stochastic models for turbulence and turbulent driven systems

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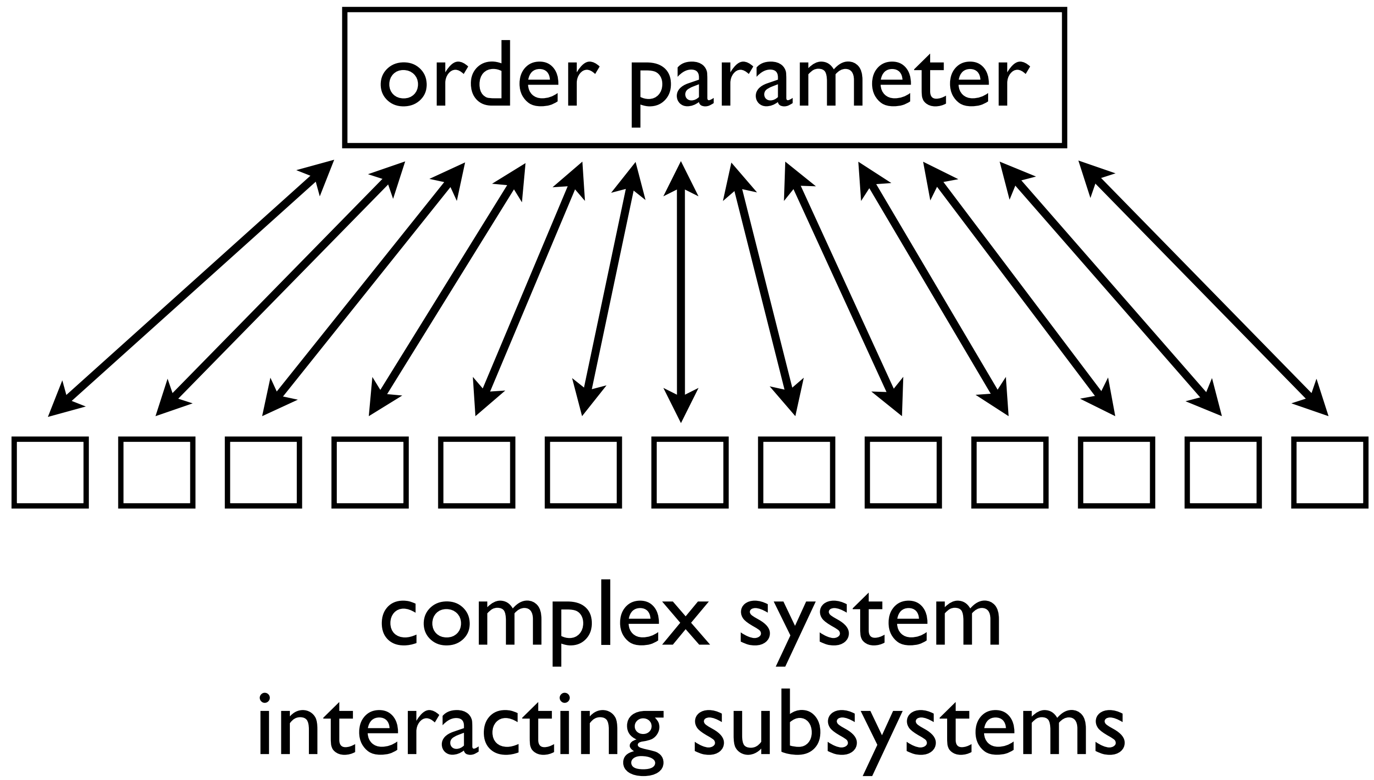
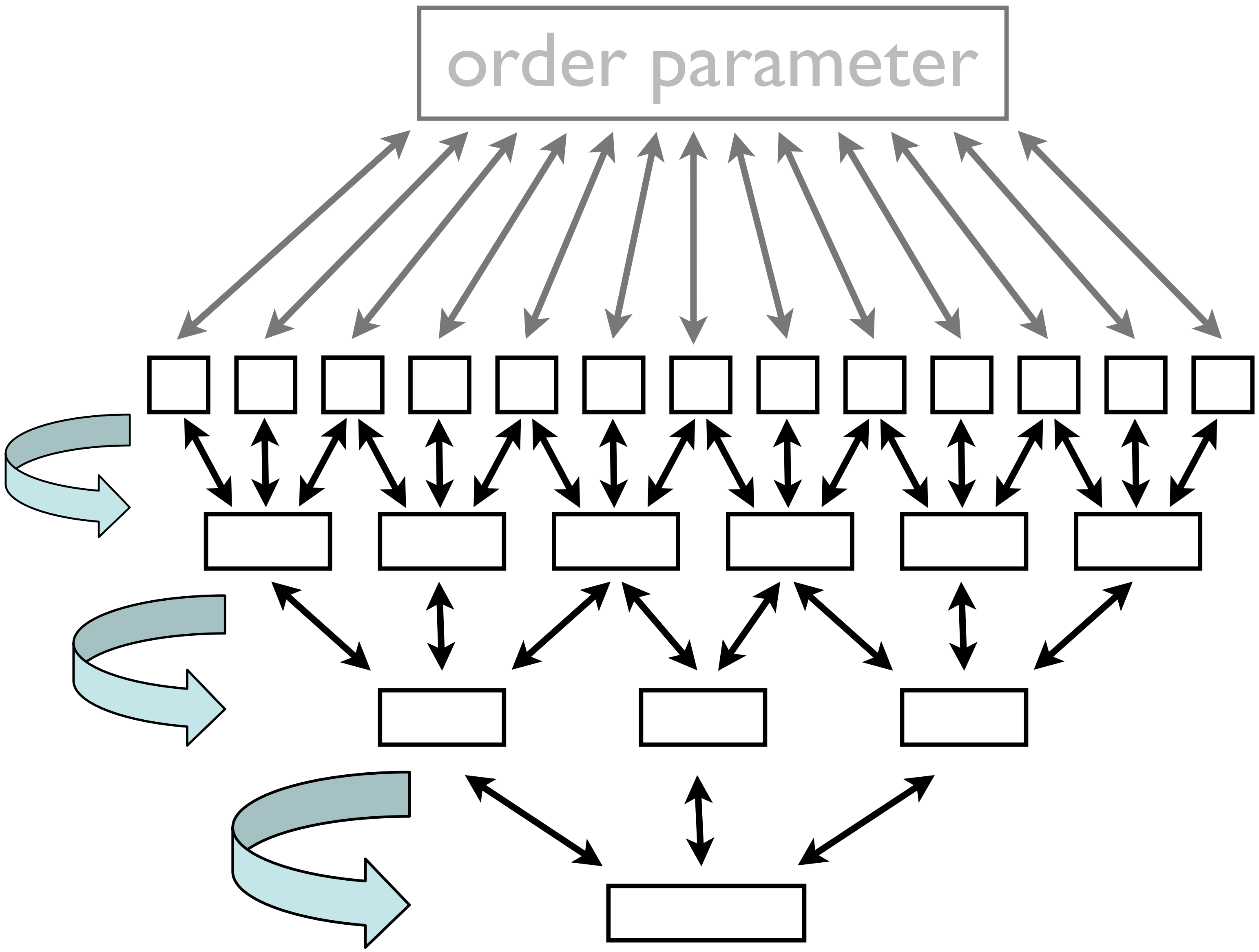
- synergetic approach to turbulence
- stochastic cascade model - n-point statistics of turbulence
- deeper insights into turbulence and turbulent driven systems

synergetic approach



slaving to
low dimensional
dynamics

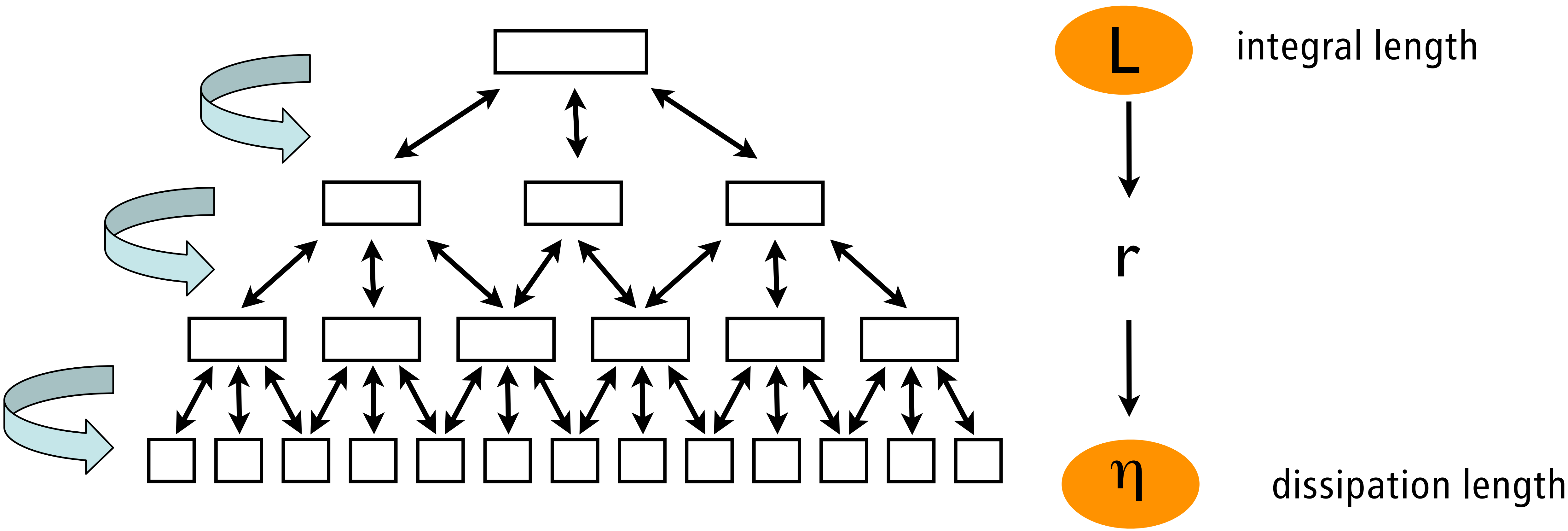
synergetics and hierarchical structures



hierarchical
cascade structure--
allows this a
simplification, too?
order parameters?

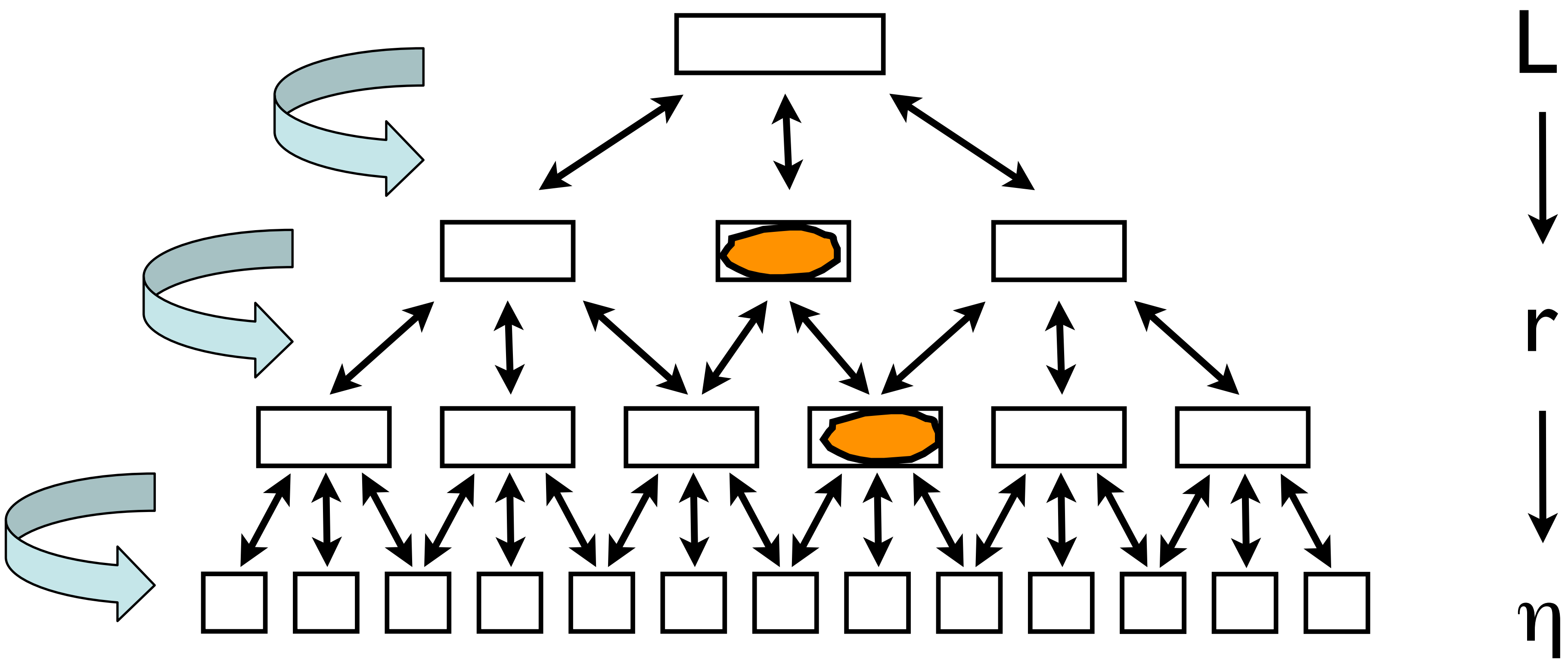
synergetics and hierarchical structures

- turbulence
  - cascade structure of interacting vorticities



synergetics and hierarchical structures

- turbulence
  - cascade structure of interacting vorticities
  - Rudolf - look at the interacting structures on different scales

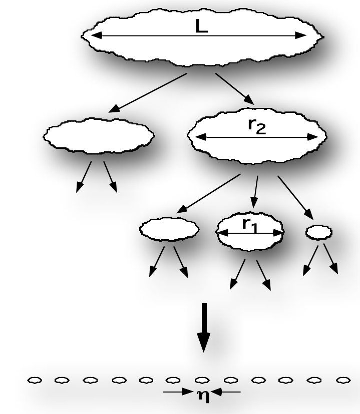


$$p(u_r, r | u_{r'}, r')$$

$$\partial_r p(u_r, r)$$

- process evolving in the cascade parameter r

- synergetic approach to turbulence
- stochastic cascade model - n-point statistics of turbulence
- deeper insights to turbulence and turbulent driven systems



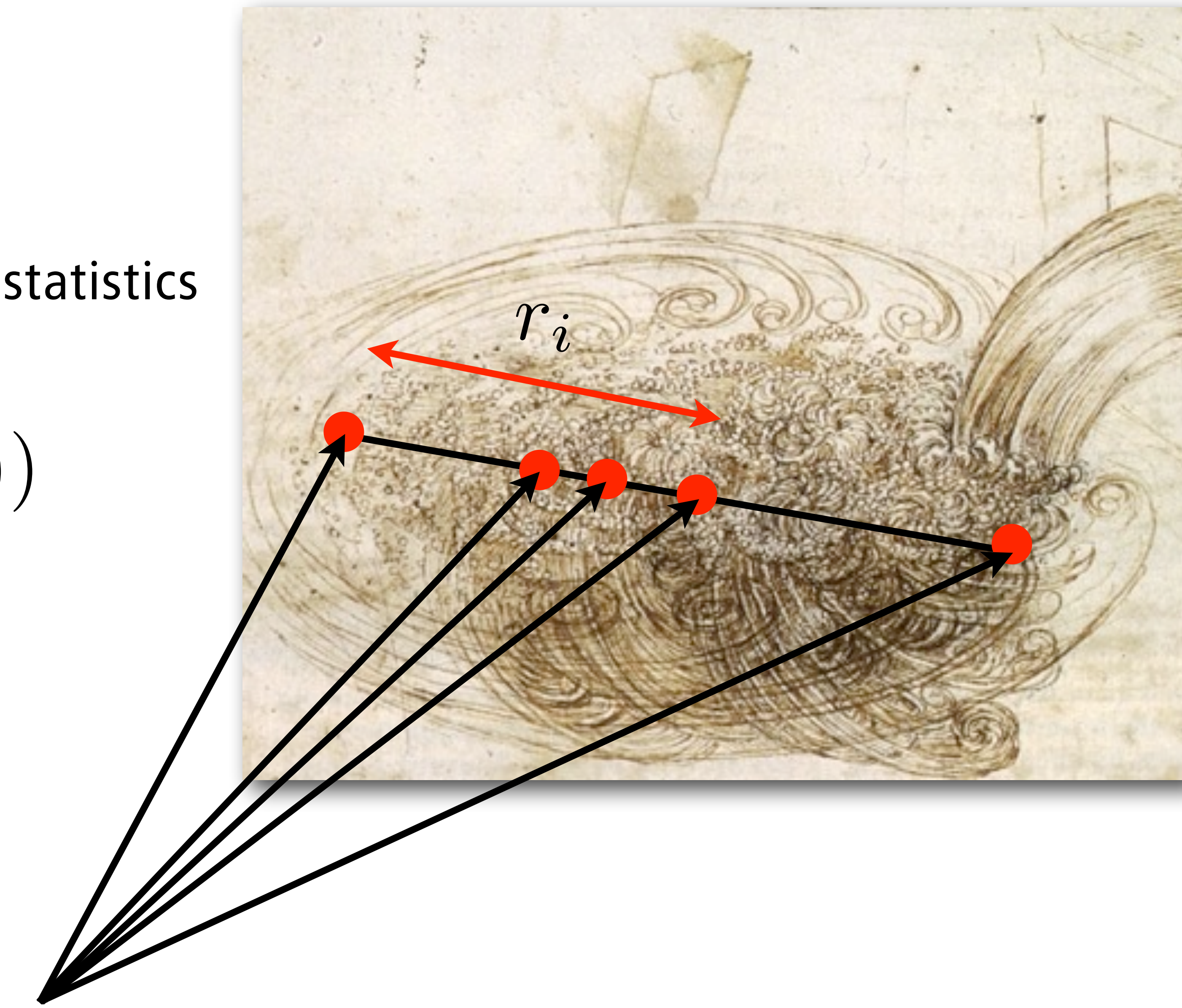
turbulence

- comprehensive description by n-point statistics

$$p(u(x_1), \ldots, u(x_{n+1}))$$

using velocity increments:

$$u_{r_i} = u(x + r_i) - u(x_i)$$



$$p(u(x_1), \ldots, u(x_{n+1})) = p(u_{r_1}, \ldots, u_{r_n}, u(x_1))$$

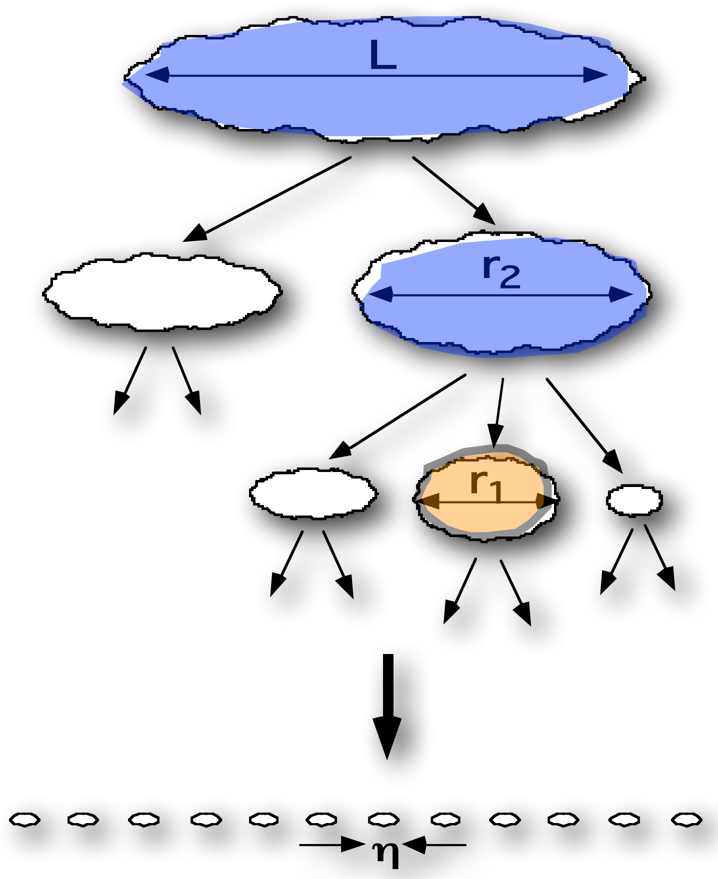
n-point statistics

$$p(u(x_1), ..., u(x_{n+1})) = p(u_{r_1}, ..., u_{r_n}, u(x_1))$$

$$= p(u_{r_1}, ..., u_{r_n} | u(x_1)) \cdot p(u(x_1))$$

Bayes theorem - joint pdf by cond. pdf

$$= p(u_{r_1} | ..., u_{r_n} u(x_1)) p(u_{r_2} | ..., u_{r_n} u(x_1)) ... \cdot p(u(x_1))$$



n-point statistics

$$
\begin{aligned}
p(u(x_1),...,u(x_{n+1})) &= p(u_{r_1},...,u_{r_n},u(x_1)) \\
&= p(u_{r_1},...,u_{r_n}|u(x_1)) \cdot p(u(x_1))
\end{aligned}
$$

Bayes theorem - joint pdf by cond. pdf

$$
= p(u_{r_1}|...,u_{r_n}u(x_1))p(u_{r_2}|...,u_{r_n}u(x_1))... \cdot p(u(x_1))
$$

can this be simplified?

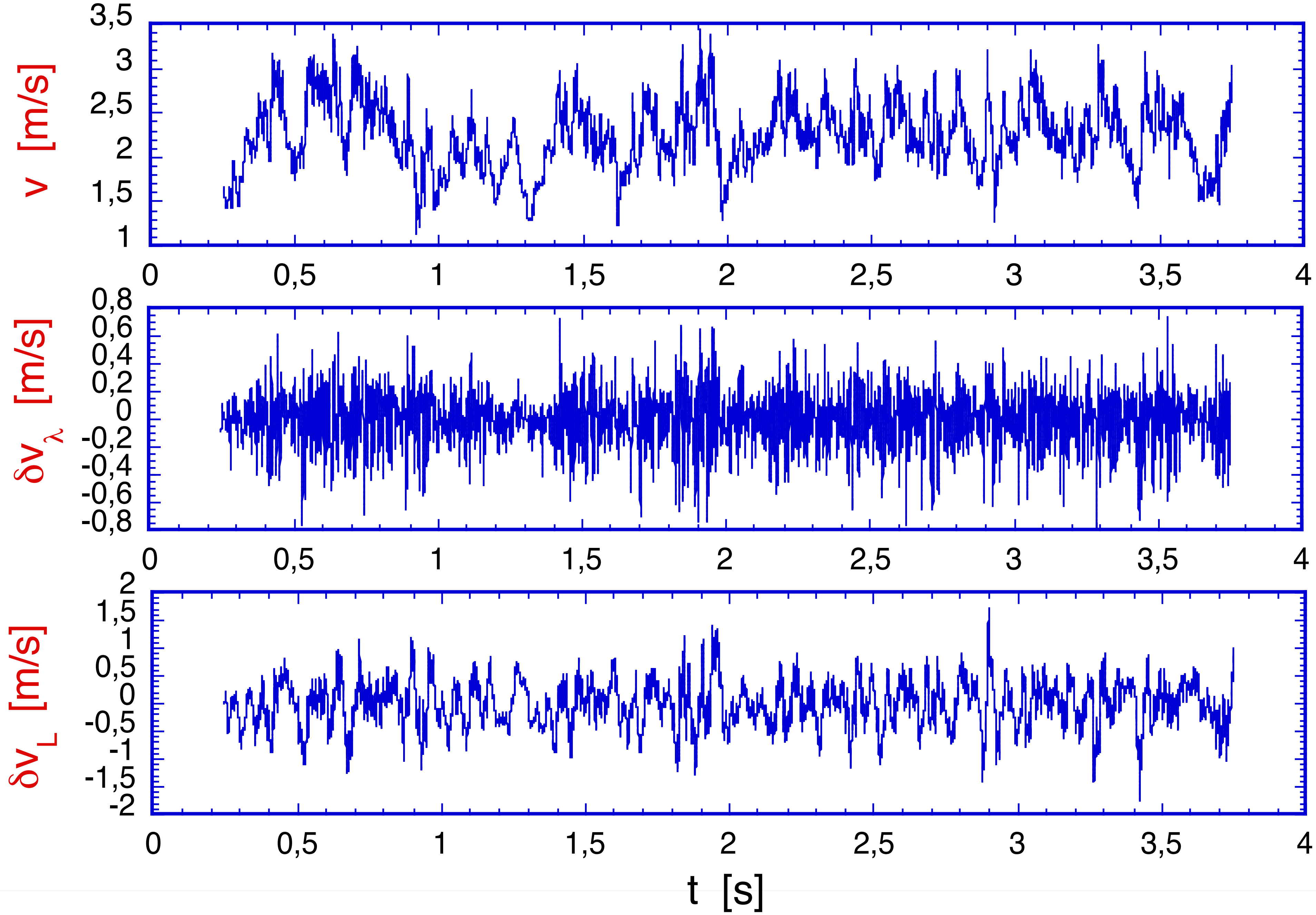
$$
= p(u_{r_1}|u_{r_2},u(x_1)) \ldots p(u_{r_{n-1}}|u_{r_n},u(x_1)) \cdot p(u(x_1))
$$

or even one increment statistics? $p(u_{r_1}) \ . \ . \ . \ p(u_{r_n})$

increment statistics measurable

- time signals, u(t) ,
- measured increments u_r for different r

$$p(u_{r_1}|\ldots,u_{r_n},u(x_1))$$



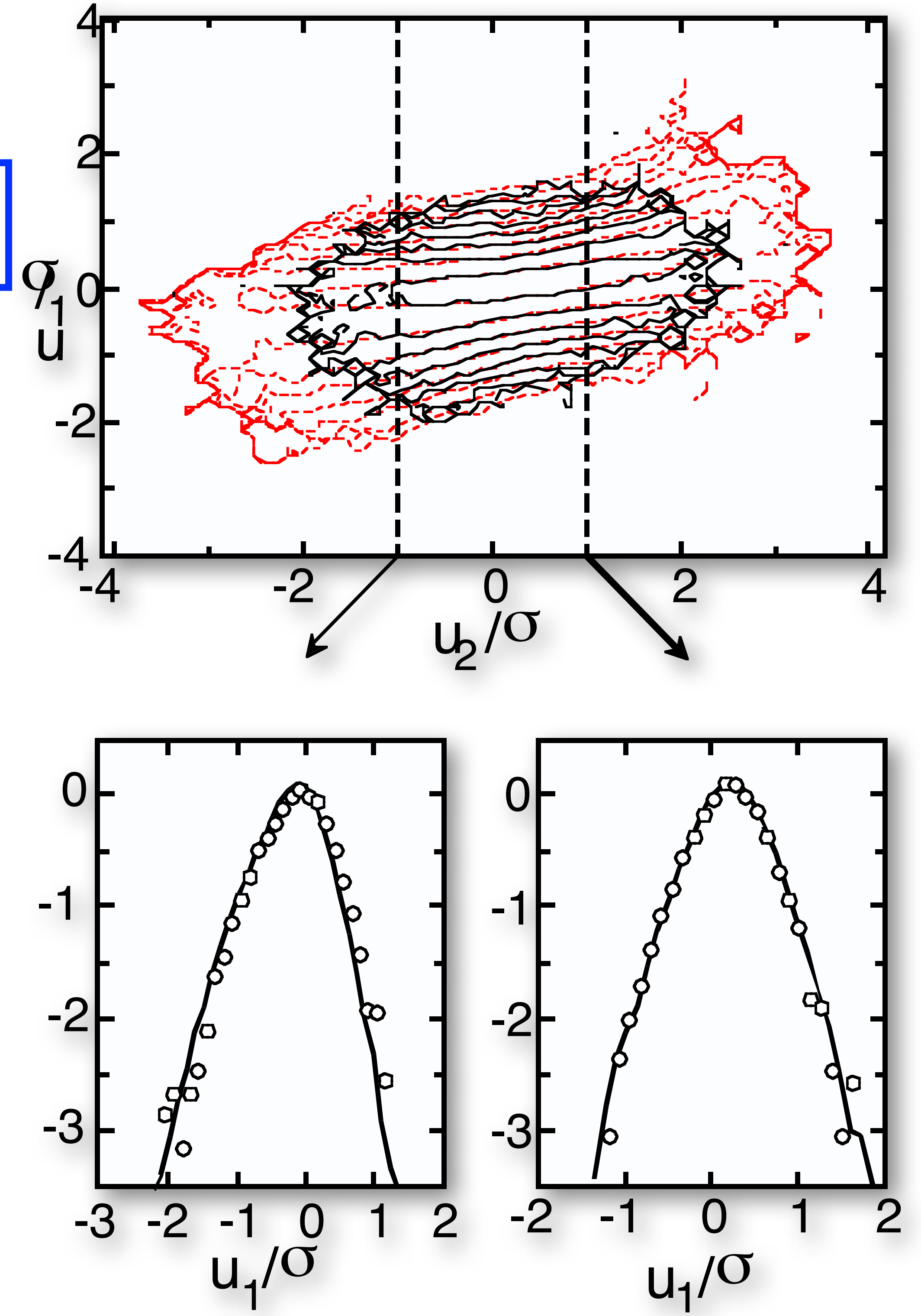
statistics of turbulence

simplification

(1) $$p(\vec{u}_1,r_1 | \vec{u}_2,r_2; \ldots; \vec{u}_n,r_n) = p(\vec{u}_1,r_1 | \vec{u}_2,r_2)$$

(2) $$p(\vec{u}_1,r_1 | \vec{u}_2,r_2; \ldots; \vec{u}_n,r_n) = p(\vec{u}_1,r_1)$$

experimental test



experimental result:

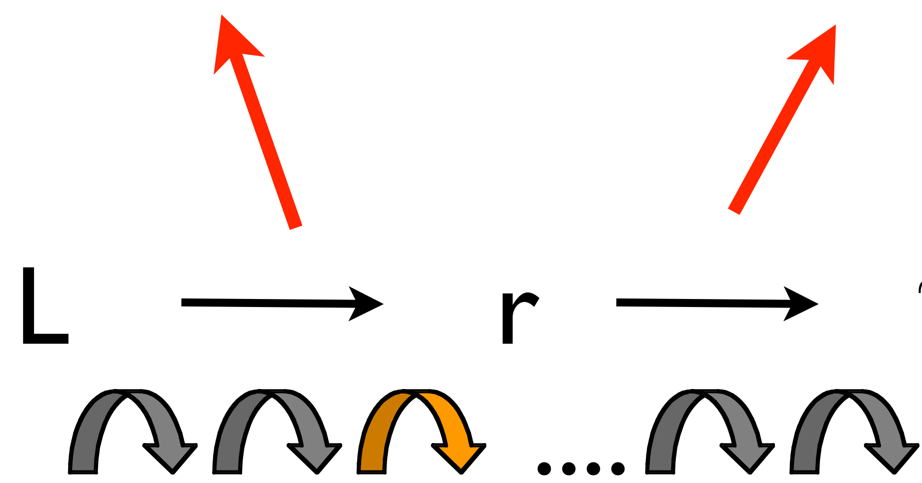
$$p(u_1|u_2,u_3) = p(u_1|u_2)$$

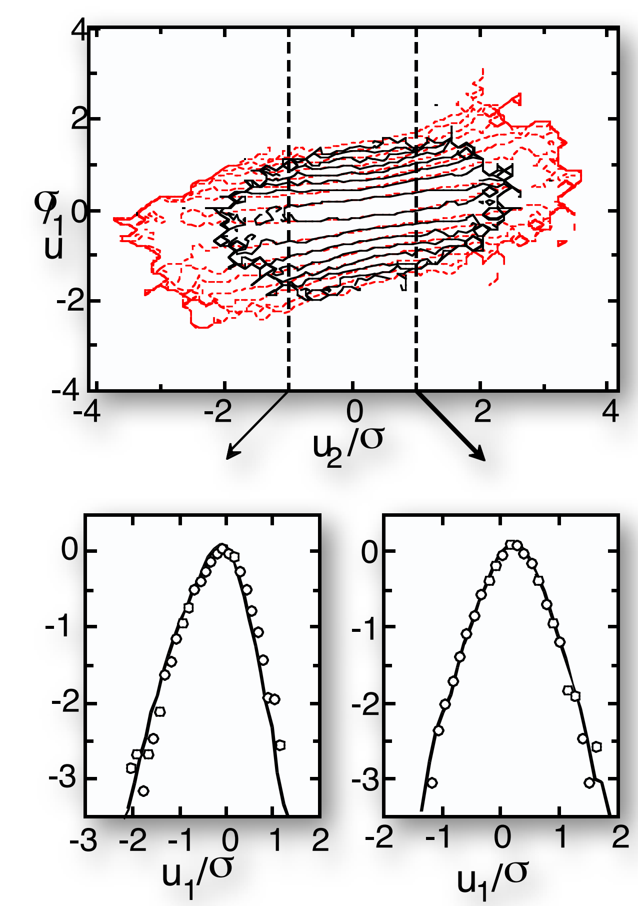
(1) holds

(2) not

n-point statistics

$$p(u(x_1), ..., u(x_{n+1}))$$
$$= p(u_{r_1}|u_{r_2}, u(x_1)) \ ... \ p(u_{r_{n-1}}|u_{r_n}, u(x_1)) \cdot p(u(x_1))$$



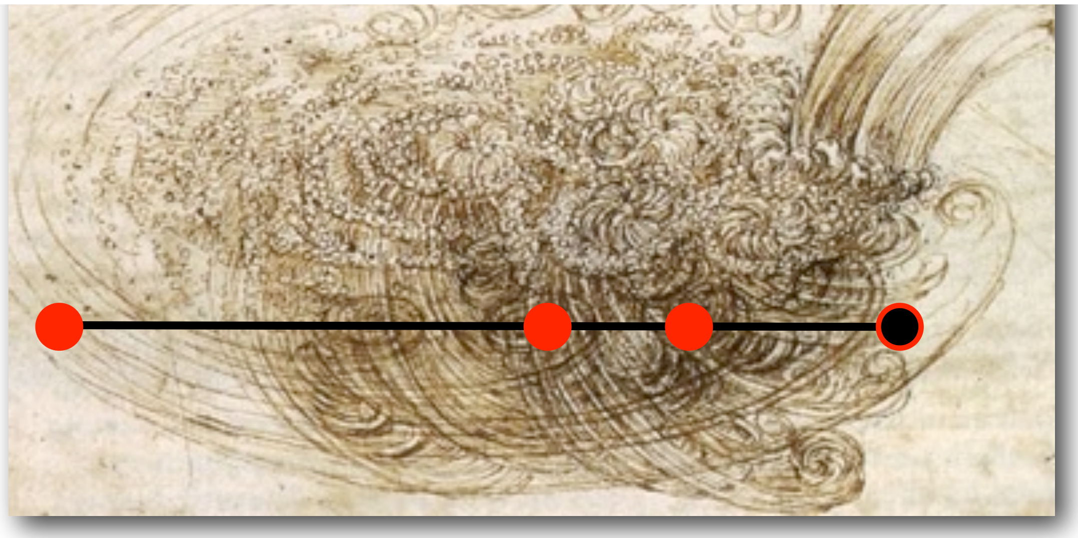


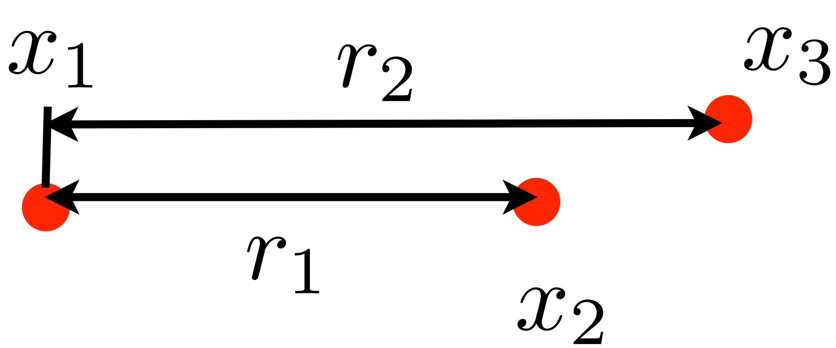
new view of cascade process :

three point closure

three point quantity

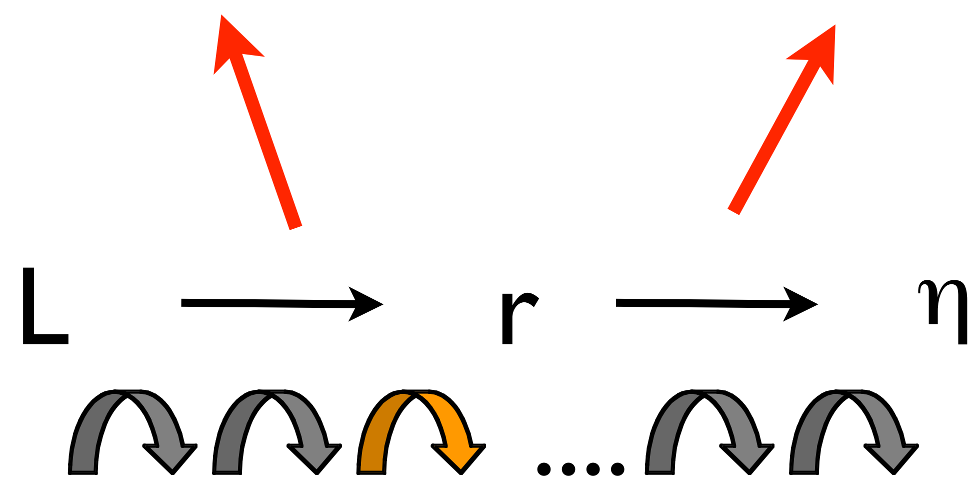
$$p(u_{r_1} | u_{r_2}, u(x_1))$$





n-point statistics

$$p(u(x_1), ..., u(x_{n+1}))$$
$$= p(u_{r_1}|u_{r_2}, u(x_1)) \; ... \; p(u_{r_{n-1}}|u_{r_n}, u(x_1)) \cdot p(u(x_1))$$



new view of cascade process:

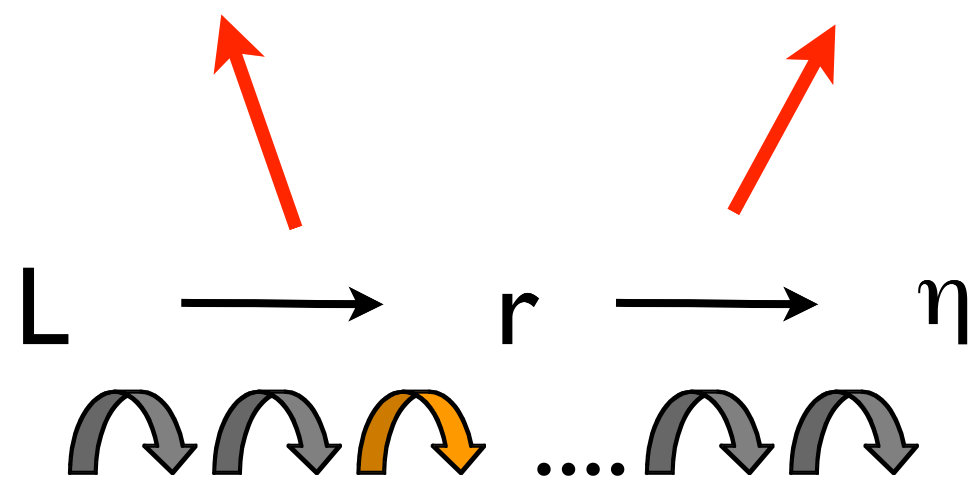
three point closure

local in the cascade means no memory

or Markow process in r

n-point statistics

$$p(u(x_1), ..., u(x_{n+1}))$$
$$= p(u_{r_1}|u_{r_2}, u(x_1)) \ldots p(u_{r_{n-1}}|u_{r_n}, u(x_1)) \cdot p(u(x_1))$$



Markow prop & cascade with Fokker-Planck Equ.

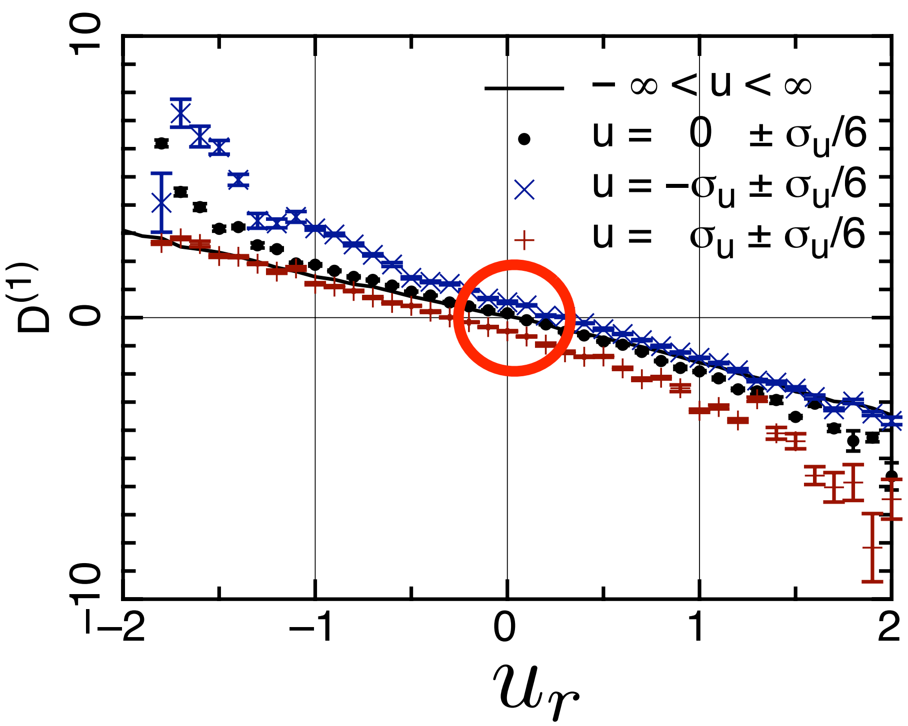
$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j}|u_{r_k}, u(x_1)) = \{-\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1))\} \, p(u_{r_j}|u_{r_k}, u(x_1))$$

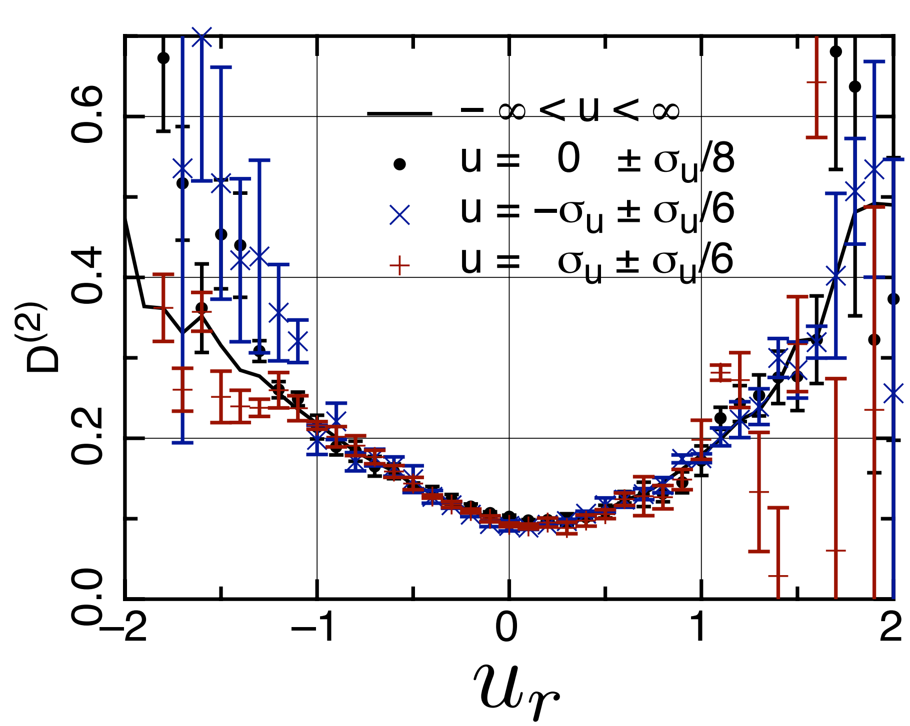
n-point statistics

$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j}|u_{r_k}, u(x_1)) = \{-\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1))\}\, p(u_{r_j}|u_{r_k}, u(x_1))$$

Markow prop. => Fokker-Planck Equ. measured by KM coefficients

$$D^{(n)}(u_r, r) = \frac{1}{n! \cdot \Delta} \lim_{\Delta \to 0} \int (\tilde{u}_r - u_r)^n \, p(\tilde{u}_{r+\Delta}, r+\Delta | u_r, r) d\tilde{u}_r$$

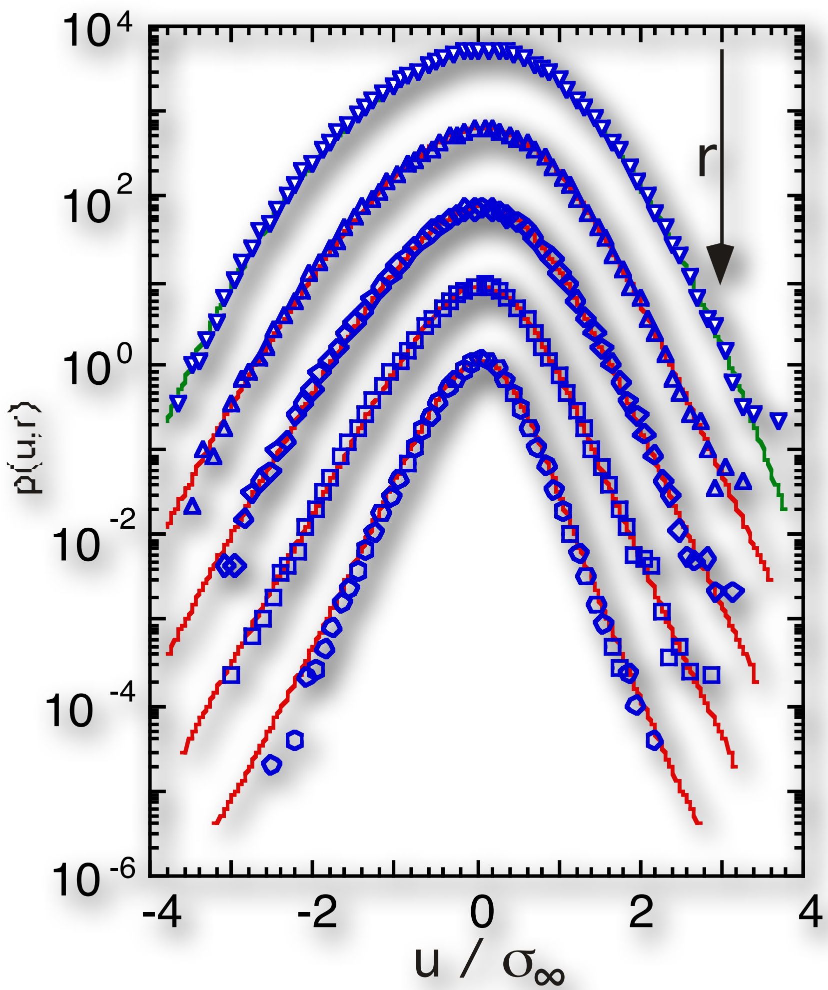




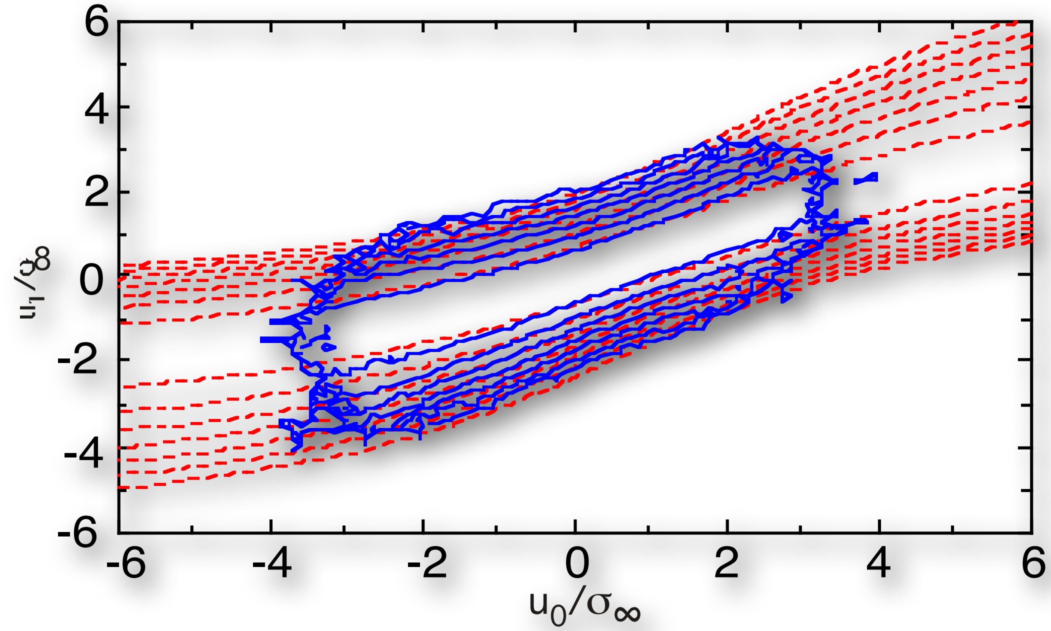
shift of drift function, no *u*-dependence of diffusion function

3. Verification of the measured Fokker-Planck equation

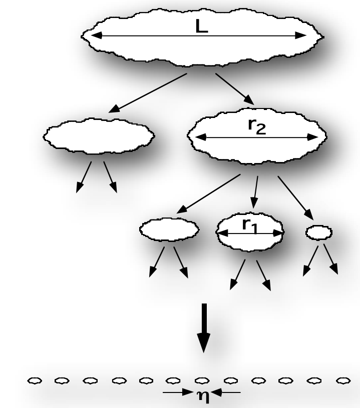
- numerical solution compared with experimental results
- => n-scale statistics



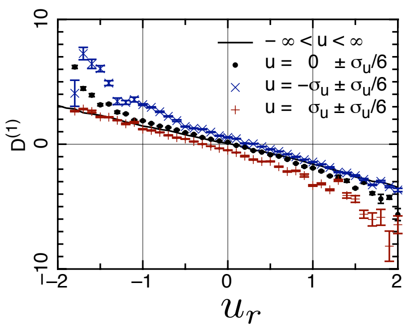
$$p(u_r, r | u_{r_0}, r_0)$$

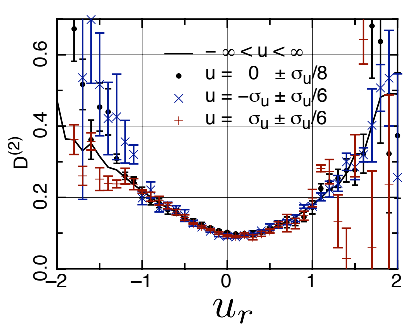


Journal of Fluid Mechanics 433 (2001)



- synergetic approach to turbulence
- stochastic cascade model - n-point statistics of turbulence
  - stochastic process in r - tremendous reduction in complexity





shift of drift function, no *u*-dependence of diffusion function

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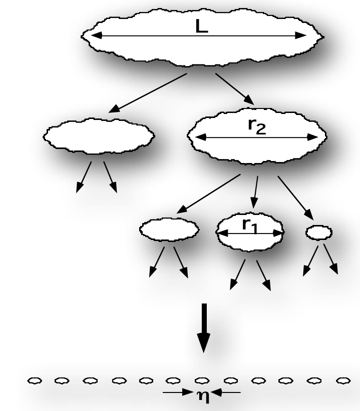

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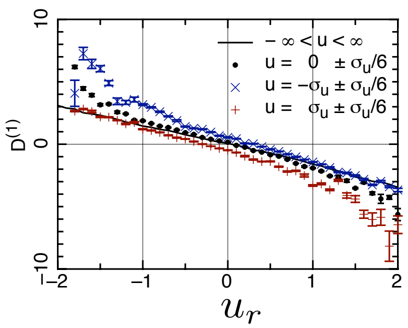
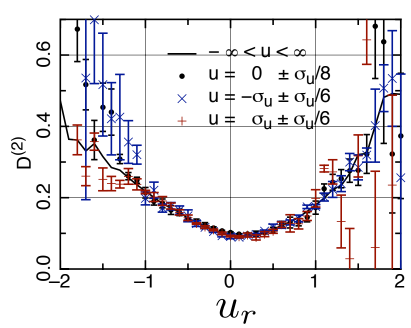
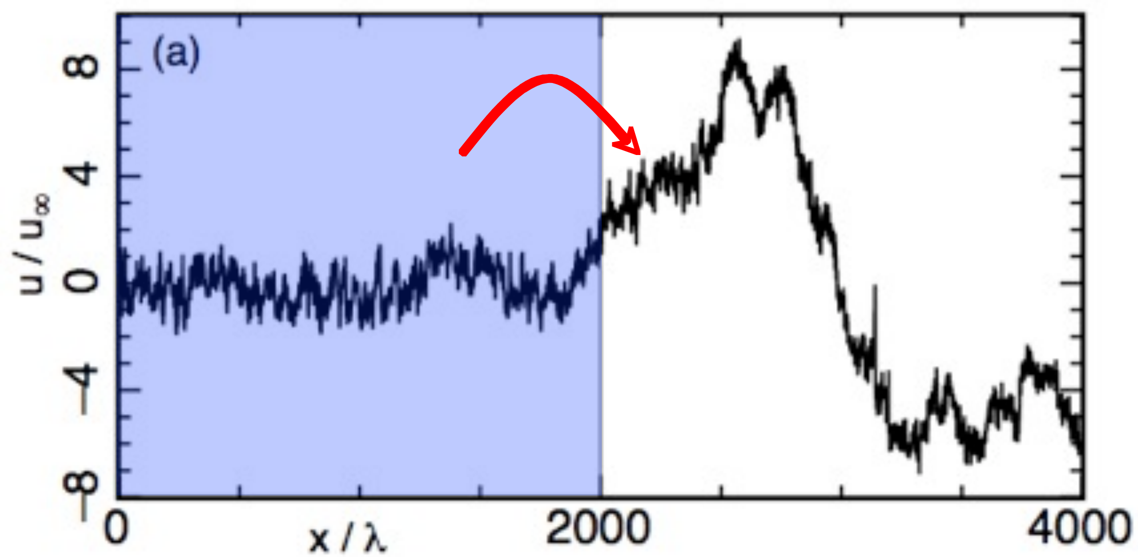


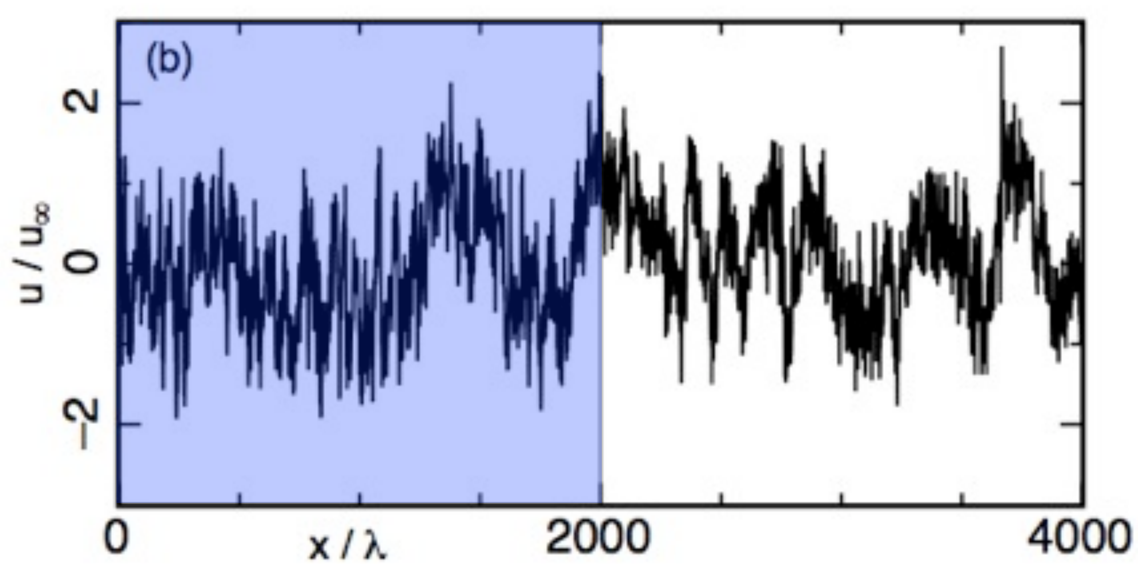


shift of drift function, no *u*-dependence of diffusion function

- deeper insights to turbulence and turbulent driven systems

reconstruction of time series

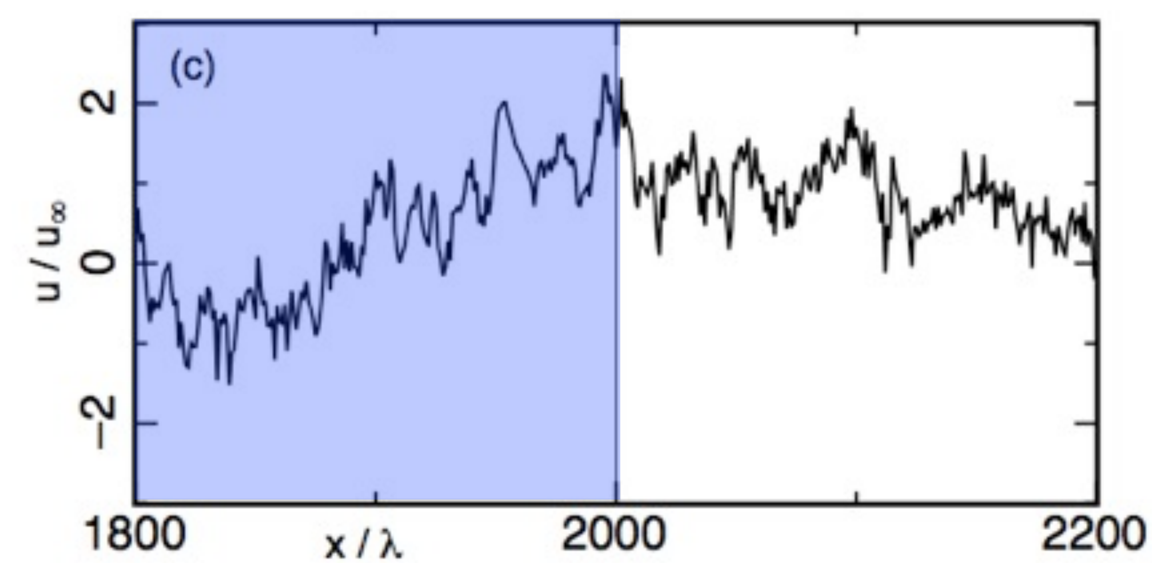




no u dependence

$$D^{(1)}(u_r, r, u) = d_{10}(r, u) - d_{11}(r) u_r$$
$$D^{(2)}(u_r, r, u) = D^{(2)}(u_r, r)$$

with u dependence

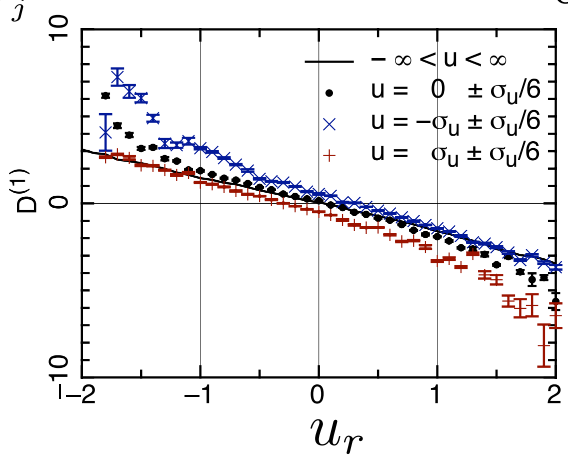


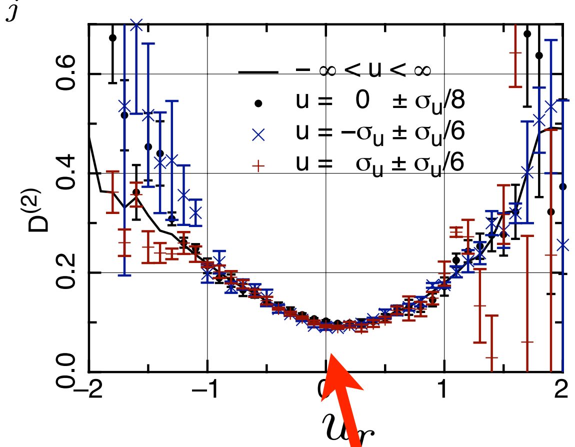
Nawroth et al Phys. Lett. (2006)

blow up

turbulence - classical theory

$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j}|u_{r_k}, u(x_1)) = \{-\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1))\}\ p(u_{r_j}|u_{r_k}, u(x_1))$$





shift of drift function, no *u*-dependence of diffusion function

Kolmogorov 41

$$D^{(1)}(u_r) = \frac{1}{3} u_r$$

Langevin equation

$$\frac{\partial}{\partial r} u_r = \frac{1}{3} \frac{u_r}{r} \qquad u_r \propto r^{1/3}$$

$$< u_r^n > \propto r^{n/3}$$

Kolmogorov 62

$$D^{(1)}(u_r,) = \gamma u_r$$

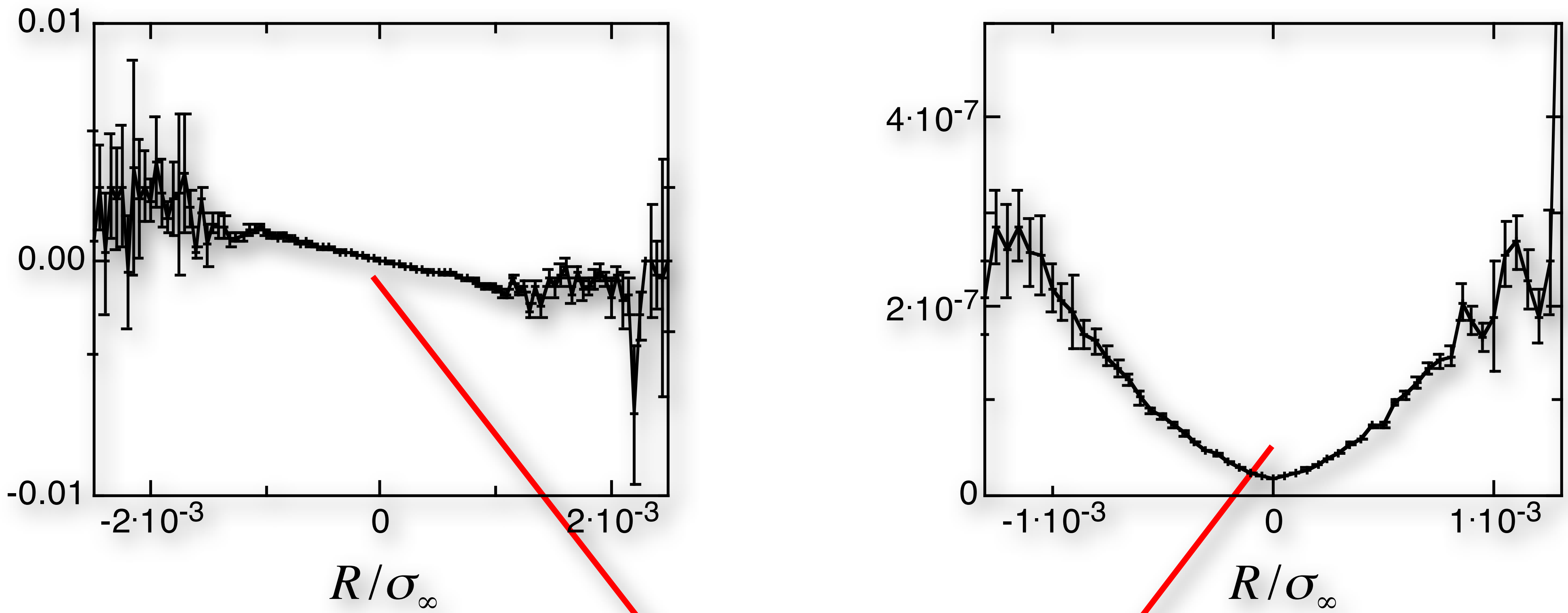
$$D^{(2)}(u_r) = \beta u_r^2$$

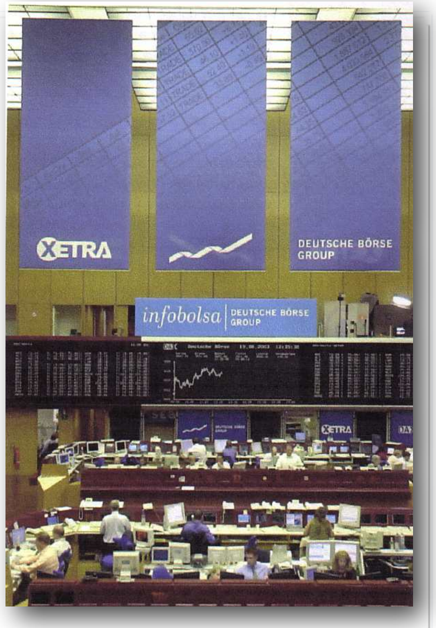
$$< u_r^n > \propto r^{n/3+\mu(n)}$$

RF & JP PRL 78 (1997)

Friedrich JP PRL (1996)

Functional form of the coefficients $D^{(1)}$ and $D^{(2)}$ is presented





$$\frac{\partial}{\partial \tau} p(R,\tau) = \left[-\frac{\partial}{\partial R} D^{(1)}(R,\tau) + \frac{\partial^2}{\partial R^2} D^{(2)}(R,\tau) \right] p(R,\tau)$$

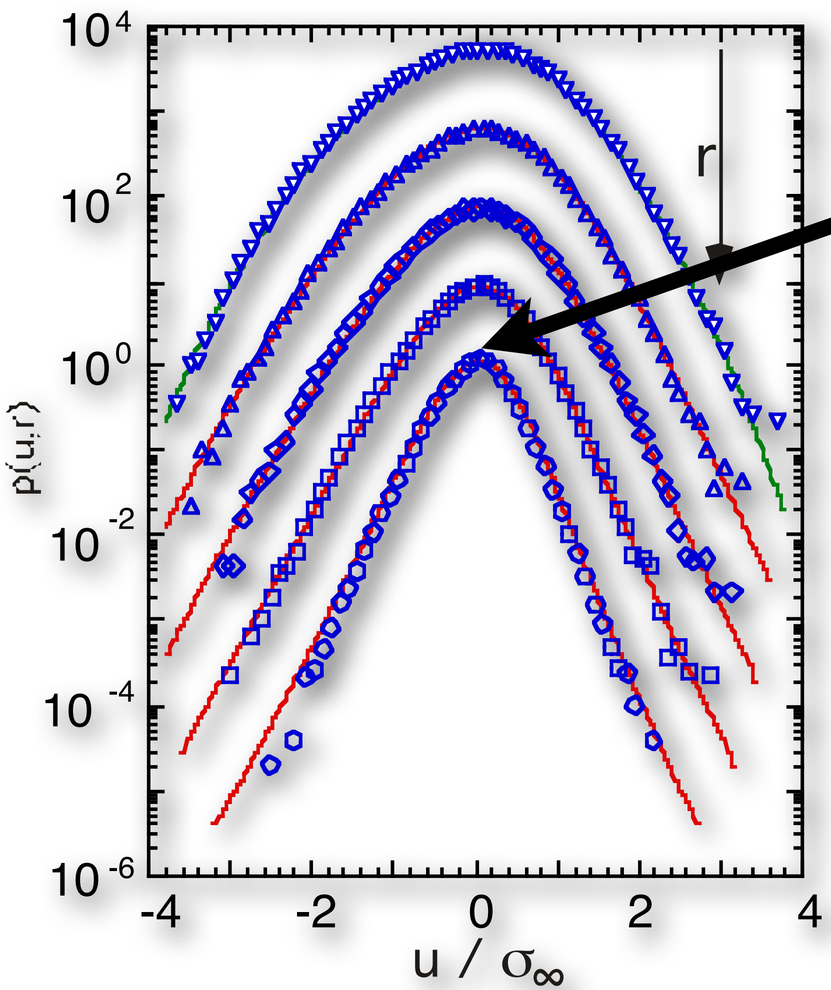
Example: Volkswagen, τ = 10 min

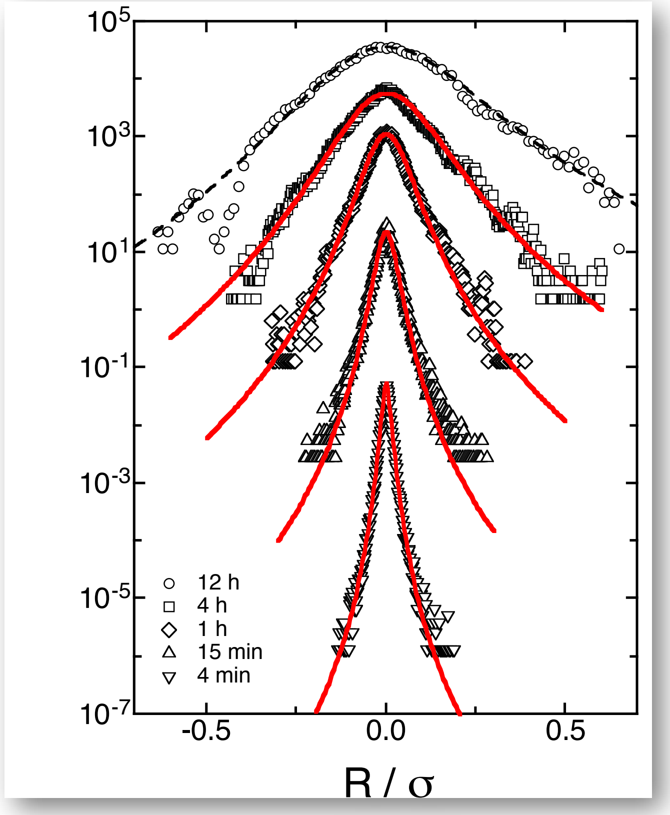
multi-scale statistics

additive term in the diffusion term: -▸ additive noise

$$D_1^{(1)}(u_r,r) = d_1^u(r)u_r$$
$$D^{(2)}(u_r,r) = d_2(r) + d_2^u(r)u_r + d_2^{uu}(r)u_r^2$$

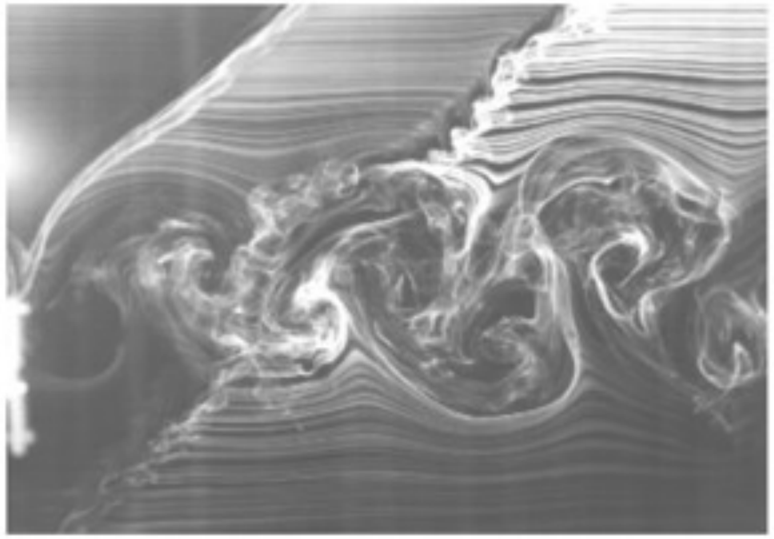
Gaussian tip

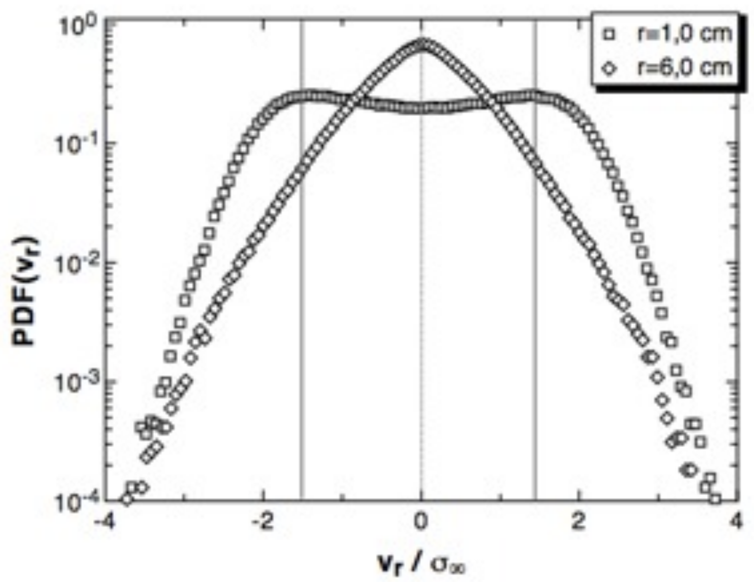


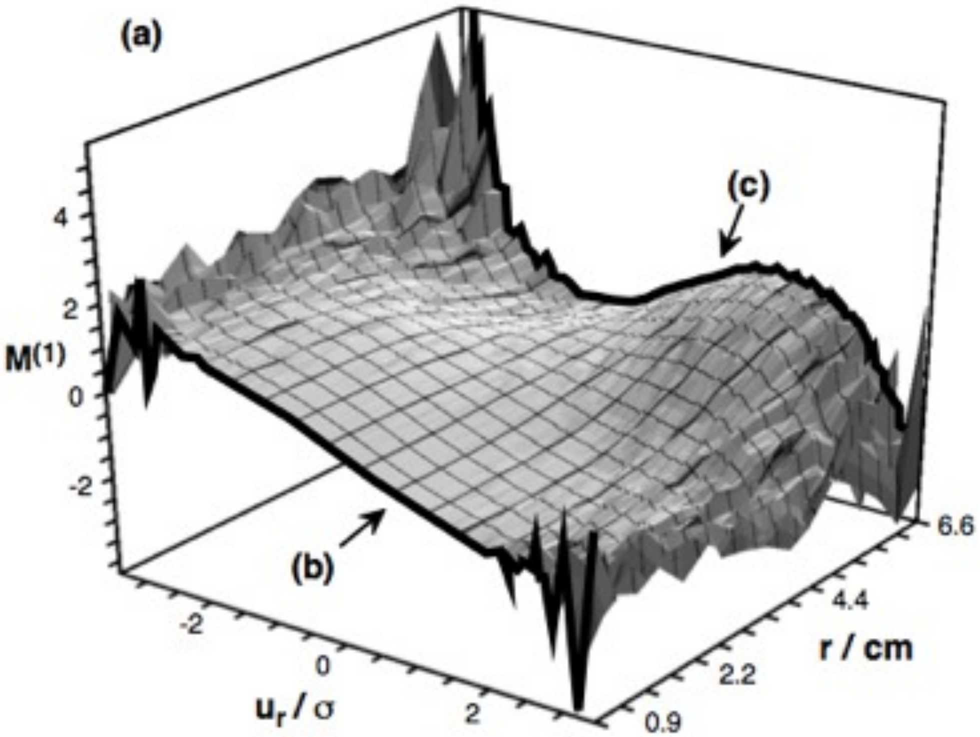


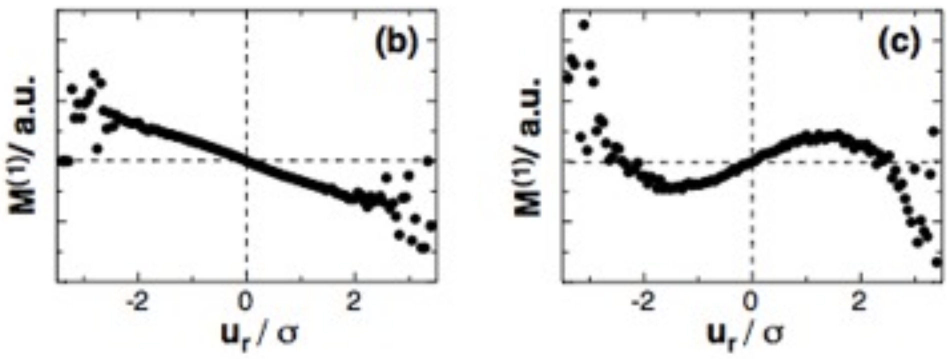
wake flow behind a cylinder - turbulent structures

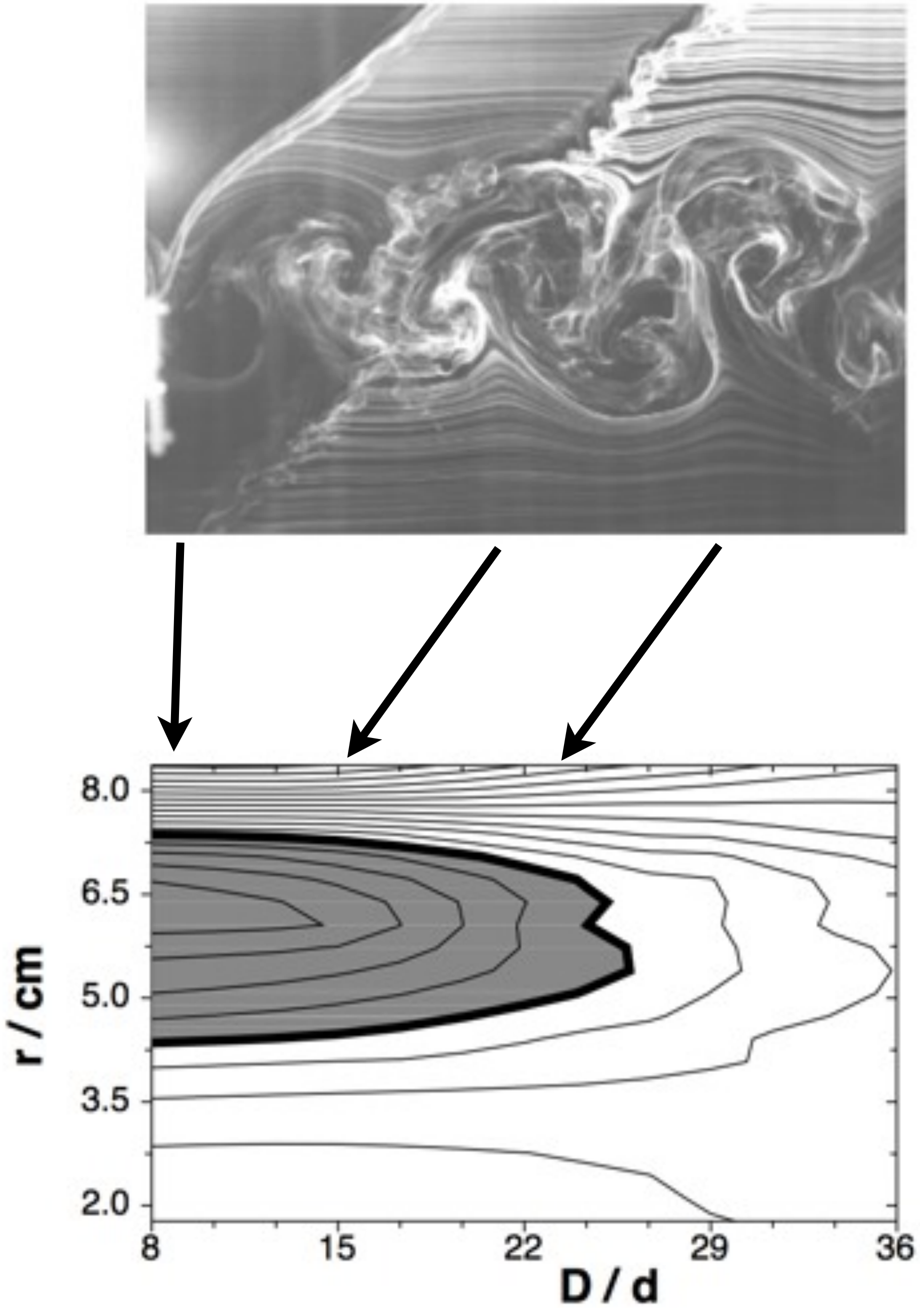
drift term as function of r



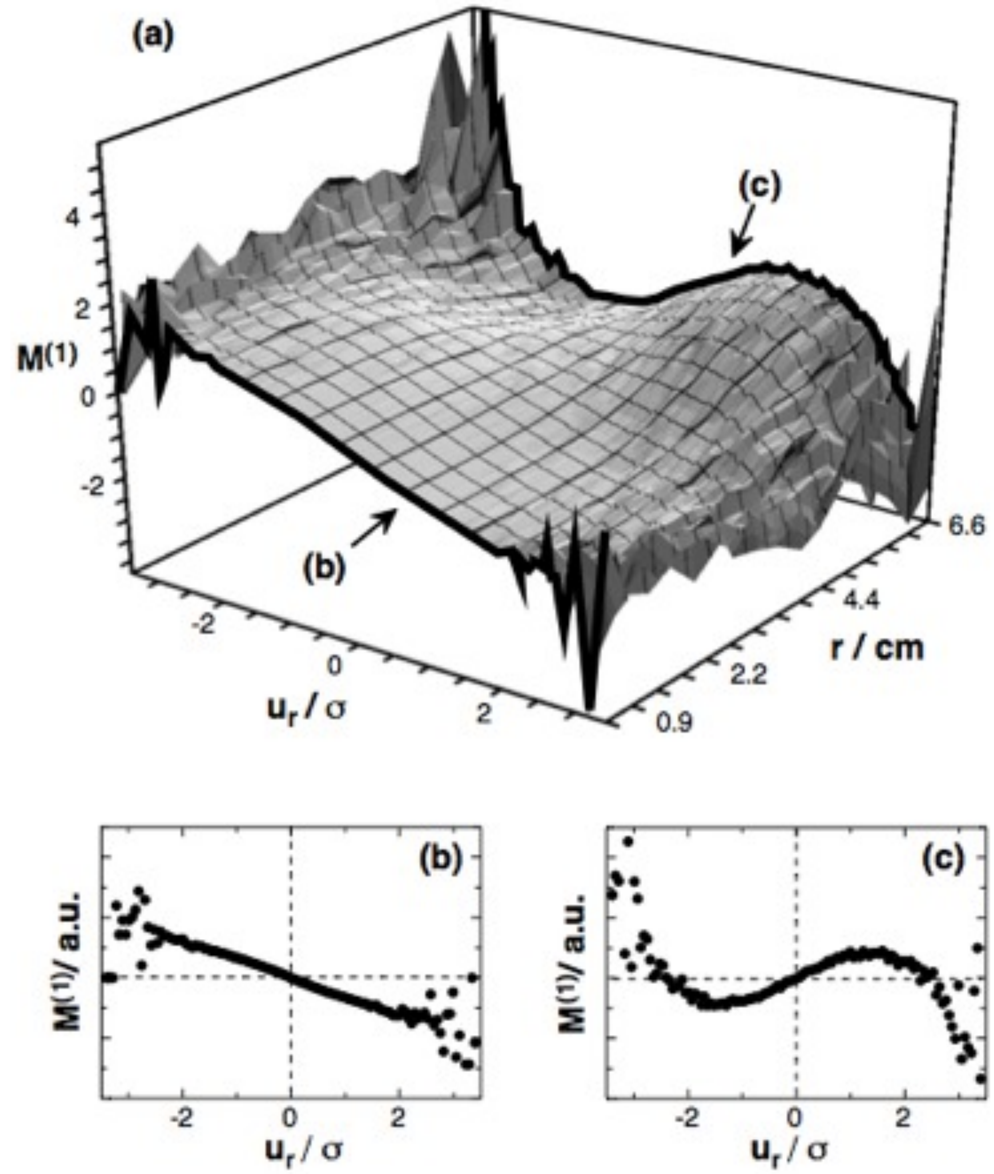








drift term as function of r

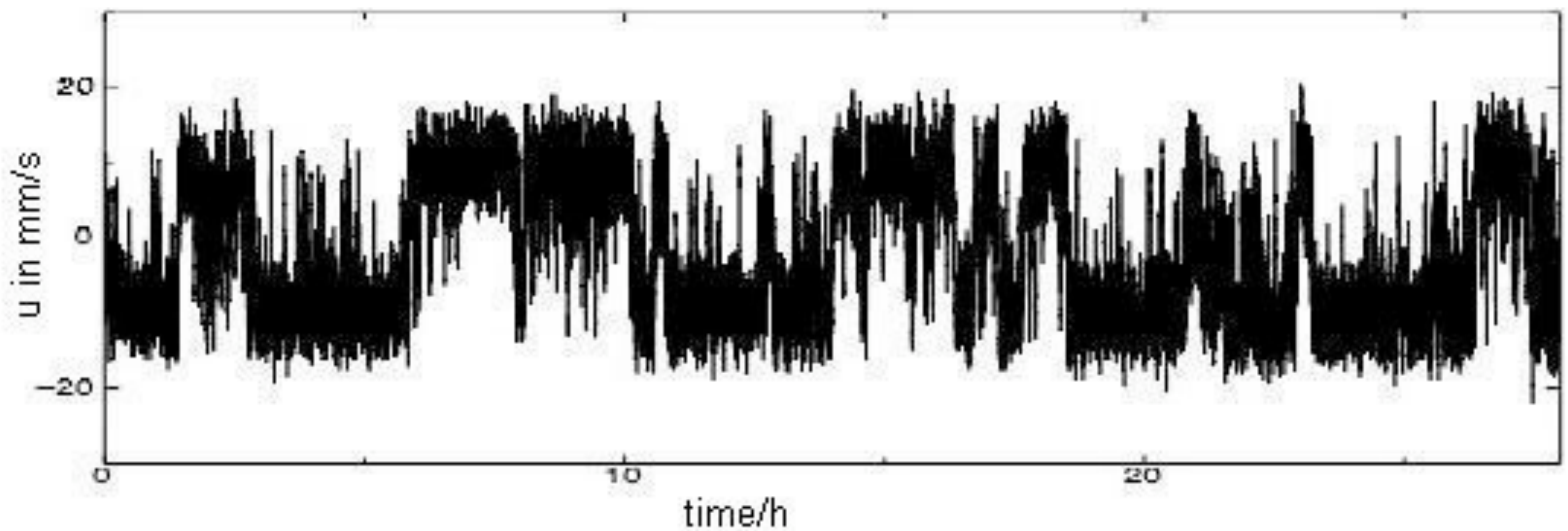


phase transition to isotropic turbulence

turbulence: new insights

- Markov-length - a coherence length
- statistics of longitudinal and transversal increments
- universality of turbulence:
- fractal grid turbulence
- role of transfered energy e_r:
- fusion rules $r_i \Rightarrow r_{i+1}$ (Davoudi, Tabar 2000; L'vov, Procaccia 1996)
- passive scalar (Tutkun, Mydlarski 2004)
- Lagrangian turbulence (Friedrich 2003,2008)

turbulent driven systems

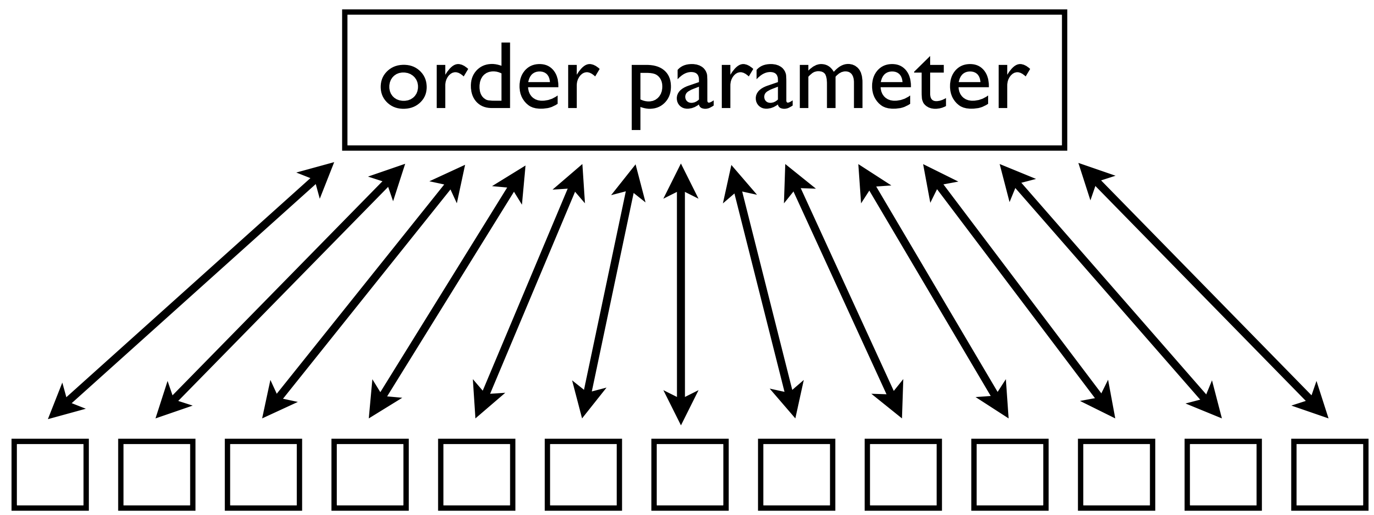


open question - what is the corresponding dynamics

$$\dot{\mathbf{x}} = ??$$

$$\mathbf{x}(t+\tau) = ??$$

synergetic approach



complex system
interacting subsystems

slaving to
low dimensional
dynamics

- deterministic part
- noisy part

$$\dot{\mathbf{x}}(t) = \mathbf{D}^{(1)}(\mathbf{x}(t_j), t_j) + \sqrt{\mathbf{D}^{(2)}(\mathbf{x}(t_j)}\,\mathbf{\Gamma}'(t_j)$$

Markov process - Langevin Equation

$$\dot{\mathbf{x}}(t) = \mathbf{D}^{(1)}(\mathbf{x}(t_j), t_j) + \sqrt{\mathbf{D}^{(2)}(\mathbf{x}(t_j)}\mathbf{\Gamma}'(t_j)$$

the basic element is the

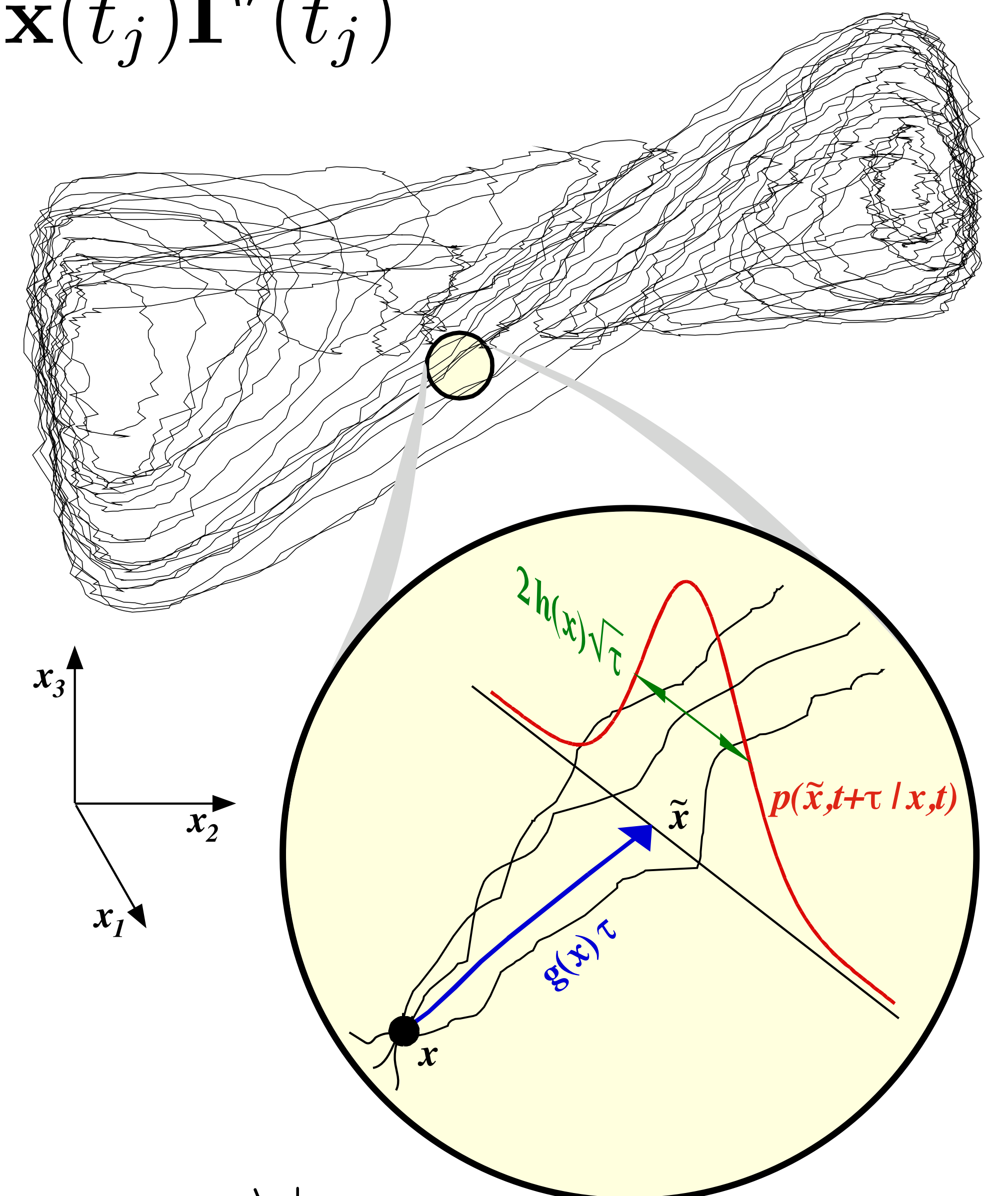
transition probability

$$p(\tilde{x}, t+\tau | x, t)$$

from this we can get

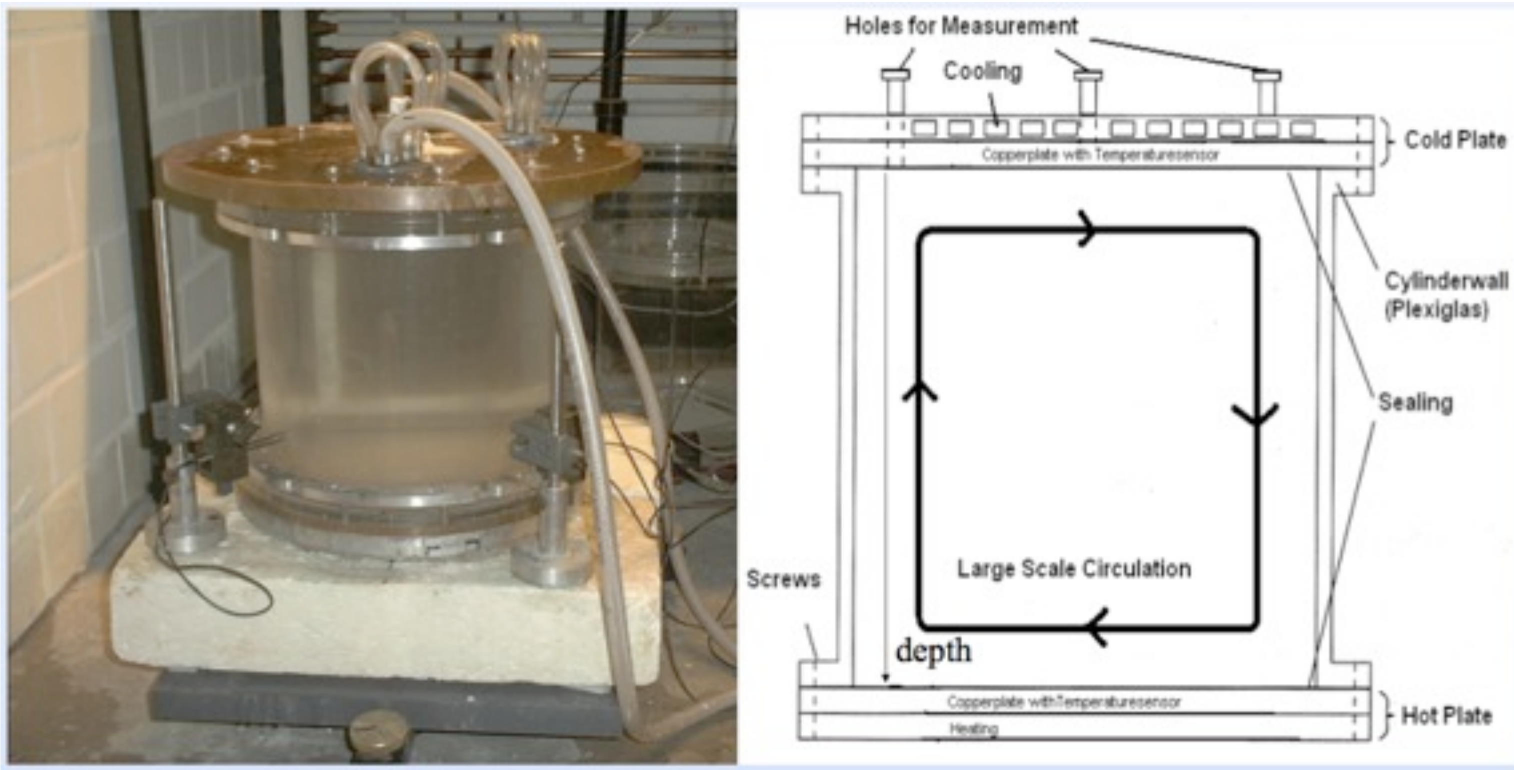
$$\mathbf{D}^{(1)}(\mathbf{x}) = \lim_{\tau \to 0} \frac{1}{\tau} \left\langle \mathbf{X}(t+\tau) - \mathbf{x} \right\rangle \Big|_{\mathbf{X}(t)=\mathbf{x}}$$

$$\mathbf{D}^{(2)}(\mathbf{x}) = \lim_{\tau \to 0} \frac{1}{\tau} \left\langle (\mathbf{X}(t+\tau) - \mathbf{x})\mathbf{x})(\mathbf{X}(t+\tau) - \mathbf{x})^{\mathrm{T}} \right\rangle \Big|_{\mathbf{X}(t)=\mathbf{x}}$$

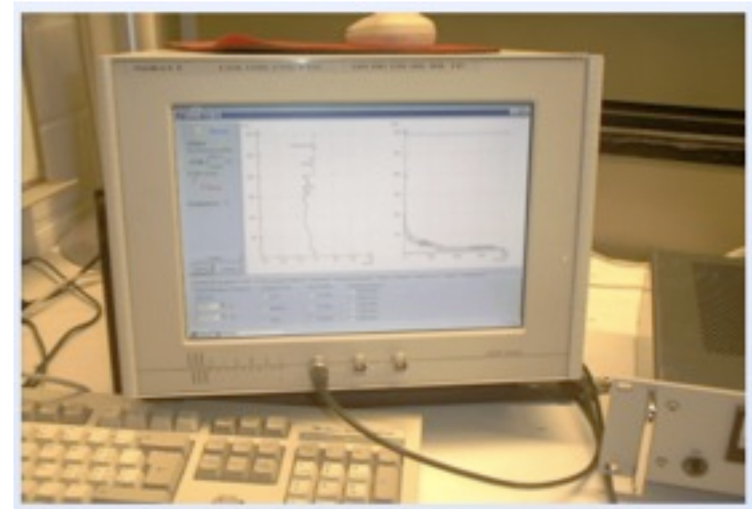


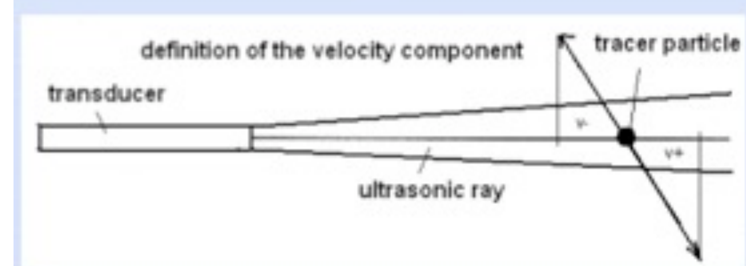
Siegert et.al. Phys. Lett. A 243, 275 (1998)

Rayleigh Benard Experiment



max. temperature difference 20°C
Ra < $9*10^9$



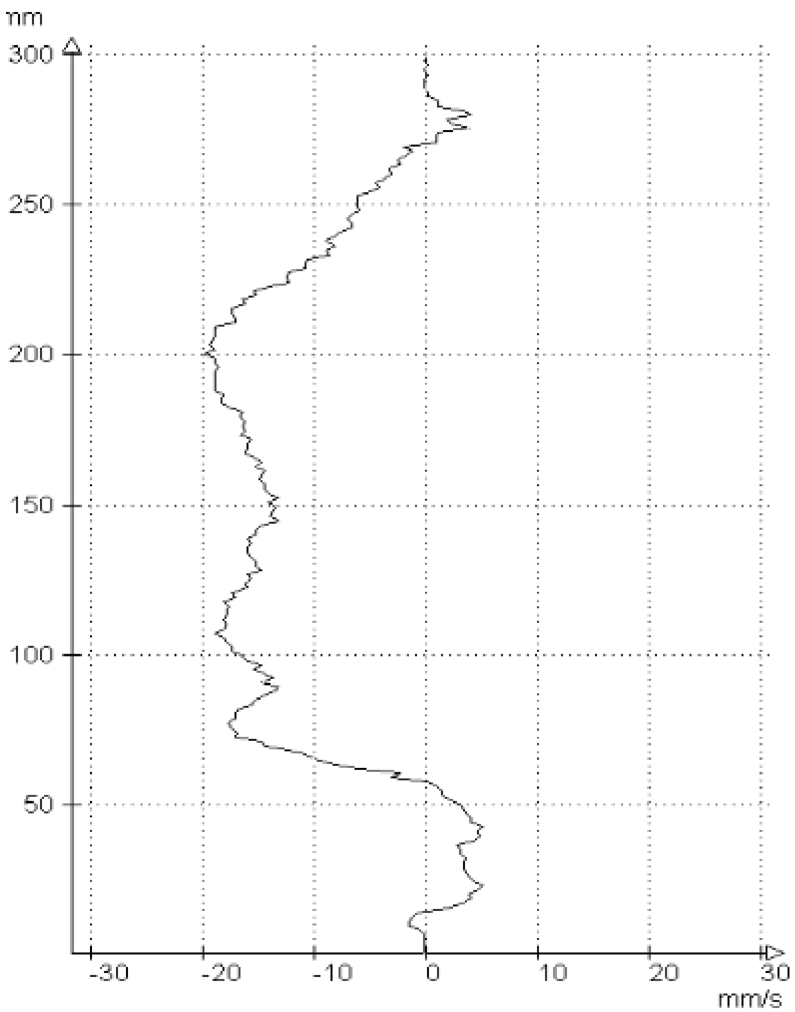


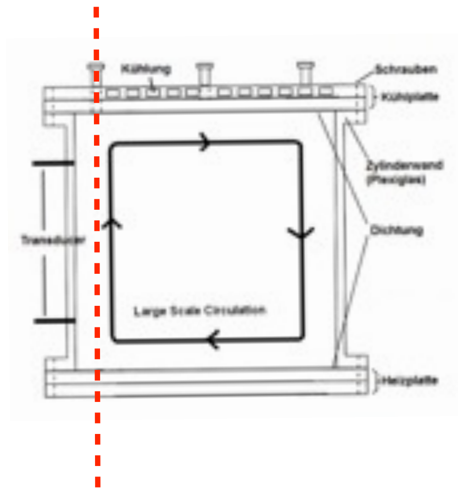
measurements with ultrasonic dopper Anemometer DOP2000ß

Rayleigh Benard Experiment

DOP2000 - profile measurements

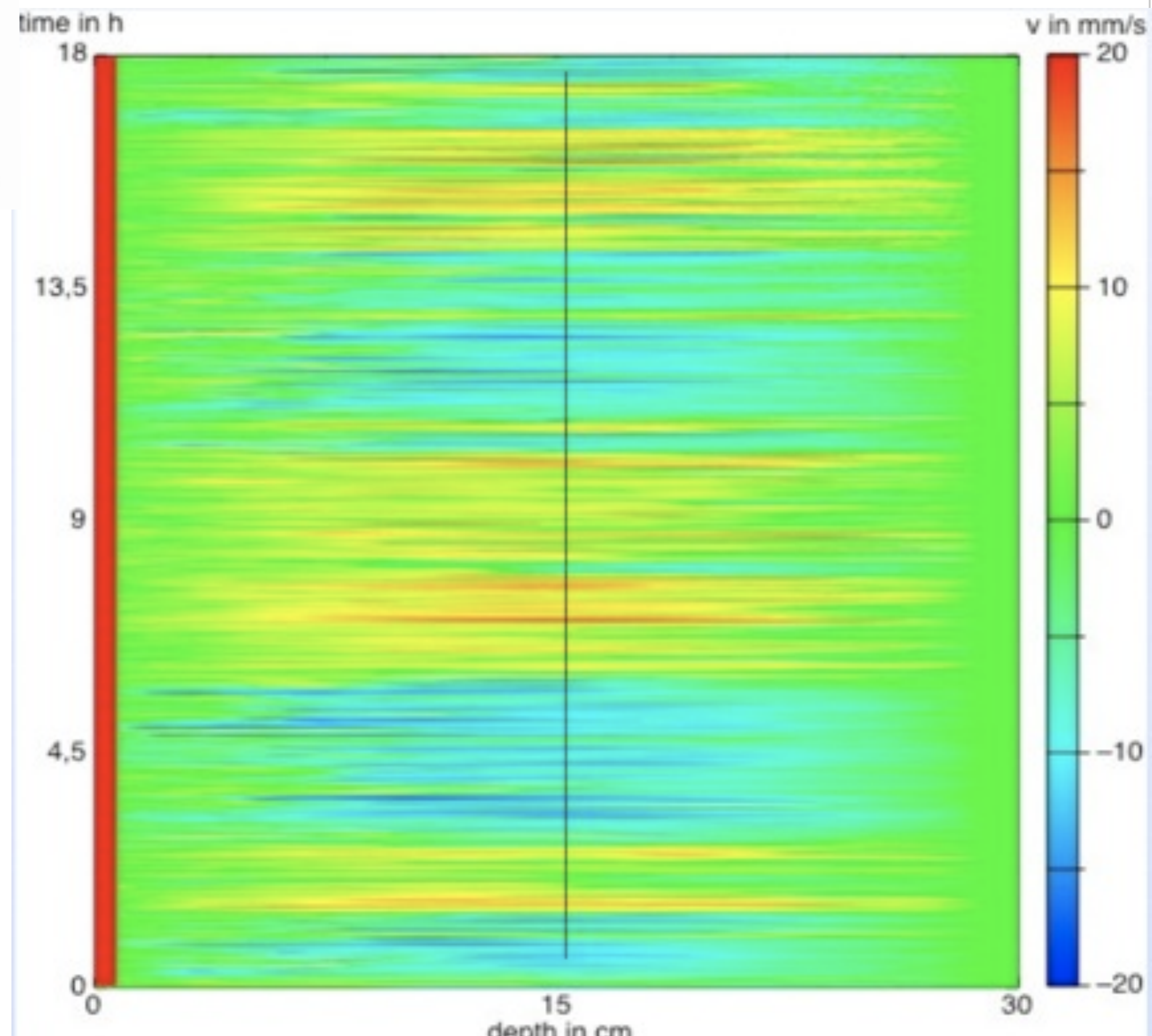
h





red rising,;
blue sinking

time



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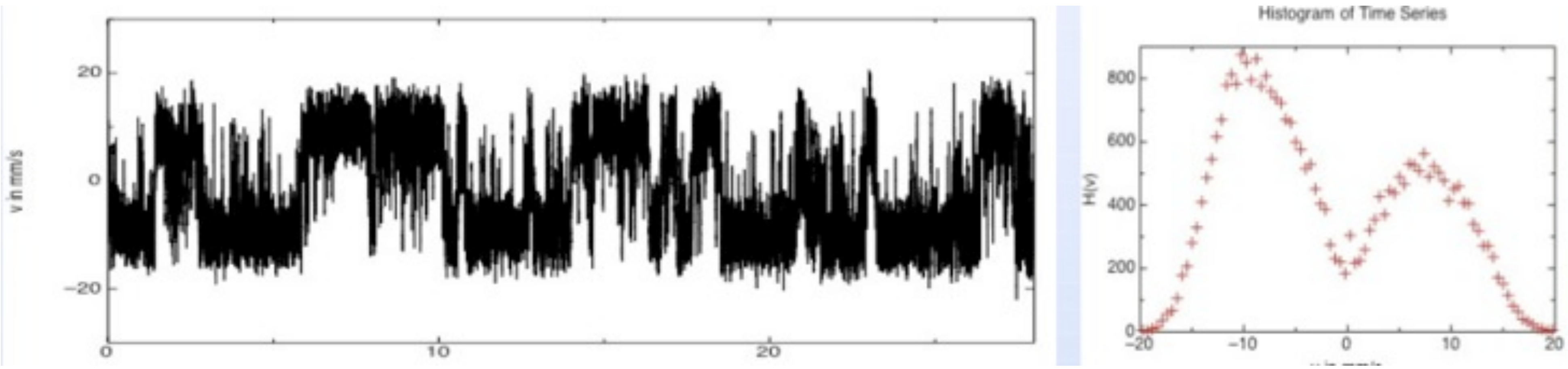
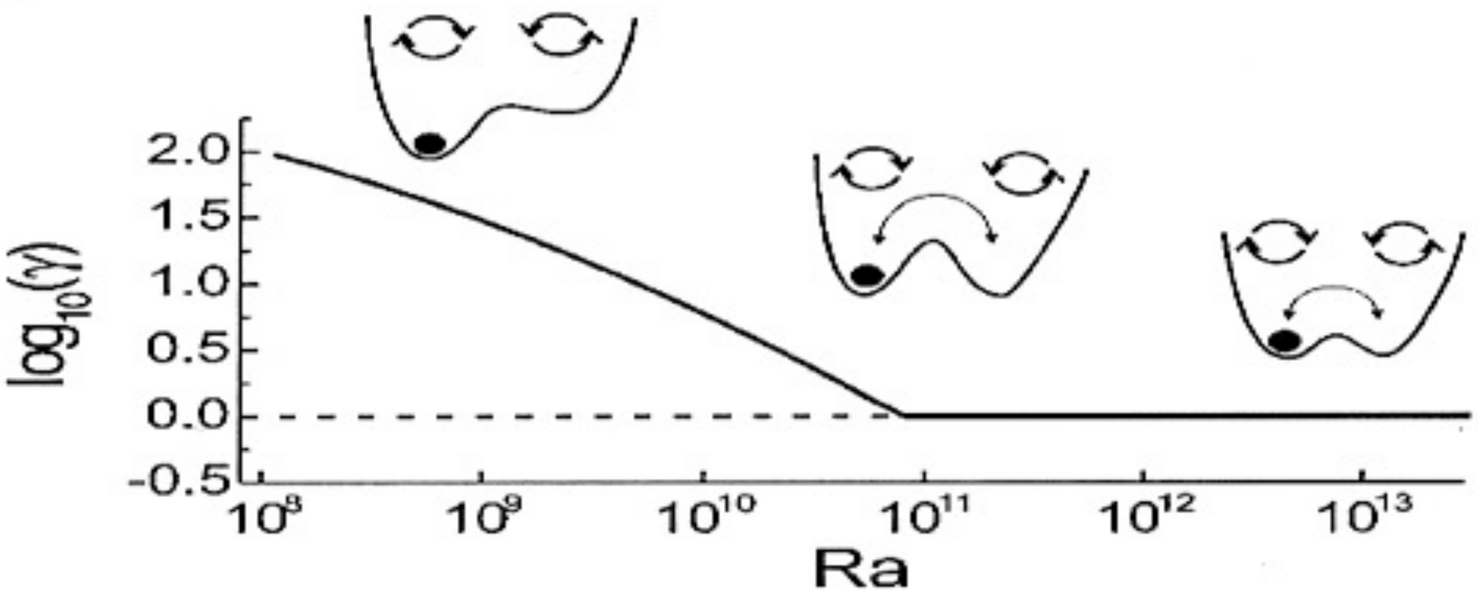
Rayleigh Benard Experiment



Ra = 10^{10}

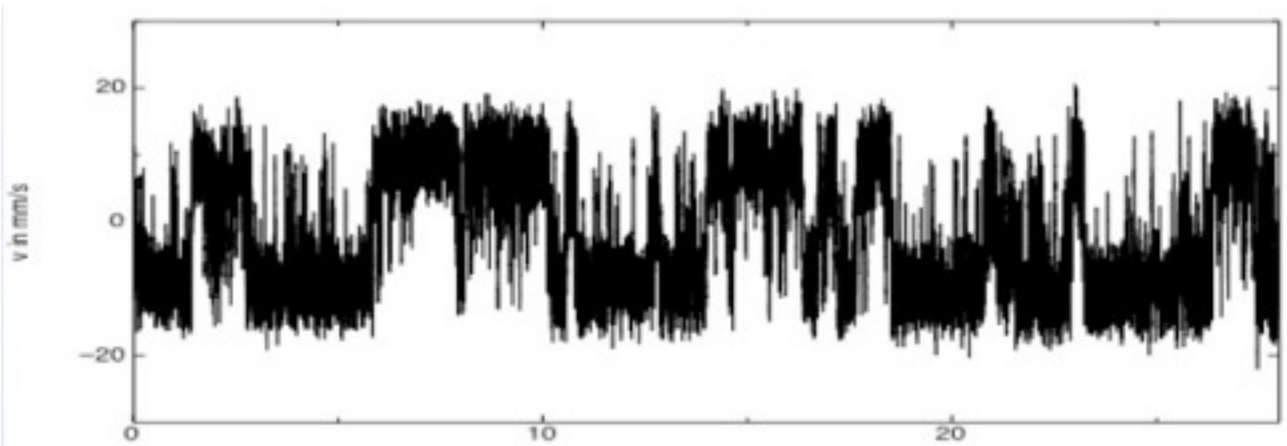
Potential

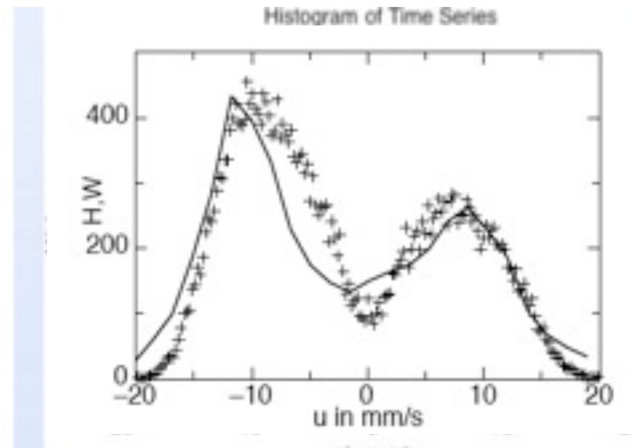
model for bistability



Sreenivasan K. R., Bershadskii A., and Niemela J. J., *Mean wind and its reversal in thermal convection*, Phys. Rev. E, 65:056306, 2002

Rayleigh Benard Experiment





PDF and its reconstruction

Ra = 10^{10}

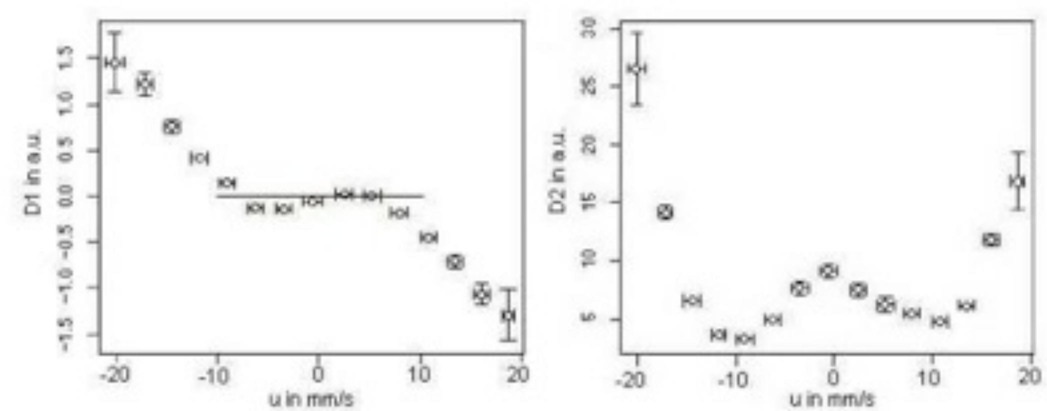


Fig. 4 Drift (left) and diffusion (right) coefficients $D1$ and $D2$, respectively; units are arbitrary.

Analysis as stochastic process in time - Langevin Equation

$$\dot{\mathbf{x}}(t) = \mathbf{D}^{(1)}(\mathbf{x}(t_j), t_j) + \sqrt{\mathbf{D}^{(2)}(\mathbf{x}(t_j)}\mathbf{\Gamma}'(t_j)$$

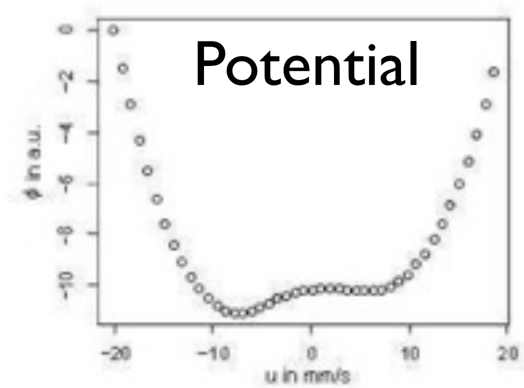
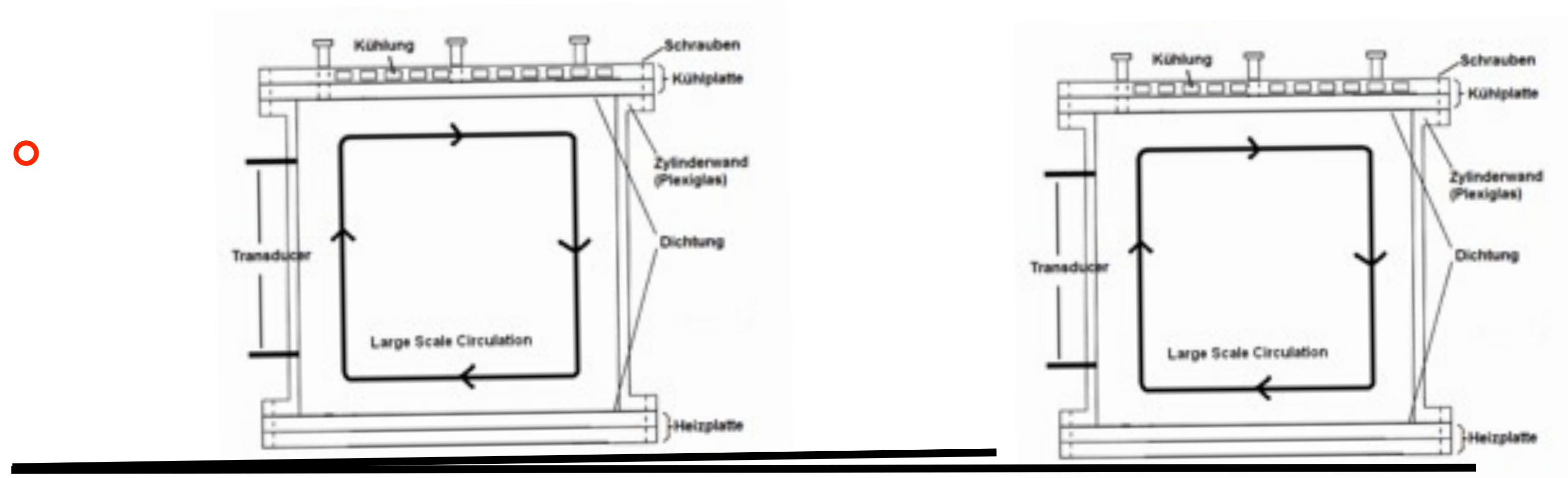
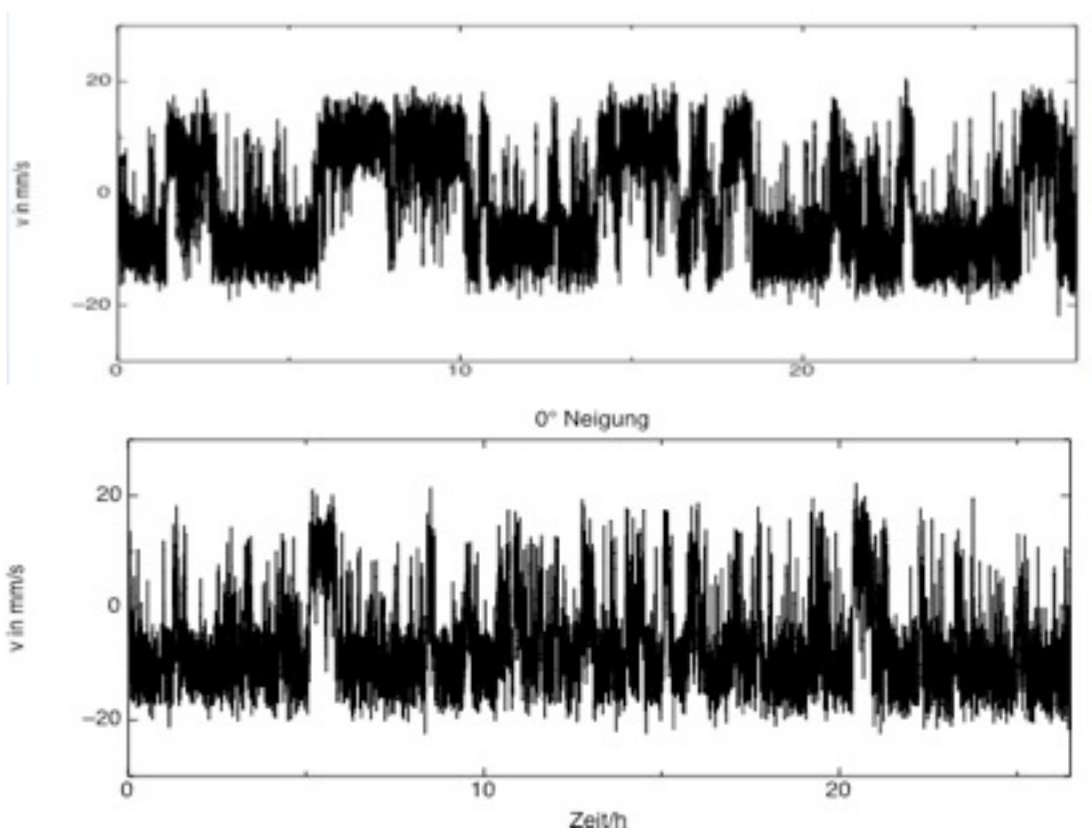


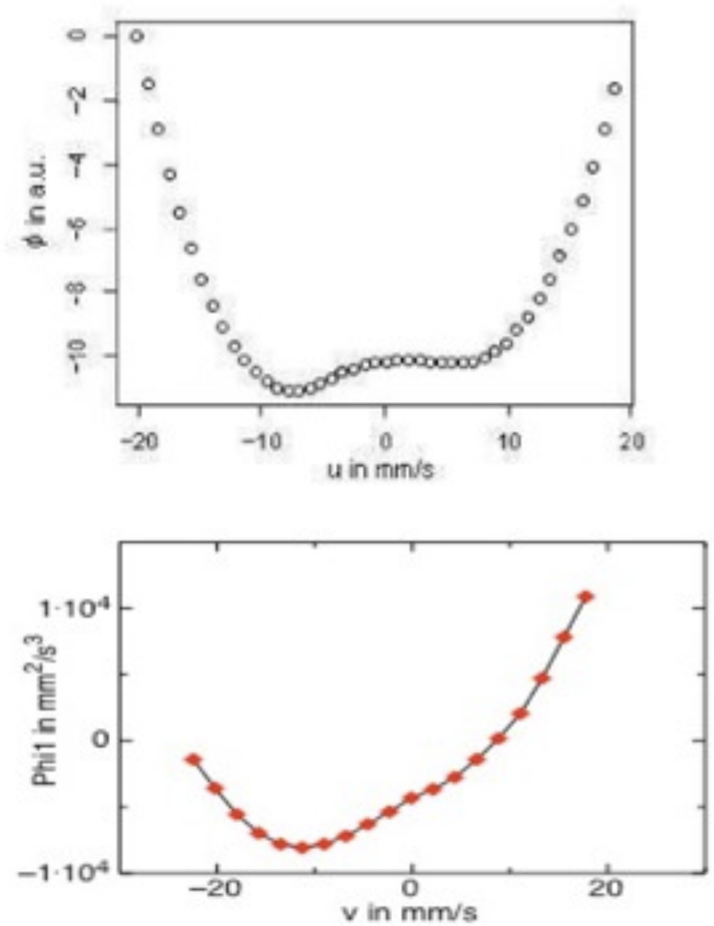
Fig. 5 Potential $\Phi(u)$ as calculated from the drift coefficient $D1$.

Rayleigh Benard Experiment

tilting - by less than 1°





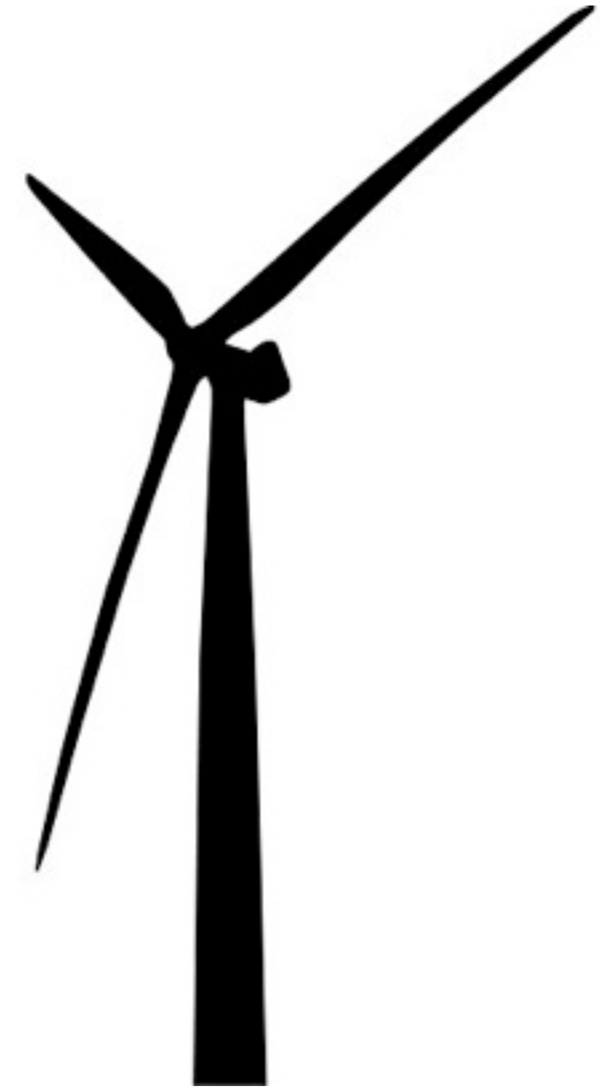


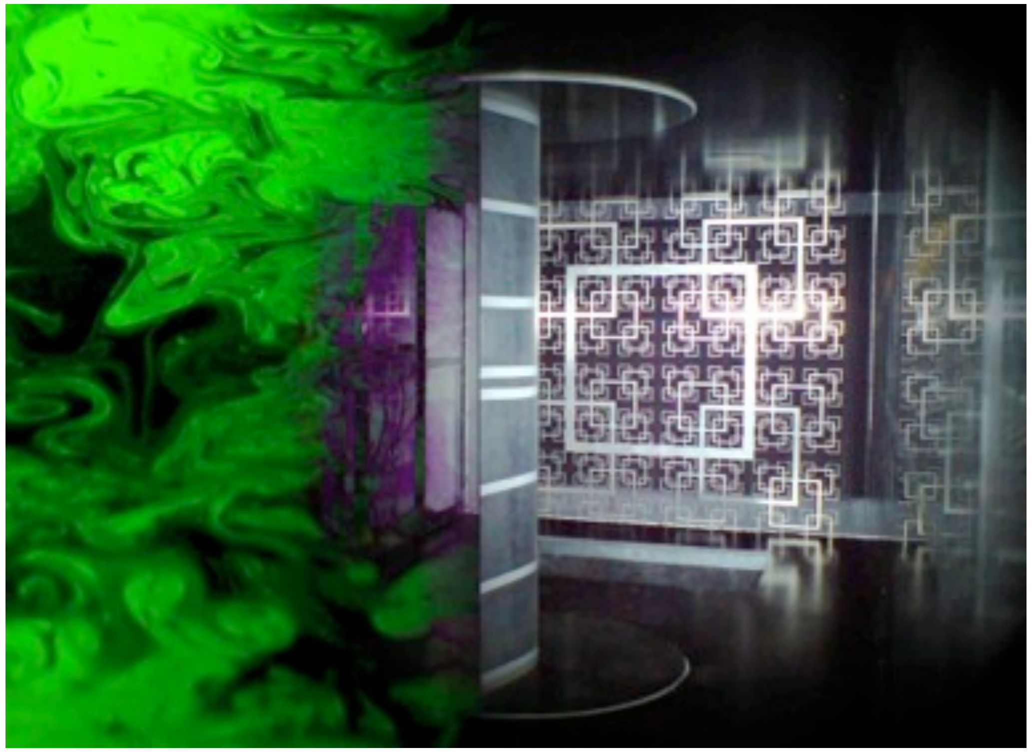
Potential

MA Thesis
M. Peters M. Langner

turbulent driven systems

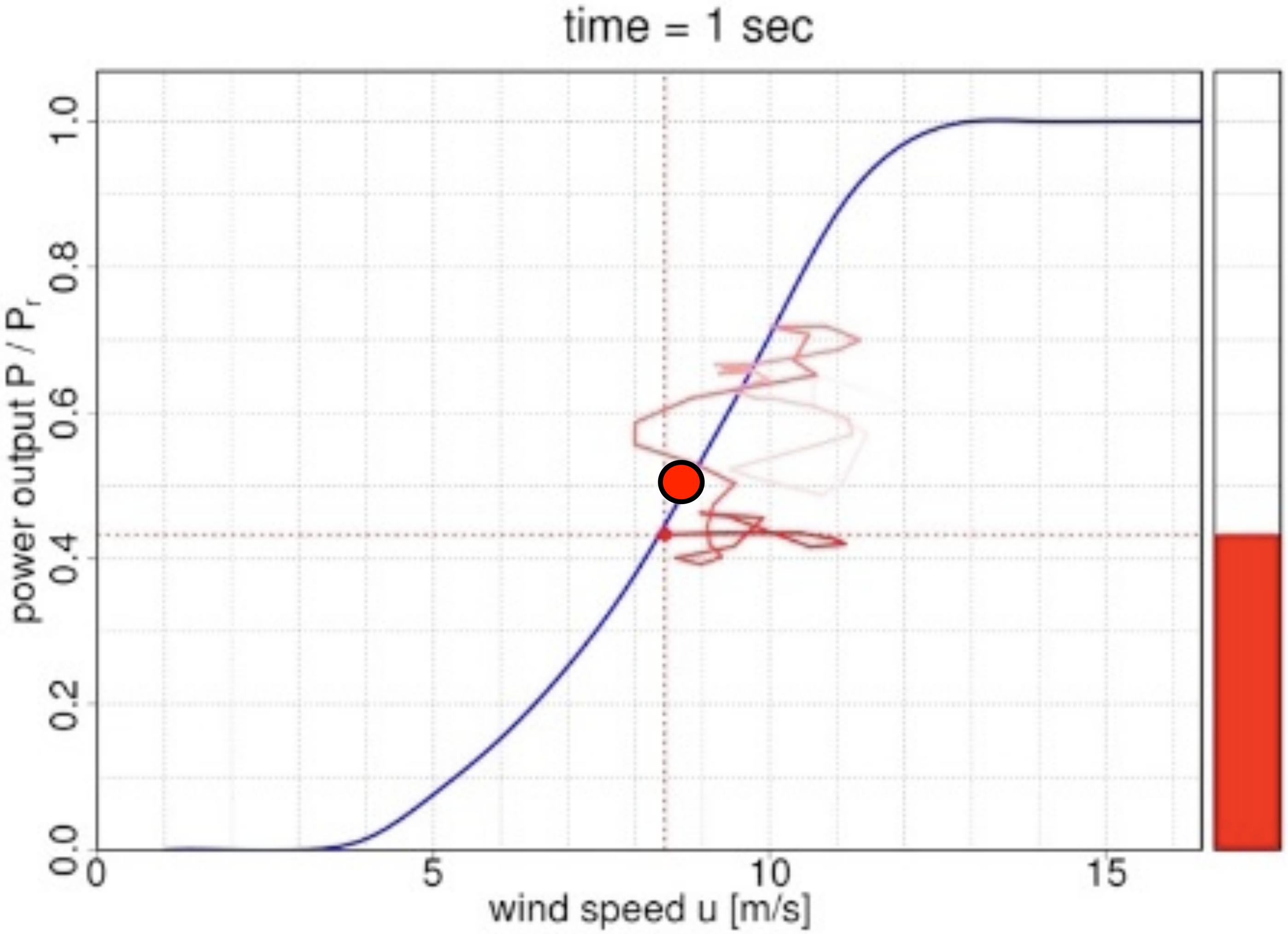
turbulent wind energy





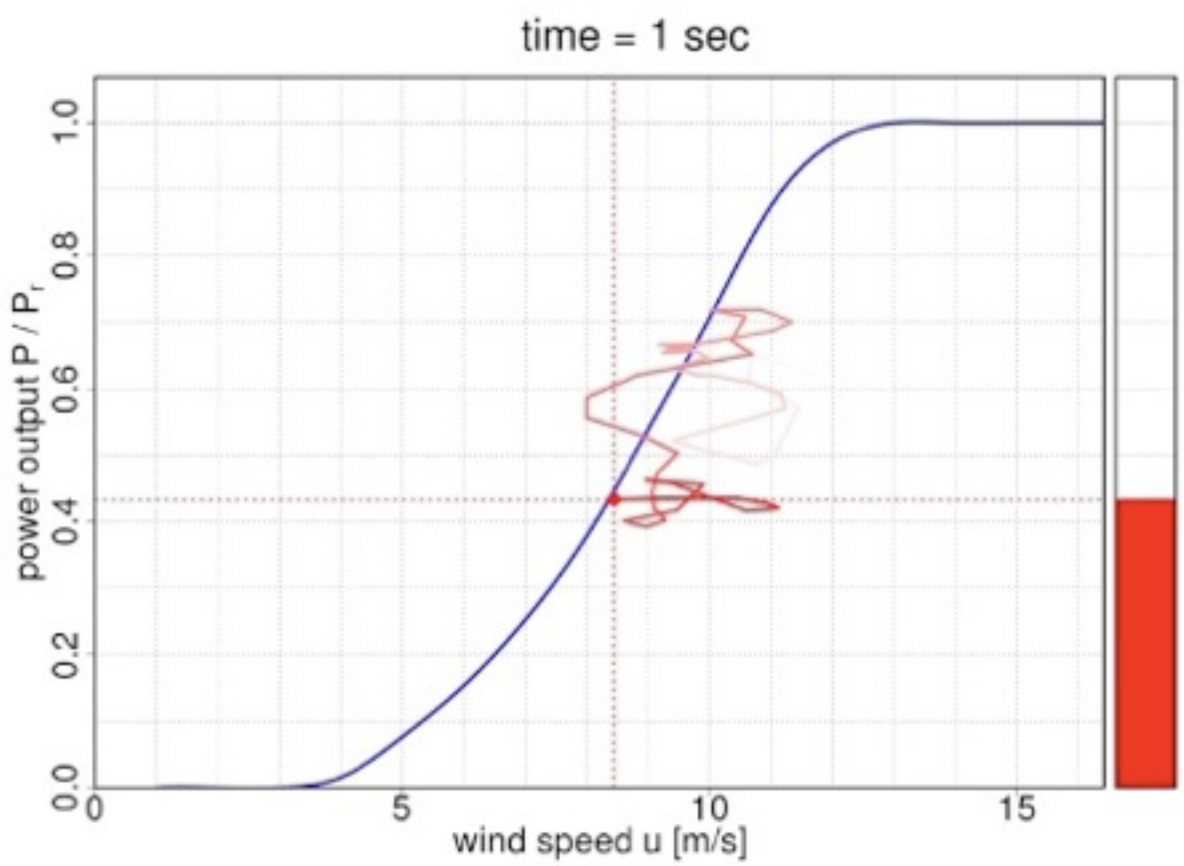
dynamics of power conversion

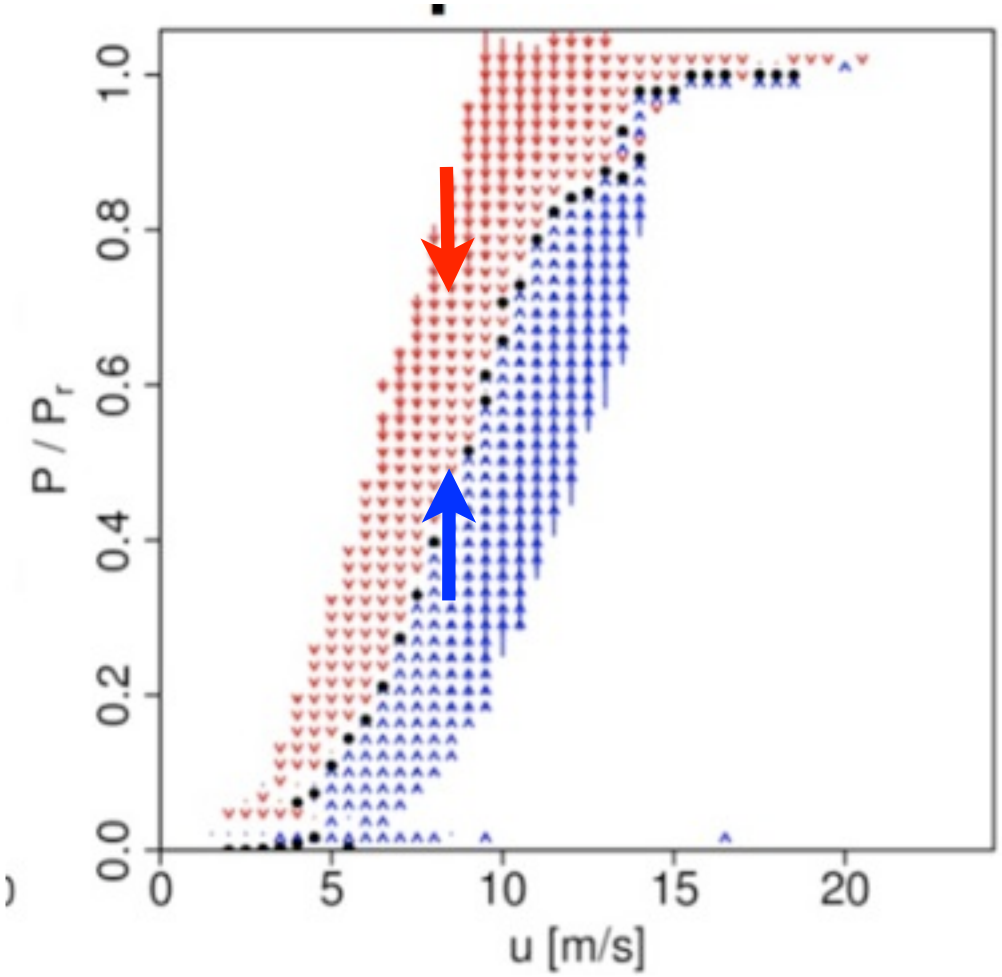
$$P_{WT} = \frac{1}{2} c_p(\lambda)\, \rho\, u_{wind}^3 \cdot A$$



working conditions for wind turbine

stochastic motion in a potential



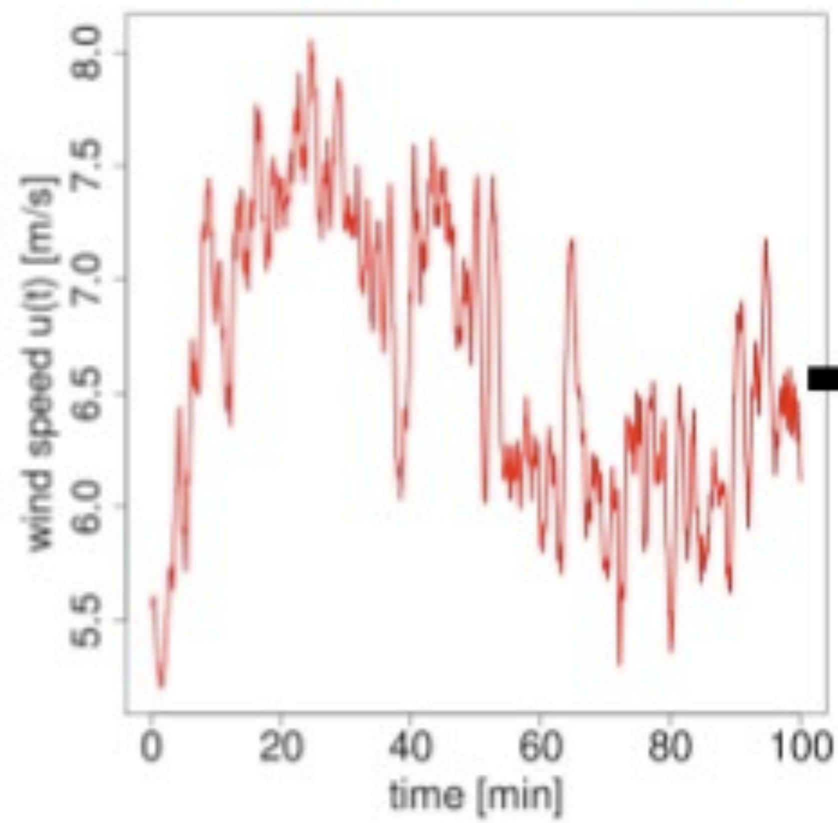


$$\dot{P} = D^{(1)}(P|u) + \sqrt{D^{(2)}(P|u)} \cdot \Gamma$$

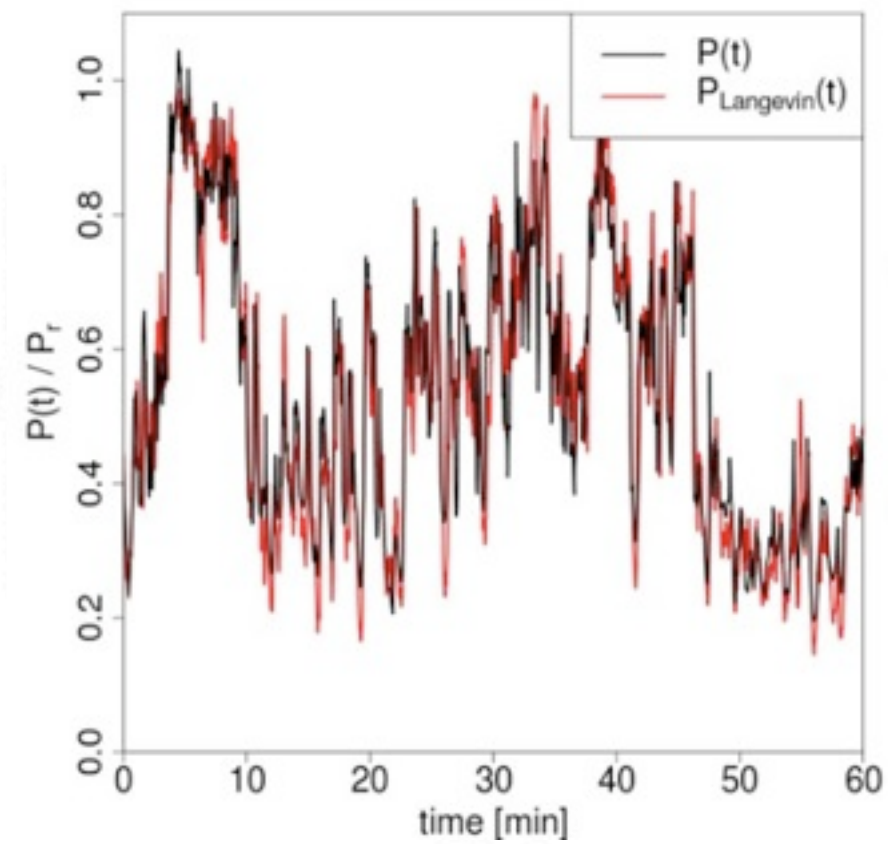
conversion of wind power a stoch. process

INPUT
wind speed **u(t)**

OUTPUT
power output **P(t)**



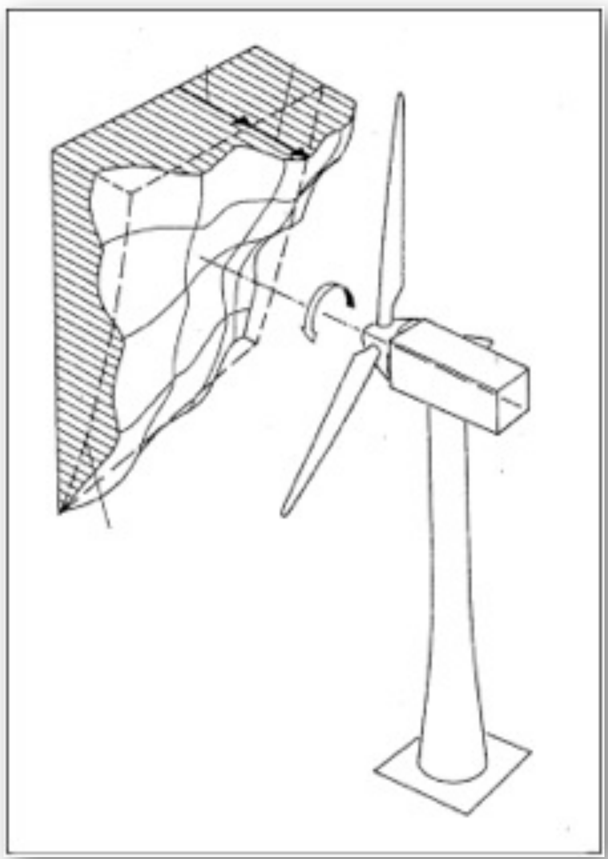
Source: D. Heißelmann

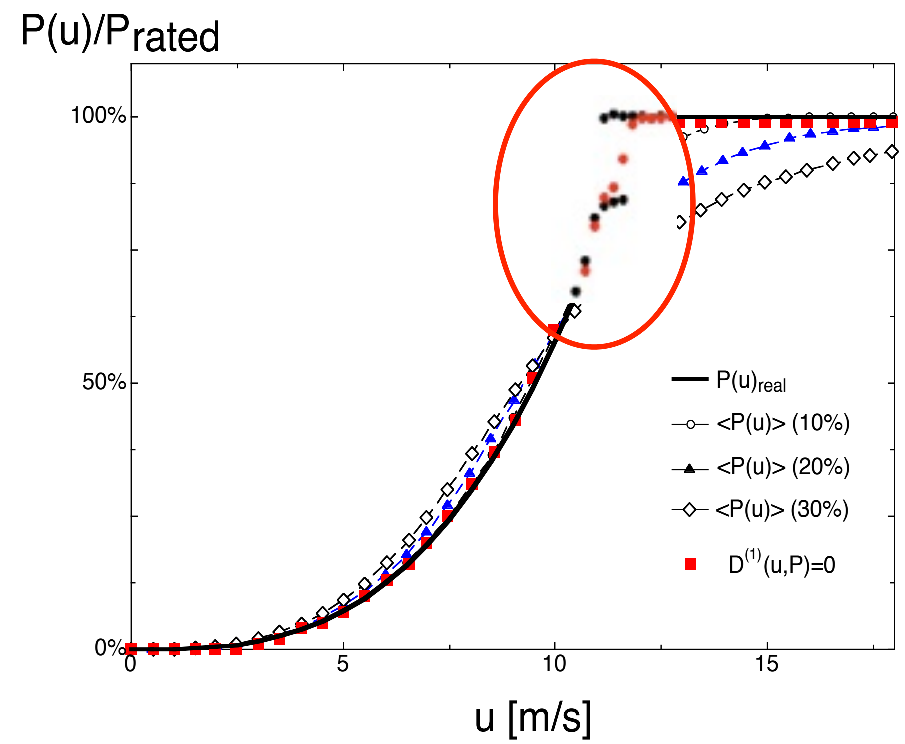


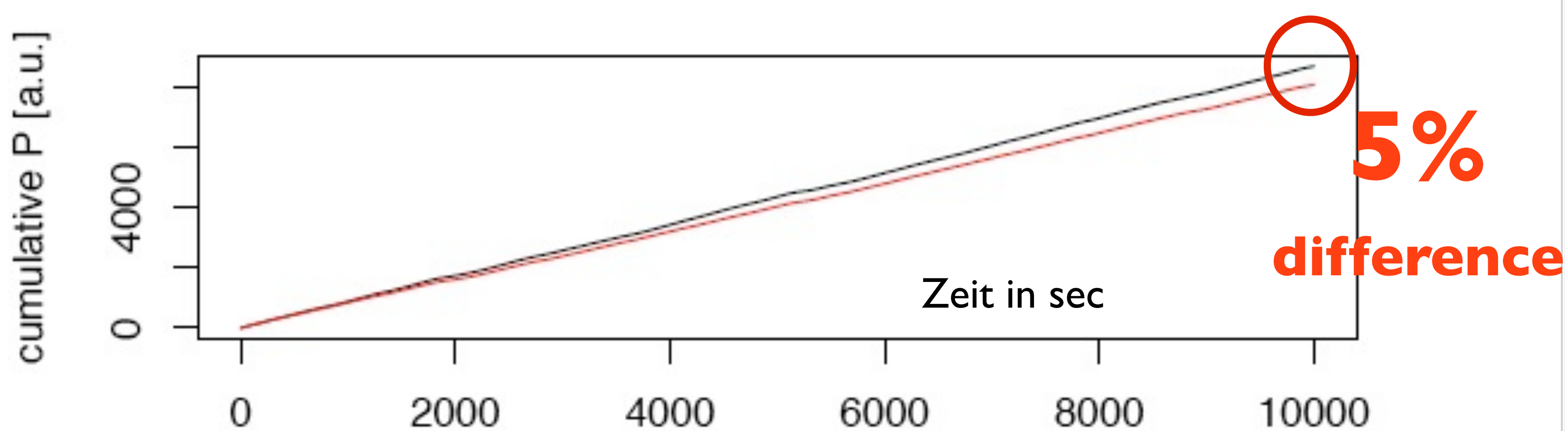
$$\dot{P} = D^{(1)}(P|u) + \sqrt{D^{(2)}(P|u)} \cdot \Gamma$$

power production

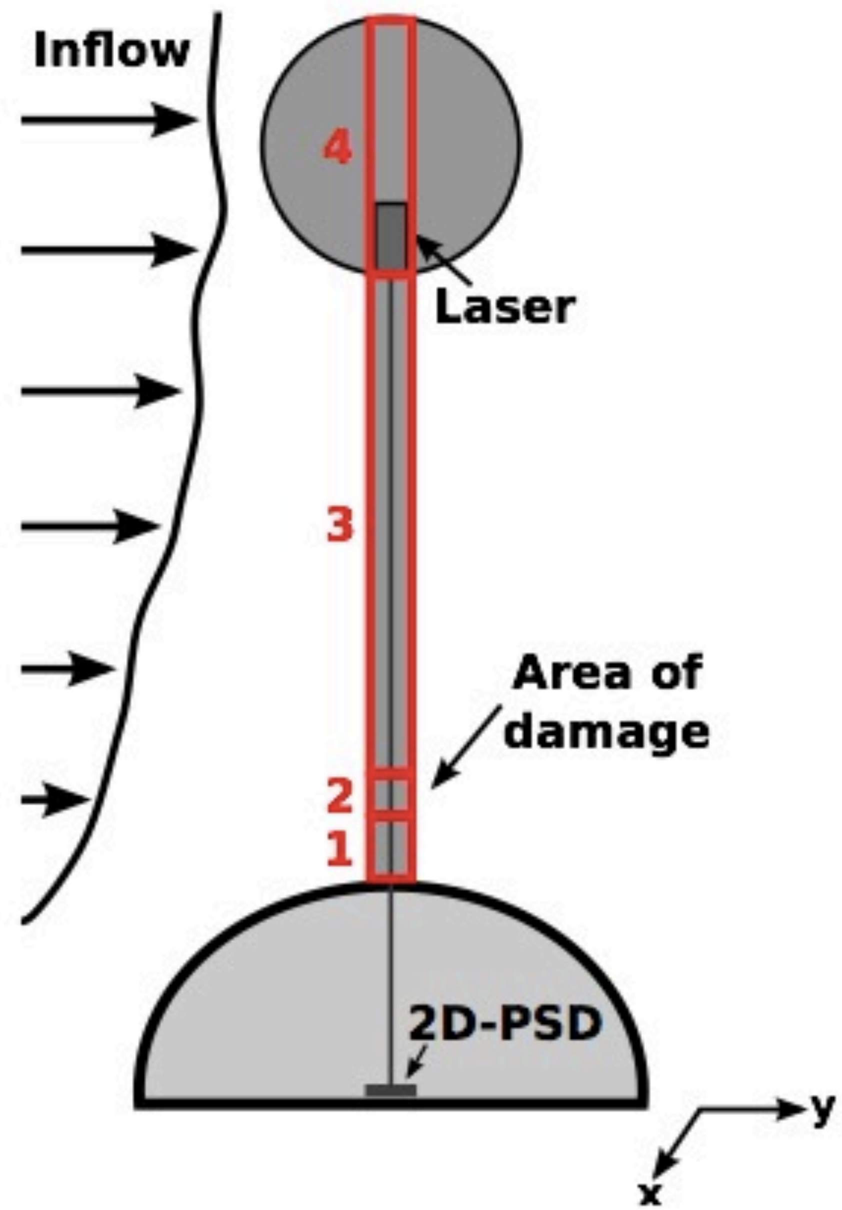
summed up difference in the power production with the same measured wind data as input







- Experimental set-up
  - Three configurations
    - undamaged
    - heated
    - cut on 40% of the circumference
  - Turbulent inflow conditions
- Numerical simulation: FEM model
  - Element 2 was "damaged"
    - Stiffness was reduced in 10% steps
    - Eight configurations: 0% to 70%

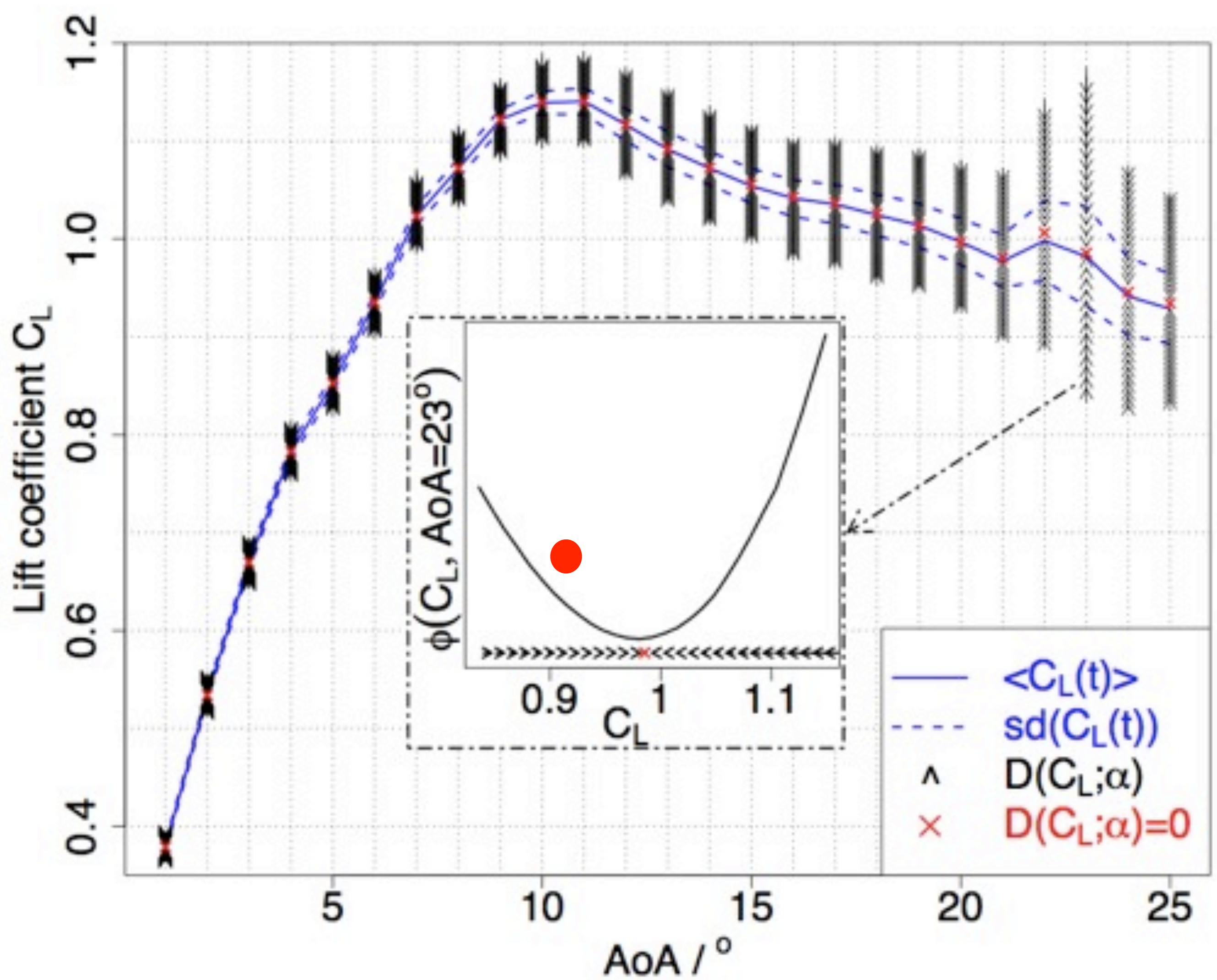


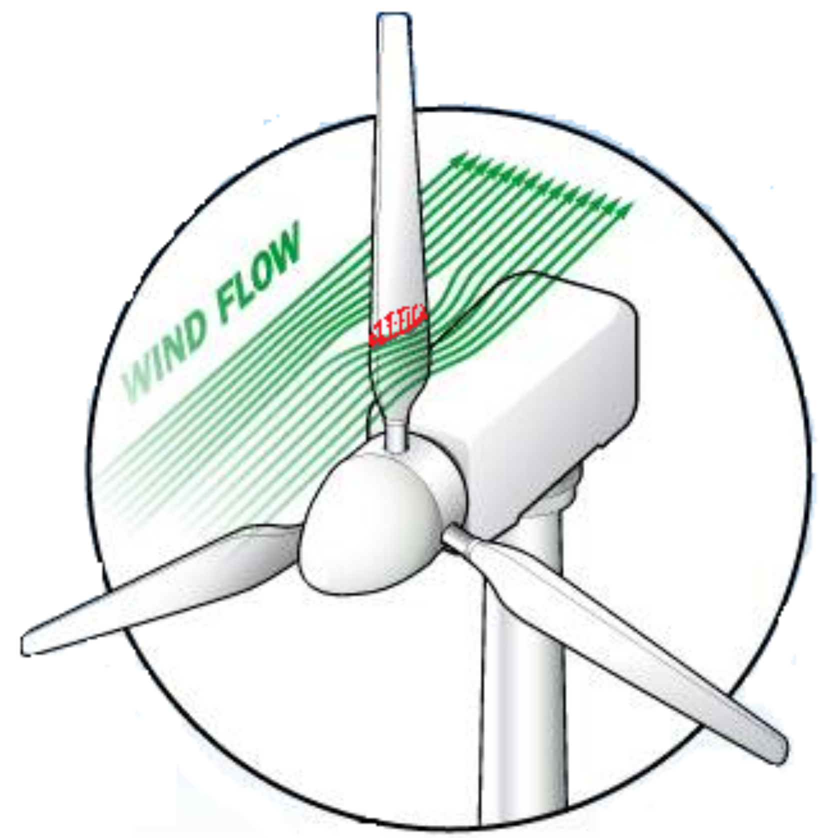
Deflection in *x*- and *y*-direction was measured

characterization of dynamic stall with turb inflow

$$\frac{dc_L(t)}{dt} = -\nabla\Phi(c_L(t), u) + g(c_L(t), u) \cdot \Gamma(t)$$

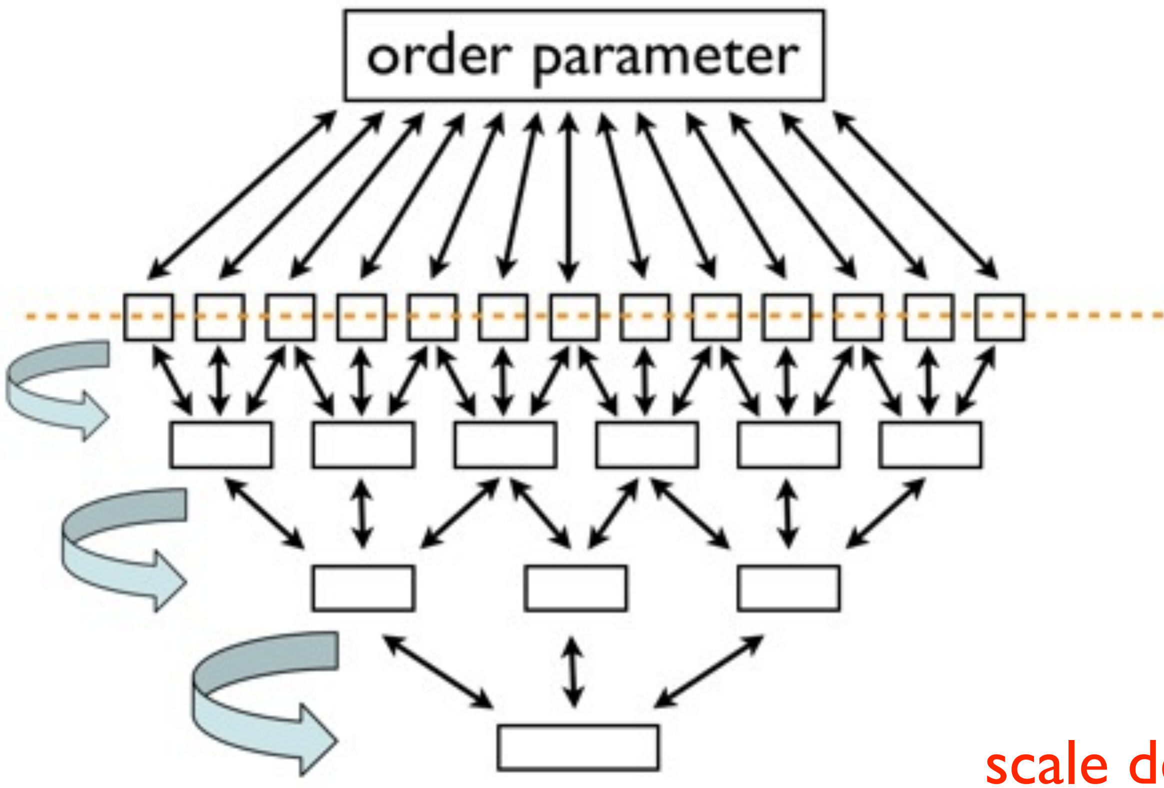
noise term





J. Schneemenn et al,
EWEC 2010;

synergetics for complexity



time dependent complexity

slaving to
low dimensional
dynamics

hierarchical
cascade structure--
allows this a
simplification, too

scale dependent complexity

- stochastic equation are measurable
  - comprehensive description of complex systems
  - deeper insights
  - high accuracy


and all best wishes