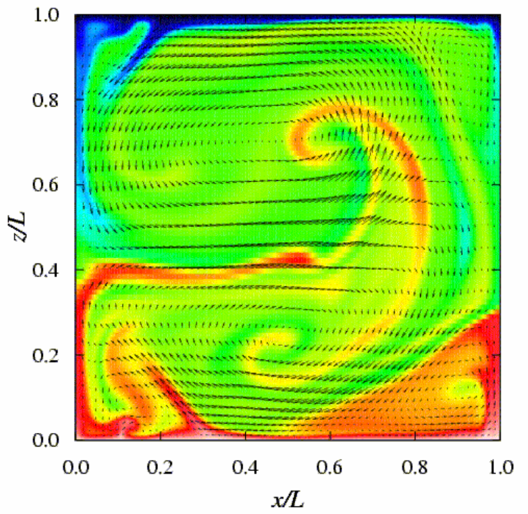
Flow organization in highly turbulent thermal convection

by



Siegfried Grossmann

Fachbereich Physik der Philipps-Universität Marburg

Ladies and Gentlemen,
dear colleagues and friends,

it is with my greatest pleasure to address our Nestor
and always stimulating academic advisor,
Professor Hermann Haken,
with my sincerest congratulations
on the occasion of his 85th birthday.
I wish to express my
warmest thanks and appreciation
for his leadership and his guidance

Thank you, Hermann,
for being our ideal over all the years,
Kindly accept all my best wishes for you!

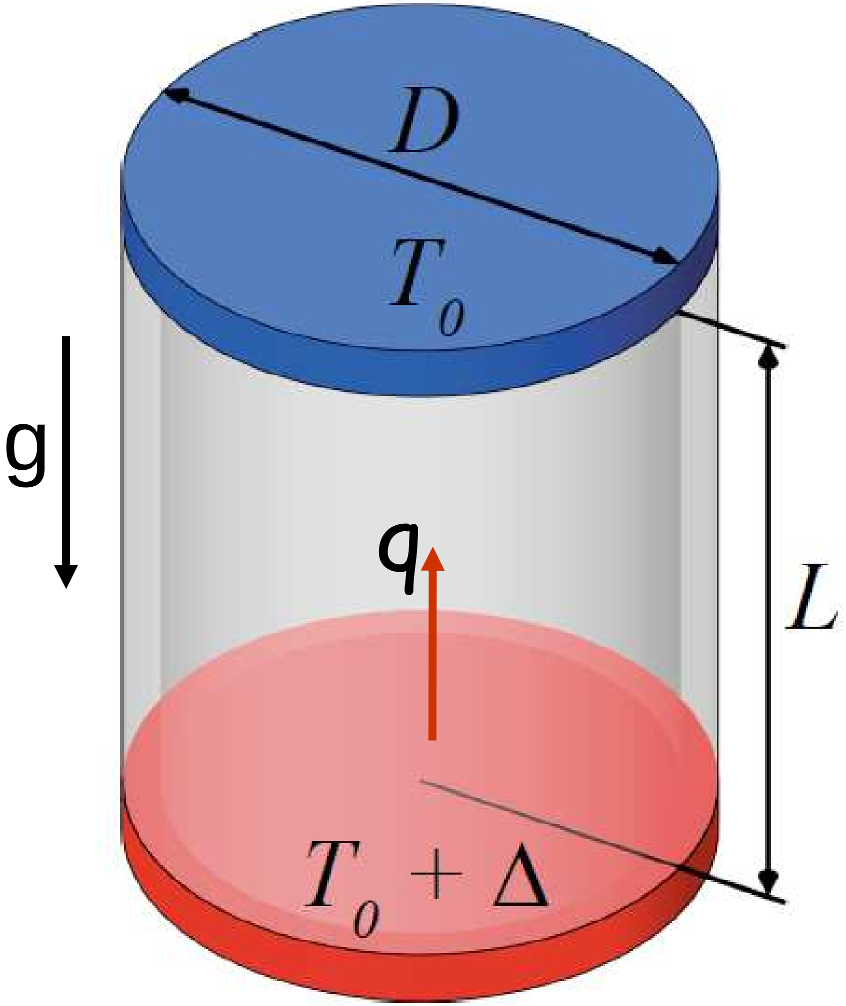
The results I want to present have been obtained in close cooperation with the following colleagues … and more

Detlef Lohse, Enschede

Kazuyazu Sugiyama, Riken
Richard J. A. M. Stevens, Maryland
Roberto Verzicco, Roma

Guenter Ahlers, Santa Barbara
Eberhard Bodenschatz, Göttingen
Denis Funfschilling, Nancy
Xiaozhou He, Göttingen
Ke-Qing Xia, Hong Kong
Quan Zhou, Shanghai

The Rayleigh-Bénard experiment



Control parameter:

$$Ra = \frac{\beta_p \, g \, L^3 \, \Delta}{\nu \kappa}$$

$$Pr = \nu / \kappa$$

Response:

$$Nu = \mathbf{q} \, / \, \Lambda \Delta L^{-1}$$

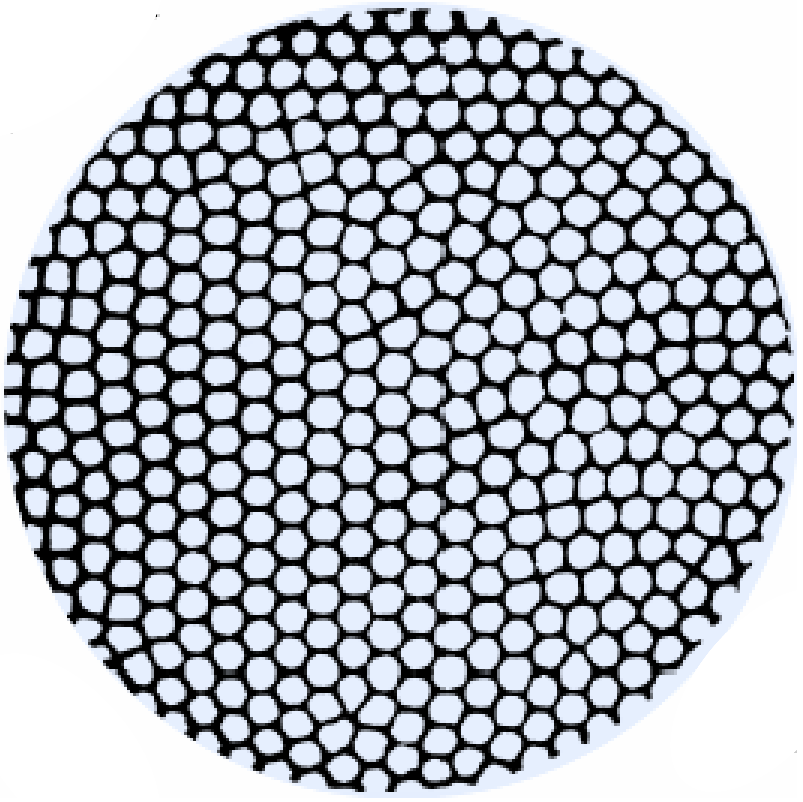
$$Re = U \, L \, / \nu$$

Pattern formation
in Bénards historical experiment
Henri Bénard (1874 – 1939)
PhD thesis in Paris 1900

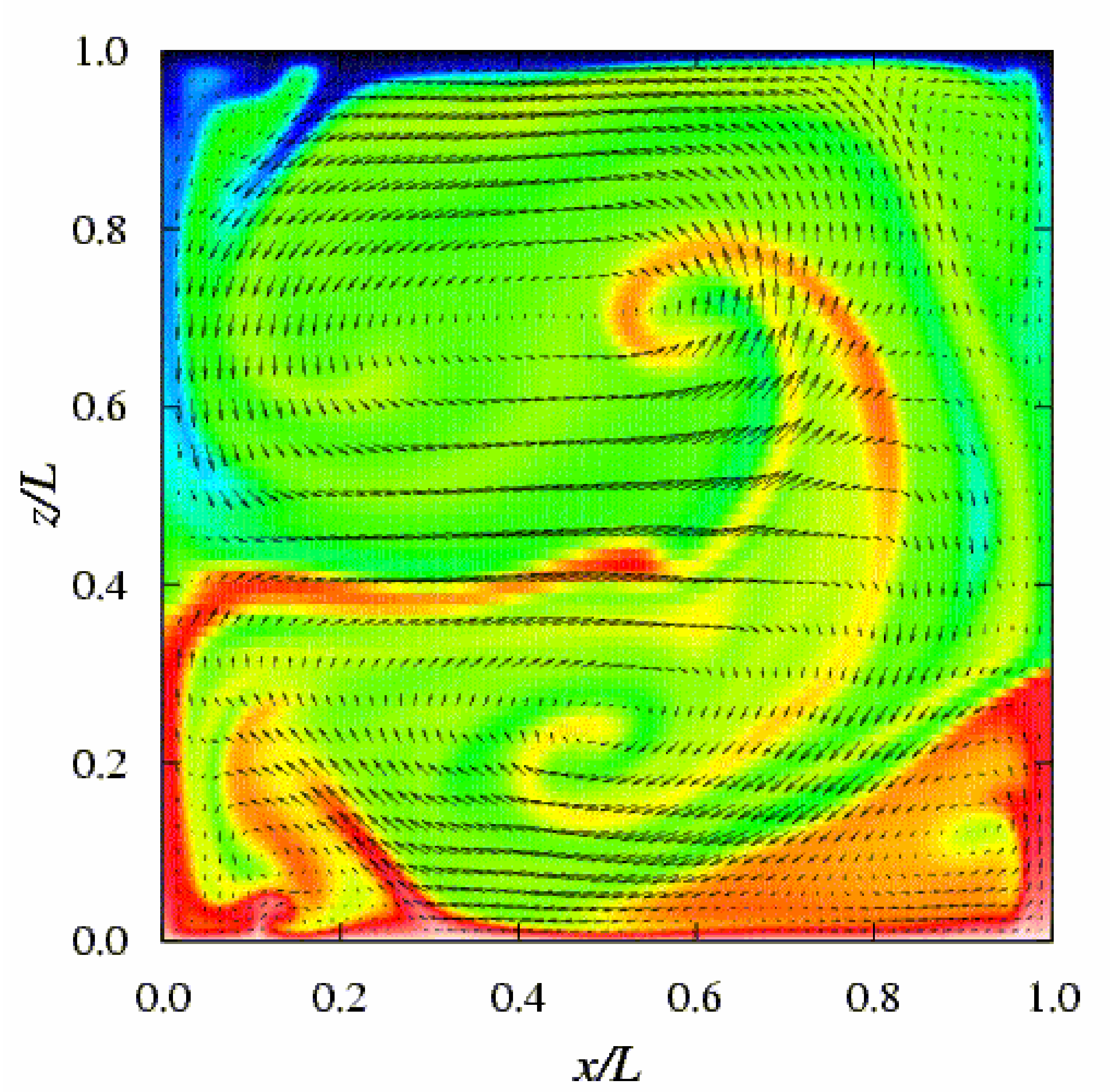
Δ = 100°C - 20°C
= 80K

L = 0.81 mm

Ra ≈ 1000
to 2000



Snapshot of the flow and thermal field



2dim DNS

$Ra = 10^8$
$Pr = 4.3$
$\Delta = 40\ K$

$Nu \approx 26$

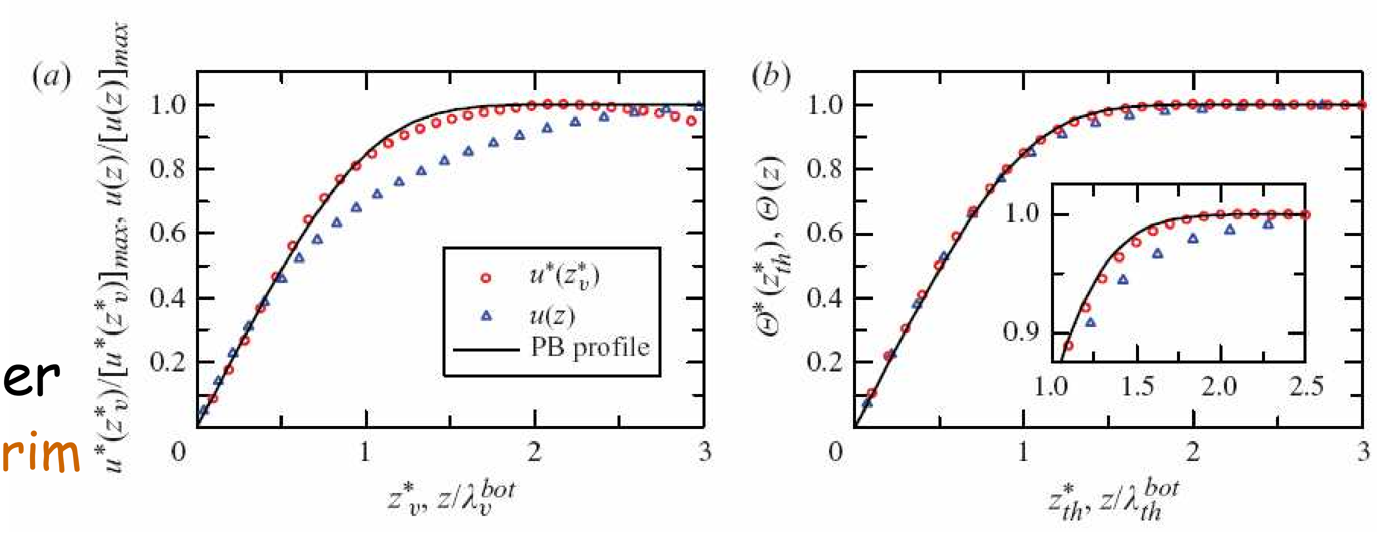
$Re \approx 1000$

- Kaz Sugiyama,E.Calzavarini,SGn,D.Lohse, JFM637(2009)105,Flow organization ...
- Same authors: ETC12 (2009) 479 Flow amplitudes
- Sugiyama,Ni,Stevens,Chen,Zhou,Sun,SGn,Xia,Lohse, PRL105(2010)034503 Flow reversals

Experimental and numerical verification of Prandtl-Blasius profile, if dynamically rescaled

water

experim



Ra = 10^8
Pr = 4.3
water

experiment

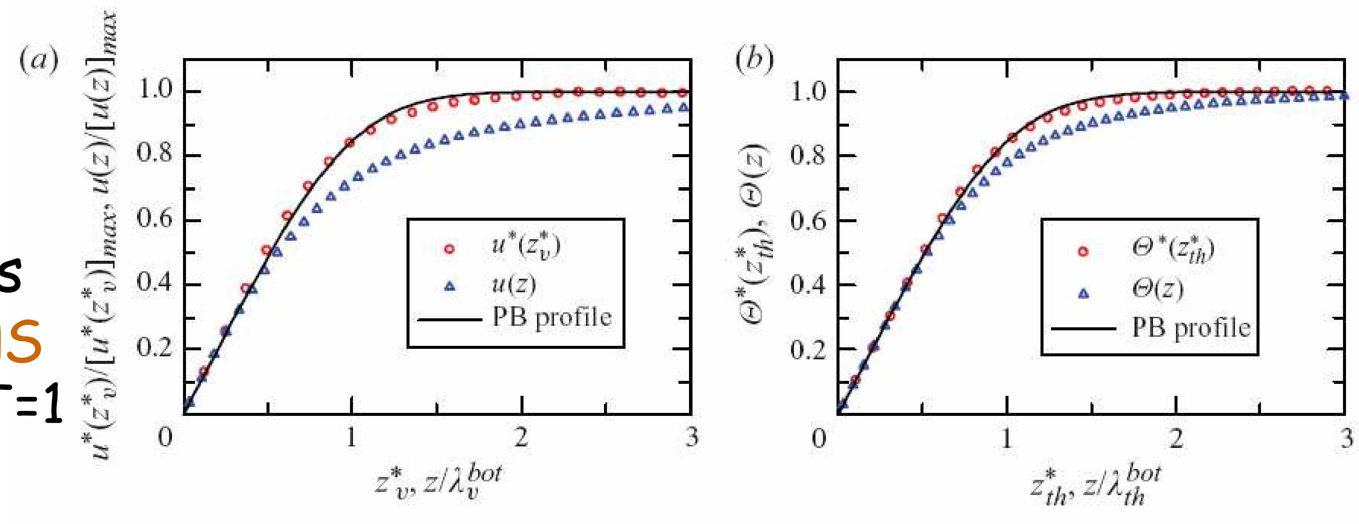
velocity

temperature

gas

DNS

2-d, Γ=1

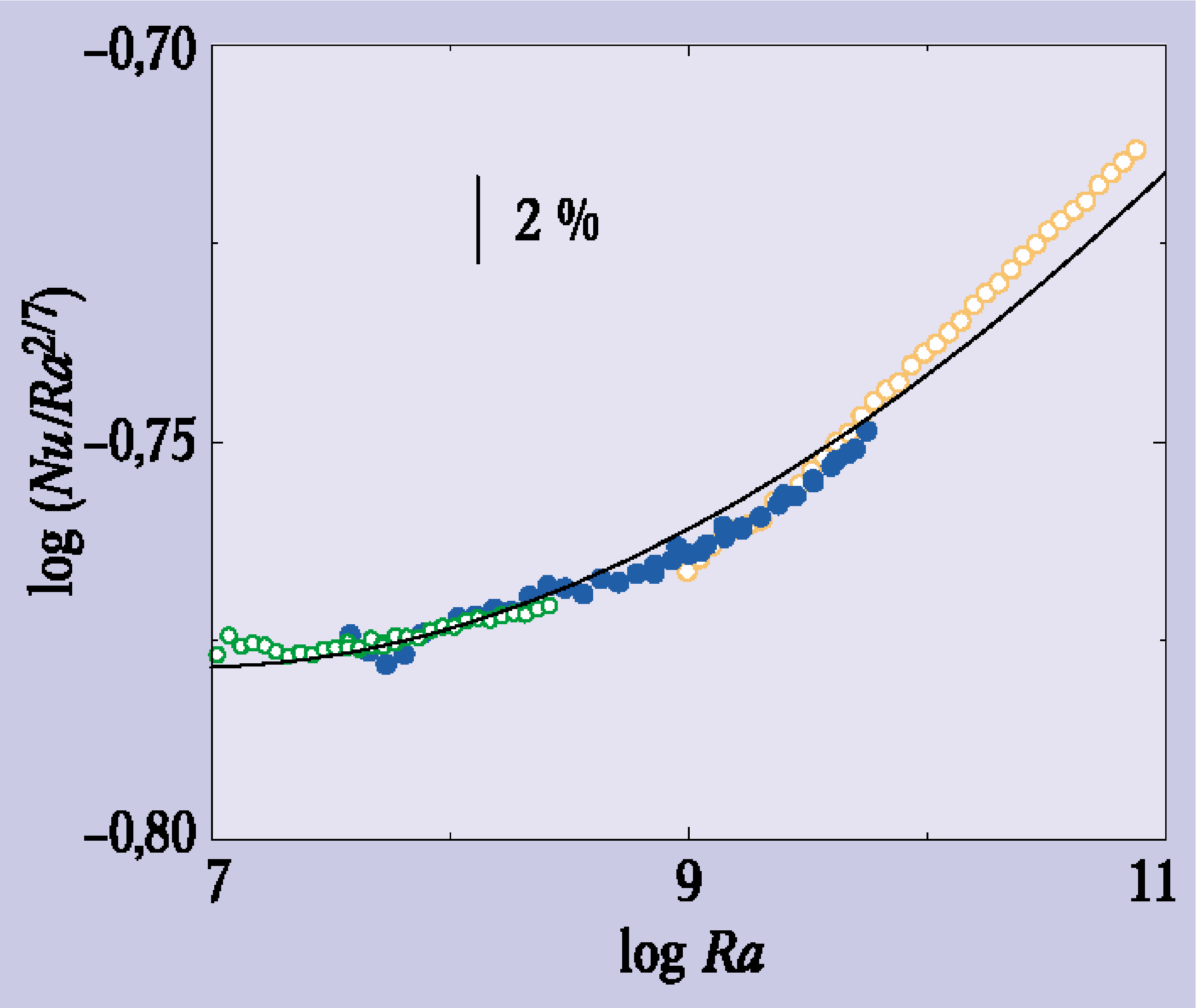


Ra = 10^9
Pr = 0.7
gas

DNS

Zhou, Stevens, Sugiyama, SGn, Lohse, Xia, JFM **664**(2010)297-312

Nu(Ra)



Theoretical prediction GL 2000 etc: no power law!

Experimental confirmation that there is no power law in Nu(Ra)

Ahlers et al., PRL2001

Why do the scaling exponents depend on Ra,Pr?

Decompose central non-equilibrium quantity into bulk and BL parts!

dissipation rate $\tilde{\varepsilon}_u = \frac{\varepsilon_u}{\nu^3 L^{-4}} = Pr^{-2} Ra (Nu - 1)$

▶ $\tilde{\varepsilon}_u = \quad \text{BL-part} \quad + \quad \text{bulk-part}$

▶ $\tilde{\varepsilon}_u = \quad \sim \nu (U/\delta)^2 \cdot \delta / L + \quad \sim U^3 / L$

take Prandtl's law for BL width: $\delta/L = a/\sqrt{Re}$ → BL part ~ $Re^{5/2}$

thermal flux J = $\kappa \Delta L^{-1} Nu = -\kappa \partial_3 \langle \theta \rangle_{A,t} + \langle u_3 \theta \rangle_{A,t}$

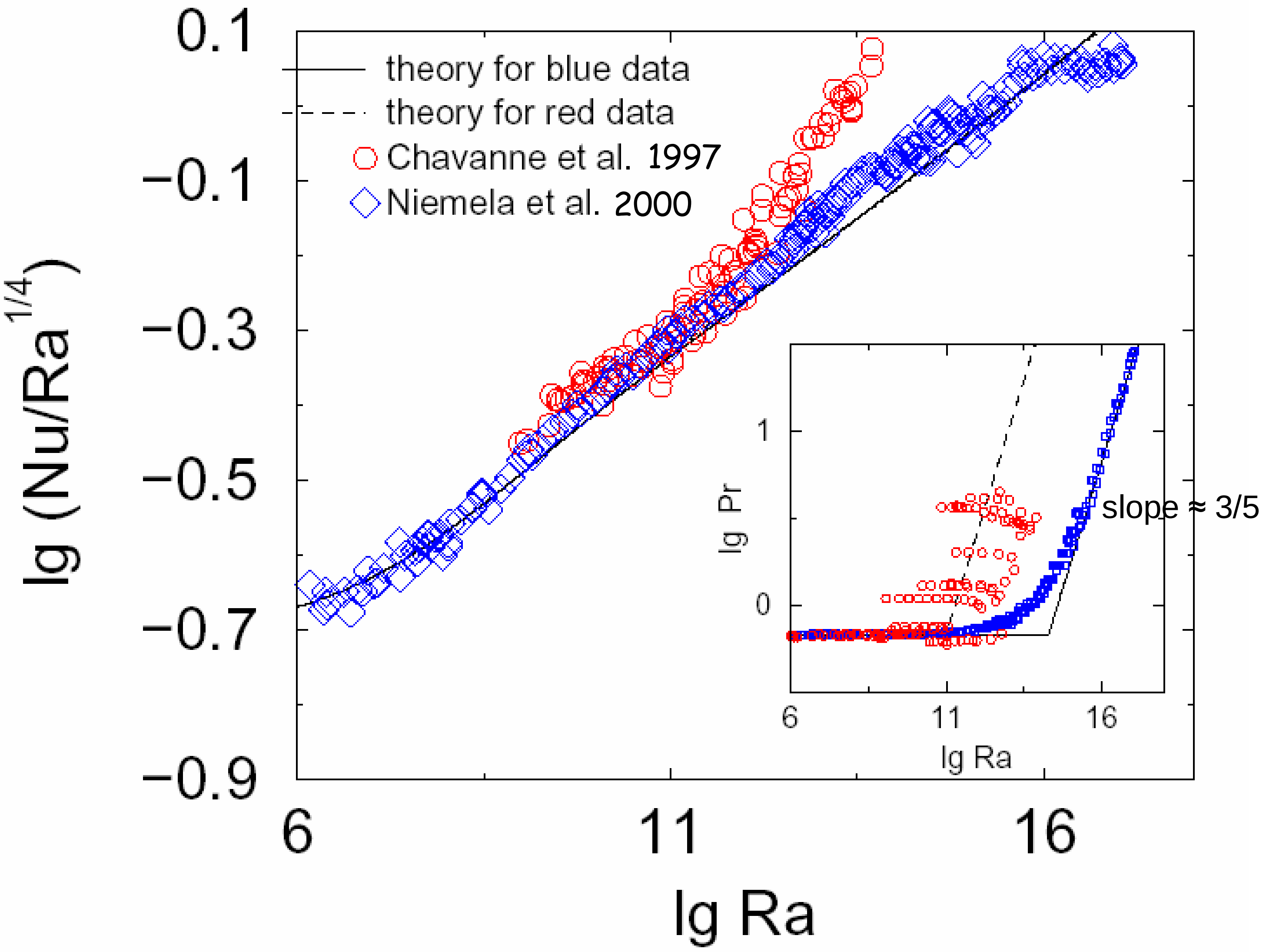
▶ $Nu \quad = \quad \text{BL-part} \quad + \quad \text{bulk-part}$

▶ $Nu \quad = \quad \sim \quad \frac{L}{\lambda} \quad + \sim \quad \frac{U}{\kappa L^{-1}}$

BL eq. $UL^{-1} \sim \kappa \lambda^{-2}$ ▶ $Nu \quad = \quad \sim \sqrt{Pr\, Re} + \sim \quad Pr\, Re$

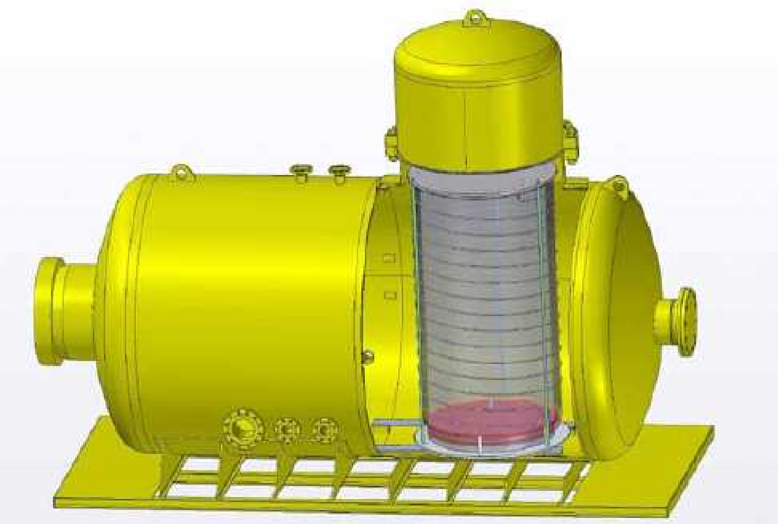
Pohlhausen Re-Pr-dependence ; “lower” or “upper” cases: U or $U \cdot \lambda / \delta$

The ultimate range mystery



Grenoble: Chavanne et al., PRL 79 (1997) 3648 and PoF 13 (2001) 1300
Oregon: Niemela et al., Nature 404 (2000) 837; J.Low.Temp.Phys.143(2006)163

“High pressure convection facility”



The Göttingen U-Boot, MPI DSO
RB-facility: M_{tot}= 2 000 kg

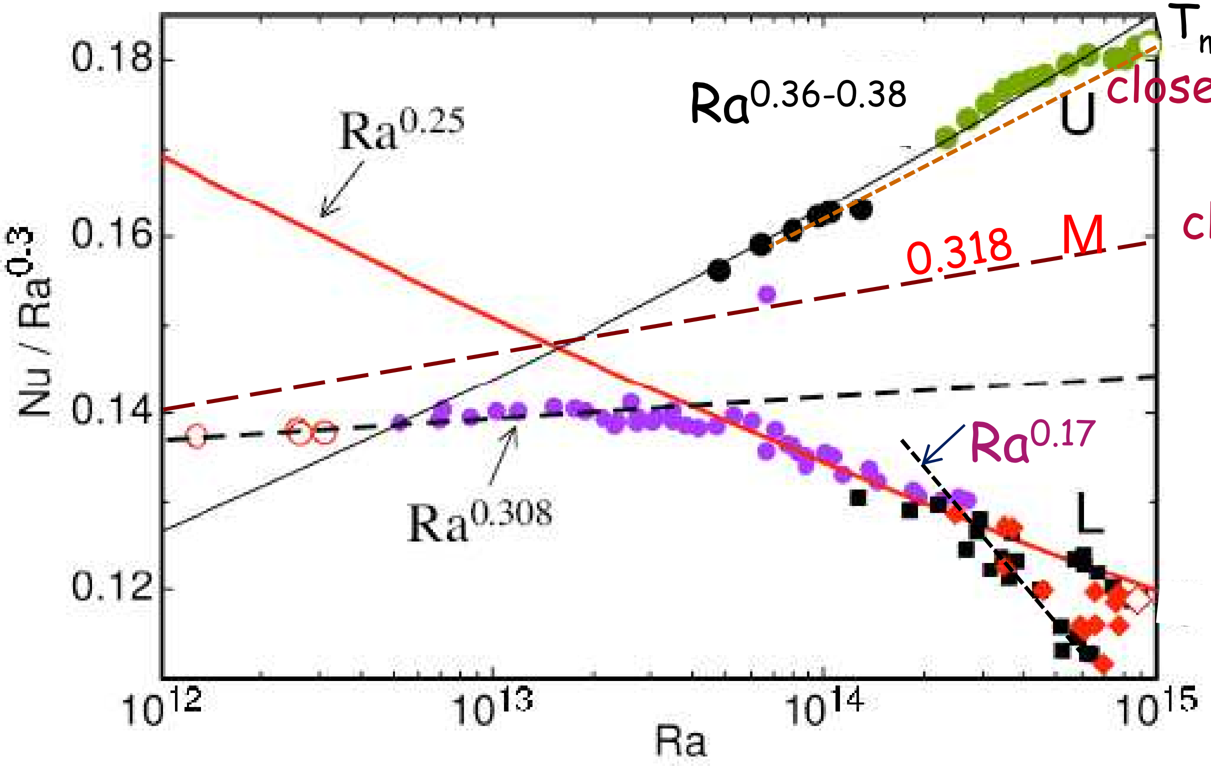
G.Ahlers, D.Funfschilling,
E. Bodenschatz,
NJP 11 (2009) 123001

height $L = 2.24$ m
diameter $D = 1.12$ m, $\Gamma = 0.5$
(He, Air) N_2; SF_6 , $Pr \approx 0.86$
pressure p up to 19 bar,
$T_m \approx 25°C$, $\Delta T \approx 10 - 12$ K

$Ra \approx 10^{15}$, $Nu \approx 5000$, $Re \approx 10^6$

$\lambda/L \approx 10^{-4}$, $\lambda_{Uboot} \approx 0.22$ mm
$\delta/L \approx 5 \times 10^{-4}$, $\delta_{Uboot} \approx 1.12$ mm

Several ultimate range states



open $T_m \approx 21°C < T_{Uboot}$

closed $T_m < T_{Uboot}$

closed $T_m > T_{Uboot}$

open $T_m > T_{Uboot}$

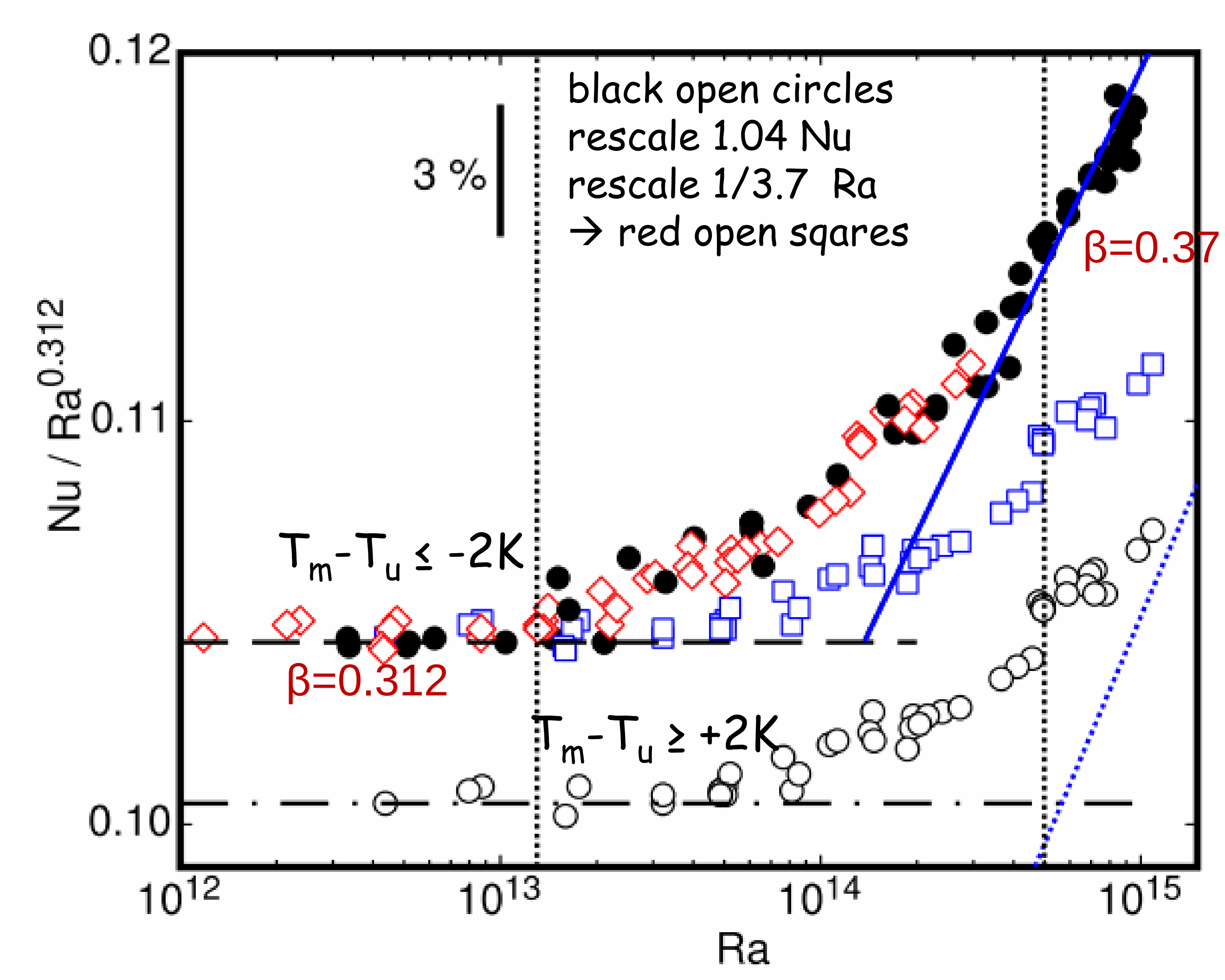
and even more, depending on ambient temperature, $T_m \neq T_{Uboot}$

G. Ahlers, D. Funfschilling, E. Bodenschatz,
Ahlers-talk at Les Houches workshop, Jan 27, 2010

www.hirac4.cnrs.fr/HIRAC4_Talks_files/Ahlers.pdf **and** NJP 13(2011)049401

Heat transport by turbulent RB convection, Pr=0.8, Γ=1/2
G. Ahlers, X.-Z. He, D. Funfschilling, E. Bodenschatz, NJP, 2012

Fig.5, page 15



black open circles
rescale 1.04 Nu
rescale 1/3.7 Ra
→ red open sqares

Laminar → turbulent transition in BL

if $Re_s = \frac{U\delta}{\nu} = a\sqrt{Re}$ large enough

turbulent if Re_s exceeds $Re_s^* = 420$; others: 320 – 150

→ $Re^* = (Re_s^*/a)^2$ exceeds 7.1×10^5; or $(4.1 - 0.9) \times 10^5$

The “wind” (in region IV_u) is $Re \approx 0.346\, Ra^{4/9}\, Pr^{-2/3}$

→ Transition expected at $Ra^* = A\, Pr^{3/2}$

for Pr = 0.86 transition near $Ra^* = 1.25 \times 10^{14}$;

or = 3.7×10^{13} to 1.22×10^{12}

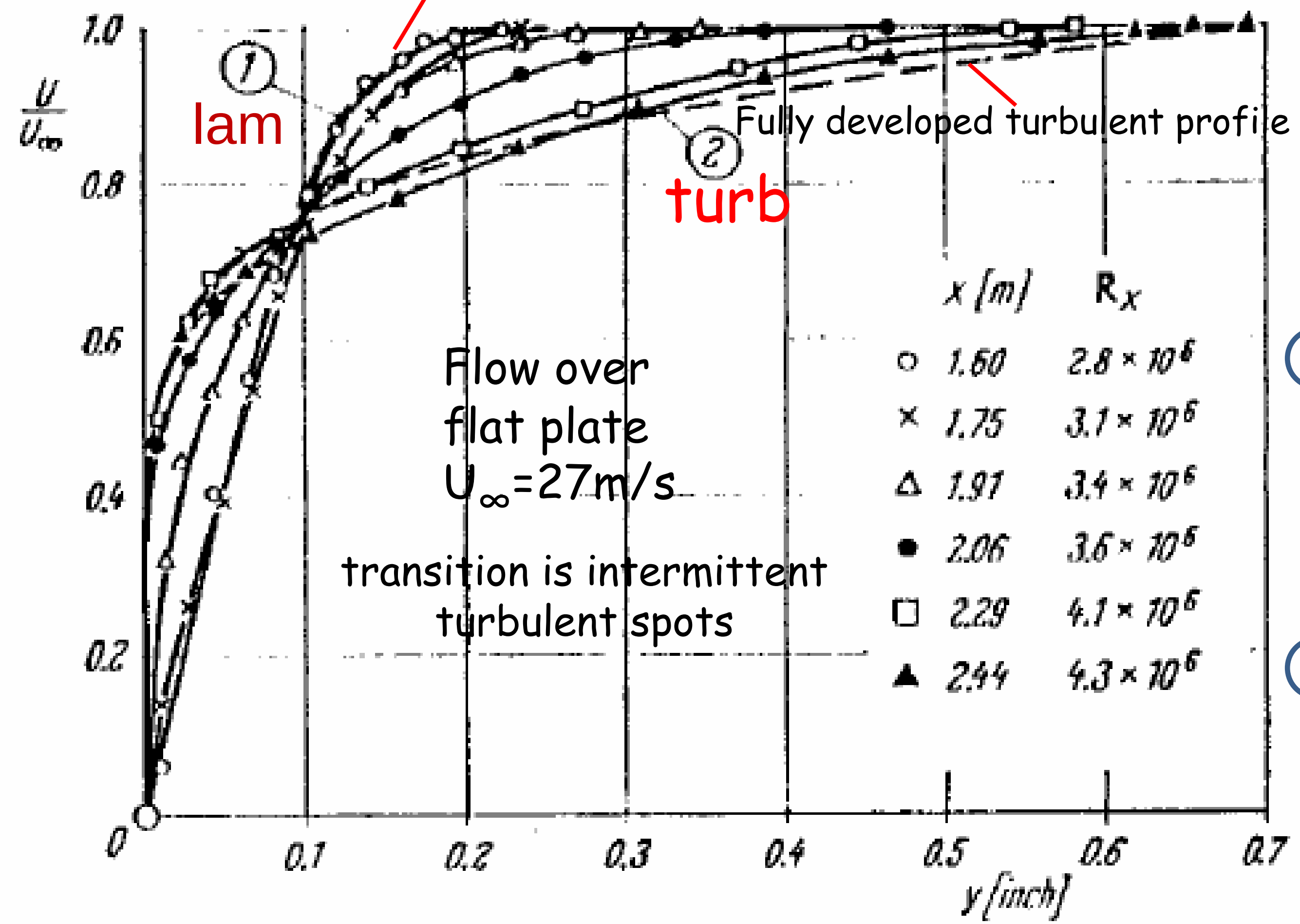
Prandtl BL thickness $\delta^*/L = a/\sqrt{Re^*} = O(6\times10^{-4}) \ldots O(8\times10^{-4})$

– $O(17\times10^{-4})$

i. e. in U-Boot: δ^* = 1.3 mm or 1.8 mm or 3.7 mm

Velocity profiles at laminar-turbulence transition

Fully developed laminar, Prandtl-Blasius profile



lam

turb

Fully developed turbulent profile

Flow over
flat plate
U_∞=27m/s

transition is intermittent
turbulent spots

① PB, lam

② turbulent

Turbulent BL MUCH steeper and MUCH thicker than laminar BL

G.B.Schubauer and P.S.Klebanoff, NACA TN 3489(1955)
Aus: Hermann Schlichting, BL Theory, 7th Edition, 1979, p. 454,Fig.16.4

The *key quantity* in turbulent U-driven flows is the fluctuation scale u_*

Represents the shear stress or drag at the wall:

$$u_*^2 = \sigma_{xz}(0) = p_{xz}(0)/\rho = \nu U_{x|z}(0)$$ ~ slope of profile

$\frac{u_*^2}{U^2}$ wall friction coefficient; u_* determines $\nu_{turb} = \bar{\kappa} u_* z$, κ_{turb}

u_* determines local turbulent dissipation rate $\varepsilon_u(z) = \frac{u_*^3}{\bar{\kappa} z}$

PoF23,045108(2011):

$$\frac{u_*}{U} = \frac{\bar{\kappa}}{\ln\left(Re \frac{u_*}{U} \frac{1}{b}\right)} = \bar{\kappa} F\left(\frac{\bar{\kappa}}{b} Re\right)$$, continued fraction solution

b empirical constant, $b = e^{-\bar{\kappa} B_u}$

Linear viscous sub-layer near wall; $z_* = \nu/u_*$,followed by buffer range, then log-profile; $\delta_{log} = O(L)$, i.e., the turbulent „BL“ occupies full bulk!

for Re ≈ 6×10^5 it is $z_*/L = 0.5 \times 10^{-4}$ instead of $\delta / L = 6.45 \times 10^{-4}$

U-Boot: $z_* \approx 0.11$ mm, while $\delta \approx 1.45$ mm and $\delta_{log} \approx 1$ m

Scaling
in the three ultimate states

(effective, local exponents)

(a_1) u-log-turb, θ-lam,plume $\quad Nu \sim Ra^{0.14}\, Pr^{0.10}$

$Re \sim Ra^{0.41}\, Pr^{-0.69}$

(a_2) u-log-turb, θ-lam,fluct $\quad Nu \sim Ra^{0.23}\, Pr^{0.16}$

$Re \sim Ra^{0.45}\, Pr^{-0.68}$

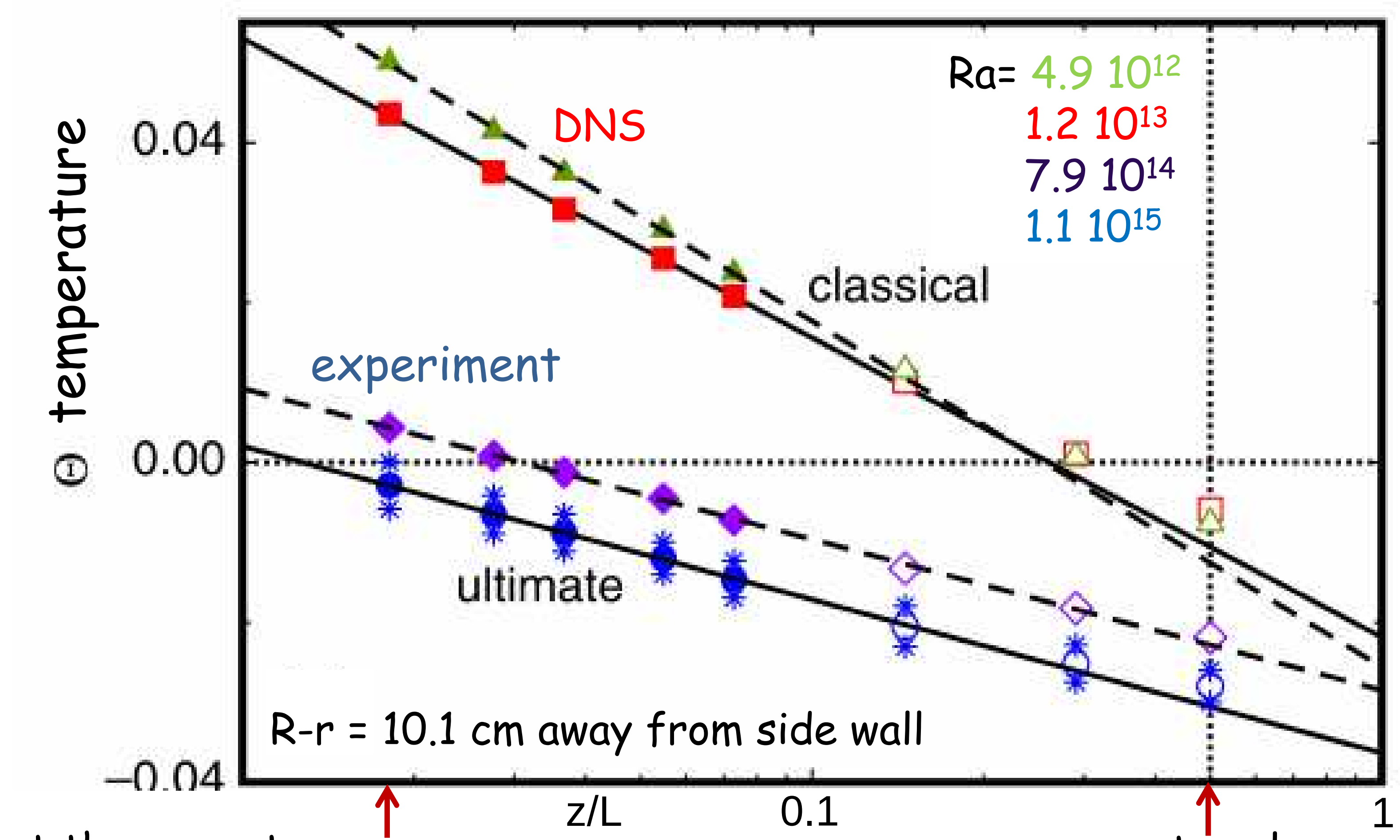
(b) u-log-turb, θ-log-turb $\quad Nu \sim Ra^{0.38}\, Pr^{0.64}$

$Re \sim Ra^{0.50}\, Pr^{-0.50}$

Predictive features of theory: i) Nu- and Pr-exponents differ in all states,
ii) Nu- and Pr-exponents depend on Ra !, iii) thermal and velocity BL profiles differ
iv) In double-turbulent state Re has NO log-correction!

“Law of the wall” - log-profile!

ABFGHLSV. Phys. Rev. Lett. 109(2012)114501



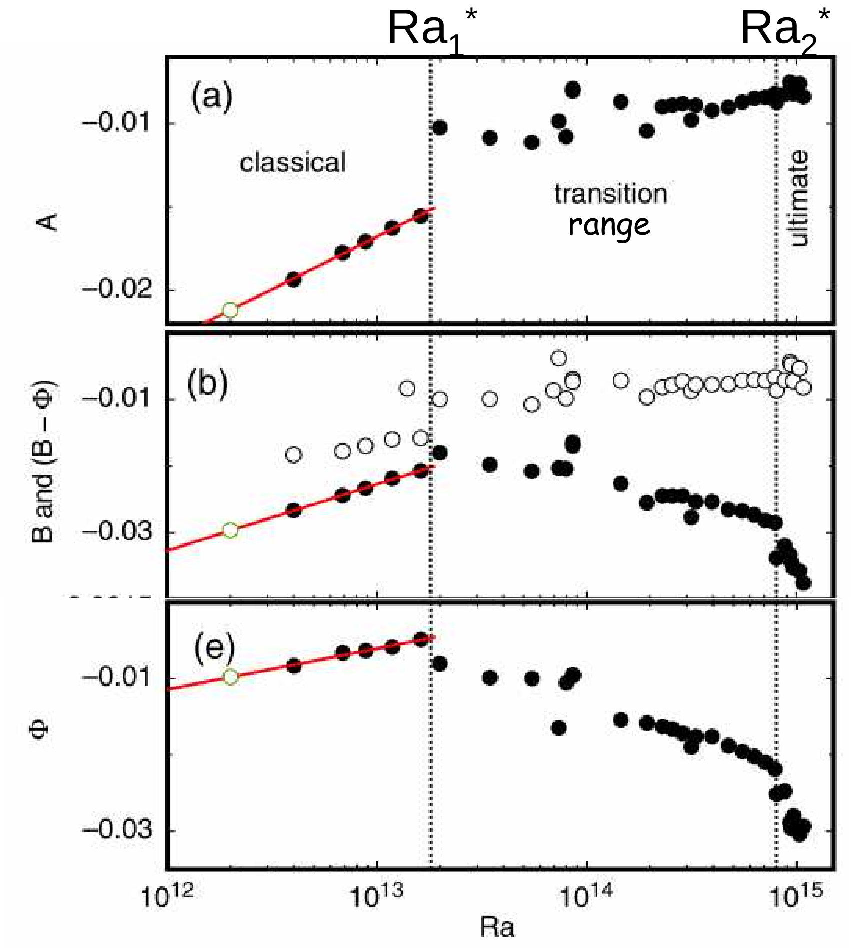
R-r = 10.1 cm away from side wall

lowest thermometer 0.018

center plane

$$(\langle T(z)\rangle - T_m)/\Delta = \Theta(z) = A \cdot \ln\left(\frac{z}{L}\right) + B$$

The profile parameters



Φ nonzero
probably due to yet
unknown NOB effects ?

Theory for ultimate state:

$A \sim Ra^{-0.043}$,
very weak decrease with Ra

B – Φ should correspond to B if OB

$$A = -\frac{\kappa\, Nu}{\bar{\kappa}_\theta L u_*} \approx -\frac{1}{2\bar{\kappa}}\frac{u_*}{U} \approx -0.038$$

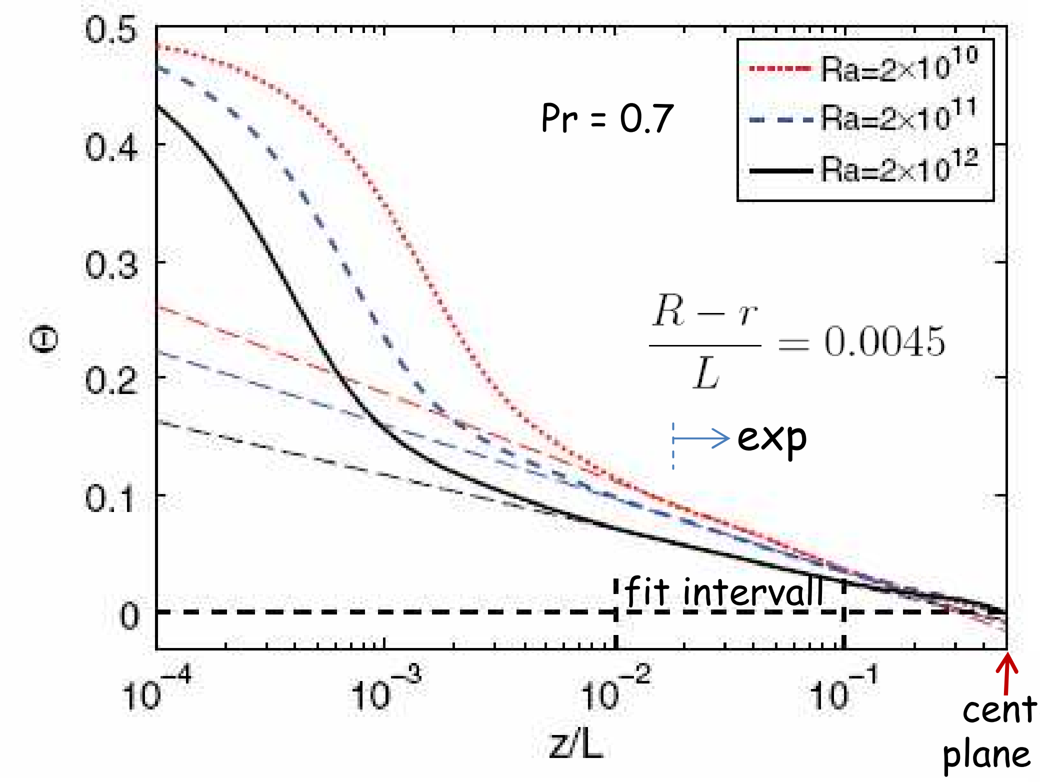
$$B = \ln 2 \cdot A \approx -0.026$$

≈70% less

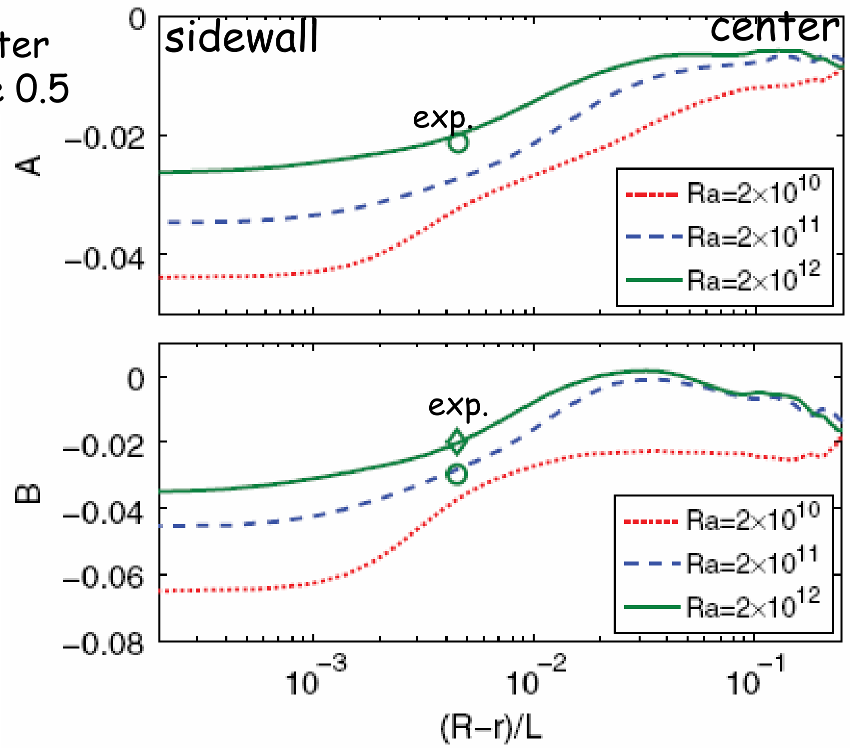
DNS

side wall distance 10.1 cm

|A(r)| increases
with distance r
from center



center
plane 0.5



time and azimuthally averaged
top and bottom profile averaged

excellent agreement
of DNS with data

The logarithmic temperature profile

$$\partial_t T = -\vec{u} \cdot \vec{\nabla} T + \kappa \Delta T,$$ T(z=0) = 0, T(z=L) = -Δ

$$0 = -\vec{U} \cdot \vec{\nabla} \Theta - \overline{\vec{u}' \cdot \vec{\nabla} \vec{\theta}'} + \kappa \Delta \Theta$$

Reynolds & mixing length ansatz: $\overline{\vec{u}'\theta'} \approx -\kappa_{turb}(z)\partial_z \Theta$

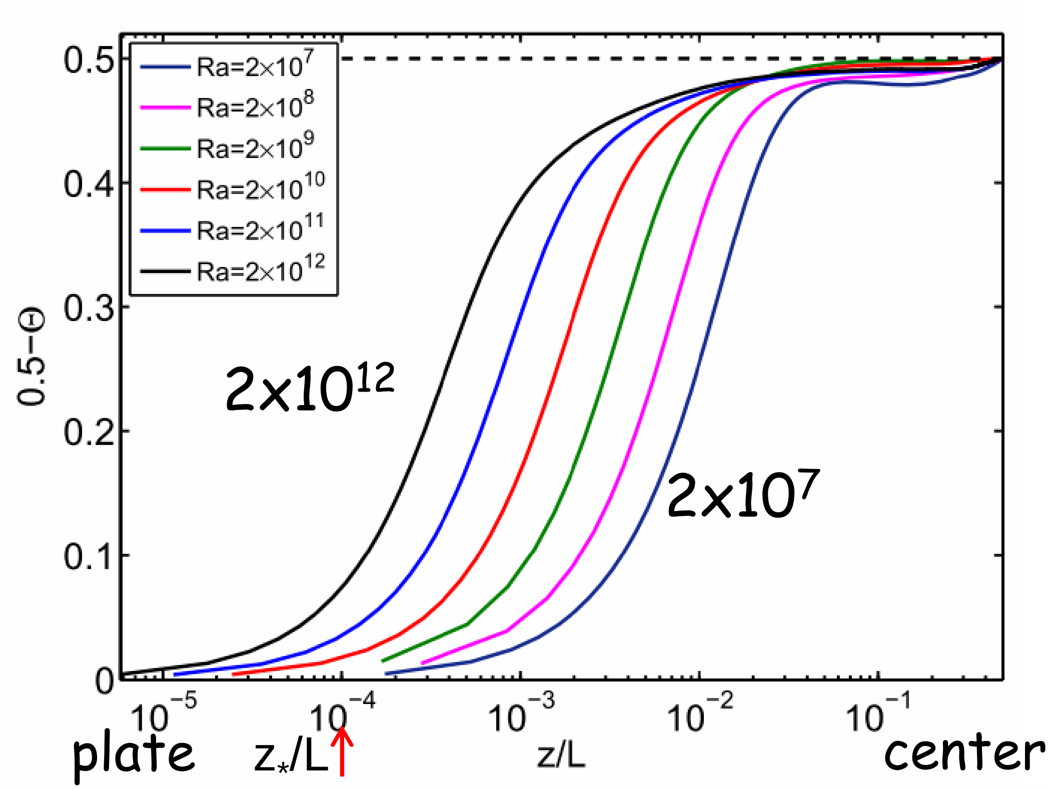
$(\kappa_{turb}(z)+\kappa)\partial_z \Theta(z) = \kappa \partial_z \Theta(0) \equiv J$ or $Nu \cdot \kappa \Delta / L$

$\kappa \gg \kappa_{turb} = \bar{\kappa}_\theta z u_*$ linear sublayer $0 \leq z \leq Pr^{-1} z_* / \bar{\kappa}_\theta$

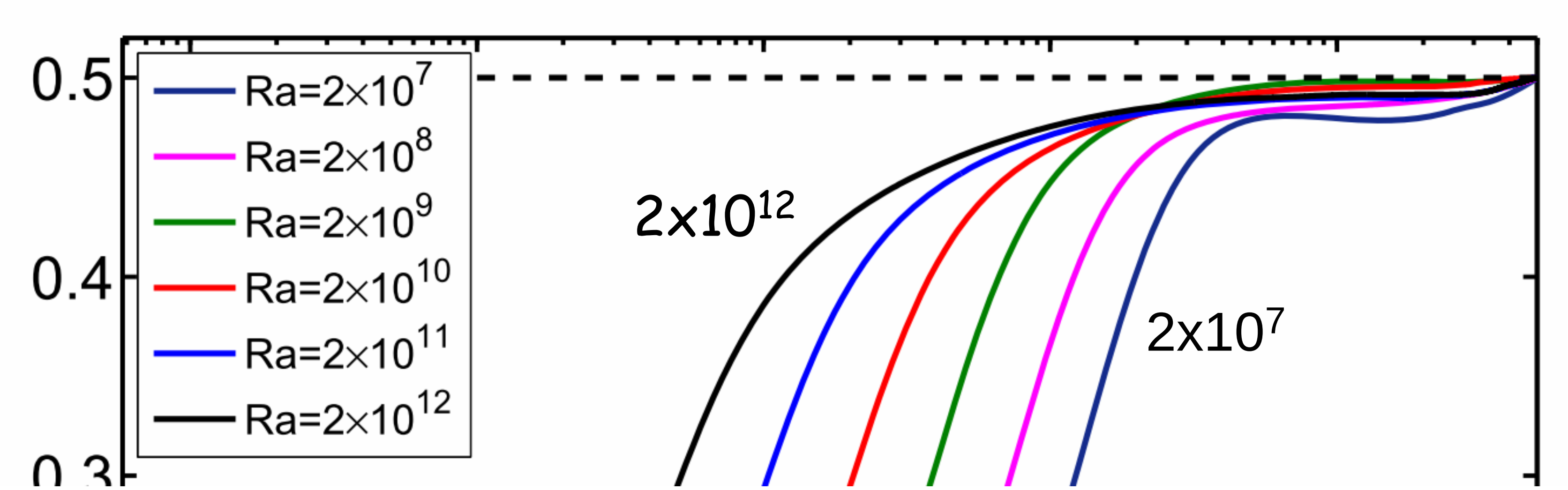
$\kappa \ll \kappa_{turb}$ "law of the wall" $\Theta(z) = -\frac{J}{\bar{\kappa}_\theta u_*}\left(\ln(\frac{z}{z_*}) + f(Pr)\right)$

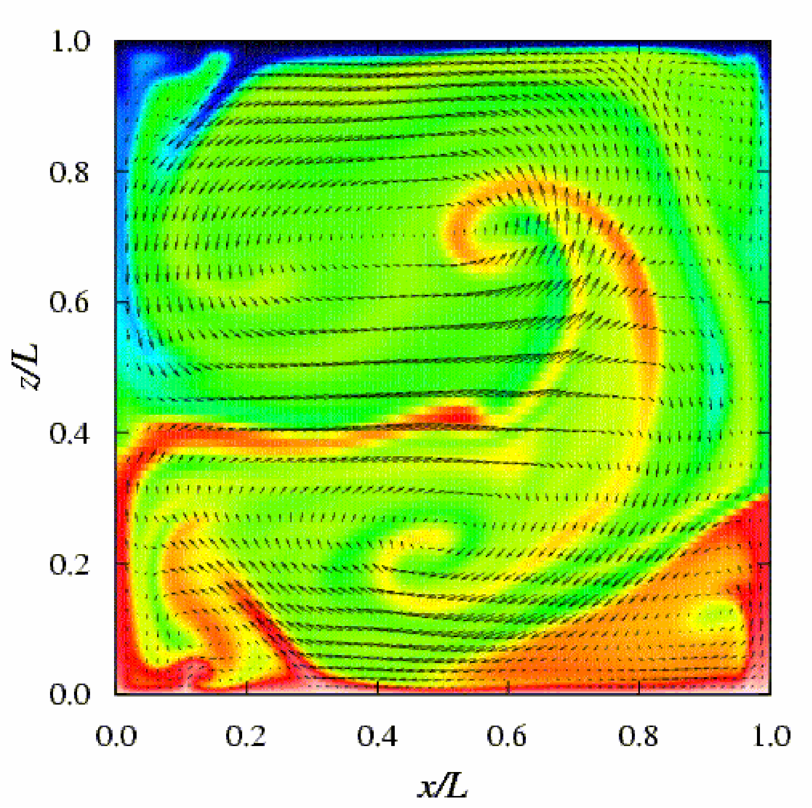
$$A = -\frac{\kappa Nu}{\bar{\kappa}_\theta u_* L} \approx -\frac{1}{2\bar{\kappa}} \cdot \frac{u_*}{U} \qquad B = \ln 2 \cdot A$$

Temperature profiles in units of Δ, measured at wall distance $z/L=0.15$



Transition to log profile: beyond 10^9





Thank you

References:

1. G. Ahlers, E. Bodenschatz, D. Funfschilling, S. Grossmann, X.-Z. He, D. Lohse, R. J. A. M. Stevens, R. Verzicco, Logarithmic Temperature Profiles in Turbulent Rayleigh-Bénard Convection, Phys. Rev. Lett. 09, 114501 (2012)
2. S. Grossmann, D. Lohse, Multiple scaling in the ultimate regime of thermal convection, Phys. of Fluids 23, 045108 (2011)
3. S. Grossmann, D. Lohse, Logarithmic temperature profiles in the ultimate range of turbulent convection, to appear in Phys. Fluids, 2012 ; ArXiV 1208.2597v1,13Aug2012
4. Q. Zhou, K. Sugiyama, R. J. A. M. Stevens, S. Grossmann, D. Lohse, K.-Q. Xia, Horizontal Dependence of Velocity and Temperature Boundary-Layers in two-dimensional Numerical Turbulent Rayleigh-Bénard Convection, Phys. of Fluids 23, 125104 (2011)
5. D. Funfschilling, E.Bodenschatz, G. Ahlers, Search for the ultimate state in turbulent Rayleigh-Bénard convection, Phys.Rev.Lett. 103, 014503 (2009)
6. G. Ahlers, D. Funfschilling, E. Bodenschatz, Transitions in heat transport by turbulent convection, New J. Phys. 11, 123001 (2009)
7. X.-Z. He, D. Funfschilling, E. Bodenschatz, G. Ahlers, Heat transport by turbulent Rayleigh-Bénard convection for $Pr = 0.8$ and $4x10^{11} < Ra < 2x10^{14}$: Ultimate state transition for aspect ratio 1.00, New J. Phys. 14, 063030 (2012)
8. X.-Z. He, D. Funfschilling, H. Nobach, E. Bodenschatz, G. Ahlers, Transition to the Ultimate State of Turbulent Rayleigh-Bénard Convetion, Phys.Rev.Lett.108,024502 (2012)
9. S. Grossmann, D. Lohse, Scaling in thermal convection: A unifying theory, J. Fluid Mech. 407,27(2000); Phys.Rev.Lett.86,3316(2001); Phys.Rev.E 66,016305(2002), PoF 16,4462(2004)