The Physics of Finance: Collective Dynamics in a Complex World

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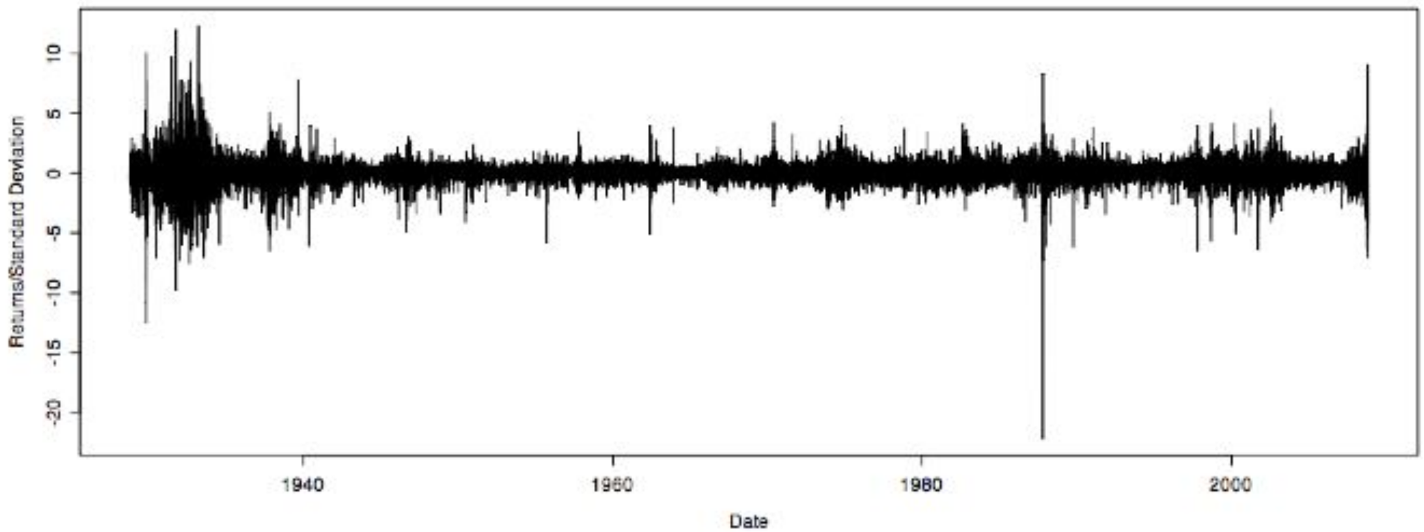
Synergetics Symposium in Honor of Professor Haken
On the Occasion of his 85th Birthday
Delmenhorst, November 2012

The price formation process: multitude of complex global interactions
Not possible to run experiments: just one single history, one realization



The price formation process: multitude of complex global interactions
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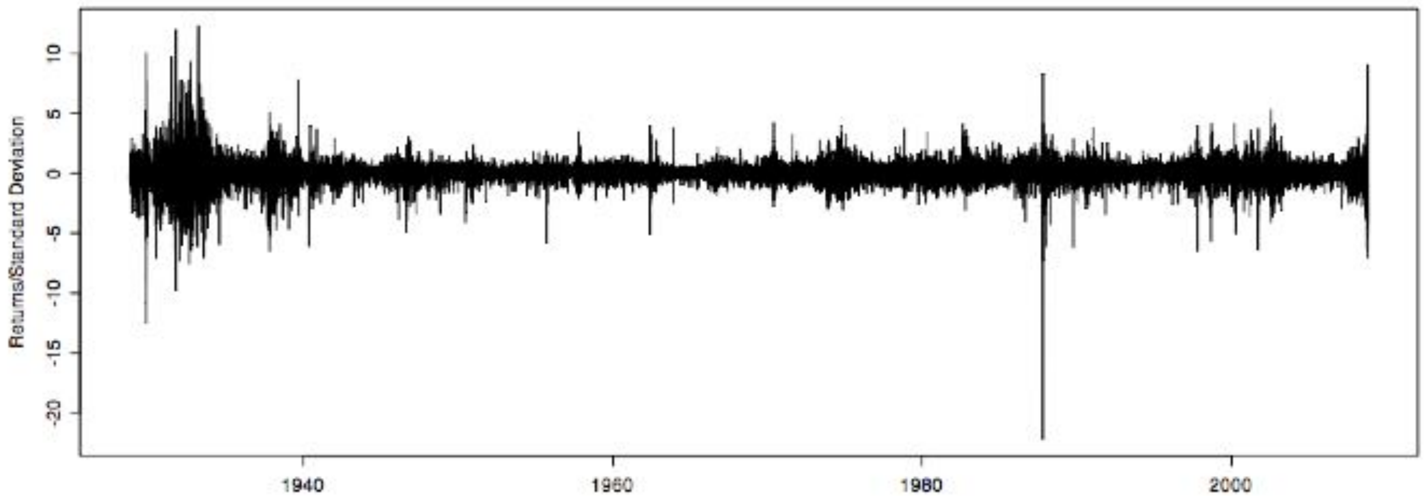
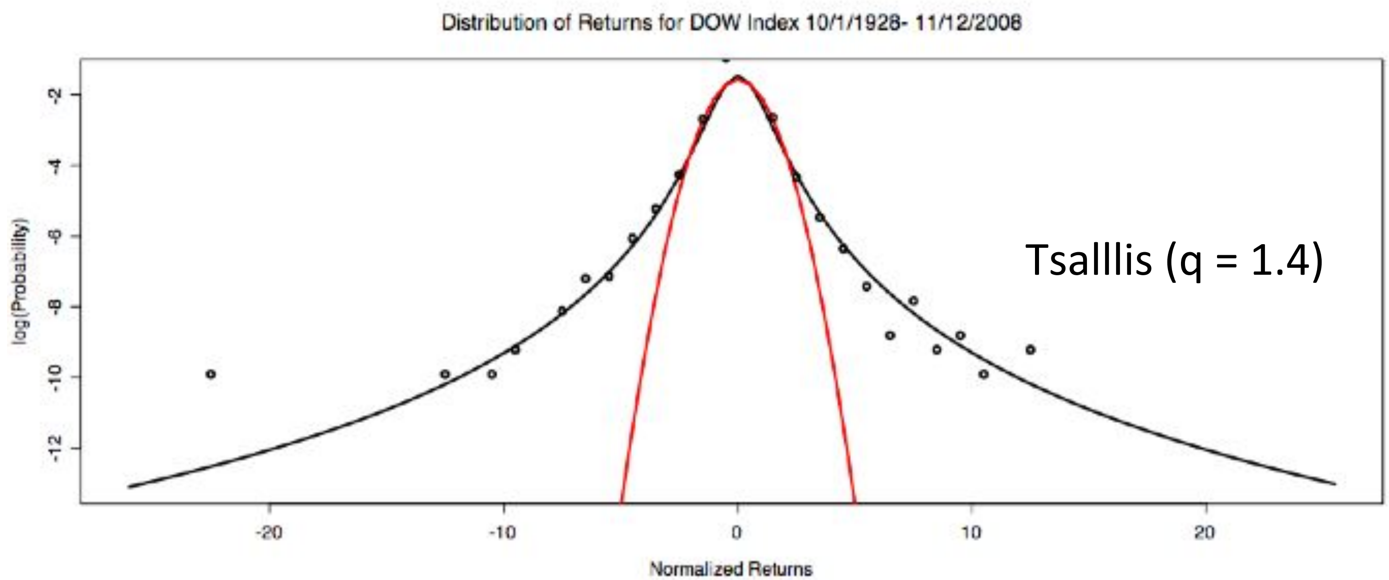




•For $q = 1.4$, a 10-sigma event has probabilty

$p = 0.028\%$ (about 1 in a decade)

•This is equally probable as a 3.45-sigma event based on a Gaussian distribution

•For a Gaussian, a 10-sigma event has probability $p = 0\%$

Exploring the joint stochastic process

What do we know about volatility?

- Across time for a given stock   Part 1

- Across stocks at a given time   Part 2

Some more properties of financial time-series

Universal features: Across time, Across the globe

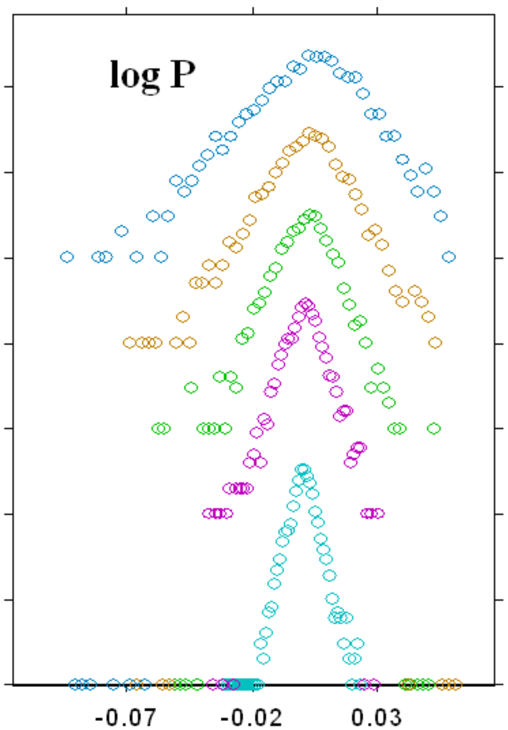
For stocks (IBM), Commodities (Oil), Currencies (EUR/USD) etc

- Power Law distributions, persistent
- Slow decay to Gaussian
- Volatility clustering
- Close-to log-normal distribution of volatility
- Time-reversal asymmetry
- Leverage effect
  (Skew: Negative returns --> Higher volatility)

Across Time:

Vast number of anomalous statistics (stylized facts)

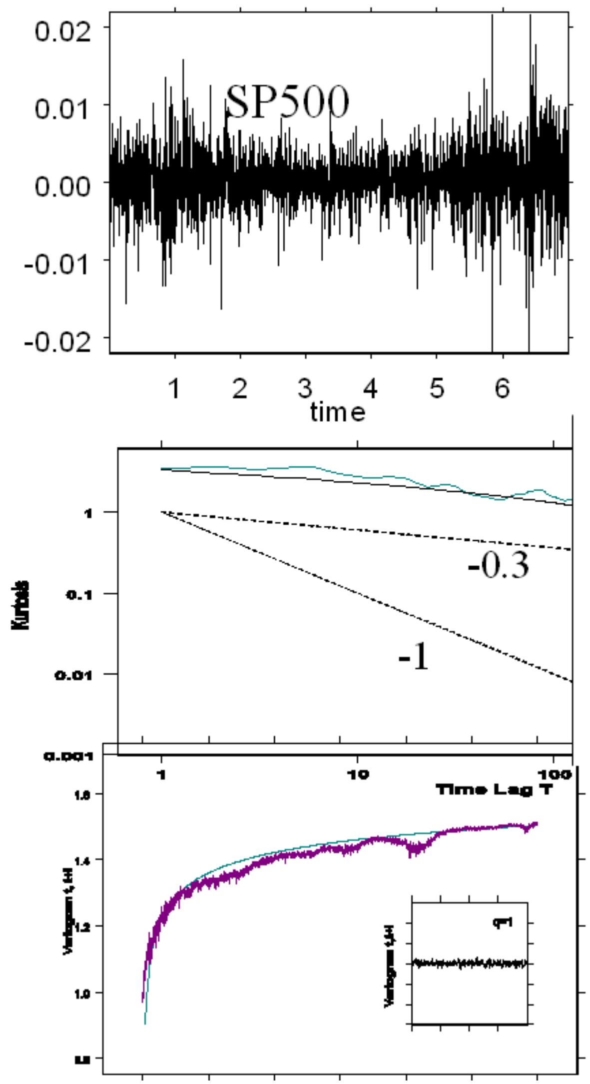
Some Stylized Facts



Y(t+tau)-Y(t)

Kurtosis decays as tau$^{-0.3}$

Leverage effect (skew)



Challenge:

A somewhat realistic model of price variations

Important for real-world reasons:

- Risk control

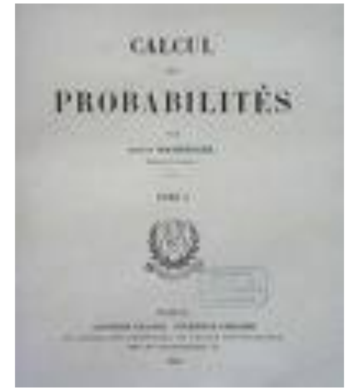
- Hedging

- Development of trading strategies

- Option pricing

- Pricing of credit risk

Century long quest:



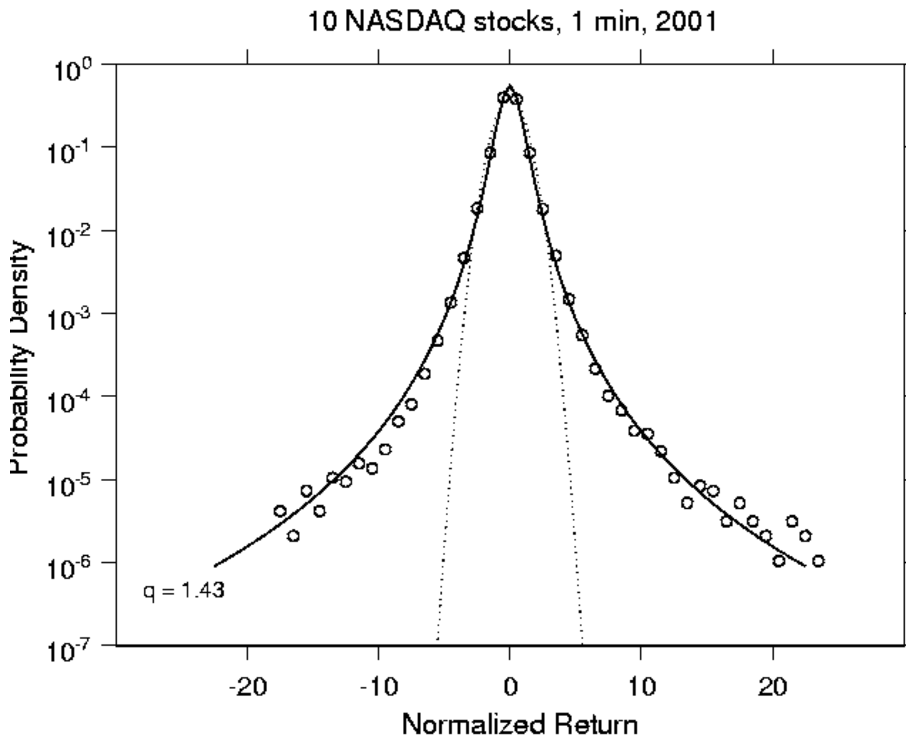
First random walk model by Bachelier in 1900

Gaussian model,
basis for celebrated 1974 Black-Scholes option pricing formula

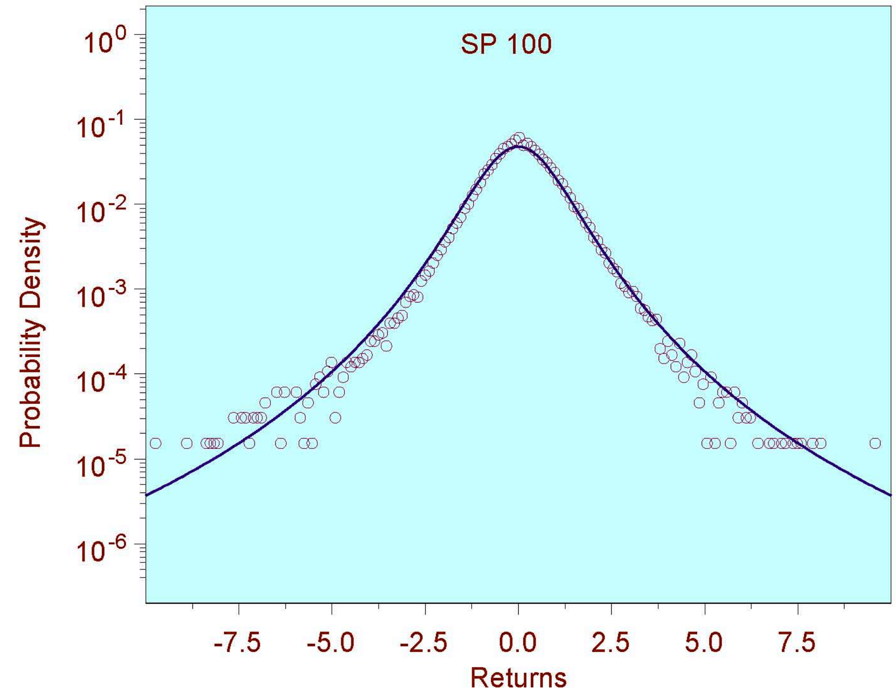
$$dS = \mu S dt + \sigma S d\omega$$

S	Price
μ	Rate of return
σ	Volatility
ω	Gaussian noise

Real log-returns not Gaussian



100 SP stocks, daily



o Empirical

--- Gaussian

___ q=1.43 Tsallis Distribution

The ideal model:

- Intuitive

- Analytically tractable

-Parsimonious

Popular models:
-Stochastic Volatility (Heston 1993)
-Levy noise
-GARCH
- Multi-fractal model (Bacry,Delour Muzy PRE 64 2001)

Exploring the Joint Stochastic Process:

Across time ….

1. A non-Gaussian model of returns
2. Options pricing incorporating fat-tails
3. Applications

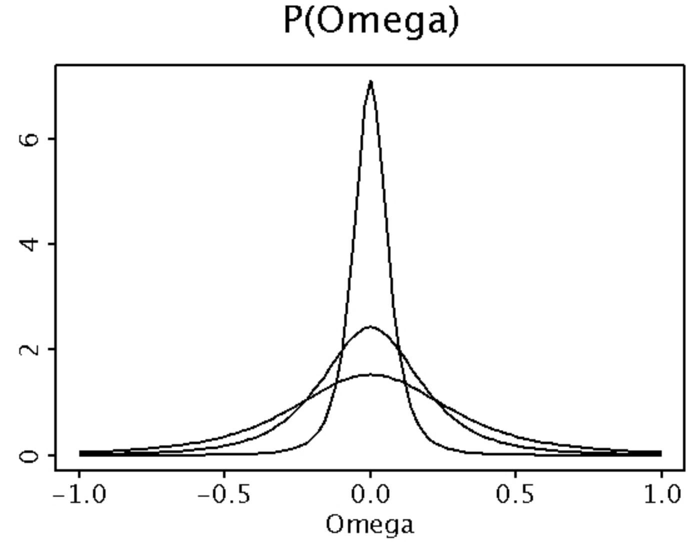
Non-Gaussian model based on statistical feedback

$$dS = \mu S dt + \sigma S d\Omega$$

Non-Gaussian model based on statistical feedback

$$dS = \mu S dt + \sigma S d\Omega$$

$$d\Omega = P(\Omega)^{\frac{1-q}{2}} d\omega$$



For stocks q = 1.4

The Halifax Chronicle

"A dependable newspaper"

FAIR AND COLD

HALIFAX, NOVA SCOTIA, WEDNESDAY, OCTOBER 30, 1929 | 16 Pages | Price Three Cents

onstituencies That Vote Today

Ontario Provincial Ridings

SELLING FRENZY MAKES NEW DRAMA IN MARKET

ONTARIO VOTERS TODAY, PICK OUT 103 LEGISLATORS

Eight Of 112 Members Already Elected — One Election Deferred

LIQUOR CONTROL ACT LAUDED BY PREMIER

Liberal Leader Calls For Plebiscite On Liquor Question

Crash Takes $1,000,000 From Halifax Traders

Local Traders Harshly Drubbed In Yesterday's Stock Market Debacle — One Man Drops $75,000—Another Has Lost $250,000 Since Decline Set In

FRANTIC SELLING BREAKS RECORDS IN STOCK MARKET

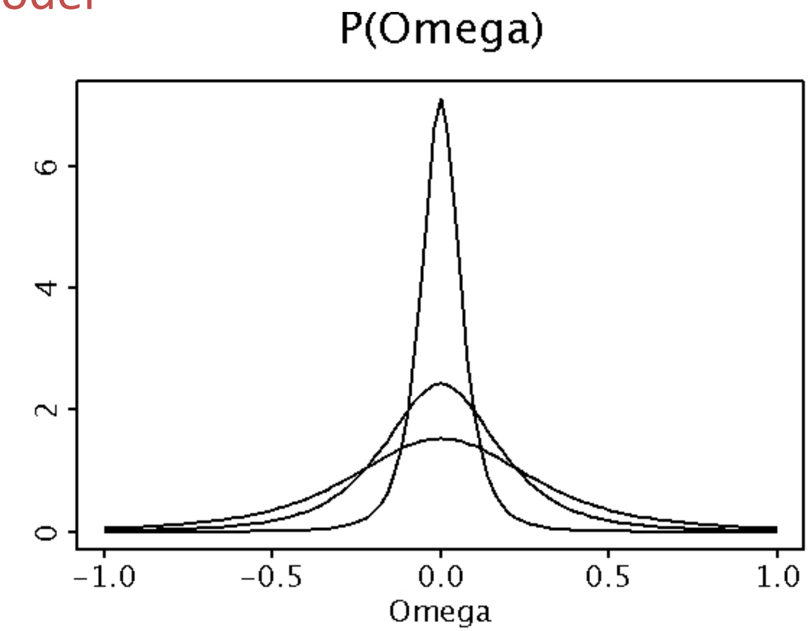
Volume Of Trading Eclipses Last Week's Record By Millions Of Shares

DAY'S TURNOVER IS 16,410,000 SHARES

Powerful Interests Combine To Halt Debacle

Borland L, Phys. Rev.Lett 89 (2002)

Mathematical details: the non-Gaussian model



$$dS = \mu S dt + \sigma S d\Omega$$

$$d\Omega = P(\Omega)^{\frac{1-q}{2}} d\omega$$

$$\frac{dP}{dt} = \frac{1}{2} \frac{d^2 P^{2-q}}{d\Omega^2}$$

Nonlinear Fokker-Planck

$$P = \frac{1}{Z(t)} (1 - (1-q)\beta(t)\Omega^2)^{\frac{1}{1-q}}$$

Tsallis Distribution (Student-t)

In other words: After inserting the expression for P -->

a state dependent deterministic model

$$d\Omega_t = [a_t + (q-1)b_t(\Omega_t - \Omega_0^2]^{\frac{1}{2}} d\omega_t$$

Work with

$$\Omega = \Omega(S)$$

as a computational tool allowing us to find the solution

Genralization: M.Vellekoop and H. Nieuwenhuis, QF, 2007

Averaging and conditioning w.r.t variable Ω_0

Not a perfect model of returns:

Well-defined starting price and time

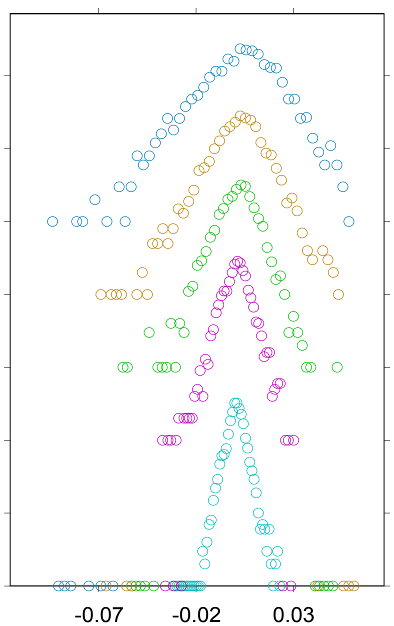
Nevertheless:

Reproduces fat-tails and volatility clustering

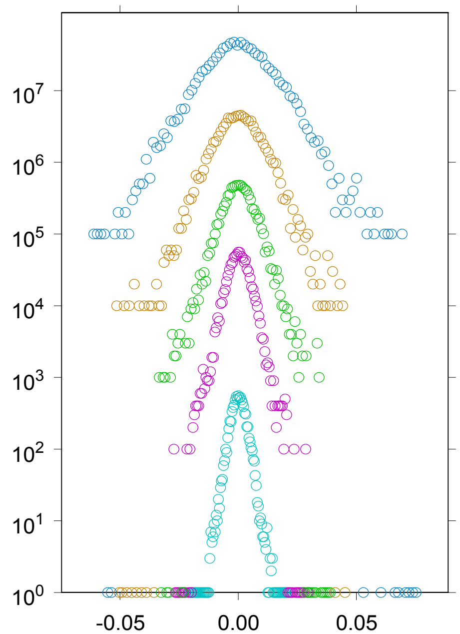
Closed form option-pricing formulae

Success for options and credit (CDS) pricing

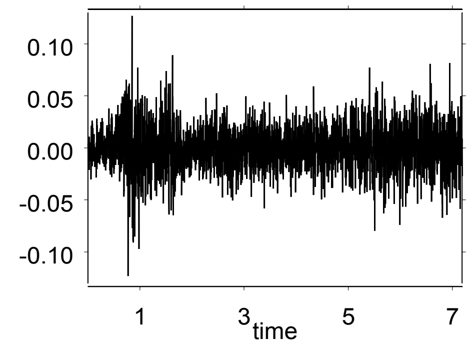
SP500



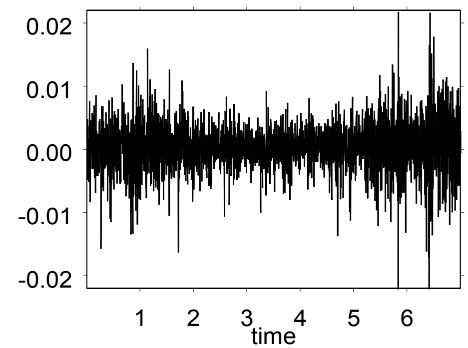
q =1.5



q=1.5



SP500



Reproduces fat-tails and volatility clustering

With a model of stock behavior, we want to study more complicated derivative markets

Some derivatives:

- Options

-Credit default swaps

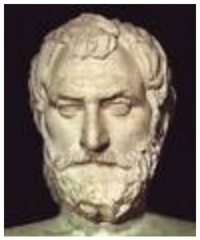
Like stocks, these are traded in large volumes, across the world

How to price them?

The price must be related to the underlying instrument, the stock

Options

The right to buy or sell a stock at a certain price (the strike) at a certain time in the future.



Greek mathematician Thales used call options on olives to make a huge profit when he foresaw a good harvest.

In Holland in the 1600s, tulip options were traded quite a bit by speculators prior to the famous tulip bubble.

1973 - the Nobel-prize winning Black-Scholes option pricing formula.

Black-Scholes Option Pricing Paradigm

S the risky stock

$F(S)$ the risky option on S

Risk-less portfolio: $\Pi = S + nF$

Return on Π must be risk-free rate r, due to no-arbitrage

This leads to the famous

Black-Scholes PDE for solving F (1973, Nobel prize 1997)

$$\frac{\partial F}{\partial t} + rS\frac{\partial F}{\partial S} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2 - rF = 0$$

Black-Scholes Option Pricing Paradigm

Equivalently:

Price options as expectations with respect to a (risk-neutral) equivalent martingale measure

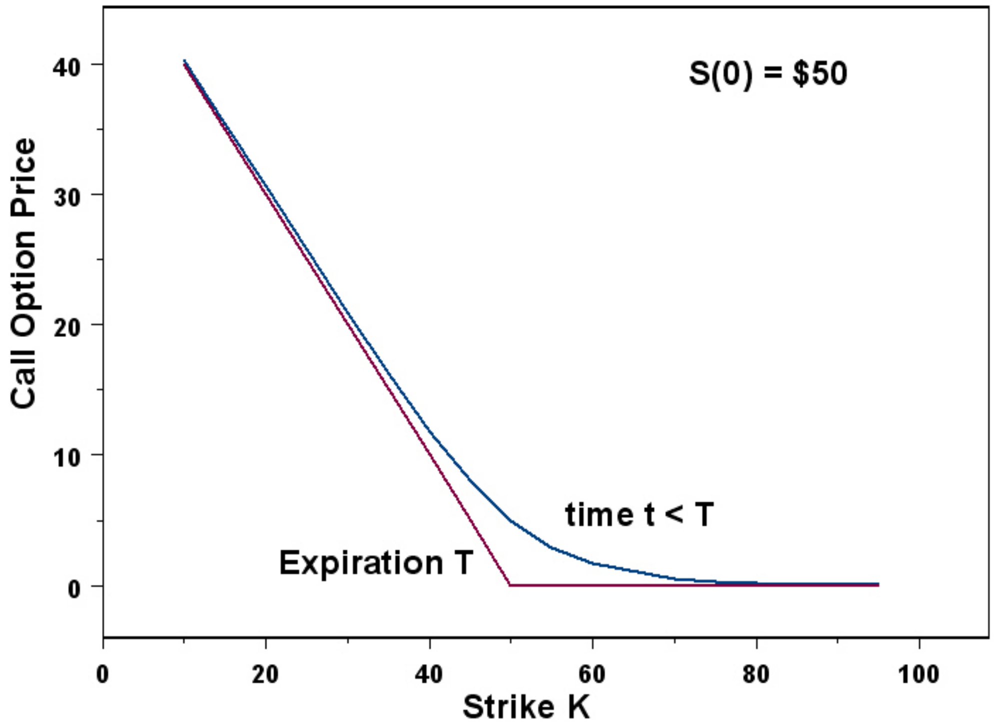
$$F = \left\langle e^{-rT} h(S) \right\rangle_Q$$

where h = the payoff of the option

(Whole field of mathematical finance based on these notions)

Example European Call Option (F=C): the right to buy at strike K at time T

$$c = \left\langle e^{-rT} \max[S(T) - K, 0] \right\rangle_Q$$

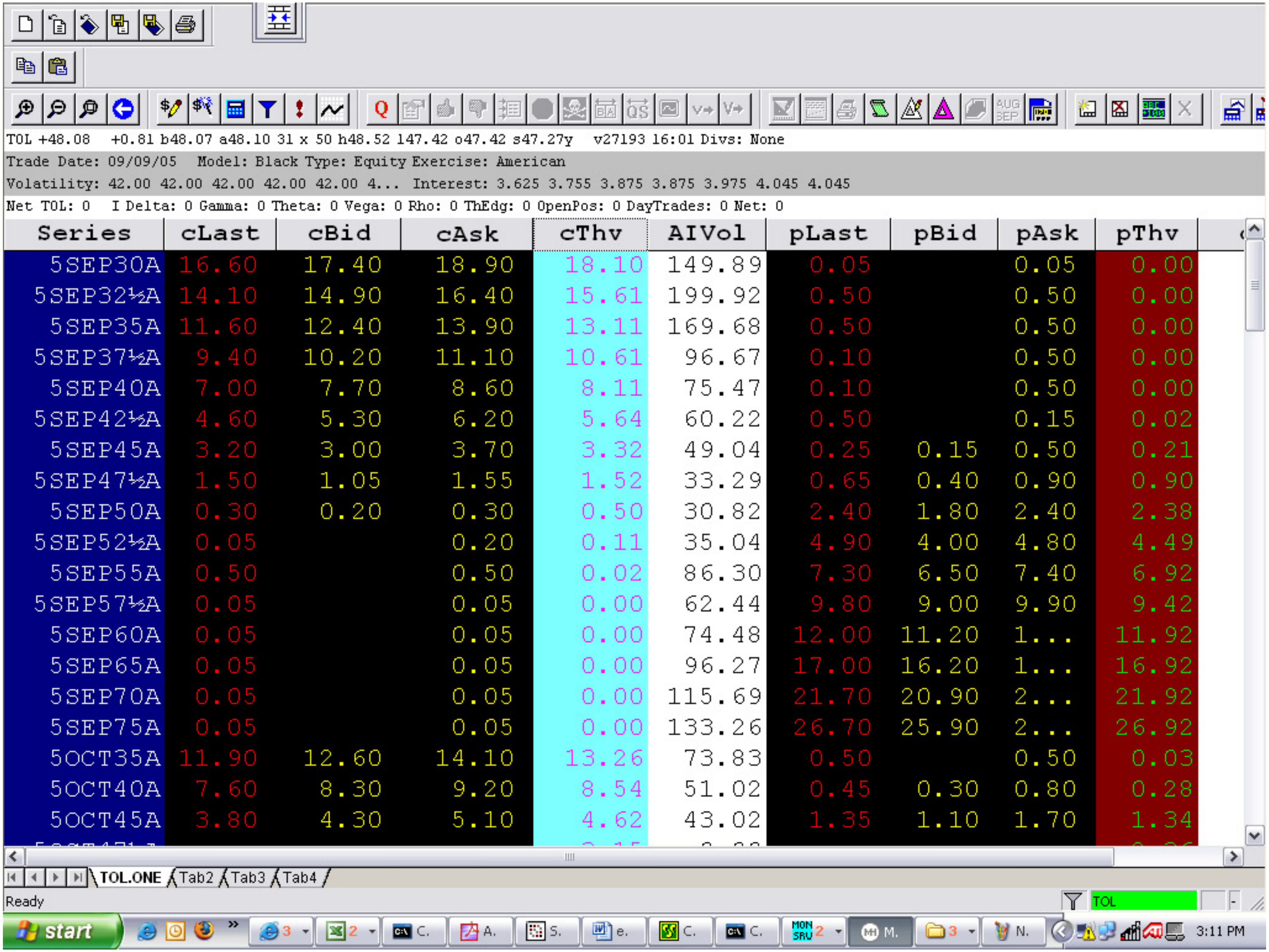


The price of the call option depends on S(0), K, T, r and σ.

Problem: Black-Scholes model must use a different σ for each K.

Volatility Smile: the plot of σ versus K.

Options: Real-time pricing and hedging

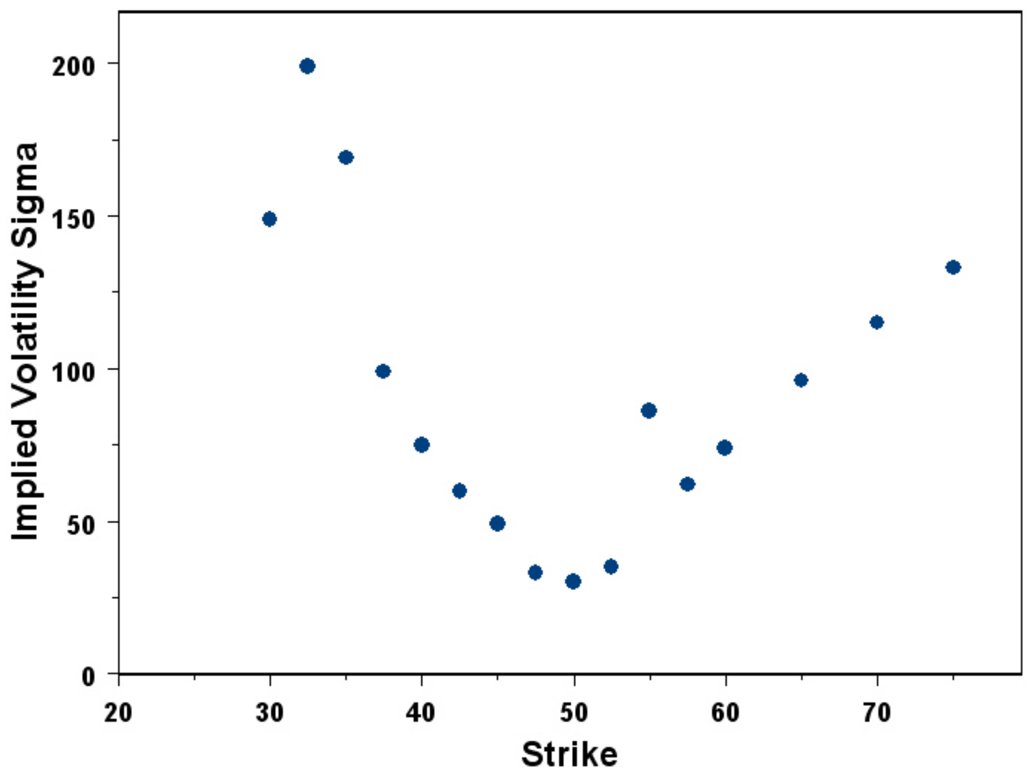


TOL +48.08 +0.81 b48.07 a48.10 31 x 50 h48.52 147.42 o47.42 s47.27y v27193 16:01 Divs: None
Trade Date: 09/09/05 Model: Black Type: Equity Exercise: American
Volatility: 42.00 42.00 42.00 42.00 42.00 4... Interest: 3.625 3.755 3.875 3.875 3.975 4.045 4.045
Net TOL: 0 I Delta: 0 Gamma: 0 Theta: 0 Vega: 0 Rho: 0 ThEdg: 0 OpenPos: 0 DayTrades: 0 Net: 0

Series	cLast	cBid	cAsk	cThv	AIVol	pLast	pBid	pAsk	pThv
5SEP30A	16.60	17.40	18.90	18.10	149.89	0.05		0.05	0.00
5SEP32½A	14.10	14.90	16.40	15.61	199.92	0.50		0.50	0.00
5SEP35A	11.60	12.40	13.90	13.11	169.68	0.50		0.50	0.00
5SEP37½A	9.40	10.20	11.10	10.61	96.67	0.10		0.50	0.00
5SEP40A	7.00	7.70	8.60	8.11	75.47	0.10		0.50	0.00
5SEP42½A	4.60	5.30	6.20	5.64	60.22	0.50		0.15	0.02
5SEP45A	3.20	3.00	3.70	3.32	49.04	0.25	0.15	0.50	0.21
5SEP47½A	1.50	1.05	1.55	1.52	33.29	0.65	0.40	0.90	0.90
5SEP50A	0.30	0.20	0.30	0.50	30.82	2.40	1.80	2.40	2.38
5SEP52½A	0.05		0.20	0.11	35.04	4.90	4.00	4.80	4.49
5SEP55A	0.50		0.50	0.02	86.30	7.30	6.50	7.40	6.92
5SEP57½A	0.05		0.05	0.00	62.44	9.80	9.00	9.90	9.42
5SEP60A	0.05		0.05	0.00	74.48	12.00	11.20	1...	11.92
5SEP65A	0.05		0.05	0.00	96.27	17.00	16.20	1...	16.92
5SEP70A	0.05		0.05	0.00	115.69	21.70	20.90	2...	21.92
5SEP75A	0.05		0.05	0.00	133.26	26.70	25.90	2...	26.92
5OCT35A	11.90	12.60	14.10	13.26	73.83	0.50		0.50	0.03
5OCT40A	7.60	8.30	9.20	8.54	51.02	0.45	0.30	0.80	0.28
5OCT45A	3.80	4.30	5.10	4.62	43.02	1.35	1.10	1.70	1.34

TOL.ONE Tab2 Tab3 Tab4
Ready TOL
3:11 PM

Volatility smile in the example of previous slide



Traders intuitively correct for the tails by bumping up the Black-Scholes volatility at far strikes

Part 1: Across time ....

1. A non-Gaussian model of returns
2. Options pricing incorporating fat-tails
3. Applications

Non-Gaussian option pricing with statistical feedback model:

There is a unique equivalent martingale measure →
can obtain closed-form option pricing formulae

Note: This is not the case for Levy noise or stochastic volatility

Parsimony:

Because the model incorporates tails for q = 1.4, we only need one value of sigma to capture all option prices across strikes

1) Exploit PDE's implied by arbitrage-free portfolios

Solve PDE to get option price

Generalization: M.Vellekoop and H. Nieuwenhuis, 2007

2) Convert prices of assets into martingales

Take expectations to get option price

$$c = \left\langle e^{-rT} h(S(T)) \right\rangle_Q$$

?Detail

Use non-Gaussian model as basis for stock price dynamics

$$S(T) = S(0)\exp\left\{\Omega_T + rT - \frac{\sigma^2}{2}\int_0^T P(\Omega_t)^{1-q}\,dt\right\}$$

Integrate using generalized Feynman-Kac

$$\approx \gamma(T)\Omega_T^2$$

- Approximation allows for closed-form solutions

- Due to the approximation:

  Martingale "breaks down" but valid for $\sigma^2 T << 1$

?Detail

European Call

$$c = \left\langle e^{-rT} \max[S(T) - K, 0] \right\rangle_Q$$

Stock Price

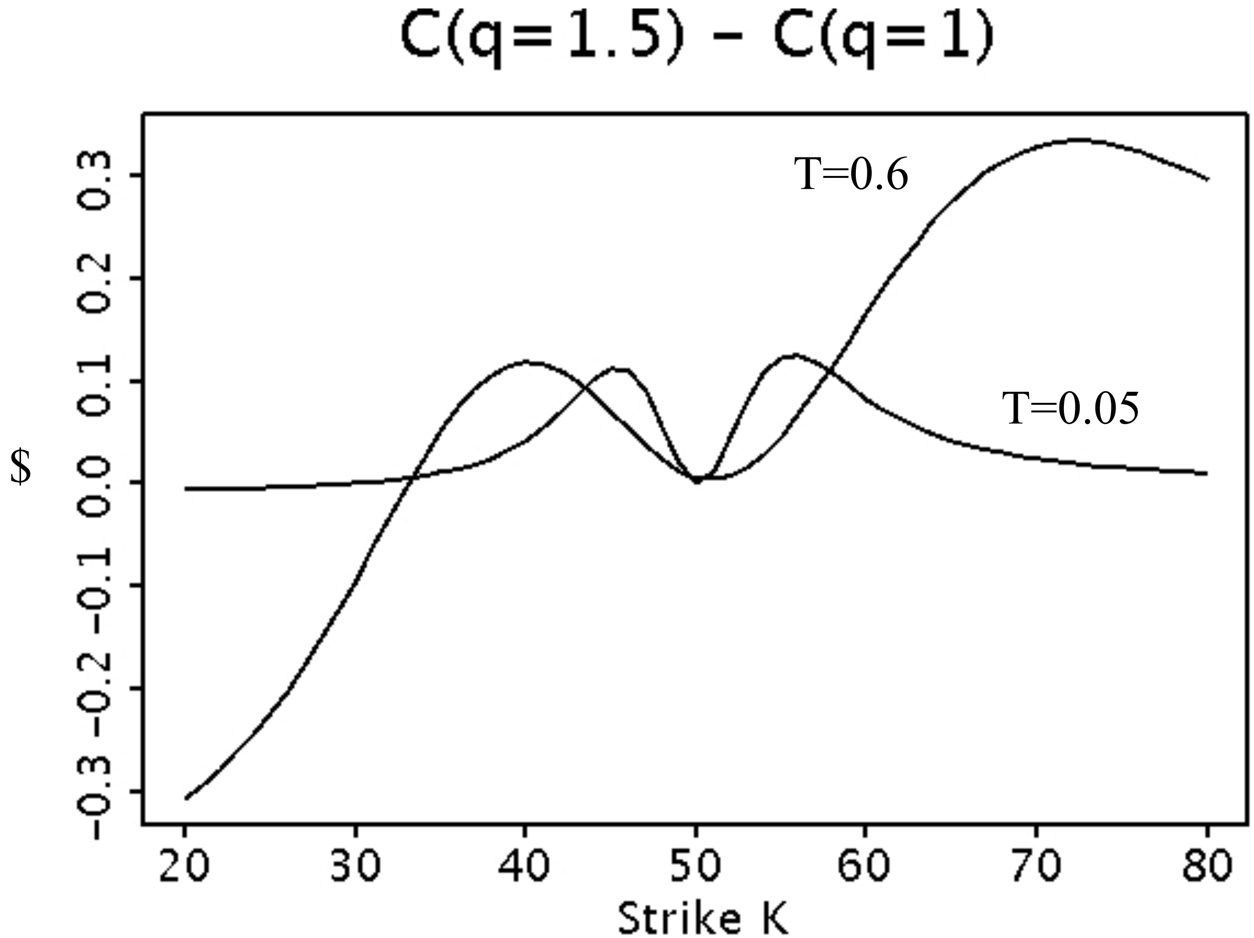
$$S(T) = S(0) \exp\left\{ \sigma\Omega_T + rT - \frac{\sigma^2}{2}\gamma_T - (1-q) g_T \Omega_T^2 \right\}$$

Payoff if $\{S(T) > K\}$ $\rightarrow$ $d_1 \leq \Omega_T \leq d_2$

$$\boxed{c = e^{-rT} \int_{d_1}^{d_2} (S(T) - K) P_q(\Omega_T) d\Omega_T}$$

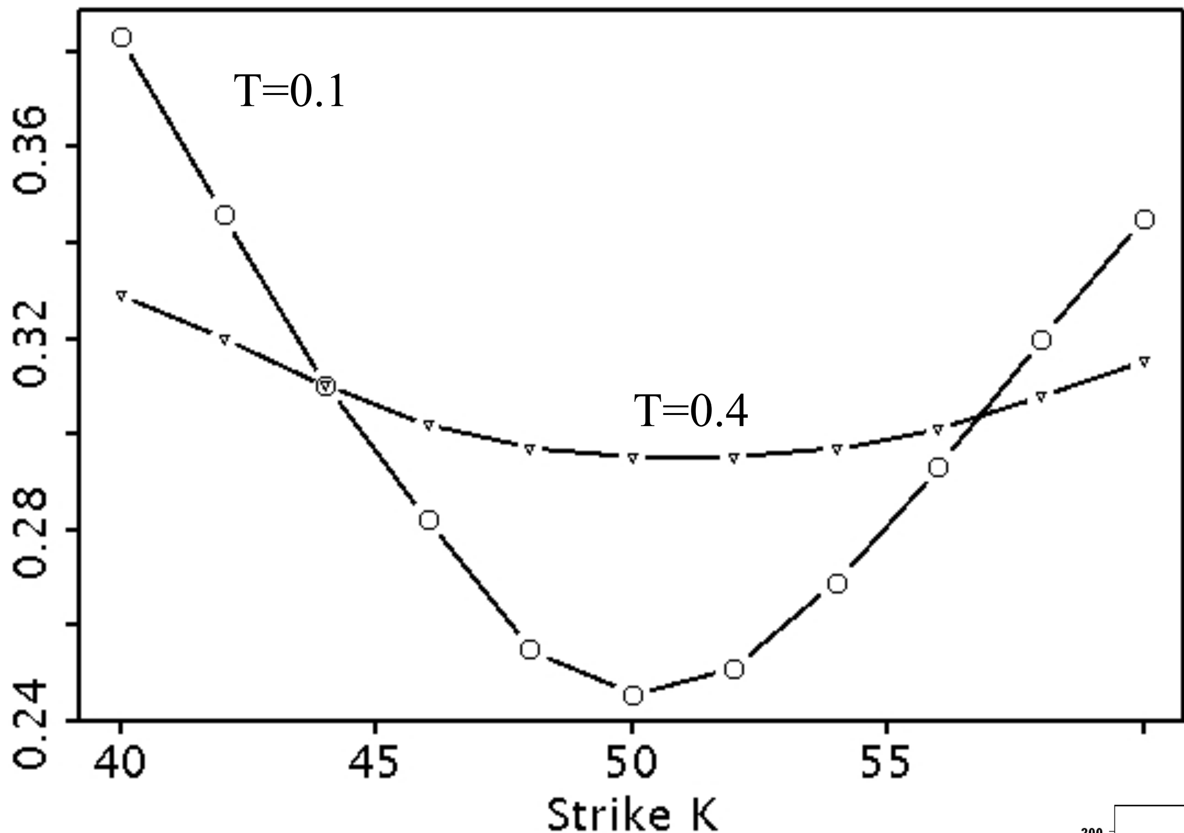
q = 1: P is Gaussian   q >1 : P is fat tailed Tsallis dist.

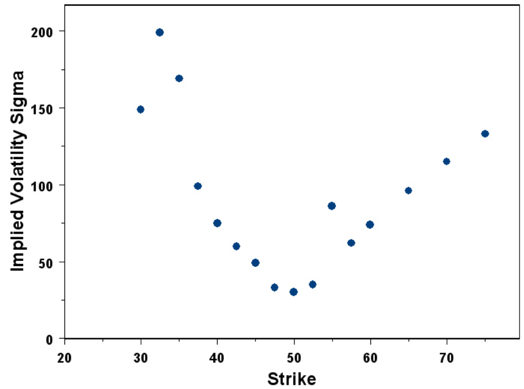
Call Price Difference



S(0) = \$50, r= 6%, σ =0.3

Implied Black-Scholes Volatility
(from q = 1.4 model with sigma = .3)

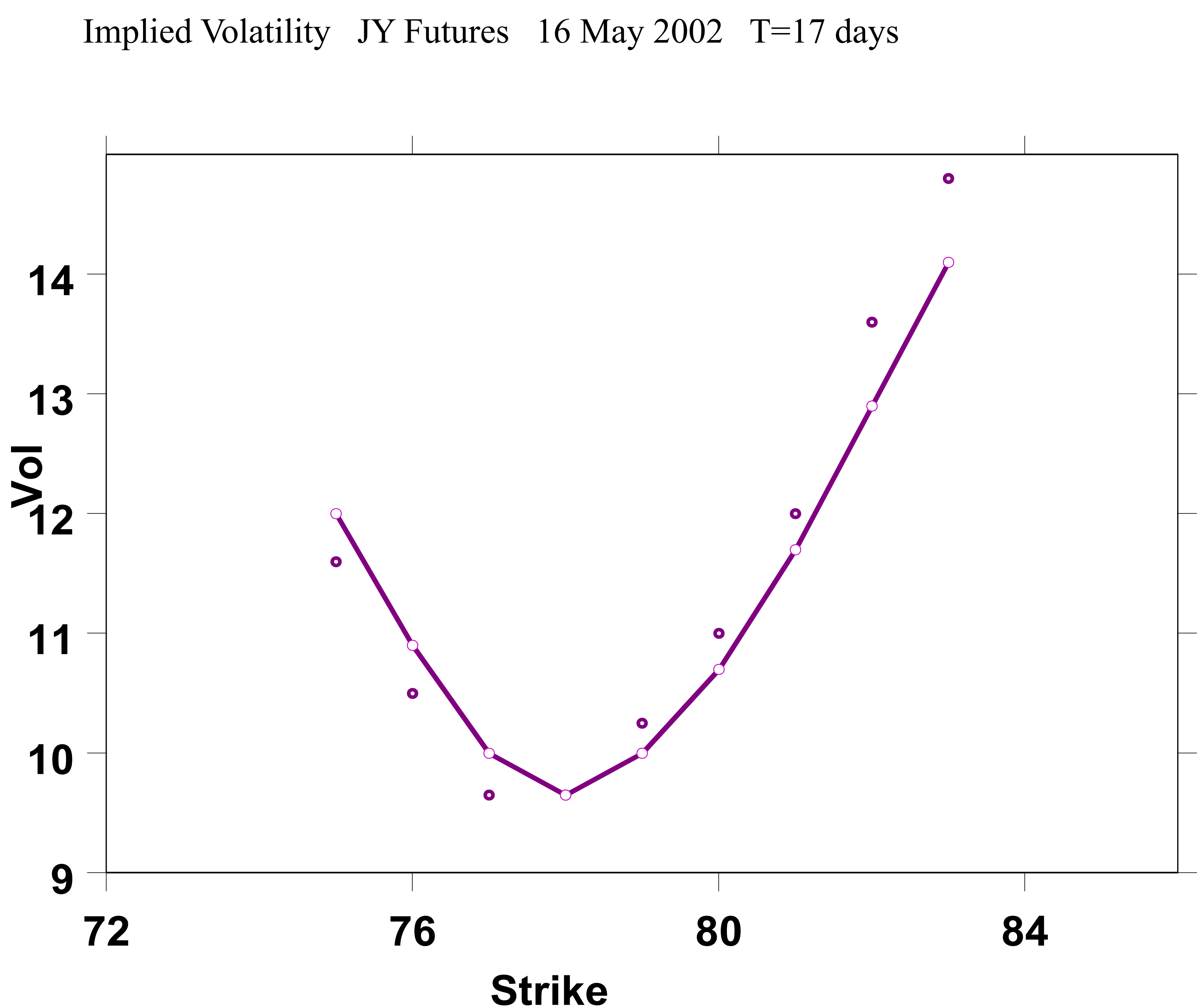


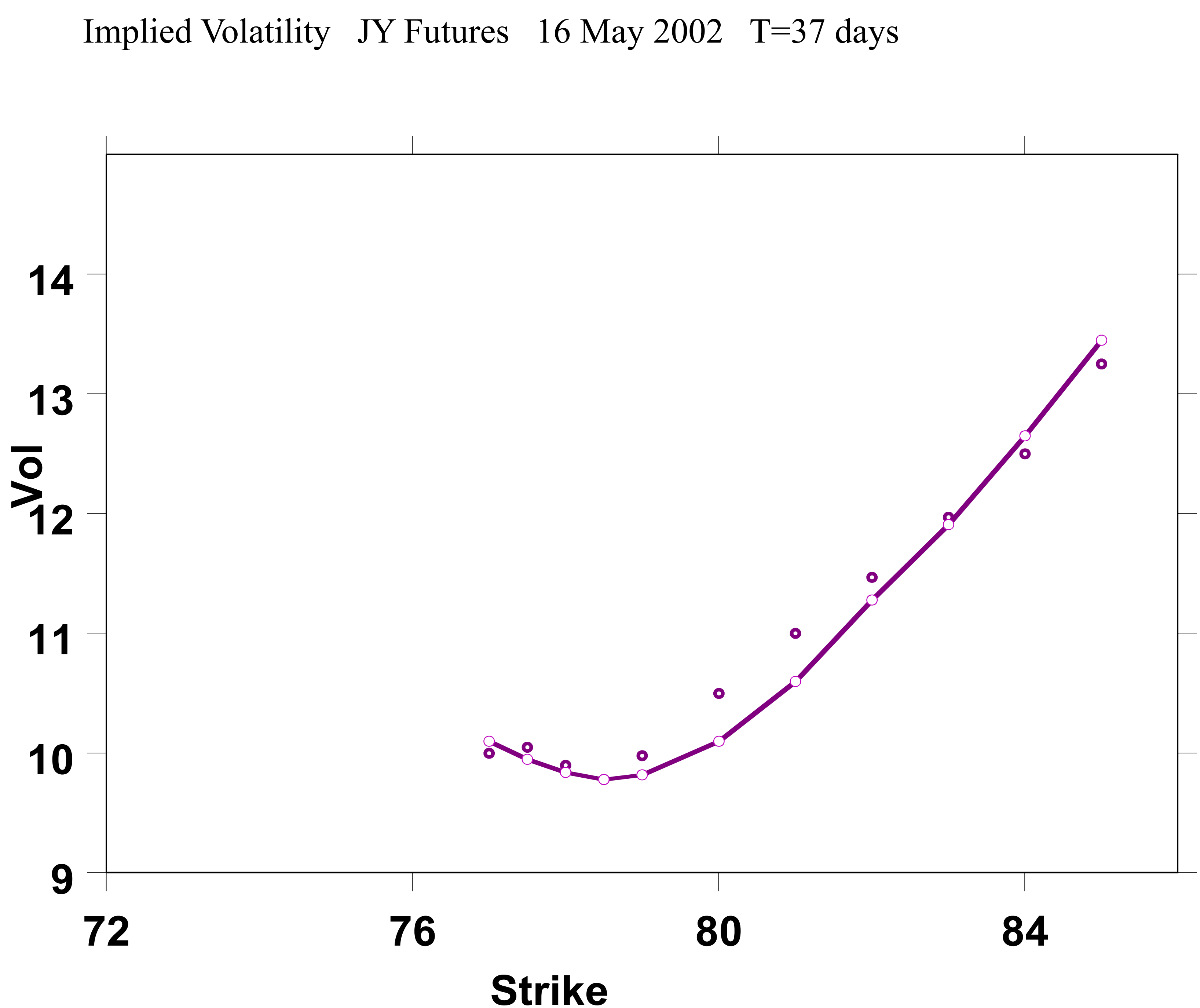


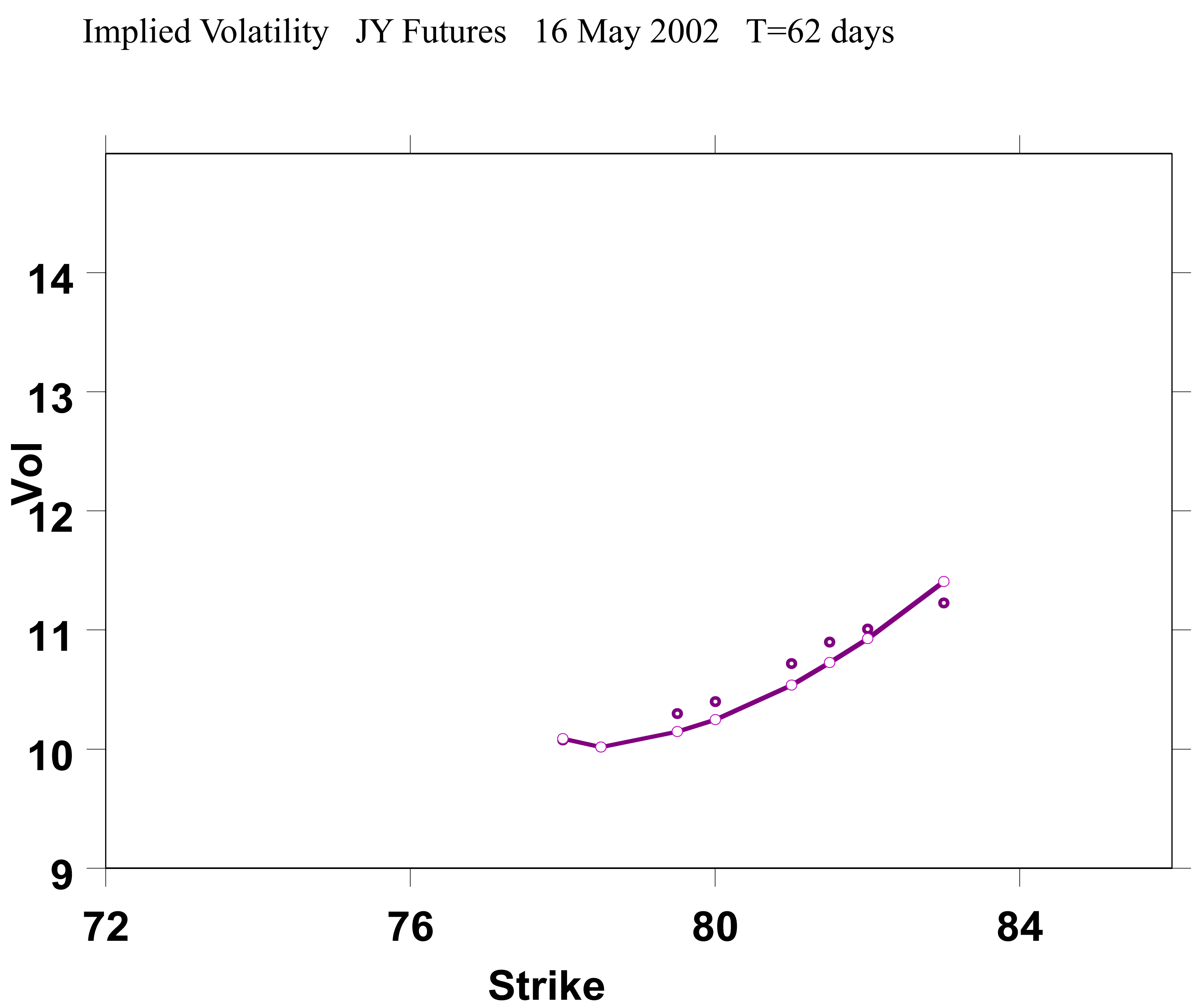
This is very similar to the volatilities that traders use

Need only one sigma across all strikes if q = 1.4

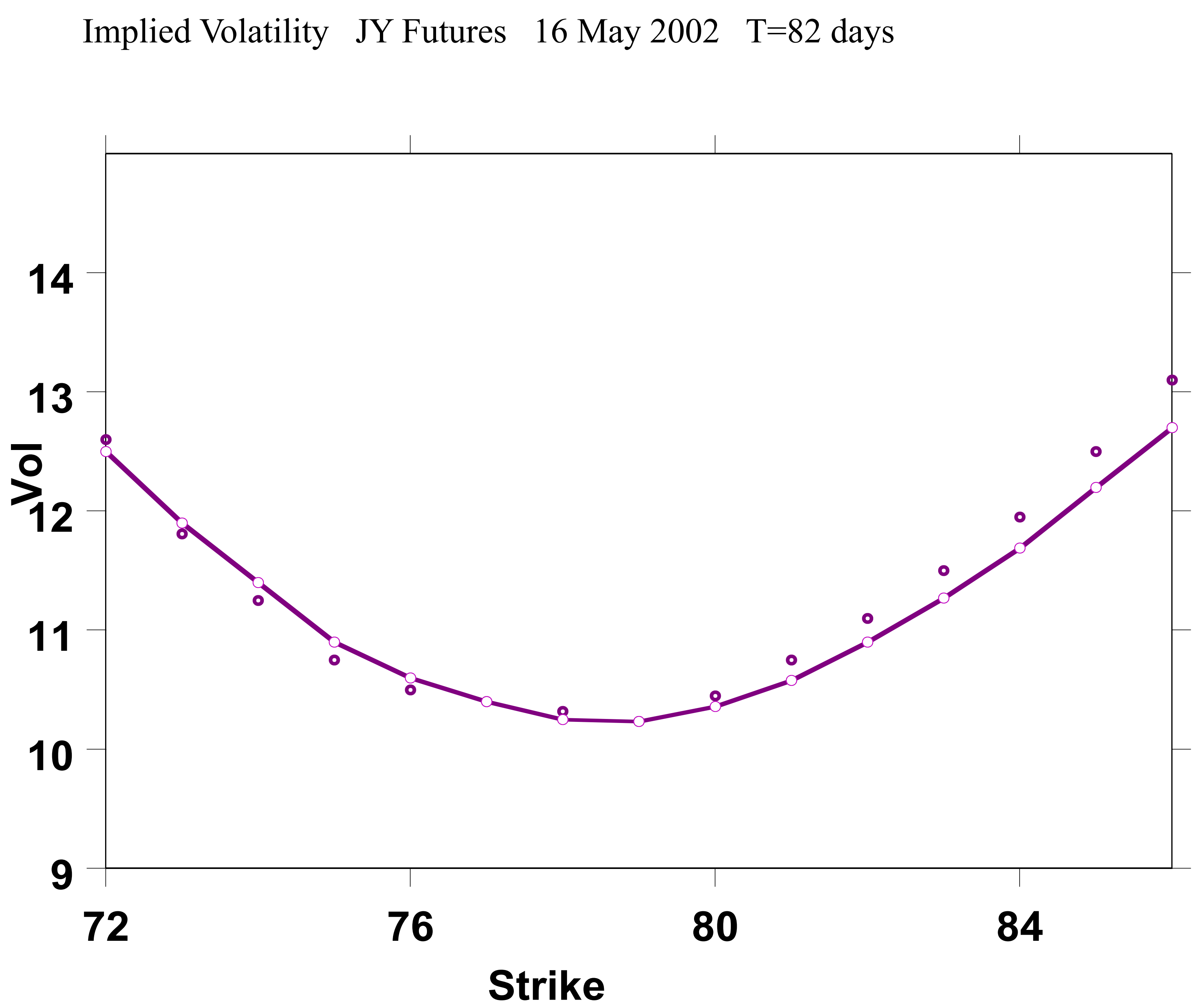
Example (Japanese Yen futures)

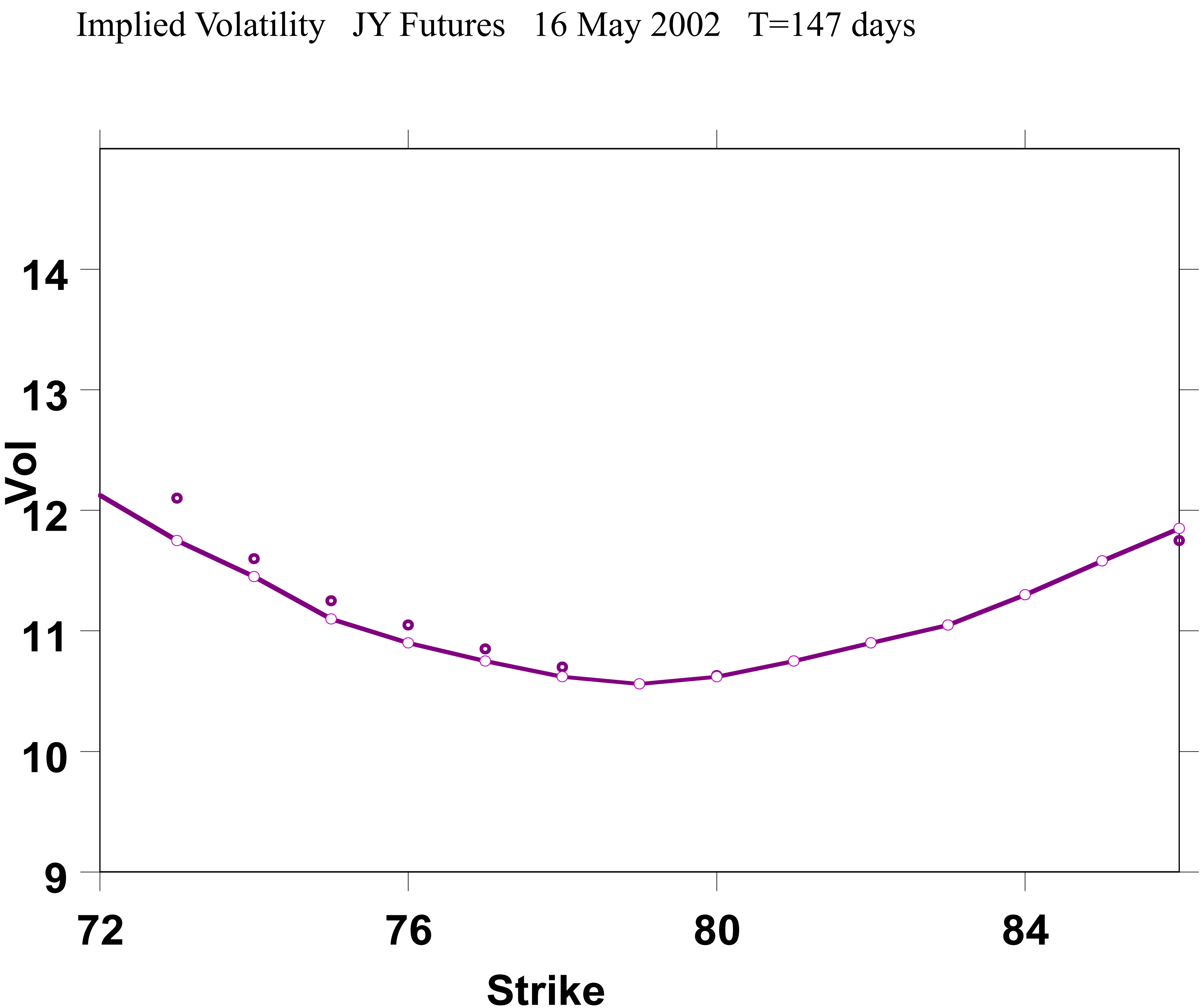

Implied Volatility   JY Futures   16 May 2002   T=17 days
Vol
14
13
12
11
10
9
72
76
80
84
Strike


Implied Volatility JY Futures 16 May 2002 T=37 days
Vol
Strike
9
10
11
12
13
14
72
76
80
84



Implied Volatility JY Futures 16 May 2002 T=62 days
Vol
14
13
12
11
10
9
72
76
80
84
Strike


Implied Volatility JY Futures 16 May 2002 T=82 days
Vol
14
13
12
11
10
9
72
76
80
84
Strike


Implied Volatility JY Futures 16 May 2002 T=147 days
Vol
14
13
12
11
10
9
72
76
80
84
Strike

Extension to include skew (asymmetry) in the model [Borland and Bouchaud, 2004]

Price crashes induce higher volatility than price rallies Leverage effect

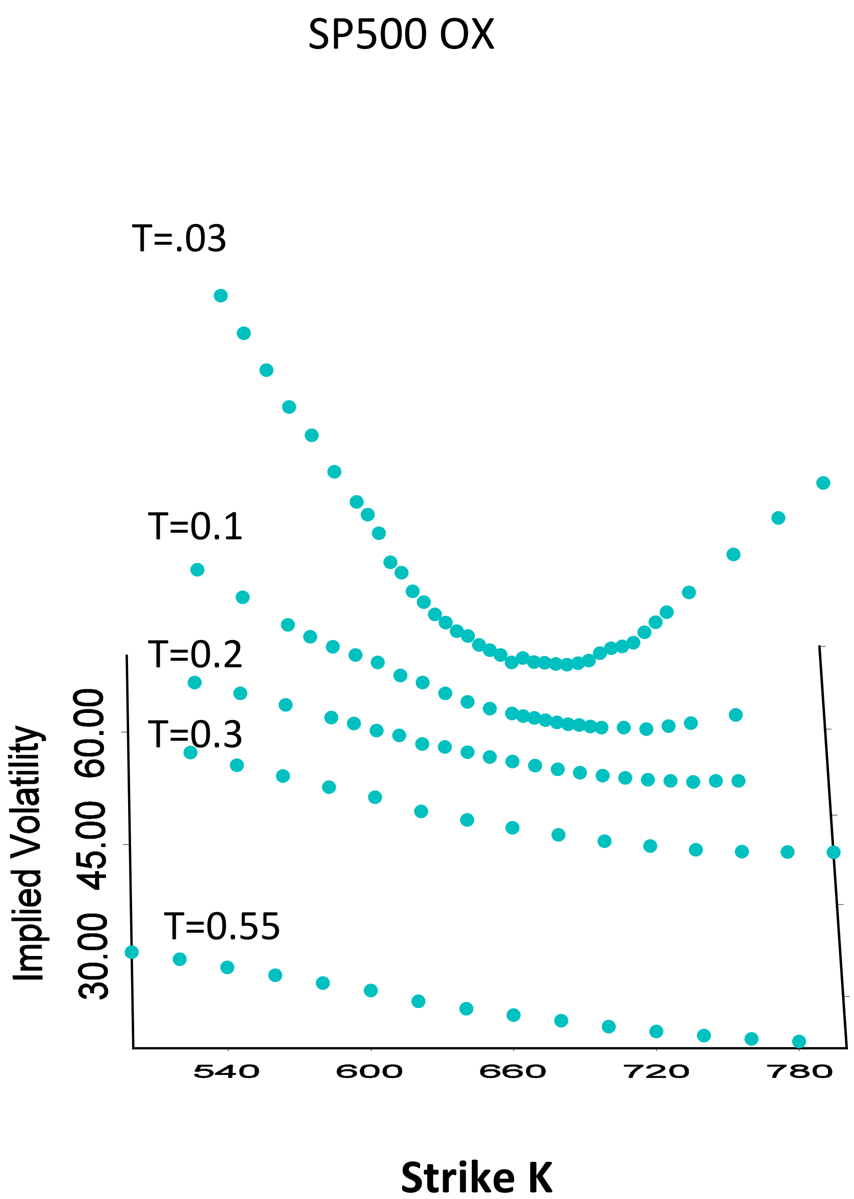
Fluctuations modeled as $$\boxed{\sigma S^{\alpha} P^{\frac{1-q}{2}} d\omega}$$ $\alpha \leq 1$

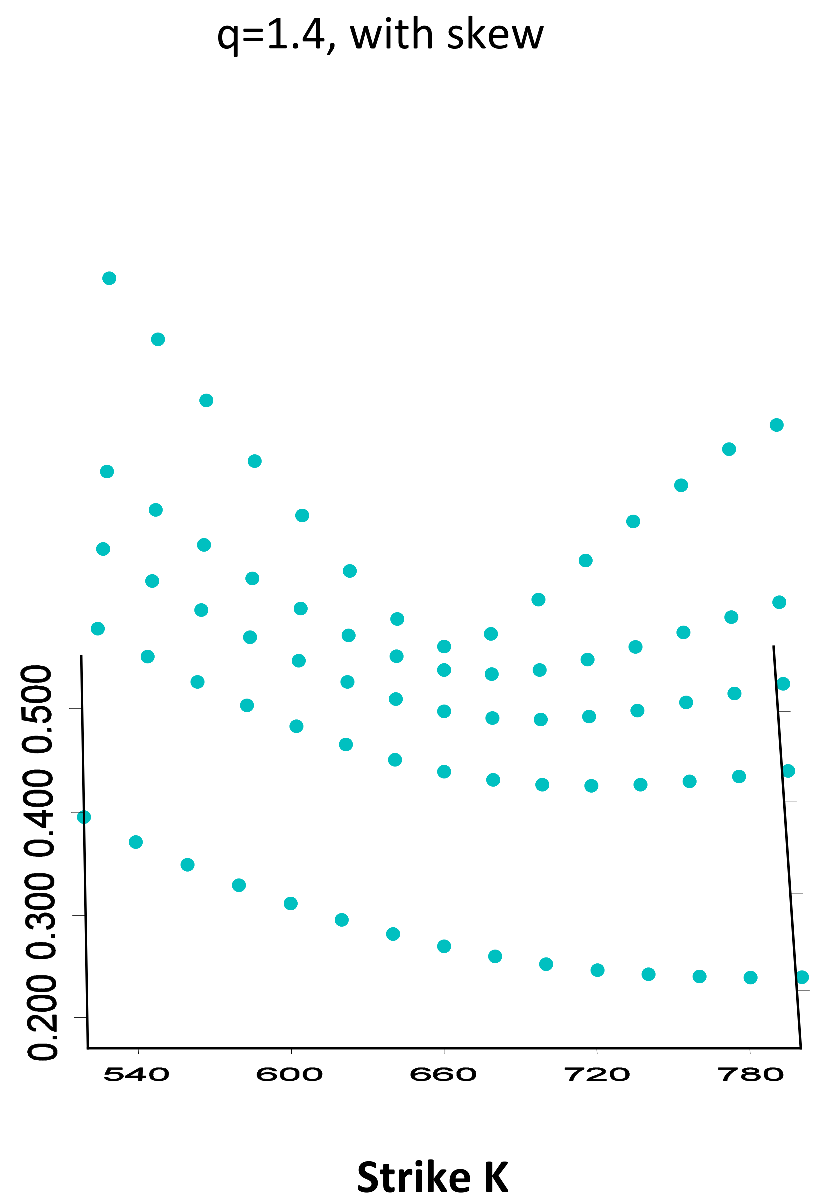
q Controls the tail

α Controls the skew

σ Volatility parameter

We obtain closed-form formulae for call options Pade expansion, Feynman-Kac


SP500 OX
T=.03
T=0.1
T=0.2
T=0.3
T=0.55
Implied Volatility
30.00 45.00 60.00
540 600 660 720 780
Strike K


q=1.4, with skew
0.200 0.300 0.400 0.500
540 600 660 720 780
Strike K

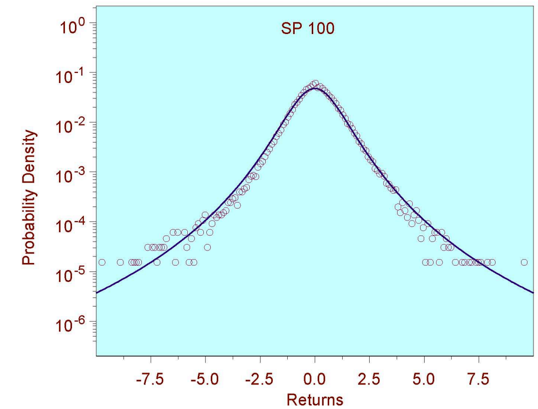
Part 1: Across time ....

1. A non-Gaussian model of returns

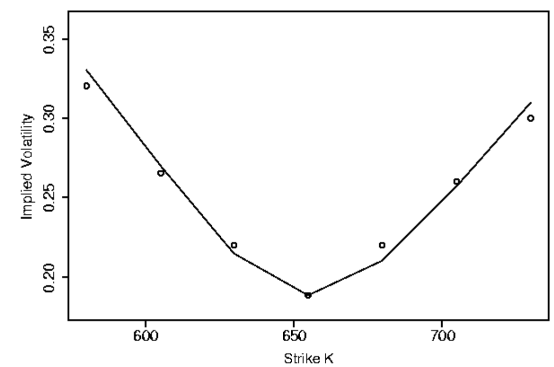
2. Options pricing incorporating fat-tails

3. Applications

-Can fit the model to historical distribution → calculate option prices



-Can imply the parameters from the option
- market

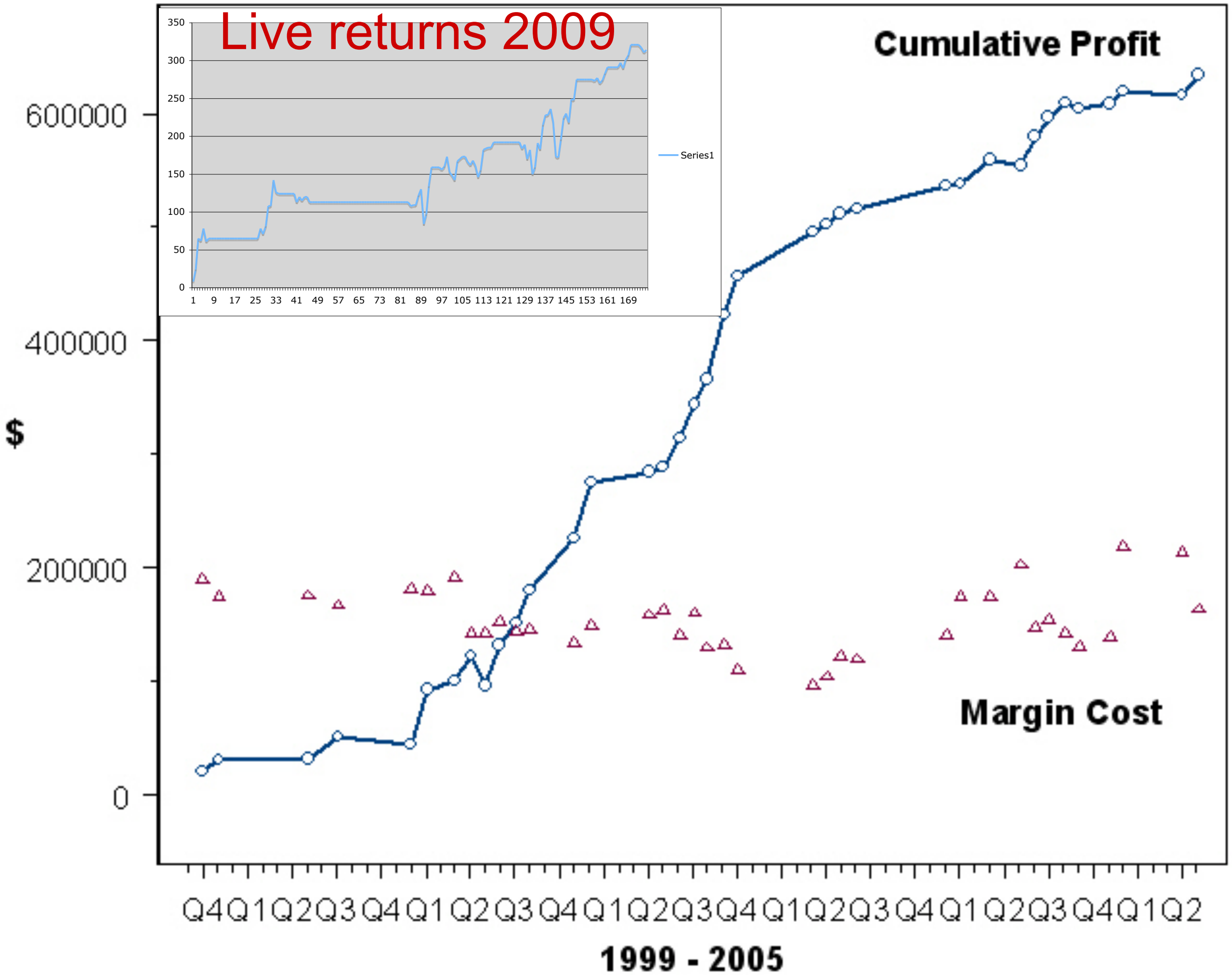


Results for stocks (top 100 stocks in S&P index'
In both stock and option markets:

- $q = 1.4$ converging slowly to Gaussian
- $\alpha \approx 0.3$
- $\sigma \approx 30\%$

Back test of trading strategy (with A. Christian Silva)

Exploit slight deviations which should converge



Options look good … what about pricing credit?

Credit Default Swaps:

Payment if the company defaults

Protection Buyer ← Protection Seller

Protection Buyer → Protection Seller

Premium (CDS price)

CDS price depends on probability to default.

We get pricing formula from the non-Gaussian model.

We implied q α σ from empirical CDS prices:

$$q = 1.2 - 1.4$$
$$\alpha = 0.2 - 0.5$$
$$\sigma = 30\%$$

Non-Gaussian model well describes many features of:

Stock Markets
Option Markets
Debt and Credit Markets

- Key ingredient of success for statistical feedback model: long-range memory (with respect to a single time)

- Random increments are uncorrelated but not independent

Motivation of statistical feedback model:

Intuition - traders react to extreme events

Generalization: Multi timescale statistical feedback model:

Intiuition -
different classes of traders react on different time-scales

For example: HARCH (Muller et al)
FIGARCH (Baillie et al)
Multi-timescale statistical feedbacl (Borland and Bouchaud)

A multi-time scale non-Gaussian model of stock return [Borland and Bouchaud 2012]

$$\Delta y_t = \sigma_t \Delta \omega_t$$

$$\sigma_t^2 = \sigma_0^2 \left[1 + g \sum_{l=1}^{\infty} \frac{1}{l^{\alpha}} \left(y_t - y_{(t-l)} \right)^2 \right]$$

Reproduces "all" stylized facts across time

Motivation: Traders act on all different time horizons

A multi-time scale non-Gaussian model of stock return [Borland and Bouchaud 2012]

y = log(Stock Price)

ω_t Gaussian noise uncorrelated in time

σ_t Volatility

g, α Parameters

$$\Delta y_t = \sigma_t \Delta \omega_t$$

$$\sigma_t^2 = \sigma_0^2 \left[1 + g \sum_{l=1}^{\infty} \frac{1}{l^\alpha} \left(y_t - y_{(t-l)} \right)^2 \right]$$

For t- l = 0 recovers Statistical feedback model

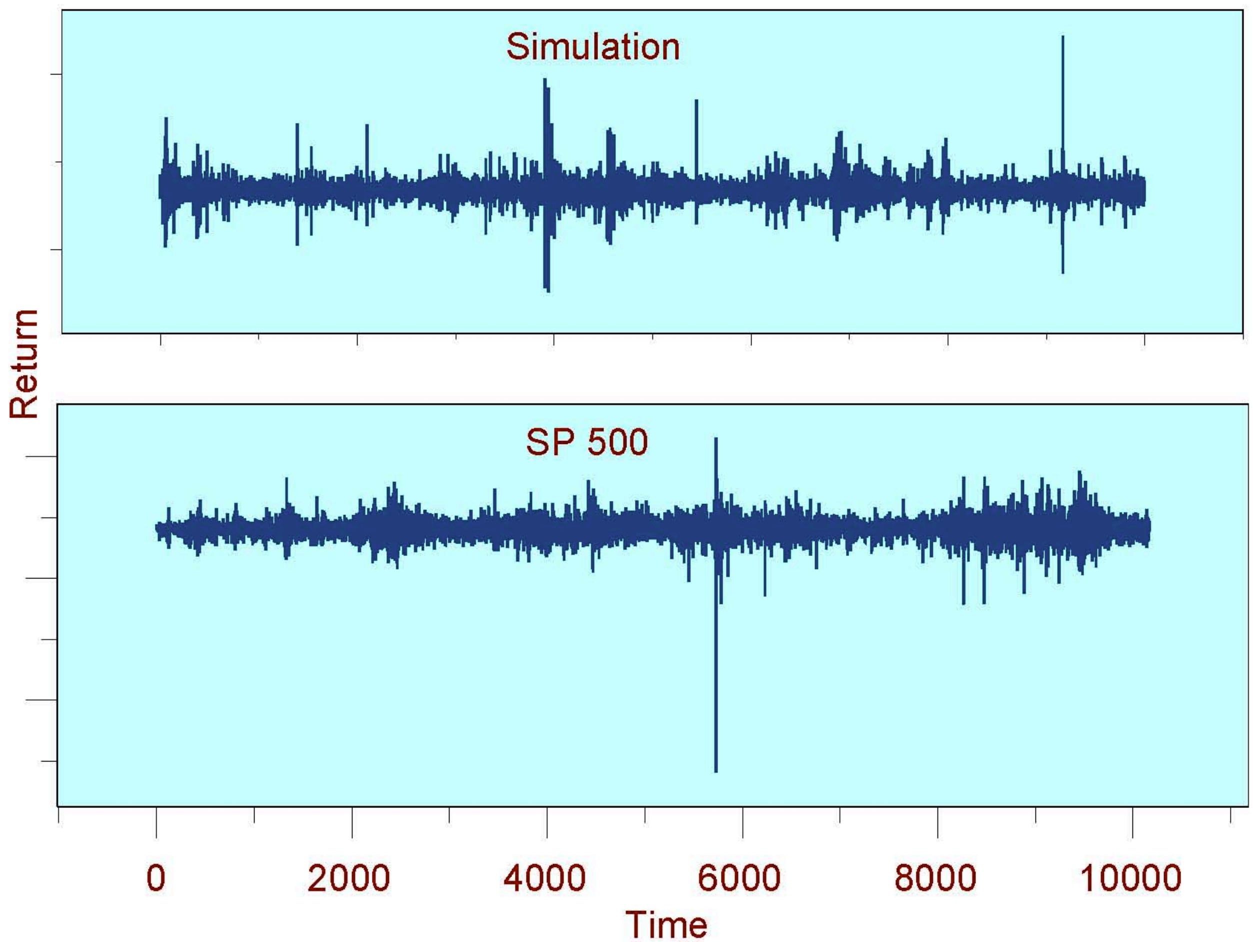
Reproduces "all" stylized facts across time

g controls the feedback

α controls the memory

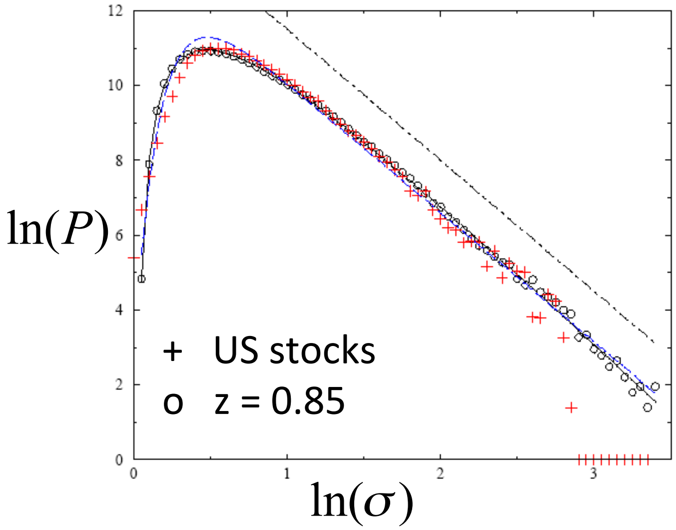
GARCH
HARCH (Muller et al)
FIGARCH (Baillie et al)

In our study: Simulations
Some analytic calculations
Calibration to stylized facts

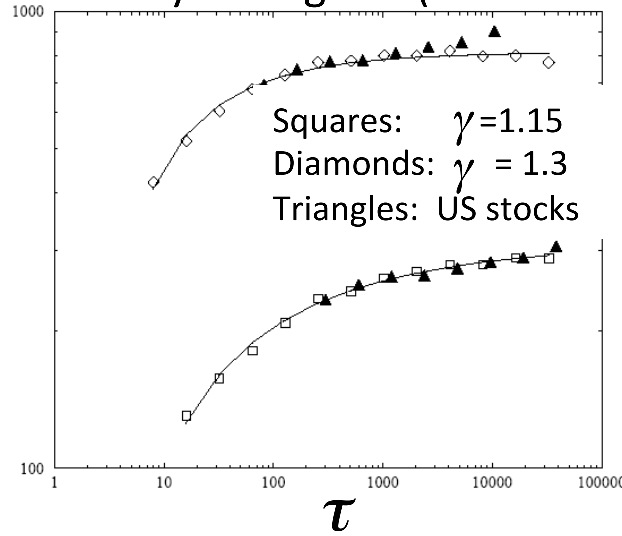


Other typical calibration methods: - Method of moment
- Maximum likelihood

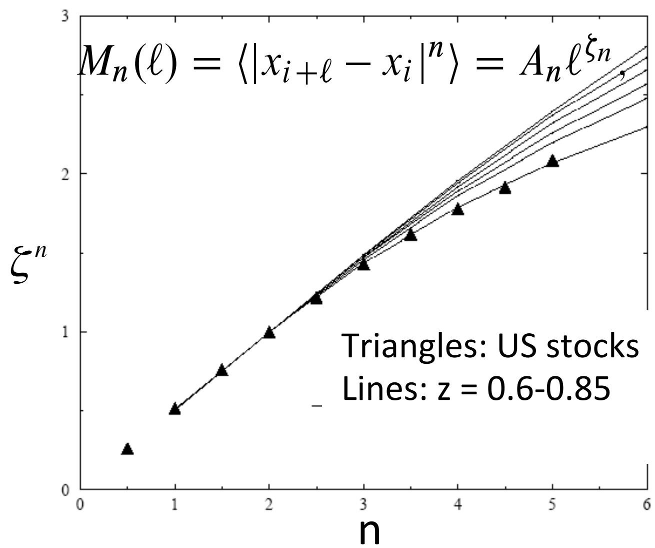
Distribution of Volatility



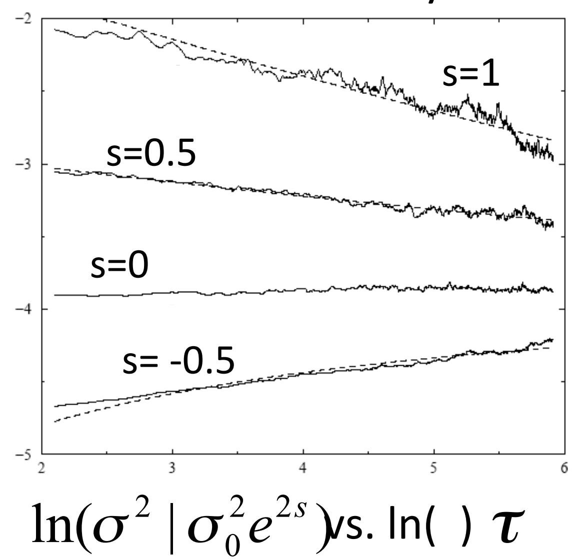
Volatility Variogram (autocorrelation)



Multi-fractal scaling



Conditional Volatility

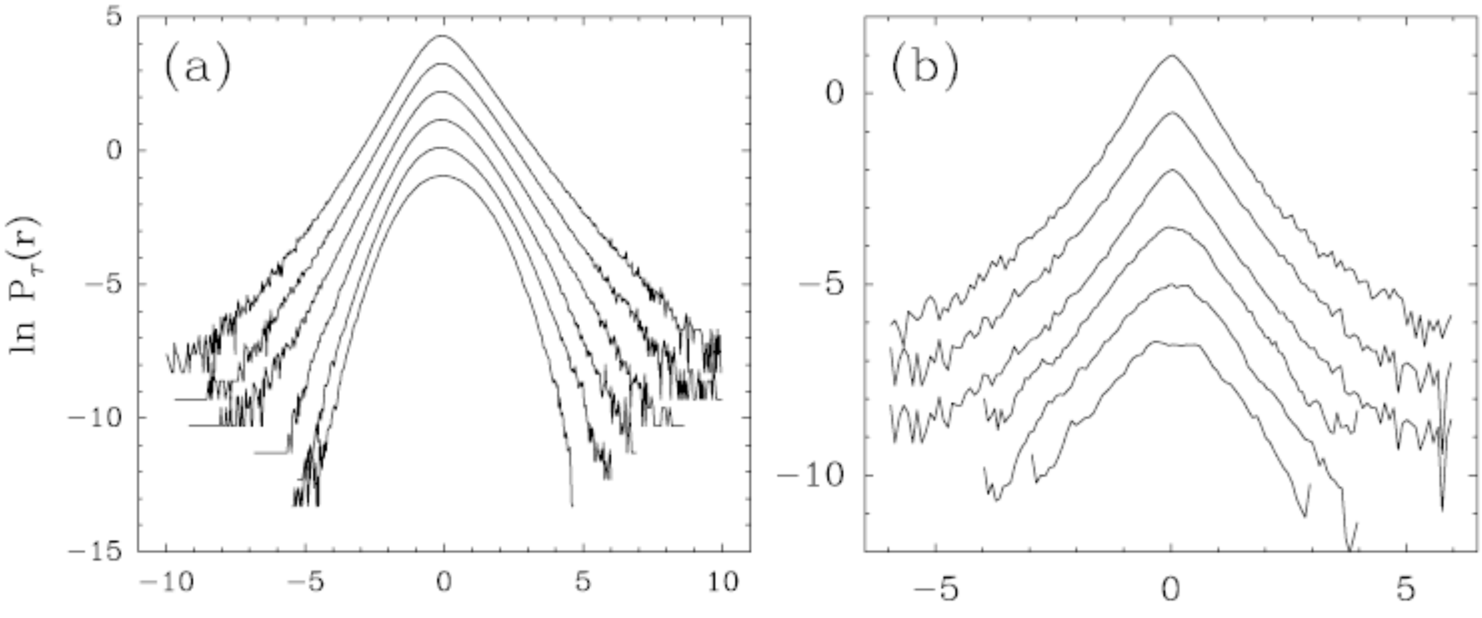


Calibration to stylized facts $z = 0.85, \gamma = 1.15$

Interesting analogy between multi-timescale models and multi-fractal (cascade) models

Turbulent velocity increments

Financial returns



Small scales (top), larger scales (bottom)

For finance, multi-timescale makes more sense because of time-reversal asymmetry

So far, we have studied models of stock prices across *time*

Now let us look at the dynamics across *stocks [Correlations]*

Exploring the joint stochastic process

What do we know about volatility?

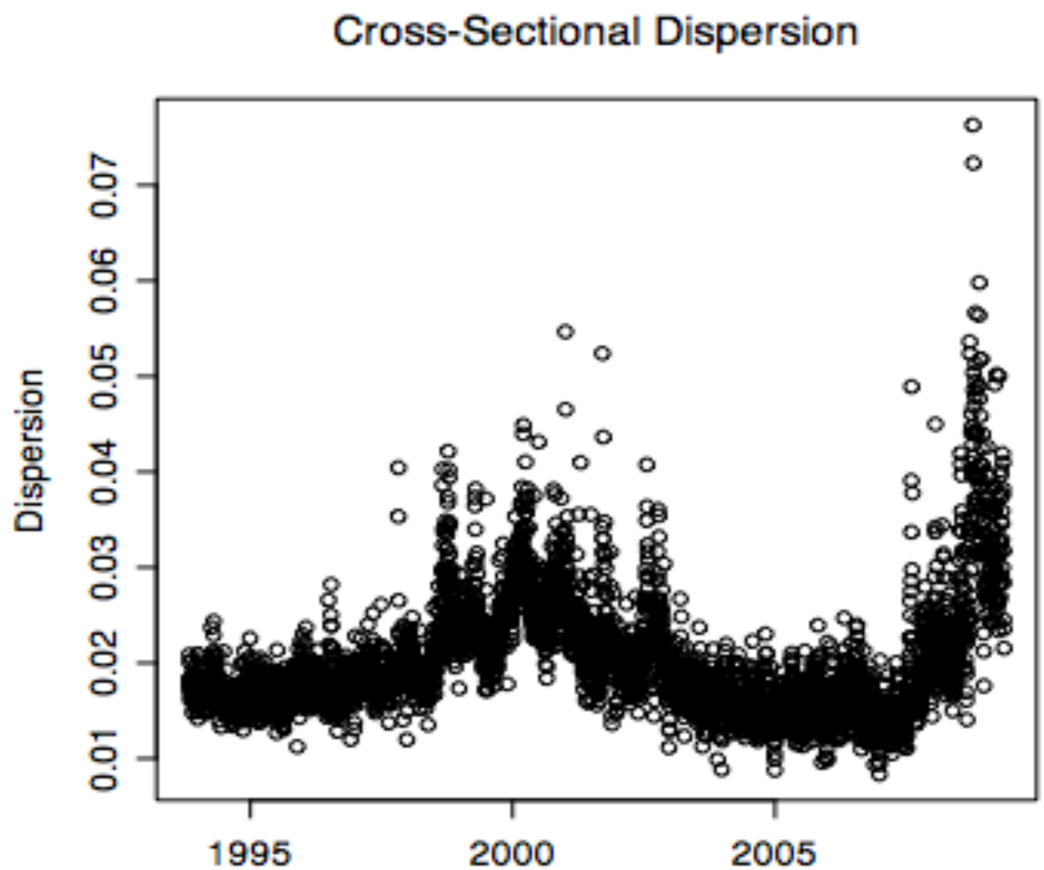
- Across time for a given stock Part 1

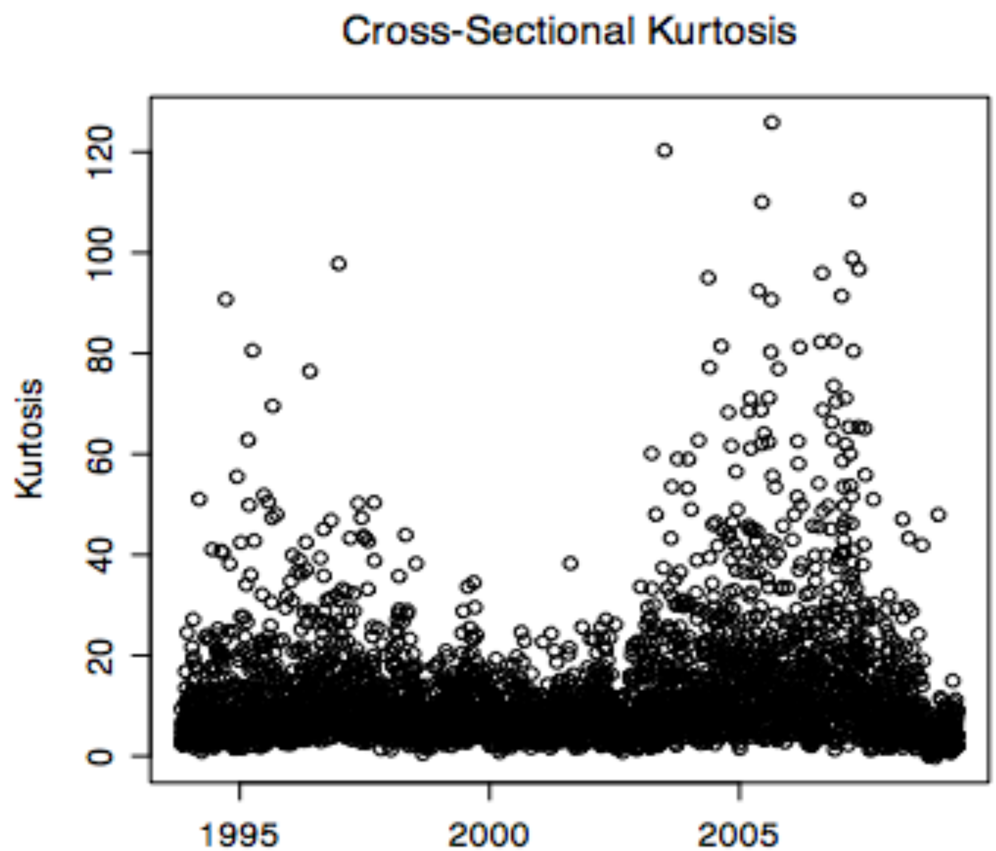
- Across stocks at a given time Part 2

Across stocks ….

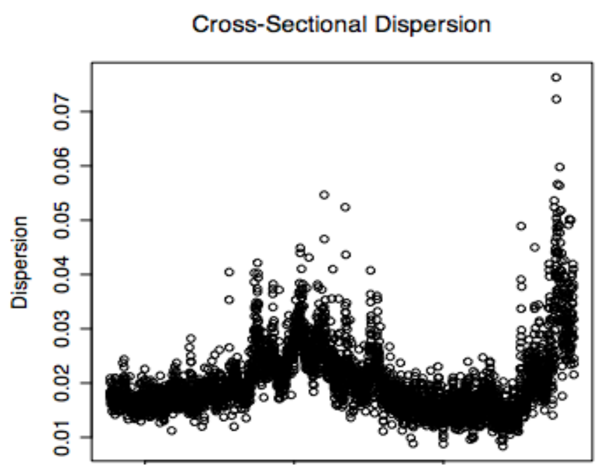
1. Cross-sectional dispersion
2. Cross-sectional kurtosis
3. Correlations
4. Do markets exhibit a phase-transition in times of panic?
5. A simple model

See also "Borland L, "Statistical signatures in times of panic: markets as a self-organizing system", (2009)

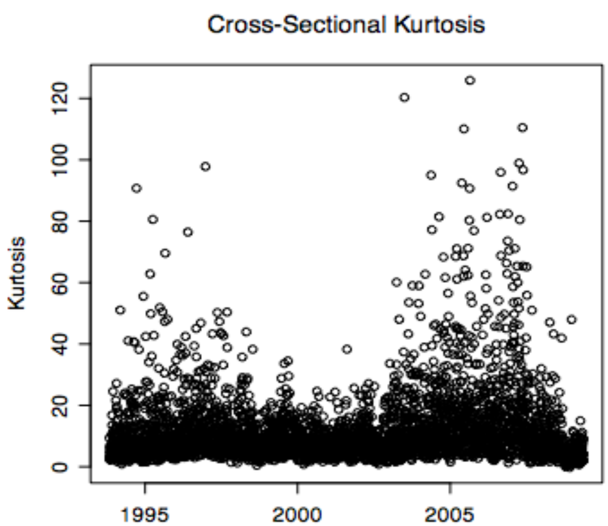

Cross-Sectional Dispersion
Dispersion
0.01
0.02
0.03
0.04
0.05
0.06
0.07
1995
2000
2005



Cross-Sectional Kurtosis
Kurtosis
0
20
40
60
80
100
120
1995
2000
2005

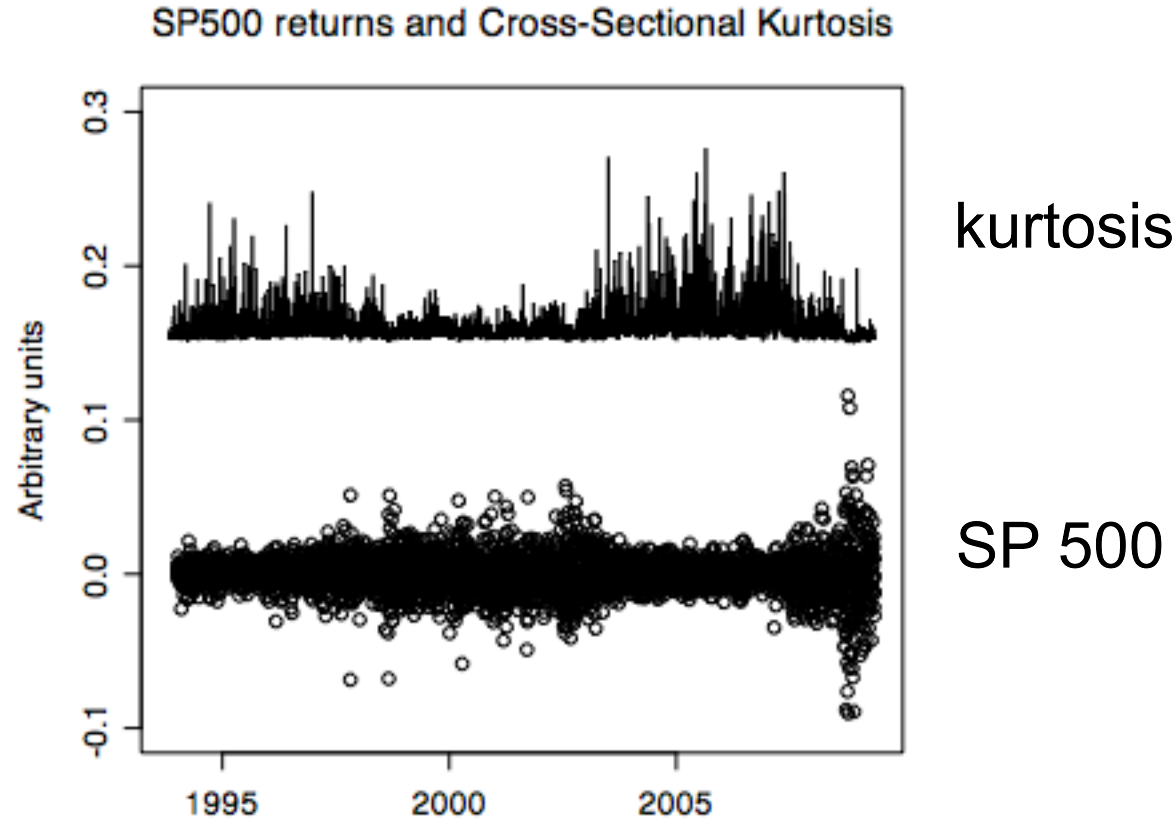


Even by eye you can see
the very high negative correlation between cross-sectional dispersion and kurtosis



<Disp(t) Kurt(t)> = - 30%

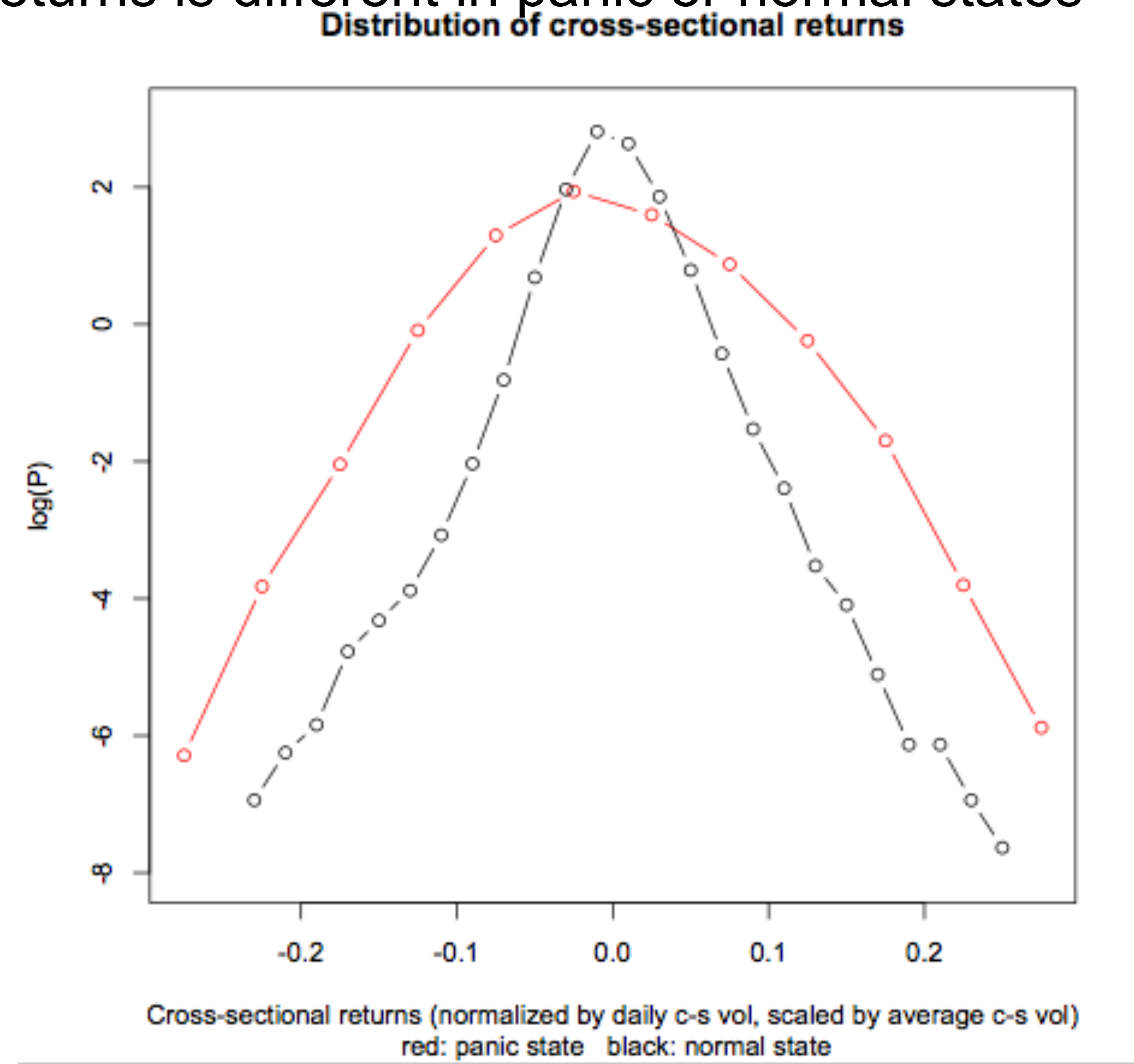
Defining Market Panic



In times of market panic:
c-s kurtosis LOW   but c-s dispersion HIGH

Some panic periods: Fall 2008, Sep 08 - April 09, 2002

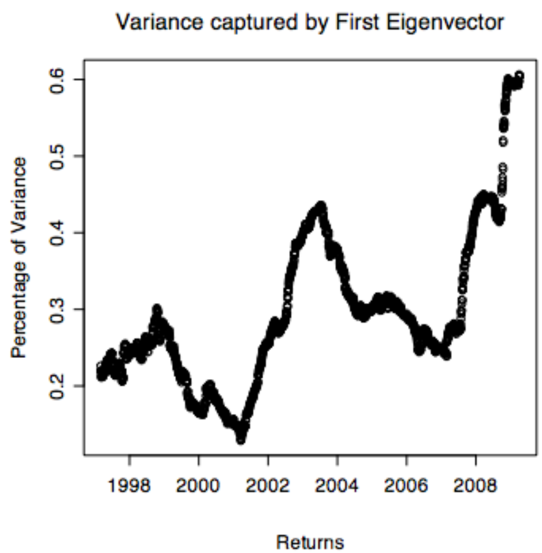
This implies that the distribution of cross-sectional returns is different in panic or normal states



Next thing to look at are correlations

- Perform rolling Principal Components Analysis

Look at percentage of variance captured by first eigenvector



Larger percentage -> More of a "market model"
-> Higher cross-sectional co-movement of stocks

Distribution becomes more Gaussian in panic times

Two possible explanations:

1) Distribution of individual stock volatilities narrows
2) Correlations among random stock moves increases

Distribution becomes more Gaussian in panic times

Two possible explanations:

1) Distribution of individual stock volatilities narrows
2) Correlations among random stock moves increases

Define on each day the quantity

$$s = \frac{s^{\uparrow} - s^{\downarrow}}{s^{\uparrow} + s^{\downarrow}}$$

Where $s^{\uparrow}$ is the number of stocks that had positive moves

$s^{\downarrow}$ is the number of stocks that had negative moves

Correlations:

Define on each day the quantity

$$s = \frac{s^{\uparrow} - s^{\downarrow}}{s^{\uparrow} + s^{\downarrow}}$$

Where $s^{\uparrow}$ is the number of stocks that had positive moves

$s^{\downarrow}$ is the number of stocks that had negative moves

If $s = 0$ no correlation, if $s \neq 0$ correlation among stocks

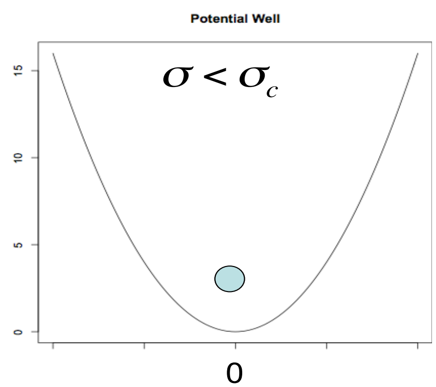
s can be seen as an Order Parameter

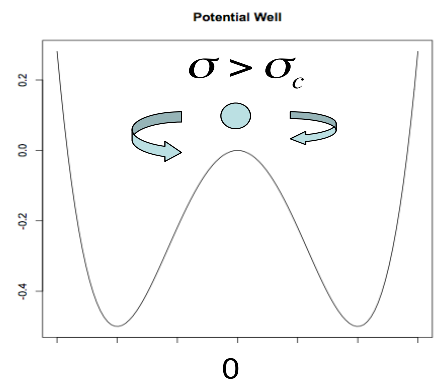
Analogy:

s is the Order Parameter

σ (external volatility perception) is the Control Parameter

$$V(s) = as^2 + bs^4$$





$$s = 0$$

$$s = \pm\sqrt{\frac{-a}{b}}$$

$$a = \alpha(\sigma_c - \sigma)$$

|s| = correlations

If $\sigma < \sigma_c$ s is in the disordered state - low correlation across stocks

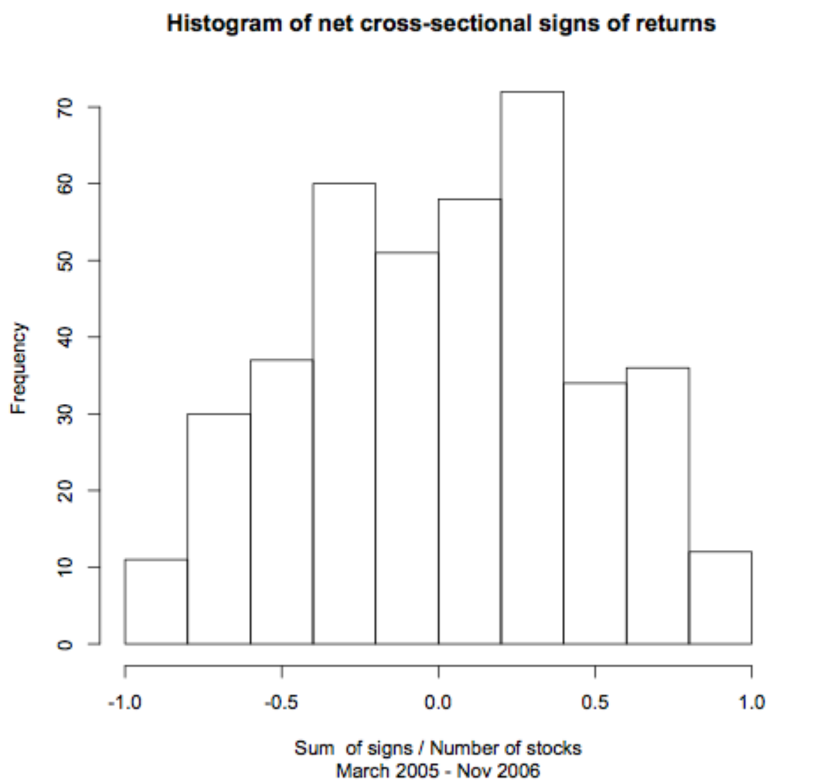
$\sigma > \sigma_c$ s is in the ordered state - high correlation
Symmetry breaking (can be positive or negative)

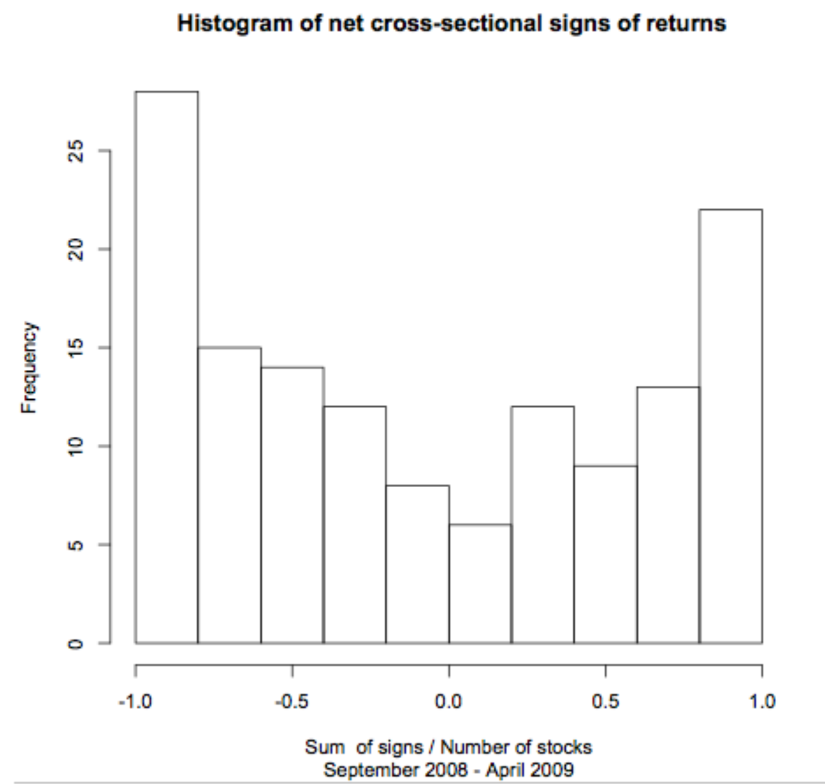
If this is plausible we expect to see

-unimodal distribution of s in normal times

-bimodal distribution of s in panic times

$s^{\uparrow}$ is the number of stocks that had positive moves
$s^{\downarrow}$ is the number of stocks that had negative moves





In times of panic, correlations are high
==> bimodal distribution of s

$$s = \frac{s^{\uparrow} - s^{\downarrow}}{s^{\uparrow} + s^{\downarrow}}$$

A simple phase transition model:

Statistical signatures in times of panic [L. Bprland. Quantitative Finance 2011]

- Log returns follow multi timescale model across time

- Log returns correlated across stocks with corr =|s|
(disorder to order)

- The transition is triggered by a volatility shock

We get:

Kurtosis down
Dispersion up
Distribution of s bimodal

Putting it all together

For each stock i across time t

y = log(stock price)

$$dy_t^i = \sigma_t^i d\omega_t^i$$

$$\sigma_t^{i2} = \sigma^2\left[1 + g\sum_{l=1}^{\infty}\frac{1}{l^{\alpha}}\left(y_t^i - y_{t-l}^i\right)^2\right]$$

Across stocks for each time

If $a > 0 \Rightarrow s = 0$: disordered state, correlations $<\omega_t^i\omega_t^j>_{i\neq j} = 0$

If $a < 0 \Rightarrow s \neq 0$: ordered state, correlations $<\omega_t^i\omega_t^j>_{i\neq j} = |s|$

$$\frac{ds}{dt} = -as - bs^3 + F_t$$

$$a = \sigma_c - \sigma_M$$

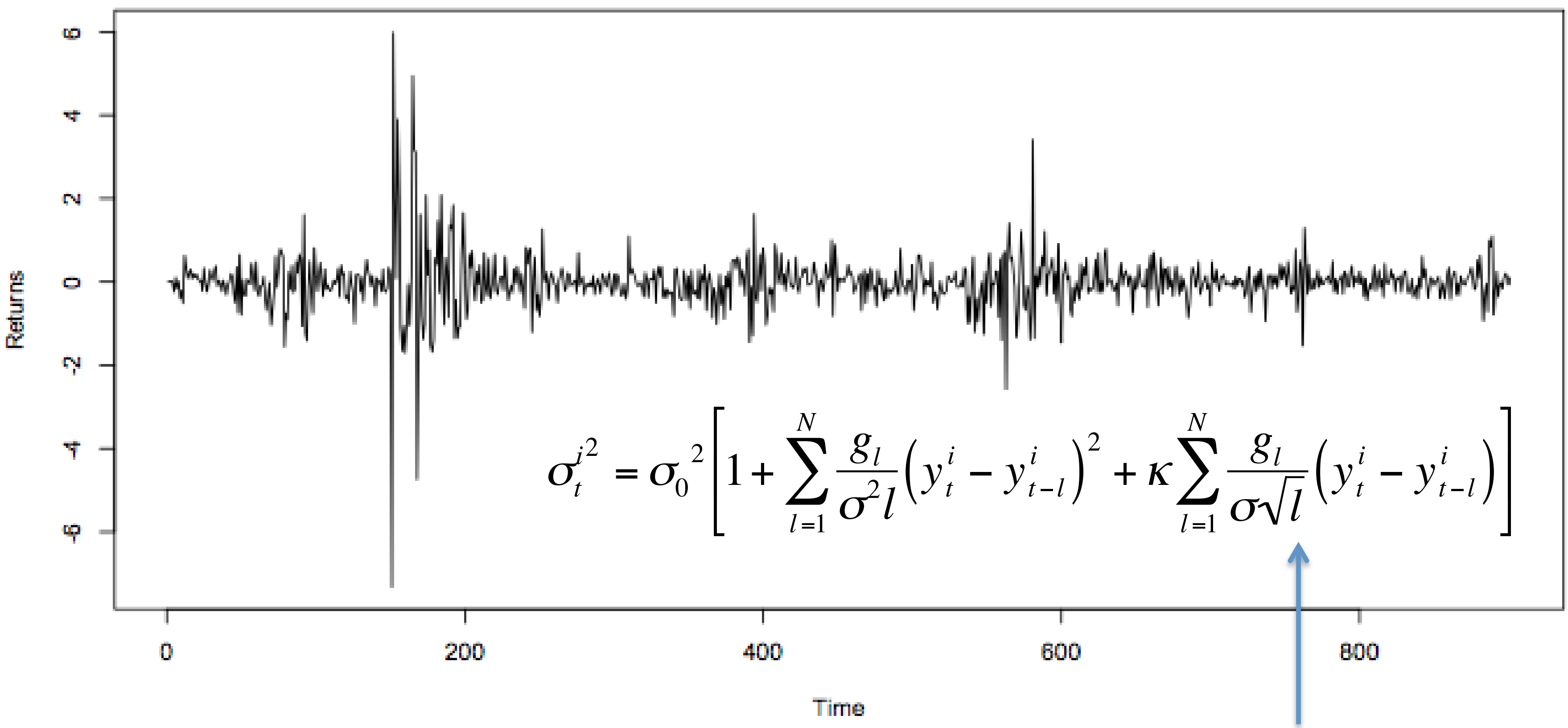
$\sigma_c =$ critical volatility

Market volatility can increase due to:

i) Exogenous jumps (news)
ii) Endogenous jumps (multi time-scale dynamics)

SIMULATIONS :

$$dy_t^i = \sigma_t^i d\omega_t^i$$



$$\sigma_t^{i2} = \sigma_0^{\ 2}\left[1 + \sum_{l=1}^{N} \frac{g_l}{\sigma^2 l}\left(y_t^i - y_{t-l}^i\right)^2 + \kappa \sum_{l=1}^{N} \frac{g_l}{\sigma\sqrt{l}}\left(y_t^i - y_{t-l}^i\right)\right]$$

Extension to include skew
[Borland and Hasaad 2010

$$a = \sigma_c - \sigma_M$$

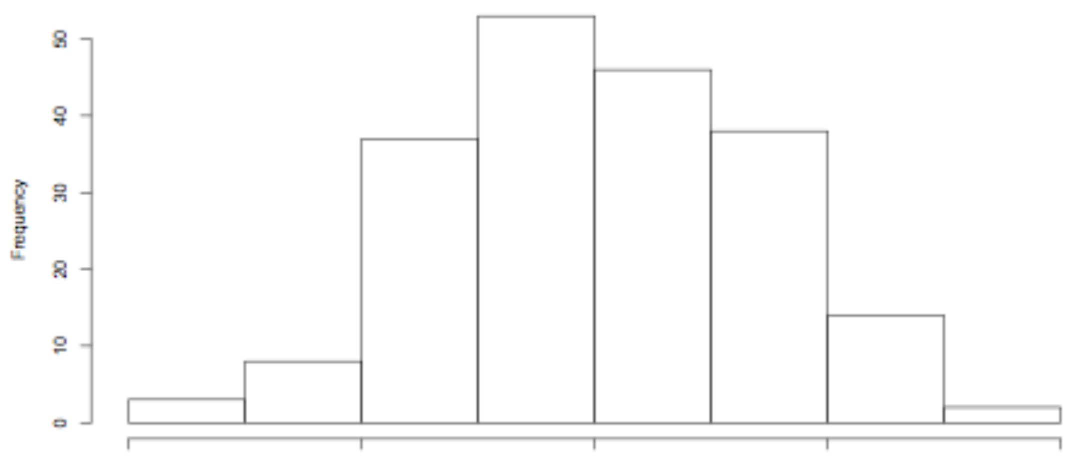
Panic: a < 0

σ_c = 2 standard deviations of recent market returns

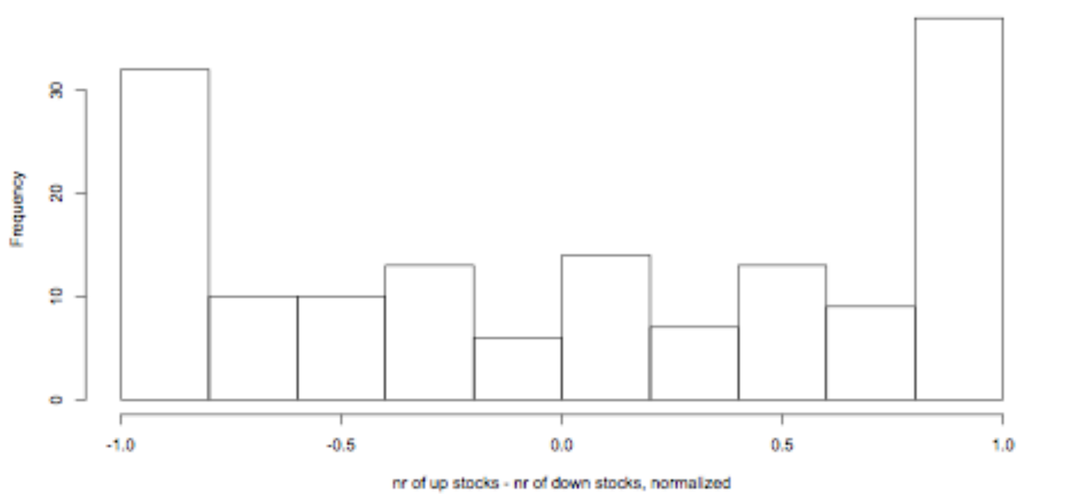
σ_M = Current market volatility

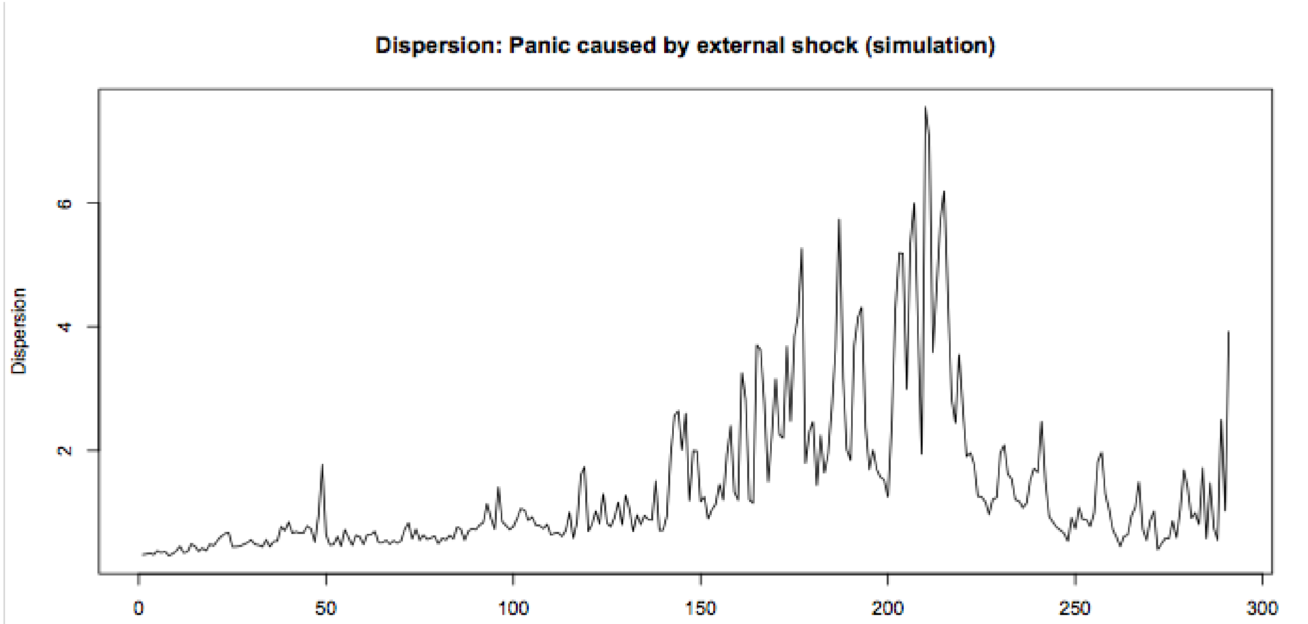
RESULTS

Histogram of s: Normal Period

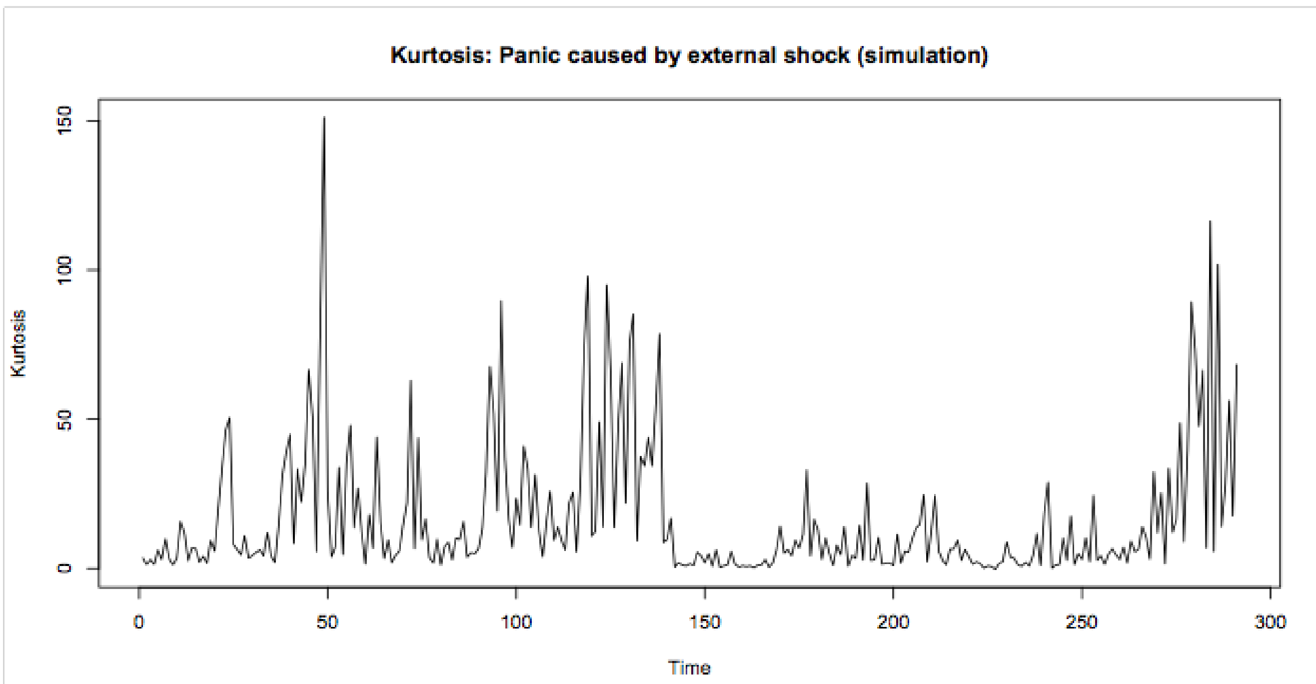


Histogram of s: Panic Period





Correlation
-15%



Summary and Open Questions

- A realistic model of the joint stochastic process of stock prices
- Dynamics across time reproduced
- Market volatility drives correlations across stocks
- In times of panic, fluctuations become more correlated
- In times of panic, cross-sectional distribution becomes more Gaussian . All stocks experience higher volatility.
- All our models have been extended to incorporate the asymmetry that negative returns lead to more panic than positive ones.
- Calibration ?
- Prediction of critical sigma ?
- Analytic solutions (e.q. for option pricing, trading strategies)?

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