



On the role of the space-time structure of couplings in synergetic networks

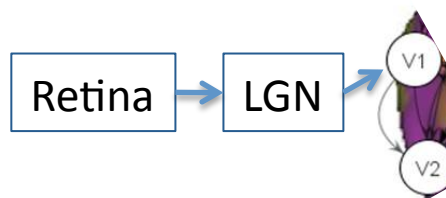
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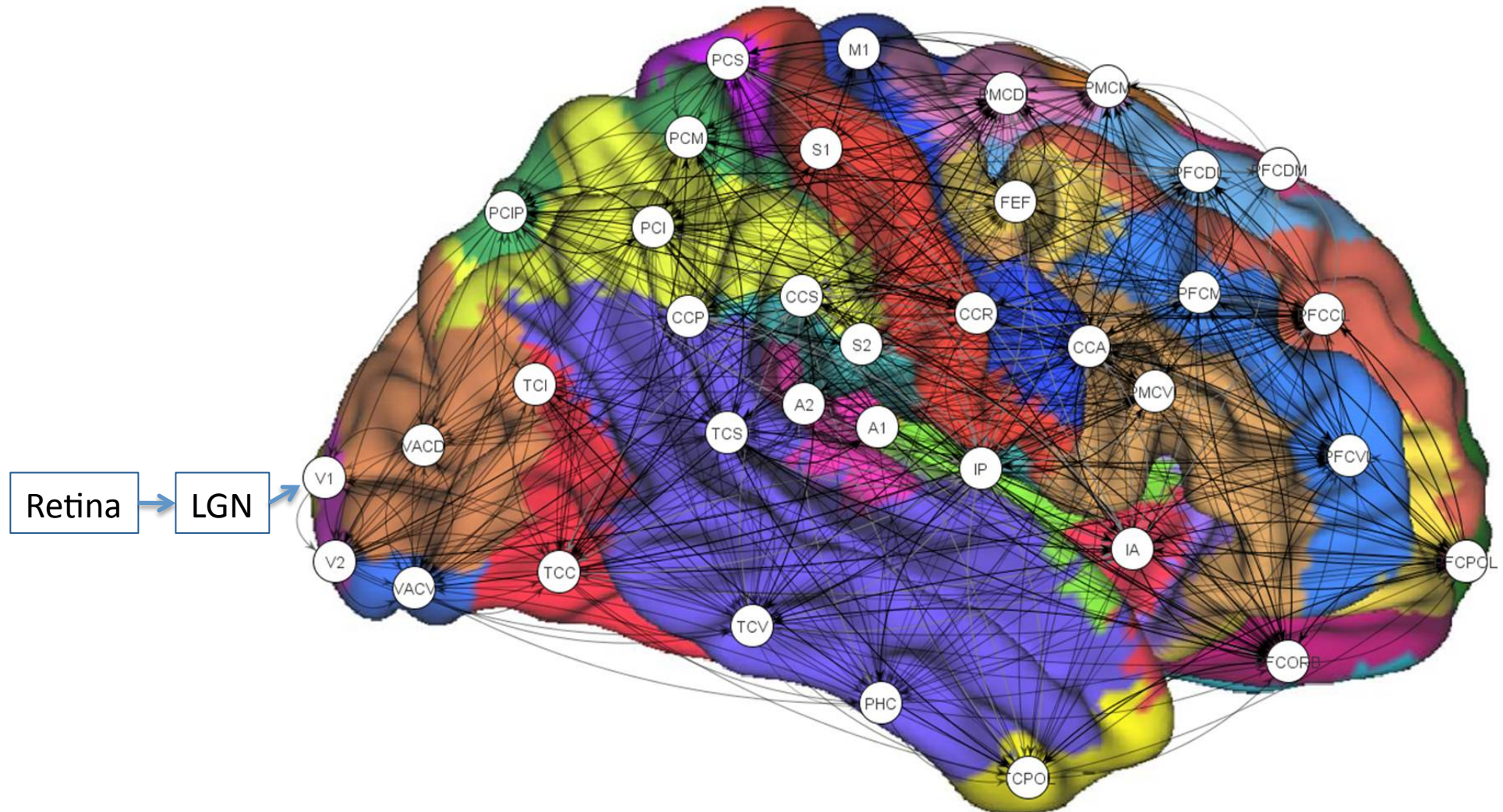
What is a full-brain approach?

Use of full connectivity matrix





Half of all connections in a given area is intracortical, the other half is corticocortical (Braitenberg & Schütz). To close the functional circuits in the brain, a full brain approach is needed.



Anatomical connectivity is a good predictor of functional connectivity.

The inverse is not true.

Line of thoughts on connectivity... a history

Nunez 1974

Jirsa-Haken PRL1996,1997,1998

Robinson et al. PRE1997,1998,2000

Hutt et al 2004, 2005

Coombes PhysicaD2010

Homogeneous approximation/assumption
on Connectivity

Review

Jirsa & Kelso PRE2000

Jirsa et al IEEE2002

Two point connection

Full brain system dynamics

Kötter & Wanke Neuroinformatics 2005

Cocomac

Ghosh et al PLoS CB2008

Cocomac based full brain dynamics at rest

Honey et al. PNAS2007

Izhikevich et al PNAS2008

Deco, Jirsa, et al PNAS2009

Bojak et al 2009

Pinotsis, Friston, Jirsa Neuroimage 2012

Deco, Jirsa, McIntosh Nat Rev Neuro 2011

“Resting state dynamics”

Review

Jirsa et al 2010; McIntosh et al. 2010

Deco & Jirsa JNS 2012

SfN 2012

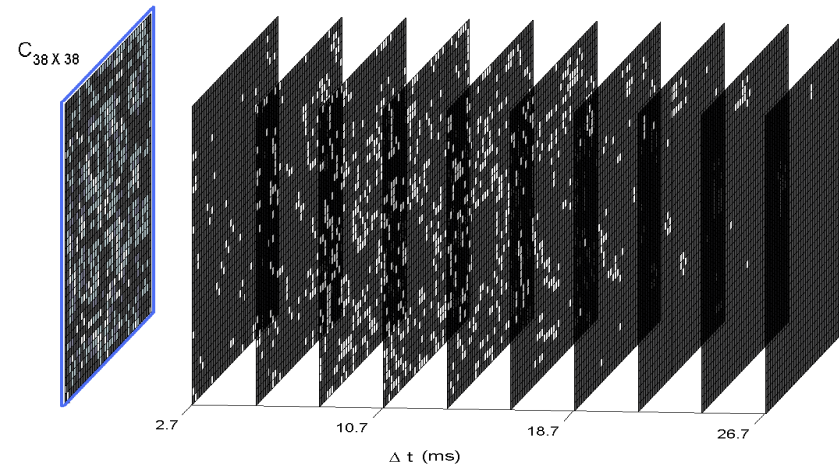
The Virtual Brain

Validation – resting state is critical

TVB release



Space-time structure of couplings



$$\dot{x}_i = -x_i + \sum_j c_{ij} x_j(t - \tau_{ij})$$

Networks with time delays

$$\dot{x}_i = -x_i + \sum_j c_{ij} x_j(t - \tau_{ij})$$

- *Networks with various connectivity, but only one time delay*

$$\dot{x}_i = -x_i + \sum_j c_{ij} x_j(t - \tau)$$

Many papers (Yeung & Strogatz PRL 1998, Chen et al IEEE 2002, Jirsa & Ding PRL 2004, ...)
Nice review by SA Campbell in Handbook of Brain Connectivity (Jirsa & McIntosh, eds)
Handbook of Chaos and Control, Schöll & Schuster (eds) 2007

- *Networks with distributed time delay* $\dot{x}_i = -x_i + \sum_j \int c_{ij}(t - \tau) x_j(\tau - \tau_{ij}) d\tau$

distribution is in time. Atay PRL2003, Qubbaj & Jirsa PRL 2007, Physica D2009

- *Networks with two time delays*

Shayer & Campbell SIAM 2000, Matsumoto et al 2011

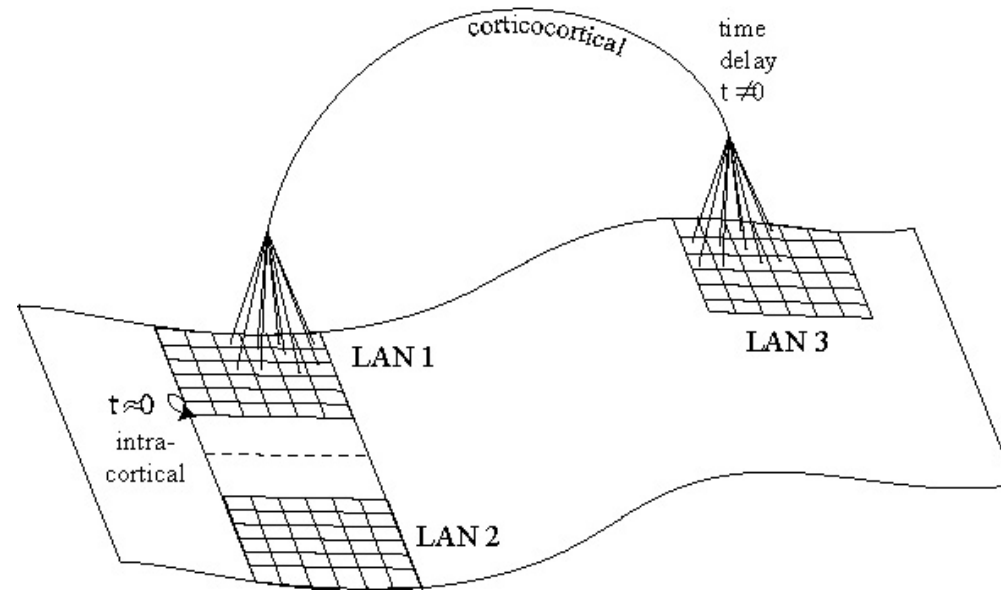
- *Networks with many time delays*

No theorems.

$$\dot{x}_i = -x_i + \sum_j c_{ij} x_j(t - \tau_{ij})$$

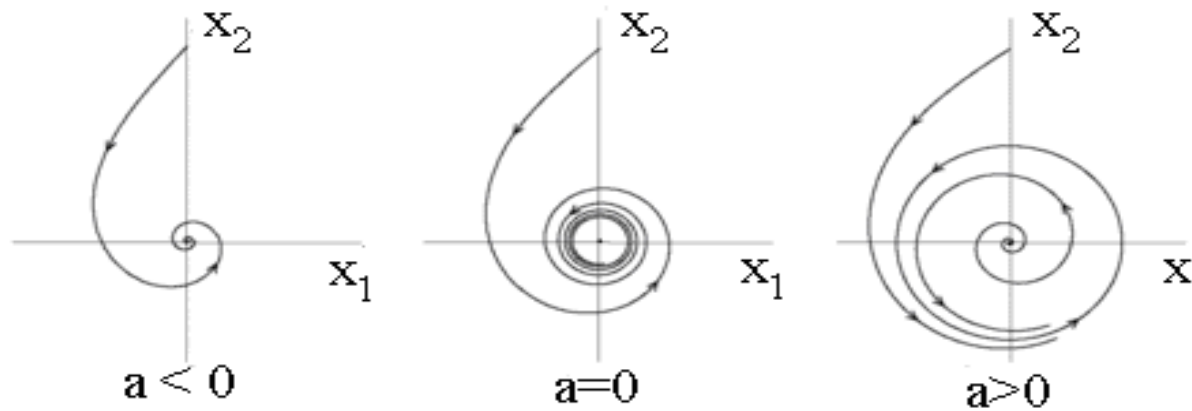
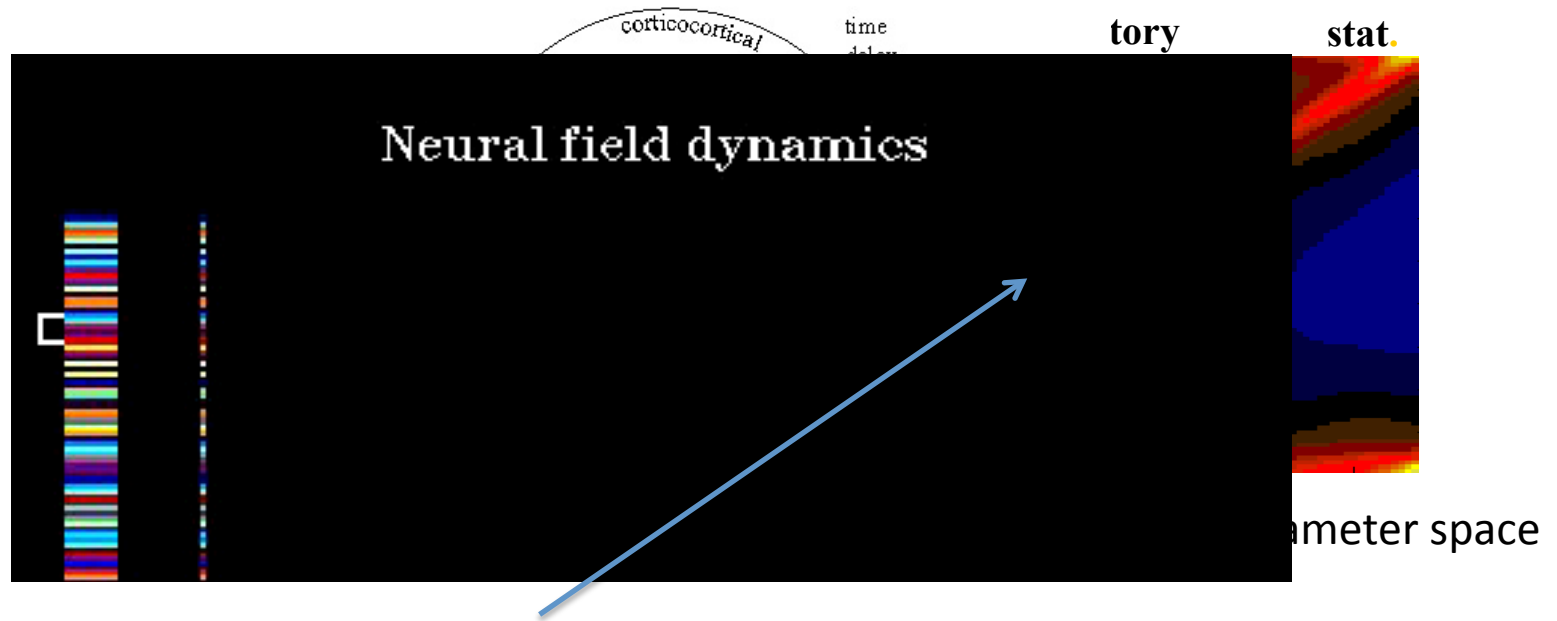


Two-point connections embedded in neural fields



$$\dot{\Psi}(x, t) = N(\Psi(x, t)) + \int_{\Gamma} f(x, X) S\left(\Psi(X, t - \frac{|x - X|}{v})\right) dX$$

Pattern formation, connectivity and time delay



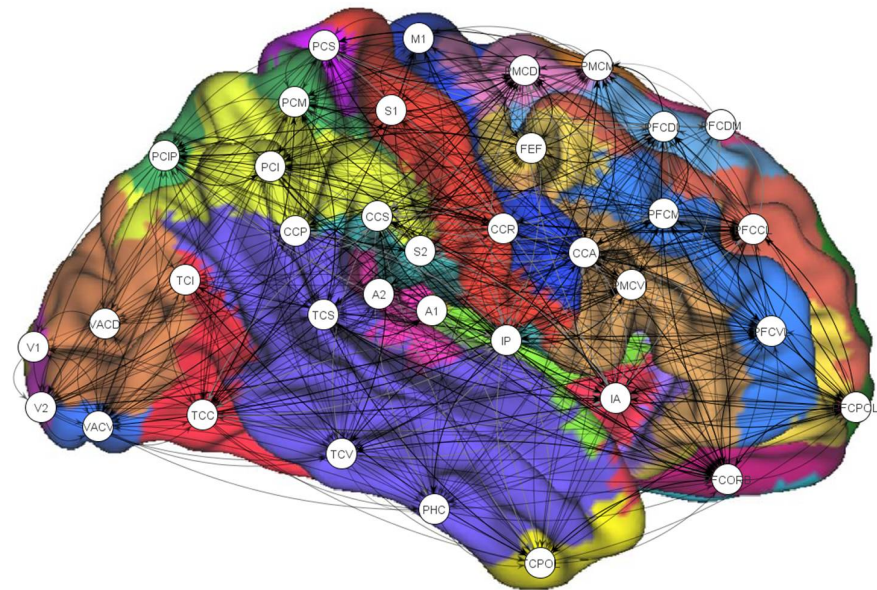


Synchronisation depends on the connectome's space-time structure

Firing rate :
$$\dot{X}_i(t) = -X_i(t) + \sum_{\substack{j=1 \\ i \neq j}}^N c_{ij} X_j(t)$$

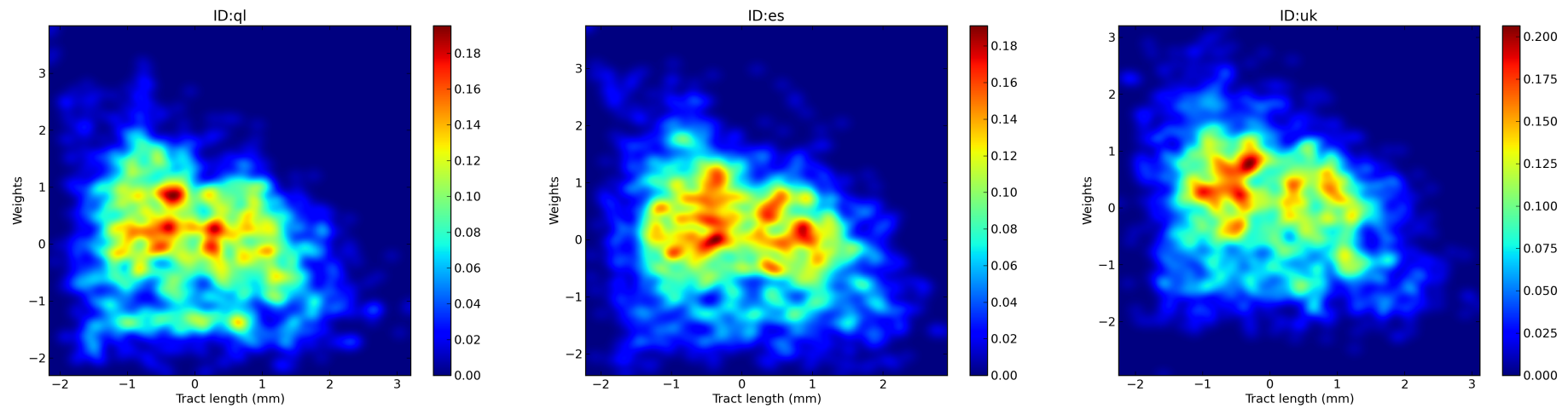
Time delays matter for oscillatory activations

$$\dot{\Theta}_i(t) = \Delta\omega_i + \sum_{\substack{j=1 \\ i \neq j}}^N c_{ij} \sin(\Theta_j(t - \tau_j) - \Theta_i(t))$$



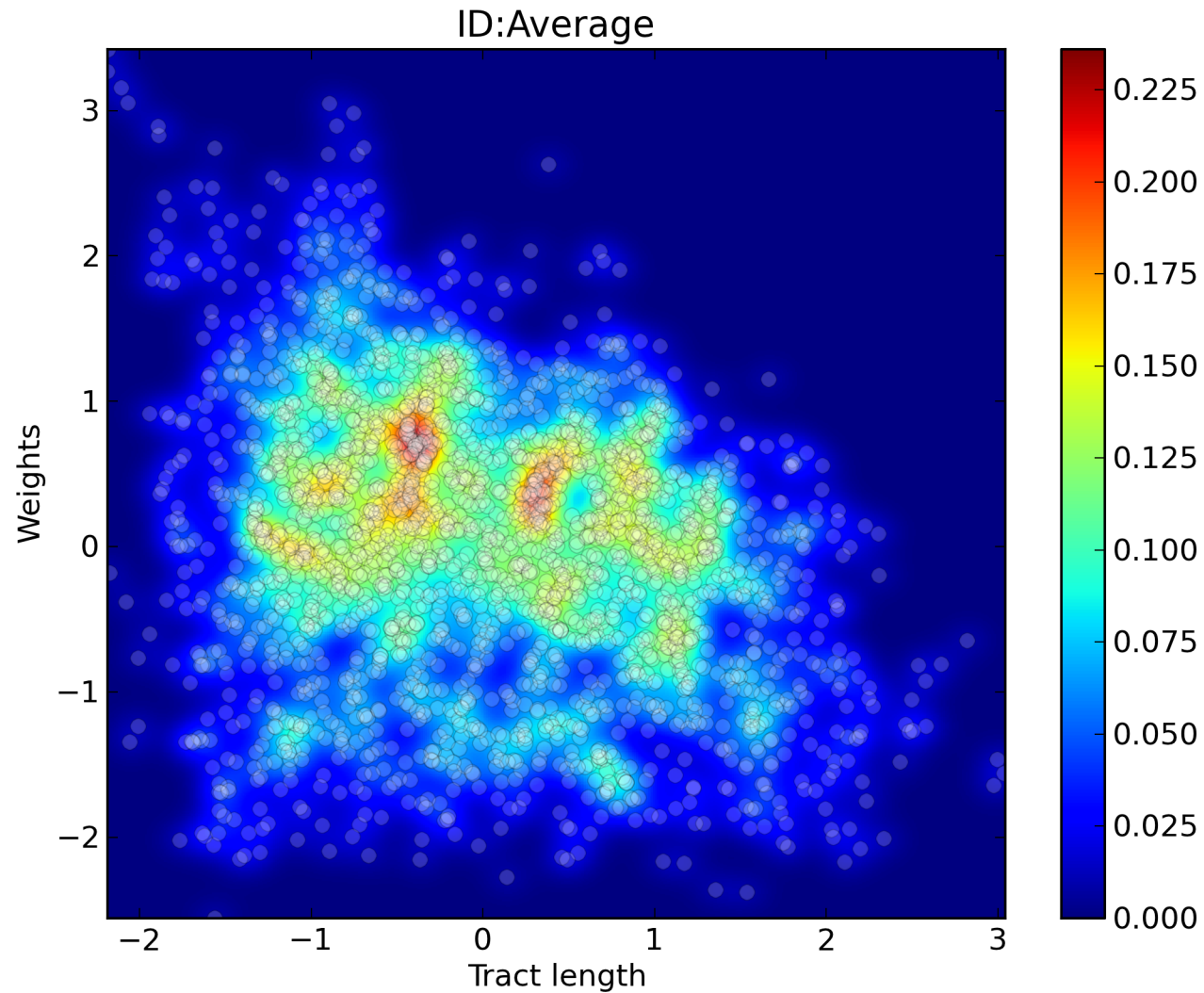


Space-time structure of three different human subjects



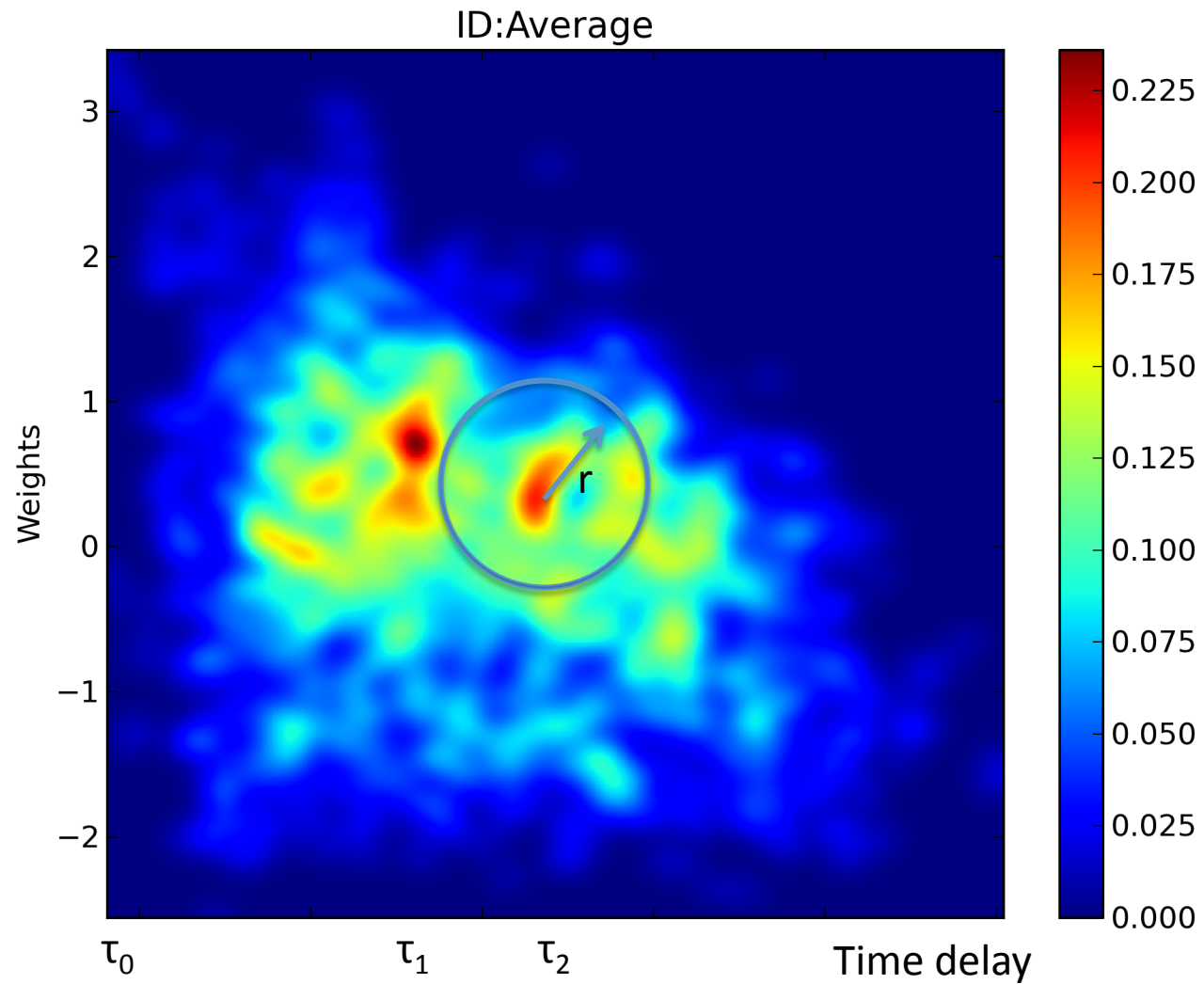


Space-time structure averaged over 10 different subjects





Space-time structure averaged over 10 different subjects





Time Delay in the Kuramoto Model of Coupled Oscillators

M. K. Stephen Yeung and Steven H. Strogatz

Department of Theoretical and Applied Mechanics, Kimball Hall, Cornell University, Ithaca, New York 14853-1503

(Received 13 July 1998)

We generalize the Kuramoto model of coupled oscillators to allow time-delayed interactions. New phenomena include bistability between synchronized and incoherent states, and unsteady solutions with time-dependent order parameters. We derive exact formulas for the stability boundaries of the incoherent and synchronized states, as a function of the delay, in the special case where the oscillators are identical. The experimental implications of the model are discussed for populations of chirping crickets, where the finite speed of sound causes communication delays, and for physical systems such as coupled phase-locked loops or lasers. [S0031-9007(98)08184-8]

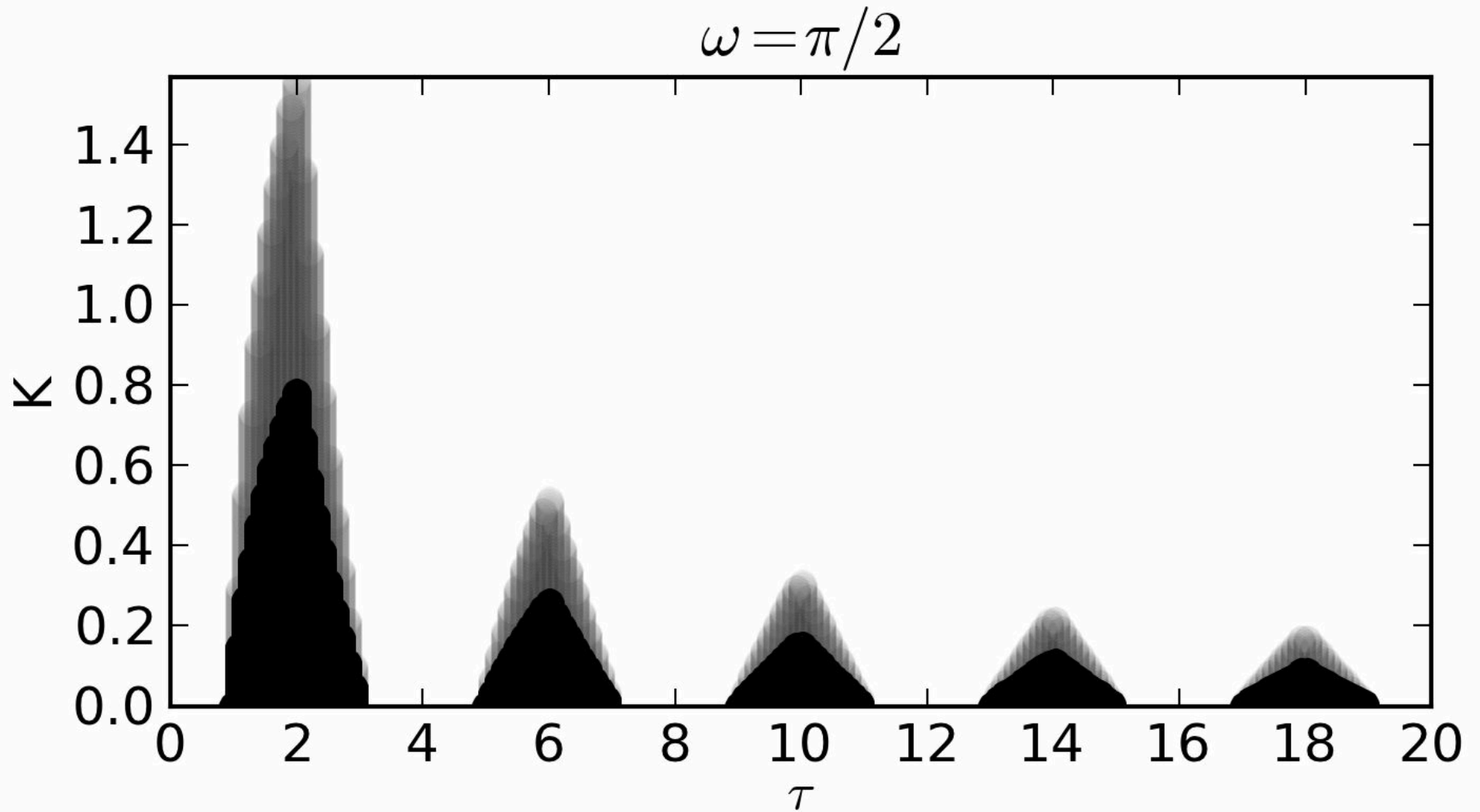
PACS numbers: 87.10.+e, 02.30.Ks, 05.45.+b

The Kuramoto model was originally developed as a tractable mean-field model of coupled biological oscillators [1], such as groups of chorusing crickets [2], flashing fireflies [3], and cardiac pacemaker cells [4]. In a beautiful analysis, Kuramoto showed that the model exhibits a spontaneous transition from incoherence to collective synchronization, as the coupling strength is increased past a certain threshold [5]. The model has since been analyzed more deeply and extended in various ways [6–10]. It has

layed mean-field coupling:

$$\dot{\theta}_i(t) = \omega_i + \xi_i(t) + \frac{K}{N} \sum_{j=1}^N \sin[\theta_j(t - \tau) - \theta_i(t) - \alpha], \quad (1)$$

for $i = 1, \dots, N$. Here $\theta_i(t)$ is the phase of the i th oscillator at time t , and ω_i is its intrinsic frequency, randomly drawn from a probability density $g(\omega)$ with mean ω_0 .



Vector-Matrix Form

$$\dot{\xi}(t) = (\epsilon I + DA)\xi(t) + B\xi(t - \tau) + \mathbf{NL}$$

I is the identity matrix, A and B have the elements A_{nk} and B_{nk}
 \mathbf{NL} represents the nonlinear terms.

The case of no diffusion $D = 0$

$$\dot{\xi}(t) = \epsilon I \xi(t) + B_0 \xi(t - \tau) \quad ; \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } B_0 = \mu \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Symmetric Coupling $\mu_{12} = \mu = \mu_{21}$

$B_0 = E^{-1} \Lambda E$ Multiply from left by E

$$E \dot{\xi}(t) = \epsilon E \xi(t) + \Lambda E \xi(t - \tau) \longrightarrow \dot{u}(t) = \epsilon u(t) + \lambda u(t - \tau)$$

Eigenvalues : $\lambda = \pm \mu$, **Eigenvectors :** \hat{e} $u(t) = \hat{e} \xi(t)$,

$$u(t) = e^{zt} \longrightarrow z = \epsilon + \lambda e^{-z\tau} \longrightarrow H(z) = z - \epsilon - \lambda e^{z\tau} = 0$$

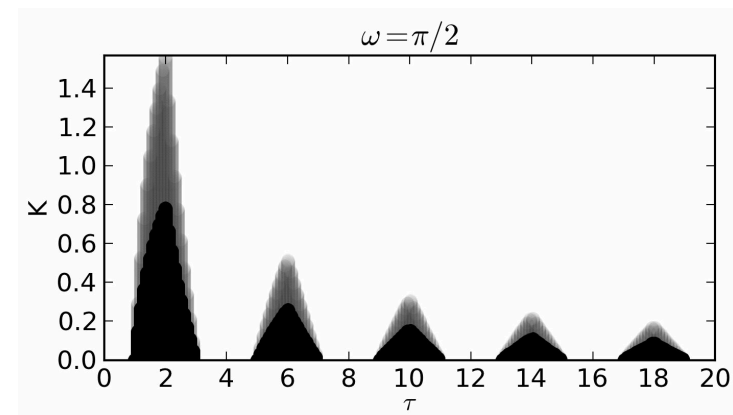
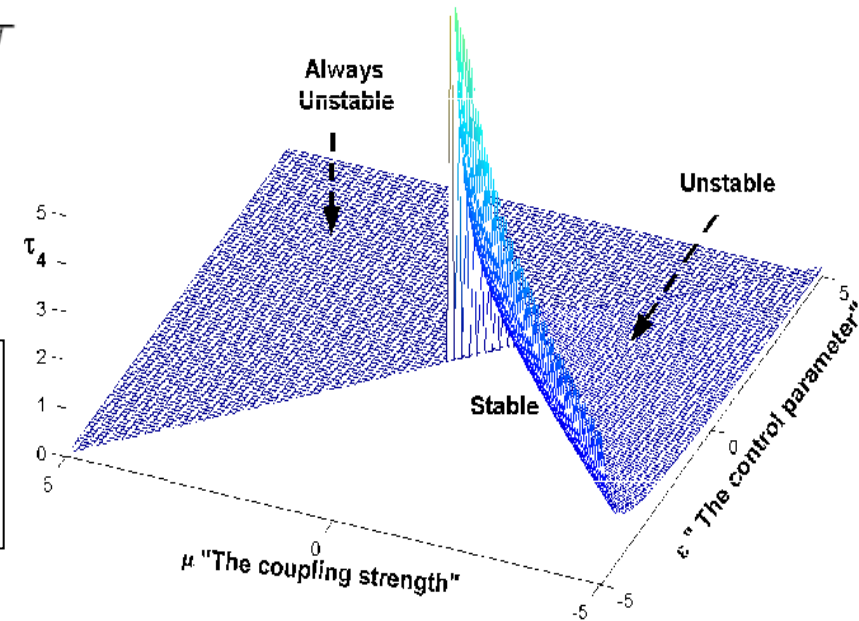
The Characteristic Eq : $z = \epsilon + \lambda e^{-z\tau}$

Critical Surface \longrightarrow Stability Change

$$\tau^* = \frac{1}{\sqrt{\lambda^2 - \epsilon^2}} \arccos\left(\frac{-\epsilon}{\lambda}\right), \quad \lambda < -|\epsilon|$$

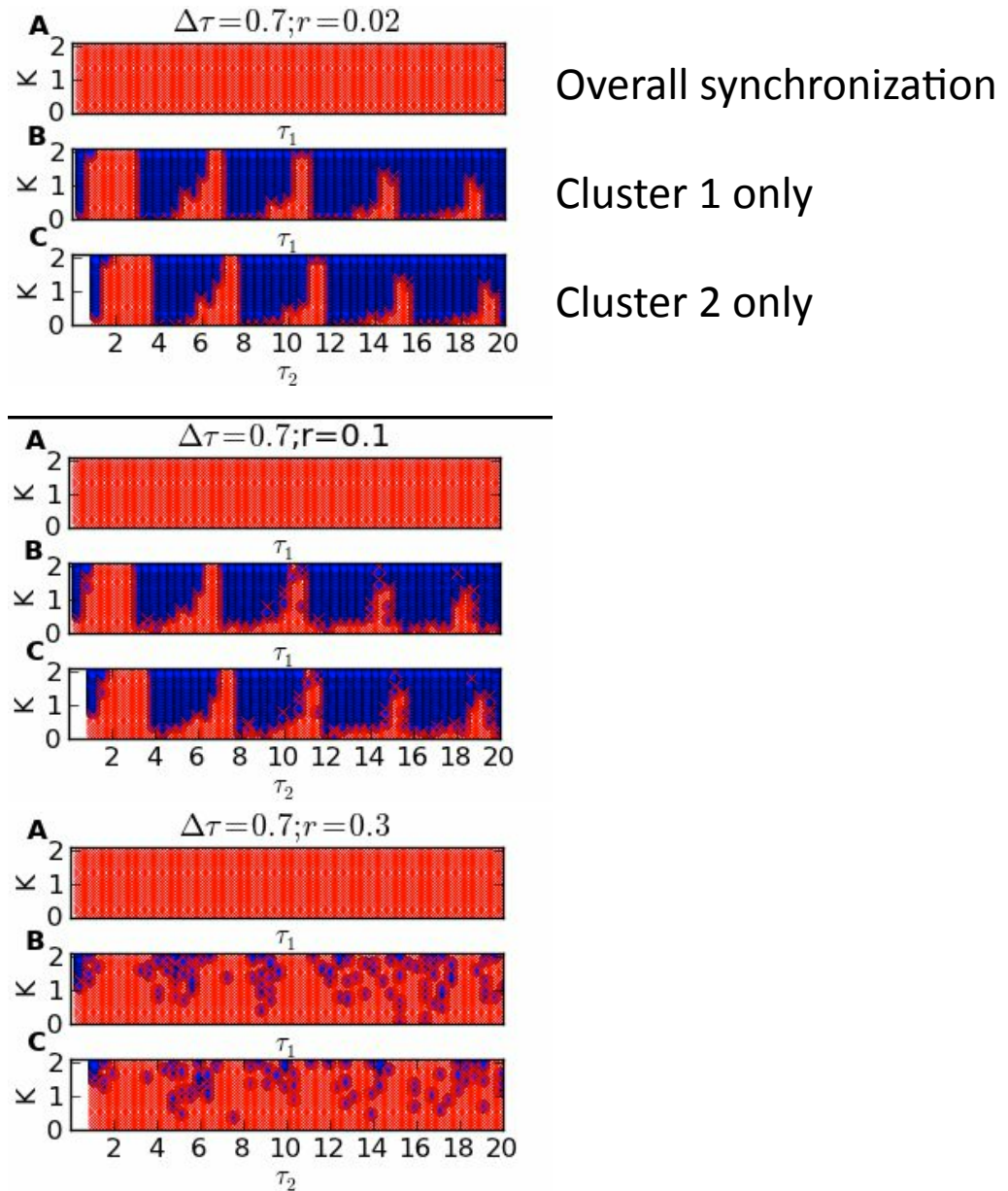
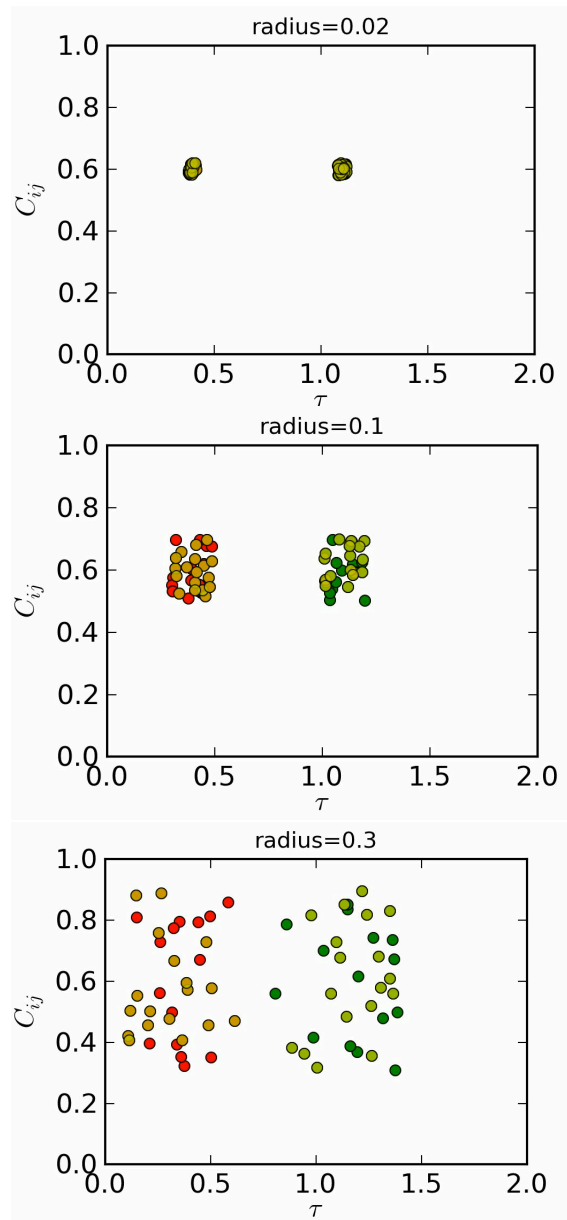
$$\begin{aligned} \left(\frac{\partial \operatorname{Re}[z]}{\partial \tau}\right)_{z=i\omega} &= -\operatorname{Re}\left[\left(\frac{\partial H(z)}{\partial \tau}\right) / \left(\frac{\partial H(z)}{\partial z}\right)\right] \\ &= \frac{\epsilon^2}{1 - 2\tau\epsilon + \lambda^2\tau^2} > 0 \end{aligned}$$

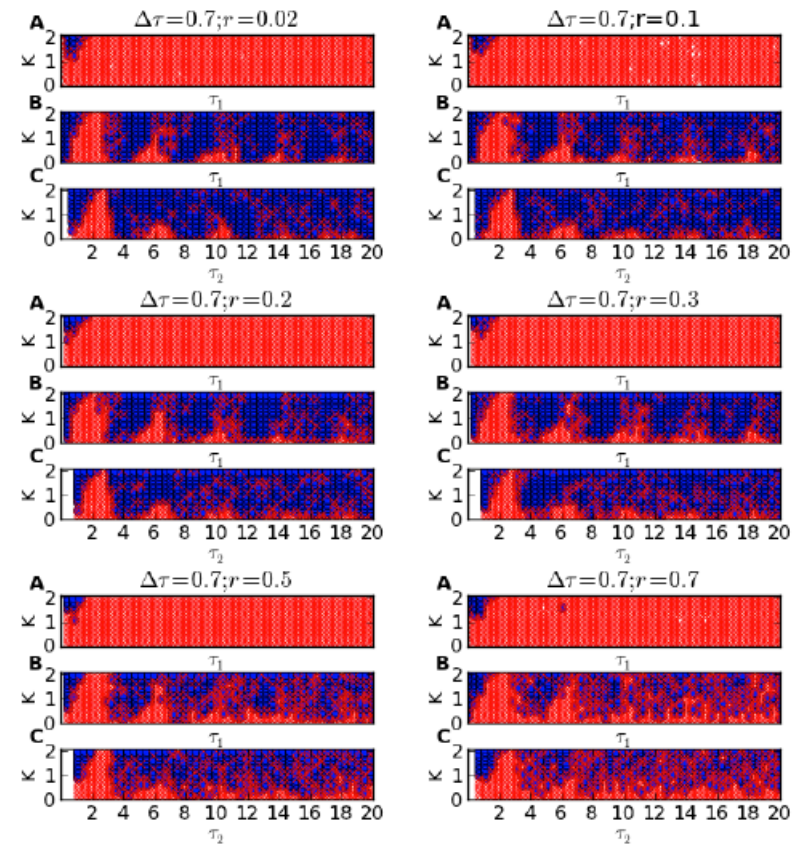
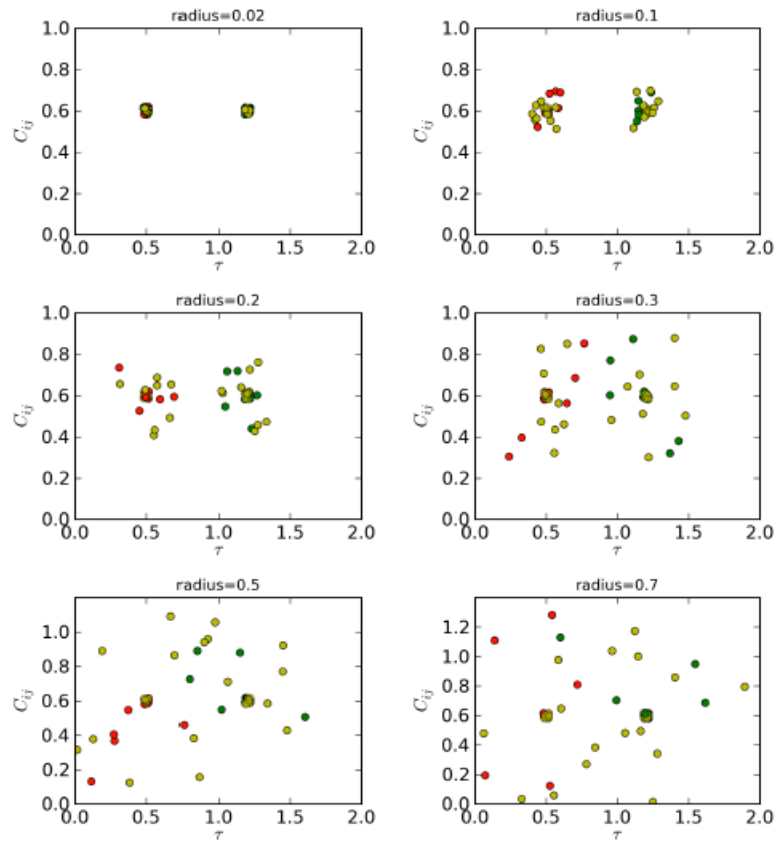
If $\operatorname{Re}[z] < 0$, then $u(t) = 0$ is **Stable**
 If $\operatorname{Re}[z] > 0$, then $u(t) = 0$ is **Unstable**



System remains stable for

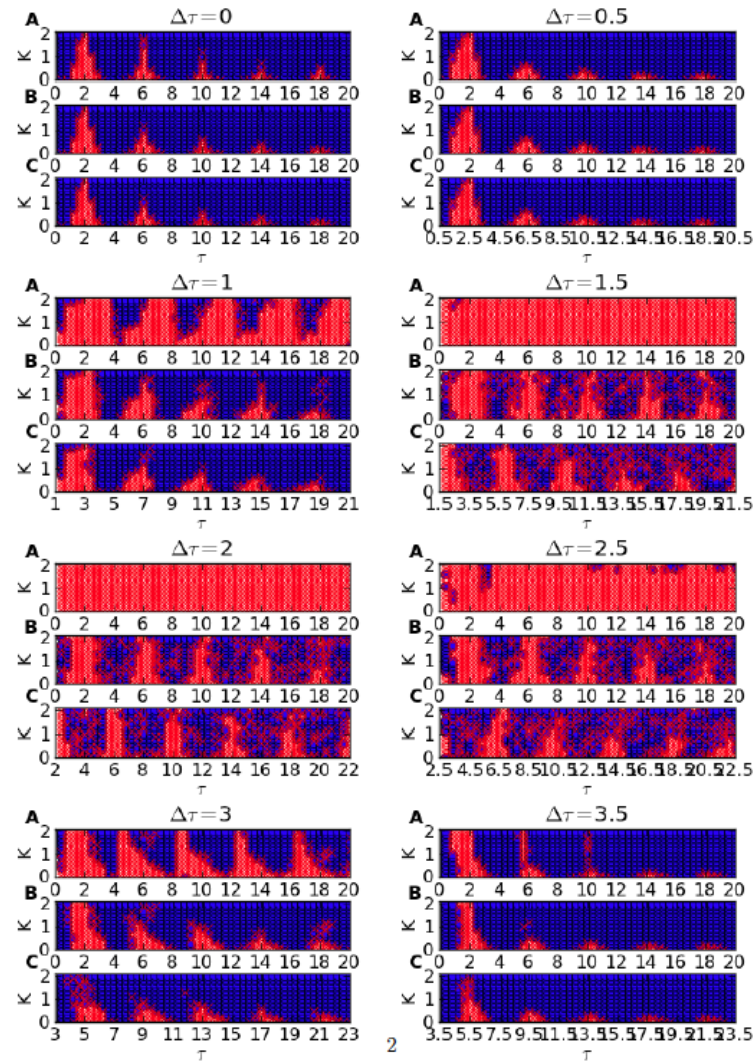
$$\tau > \tau^*$$





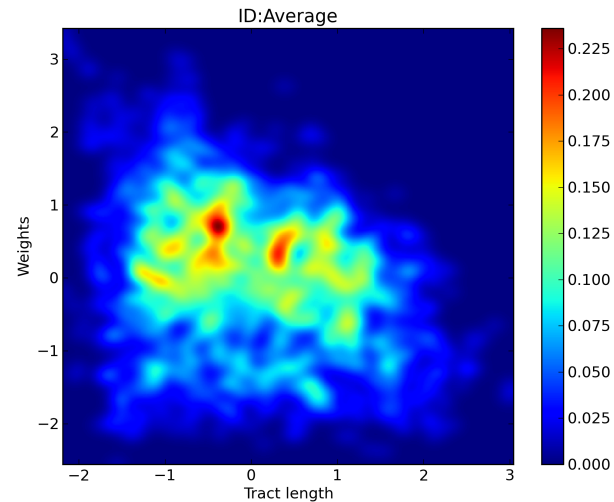


Systematic change of $\Delta\tau$

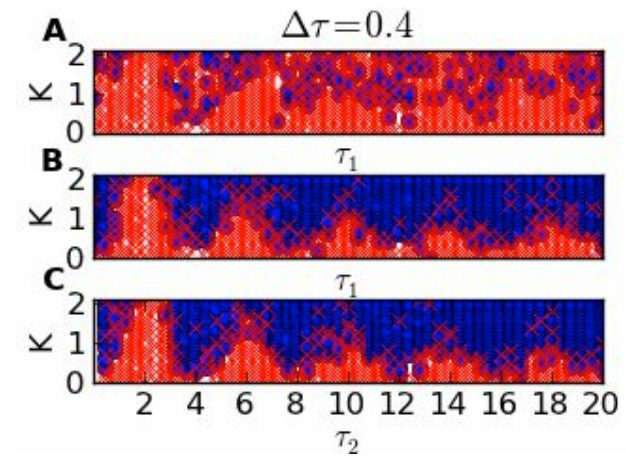
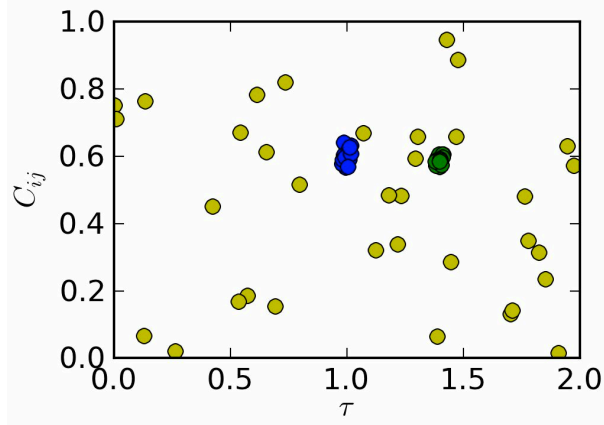




Space-time structure of the human connectome



Synchronizability in network with connectivity matrix averaged across 10 subjects

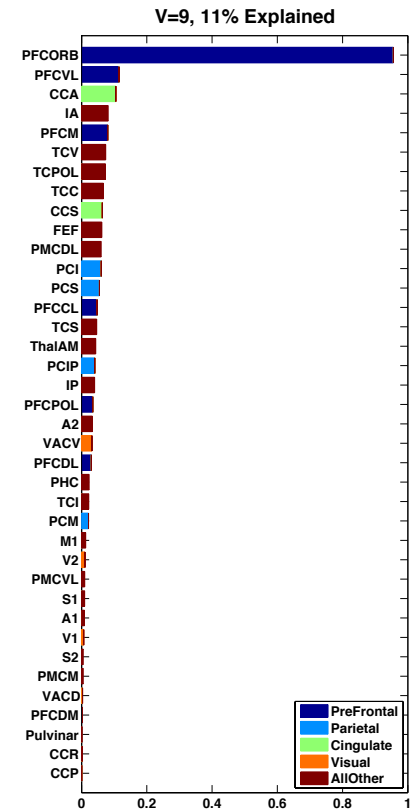
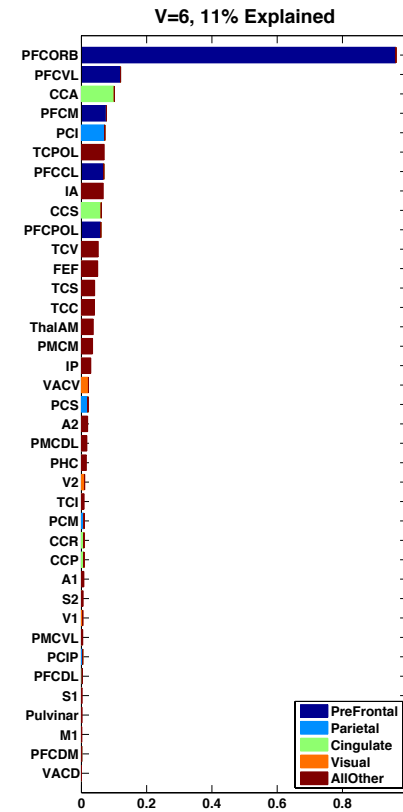
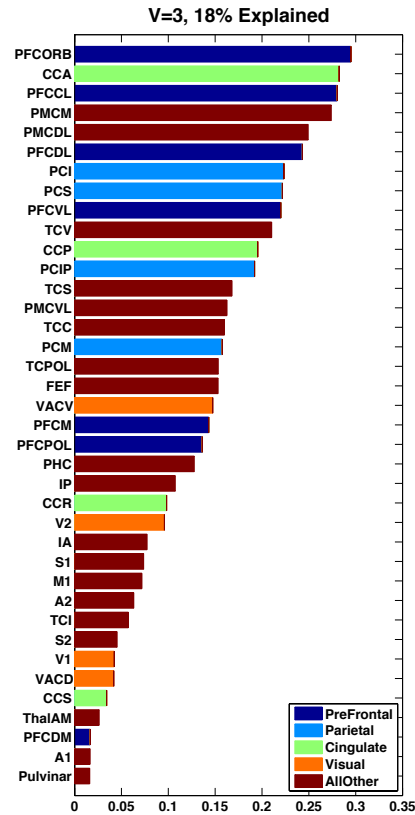
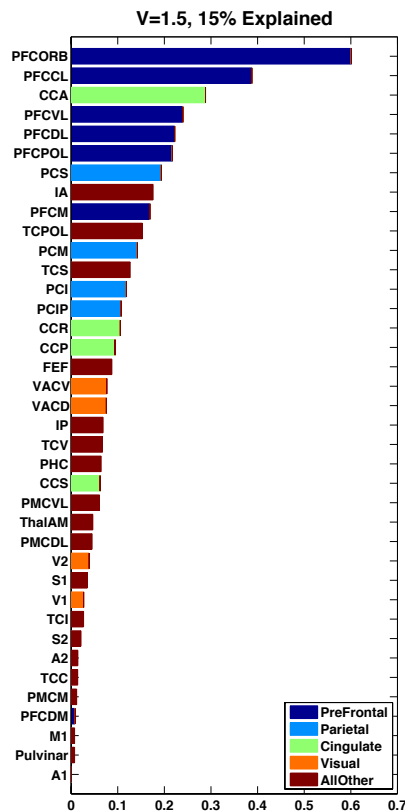
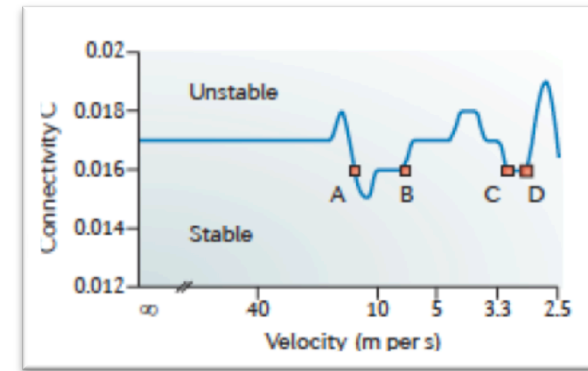




Ghosh et al. PLoS CB (2008)

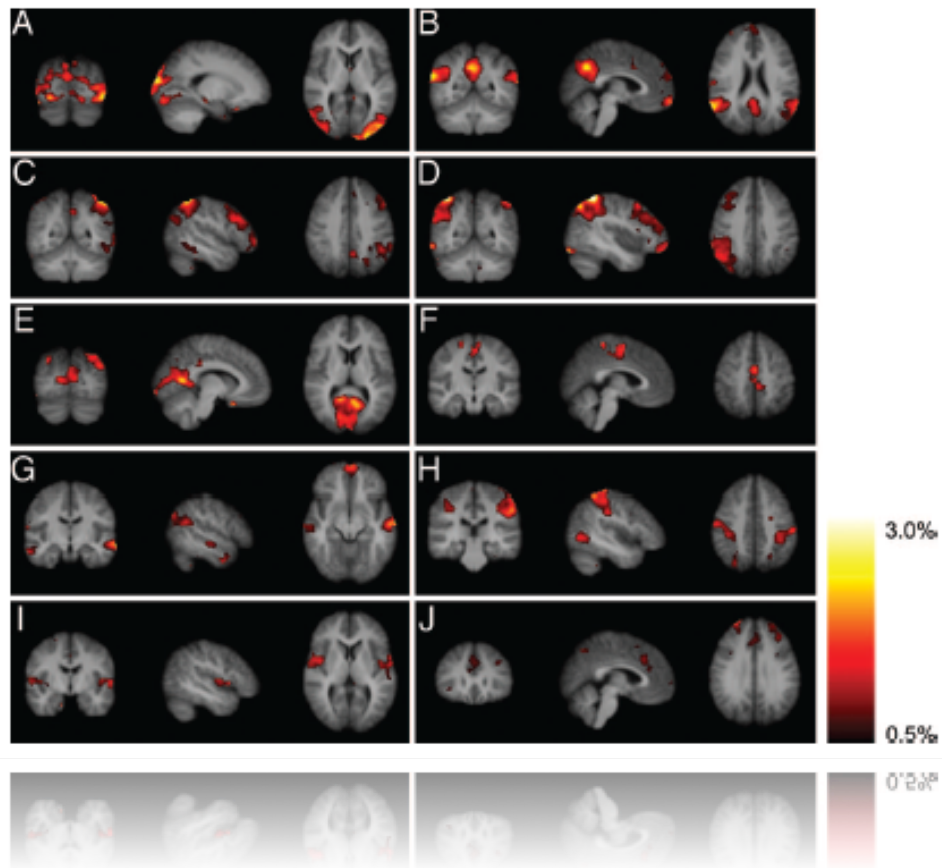
Knock et al. JNSM (2009)

Stability of the Equilibrium point – the human brain at rest Principal Component Analysis



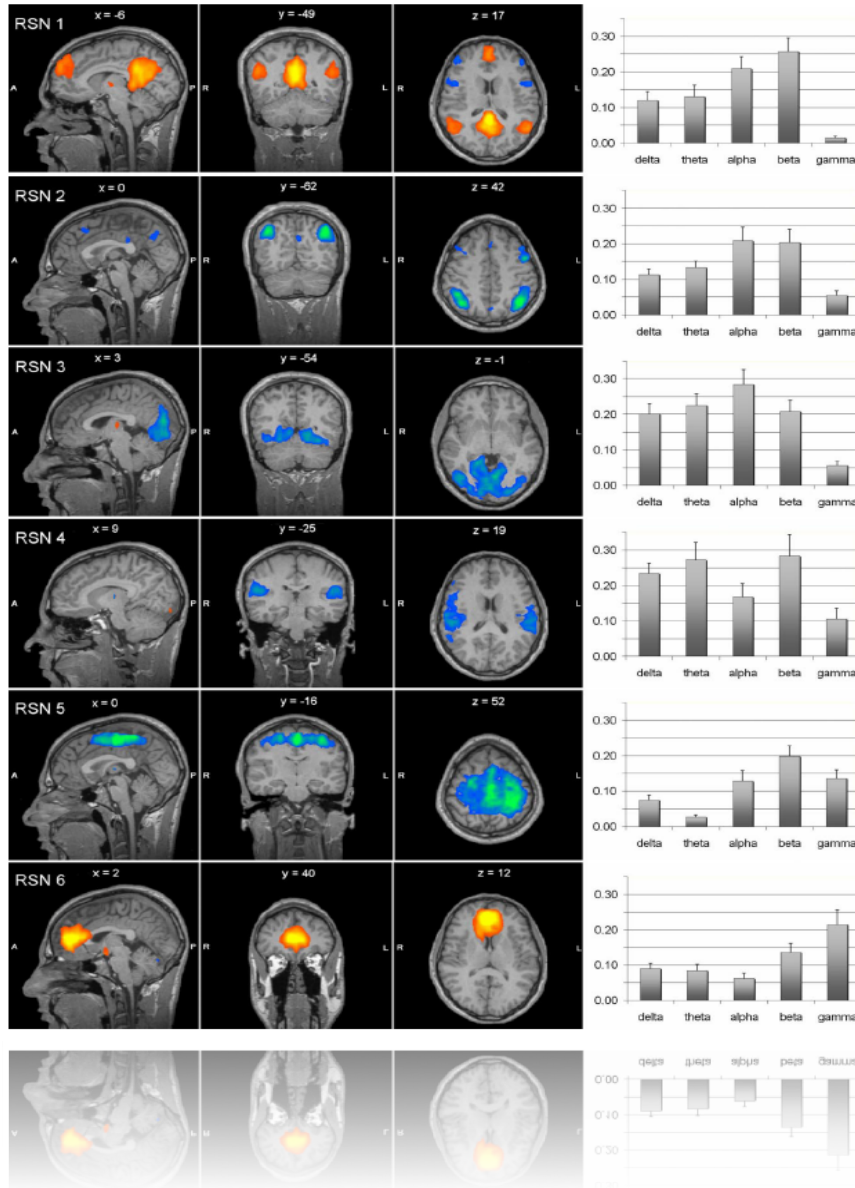
Resting state dynamics of the human brain

Independent component analysis demonstrates the existence of robust various resting state networks



Damoiseaux et al PNAS 2006

Resting state activity displays ultraslow fluctuations



Simultaneous EEG and fMRI study finds cross-correlations between BOLD signal and the power fluctuations in each frequency band.

Mantini et al. PNAS 2007

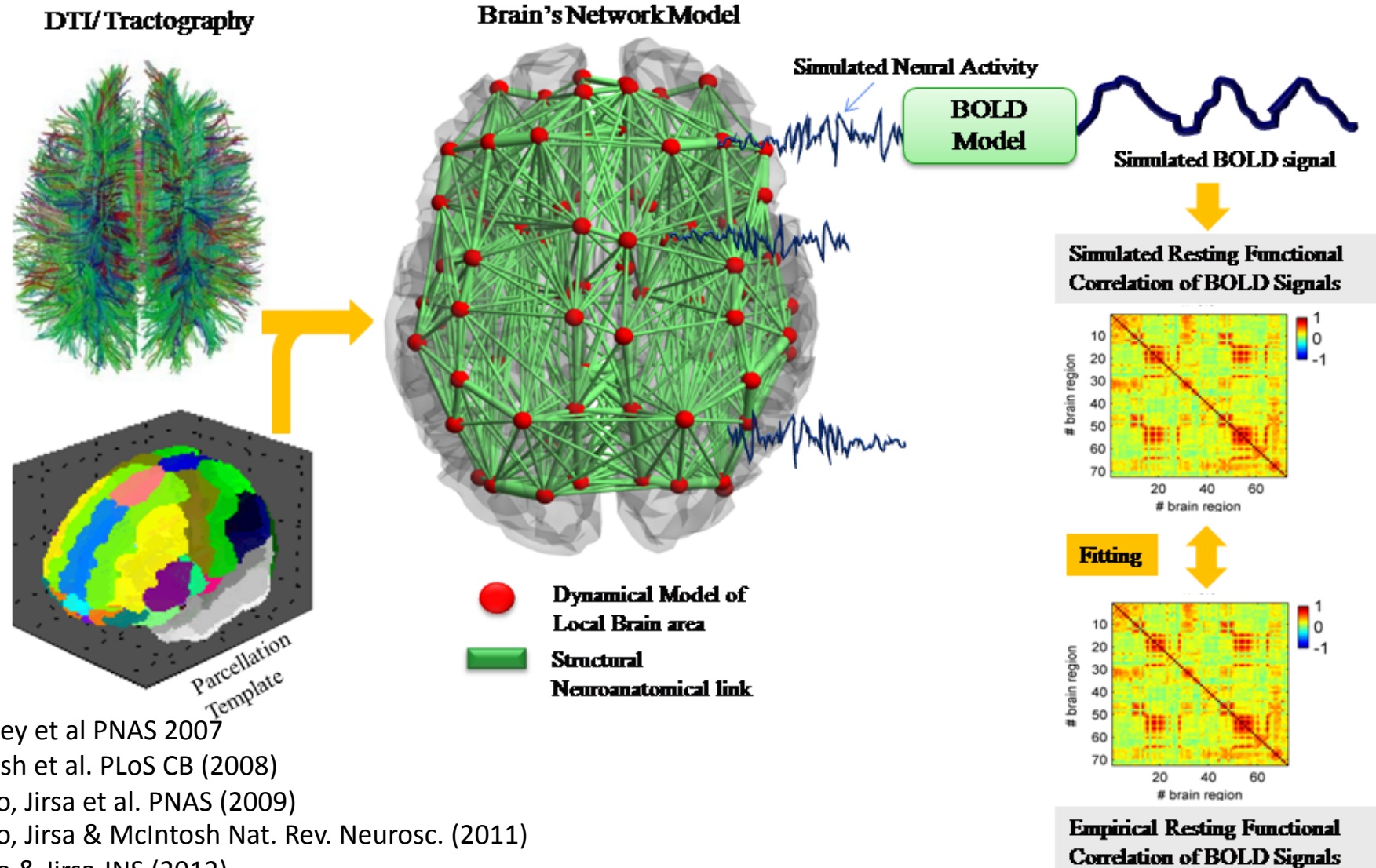
Resting state networks (RSN) as a biomarker of...

- Schizophrenia and autism (Uhlhaas & Singer 2011)
- Obesity (Dubbelink et al 2008, Garcia-Garcia et al 2012, Babiloni et al 2011, Kullmann et al 2011)
- Epilepsy (Bettus et al PLoS One 2011)
- Ageing and Alzheimer (Koch et al 2010, Agosta et al 2011)
- ...

Can we use the RSN dynamics for the testing and validation of hypothesized physiological mechanism?



Model of Resting-State Activity



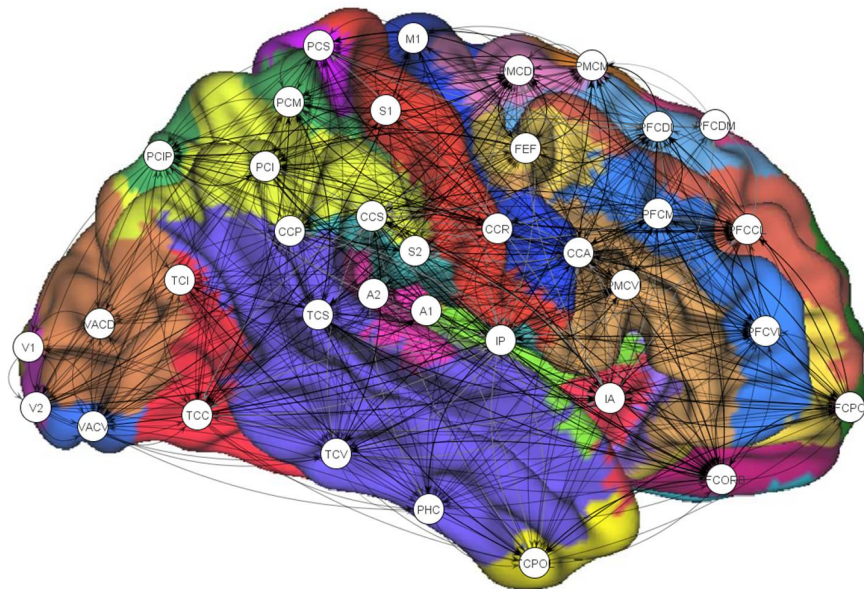
- Honey et al PNAS 2007
- Ghosh et al. PLoS CB (2008)
- Deco, Jirsa et al. PNAS (2009)
- Deco, Jirsa & McIntosh Nat. Rev. Neurosc. (2011)
- Deco & Jirsa JNS (2012)

Implementation of biologically realistic large scale model

$$\dot{u}_i(t) = \tau \left[g(u_i, v_i) - c \sum_{j=1}^N f_{ij} u_j(t - \Delta t_{ij}) \right]$$

$$\dot{v}_i(t) = -(1/\tau) h(u_i, v_i)$$

$$\Delta t_{ij} = \frac{d_{ij}}{v}$$

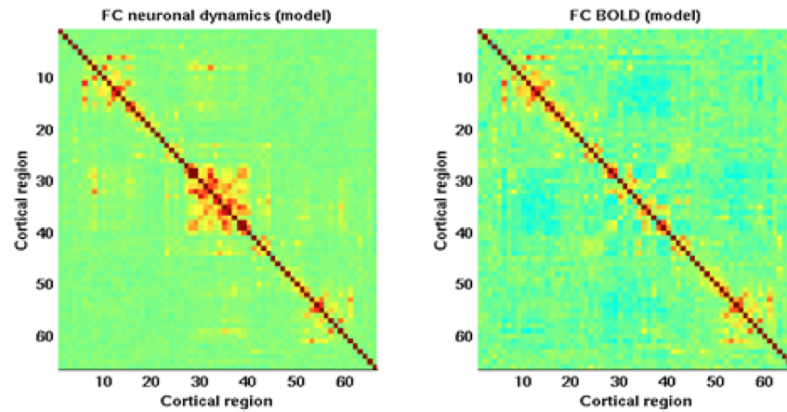


- Number of regions: $N=38$
- Connectivity matrix, $f(i,j)$, gives the weight of link $i-j$
- f is an asymmetric matrix

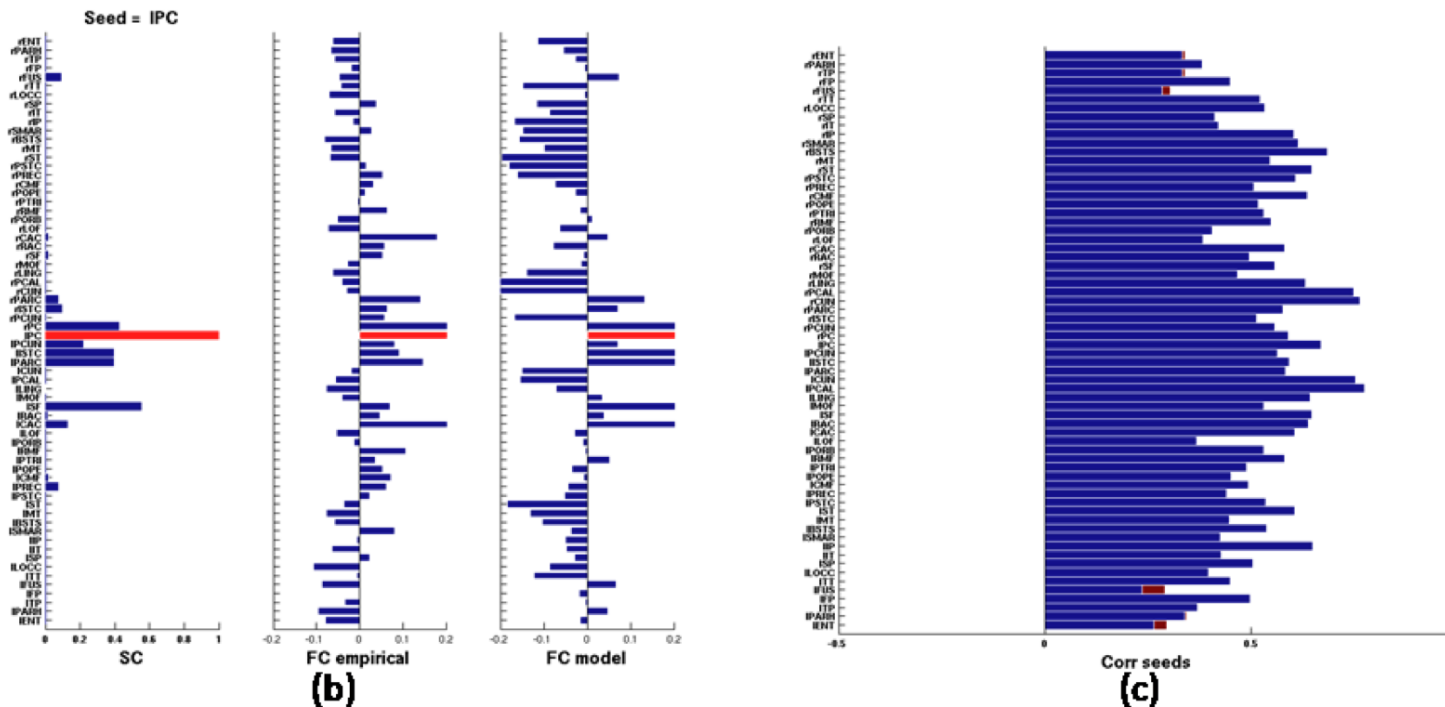
Honey et al PNAS 2007
Ghosh et al. PLoS CB (2008)
Deco, Jirsa et al. PNAS (2009)
Deco, Jirsa & McIntosh Nat. Rev. Neurosc. (2011)
Deco & Jirsa JNS (2012)



(a)

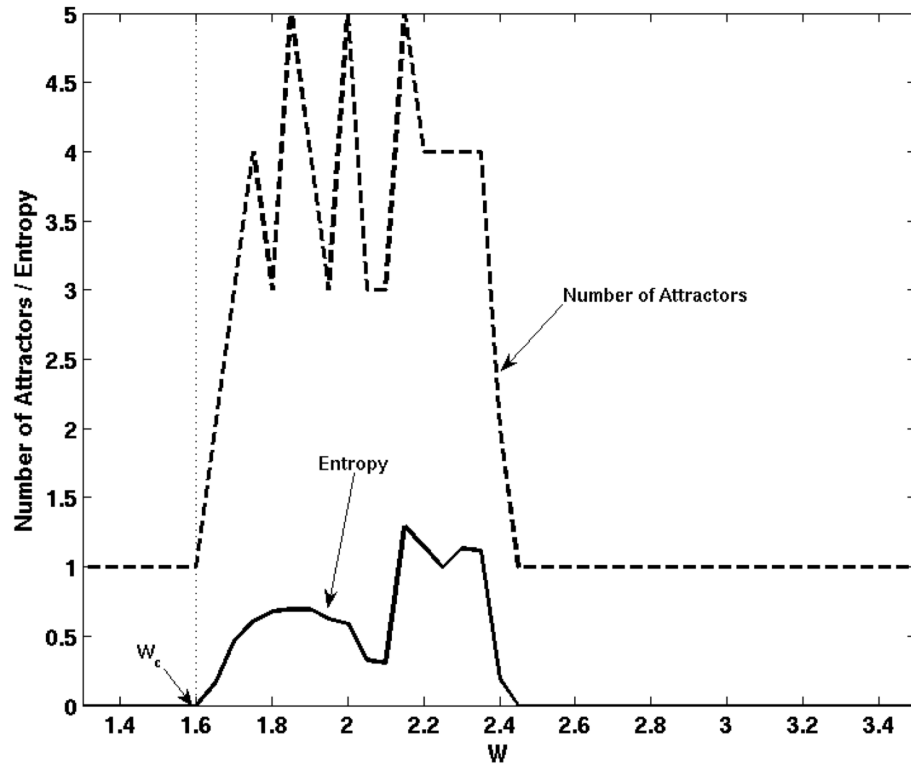


Experimental Validation

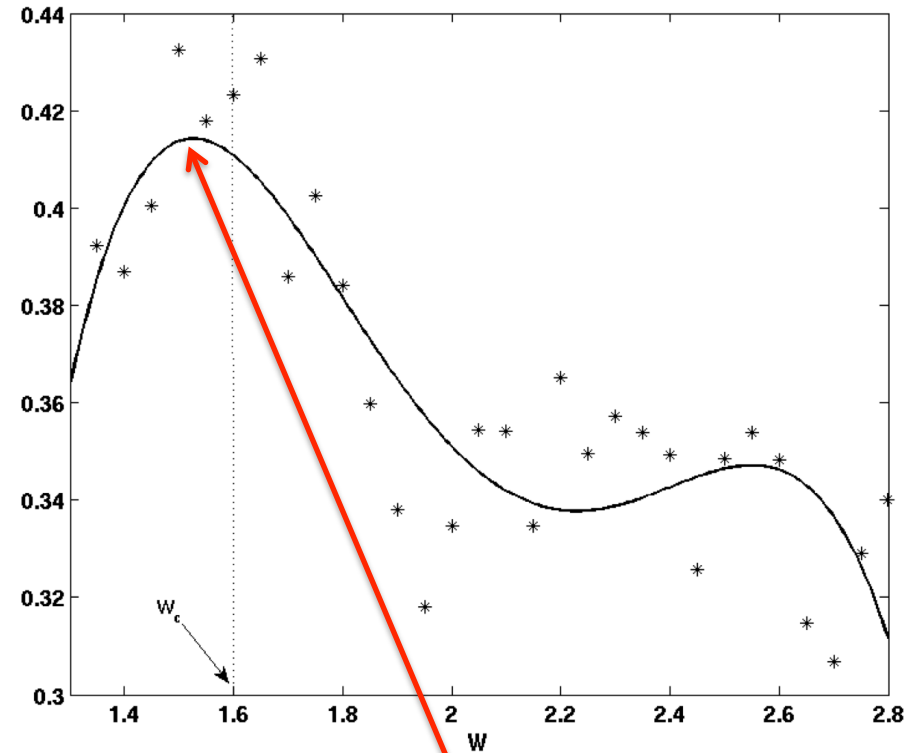




Number of Attractors

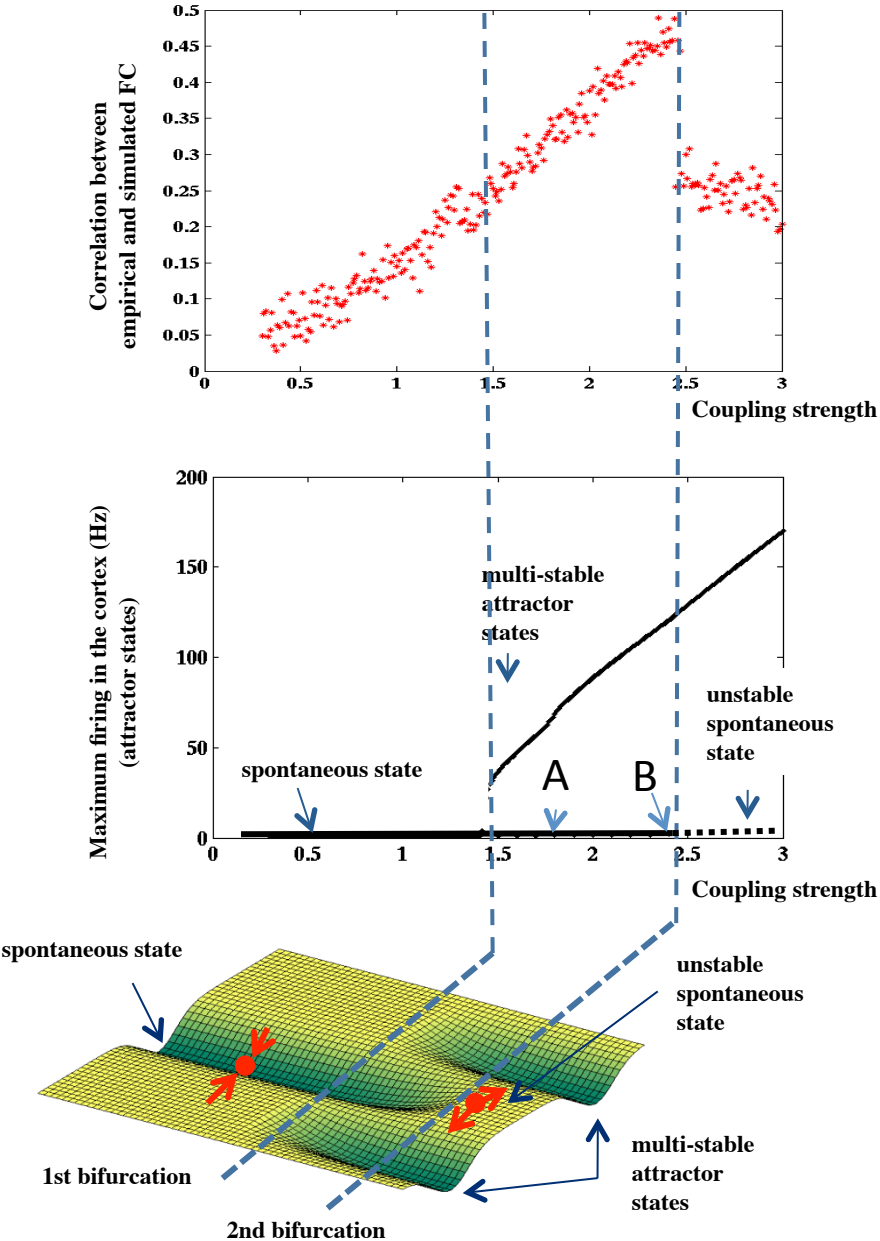


Correlation with empirical data



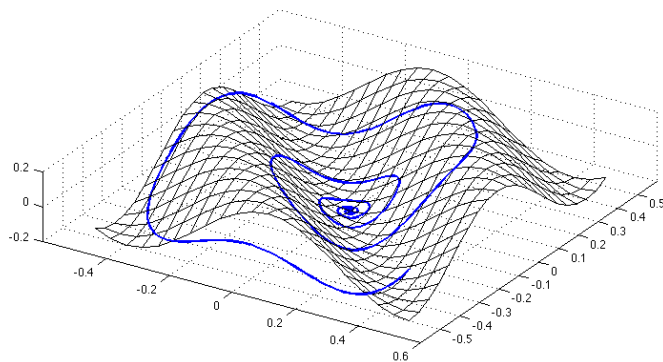
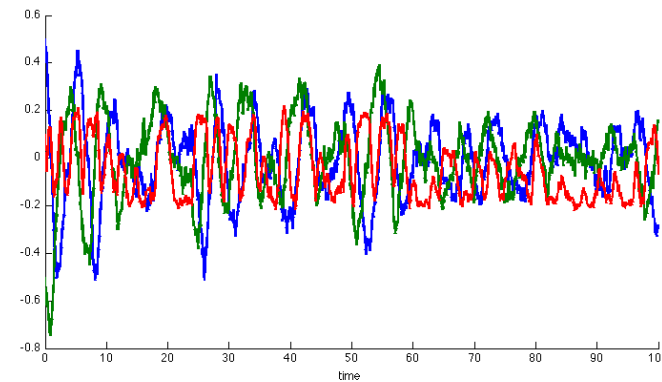
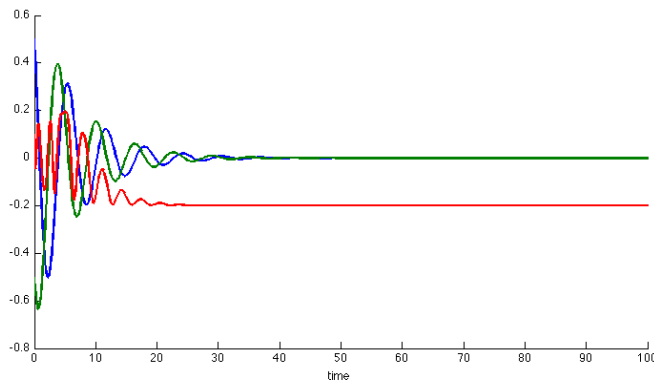
During the resting state, the brain operates close to “criticality”

Resting-State: Exploration of the Dynamical Repertoire

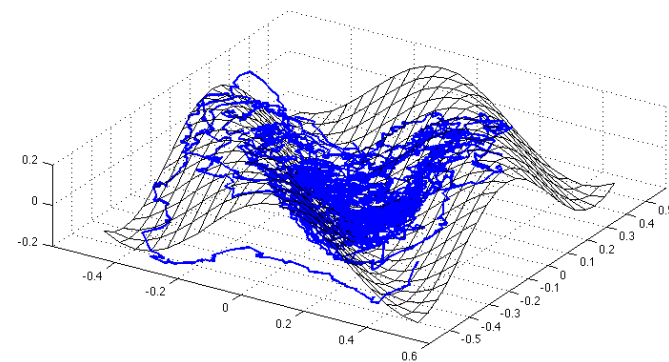




At criticality noise aids in the exploration of the dynamic neighborhood



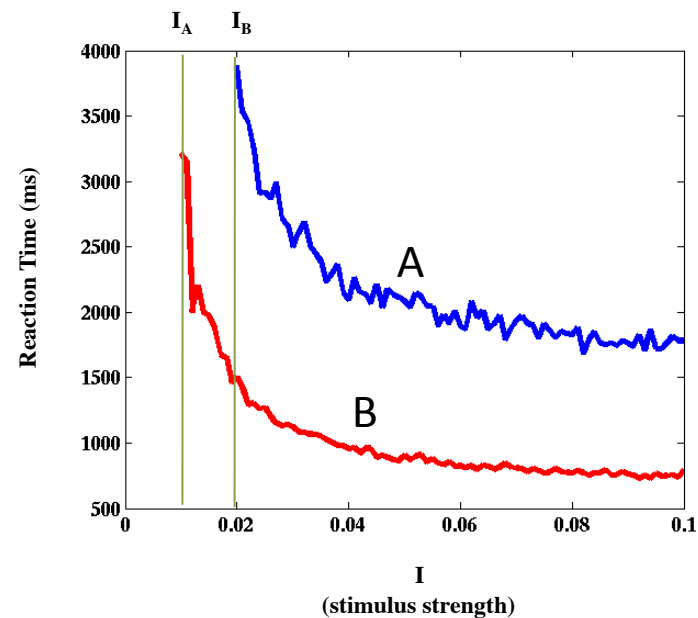
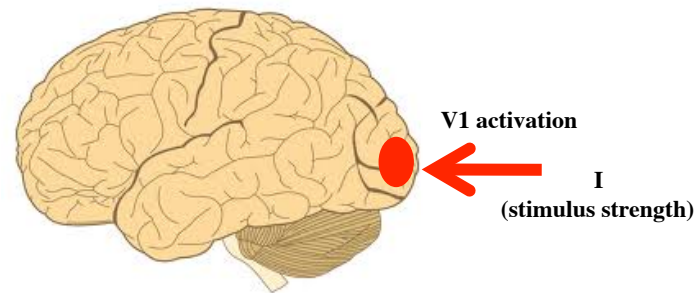
no noise



with noise



Stimulation paradigm

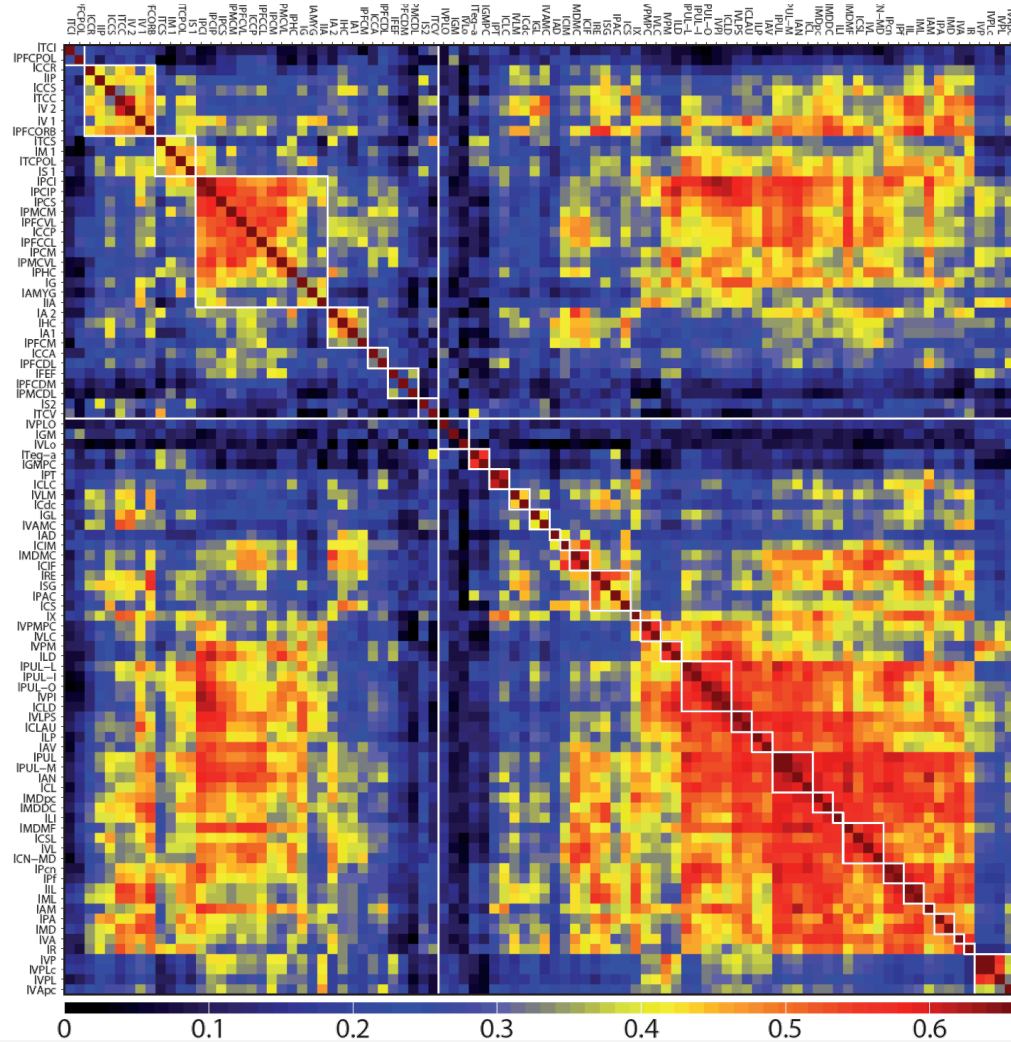




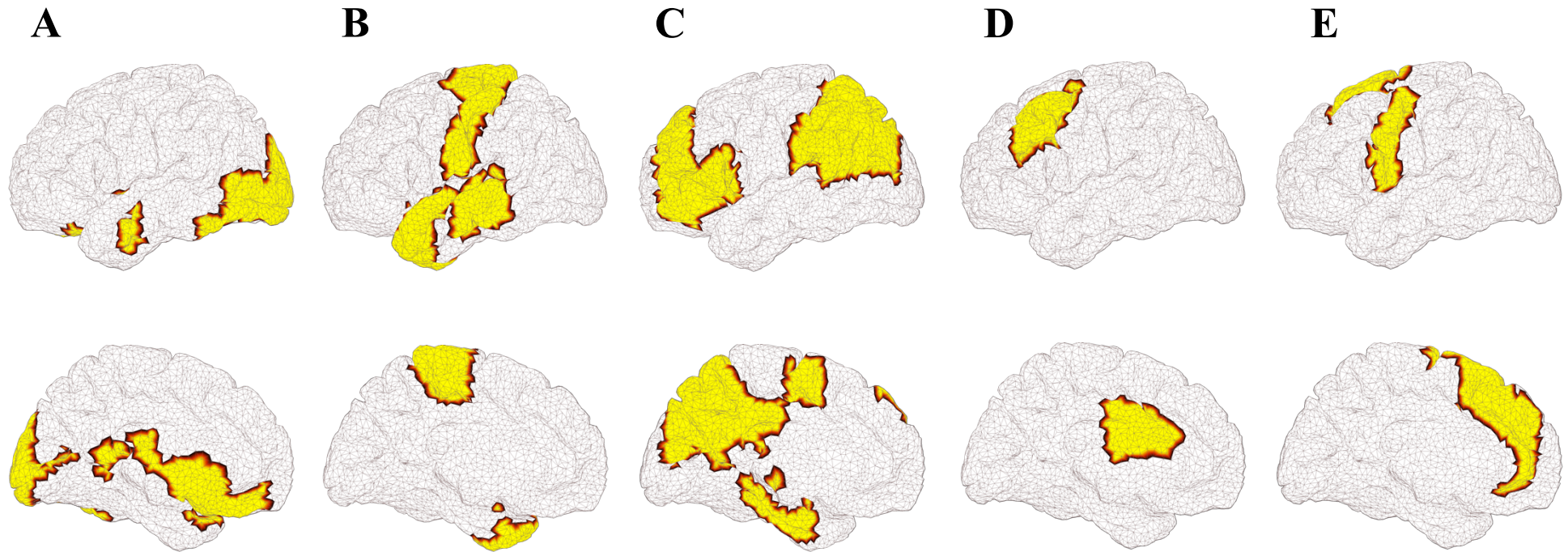
Left side $C_{(R \times R)}$
($\alpha = .4, \sigma = 15 \text{ mm}$)

Cortex

Subcortex



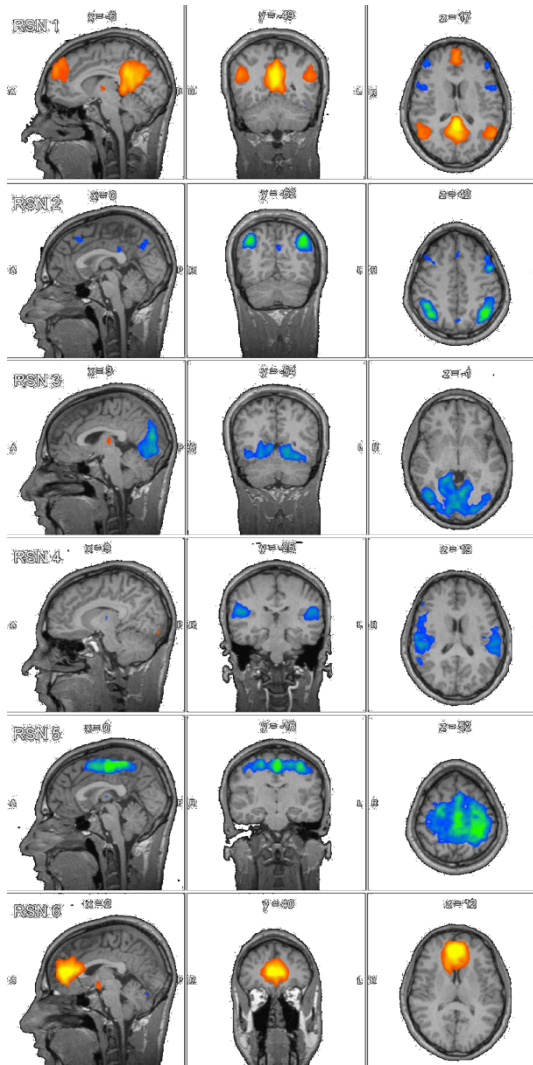
Clusters – Stimulation sites



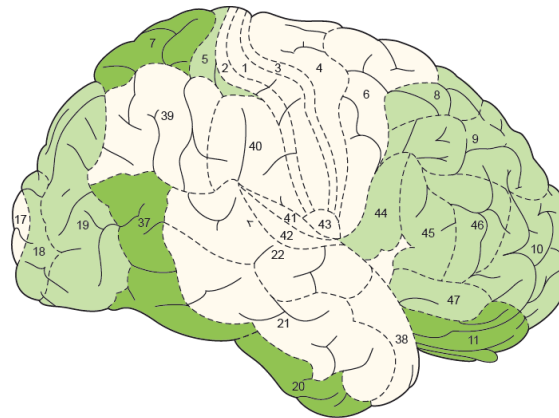
Ten clusters are consistent over (α, σ) .

Stimulation sites that share one network are distributed.

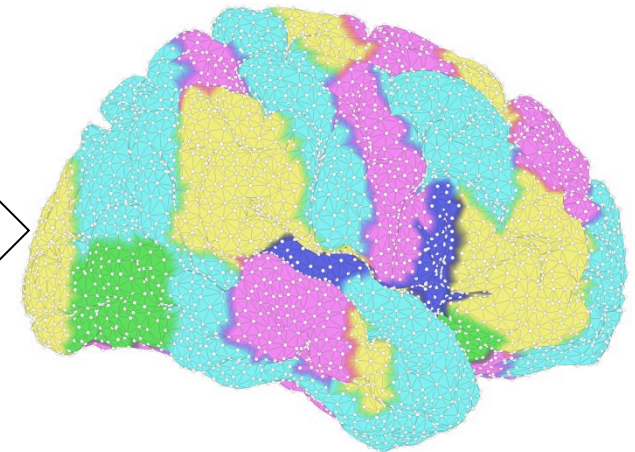
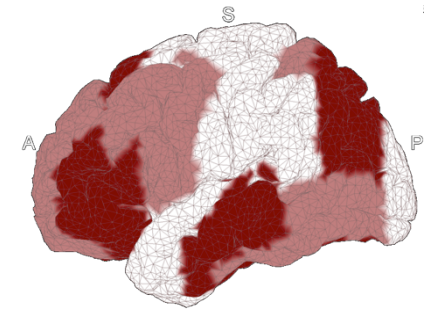
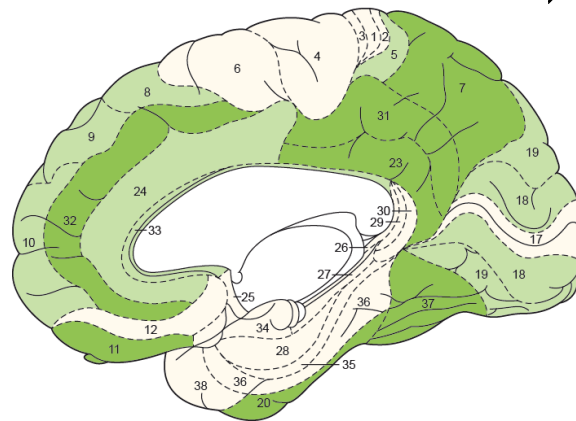
Mantini et al. 2007



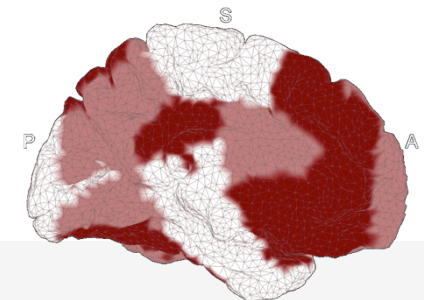
Comparison – Masks



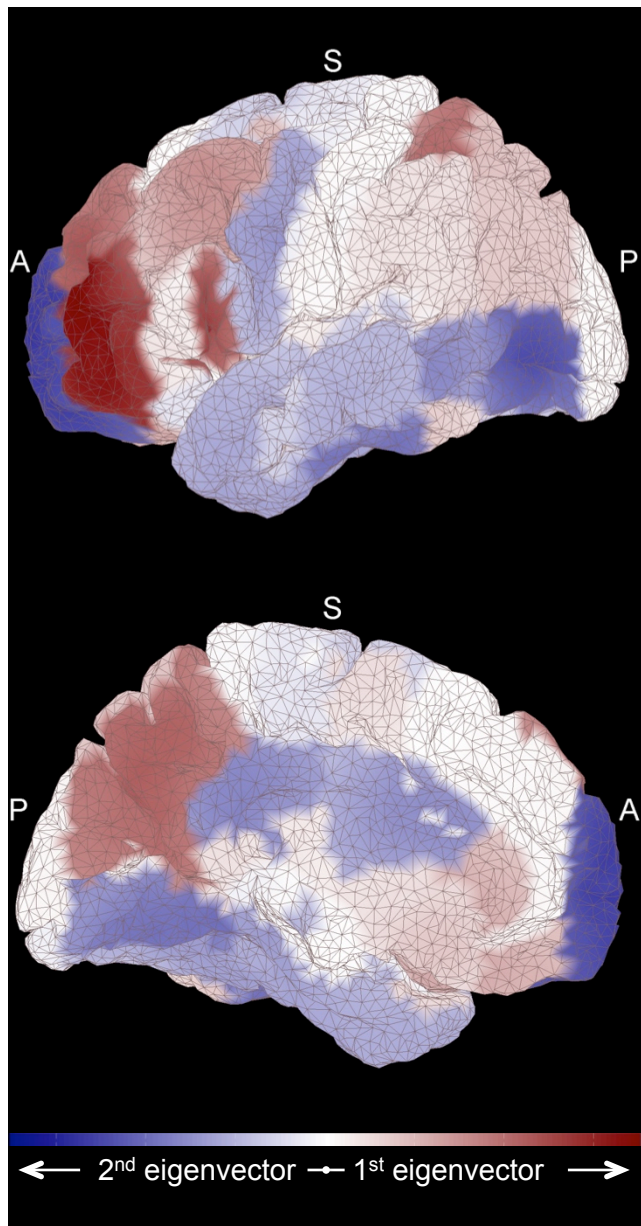
Brodmann



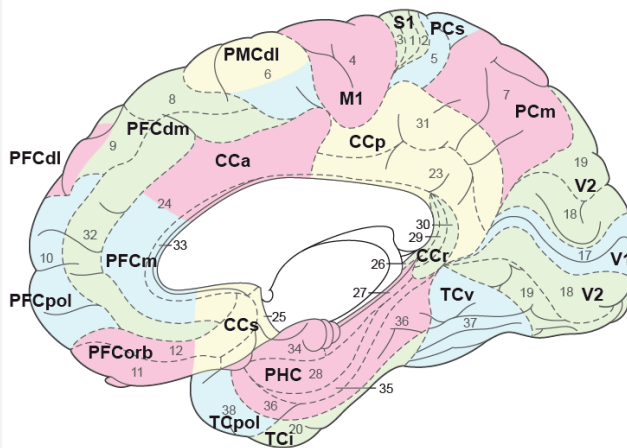
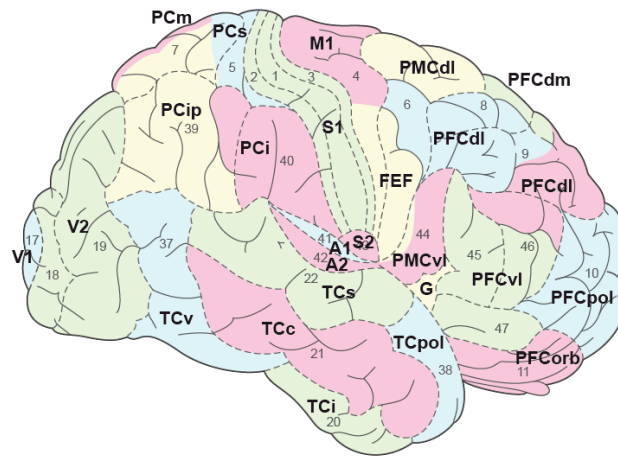
TVB: *reg_13*



Simulation

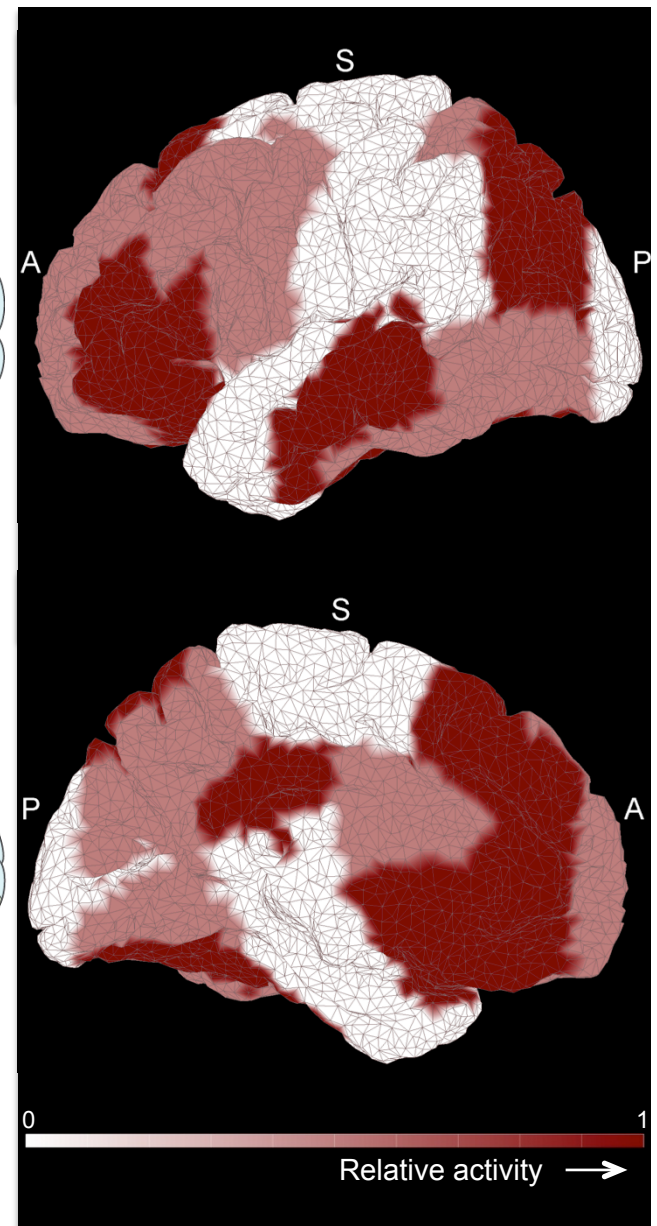


Mapping

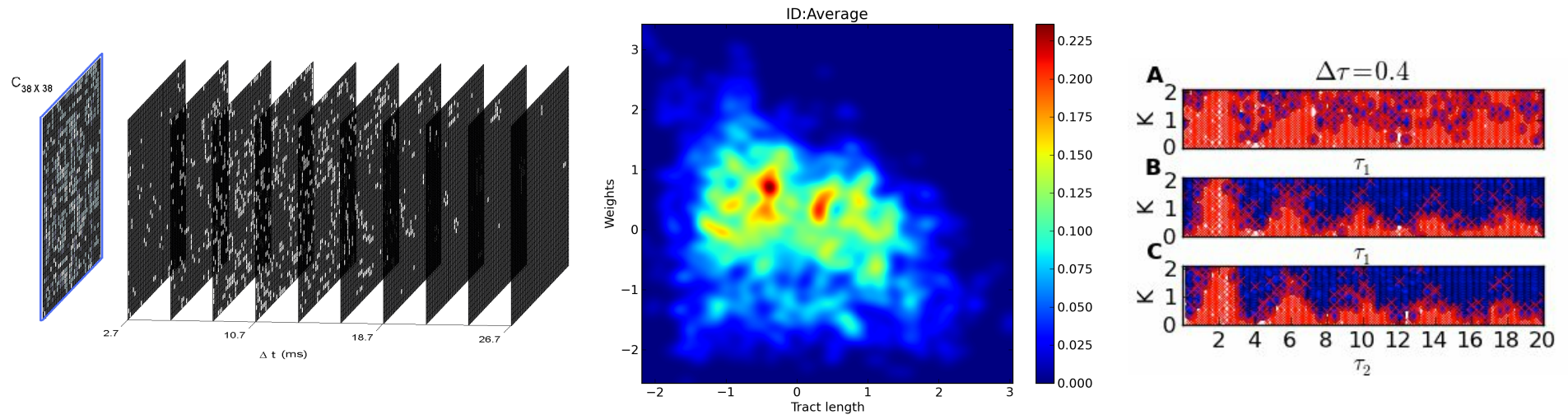


| Parcellation | No. of areas |
|--------------|--------------|
| Brodmann | 52 |
| TVB | 37 |

Default-mode



Damoiseaux et al. (PNAS) 2006



Structure within the space-time of the couplings of the full brain matrix exists (two clusters)

The clusters generate two dominant time delays with a preference to synchronize in well defined regions of the parameter space

Not only time-scale separation can decouple subnetworks, but also structure in the space-time of the couplings

Thank you

Codebox Research

BrainScaleS

James S. McDonnell Foundation

Parham Aram

Randy McIntosh

Andreas Spiegler

Gustavo Deco

Marmaduke Woodman

Enrique Hansen