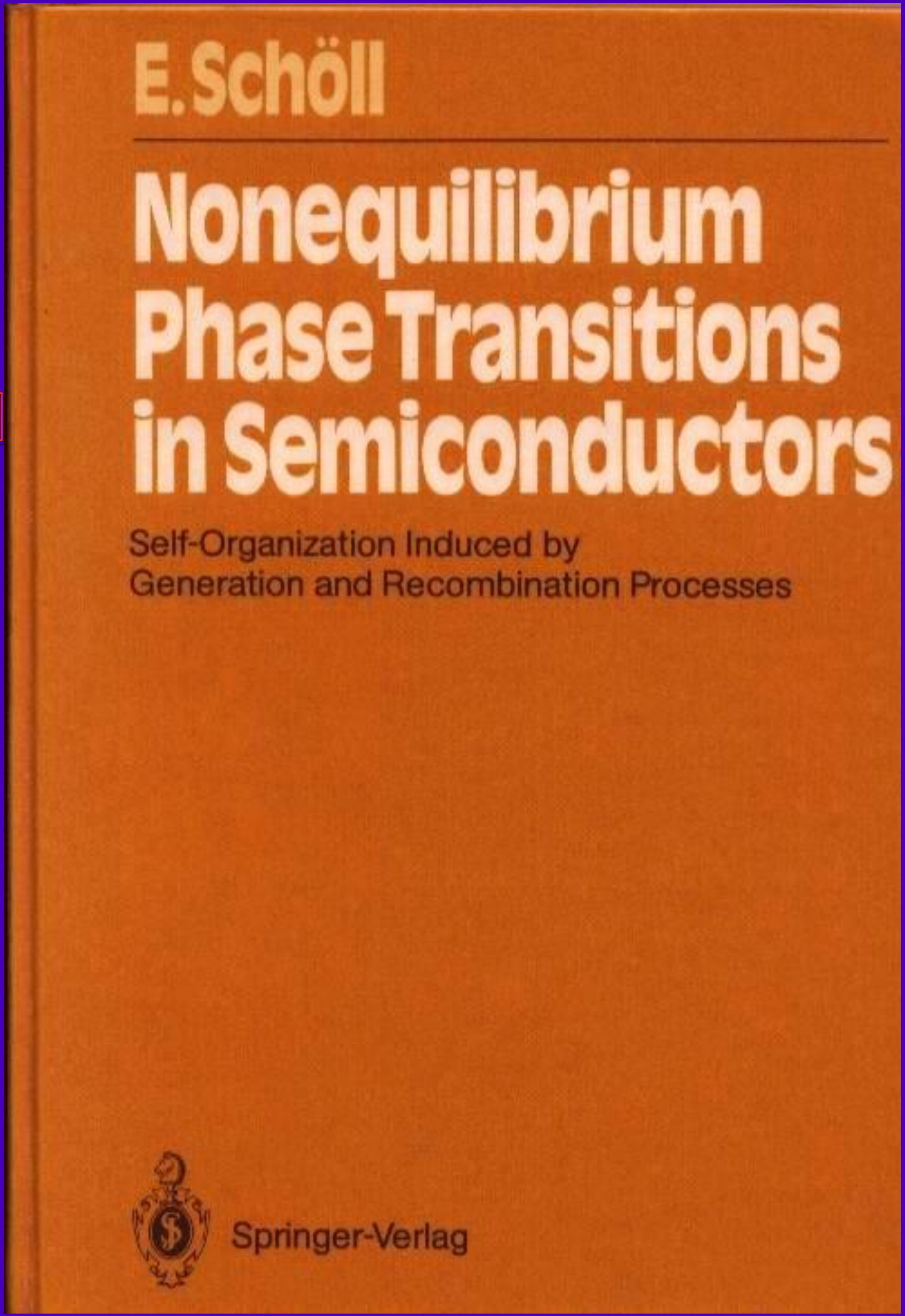
Synergetics

Brain

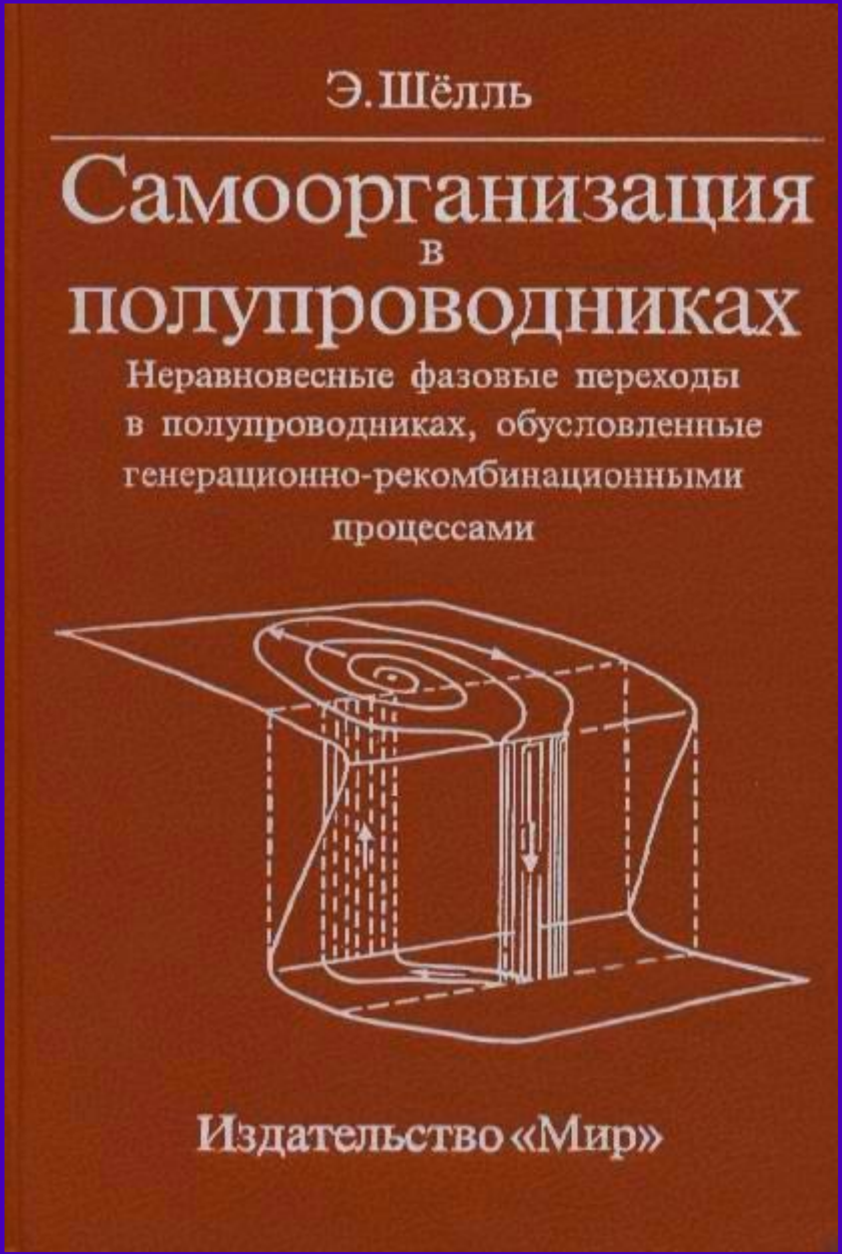




Springer Series in Synergetics Vol. 35 (1987)



Russian translation (1991):



Outline

- Introduction: Delay in nonlinear systems and networks
- Universal classification of delay-coupled networks for large time delay: stability of synchronization
- Application to neural networks and coupled lasers
- Controlling synchrony in delay-coupled networks: from in-phase to splay and cluster states
- Adaptive control

Delay in nonlinear systems is ubiquitous

- **mechanical systems: balancing, segway**
- **electronic systems: capacitive effects (τ=RC)**
  latency time due to processing
- **optical systems: signal transmission times**
  travelling waves + reflections
  - **laser coupled to external cavity (Fabry-Perot)**
  - **laser with optical injection or feedback (mirror)**
  - **optically coupled lasers**
- **biological systems: cell cycle time**
  biological clocks
  - **neural networks: delayed coupling, delayed feedback**

Why is delay interesting in dynamics?

- Delay increases the dimension of a differential equation to infinity:

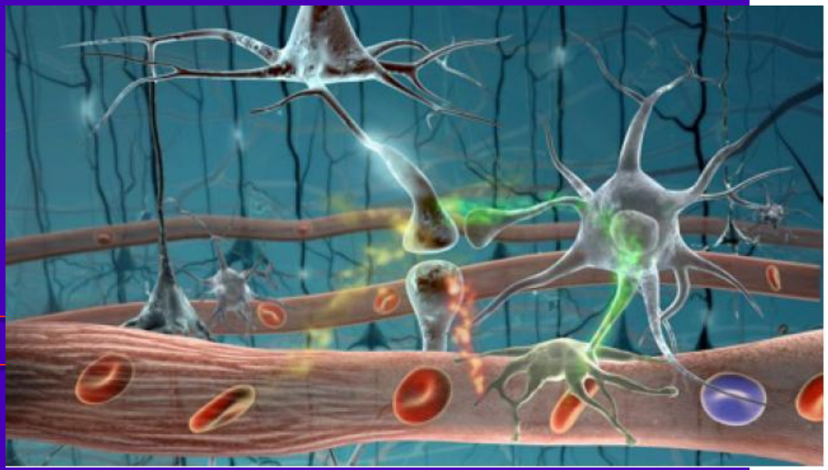
$$\dot{x}(t) = -ax(t) + bx(t-\tau)$$

delay τ generates infinitely many eigenmodes

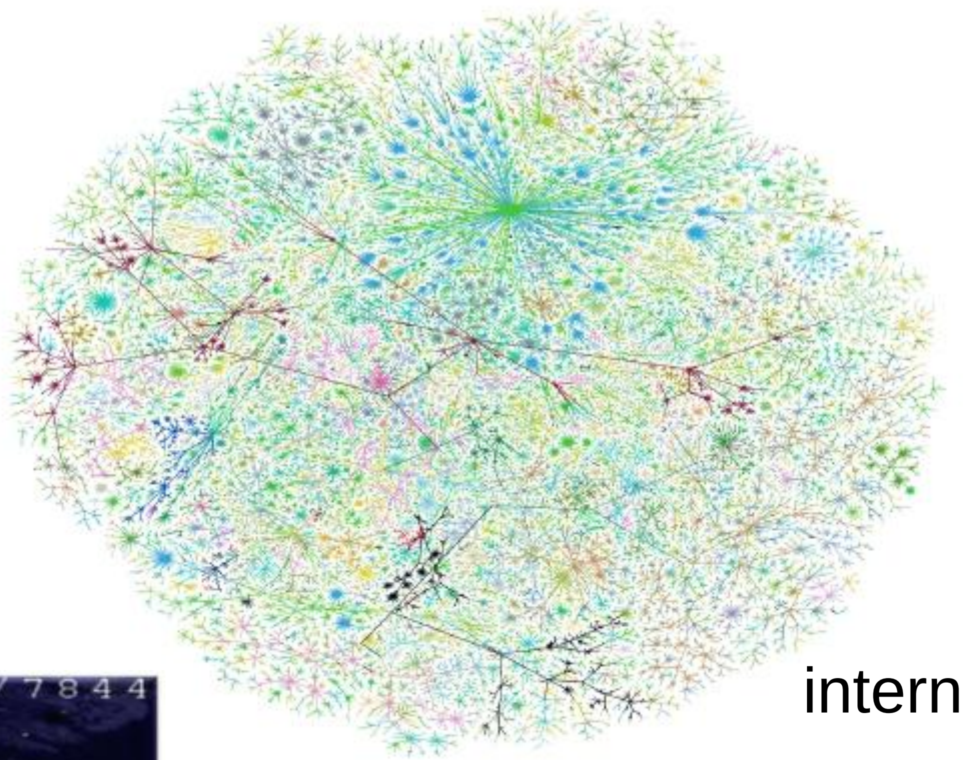
- Simple equations produce very complex behavior:
- delay-induced bifurcations
- delay-induced multistability
- stabilization of unstable periodic or stationary states

Examples of complex systems (networks)

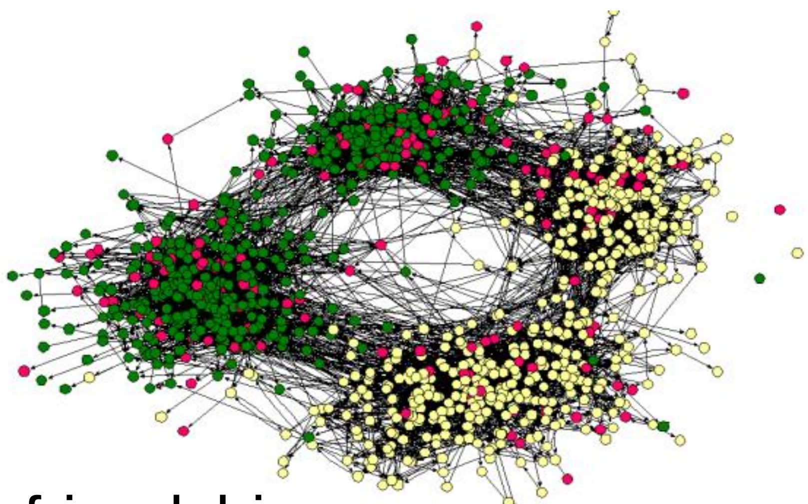
brain



internet



power grid



friendships

Synchronization in complex networks

- Synchronization and Desynchronization
  - Constructive role for strongly coherent fields:
    - Laser system, …

      → **Synchronization**

      - **A. Pikovsky, *et al.*, *Synchronization*, Cambridge, 2001**
  - On occasion, undesirable phenomenon:
    - Parkinsonian tremor
    - Swaying motion of London's Millennium Bridge

      → **Desynchronization**

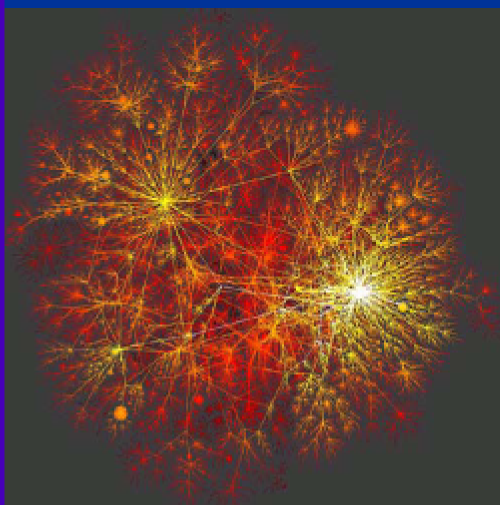
Tass, *Biol. Cybern.*, 89, 81 (2003)

. Rosenblum *et al.*, *Phys. Rev. Lett.* 92, 114102 (2004)

Popovych *et al.*, *Phys. Rev. Lett.* 94, 164102 (2005)

Stability of synchronous solutions

- Master Stability Function (MSF)
  - *L. M. Pecora and T. L. Carroll: Phys. Rev. Lett. 80, 2109 (1998).*



Network System

$$\dot{x}_i = F(x_i) + \sigma \sum_j G_{ij} H(x_j)$$

$$i = 1, \cdots, N$$

ν_k : Eigenvalue of G

Topological Part

$$\dot{\xi} = [DF + (\alpha + i\beta) DH]\xi$$

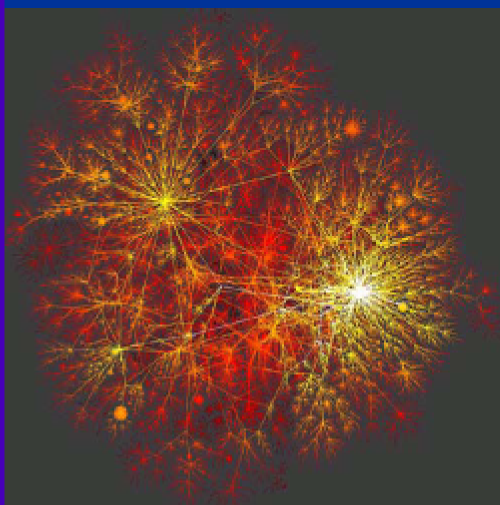
Local Dynamics

LyapunovExp. $\lambda(\nu_k)$

MSF Method

Stability of synchronous solutions

- Master Stability Function (MSF)
  - *L. M. Pecora and T. L. Carroll: Phys. Rev. Lett. 80, 2109 (1998).*



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Topological Part

ν_k : Eigenvalue of G

Local Dynamics

$$\dot{\xi} = \left[DF + (\alpha + i\beta)DH\right]\xi$$

MSF Method

$$\text{LyapunovExp.}\ \lambda(\nu_k)$$

Extend to delayed coupling:

$$\dot{\mathbf{x}}_i = \mathbf{F}(\mathbf{x}_i(t)) + \sigma \sum_j G_{ij} \mathbf{H}\left[\mathbf{x}_j(t-\tau) - \mathbf{x}_i(t)\right]$$

Synchronization in delay-coupled networks

Synchronization and delay

Synchronization in networks:

- Synchronization of neurons in the brain:
  cognition, learning, Parkinson's, epilepsy
- Chaos synchronization of lasers
  chaos communication, encryption

Delays play a crucial role in realistic networks:

delay $\longleftrightarrow$ synchronization

Synchronization in delay-coupled networks

- M. Dhamala, V. K. Jirsa, M. Ding, PRL 92, 074104 (2004)
- I. Fischer, R. Vicente, J.M. Buldu, M. Peil, C. Mirasso, M. Torrent, J. Garcia-Ojalvo: PRL 97, 123902 (2006)
- W. Kinzel, A.Englert, G.Reents, M.Zigzag, I. Kanter, PRE 79, 056207 (2009)
- V. Flunkert, S.Yanchuk,T. Dahms, E.Schöll: PRL 105, 254101 (2010)

….

Theme Issue on Delayed Complex Systems
W. Just, A. Pelster, M. Schanz, and E. Schöll (Eds.):
Phil. Trans. Royal Soc. A 368, 301 (2010)

Synchronization in delay-coupled networks

Master stability function (MSF)

- Network of delay-coupled elements (for simplicity use maps):

$$x_{t+1}^i = f(x_t^i) + \sum_{j=1}^{N} g_{ij}\, h(x_{t-\tau}^j) \qquad (x^i \in \mathbb{R}^d)$$

$\sigma = \sum_{j=1}^N g_{ij}$ row sum of the coupling matrix G

$\gamma_1 \dots \gamma_{N-1}$ transverse eigenvalues of G

- Small perturbations ξ^i around synchronized solution: $x_t^i = \overline{x}_t + \xi_t^i$
- Linearization and diagonalization of G $\rightarrow$ variational equations

$$\xi_{t+1} = Df(\overline{x}_t)\,\xi_t + \sigma\, Dh(\overline{x}_{t-\tau})\,\xi_{t-\tau},$$

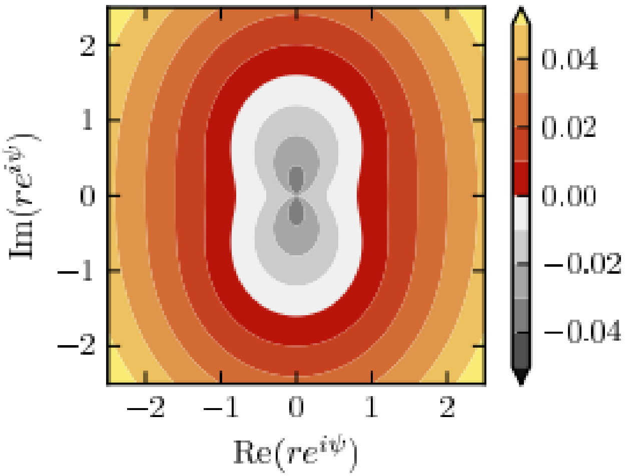
$$\xi_{t+1} = Df(\overline{x}_t)\,\xi_t + \gamma_n\, Dh(\overline{x}_{t-\tau})\,\xi_{t-\tau}$$

Synchronization in delay-coupled networks

Master stability function (MSF) II

MSF = largest Lyapunov exponent $\lambda_{\max}(r\,e^{i\psi})$ that arises from the variational equation

$$\xi_{t+1} = Df(\overline{x}_t)\,\xi_t \;+\; r\,e^{i\psi}\,Dh(\overline{x}_{t-\tau})\,\xi_{t-\tau}$$



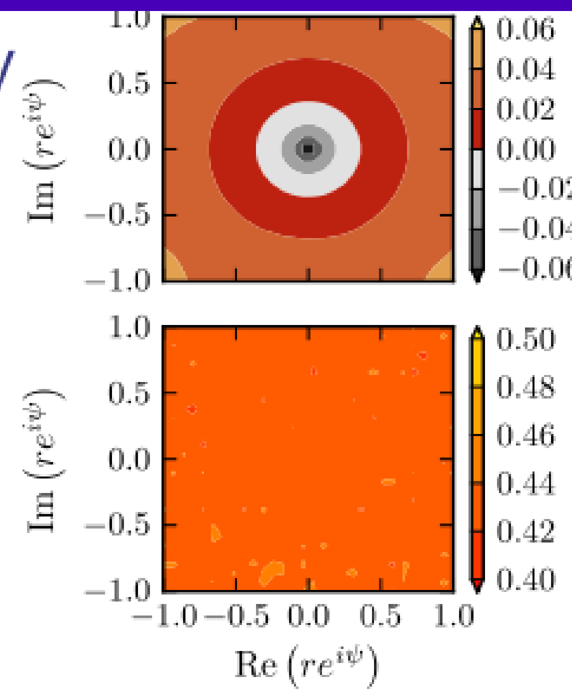
$\lambda_{\max}(\sigma) > 0$	system chaotic
$\lambda_{\max}(\sigma) < 0$	system non-chaotic
$\lambda_{\max}(\gamma_n) < 0$	synchronization stable
$\lambda_{\max}(\gamma_n) > 0$	synchronization unstable

V. Flunkert, S. Yanchuk, T. Dahms, E. Schöll: PRL 105, 254101 (2010)

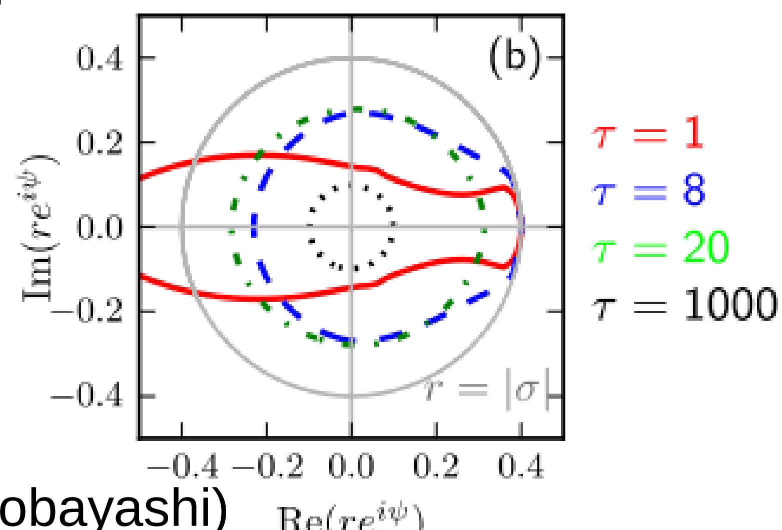
Synchronization in networks with large delay

Structure of the MSF for large delay

$$\lambda_{\max}(re^{i\psi})$$



- Rotationally symmetric around the origin, i.e. independent of ψ
- Either negative at the origin and monotonically increasing with r $\rightarrow$ $\lambda_{\max}$ changes sign at a crictical radius r_0
- Or positive at the origin and then constant everywhere
  [Local Lyapunov exponent]



Coupled semiconductor lasers (Lang-Kobayashi)

V. Flunkert, S. Yanchuk, T. Dahms, E. Schöll: PRL 105, 254101 (2010)

Synchronization in networks with large delay

Universal classification of networks

class		chaotic synchronized dynamics ($r_0 < \|\sigma\|$)	non-chaotic synchronized dynamics ($r_0 > \|\sigma\|$)
(A)	$\|\gamma_{\max}\| < \|\sigma\|$	synchr. stable iff $\|\gamma_{\max}\| < r_0$	synchr. stable
(B)	$\|\gamma_{\max}\| = \|\sigma\|$	synchr. unstable	synchr. stable
(C)	$\|\gamma_{\max}\| > \|\sigma\|$	synchr. unstable	synchr. stable iff $\|\gamma_{\max}\| < r_0$

$\sigma = \sum_{j=1}^{N} g_{ij}$ row sum of the coupling matrix G

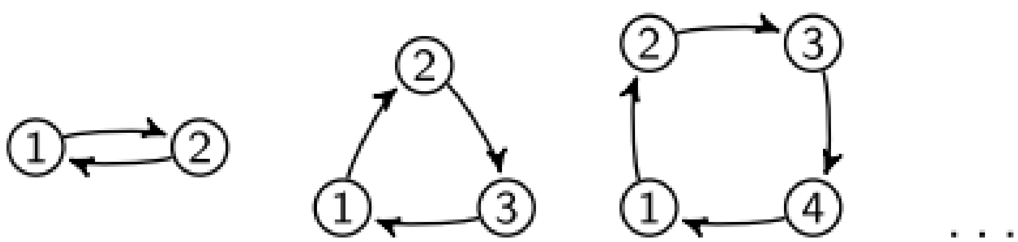
$\gamma_1 \dots \gamma_{N-1}$ transverse eigenvalues of G

V. Flunkert, S. Yanchuk, T. Dahms, E. Schöll: PRL 105, 254101 (2010)

Synchronization in networks with large delay

Class B and C – no chaos synchronization possible

- Rings of unidirectionally coupled elements



- Networks with zero row sum ($\sigma = 0$)

  [W. Kinzel *et al.*, Phys. Rev. E **79**, 056207 (2009)]

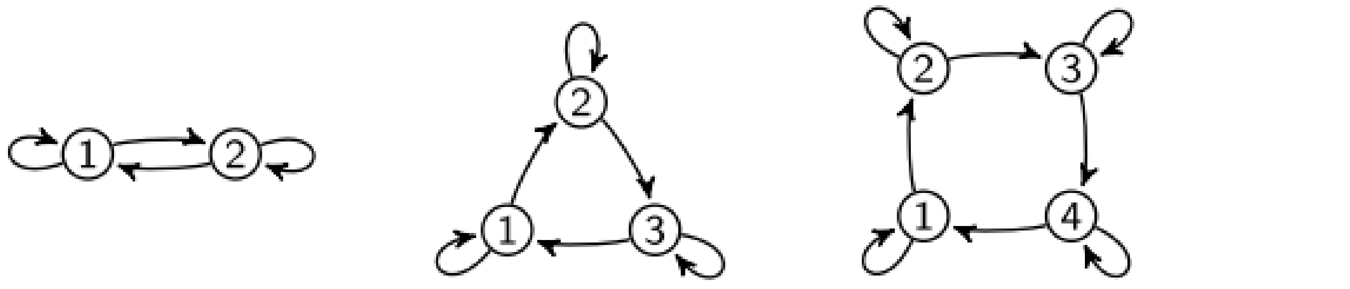
Lasers (including role of phases):

V. Flunkert, E. Schöll: New J. Phys. 14, 033039 (2012)

Synchronization in networks with large delay

Class A – chaos synchronization possible,
non-chaotic synchronization stable

- Rings of unidirectionally coupled elements with self-feedback



- Rings with bidirectional coupling
- Mean-field coupled systems
- Networks with only inhibitory or only excitatory coupling (class A or B)

V. Flunkert, E. Schöll: New J. Phys. 14, 033039 (2012)

Networks of chaotic lasers

$$\dot{\mathbf{x}}_i^{(k)} = \mathbf{F}(\mathbf{x}_i^{(k)}) + \sigma \sum_{n=1}^{M} \sum_{j=1}^{N_n} A_{ij}^{(kn)} \mathbf{H}^{(kn)} \mathbf{x}_j^{(n)}(t - \tau^{(kn)})$$

The Lang-Kobayashi model for semiconductor lasers

with carrier inversion n, electric field $E = x + iy$: $\mathbf{x} = (n, x, y)^T$

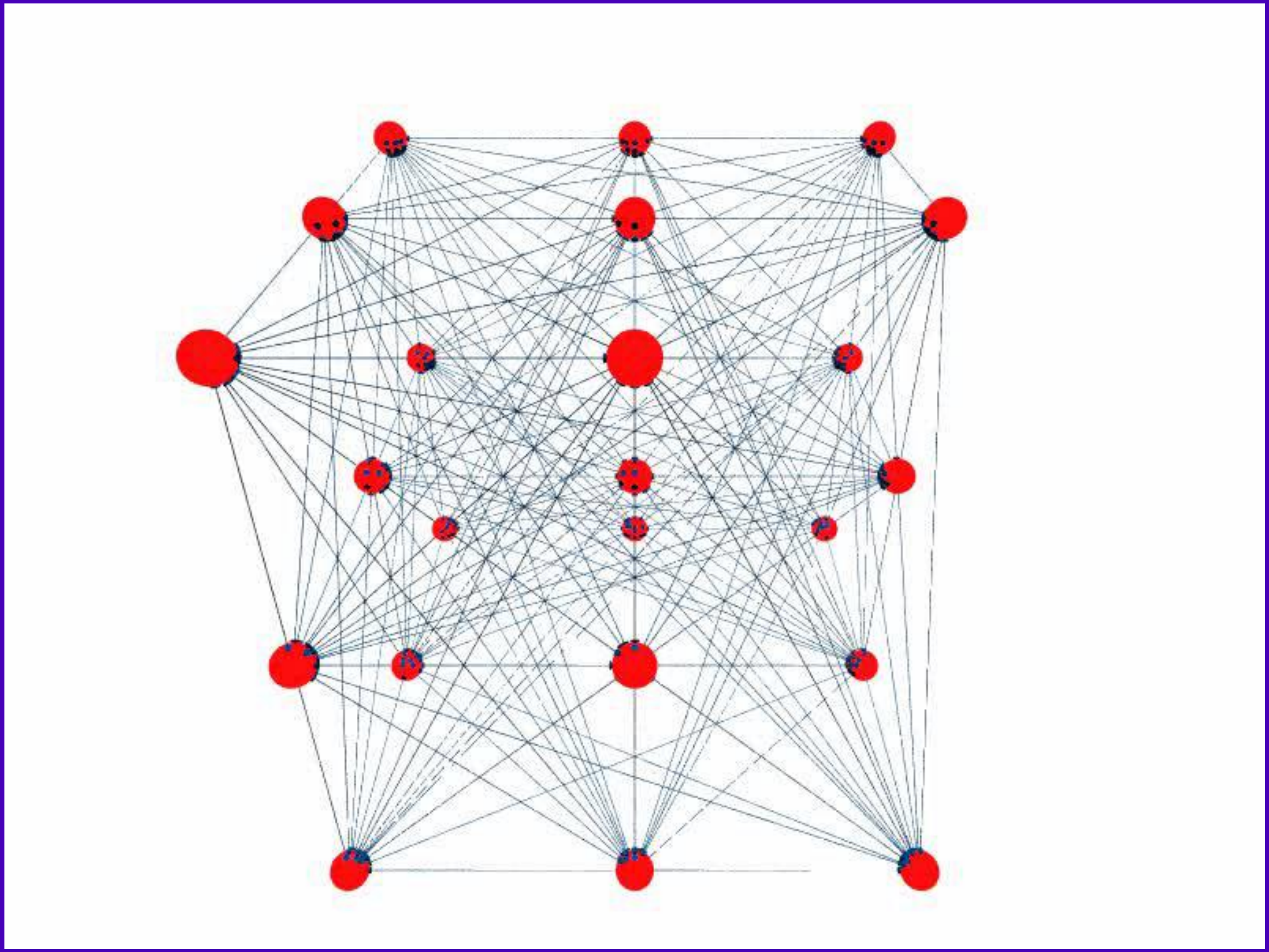
$$\mathbf{F}(\mathbf{x}) = \begin{pmatrix} \frac{1}{T}\left[I - n - (1+n)\left(x^2 + y^2\right)\right] \\ \frac{n}{2}(x - \alpha y) \\ \frac{n}{2}(\alpha x + y) \end{pmatrix}, \quad \mathbf{H} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Why networks of lasers?

- Chaotic dynamics as information carrier
- Secure communication
- Lang-Kobayashi as paradigmatic model in nonlinear dynamics

T. Dahms, J. Lehnert, E. Schöll: Phys. Rev. E 86, 016202 (2012)

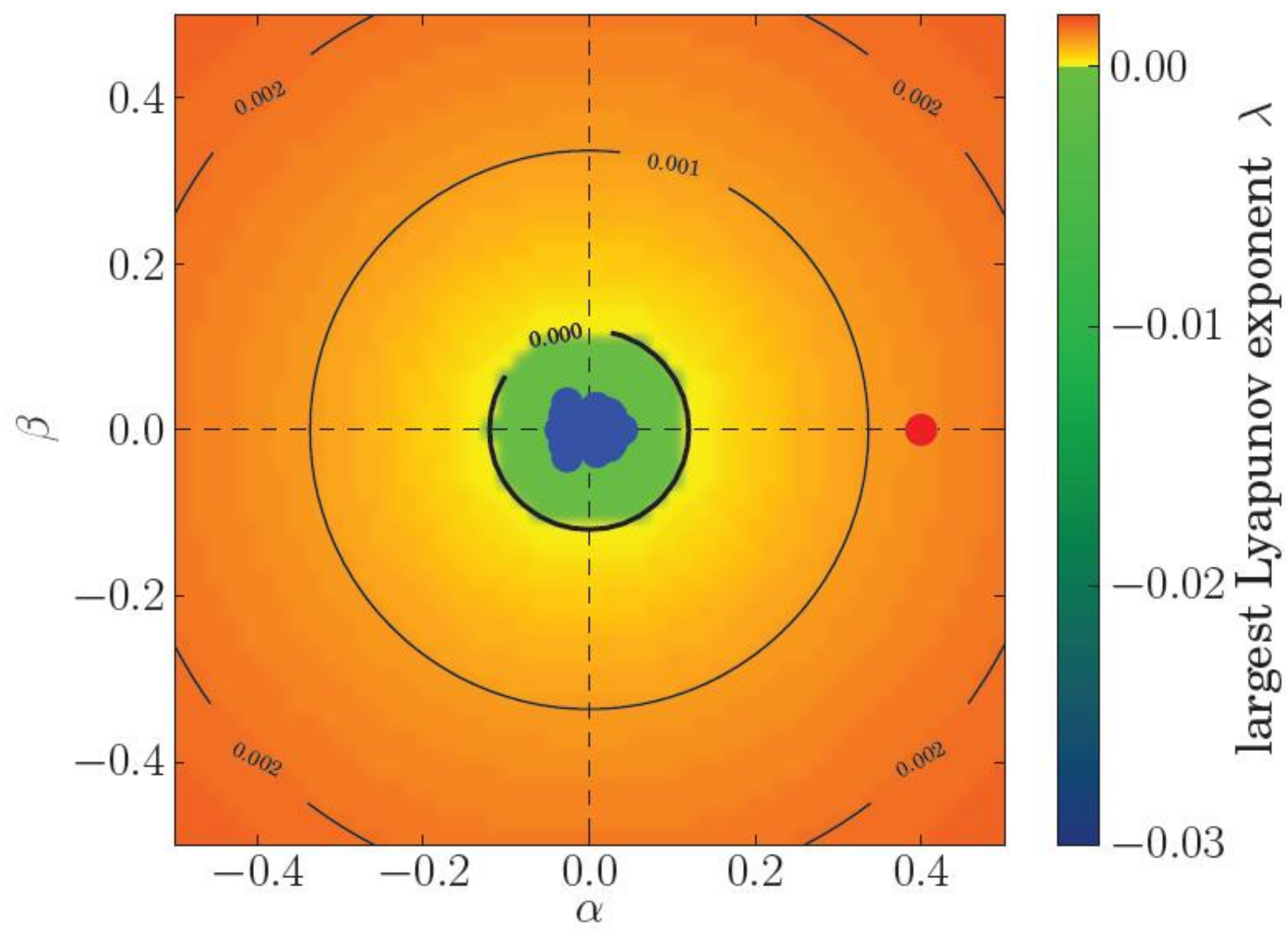
Isochronous chaotic synchronization



Example: isochronous synchronization in a random network, $N = 25$ lasers, $p = 0.8$ τ=1000 (1 ns)

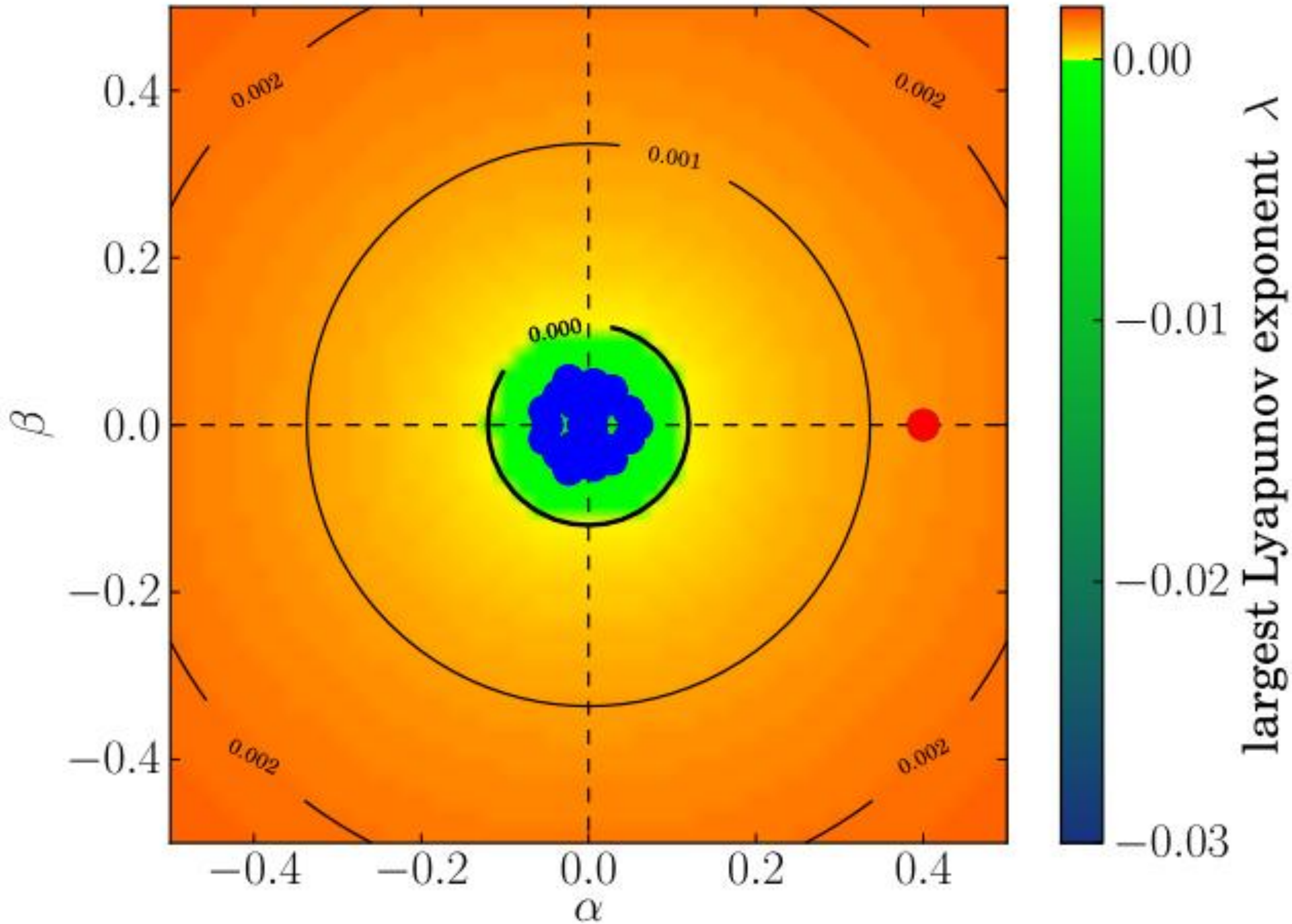
Master stability function

Example: random network: $p = 0.8$

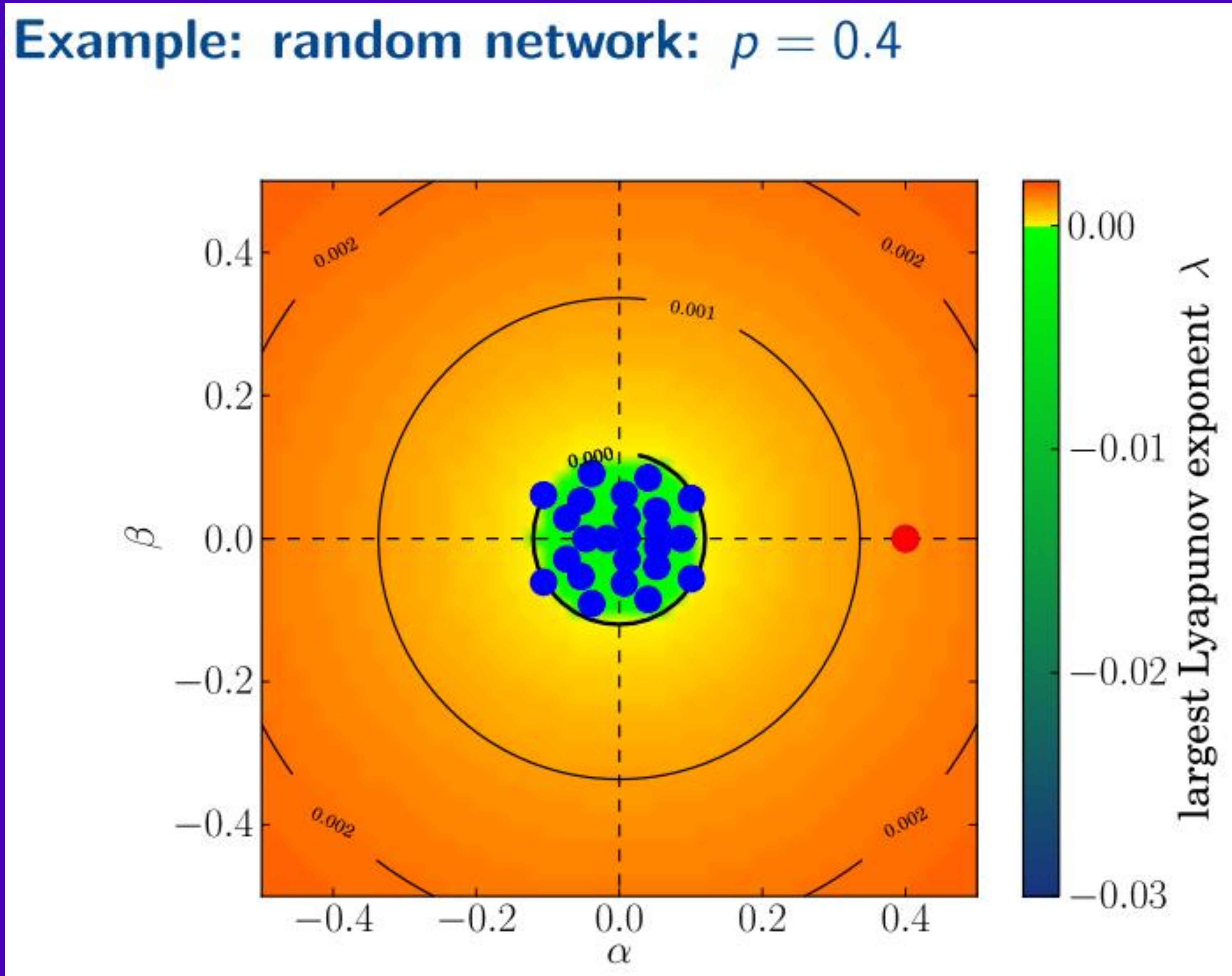


Master stability function

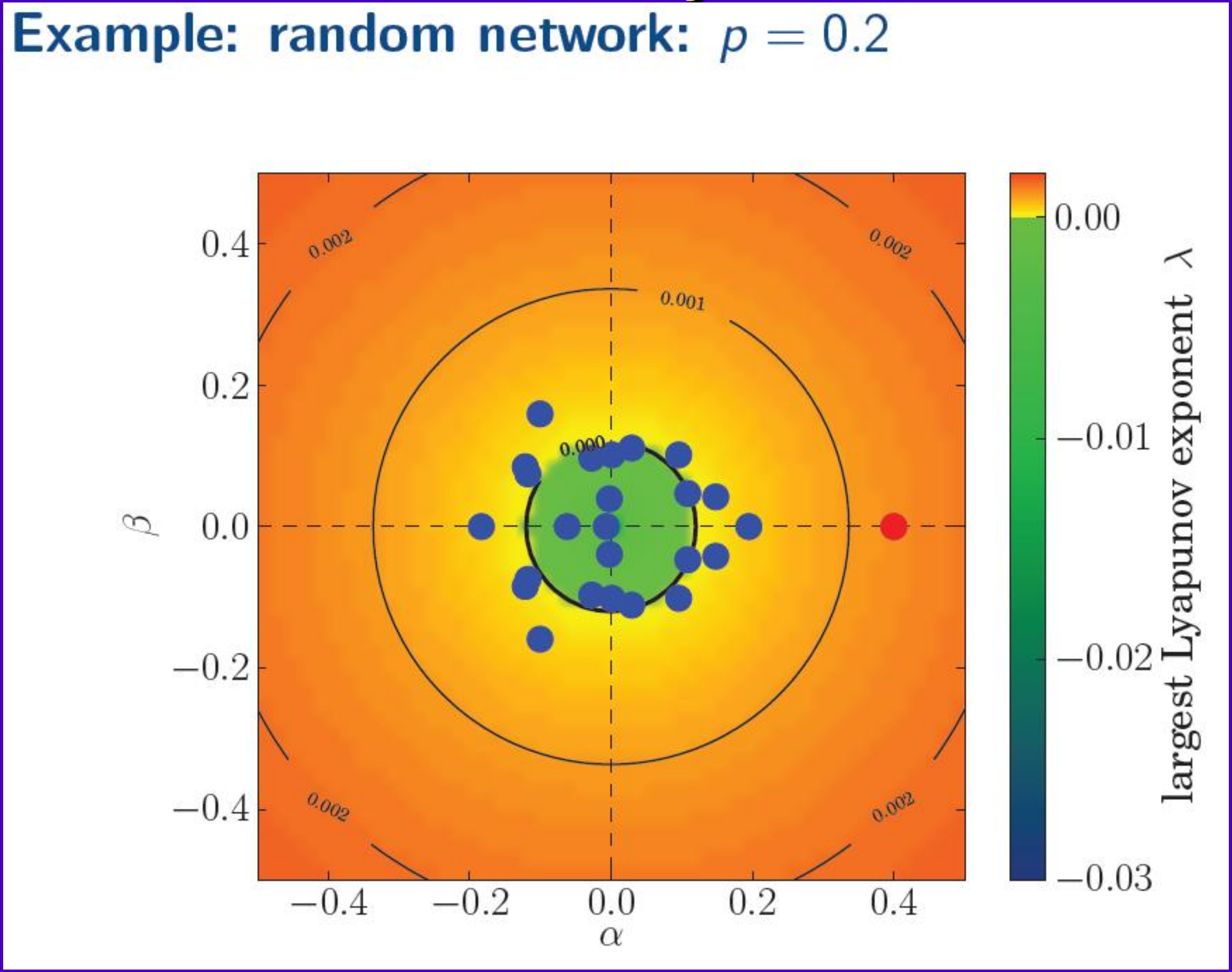
Example: random network: $p = 0.6$



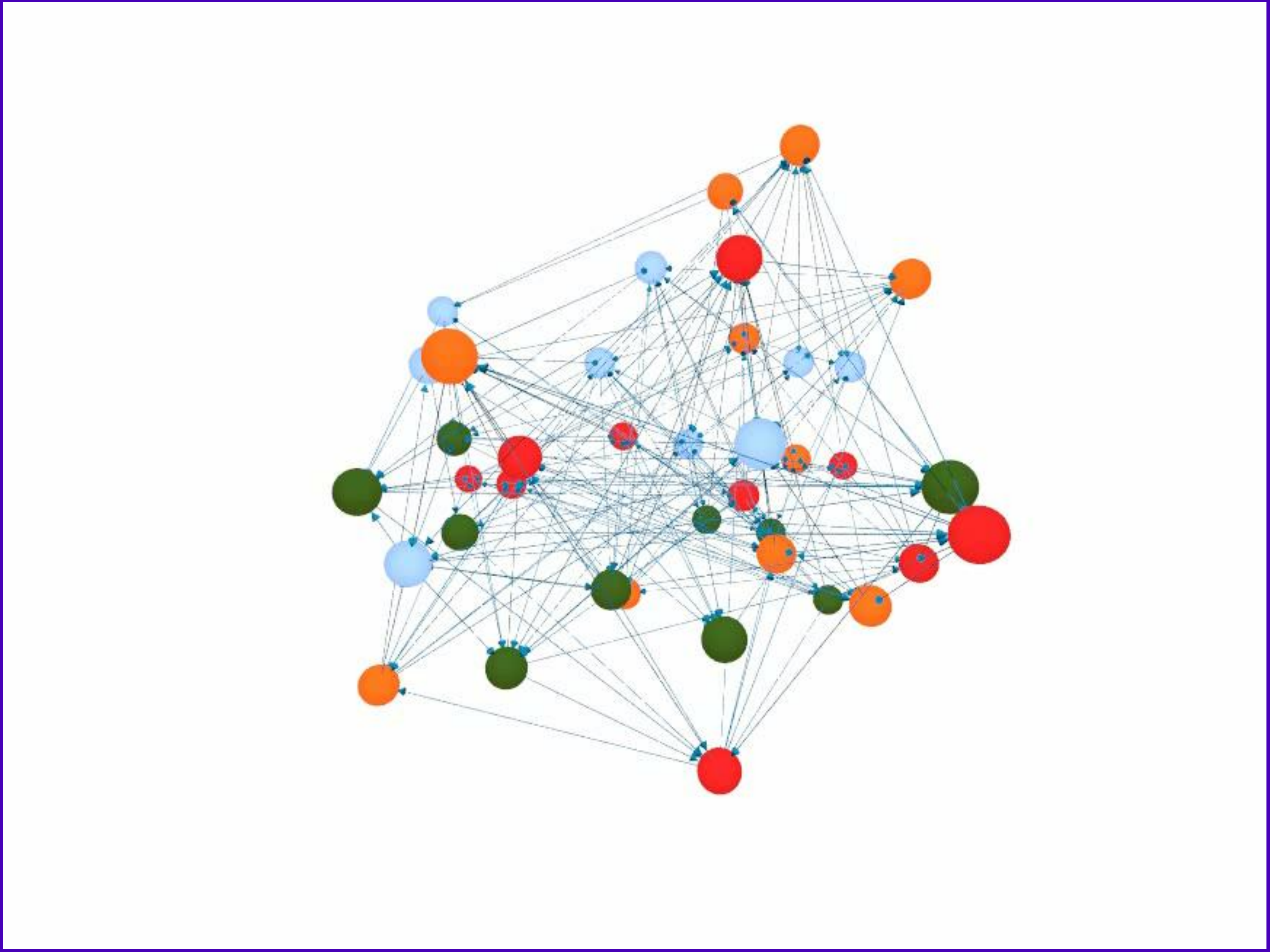
Master stability function



Master stability function



Group synchrony



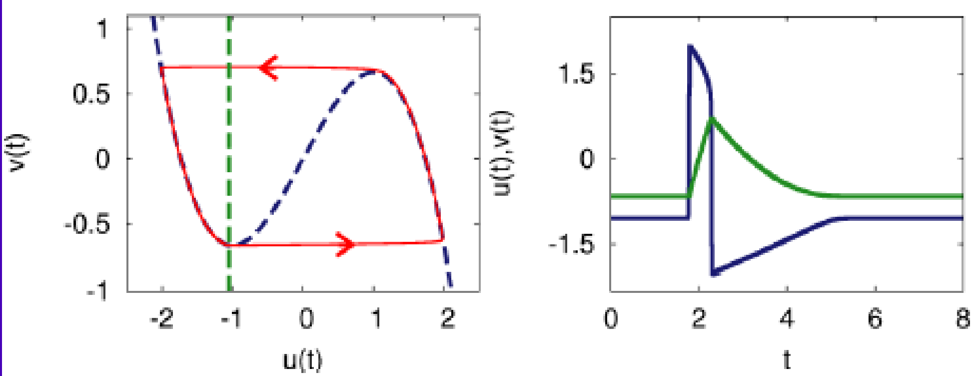
T. Dahms, J. Lehnert, E. Schöll: Phys. Rev. E 86, 016202 (2012)

Synchronization in neural networks

The FitzHugh-Nagumo model for neuronal activity

with activator u, inhibitor v: $\mathbf{x} = (u, v)^T$

$$\mathbf{F}(\mathbf{x}) = \begin{pmatrix} \frac{1}{\epsilon}\left(u - \frac{u^3}{3} - v\right) \\ u + a \end{pmatrix}, \quad \mathbf{H} = \begin{pmatrix} 1/\epsilon & 0 \\ 0 & 0 \end{pmatrix}$$



excitability type-II

- operation in the excitable regime
- uncoupled neurons rest in fixed point
- network coupling induces periodic spiking
- time delay sets period

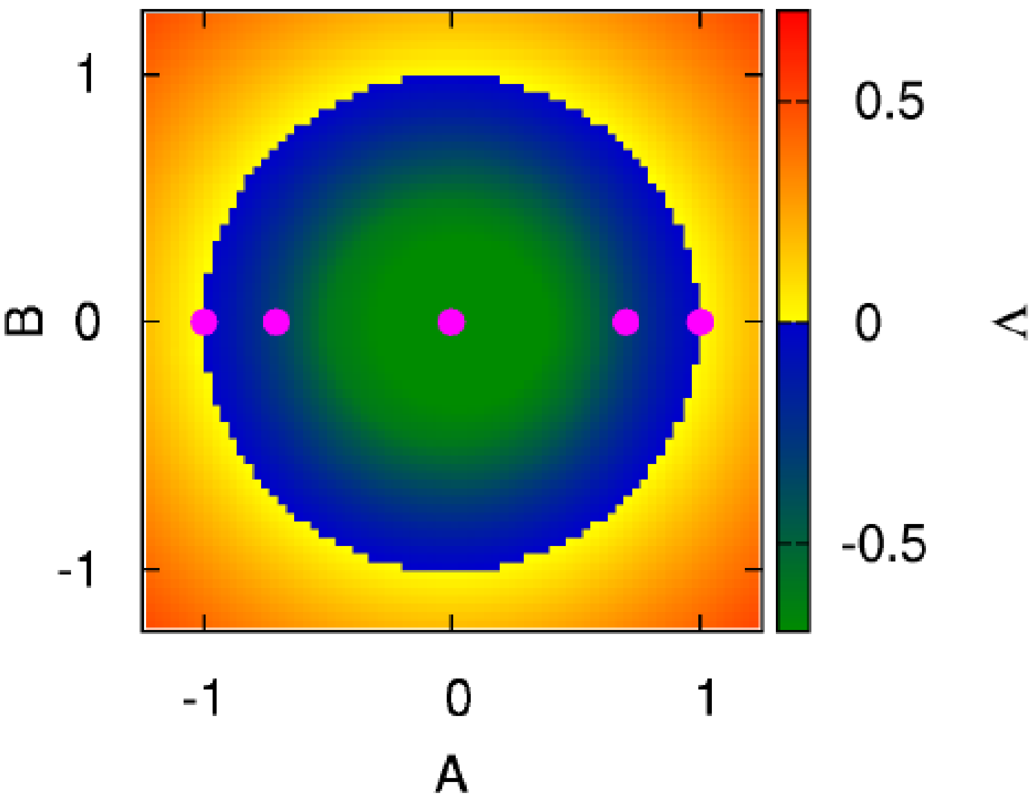
$$\dot{\mathbf{x}}_i = \mathbf{F}(\mathbf{x}_i(t)) + \sigma \sum_j G_{ij}\mathbf{H}\left[\mathbf{x}_j(t-\tau) - \mathbf{x}_i(t)\right]$$

In-phase synchronization of FHN model

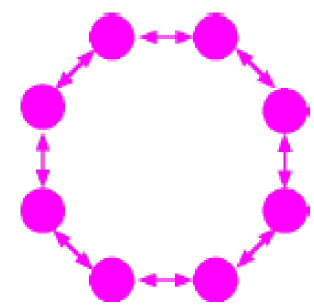
Master stability function

$$\dot{\xi} = (D\mathbf{F} + gC)\xi + C(A + iB)\mathbf{H}\xi_\tau$$

Stability does not depend on τ or C but exclusively on the topology!!!



real coupling C



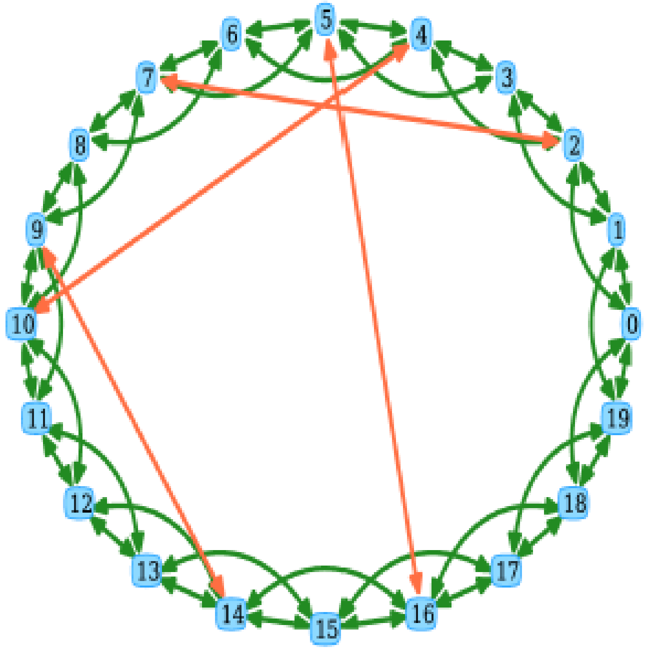
Class B: stable periodic sync

Maximum Lyapunov exponents as a continuous function of the eigenvalues (topology!)

J. Lehnert, Th. Dahms, P. Hövel, E. Schöll: Europhys. Lett. 96, 60013 (2011)

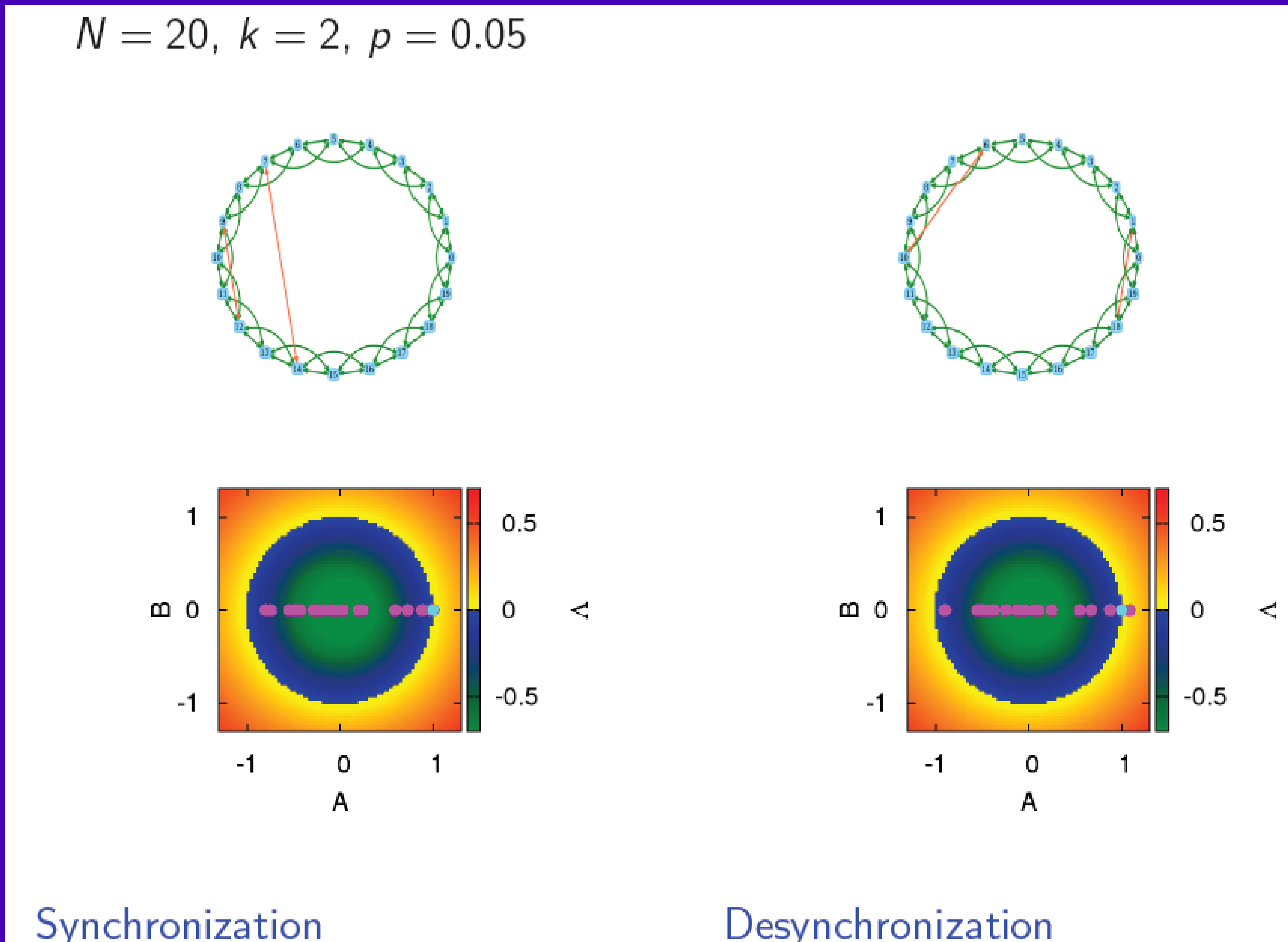
Small-world network with inhibitory coupling

- Regular network of excitatory coupled neurons
- Add randomly for each excitatory link with probability p an inhibitory one (no rewiring!)
- Example: $N = 20$, next-nearest neighbors $k = 2$, $p = 0.1$



M. E. J. Newman and D. J. Watts, Phys. Lett. A 263, 341 1999

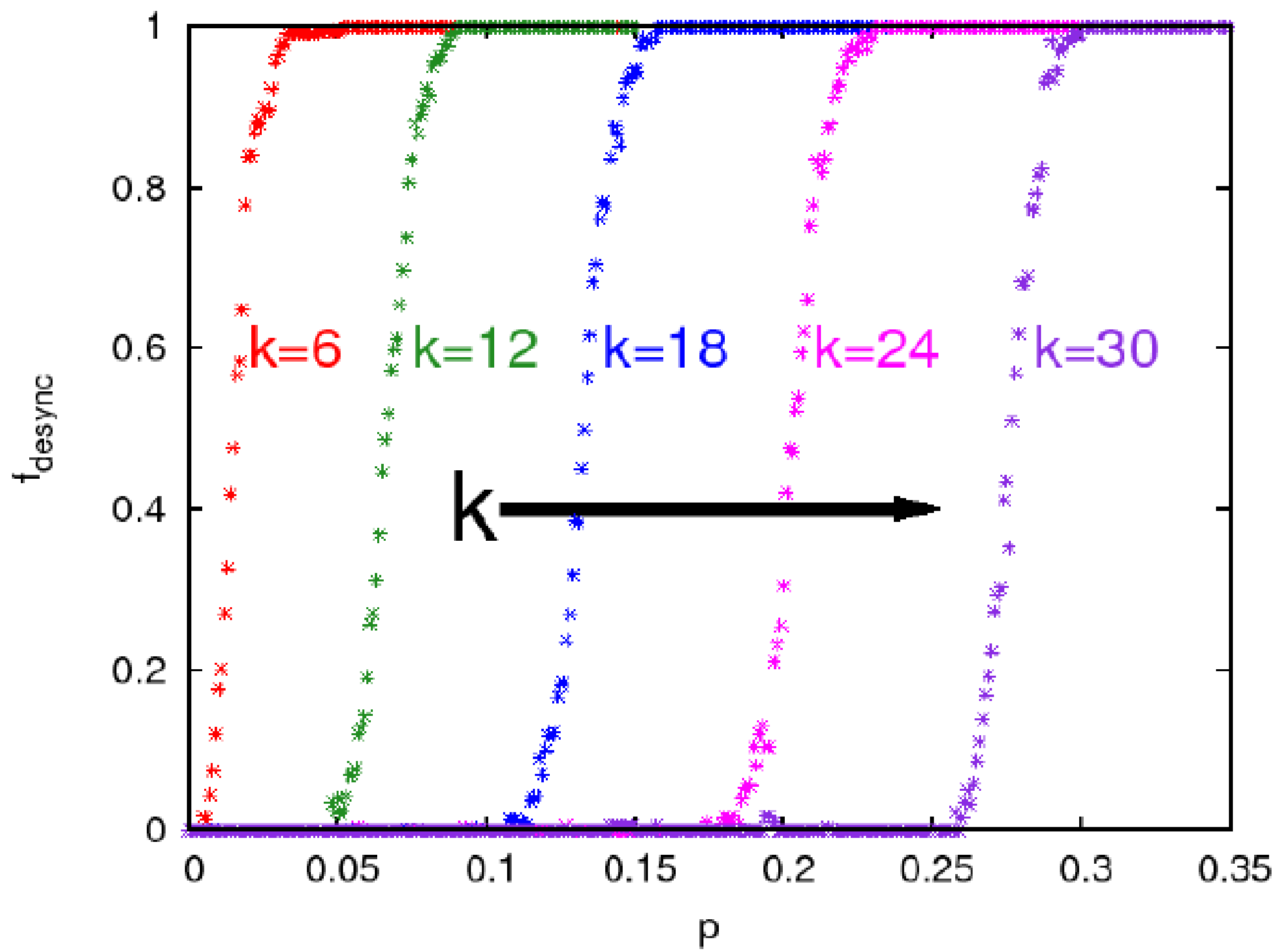
Eigenvalue spectrum of coupling matrix



J. Lehnert, Th. Dahms, P. Hövel, E. Schöll: Europhys. Lett. 96, 60013 (2011)

Desynchronization phase transition

Fraction of desynchronized networks ($N = 100$)

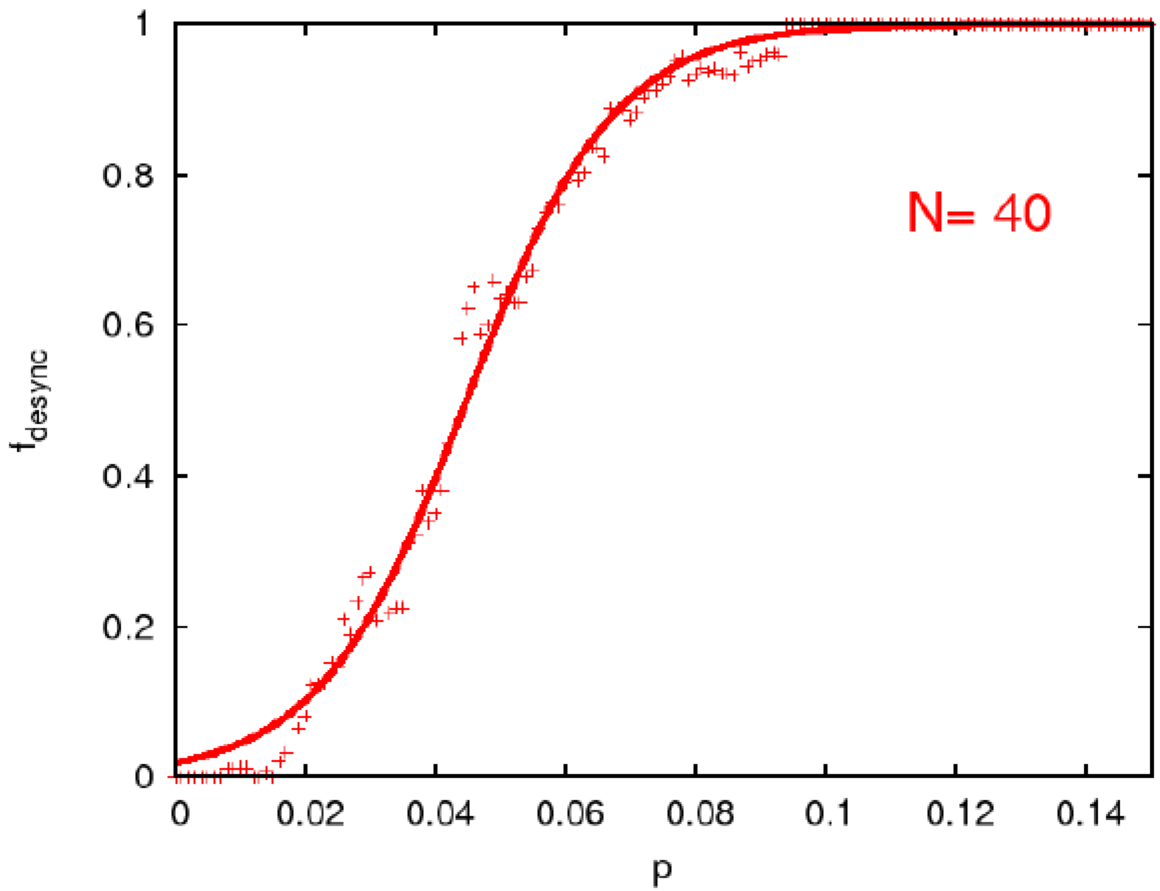


Inhibitory links introduce desynchronization

Thermodynamic limit of phase transition

Fraction of desynchronized networks:

$N/k = 10$



Thermodynamic limit of phase transition

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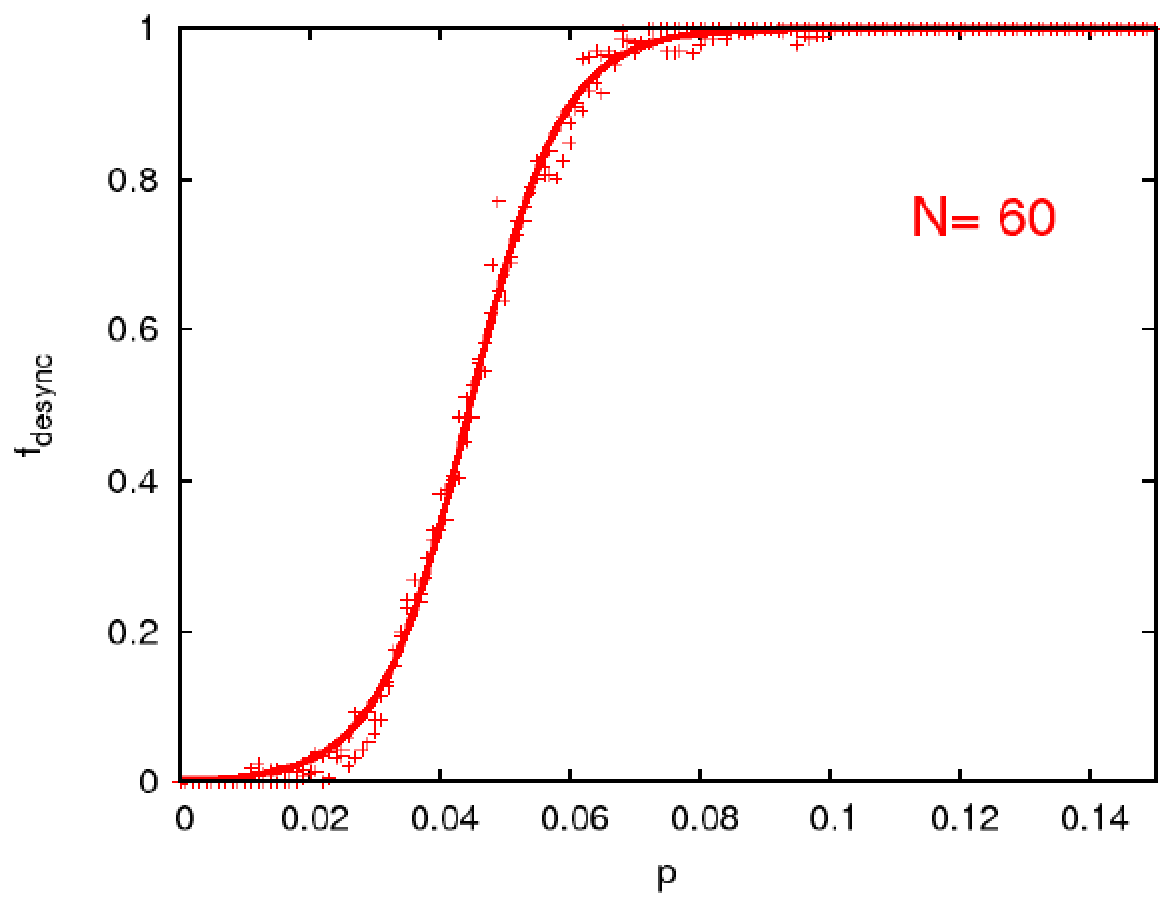

Thermodynamic limit of phase transition

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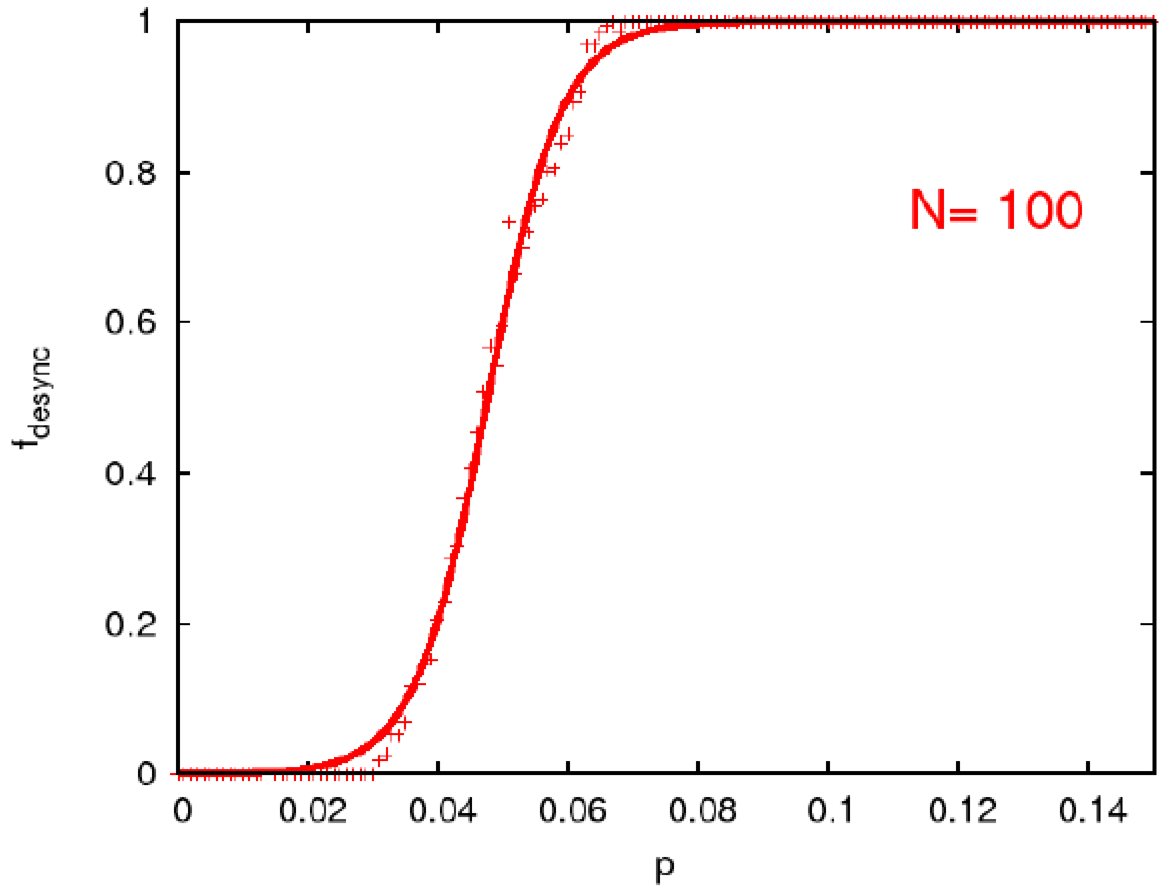

Thermodynamic limit of phase transition

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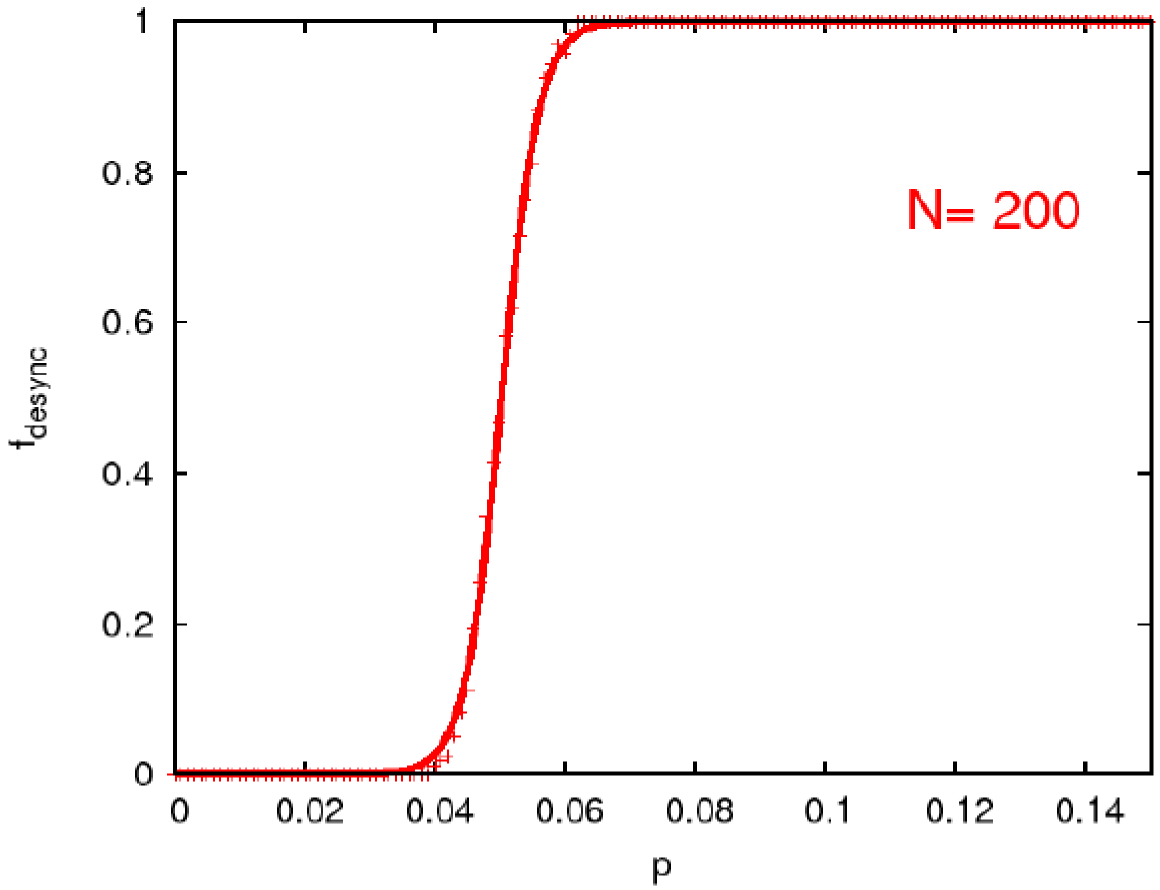

Thermodynamic limit of phase transition

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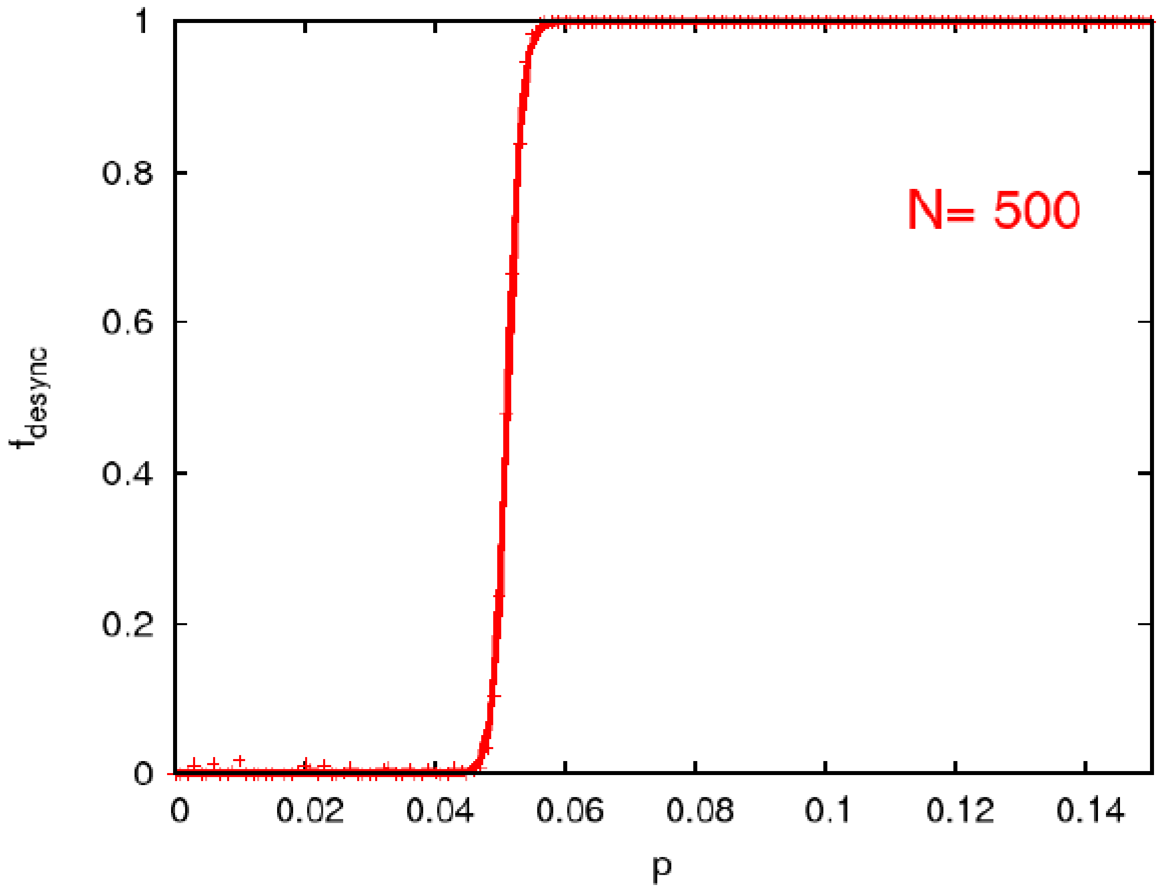

Thermodynamic limit of phase transition

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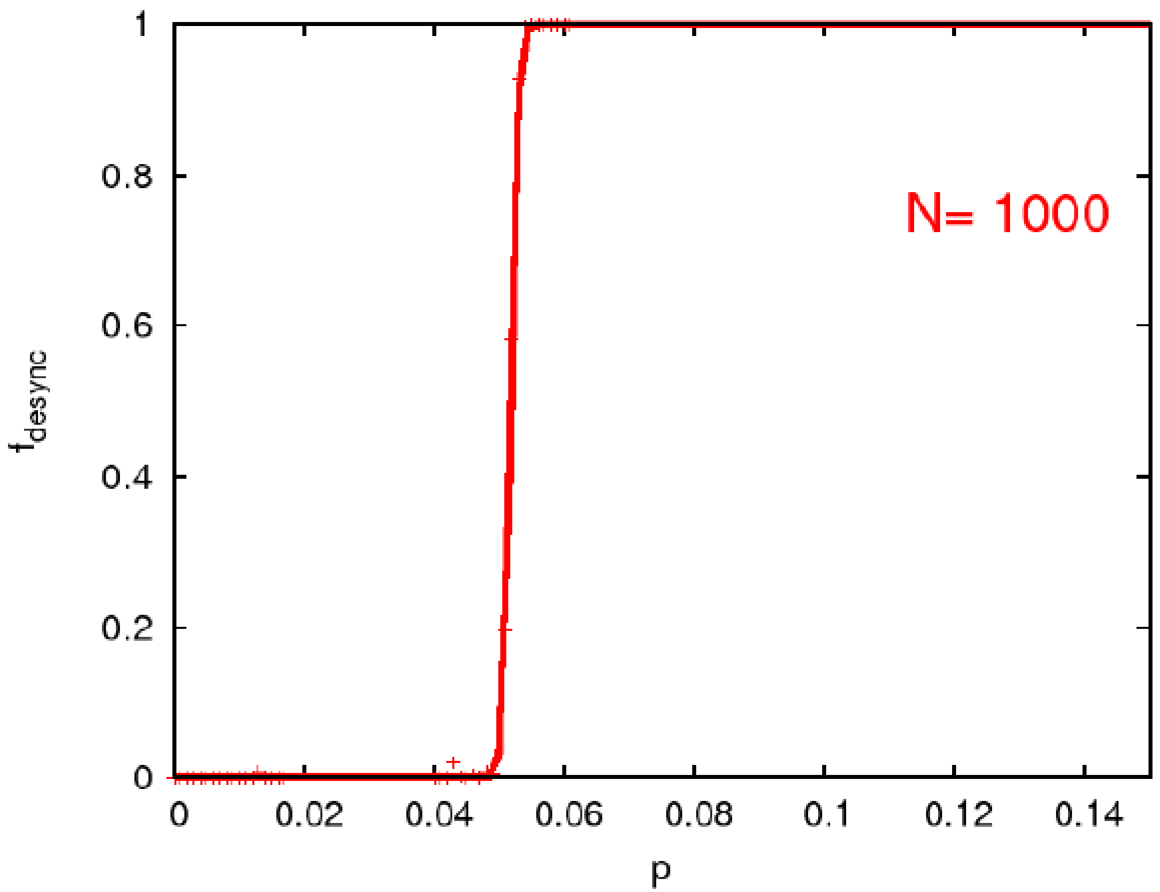
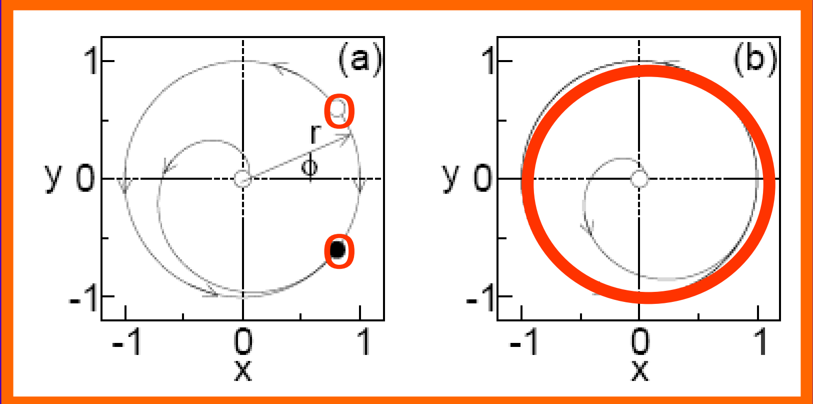


Excitability type-I: SNIPER or SNIC (saddle-node infinite period bifurcation)

Global bifurcation of limit cycle for b=1 (SNIPER)
frequency of limit cycle scales as $f \sim (b-1)^{1/2}$

$$\dot{x} = x(1 - x^2 - y^2) + y(x - b)$$
$$\dot{y} = y(1 - x^2 - y^2) - x(x - b)$$

b < 1 b > 1



$$\dot{r} = r\left(1 - r^2\right)$$
$$\dot{\varphi} = b - r\cos\varphi$$

Gang Hu, Ditzinger, Ning, Haken, PRL 71, 807 (1993)

Excitable system exhibiting coherence resonance for b<1
(similar: Hindmarsh-Rose model for neurons)

Coherence Resonance

Gang Hu, Ditzinger, Ning, Haken, PRL 71, 807 (1993)
Pikovsky, Kurths, PRL 78, 775 (1997)

PHYSICAL REVIEW
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Stochastic Resonance without External Periodic Force

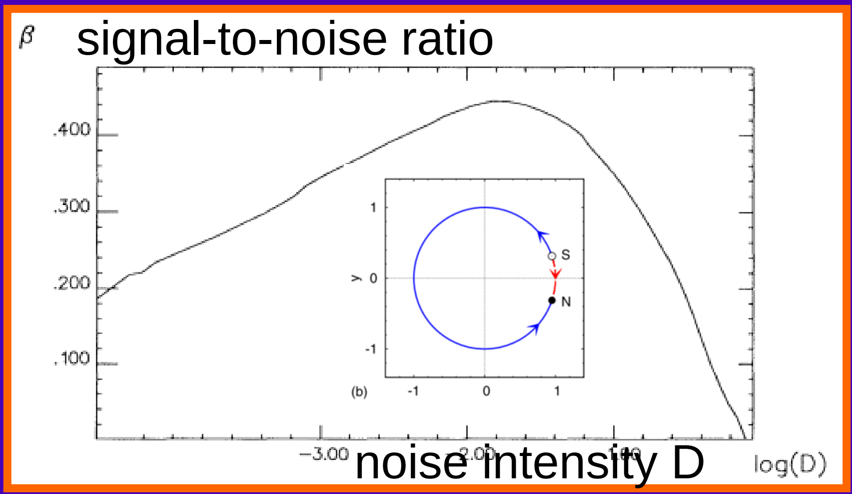
Hu Gang
International Centre for Theoretical Physics, Trieste 34100, Italy
and Physics Department, Beijing Normal University, Beijing 100875, People's Republic of China

T. Ditzinger,* C. Z. Ning, and H. Haken
Institut für Theoretische Physik und Synergetik, Universität Stuttgart, Pfaffenwaldring 57/IV,
D-7000 Stuttgart 80, Federal Republic of Germany

$$\dot{x} = x\left(1 - x^2 - y^2\right) + y\,(x - b) + D\xi$$

$$\dot{y} = y\left(1 - x^2 - y^2\right) - x\,(x - b) + D\xi$$

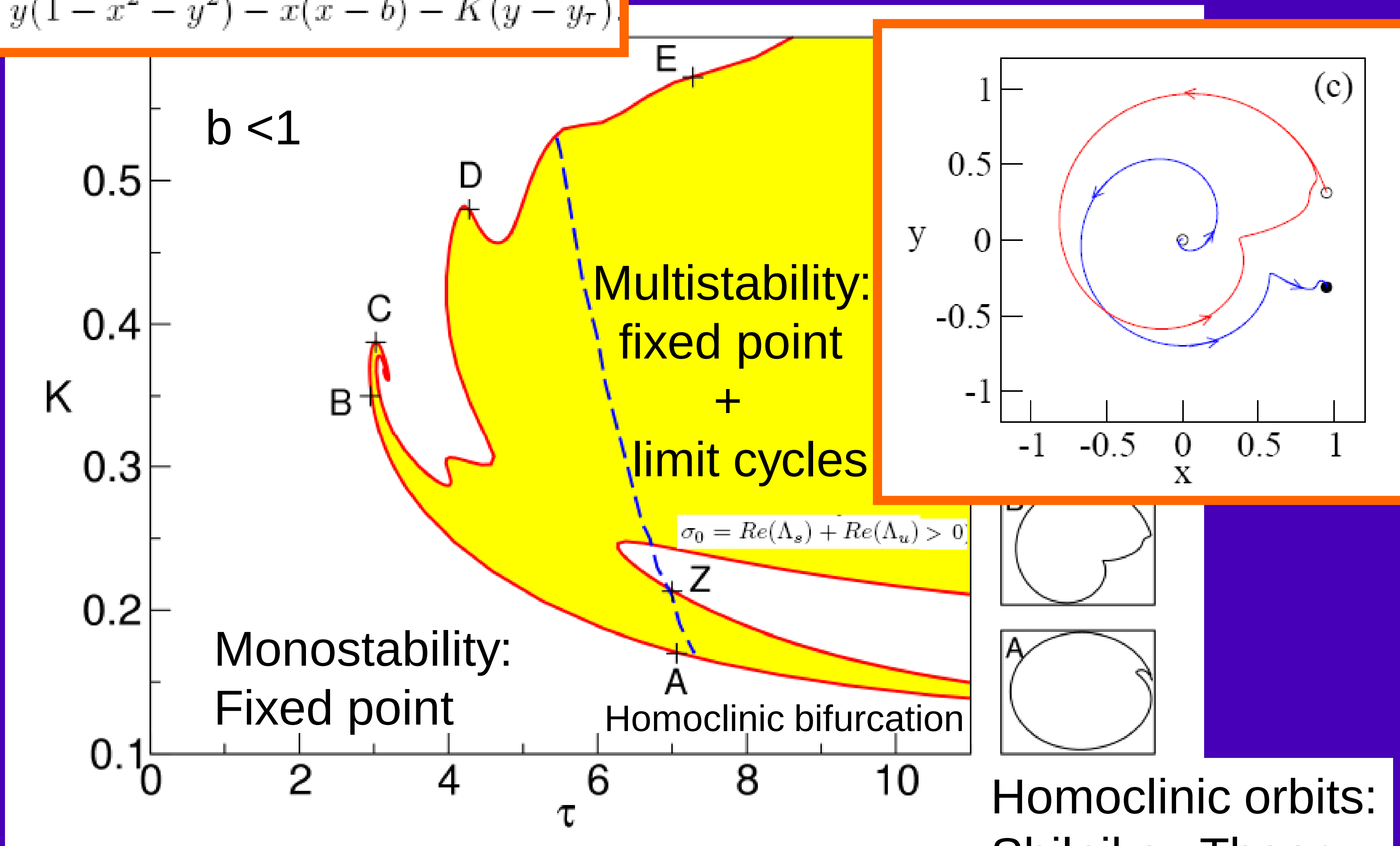
Optimal coherence at finite noise D



Delay-induced multistability near SNIPER

$$\dot{x} = x(1 - x^2 - y^2) + y(x - b) - K(x - x_\tau)$$
$$\dot{y} = y(1 - x^2 - y^2) - x(x - b) - K(y - y_\tau).$$

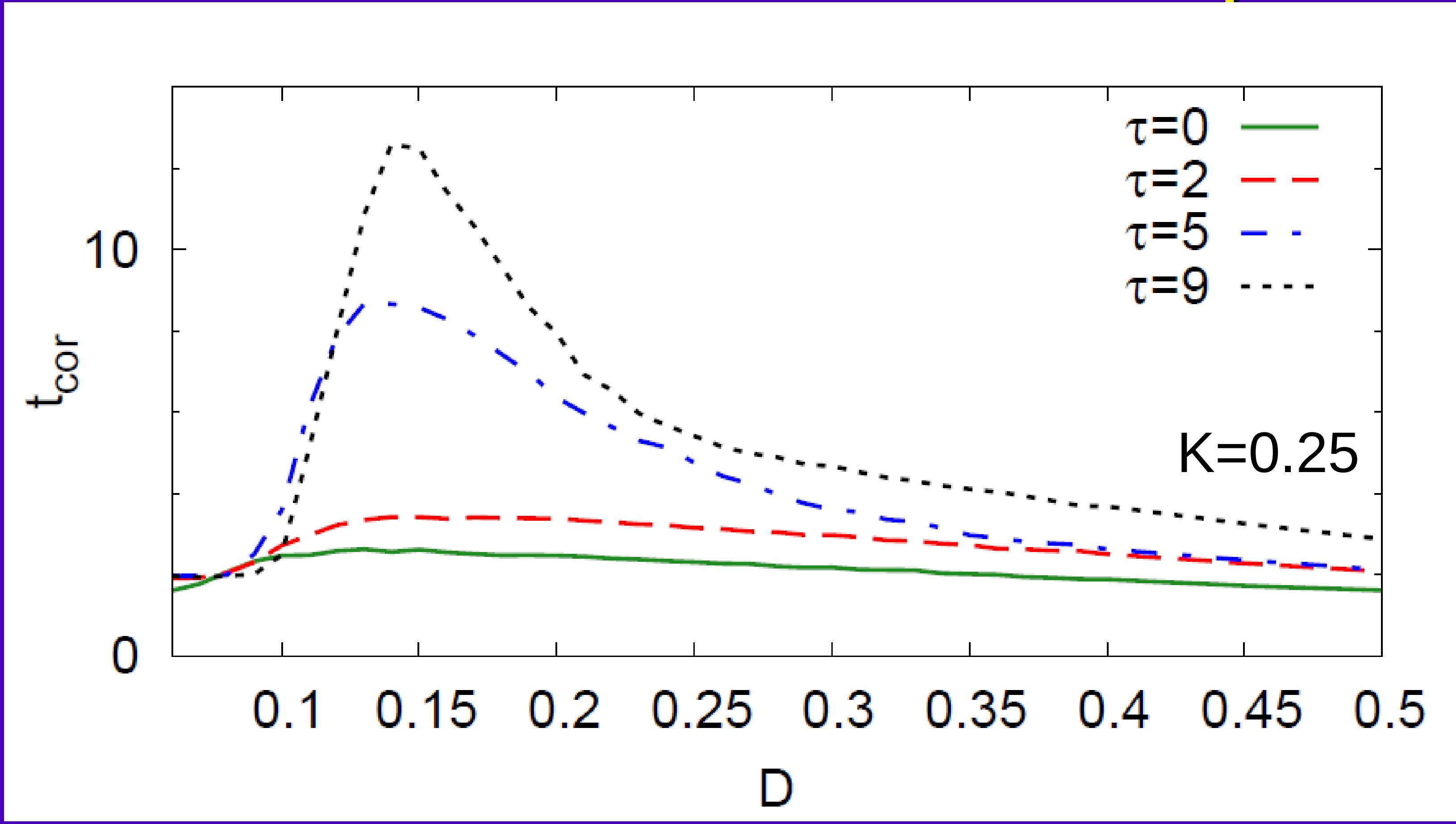
K=0: Global bifurcation (SNIPER)



Homoclinic orbits:
Shilnikov Theory

J. Hizanidis, R. Aust and E. Schöll: IJBC 18, 1759 (2008)

Delayed feedback control of coherence resonance: enhancement for optimal τ



$$\dot{x} = x\left(1 - x^2 - y^2\right) + y\left(x - b\right) + D\xi + K\left(x_\tau - x\right)$$
$$\dot{y} = y\left(1 - x^2 - y^2\right) - x\left(x - b\right) + D\xi + K\left(y_\tau - y\right)$$

R. Aust, P. Hövel, J. Hizanidis, and E. Schöll: EPJ-ST 187, 77 (2010)

Many other examples: lasers, neural systems, semiconductor nanostructures (Handbook of Chaos Control (Eds. E. Schöll, H.G. Schuster), Wiley 2008)

Networks of type-I excitable systems

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) + \sigma \sum_{j=1}^{N} G_{ij} \mathbf{H}(\mathbf{x}_j(t-\tau) - \mathbf{x}_i(t))$$

$$\sum_j G_{ij} = 1$$

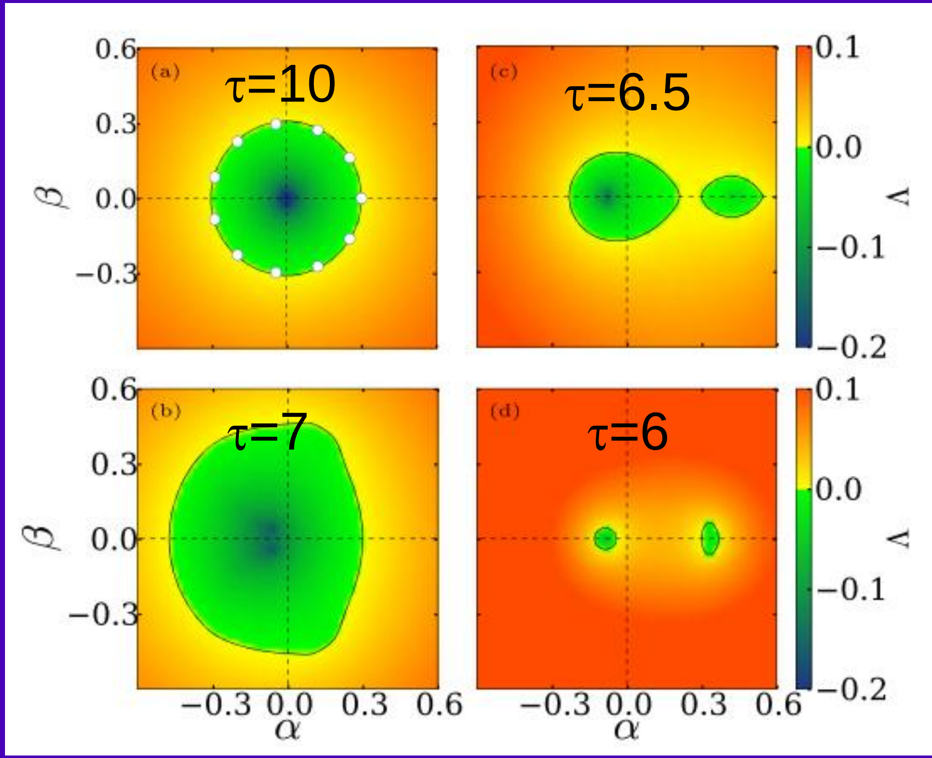
Synchronous dynamics:

$$\dot{\mathbf{x}}_s = \mathbf{f}(\mathbf{x}_s) + \sigma \mathbf{H}(\mathbf{x}_s(t-\tau) - \mathbf{x}_s(t))$$

Master stability function Λ:
Stability of zero-lag sync

$$\delta\dot{\mathbf{x}}(t) = [D\mathbf{f}(\mathbf{x}_s) - \sigma\mathbf{H}]\delta\mathbf{x}(t) + (\alpha + i\beta)\mathbf{H}\delta\mathbf{x}(t-\tau)$$

unidirectional ring:
Stable delay-induced periodic sync at τ=10 for excitatory coupling



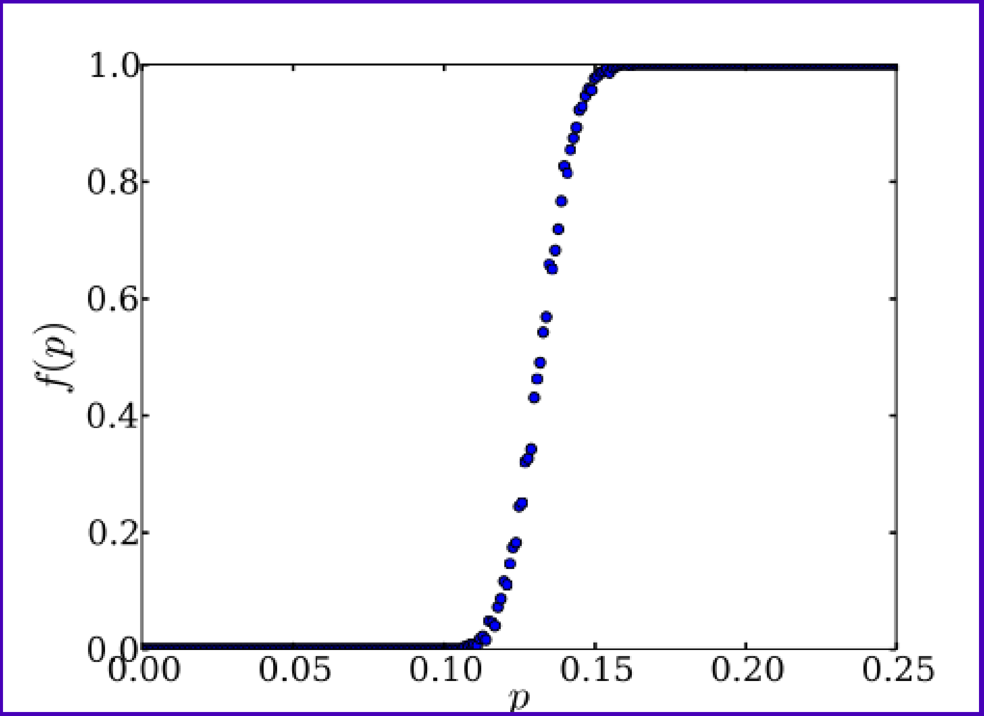
$\sigma = 0.3$

A. Keane, T. Dahms, J. Lehnert, S. Suryanarayana, P. Hövel, E. Schöll: EPJB (2012)

Desynchronization by inhibitory couplings

Start with regular ring with excitatory coupling (k nearest neighbors)

Introducing long-range inhibitory links in a small-world like fashion with probability p can lead to desynchronization



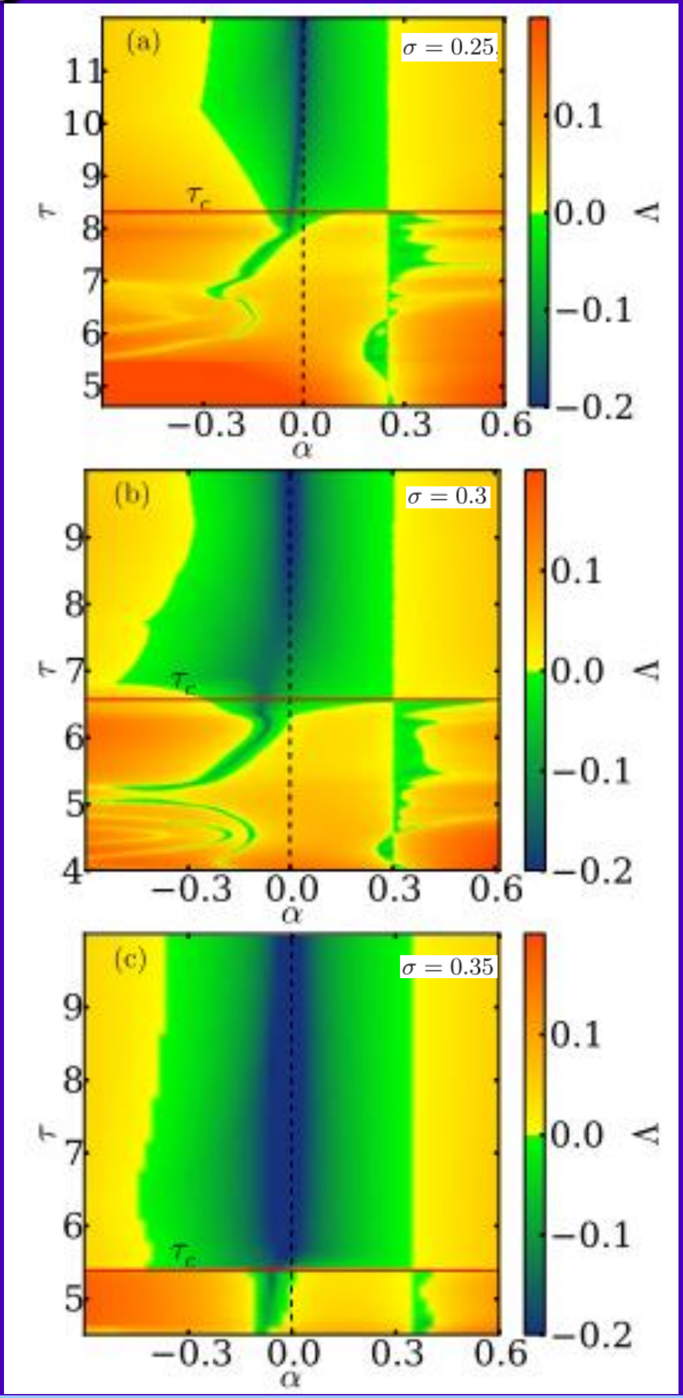
$N = 100$ and $k = 24$ for $\sigma = 0.3$, $\tau = 10$, $b = 0.95$.

Small delay: Master stability function

Behavior different from FHN:

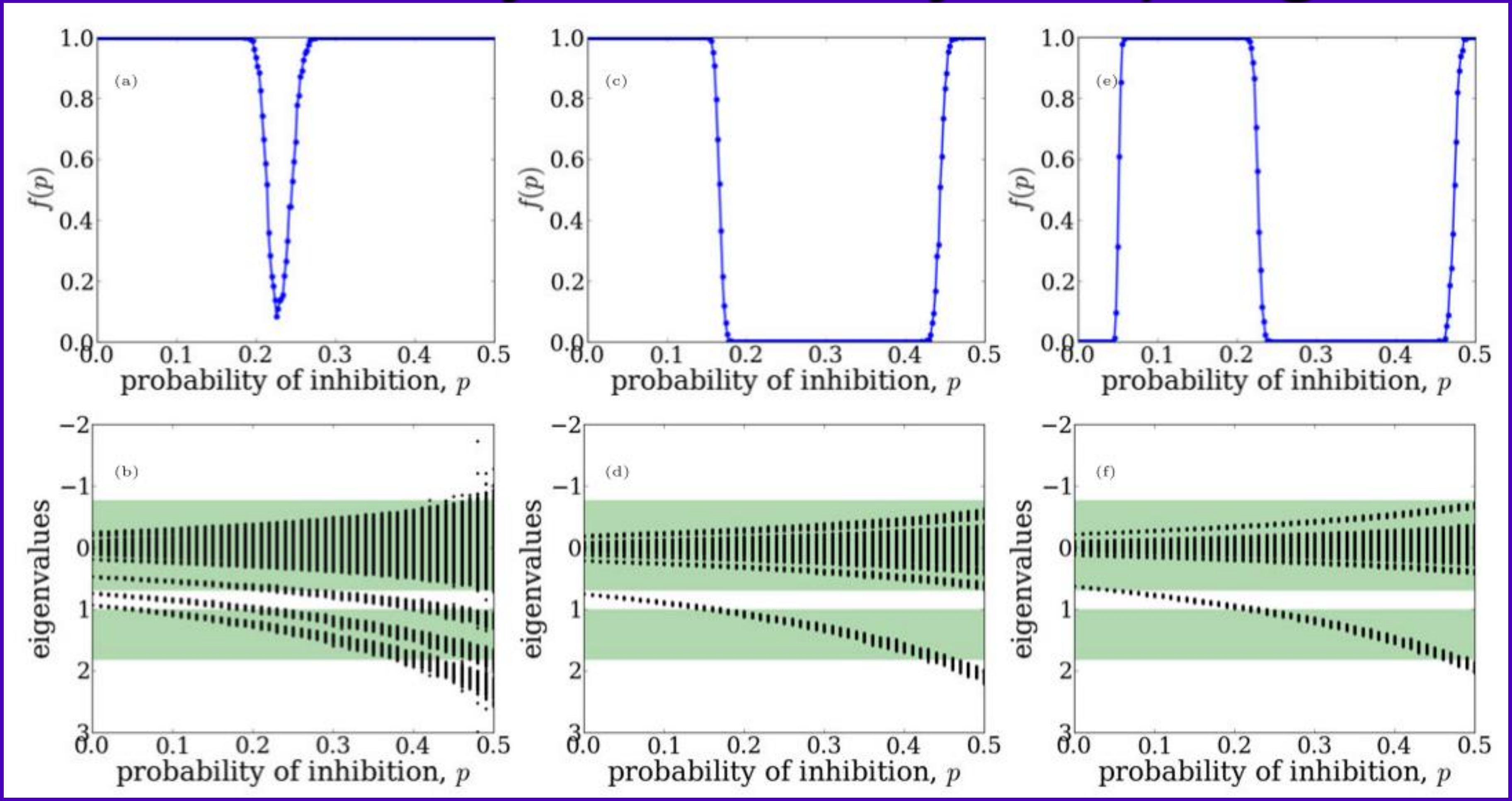
rotational symmetry breaks down

disconnected stability islands



$(\beta = 0)$

Control of synchronization by balance of excitatory / inhibitory coupling



(a) $k = 20$, (c) $k = 40$, and (e) $k = 50$

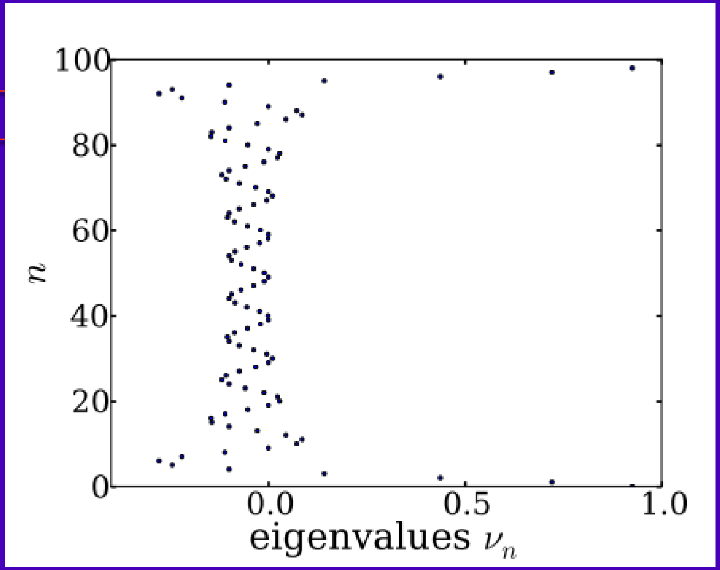
500 realisations of networks of $N = 200$

$\sigma = 0.3$ and $\tau = 6.5$.

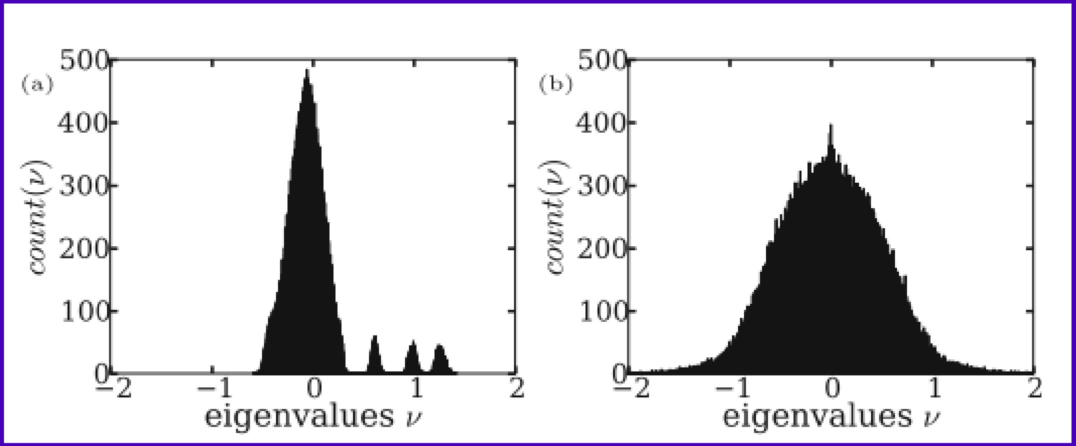
Multiple synchronization/desynchronization transitions: increasing inhibition can also re-synchronize!

Mechanism for multiple sync/desync transitions

Eigenvalue spectrum
N=100, k=10, p=0



Histogram of eigenvalue spectrum
N=100, k=10, p=0.2 (1000 realizations)
small-world random network



Matching the instability regions with the gaps in the eigenvalue spectrum (small-world)

A. Keane, T. Dahms, J. Lehnert, S. Suryanarayana, P. Hövel, E. Schöll: EPJB (2012)

Delay-coupled network of Stuart-Landau oscillators

C.U.Choe, T. Dahms, P. Hövel, E. Schöll: Phys. Rev. E 81, 025205 (R) (2010):

Model: Delayed networks of super- and subcritical Hopf normal forms

$$\dot{z}_j = \left[\lambda + i\omega \mp (1 + i\gamma) |z_j(t)|^2 \right] z_j(t) + \sigma \sum_{n=1}^{N} a_{jn} \left[z_n(t-\tau) - z_j(t) \right]$$

$$z_j = r_j e^{i\varphi_j} \in \mathbb{C}$$

'-': supercritical Hopf bifurcation
'+': subcritical Hopf bifurcation

$\sigma = K e^{i\beta}$ complex coupling strength
β coupling phase
$j = 1, \cdots, N$

Constant row sum condition: $-\sum_n a_{jn} =: d_0$

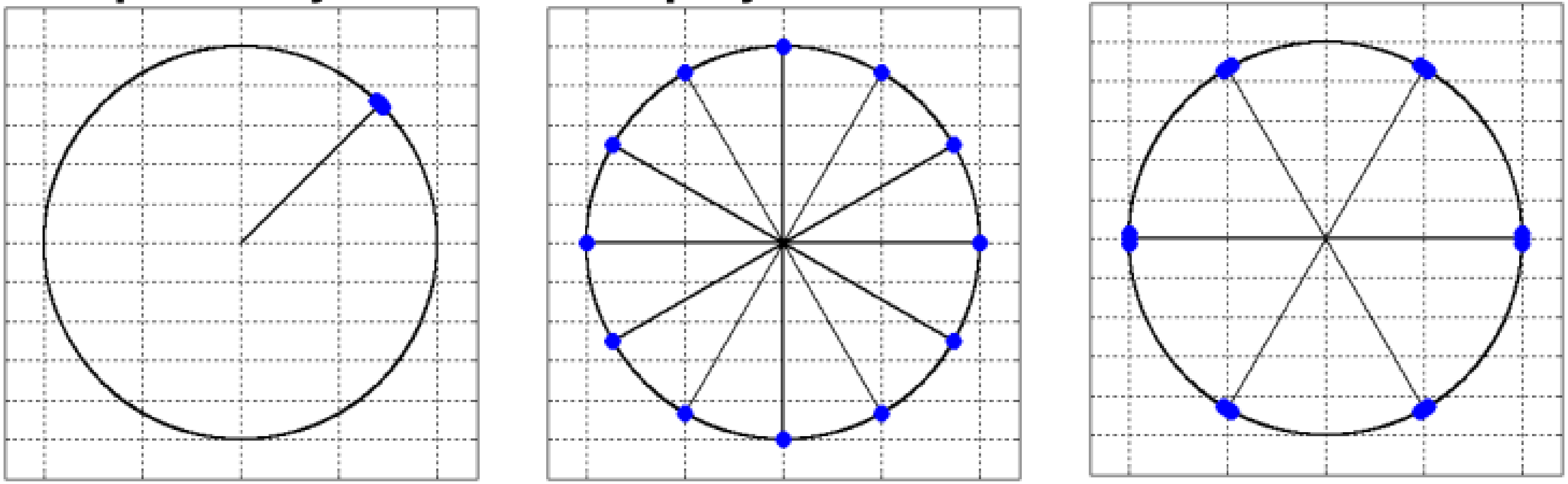
Tune coupling phase β for in-phase / splay / cluster state

By changing the value of β one can provide stability of different states:

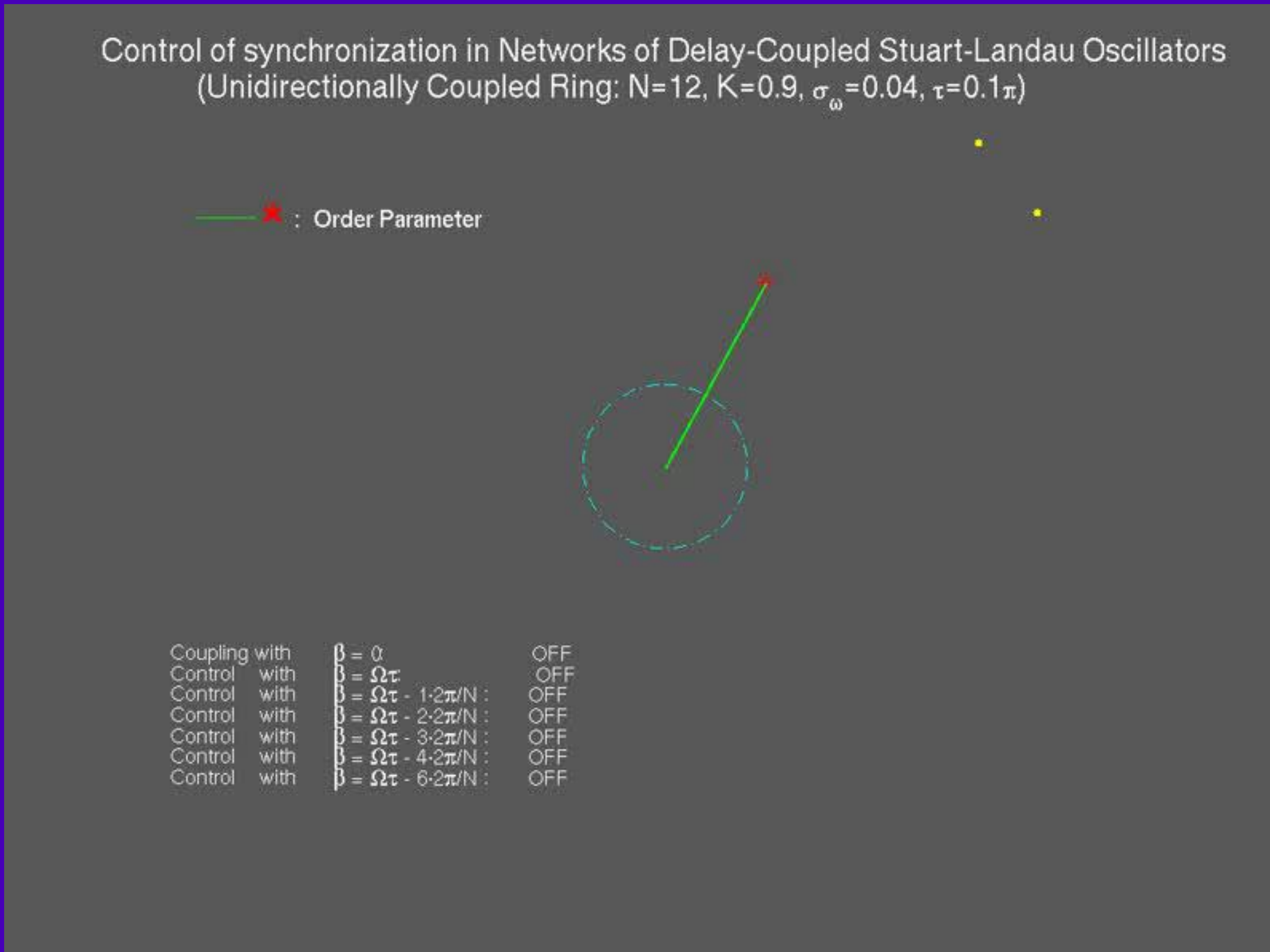
$\beta = \Omega_0 \tau$ **In-phase sync**

$\beta = \Omega_1 \tau - \frac{2\pi}{N}$ **Splay state**

$\beta = \Omega_m \tau - \frac{2\pi m}{N}$ **Cluster state**



Simulation of 12 non-identical oscillators



in-phase
splay
6-cluster
4-cluster
3-cluster
2-cluster

order parameter $Re^{i\Theta} \equiv \frac{1}{N}\sum_j \frac{z_j}{|z_j|}$

Adaptive control

Find phase β by adaptive algorithm:

Minimize goal function $Q(x(t),t)$ to find optimum coupling phase β

Speed-gradient method of control theory: (along trajectory)

control variable u $\frac{du}{dt} = -\Gamma \nabla_u \omega(x,u,t)$ $\dot{Q} = \omega(x,u,t)$ $\dot{x} = F(x,u,t)$

u=β:

$$\frac{d\beta}{dt} = -\Gamma \frac{\partial}{\partial \beta} \omega(x,\beta,t) = -\Gamma \left(\frac{\partial F}{\partial \beta} \right)^{\mathrm{T}} \nabla_x Q(x,t)$$

Possible goal functions for in-phase synchronous state: $Q(x,t) \geq 0$

$$Q_2 = 1 - \frac{1}{N^2} \sum_{j=1}^{N} e^{i\varphi_j} \sum_{k=1}^{N} e^{-i\varphi_k}$$

A. Selivanov, J. Lehnert, Th. Dahms, P. Hövel, A. Fradkov, E. Schöll: Phys. Rev. E 85, 016201 (2012)

Adaptive in-phase synchronization

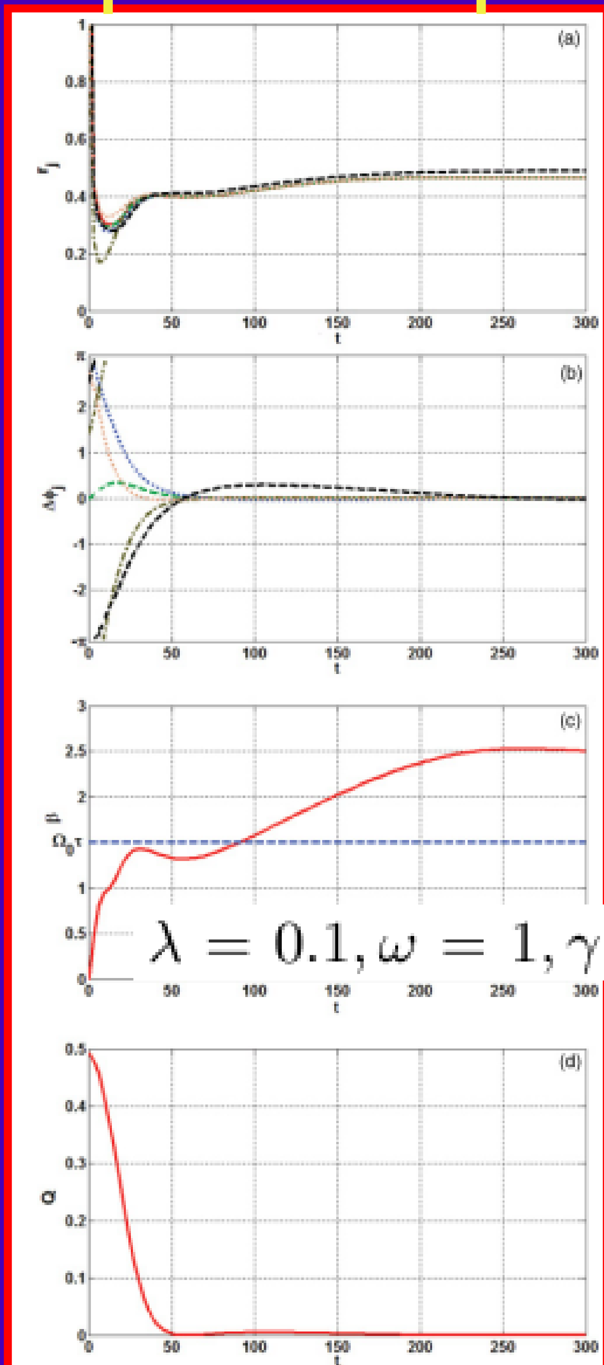
in-phase state

radius r_j

phase difference $\Delta\phi_j$

coupling phase β

goal function



$$Q_2 = 1 - \frac{1}{N^2} \sum_{j=1}^{N} e^{i\varphi_j} \sum_{k=1}^{N} e^{-i\varphi_k}$$

Γ=1, N=6 nodes
Erdös-Rényi random network

different from MSF value $\Omega\tau$

$$\lambda = 0.1, \omega = 1, \gamma = 0,\ K = 0.08,\ \tau = 0.52\pi,\ N = 6.$$

A. Selivanov, J. Lehnert, Th. Dahms, P. Hövel, A. Fradkov, E. Schöll: Phys. Rev. E 85, 016201 (2012)

Adaptive cluster synchronization

Choose different goal function to distinguish splay and d-cluster states:

$$Q_4 = 1 - f_d(\varphi)$$

$$f_p(\varphi) = \frac{1}{N^2} \sum_{j=1}^{N} e^{pi\varphi_j} \sum_{k=1}^{N} e^{-pi\varphi_k}$$

=1 for p-cluster state

Motivation by generalized order parameter:

$$R_d = \frac{1}{N} \left| \sum_{k=1}^{N} e^{di\varphi_k} \right|$$

Adaptive cluster synchronization

Choose different goal function to distinguish splay and d-cluster states:

$$Q_4 = 1 - f_d(\varphi)$$

all divisors p of d also satisfy $f_d = 1$
-> add penalty

$$f_p(\varphi) = \frac{1}{N^2} \sum_{j=1}^{N} e^{pi\varphi_j} \sum_{k=1}^{N} e^{-pi\varphi_k}$$

=1 for p-cluster state

Motivation by generalized order parameter:

$$R_d = \frac{1}{N} \left| \sum_{k=1}^{N} e^{di\varphi_k} \right|$$

Adaptive cluster synchronization

Choose different goal function to distinguish splay and d-cluster states:

$$Q_4 = 1 - f_d(\varphi) + \frac{N^2}{2} \sum_{p|d, 1 \le p < d} f_p(\varphi)$$

all divisors p of d also satisfy $f_d = 1$
-> add penalty

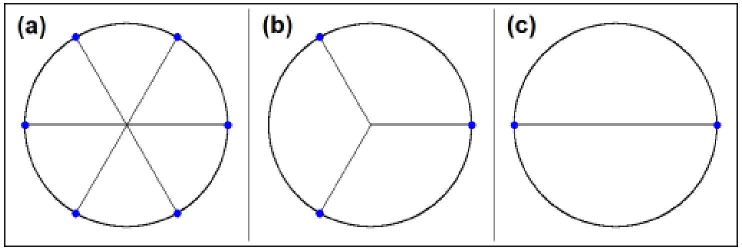
$$f_p(\varphi) = \frac{1}{N^2} \sum_{j=1}^{N} e^{p i \varphi_j} \sum_{k=1}^{N} e^{-p i \varphi_k}$$

=1 for p-cluster state

Motivation by generalized order parameter:

$$R_d = \frac{1}{N} \left| \sum_{k=1}^{N} e^{d i \varphi_k} \right|$$

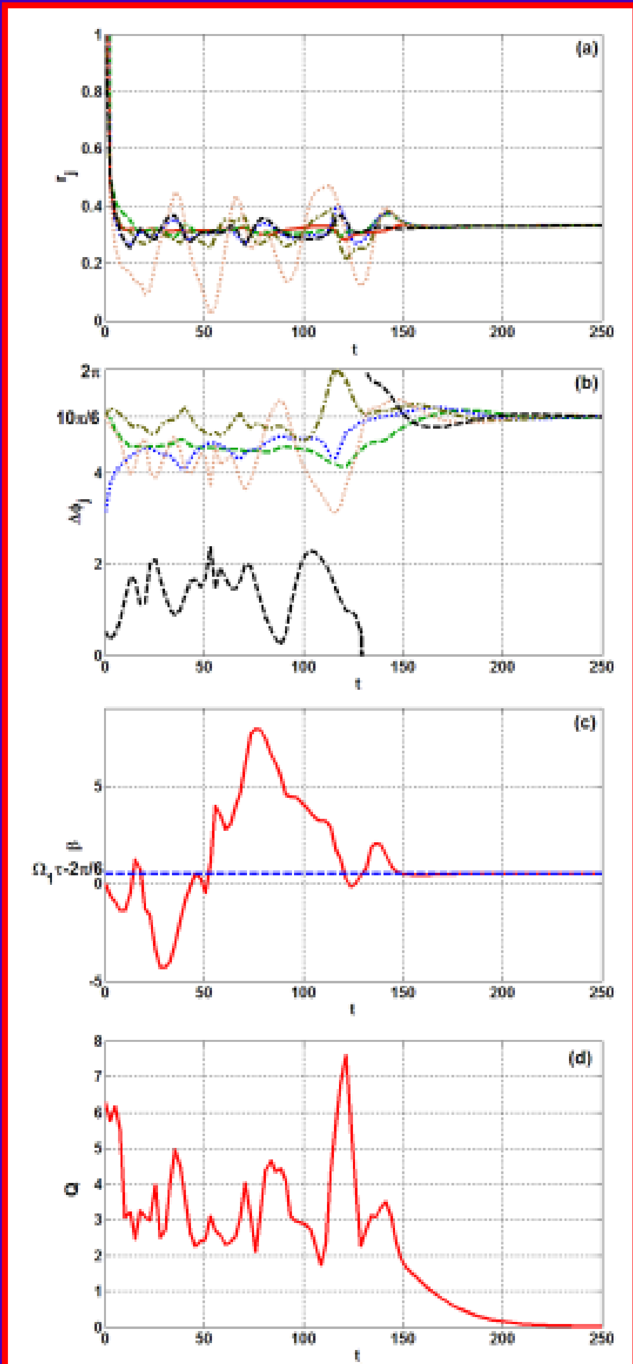
Splay state 3-cluster 2-cluster



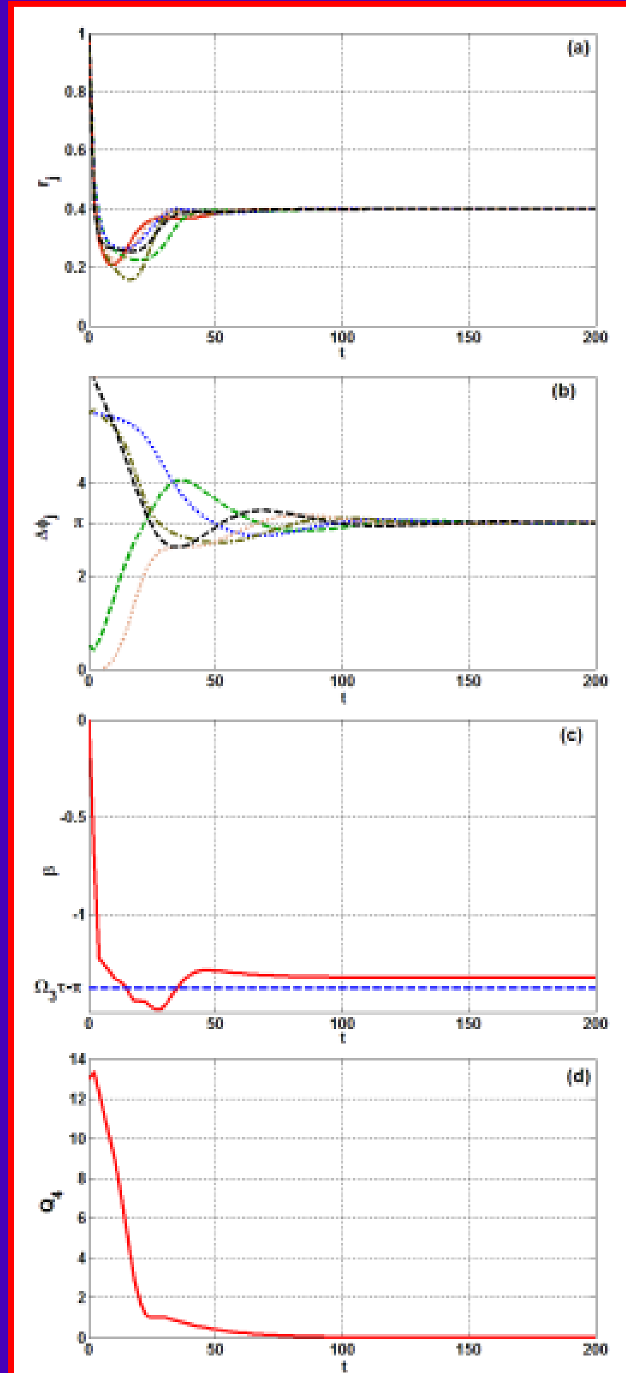
$f_1=f_2=f_3=0$ $f_1= f_2=0, f_3=1$ $f_1=f_3=0, f_2=1$

(N=6)

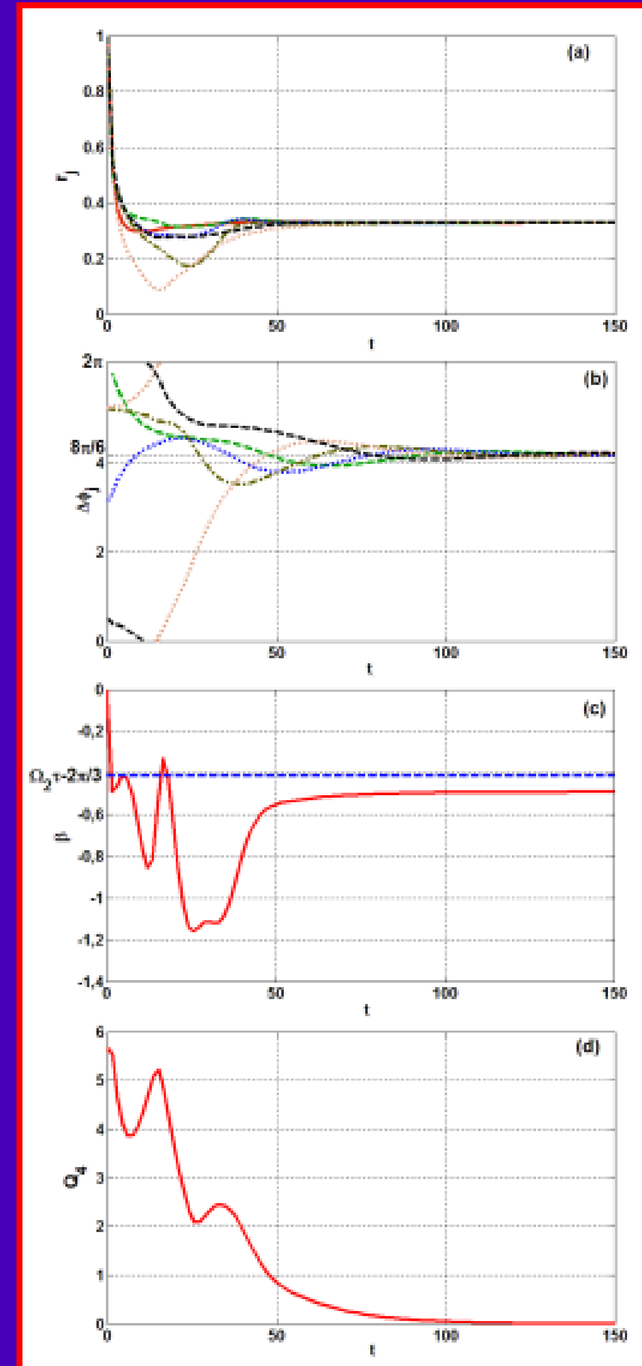
splay state



2-cluster



3-cluster



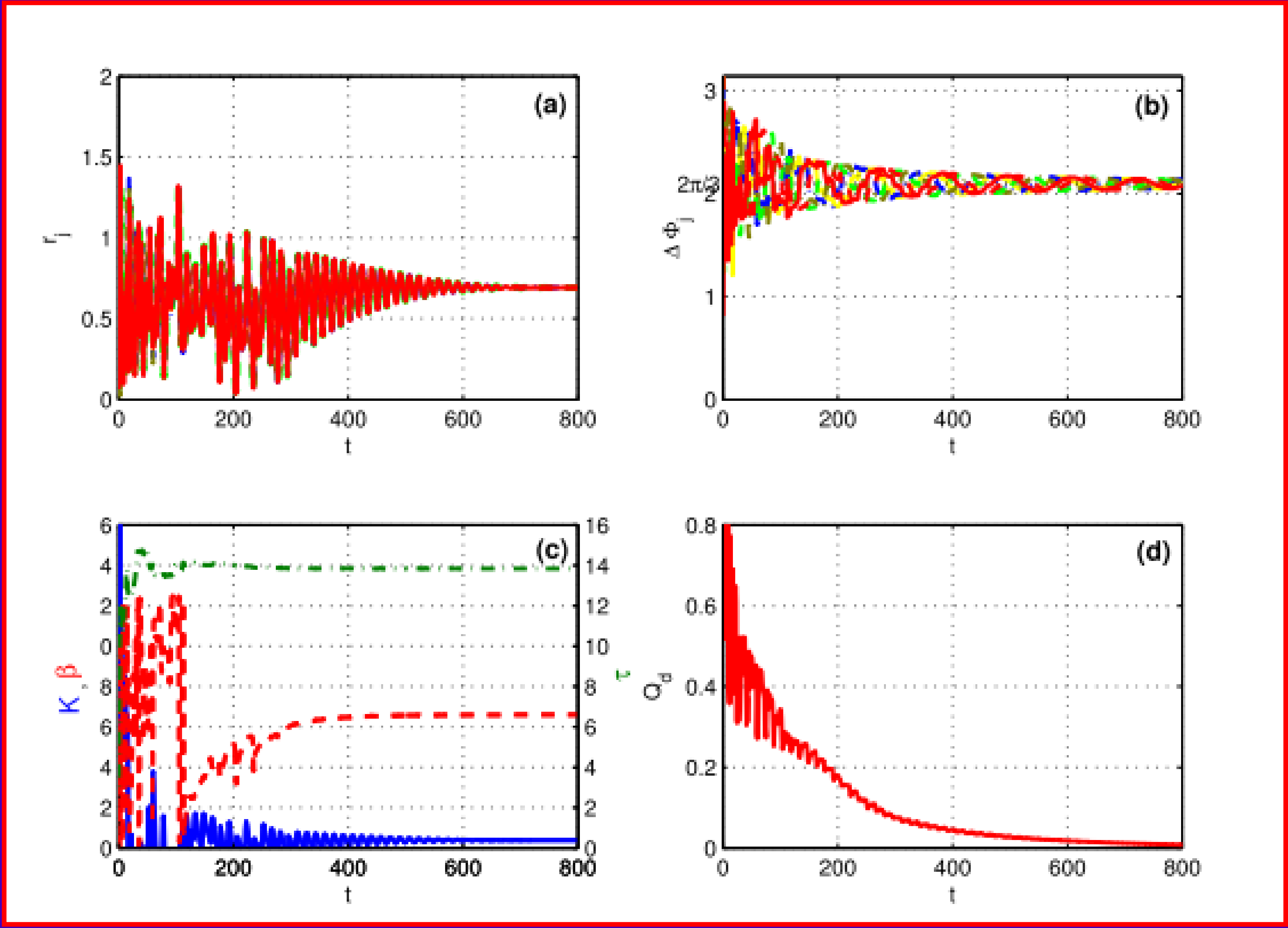
Controlling K, β, τ simultaneously: 3-cluster

r_j

$\Delta\phi_j$

K, β, τ

Q_4



E. Schöll, A. Selivanov, J. Lehnert, Th. Dahms, P. Hövel, A. Fradkov: Int. J. Mod. Phys. B 26, 1246007 (2012)

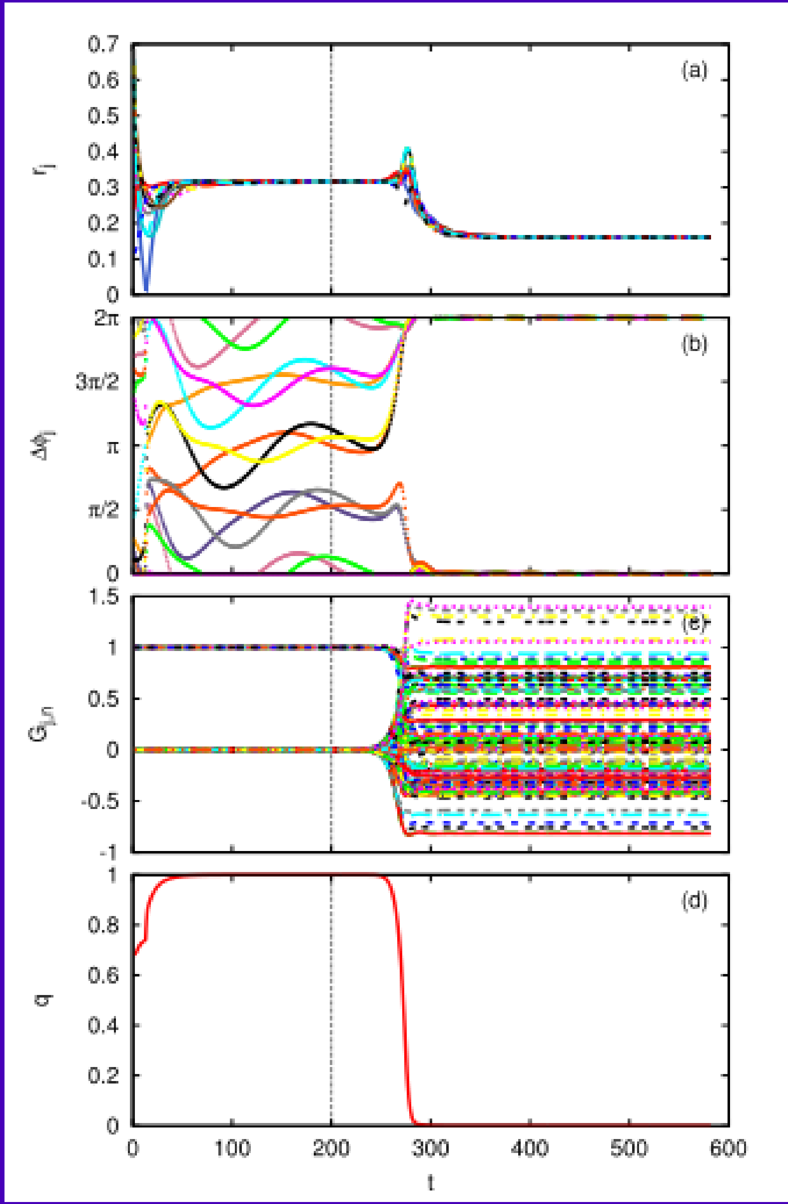
Self-adaptive control of network topology

Use speed-gradient algorithm to self-adaptively adjust the coupling matrix G_{ij} for a desired synchronization state (zero-lag or cluster)

Goal function for d-cluster state:

$$Q_d = 1 - f_d(\varphi) + \frac{N^2}{2} \sum_{p|d, 1 \leq p < d} f_p(\varphi) + \frac{c}{2} \int_0^t \sum_k \left(\sum_i G_{ki} - 1\right)^2 dt$$

previous goal function q   constant row sum



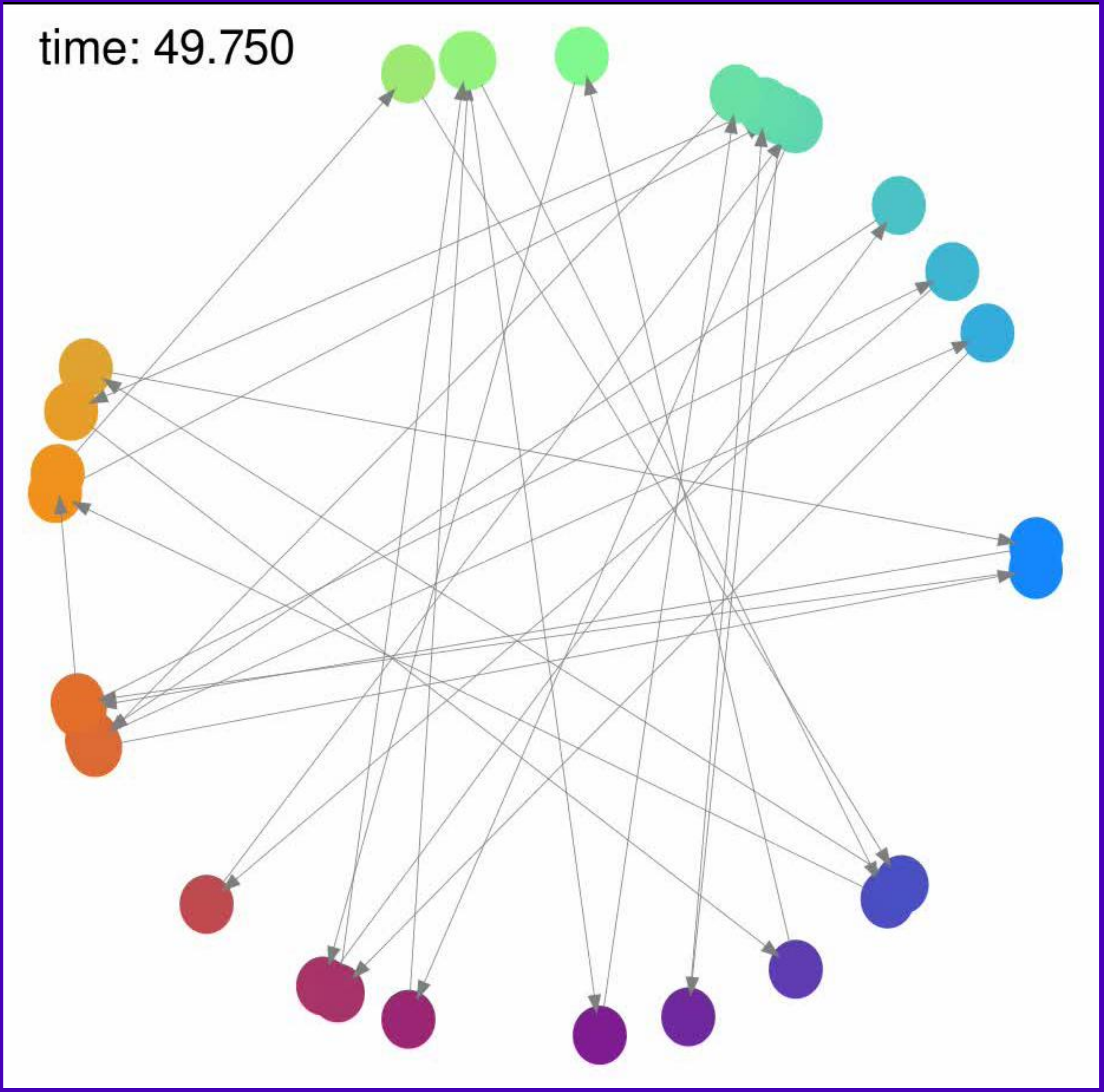
Result for zero-lag synchronization (d=1) :
Initial condition: unidirectional ring,
zero-lag solution unstable,
4-cluster-synchronization stable

Control switched on at t=200

J. Lehnert, unpublished (2012)

Self-adaptive control of network topology

Time evolution of network topology (30 nodes): 6-cluster state



J. Lehnert, unpublished (2012)

Conclusions

- Delay-coupled networks of oscillators
  - Master Stability Function for delay-coupled networks: universal classification of stable sync for large delay
  - Application to neural networks and laser networks
  - Control of synchronization/desynchronization transitions by balance of excitatory and inhibitory couplings
  - Adaptive synchronization: speed gradient method helps to find suitable coupling phase and coupling strength for in-phase, splay, and cluster states
  - Adaptive control of network topology

Postdocs

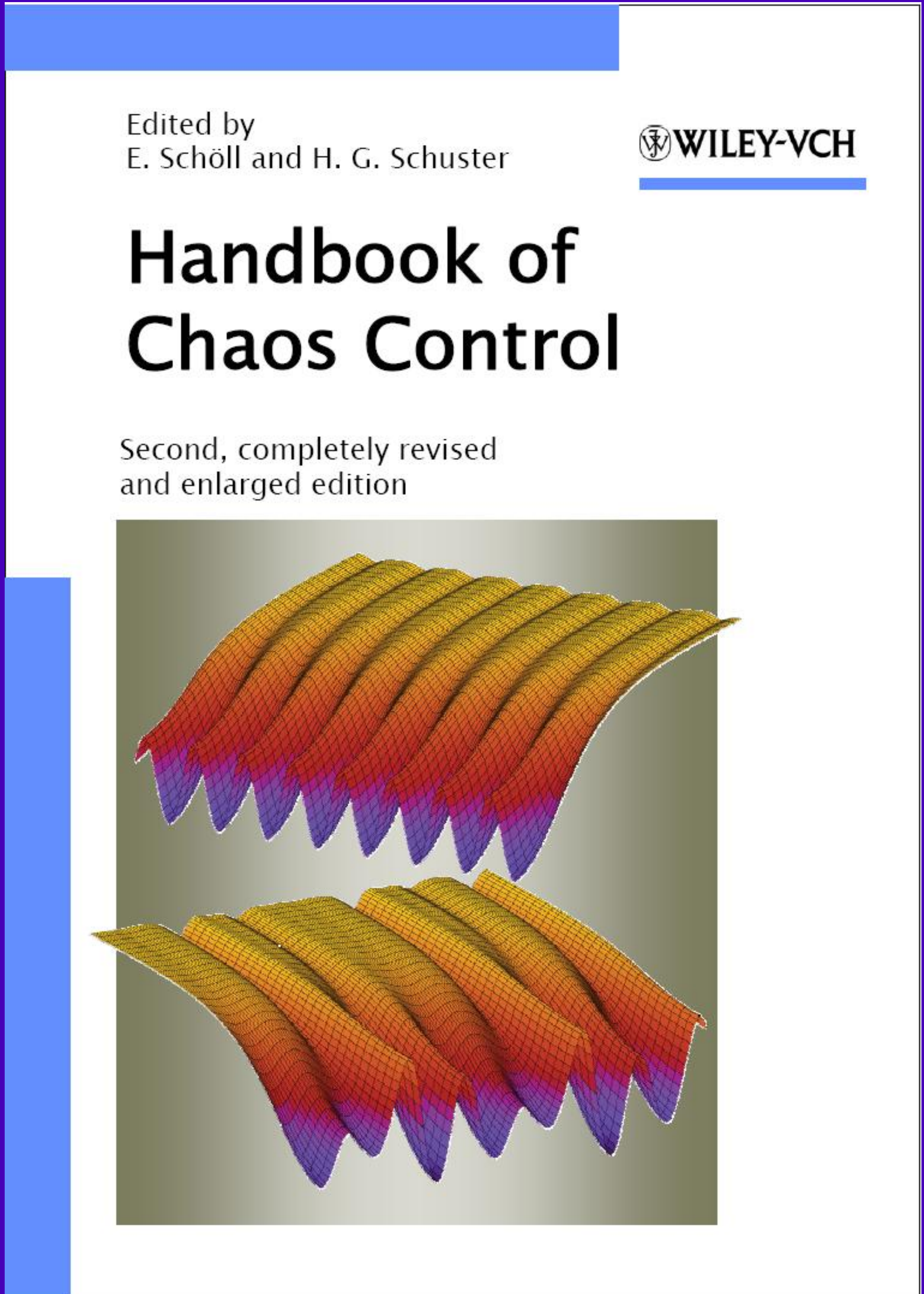
Thomas Dahms
Valentin Flunkert
Philipp Hövel
Kathy Lüdge
Anna Zakharova

Students

- Andrew Keane
- Thomas Isele
- Judith Lehnert
- Christian Otto
- David Rosin
- Andrea Vüllings

Collaborators

Andreas Amann
Kostya Blyuss
Chol-Ung Choe
Bernold Fiedler
Ingo Fischer
Alexander Fradkov
Yuliya Kyrychko
Yuri Maistrenko
Iryna Omelchenko
Anastasiia Panchuk
Anton Selivanov
Serhiy Yanchuk



Published 2008

Time-delayed feedback control of deterministic and stochastic systems

Synchronization in networks with large delay

Sketch of the proof I - fixed point

Variational equation

$$\xi_{k+1} = Df(\overline{x}_k)\,\xi_k \;+\; r\,e^{i\psi}\,Dh(\overline{x}_{k-\tau})\,\xi_{k-\tau}.$$

Simplest case:

Fixed point $\overline{x}_k = \overline{x}$ in the synchronization manifold

$$\xi_{k+1} = A\,\xi_k \;+\; r\,e^{i\psi}\,B\,\xi_{k-\tau}.$$

Ansatz $\xi_k = z^k\,\xi_0$ yields eigenvalue equation

$$\det[A - z\,I + re^{i\psi}Bz^{-\tau}] = 0$$

Synchronization stable iff all solutions z have $|z| < 1$.

V. Flunkert, S. Yanchuk, T. Dahms, E. Schöll: PRL 105, 254101 (2010)

Synchronization in networks with large delay

Sketch of the proof II – eigenvalue spectrum

$$\det[A - z\,I + re^{i\psi}Bz^{-\tau}] = 0 \qquad (1)$$

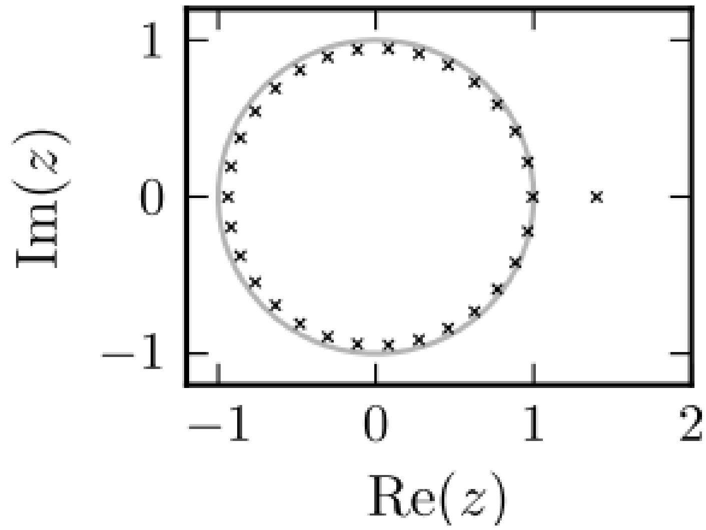
For large τ two types of solutions:

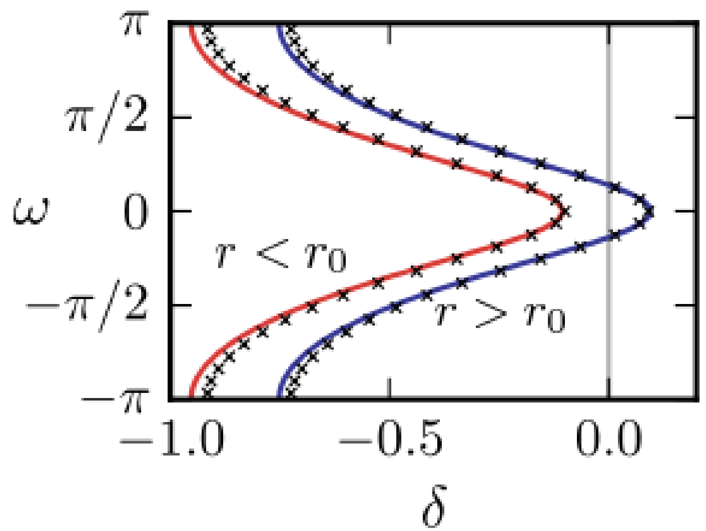
1. strongly unstable spectrum
   Solutions of $\det[A - z\,I] = 0$ with $|z| > 1$ are also solutions of Eq. (1).
2. pseudo-continuous spectrum
   Ansatz

   $$z = (1 + \delta/\tau)e^{i\omega}$$

   gives solution branches $\delta(\omega)$ on which the solutions lie densely.

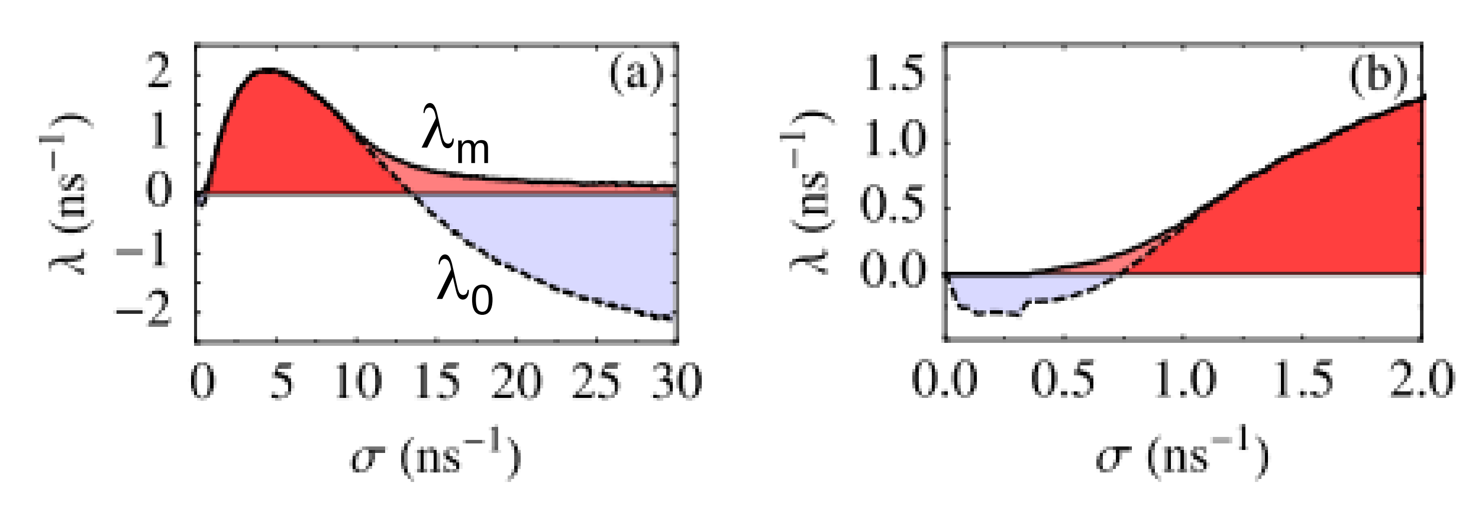
- r shifts branches right / left
- ψ shifts branches up / down





See also: S. Yanchuk, M. Wolfrum, P. Hövel, E. Schöll: PRE 74, 026201 (2006);
S. Yanchuk and P. Perlikowski: PRE 79, 046221 (2009)

Weak and strong chaos



Lang-Kobayashi laser model

FIG. 2 (color online). (a) Maximum Lyapunov exponents λ_m (solid line) and λ_0 (dashed line) of the SM for $\tau = 10$ ns vs coupling strength σ. (b) Enlarged view for small coupling strengths σ.

Maximum Lyapunov exponent within SM: λ_m
Instantaneous Lyapunov exponent (SM without delay term): $\lambda_0 = \lambda_{max}(0)$
Weak chaos ($\lambda_0 < 0$): stable chaotic synchronization ($\lambda_m = \eta/\tau$)
strong chaos ($\lambda_0 > 0$): no chaotic synchronization ($\lambda_m \sim \lambda_0$)

$$r_0 = \sigma e^{-\lambda_m \tau}.$$

S. Heiligenthal, T. Dahms, S. Yanchuk, T. Jüngling, V. Flunkert,
I. Kanter, E. Schöll, W. Kinzel: PRL 107, 234102 (2011)

Conclusions

- Delay-coupled networks of oscillators
  - Generalization of Master Stability Function for general networks with constant row sum
  - Control of synchrony by coupling phase: syn- or desynchronization
  - Unidirectional ring: in-phase / splay / cluster states controlled by coupling phase
  - Results are robust for slightly non-identical oscillators
- Synchronization in neural and laser networks
  - Desynchronization transition in small-world networks with inhibitory coupling
  - Cluster synchronization in chaotic laser networks