Quantum Thermodynamics: A Case Study for Emergent Behavior

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Overview

1. Synerg. & q-thermo
2. Contextuality
3. Hilbert-space
4. HAM
5. Thermal properties
6. A complete H-model
7. Thermalization-dyn.
8. Environment under observation
9. Summary

1. Synergetics & q-thermodyn.

Methods – Contents – **Concepts**

- Slow/fast variables
- Control parameters
- Micro/macro-description
- Contextuality
- Coarse-graining
- Effective description
- Emergence
- Universality

2. Contextual quantum description

Pure state: vector in $\mathcal{H}$
Closed system: H-model
Unitary dynamics

Abstract quantum description

Operational aspects
Partition: classical index; indistinguishable particles; frozen-in structure
Q-observables: select/register; back-action (measurement)

Partitioned state
System + environment
Effective dynamics

Contextual quantum description

3. Hilbert-space: a. Partitioning

Virtual partitioning: Prime factorization (d = dim., q = prime)

$$d = \prod_{j=1}^{r} q_j^{n_j} \qquad N_d = \sum_{j=1}^{r} n_j$$

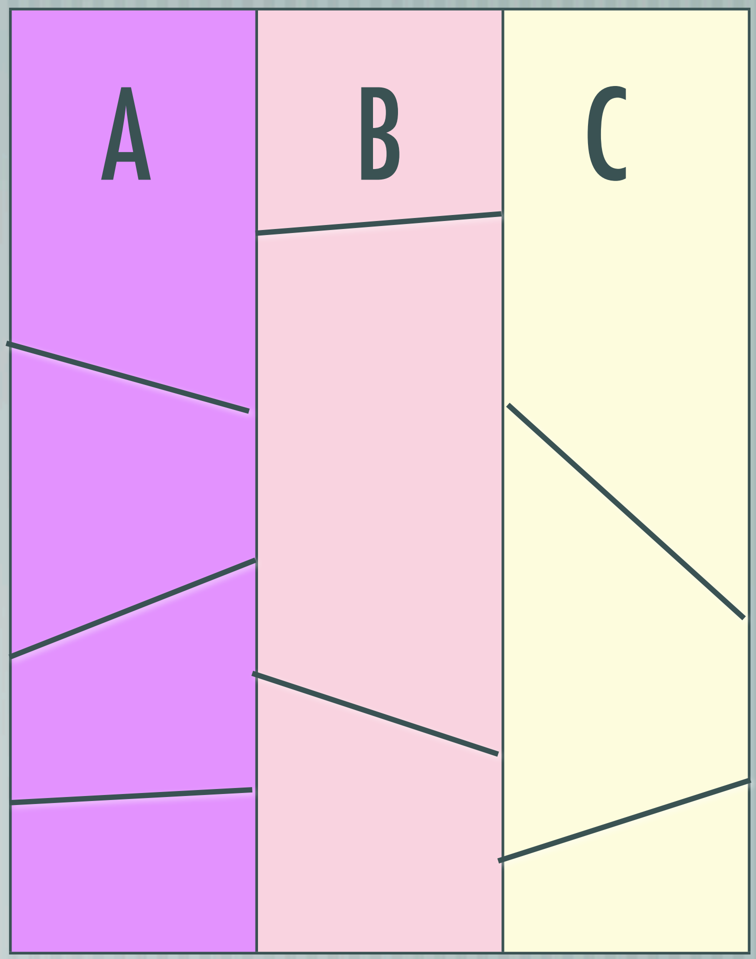
Operationally accessible Partitioning:

Number $N \ll N_d$

Classical index (no superposition): $\mu = A, B, C, \ldots$

Example N = 3: Alternative perspectives

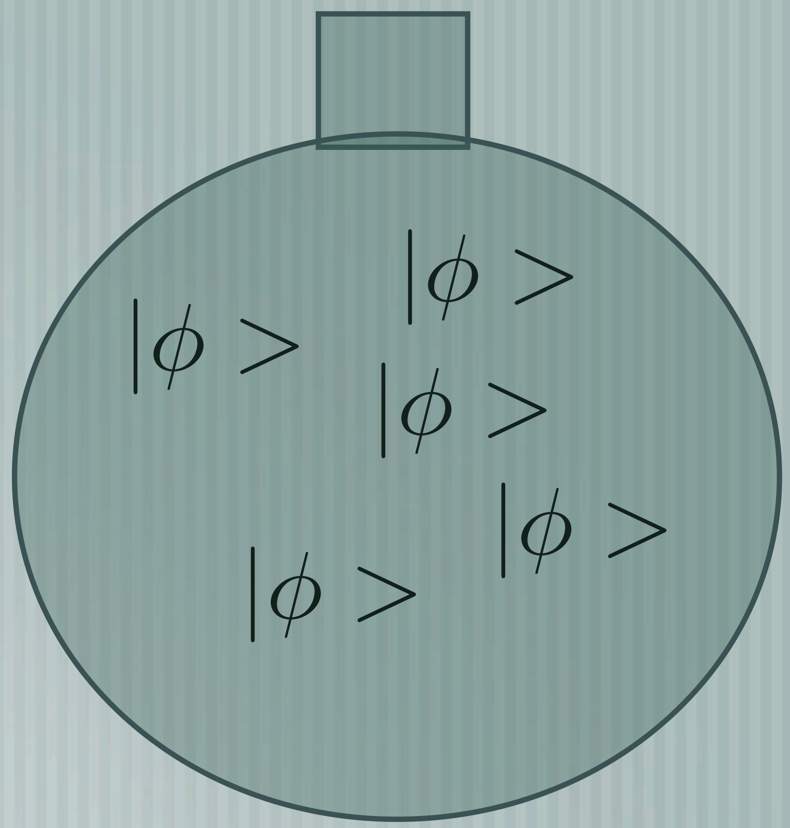
N=1: (ABC) N=2: (AB)C N=3: (A)(B)(C)



3. Hilbert-space: b. Statistics

Parametrization:

$$|\phi >= \sum_{j=1}^{d} (\eta_j + i\xi_j)|j >$$



$$w(\eta_1, \xi_1, \ldots \eta_d, \xi_d) = \left(\frac{1}{\sqrt{\pi\gamma}}\right)^{2d} \exp\left(-G(\eta_1, \xi_1, \ldots \eta_d, \xi_d)/\gamma\right)$$

$$G(\eta_1, \xi_1, \eta_2, \xi_2, \ldots \eta_d, \xi_d) = \sum_{j=1}^{d} (\eta_j^2 + \xi_j^2) \quad \gamma = 1/d$$

Hilbert-space function:

$$h(|\phi >) = h(\eta_1, \xi_1, \ldots \eta_d, \xi_d)$$

$$w(h) = \int w(\eta_1, \xi_1, \ldots \eta_d, \xi_d)\delta(h - h(\eta_1, \xi_1, \ldots \eta_d, \xi_d))d^d\eta d^d\xi$$

$$\overline{h} = \int w(\eta_1, \xi_1, \ldots \eta_d, \xi_d)h(\eta_1, \xi_1, \ldots \eta_d, \xi_d)d^d\eta d^d\xi$$

3. Hilbert-space: c. Landscapes

h - Landscapes: „Netherlands" or „Switzerland"?

$$\Delta^2(h) = \overline{h^2} - (\overline{h})^2$$

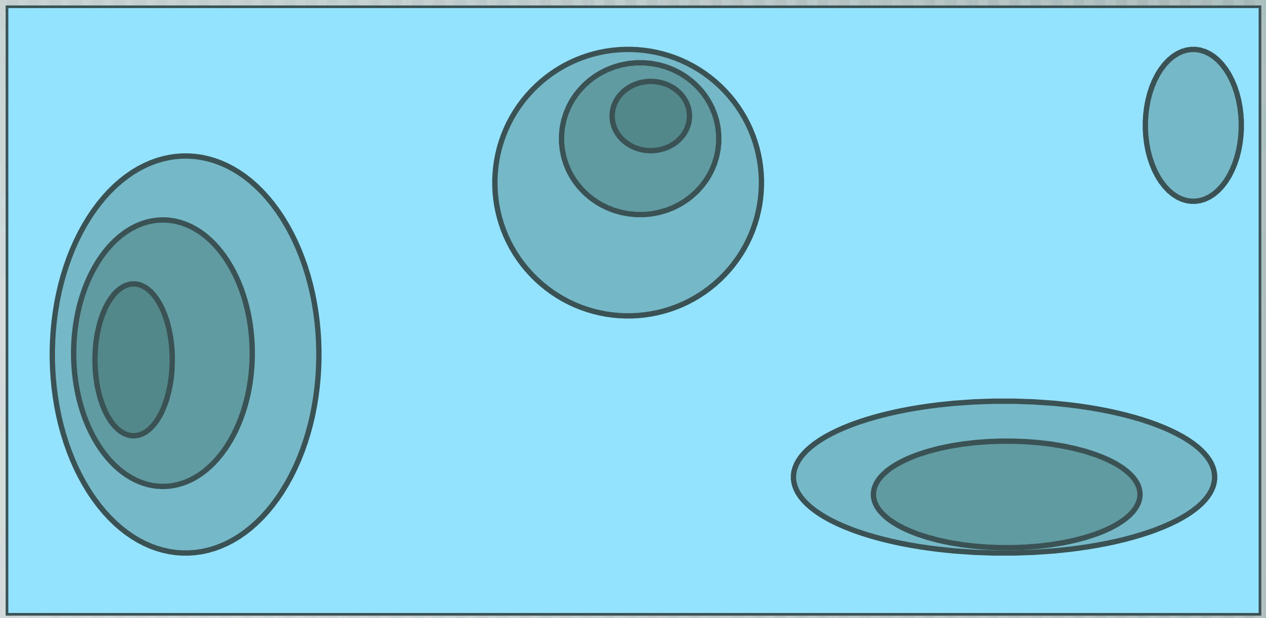
$$\frac{\Delta(h)}{\overline{h}} \ll 1$$

Hilbert-space average method (HAM): Good approximation in flatland:

$$h(\eta_1, \xi_1, \ldots \eta_d, \xi_d) \rightarrow \overline{h}$$

= typical for randomly chosen position

d = 2



4. HAM: a. for state measures

Reduced state:

$$\hat{\rho}(A) = \mathtt{Tr}_B\{\hat{\rho}(AB)\}$$

$$S_{vN}(\nu) = -\mathtt{Tr}\{\hat{\rho}(\nu)\ln\hat{\rho}(\nu)\} \quad \nu = A, B$$

Bi-partite system:

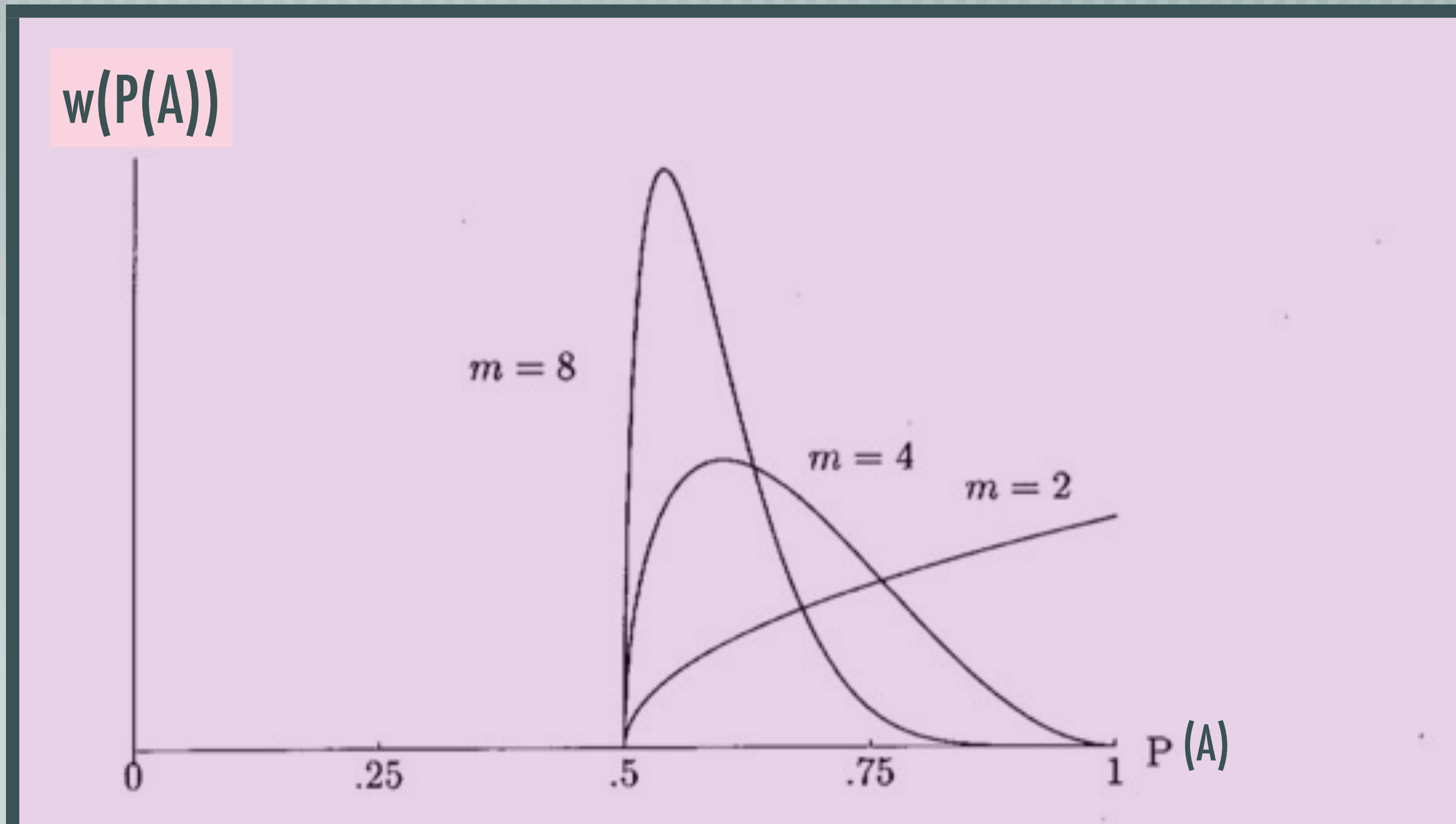
$$d = d(A) \cdot d(B)$$

i.) $h = S_{vN}(A) \rightarrow$

$$\overline{S}_{vN}(A) \approx \ln d(A) - \frac{d(A)}{2d(B)}$$

ii.) $P(\nu) = \mathtt{Tr}\{\hat{\rho}^2(\nu)\}$

w(P(A)) Purity distribution:



d(A) = 2   m = d(B)

4. HAM: b. for observables

Consider operator $\hat{F} = \sum_k F_k |f^{(k)} >< f^{(k)}|$

$$\overline{h} = \overline{< \phi|\hat{F}|\phi >} = \overline{< F >} = \sum_j F_j (\overline{\eta_j^2} + \overline{\xi_j^2}) = \frac{1}{d}\mathtt{Tr}\{\hat{F}\}$$

$$\Delta^2(< F >) = \frac{1}{d+1}\left(\mathtt{Tr}\{\hat{F}^2\}/d - \mathtt{Tr}\{\hat{F}\}^2/d^2\right)$$

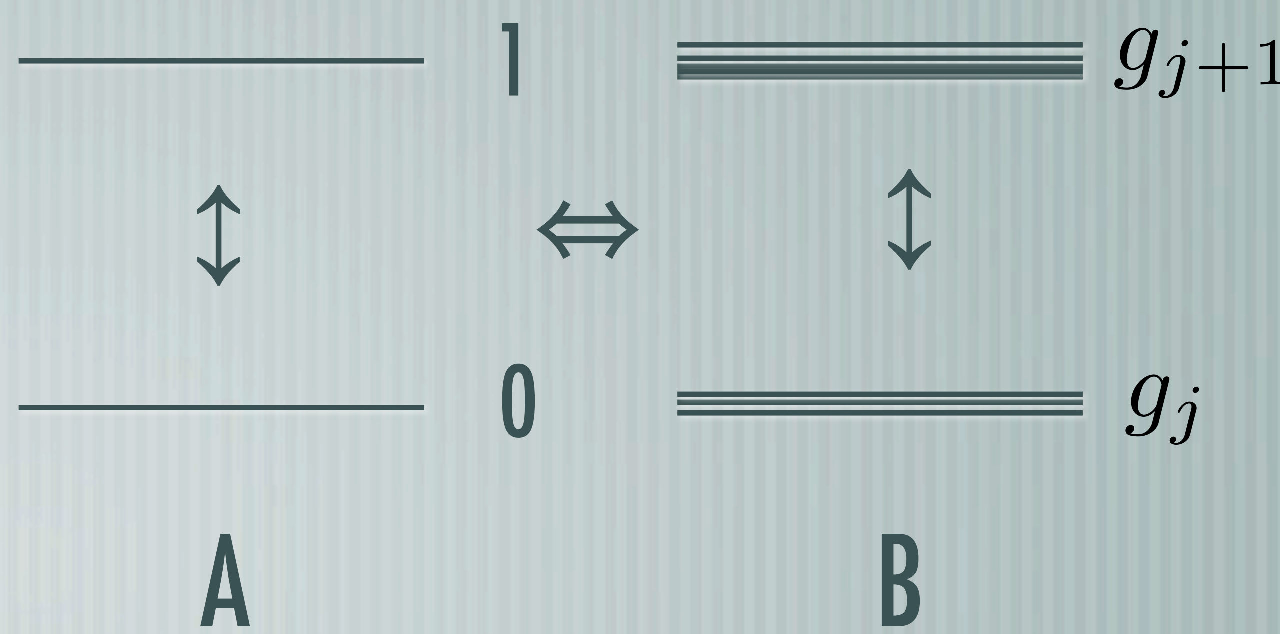
5. Thermal prop. : a. Constraints

A = Spin; B = environment

Bi-partite system with energy exchange: Total energy $\Delta E = E_1 - E_0$

$$\overline{< P_0 >}_j = \frac{\texttt{Tr}\{\hat{P}_0\}}{d(B)} = \frac{g_{j+1}}{g_j + g_{j+1}}$$

$$\overline{< P_1 >}_j = \frac{\texttt{Tr}\{\hat{P}_1\}}{d(B)} = \frac{g_j}{g_j + g_{j+1}}$$



5. Therm. prop.: b. Temperature

$$\frac{\Delta E}{k_B T(A)} = \ln\left(\frac{\overline{< P_0 >_j}}{\overline{< P_1 >_j}}\right) = \ln\left(\frac{g_{j+1}}{g_j}\right)$$

$$g_j = g_{j+1} \qquad k_B T(A) = \infty$$

$$g_j \ll g_{j+1} \qquad k_B T(A) = 0$$

Thermal averaging over the environmental states of B: $E_j = j\Delta E$

$$\hat{\rho}_{equ}(A) = \frac{1}{Z(B)} \sum_j g_j \exp\left(-\beta(B) E_j\right) \times$$

$$\{\overline{< P_0 >}_j |0(A) >< 0(A)| + \overline{< P_1 >}_j |1(A) >< 1(A)|\}$$

$$g_j = \binom{n}{j} = \frac{n!}{(n-j)!j!}$$

$$n \to \infty \qquad \beta(A) \approx \beta(B)$$

6. A concrete Hamilton-model

$$\hat{H}(AB) = \frac{\delta(A)}{2}\hat{\sigma}_3(A) + \hat{H}_0(B) + \hat{V}(BB) + \lambda\hat{V}(AB)$$

Environment B: n spins, weak mutual interaction; binomial degeneracy:

$$E_k = k \cdot \delta(B)$$

Choose k_0

$$g_k \approx g_0 \exp\left(\beta(k_0)E_k\right)$$

$$\beta(k_0) \approx \frac{1}{\delta(B)} \ln\left(\frac{n}{k_0} - 1\right)$$

System-environment A-B:

$$\hat{V}(AB) = \hat{\sigma}_1(A) \otimes \hat{I}(AB)$$

$$\hat{I}(AB) = \sum_k \sum_{n_k, m_{k+1}} C_{k+1,k}(n_k, m_{k+1}) |n_k> < m_{k+1}| + c.c.$$

$$\hat{\rho}(AB) = \hat{\rho}(A) \otimes \hat{\rho}(B) - \hat{C}(AB)$$

7. Thermalization - dynamics

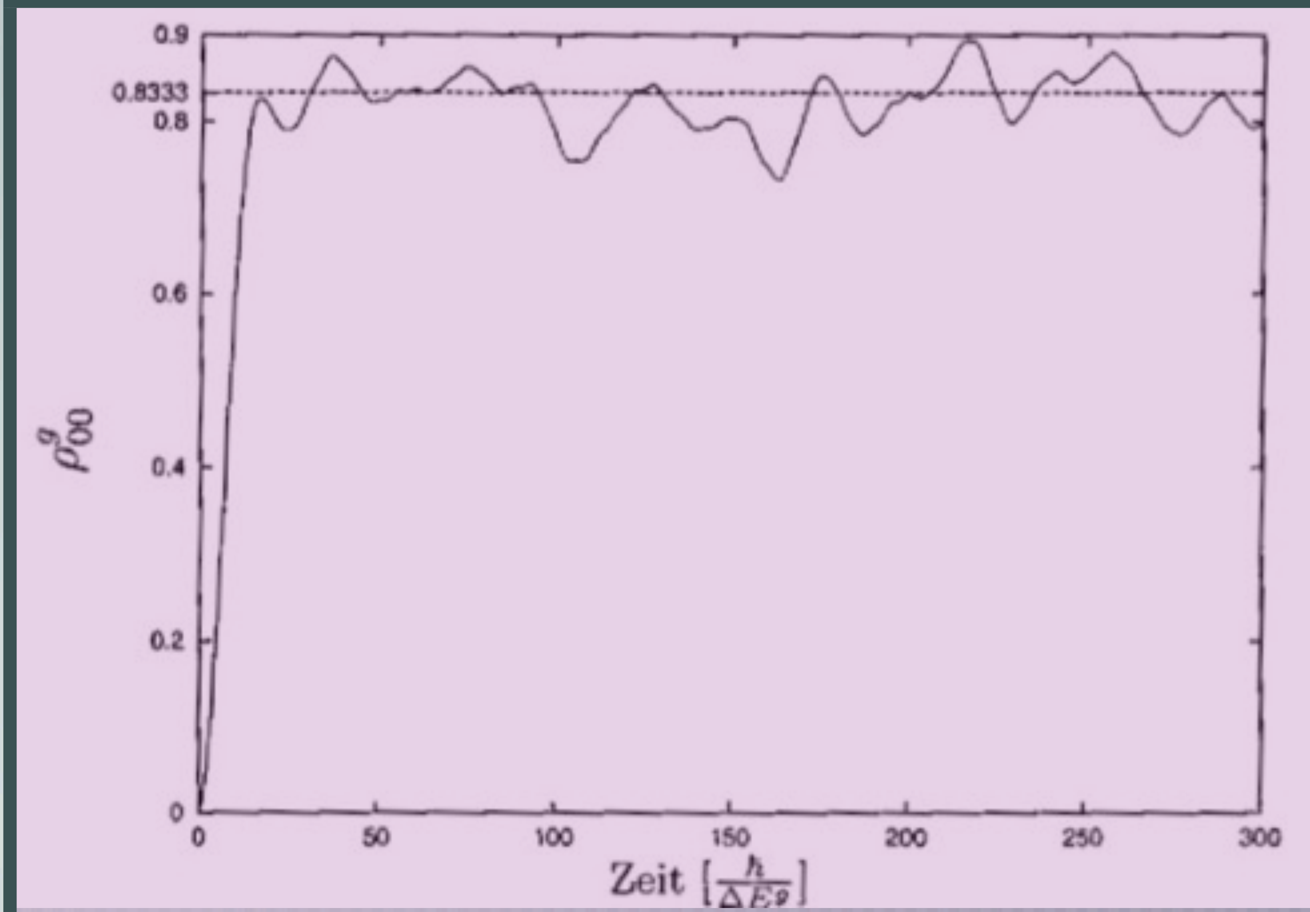
$$\hat{\rho}(AB;t_0) = \hat{\rho}(A;t_0) \otimes \hat{\rho}(B;k_0)$$

$$\hat{\rho}(B;k_0) \equiv \frac{1}{g_{k_0}} \sum_{n_{k_0}=1}^{g_{k_0}} |n_{k_0} > < n_{k_0}|$$

$$\frac{1}{k_B T(A)} = \beta(A) = \beta(k_0)$$

apparently irreversible

$t > t_0$: correlations built up



8. Environment under observation

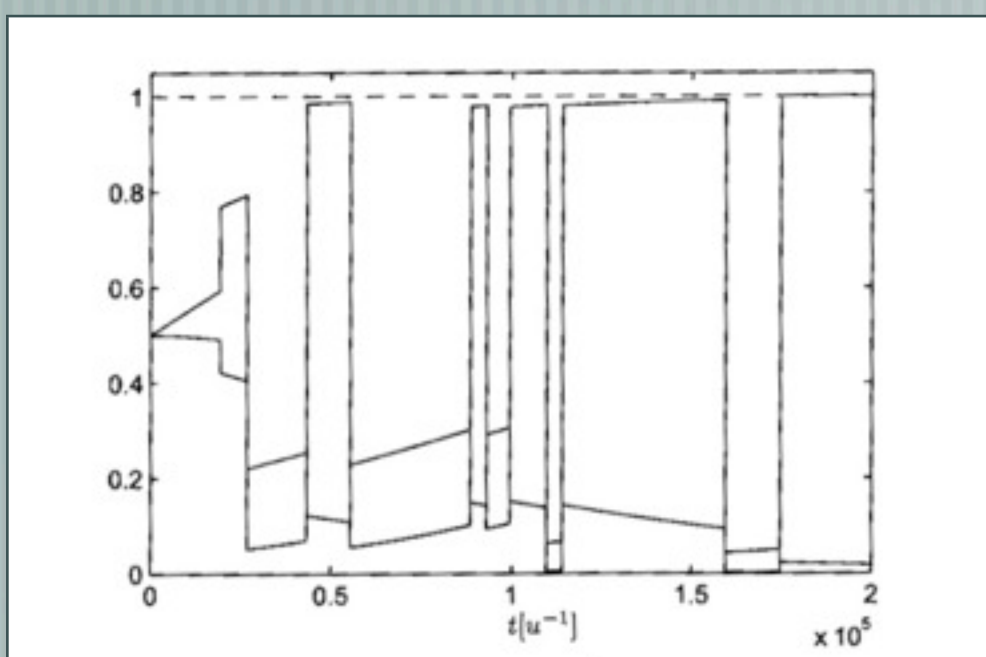
Periodic measurements:

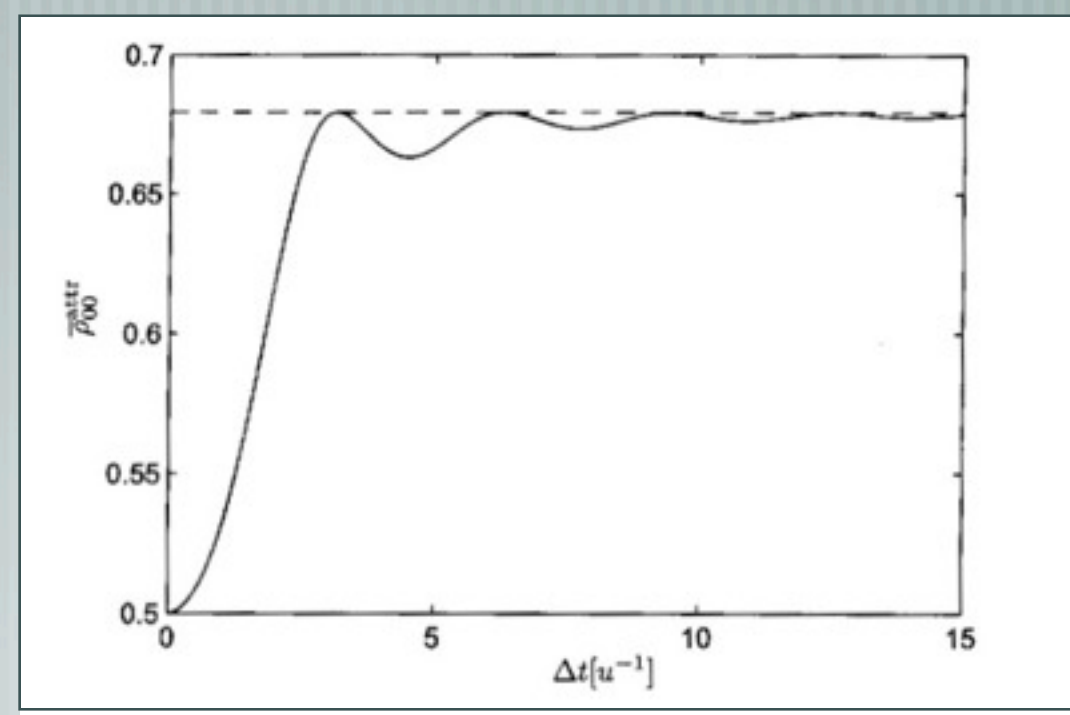
$$t_1 = t_0 + \Delta t$$

$$\hat{\rho}(AB; t_1) \approx \hat{\rho}'(A; t_1) \otimes \hat{\rho}(B; k_1)$$

$$k_1 = \{k_0 + 1, k_0, k_0 - 1\}$$

Attractor for A:





Pure-state trajectory for A

$$\lim_{\Delta t \to 0} \bar{\rho}_{00}(\Delta t) = 1/2 \qquad T(A) = \infty$$

$$\Delta t = \frac{n\pi}{\delta} \quad n = 1, 2, \ldots \qquad T(A) = 1/\beta(k_0)$$

9. Summary

1. Partition as reference frame ($\rightarrow$ classical indices)
2. Hilbert-space statistics $\rightarrow$ typicality (landscapes)
3. Entanglement typical (for pure states in bi-partite System)
4. Accessible region constrained by H-model (conservation laws)
5. Fundamental dynamics $\rightarrow$ effective dynamics (irreversible)
6. Thermalizing environment: Embedding temperature
7. Observation = link between class/qm-description