stochastic models for turbulence and turbulent driven systems

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synergetic approach to turbulence

stochastic cascade model - n-point statistics of turbulence

deeper insights into turbulence and turbulent driven systems
synergetic approach

order parameter

complex system
interacting subsystems

\{ slaving to low dimensional dynamics

synergetics and hierarchical structures

hierarchical cascade structure--allows this a simplification, too? order parameters?
synergetics and hierarchical structures

• turbulence
  • cascade structure of interacting vorticities

\[
\begin{align*}
L & \quad \text{integral length} \\
r & \quad \eta & \quad \text{dissipation length}
\end{align*}
\]
synergetics and hierarchical structures

• turbulence
  • cascade structure of interacting vorticities
  • Rudolf - look at the interacting structures on different scales

\[ p(u_r, r | u_{r'}, r') \]
\[ \partial_r p(u_r, r) \]

\( r \) process evolving in the cascade parameter
synergetic approach to turbulence

stochastic cascade model - n-point statistics of turbulence

deep insights to turbulence and turbulent driven systems
turbulence

- comprehensive description by n-point statistics

\[ p(u(x_1), \ldots, u(x_{n+1})) \]

using velocity increments:

\[ u_{r_i} = u(x + r_i) - u(x_i) \]

\[ p(u(x_1), \ldots, u(x_{n+1})) = p(u_{r_1}, \ldots, u_{r_n}, u(x_1)) \]
n-point statistics

\[ p(u(x_1), ..., u(x_{n+1})) = p(u_{r_1}, ..., u_{r_n}, u(x_1)) \]

\[ = p(u_{r_1}, ..., u_{r_n} | u(x_1)) \cdot p(u(x_1)) \]

Bayes theorem - joint pdf by cond. pdf

\[ = p(u_{r_1} | ..., u_{r_n} u(x_1)) p(u_{r_2} | ..., u_{r_n} u(x_1)) \cdot p(u(x_1)) \]

Mittwoch, 14. November 12

ForWind
Center for Wind Energy Research

HWK 2012
n-point statistics

\[ p(u(x_1), \ldots, u(x_{n+1})) = p(u_{r_1}, \ldots, u_{r_n}, u(x_1)) \]
\[ = p(u_{r_1}, \ldots, u_{r_n} | u(x_1)) \cdot p(u(x_1)) \]

Bayes theorem - joint pdf by cond. pdf

\[ = p(u_{r_1} | \ldots, u_{r_n} u(x_1)) p(u_{r_2} | \ldots, u_{r_n} u(x_1)) \ldots \cdot p(u(x_1)) \]

can this be simplified?

\[ = p(u_{r_1} | u_{r_2}, u(x_1)) \ldots p(u_{r_{n-1}} | u_{r_n}, u(x_1)) \cdot p(u(x_1)) \]

or even one increment statistics?

\[ p(u_{r_1}) \ldots p(u_{r_n}) \]
\( \nabla \) time signals, \( u(t) \),

\( \nabla \) measured increments \( u_r \) for different \( r \)

\[ p(u_{r1} | \ldots, u_{rn}, u(x_1)) \]
simplification
(1) \[ p(\tilde{u}_1, r_1 | \tilde{u}_2, r_2; \ldots; \tilde{u}_n, r_n) = p(\tilde{u}_1, r_1 | \tilde{u}_2, r_2) \]
(2) \[ p(\tilde{u}_1, r_1 | \tilde{u}_2, r_2; \ldots; \tilde{u}_n, r_n) = p(\tilde{u}_1, r_1) \]

experimental test

experimental result:
\[ p(u_1 | u_2, u_3) = p(u_1 | u_2) \]
(1) holds
(2) not
\[ p(u(x_1), ..., u(x_{n+1})) \]
\[ = p(u_{r_1} | u_{r_2}, u(x_1)) \cdots p(u_{r_{n-1}} | u_{r_n}, u(x_1)) \cdot p(u(x_1)) \]

new view of cascade process: three point closure
three point quantity

\[ p(u_{r_1} | u_{r_2}, u(x_1)) \]
\[ p(u(x_1), \ldots, u(x_{n+1})) = p(u_{r_1} | u_{r_2}, u(x_1)) \ldots p(u_{r_{n-1}} | u_{r_n}, u(x_1)) \cdot p(u(x_1)) \]

new view of cascade process:
three point closure
local in the cascade means no memory
or Markow process in \( r \)
\[ p(u(x_1), \ldots, u(x_{n+1})) = p(u_{r_1} | u_{r_2}, u(x_1)) \ldots p(u_{r_{n-1}} | u_{r_n}, u(x_1)) \cdot p(u(x_1)) \]

Markow prop & cascade with Fokker-Planck Equ.

\[-r_j \frac{\partial}{\partial r_j} p(u_{r_j} | u_{r_k}, u(x_1)) = \left\{ -\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1)) \right\} p(u_{r_j} | u_{r_k}, u(x_1))\]
\[-r_j \frac{\partial}{\partial r_j} p(u_{r_j} | u_{r_k}, u(x_1)) = \{- \frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1))\} p(u_{r_j} | u_{r_k}, u(x_1))\]

Markow prop. $\Rightarrow$ Fokker-Planck Equ. measured by KM coefficients

\[D^{(n)}(u_r, r) = \frac{1}{n!} \cdot \lim_{\Delta \to 0} \int (\tilde{u}_r - u_r)^n p(\tilde{u}_{r+\Delta}, r + \Delta | u_r, r) d\tilde{u}_r\]

- shift of drift function, no $u$-dependence of diffusion function


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n-point statistics

3. Verification of the measured Fokker-Planck equation
- numerical solution compared with experimental results
- $\Rightarrow$ n-scale statistics

$\rho(u,r)$

$p(u_r,r|u_{r_0},r_0)$

$\frac{u_0}{\sigma_\infty}$

synergetic approach to turbulence

stochastic cascade model - n-point statistics of turbulence
  - stochastic process in r - tremendous reduction in complexity

shift of drift function, no u-dependence of diffusion function
synergetic approach to turbulence

stochastic cascade model - n-point statistics of turbulence

• stochastic process in r - tremendous reduction in complexity

deeper insights to turbulence and turbulent driven systems

shift of drift function, no u-dependence of diffusion function
reconstruction of time series

no u dependence

\[ D^{(1)}(u_r, r, u) = d_{10}(r, u) - d_{11}(r)u_r \]

\[ D^{(2)}(u_r, r, u) = D^{(2)}(u_r, r) \]

with u dependence


blow up
turbulence - classical theory

\[-r_j \frac{\partial}{\partial r_j} p(u_{r_j} | u_{r_k}, u(x_1)) = \left\{ -\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1)) \right\} p(u_{r_j} | u_{r_k}, u(x_1)) \]

Kolmogorov 41

\[D^{(1)}(u) = \frac{1}{3} u \]

Langevin equation

\[\frac{\partial}{\partial r} u_r = \frac{1}{3} \frac{u_r}{r} \quad u_r \propto r^{1/3} \]

\[\langle u_r^n \rangle \propto r^{n/3} \]

Kolmogorov 62

\[D^{(1)}(u_r) = \gamma u_r \quad D^{(2)}(u_r) = \beta u_r^2 \]

\[\langle u_r^n \rangle \propto r^{n/3 + \mu(n)} \]

RF & JP PRL 78 (1997)
Functional form of the coefficients $D^{(1)}$ and $D^{(2)}$ is presented

\[
\frac{\partial}{\partial \tau} p(R, \tau) = \left[ -\frac{\partial}{\partial R} D^{(1)}(R, \tau) + \frac{\partial^2}{\partial R^2} D^{(2)}(R, \tau) \right] p(R, \tau)
\]

Example: Volkswagen, $\tau = 10$ min
multi-scale statistics

additive term in the diffusion term: \(\rightarrow\) additive noise

\[
D^{(1)}_{1}(u_r, r) = d^u_1(r) u_r \\
D^{(2)}(u_r, r) = d_2(r) + d^u_2(r) u_r + d^{uu}_2(r) u_r^2
\]

Gaussian tip

Comparison of data with numerical solution of the Kolmogorov equation
wake flow behind a cylinder - turbulent structures

drift term as function of r
wake flow behind a cylinder - turbulent structures

phase transition to isotropic turbulence

drift term as function of r
Markov-length - a coherence length

statistics of longitudinal and transversal increments

universality of turbulence:

fractal grid turbulence

role of transferred energy $\varepsilon_r$:

fusion rules $r_i \Rightarrow r_{i+1}$ (Davoudi, Tabar 2000; L’vov, Procaccia 1996)

passive scalar (Tutkun, Mydlarski 2004)

Lagrangian turbulence (Friedrich 2003, 2008)
turbulent driven systems
open question - what is the corresponding dynamics

\[ \dot{x} = ?? \]

\[ x(t + \tau) = ?? \]
synergetic approach

order parameter

complex system
interacting subsystems

{ }

slaving to
low dimensional

dynamics

• deterministic part

\[ \dot{x}(t) = D^{(1)}(x(t_j), t_j) + \sqrt{D^{(2)}(x(t_j))\Gamma'(t_j)} \]
Markov process - Langevin Equation

\[ \dot{x}(t) = D^{(1)}(x(t), t) + \sqrt{D^{(2)}(x(t))} \Gamma'(t) \]

the basic element is the transition probability

\[ p(\tilde{x}, t + \tau | x, t) \]

from this we can get

\[ D^{(1)}(x) = \lim_{\tau \to 0} \frac{1}{\tau} \left\langle X(t + \tau) - x \right\rangle \Big|_{X(t) = x} \]

\[ D^{(2)}(x) = \lim_{\tau \to 0} \frac{1}{\tau} \left\langle (X(t + \tau) - x)x(X(t + \tau) - x)^T \right\rangle \Big|_{X(t) = x} \]

Rayleigh Benard Experiment

max. temperature difference 20°C
Ra < 9*10⁹

measurements with ultrasonic dopper
Anemometer DOP200β
Rayleigh Benard Experiment

DOP2000 - profile measurements

red rising, blue sinking

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Rayleigh Benard Experiment


\[ Ra = 10^{10} \]
Rayleigh Benard Experiment

Analysis as stochastic process in time - Langevin Equation

\[ \dot{x}(t) = D^{(1)}(x(t_j), t_j) + \sqrt{D^{(2)}(x(t_j))} \Gamma'(t_j) \]

Ra = 10^{10}

PDF and its reconstruction

Potential
Rayleigh Benard Experiment

tilting - by less than 1°

Potential

MA Thesis
M. Peters M. Langner

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turbulent driven systems

\[ \text{turbulent wind energy} \]
dynamics of power conversion

\[ P_{WT} = \frac{1}{2} c_p(\lambda) \, \rho \, u_{wind}^3 \cdot A \]
working conditions for wind turbine
stochastic motion in a potential

\[ \dot{P} = D^{(1)}(P|u) + \sqrt{D^{(2)}(P|u)} \cdot \Gamma \]
conversion of wind power a stoch. process

\[ \dot{P} = D^{(1)}(P|u) + \sqrt{D^{(2)}(P|u)} \cdot \Gamma \]
summed up difference in the power production with the same measured wind data as input
Experimental set-up
- Three configurations
  - undamaged
  - heated
  - cut on 40% of the circumference
- Turbulent inflow conditions

Numerical simulation: FEM model
- Element 2 was “damaged”
  - Stiffness was reduced in 10% steps
  - Eight configurations: 0% to 70%

Deflection in x- and y-direction was measured
characterization of dynamic stall with turb inflow

\[
\frac{dc_L(t)}{dt} = -\nabla \Phi(c_L(t), u) + g(c_L(t), u) \cdot \Gamma(t)
\]

J. Schneemenn et al, EWEC 2010;
synergetics for complexity

- time dependent complexity
  - slaving to low dimensional dynamics
  - hierarchical cascade structure-- allows this a simplification, too
- scale dependent complexity

♫ stochastic equation are measurable
  - comprehensive description of complex systems
  - deeper insights
  - high accuracy
and all best wishes