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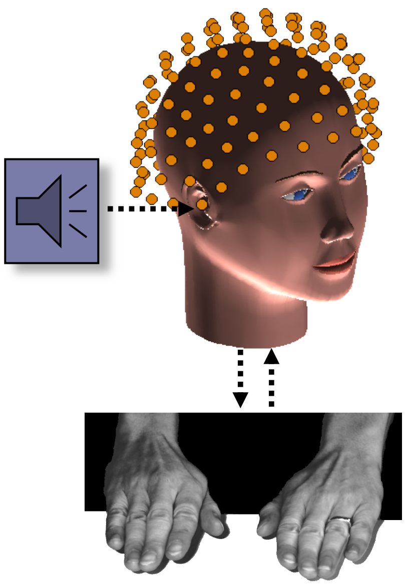
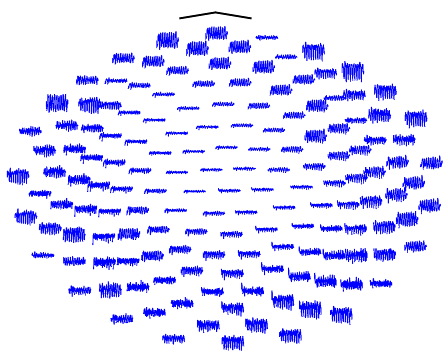
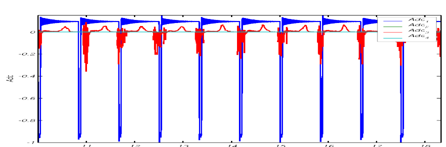
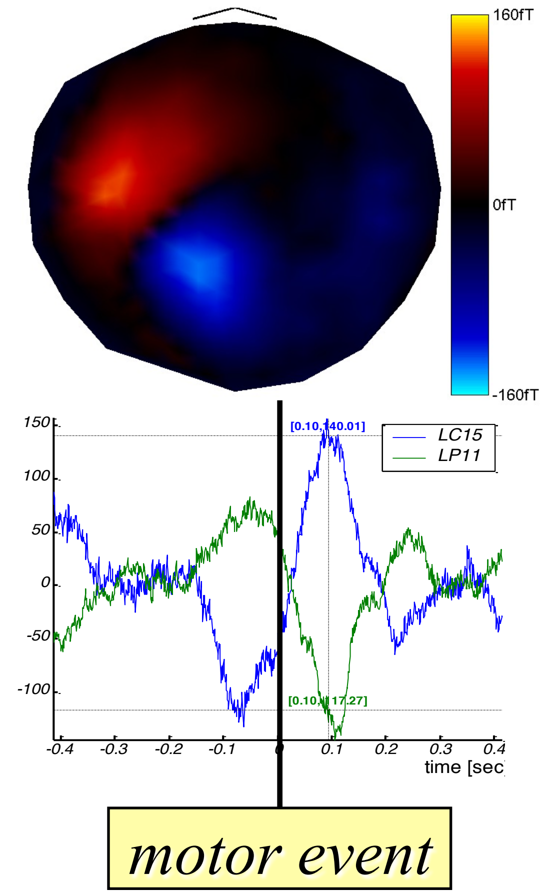


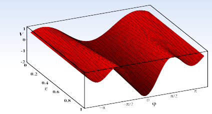


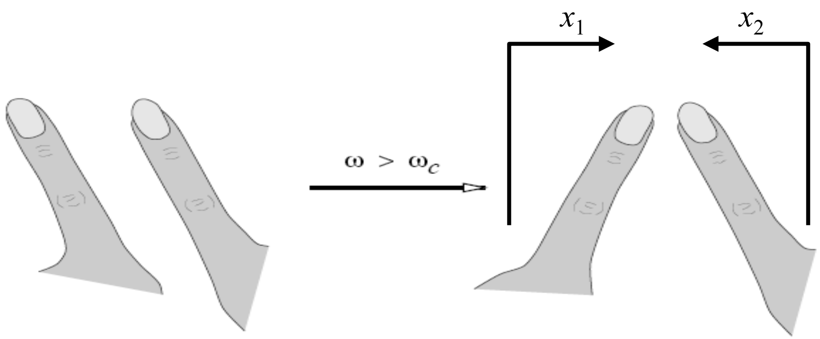


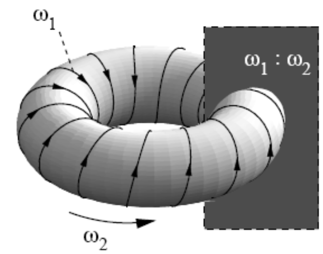


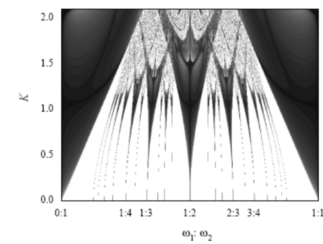
- Dorothy: But how can you talk without a brain?
- Scarecrow: Well, I don't know... but some people without brains do an awful lot of talking.

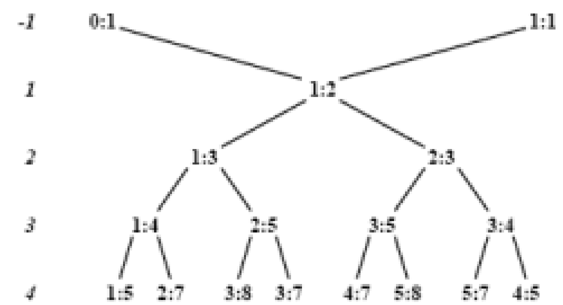
-- The Wizard of Oz

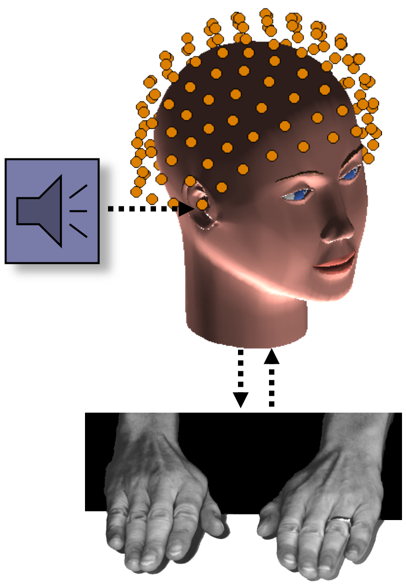


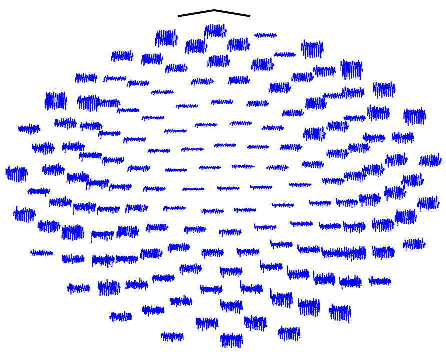


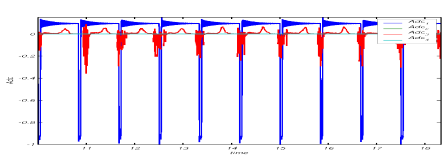






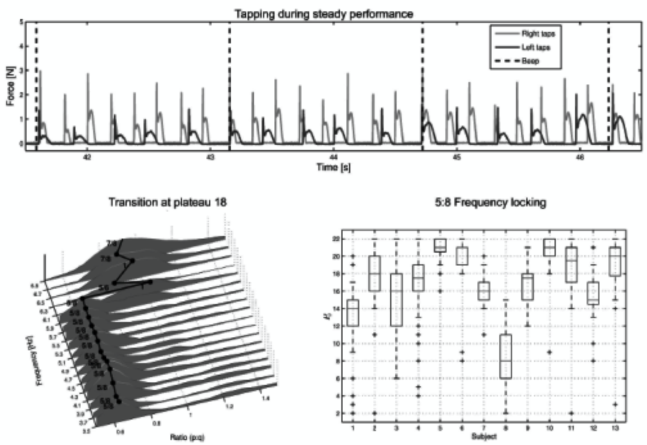


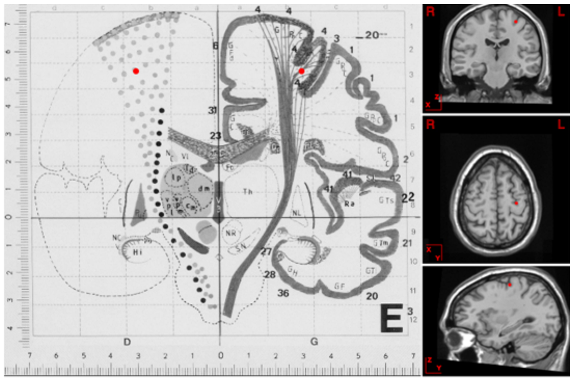


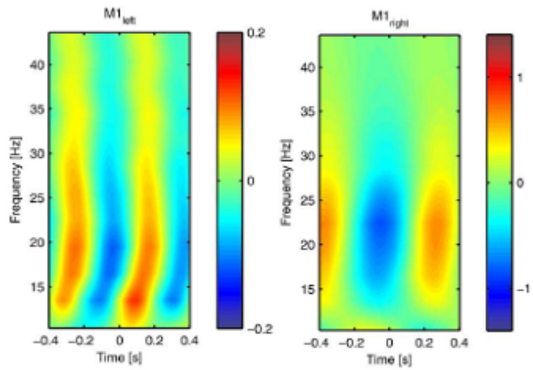


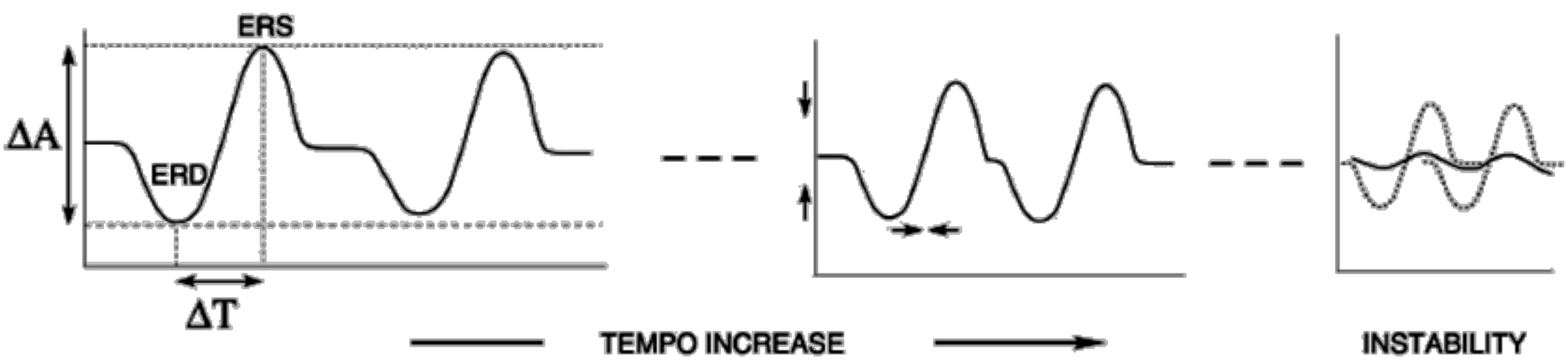
Haken et al., Biolog. Cybern. 1985
Schöner et al., Biolog. Cybern. 1986
Kay et al., J. Exp Psychol. 1987, 1991
Fuchs et al., Biolog. Cybern. 1996
Daffertshofer et al., Physica D 1999
Beek et al., Brain & Cogn. 2002 …

Instabilities in rhythmic bimanual tapping









Houweling et al., *Cerebral Cortex* 2010

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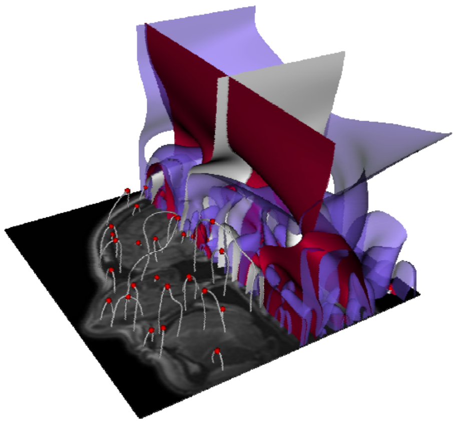

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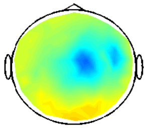
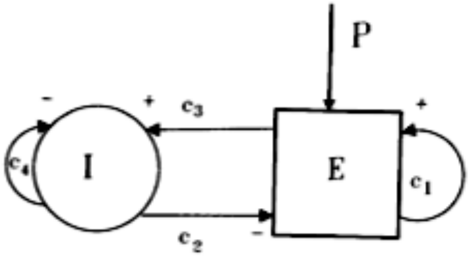


Neural activity

$$\frac{d}{dt}E = -E + S\left(a_e\left[c_1 E - c_2 I - \Theta^e + P\right]\right)$$

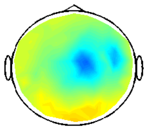
$$\frac{d}{dt}I = -I + S\left(a_i\left[c_3 E - c_4 I - \Theta^i\right]\right)$$



E = (mean) firing rate of a population of excitatory neurons

I = (mean) firing rate of a population of inhibitory neurons

Wilson & Cowan, *Biophys. J.* 1972, Schuster & Wagner, *Biolog. Cybern.* 1990



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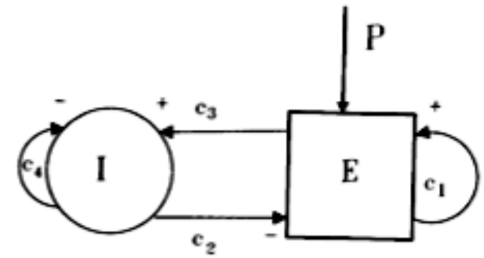
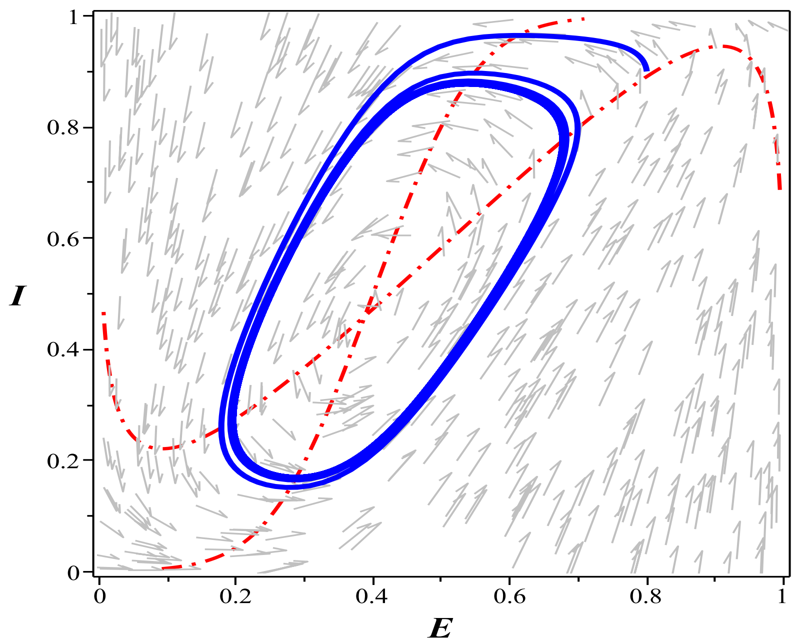
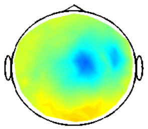




Wilson & Cowan, *Biophys. J.* 1972, Schuster & Wagner, *Biolog. Cybern.* 1990



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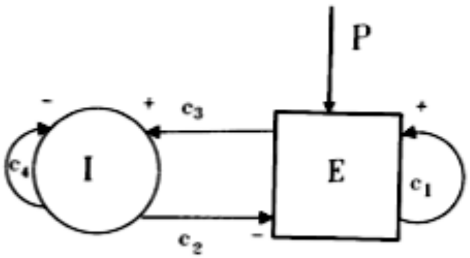
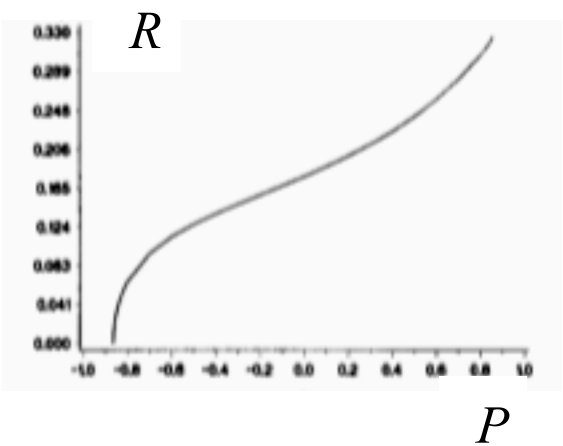
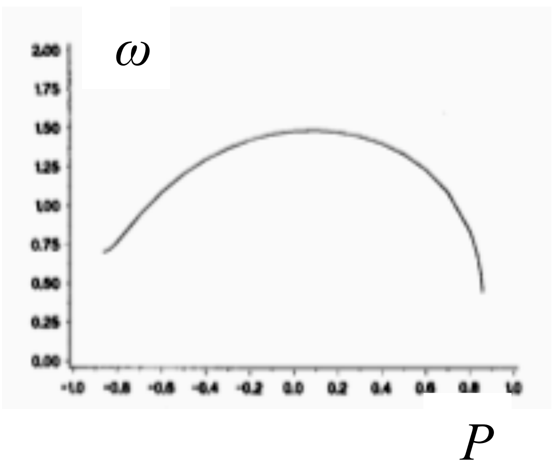
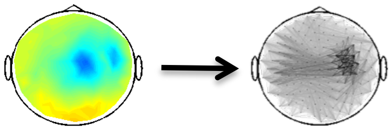




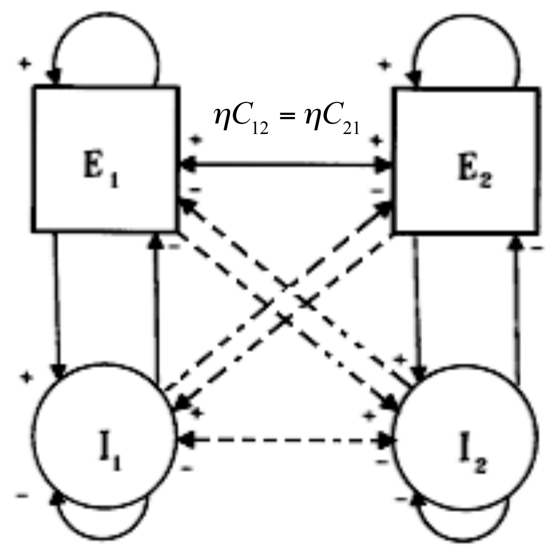


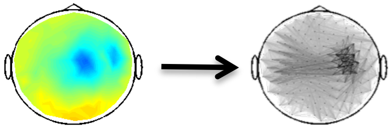


Neural activity

$$\frac{d}{dt}E_k = -E_k + S\left(a_e\left[c_1E_k - c_2I_k - \Theta^e + P_k + \frac{\eta}{N}\sum_{l=1}^{N}C_{kl}E_l\right]\right)$$

$$\frac{d}{dt}I_k = -I_k + S\left(a_i\left[c_3E_k - c_4I_k - \Theta^i\right]\right)$$





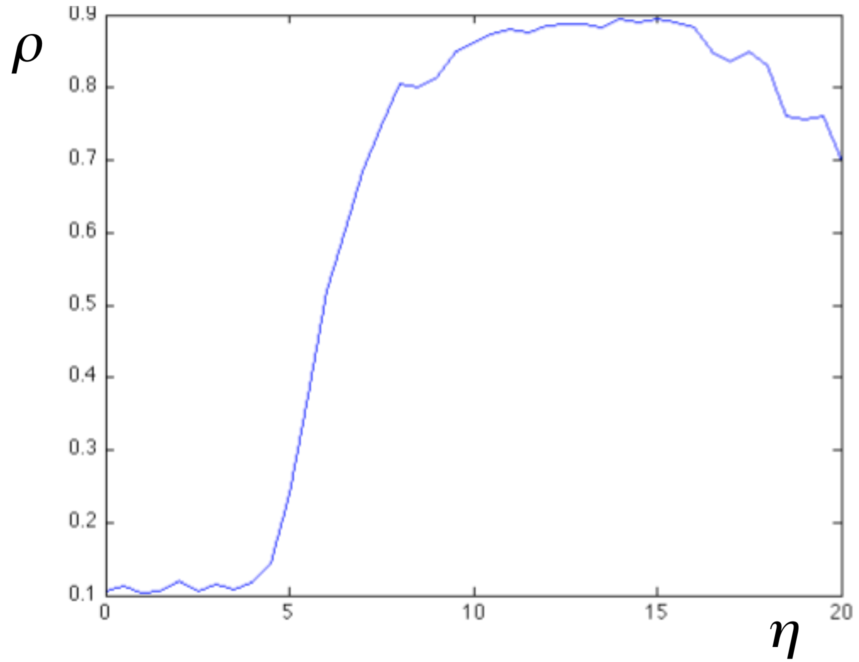
Neural synchronization

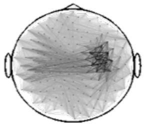
$$\frac{d}{dt}E_k = -E_k + S\left(a_e\left[c_1E_k - c_2I_k - \Theta^e + P_k + \frac{\eta}{N}\sum_{l=1}^{N} C_{kl}E_l\right]\right)$$

$$\frac{d}{dt}I_k = -I_k + S\left(a_i\left[c_3E_k - c_4I_k - \Theta^i\right]\right)$$

phase $\phi_k = \arctan'\left(\frac{E_k}{I_k}\right)$

uniformity $\rho = \frac{1}{N}\left|\sum_{k=1}^{N} e^{i\phi_k}\right|$

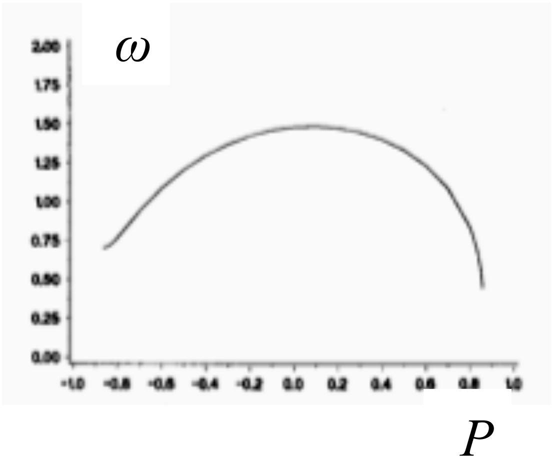


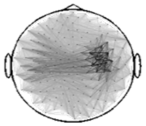


Kuramoto network

$$\dot{\phi}_k = \omega_k + \frac{\eta}{N}\sum_{l=1}^{N} D_{kl}\sin\left(\phi_l - \phi_k\right)$$

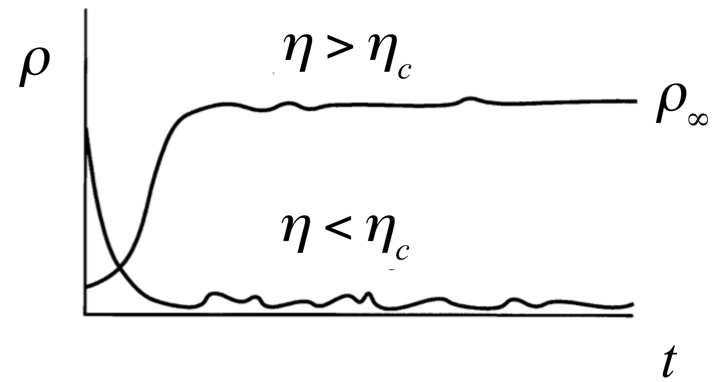
$$\frac{d}{dt}E_k = -E_k + S\left(a_e\left[\cdots + P_k + \cdots\right]\right)$$





Kuramoto network

$$\dot{\phi}_k = \omega_k + \frac{\eta}{N}\sum_{l=1}^{N} D_{kl}\sin\left(\phi_l - \phi_k\right)$$

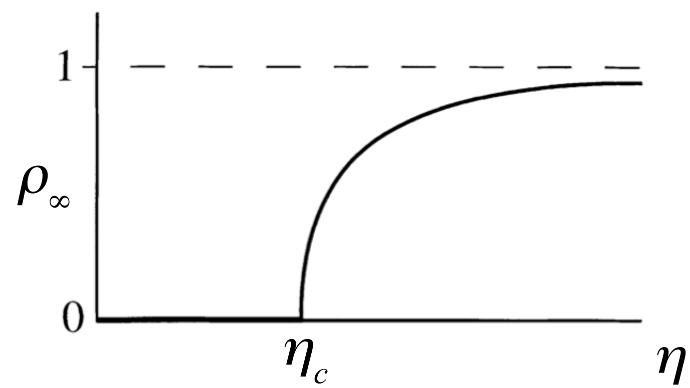


mean field approximation for $D_{kl} = 1$

order parameter $\rho e^{i\psi} = \frac{1}{N}\sum_{k=1}^{N} e^{i\phi_k}$

$$\left[\ \rho = \frac{1}{N}\left|\sum_{k=1}^{N} e^{i\phi_k}\right|\ \right]$$

$$\dot{\phi}_k = \omega_k + \eta\rho\sin\left(\psi - \phi_k\right)$$



Kuramoto, *Chemical Oscillations…*, Springer, 1984, Strogatz, *Physica D*, 2000

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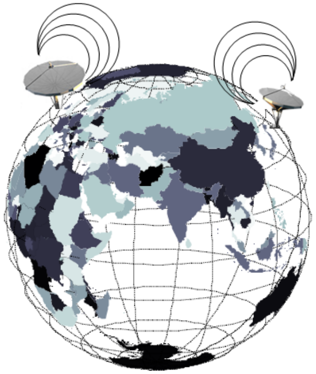

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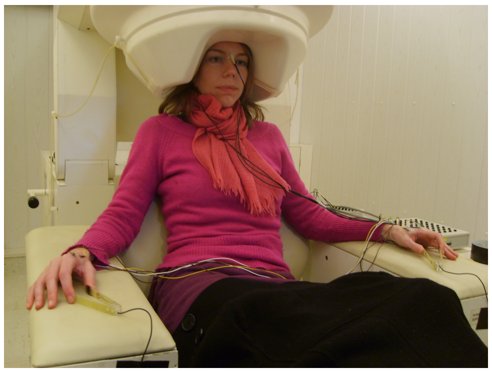
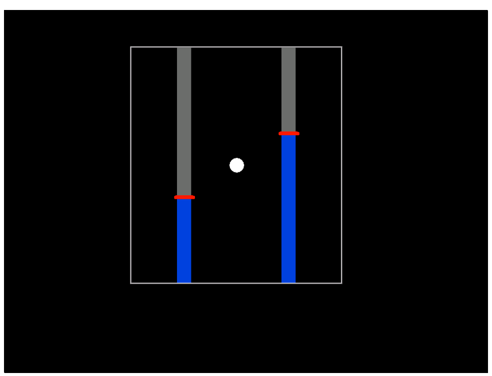
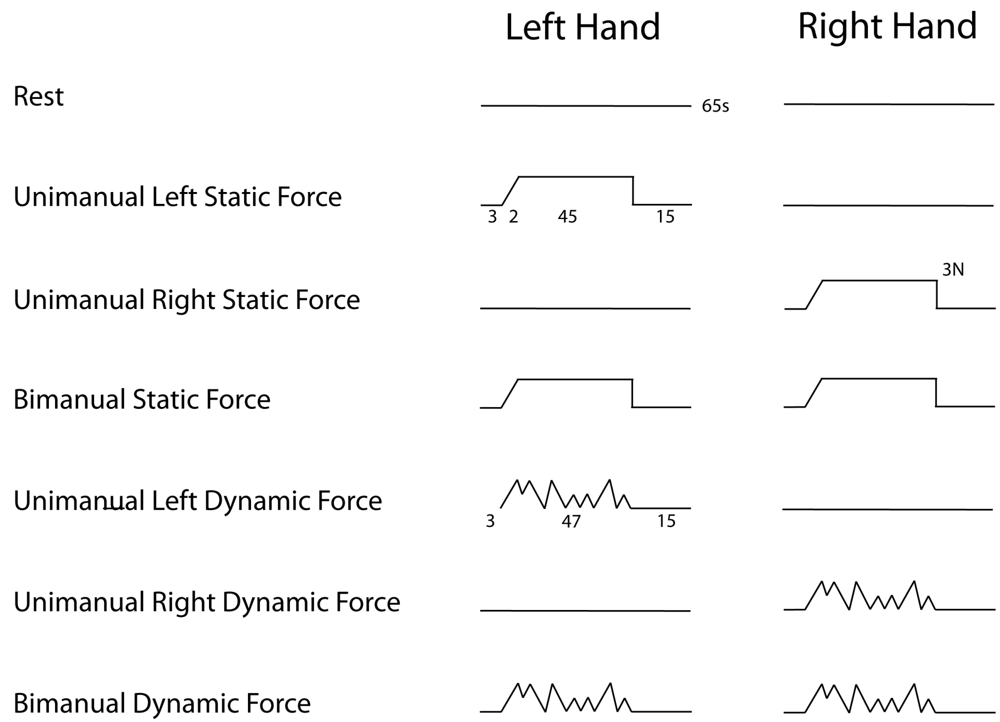
Do static and dynamic performances reflect distinct networks?

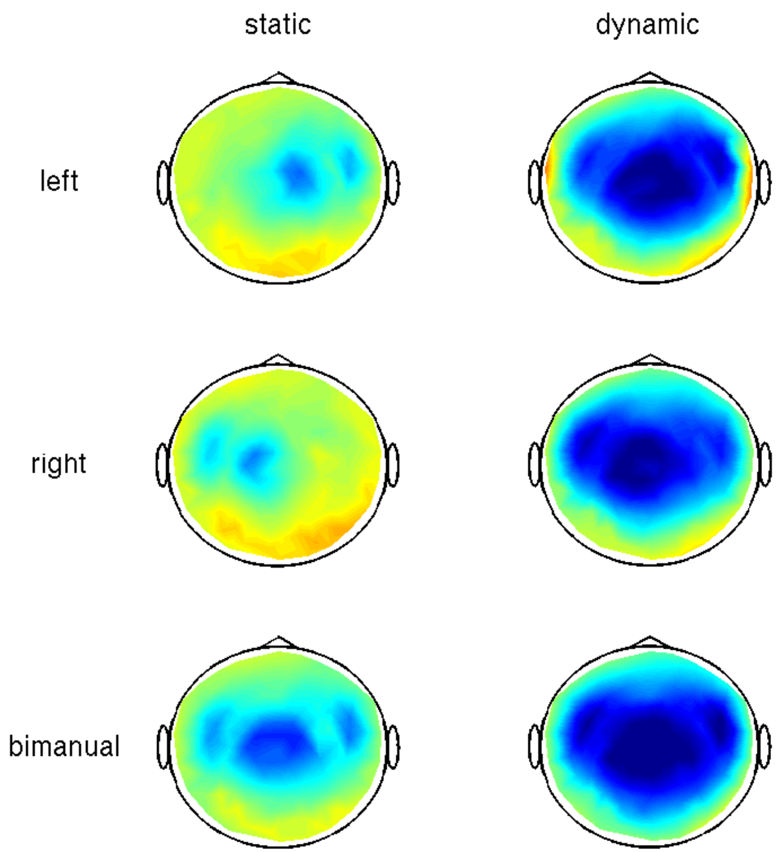




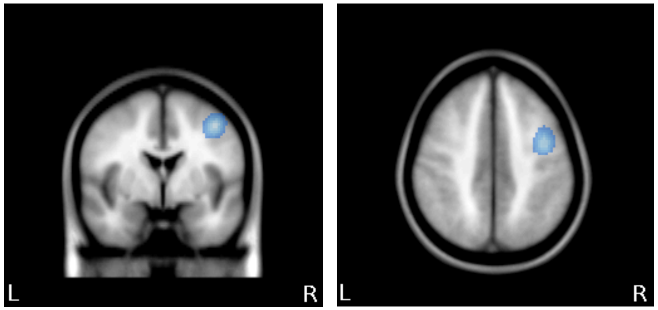


Van Wijk et al., *Europ. J.* Neurosci. 2012

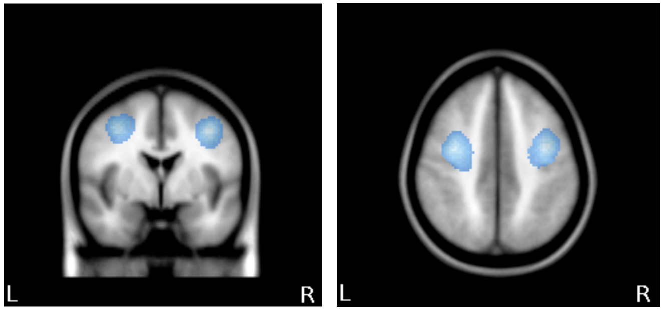
Beta power changes in M1s



Unimanual static: contralateral M1

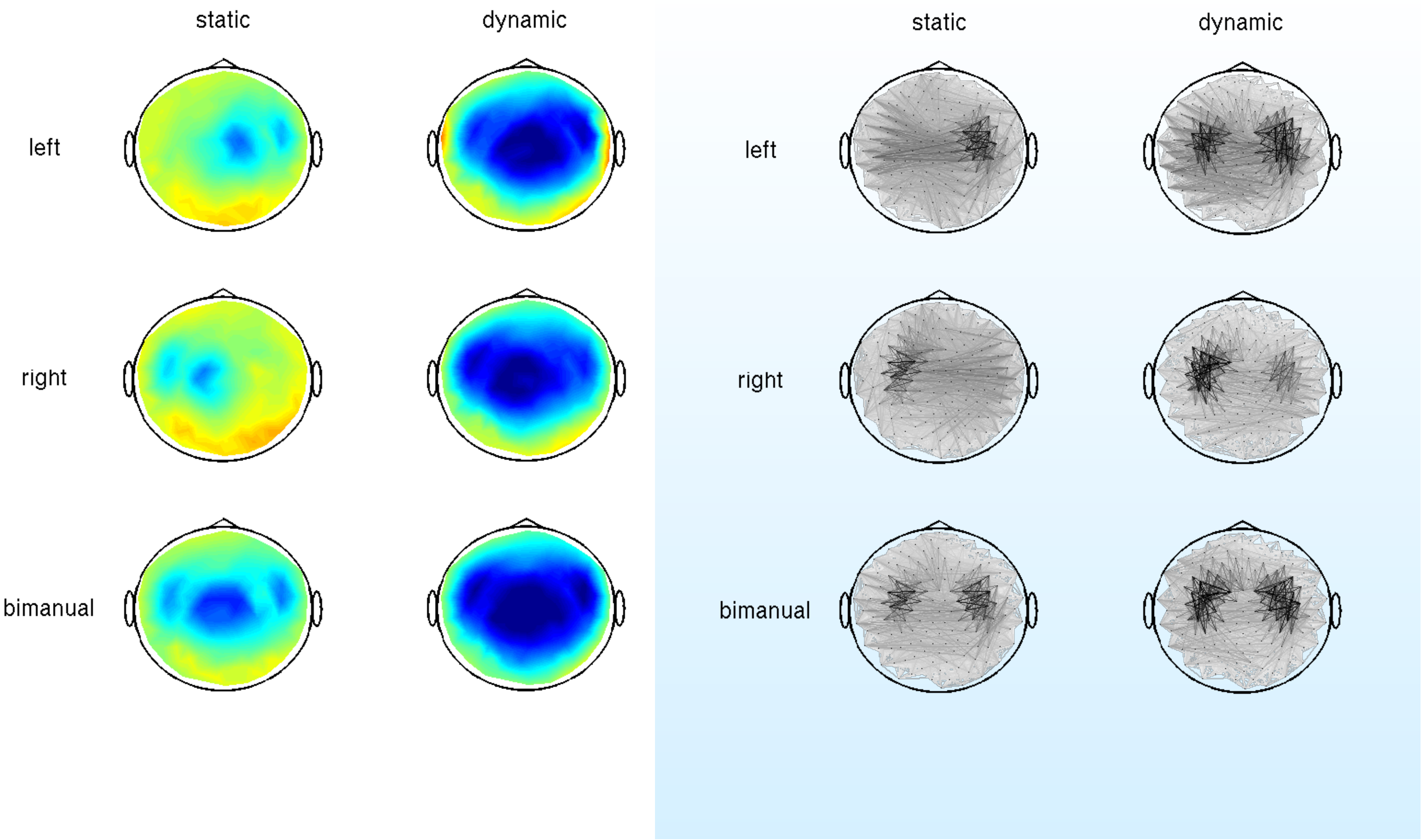


Unimanual dynamic: ipsi & contralateral M1



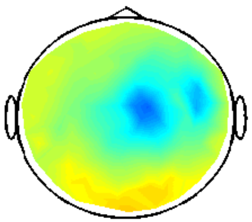
Van Wijk et al., *Europ. J.* Neurosci. 2012

Is cortico-cortical phase synchrony affected by movement type?



Van Wijk et al., *Europ. J.* Neurosci. 2012

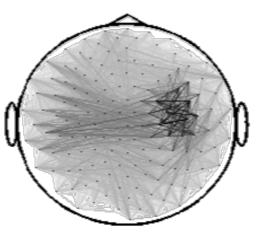
Is phase connectivity affected by amplitude?



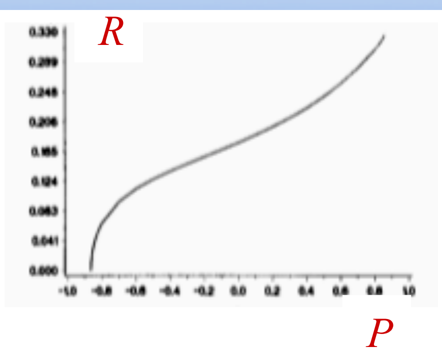
$$\dot{E}_k = -E_k + S\left(a_e\left[c_1 E_k - c_2 I_k - \Theta^e + P_k + \frac{\eta}{N}\sum_{l=1}^{N} C_{kl} E_l\right]\right)$$

$$\dot{I}_k = -I_k + S\left(a_i\left[c_3 E_k - c_4 I_k - \Theta^i\right]\right)$$

$$D_{kl} = \tfrac{1}{2} a_E S'\left[\chi_{E,k}^{(0)}\right] \frac{R_l}{R_k} C_{kl}$$



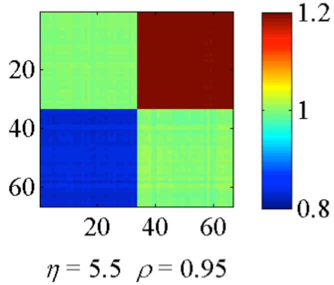
$$\dot{\phi}_k = \omega_k + \frac{\eta}{N}\sum_{l=1}^{N} D_{kl} \sin\left(\phi_l - \phi_k\right)$$



Daffertshofer, van Wijk, *Frontiers Neuroinf,*, 2011

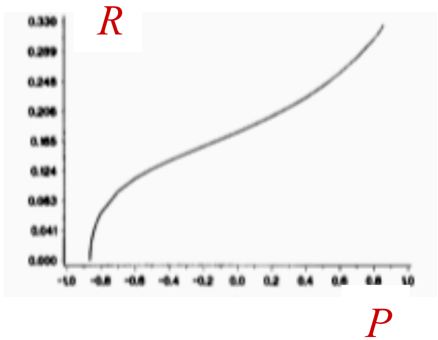
Is phase connectivity affected by amplitude?

R_l / R_k



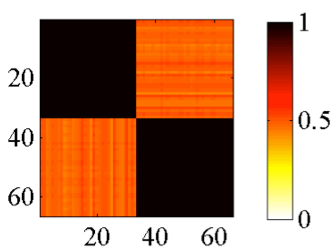
$$\dot{E}_k = -E_k + S\left(a_e\left[c_1 E_k - c_2 I_k - \Theta^e + P_k + \frac{\eta}{N}\sum_{l=1}^{N} \cancel{C_{kl} E_l}\right]\right)$$

$$\dot{I}_k = -I_k + S\left(a_i\left[c_3 E_k - c_4 I_k - \Theta^i\right]\right)$$



phase $\phi_k = \arctan'\left(\frac{E_k}{I_k}\right)$

ρ_{kl}



uniformity rel. phase $\rho_{kl} = \frac{1}{N}\left|\sum_{k=1}^{N} e^{i(\phi_k - \phi_l)}\right|$

Daffertshofer, van Wijk, *Frontiers Neuroinf,*, 2011

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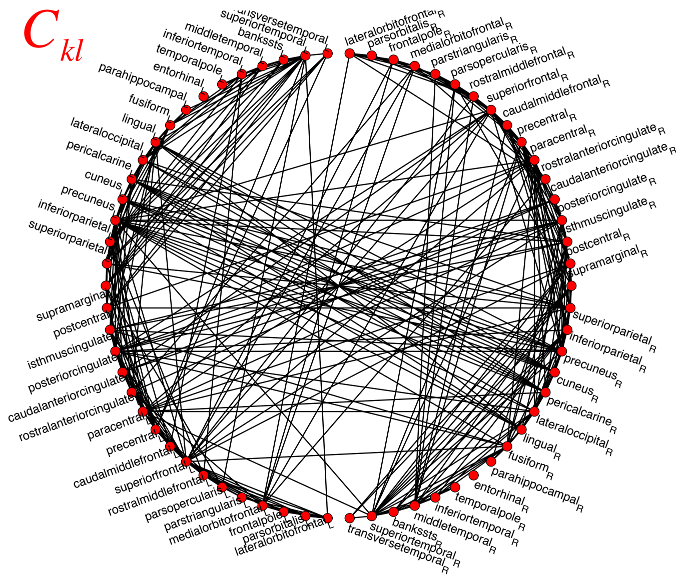

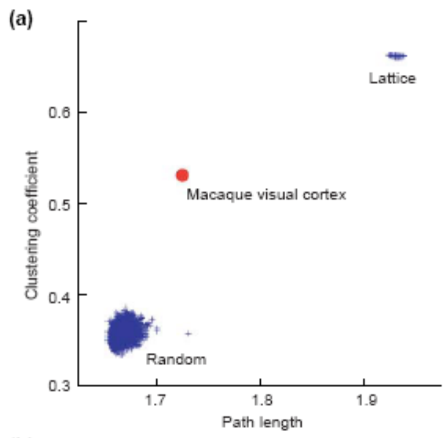

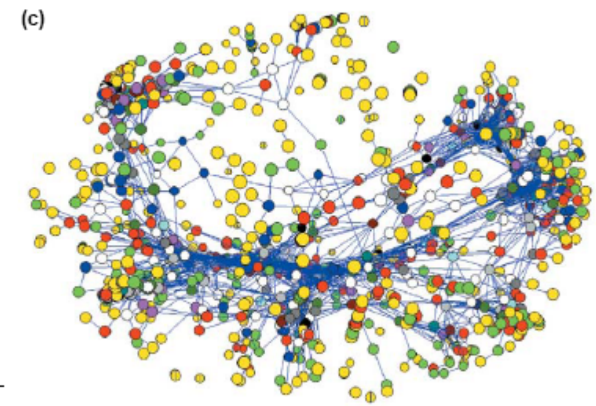

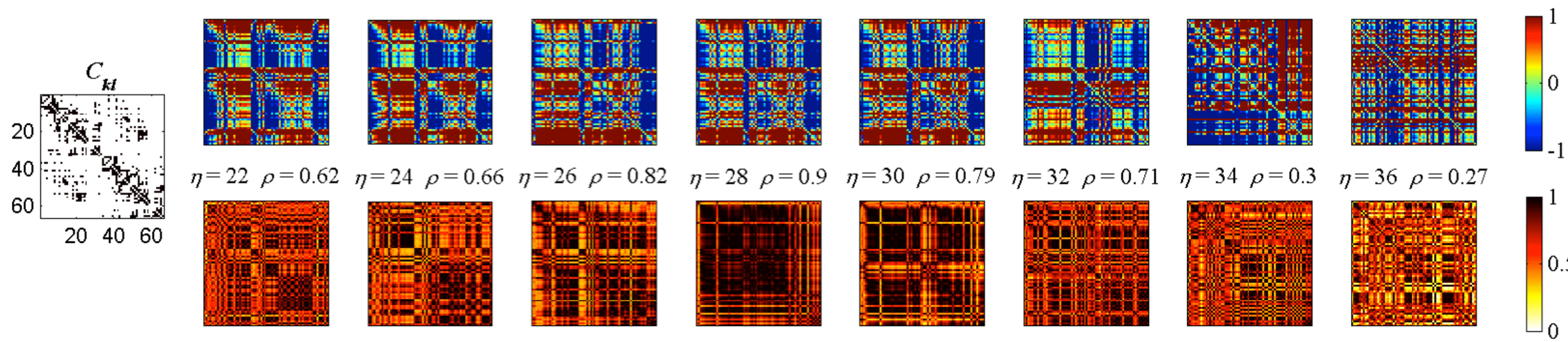

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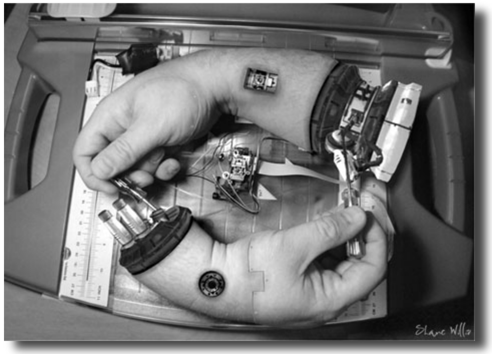

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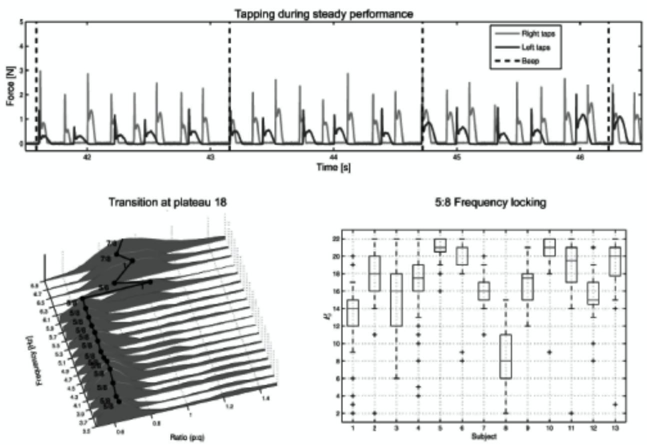
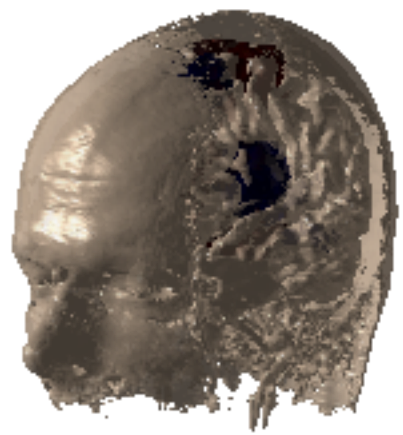
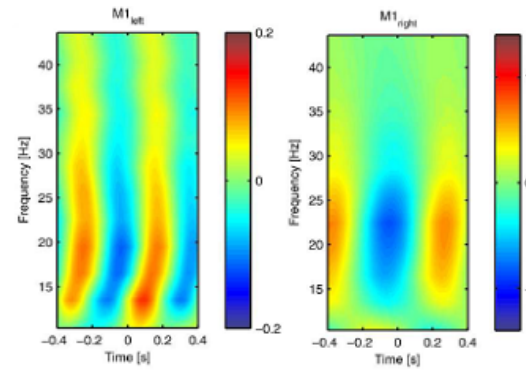
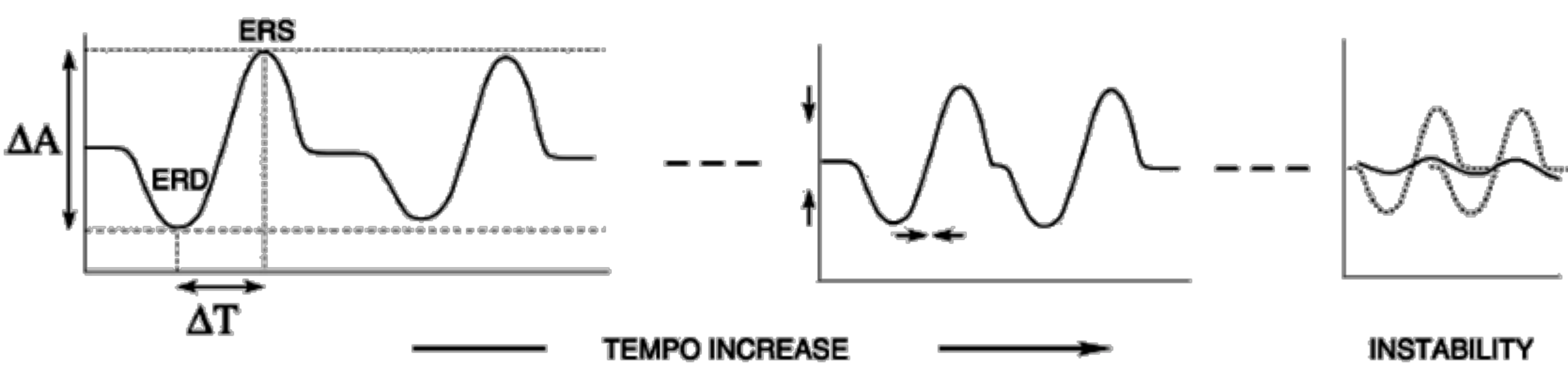
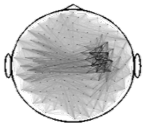




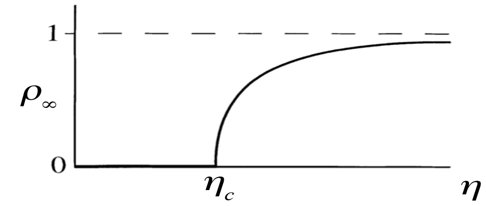




Houweling et al., *Cerebral Cortex* 2010



Kuramoto network

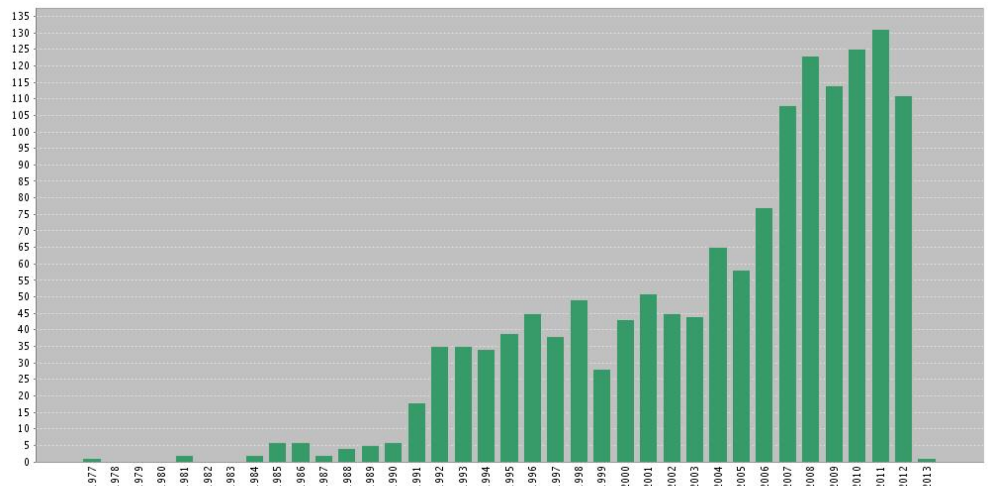


$$\dot{\phi}_k = \omega_k + \frac{\eta}{N}\sum_{l=1}^{N}\sin\left(\phi_l - \phi_k\right)$$

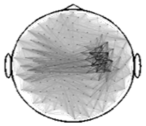
order parameter $\rho = \frac{1}{N}\left|\sum_{k=1}^{N} e^{i\phi_k}\right|$

$$\dot{\phi}_k = \omega_k + \eta\rho\sin\left(\psi - \phi_k\right)$$

$$\dot{\rho} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots??$$

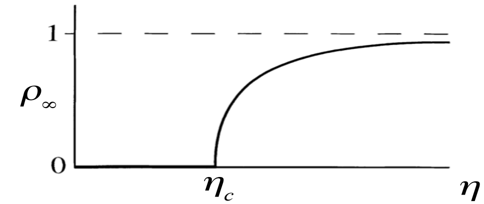


Kuramoto, *Chemical Oscillations…*, Springer, 1984, Strogatz, *Physica D*, 2000



Kuramoto network

$$\dot{\phi}_k = \omega_k + \frac{\eta}{N}\sum_{l=1}^{N}\sin\left(\phi_l - \phi_k\right)$$



order parameter $\rho = \frac{1}{N}\left|\sum_{k=1}^{N} e^{i\phi_k}\right|$

Kuramoto/Nishikawa $\dot{\rho} \propto \frac{\eta - \eta_c}{\eta_c}\rho^2 - \rho^4$

Crawford $\dot{\rho} = \frac{\eta - \eta_c}{2}\rho - \frac{8\pi}{K_c^3}\rho^3 + O^5\left(\rho\right)$

Kuramoto, *Chemical Oscillations…*, Springer, 1984, Strogatz, *Physica D*, 2000

Kramers-Moyal expansion

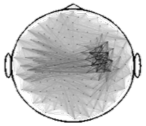
$$\frac{\partial}{\partial t}P(x,t)=\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{\partial}{\partial x}\right)^{k}D^{(k)}(x,t)P(x,t)$$

$$D^{(k)}(x,t)=\lim_{\Delta t\to 0}\frac{1}{\Delta t}\left\langle\left[\xi(t+\Delta t)-\xi(t)\right]^{k}\right\rangle$$

Gauss process: $\forall_{k>2}:D^{(k)}(x,t)=0$

Langevin system: $\dot{\xi}=N(\xi,t)+G(\xi,t)\cdot\Gamma_t$

$$\dot{\xi}=D^{(1)}(\xi,t)+\sqrt{2D^{(2)}(\xi,t)}\Gamma_t$$



Kramers-Moyal expansion

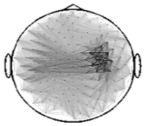
$$\frac{\partial}{\partial t}P(x,t)=\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{\partial}{\partial x}\right)^{k}D^{(k)}(x,t)P(x,t)$$

$$D^{(k)}(x,t)=\tfrac{1}{k!}\lim_{\Delta t\to 0}\frac{1}{\Delta t}\int (x'-x)^{k}P(x',t+\Delta t\mid x,t)\,dx'$$

Gauss process: $\forall_{k>2}: D^{(k)}(x,t)=0$

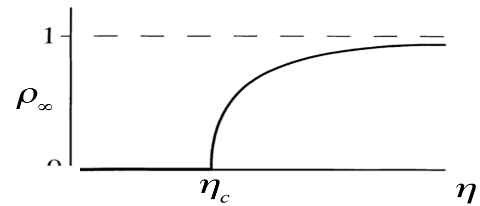
Langevin system: $\dot{\xi}=N(\xi,t)+G(\xi,t)\cdot \underset{\sim}{\Gamma}_{t}$

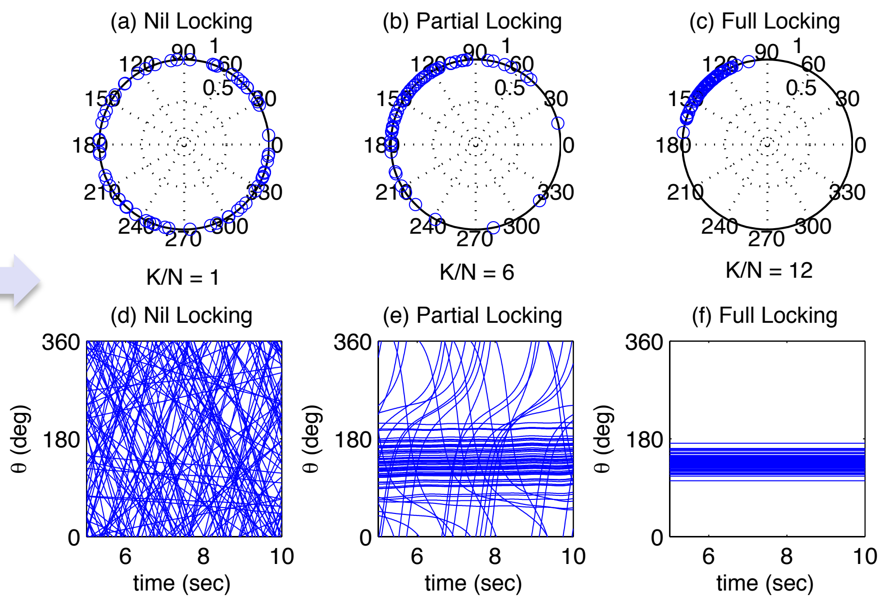
$$\dot{\xi}=D^{(1)}(\xi,t)+\sqrt{2D^{(2)}(\xi,t)}\,\Gamma_{t}$$



Kuramoto network

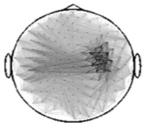
$$\dot{\phi}_k = \omega_k + \frac{\eta}{N}\sum_{l=1}^{N}\sin\left(\phi_l - \phi_k\right)$$





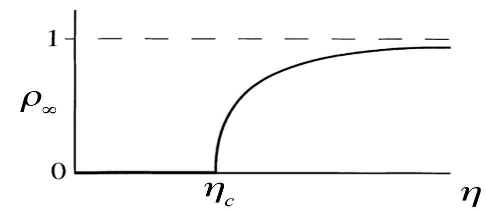
$$\rho = \frac{1}{N}\left|\sum_{k=1}^{N} e^{i\phi_k}\right|$$

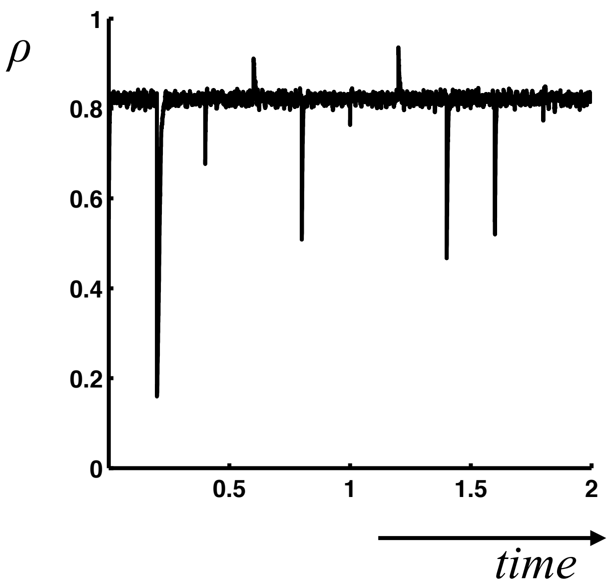
$$\dot{\rho} = D^{(1)}(\rho) + \sqrt{2D^{(2)}(\rho)}\,\Gamma_t$$

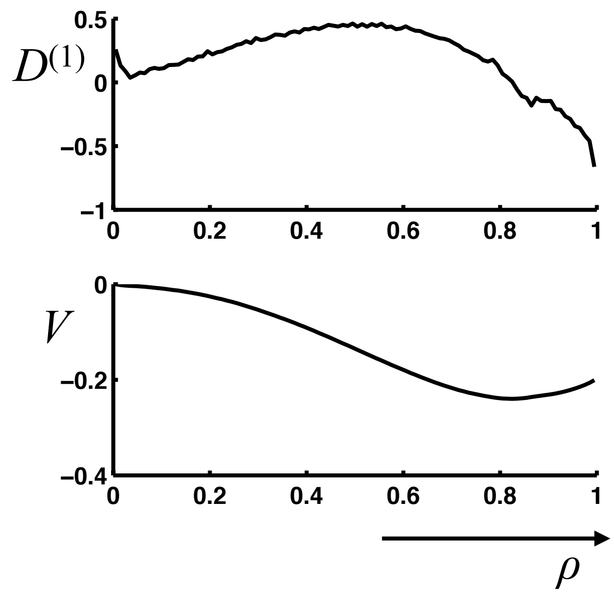


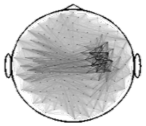
Kuramoto network

$$\dot{\rho} = \ldots\ldots\ldots\ldots\ldots\ldots + \sqrt{2\tilde{Q}}\,\Gamma_t$$





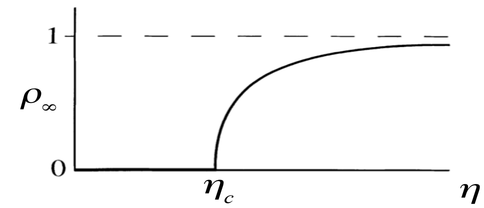


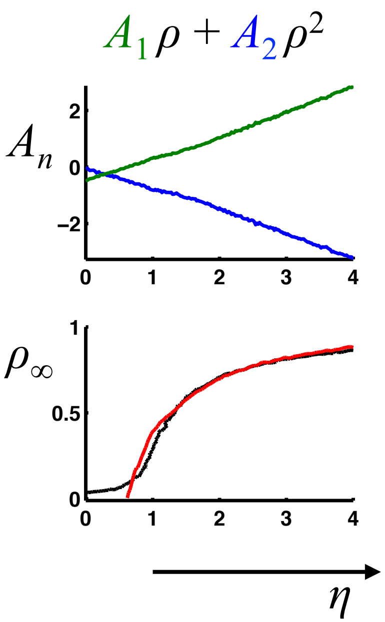


Kuramoto network

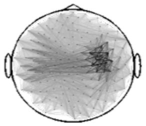
$$\dot{\rho} = A_1\rho + A_2\rho^2 + \sqrt{2\tilde{Q}}\Gamma_t$$

$$g(\omega) = \tfrac{1}{\pi}\frac{\gamma}{\gamma^2+\omega^2} \Rightarrow \eta_c = 2\gamma$$





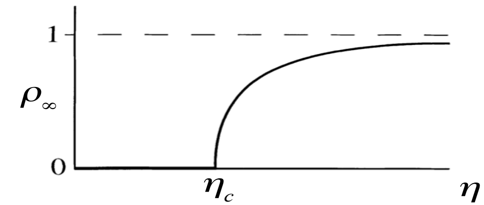
Kuramoto, *Chemical Oscillations…*, Springer, 1984, Strogatz, *Physica D*, 2000

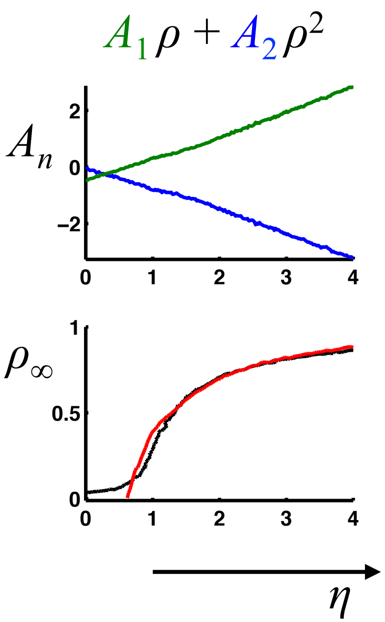


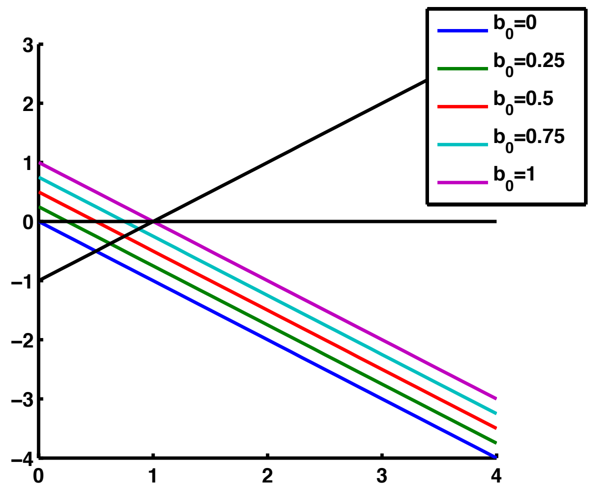
Kuramoto network

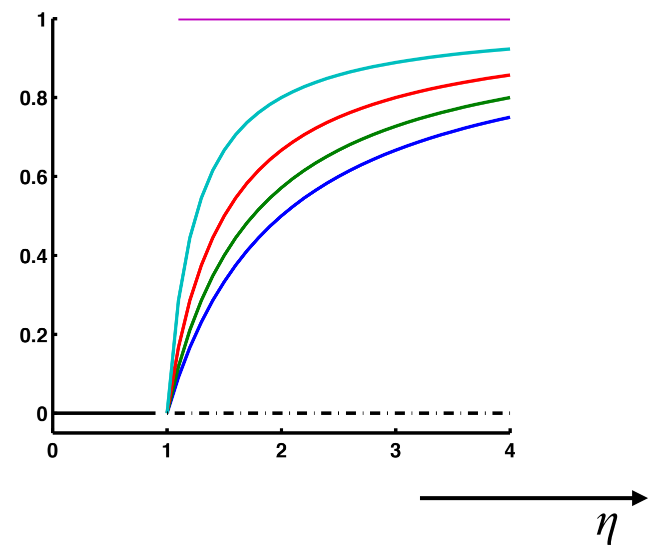
$$\dot{\rho} = (\eta - 1)\rho + (b_0 - \eta)\rho^2 + \sqrt{2\tilde{Q}}\,\Gamma_t$$

$$g(\omega) = \tfrac{1}{\pi}\frac{\gamma}{\gamma^2 + \omega^2} \Rightarrow \eta_c = 2\gamma$$

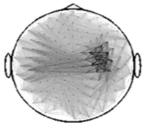






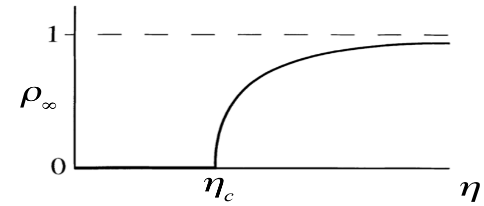


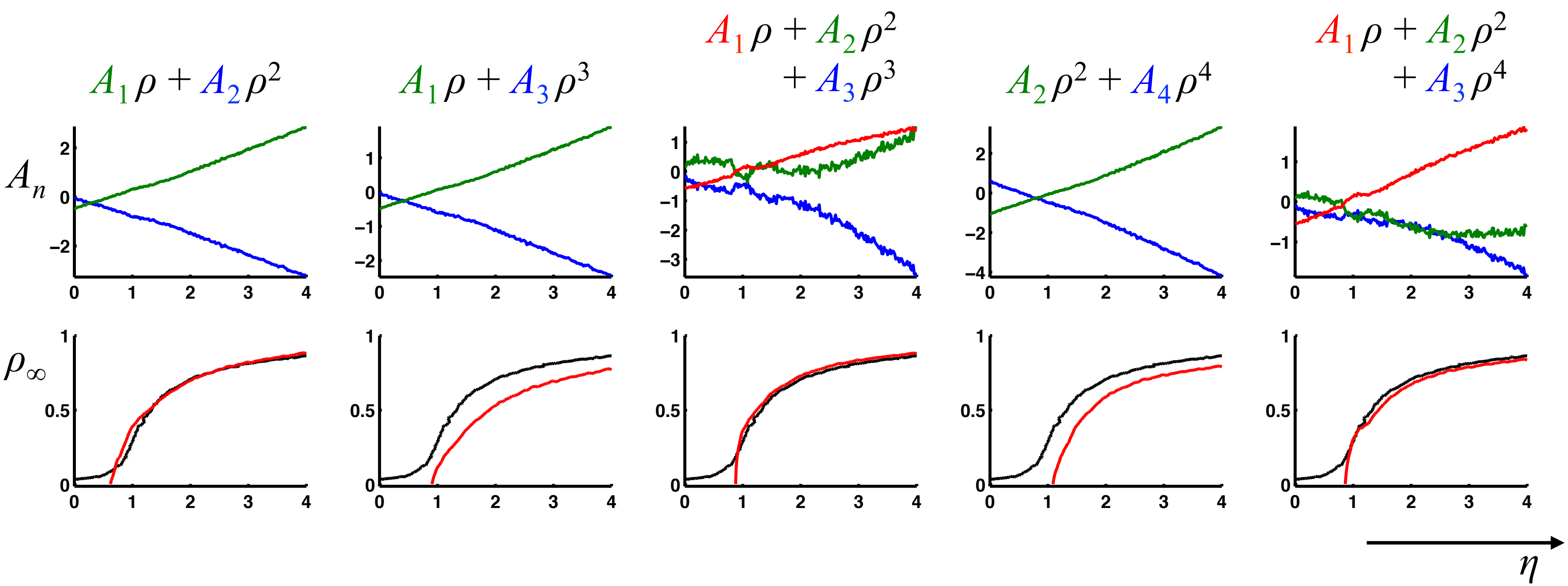
Kuramoto, *Chemical Oscillations…*, Springer, 1984, Strogatz, *Physica D*, 2000



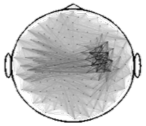
Kuramoto network

$$\dot{\rho} = \sum_n A_n \rho^n + \sqrt{2\tilde{Q}}\Gamma_t$$





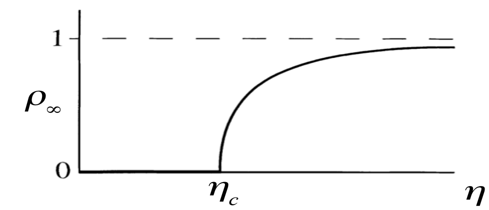
Kuramoto, *Chemical Oscillations…*, Springer, 1984, Strogatz, *Physica D*, 2000



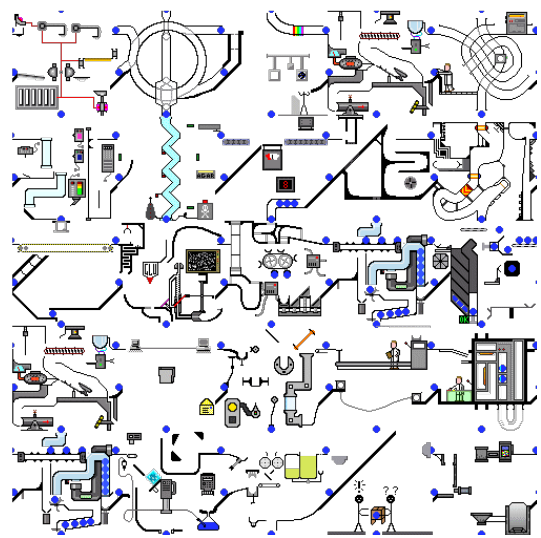
Kuramoto network

$$\dot{\phi}_k = \omega_k + \frac{\eta}{N}\sum_{l=1}^{N}\sin\left(\phi_l - \phi_k\right) + \sqrt{2Q}\Gamma_{k,t}$$

$$g\left(\omega\right) = \frac{1}{\pi}\frac{\gamma}{\gamma^2+\omega^2} \Rightarrow \eta_c = 2\gamma + 2Q$$



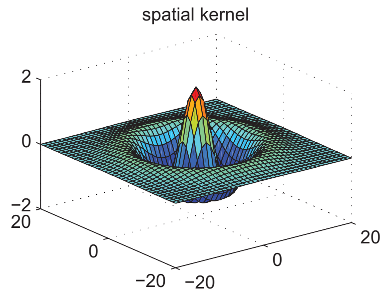
$$\frac{d}{dt}\rho = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \sqrt{2\tilde{Q}}\Gamma_t$$

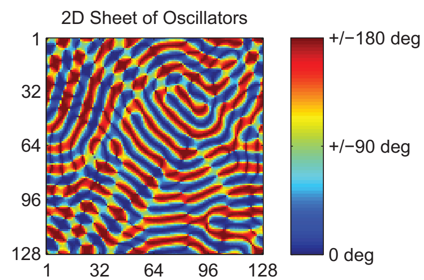


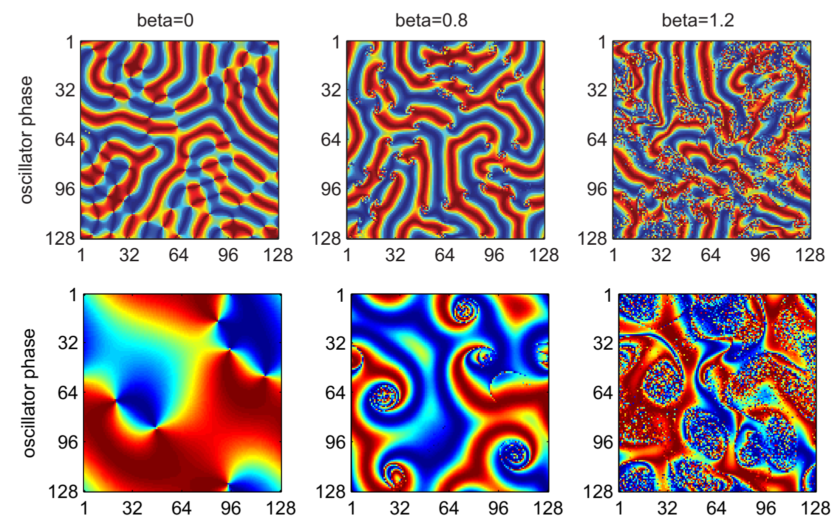
Strogatz & Mirollo, *J. Stat. Phys.*, 1991

Kuramoto network

$$\dot{\phi}_k = \omega_k + \frac{\eta}{N}\sum_{l=1}^{N} D\left(\left|x_k - x_l\right|\right)\left[\sin\left(\phi_l - \phi_k\right) + \beta \sin 2\left(\phi_l - \phi_k\right)\right]$$







Breakspear, Heitman, Daffertshofer, *Frontiers Hum. Neurosci*, 2011

Dynamics of the rotating phase of each oscillator

$$\frac{d}{dt}\Phi_k^{(\text{rot})} = \omega_k + h_k\left(\Phi_k^{(\text{rot})} - \Omega t - \alpha_k;\{\mu\}\right) - \frac{\eta}{N}\sum_{l=1}^{N} A_{kl}\sin\left(\Phi_k^{(\text{rot})} - \Phi_l^{(\text{rot})}\right) + \sqrt{2Q_k}\,\Gamma_{k,t}$$

$$K_{kl} = v_k v_l\,,\quad v_j = \begin{cases} 1 & \text{for } l = 1,\ldots,M \\ -1 & \text{otherwise} \end{cases}$$

heterogeneity

$$\phi_k = \begin{cases} \Phi_k^{(\text{rot})} - \tfrac{1}{2}\pi - \Omega t & \text{for } k = 1,\ldots,M \\ \Phi_k^{(\text{rot})} - \tfrac{3}{2}\pi - \Omega t & \text{otherwise} \end{cases}$$

non-rotating phase
(unimodal freq. dist.)

$$h = -\frac{dV}{d\Phi} \quad \text{with} \quad V(\Phi) = V(\Phi + 2\pi)$$

external force
attracts phase

Frank, Daffertshofer et al., *Physica D* 2000

Phase oscillators

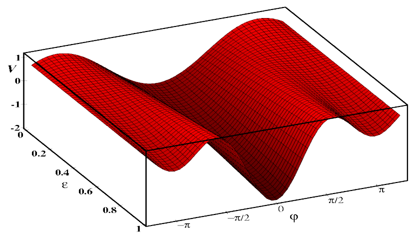
$$\frac{d}{dt}\phi_k = \tilde{h}\left(\phi_k;\{\mu\}\right) - \frac{\eta}{N}\sum_{l=1}^{N}\sin\left(\phi_k - \phi_l\right) + \sqrt{2Q}\,\Gamma_{k,t}$$

Mean field approximation yields a Fokker-Planck equation…

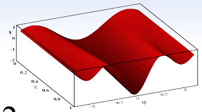
$$\frac{\partial}{\partial t}P(\phi,t) = -\frac{\partial}{\partial\phi}\left[\tilde{h}\left(\phi;\{\mu_j\}\right) - K\int_0^{2\pi}\sin(\phi-\chi)P(\chi,t)\,d\chi\right]P(\phi,t) + Q\frac{\partial^2}{\partial\phi^2}P(\phi,t)$$

…that is non-linear in the probability density

$$V\left(\phi;\{\alpha,\beta\}\right) = -\alpha\left[\cos(\phi) + \tfrac{1}{4}\beta\cos(2\phi)\right]$$



Frank, Daffertshofer et al., *Physica D* 2000



$$\frac{\partial}{\partial t}P(\phi,t) = -\frac{\partial}{\partial \phi}\left[\tilde{h}(\phi;\{\mu\}) - \eta\int_0^{2\pi}\sin(\phi-\chi)P(\chi,t)\,d\chi\right]P(\phi,t) + Q\frac{\partial^2}{\partial\phi^2}P(\phi,t)$$

Nonlinear Fokker-Planck Equations

- can be related to non-extensive entropies via MaxEnt principles (Frank & Daffertshofer, *Physica* A, 1999);

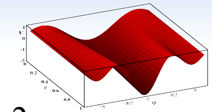
  e.g., for Tsallis generalized entropy we find

$$S_q(p) = \frac{1}{q-1}\left(1-\int p^q(x)\,dx\right) \quad \rightleftharpoons$$

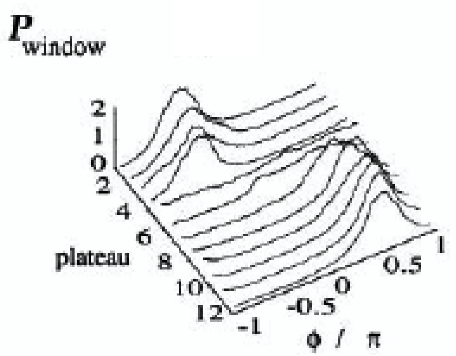
$$\frac{\partial}{\partial t}P(x,t) = -\frac{\partial}{\partial x}N(x)P(x,t) + Q\frac{\partial^2}{\partial x^2}\left[P(x,t)\right]^q$$

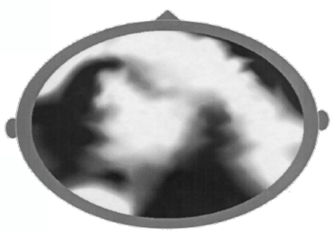
- often obey power-laws, i.e. they mimic long-range correlations

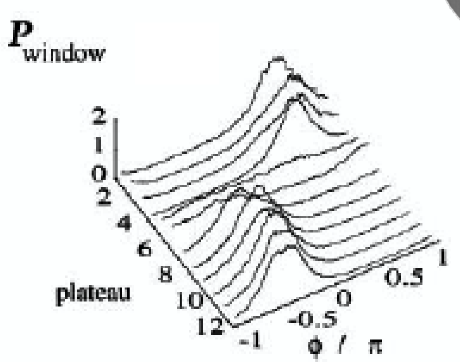
Frank, *Non-linear Fokker-Planck equations* Springer 2004

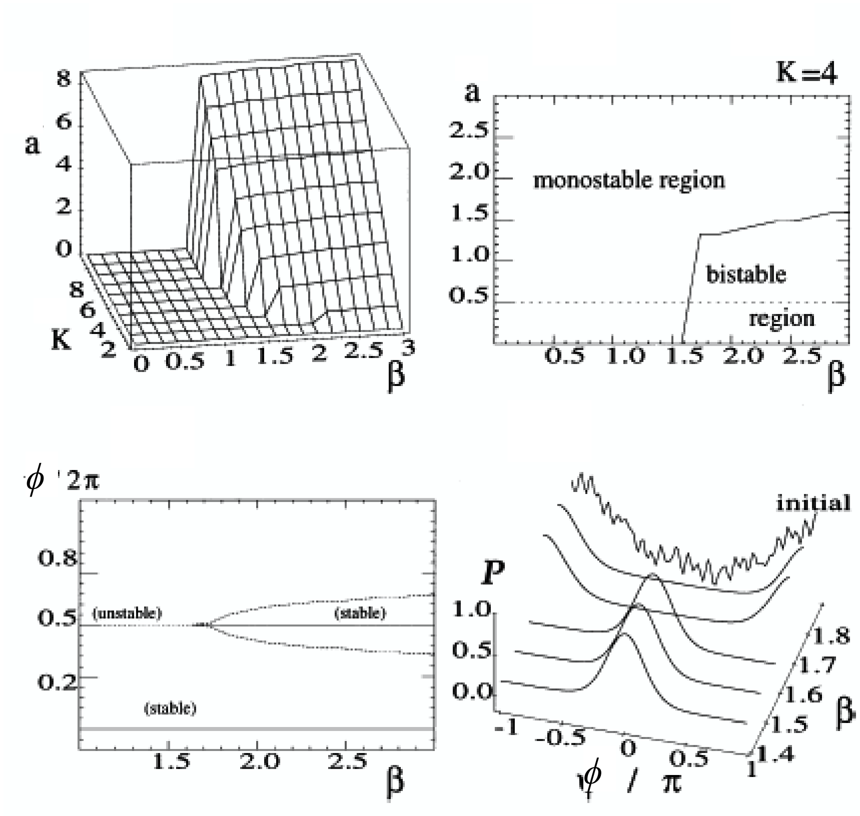


$$\frac{\partial}{\partial t}P(\phi,t) = -\frac{\partial}{\partial\phi}\left[\tilde{h}(\phi;\{\mu\}) - \eta\int_0^{2\pi}\sin(\phi-\chi)P(\chi,t)\,d\chi\right]P(\phi,t) + Q\frac{\partial^2}{\partial\phi^2}P(\phi,t)$$







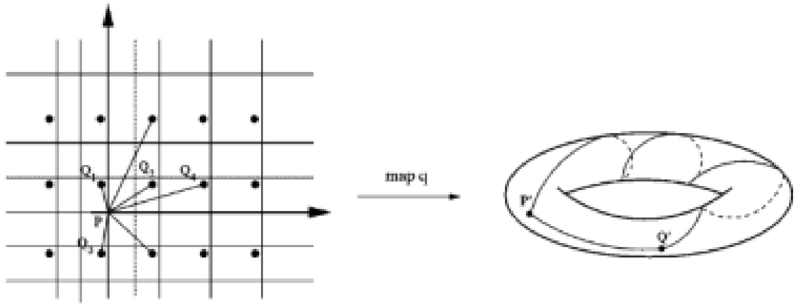


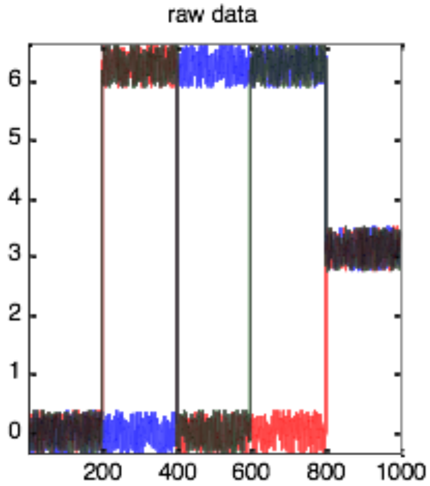
Frank, Daffertshofer et al., *Physica D* 2000

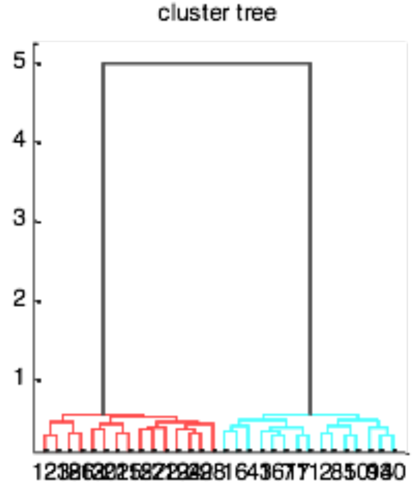
Phase clustering

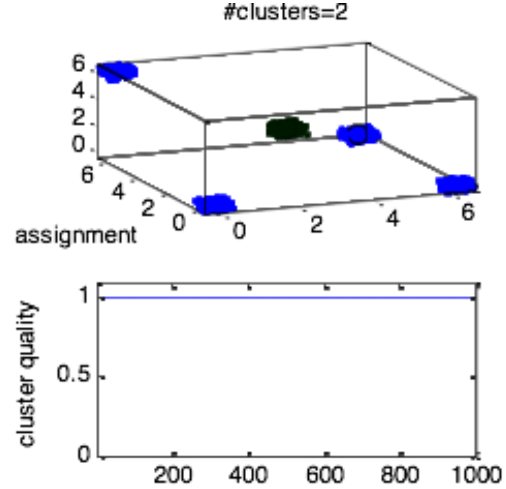
PHYSICAL REVIEW E **68**, 036219 (2003)

Detection of mutual phase synchronization in multivariate signals and application to phase ensembles and chaotic data

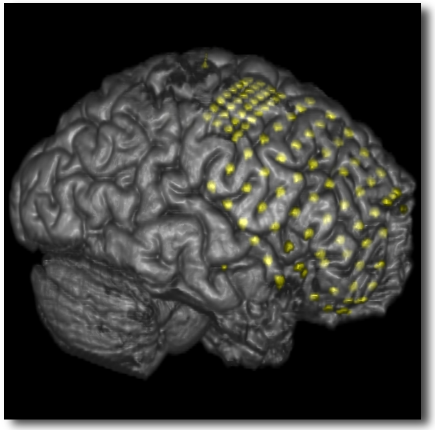


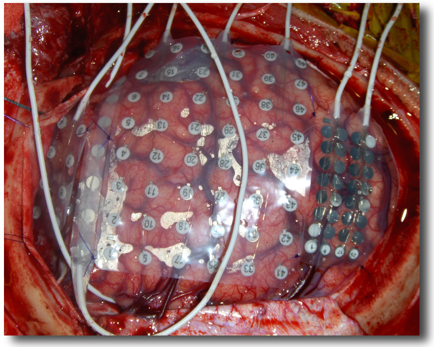


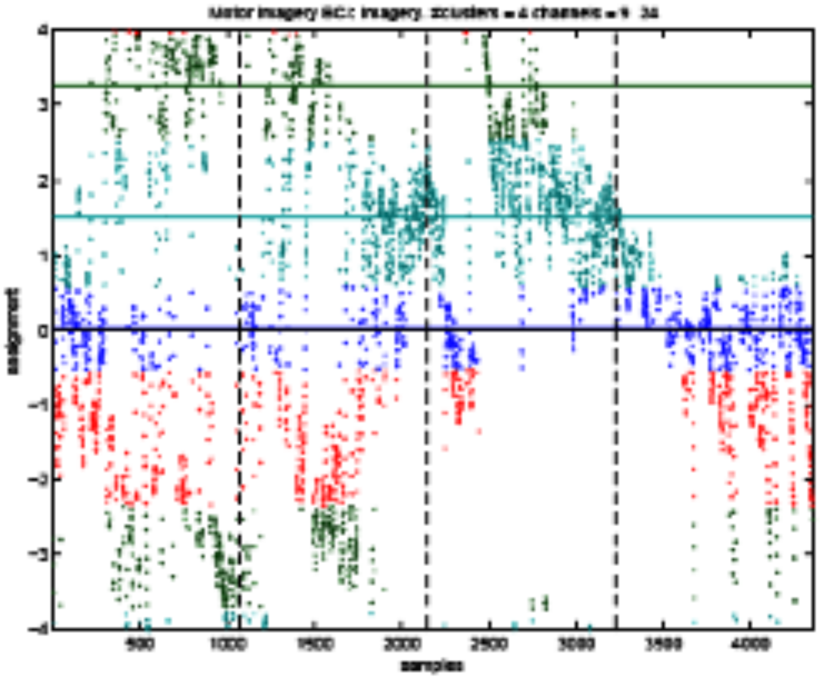




ECoG during decision making and movement imagination…

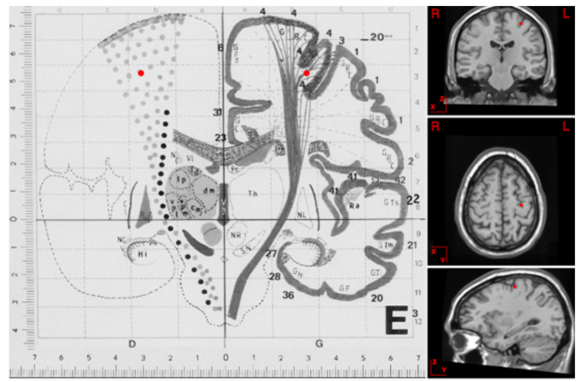


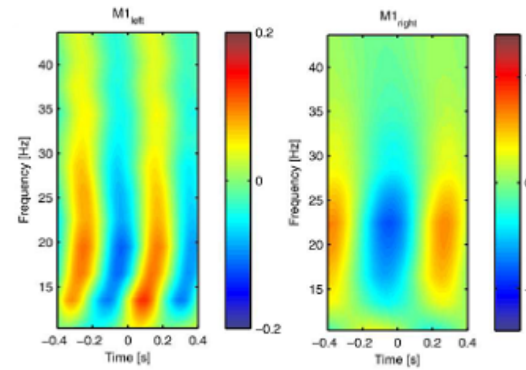


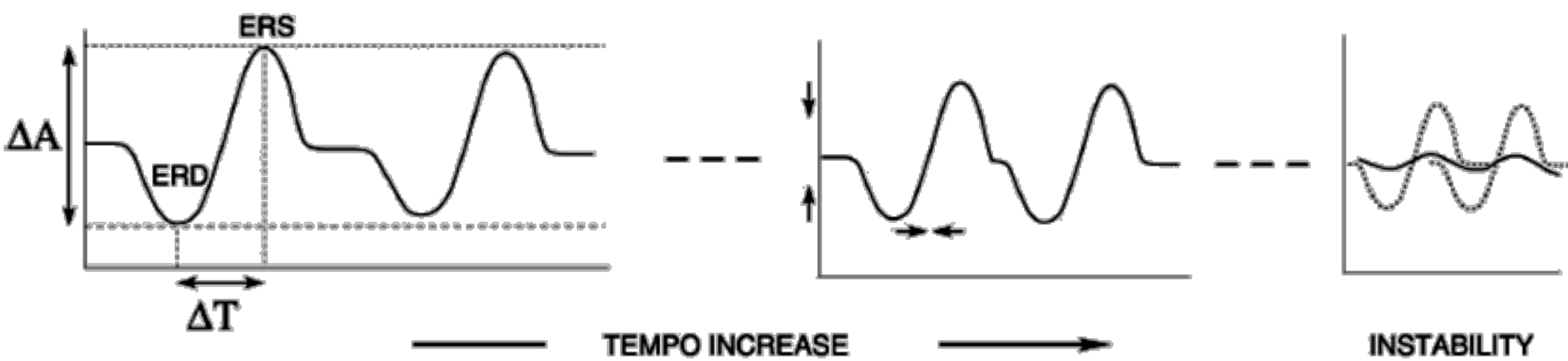


Torres-Valderama, Ramsey, *unpublished*

Instabilities in rhythmic bimanual tapping







Houweling et al., *Cerebral Cortex* 2010

Thanks for your attention

Peter Beek
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Alan Wing

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