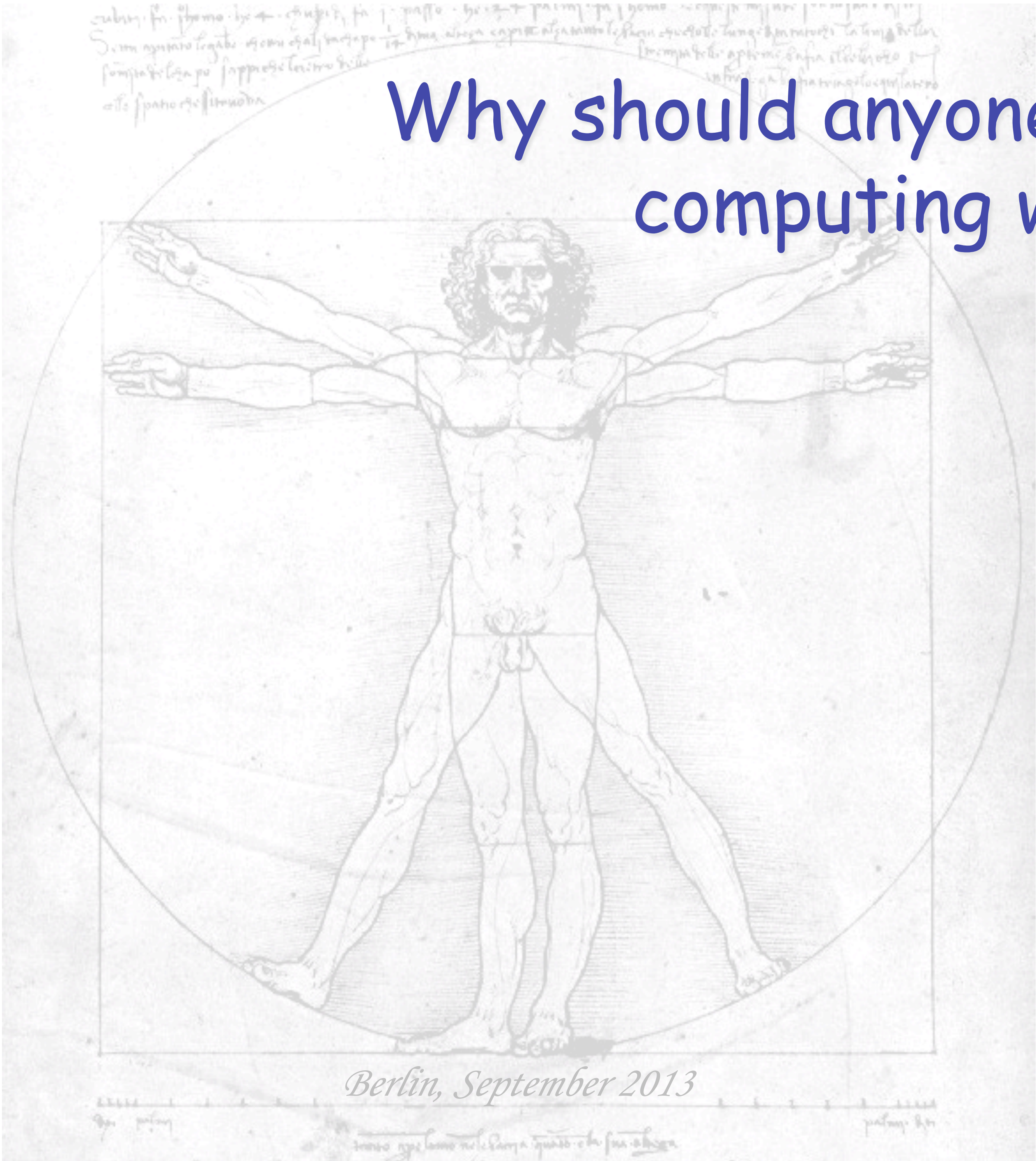
Why should anyone care about computing with anyons?

Jiannis K. Pachos

Jones Polynomials



Berlin, September 2013

EPSRC
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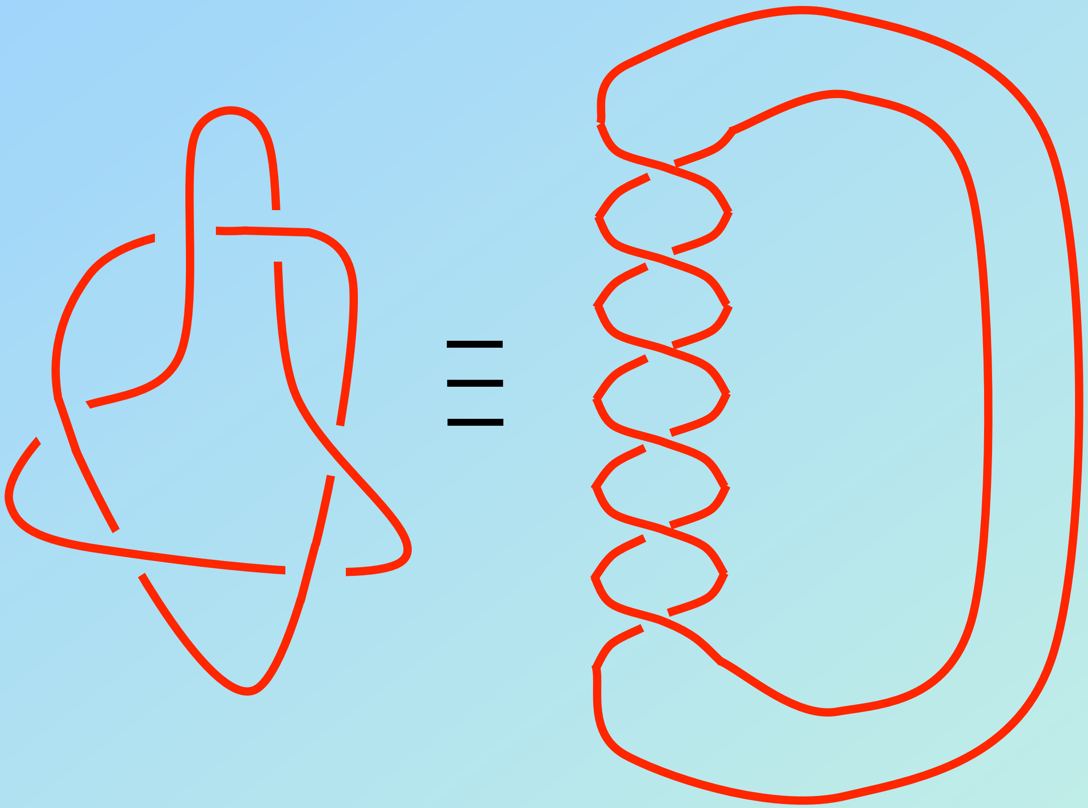
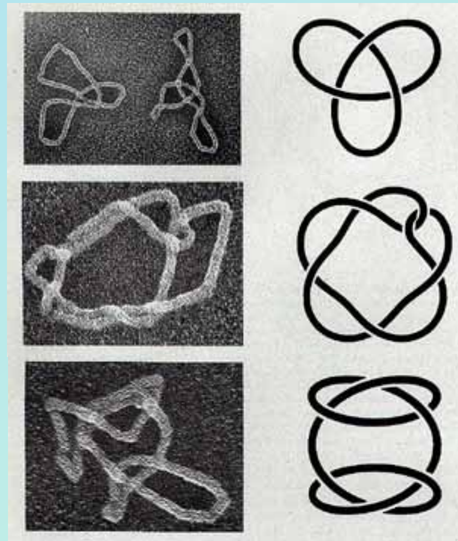
Jones polynomials



DNA folding



Is it possible to check if **two links are equivalent or not**?
The **Jones polynomial** is a topological invariant:
    if it differs, links are not equivalent. [Jones (1985)]
Exponentially hard to evaluate classically. **Applications**:
DNA reconstruction, integrable statistical physics…

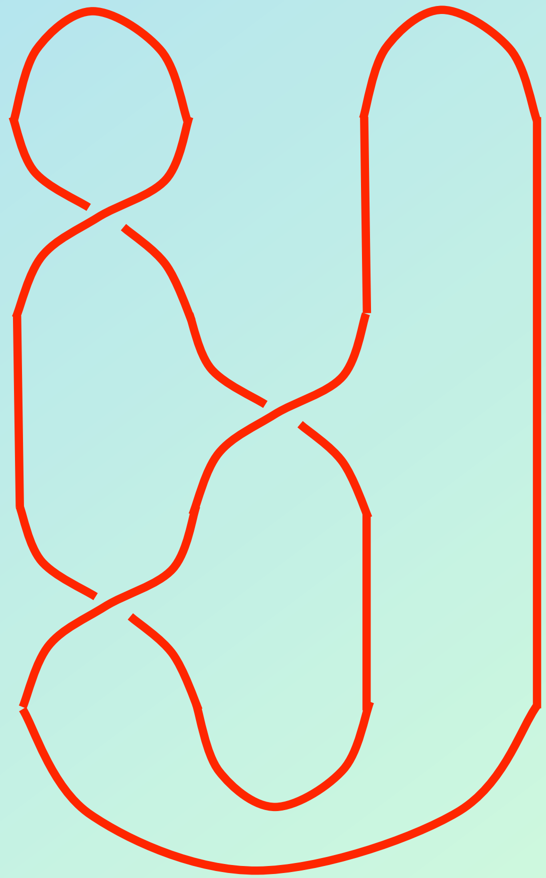
Anyons and links

The state of anyons is efficiently described by their **world lines**.

Creation, braiding, fusion.

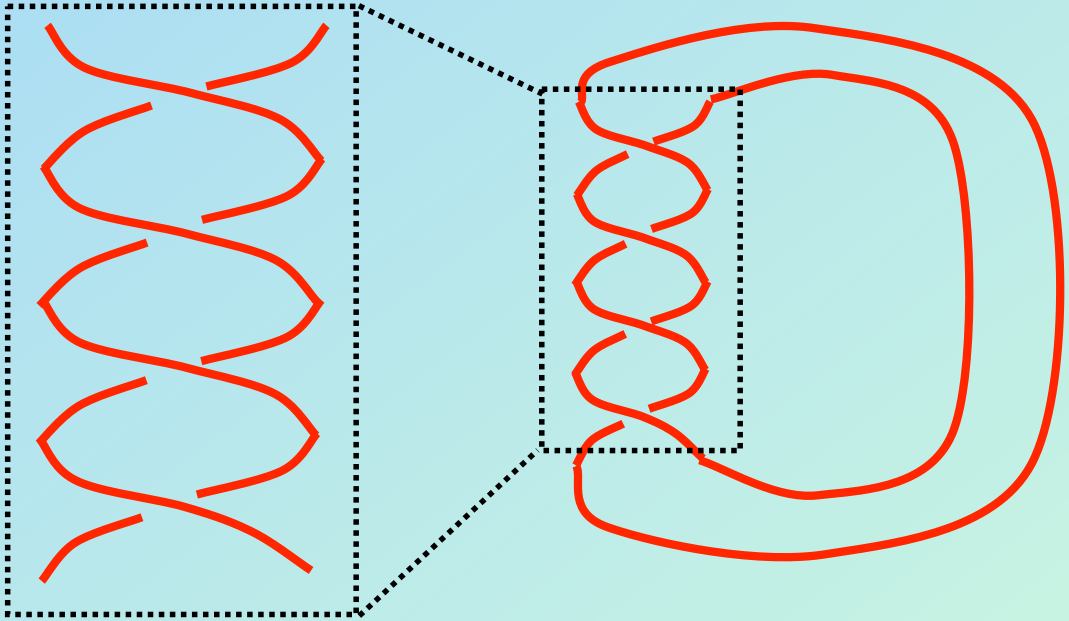
Consider **one type of anyons** so one type of strings is necessary.

Associate **anyons** with **links** and knots.



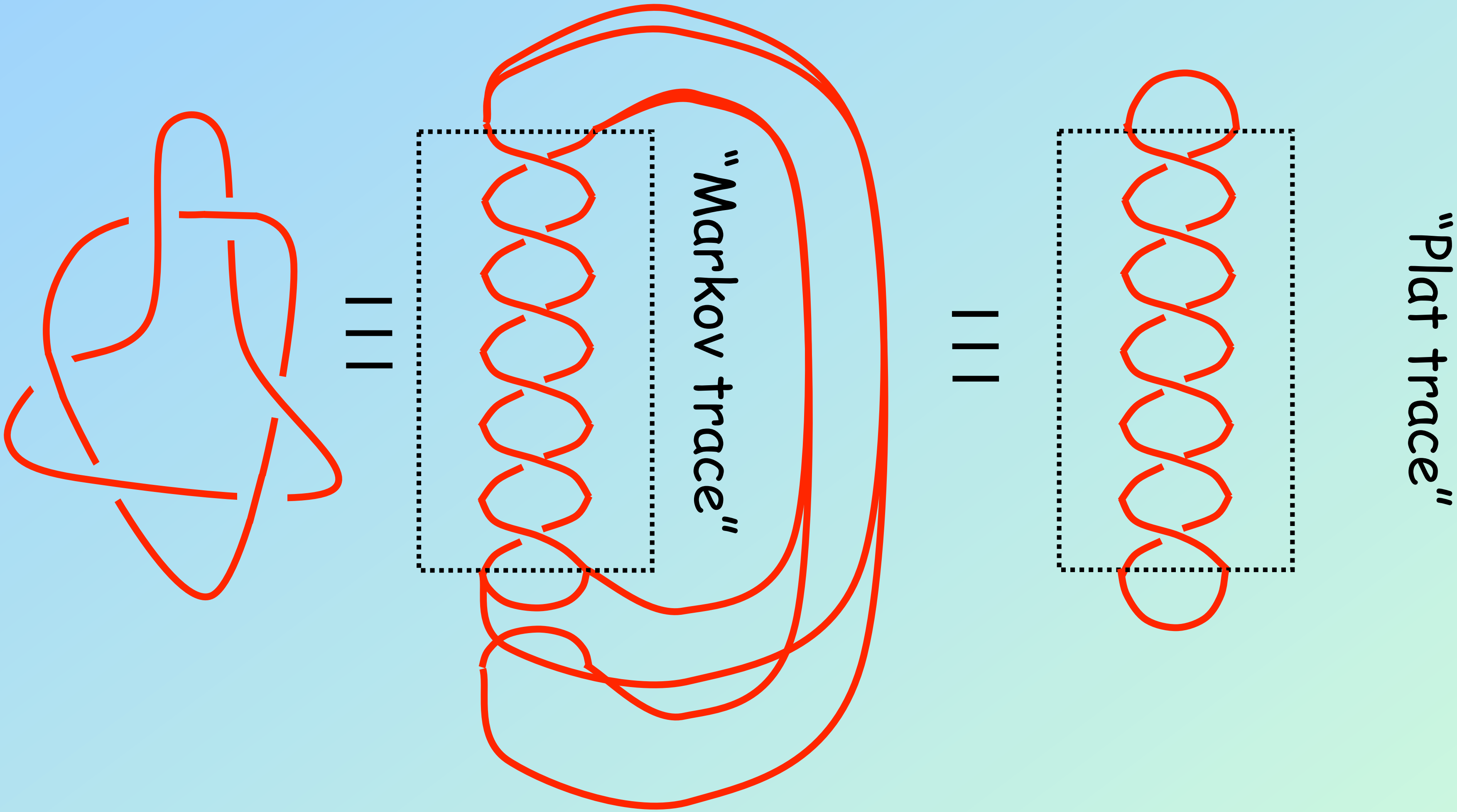
Anyons and links

Consider a **braidword** associated to certain number of strands. Define a unique relation to **links**, **L**.



Quantum simulation: determines hard to calculate classical quantities.

The trace of braidwords



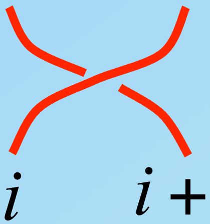
Links are equivalent to braids with a "trace".
Markov trace and **Plat trace.**
[Markov, Alexander theorems]

The braid group B_n

The braid group B_n has elements $b_1, b_2, \ldots, b_{n-1}$ that satisfy:

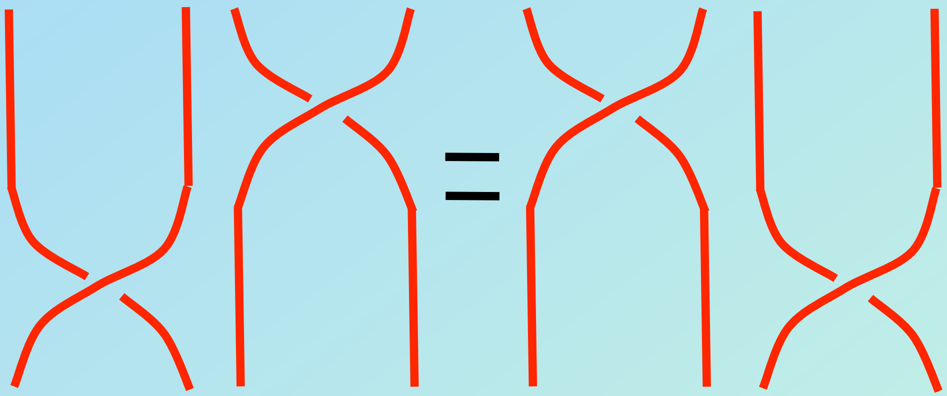
$$b_i b_j = b_j b_i, \quad for\ |i-j| \geq 2$$

$$b_i b_{i+1} b_i = b_{i+1} b_i b_{i+1} \quad for\ 1 \leq i < n$$



$\equiv b_i$

Pictorially:



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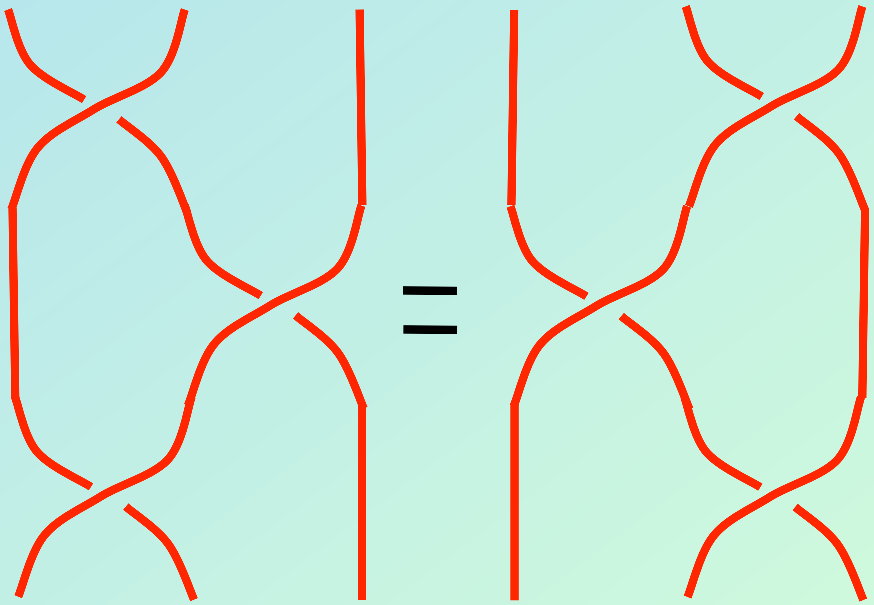

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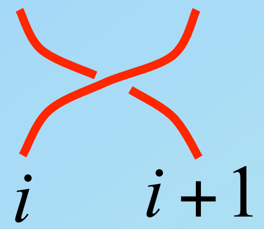


$$\equiv b_i$$

$$b_i b_{i+1} b_i = b_{i+1} b_i b_{i+1} \quad for\ 1 \leq i < n$$

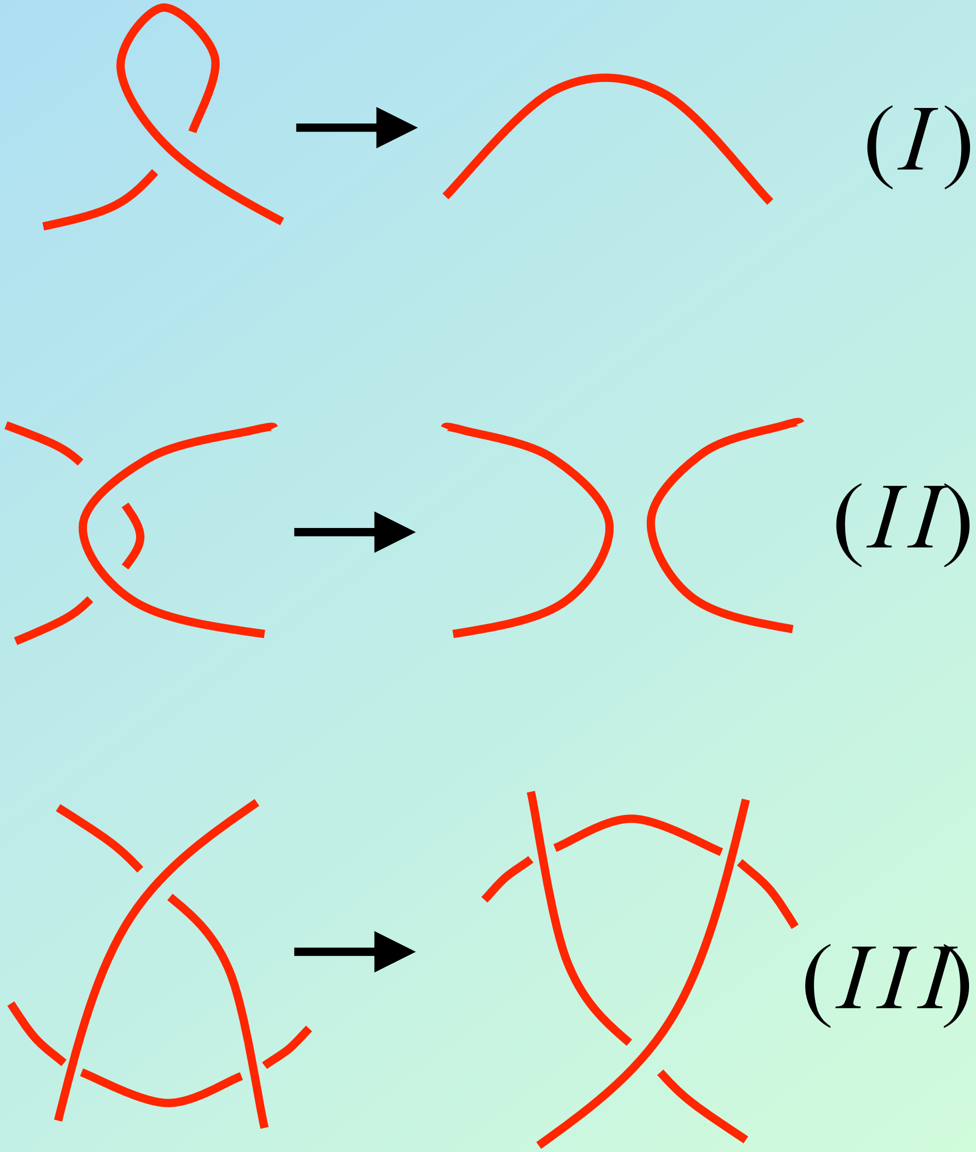
A given **type** of anyons corresponds to a certain **representation** of the braid group.

Hence, there is a correspondence between **anyons** and **unitary** braiding matrices.

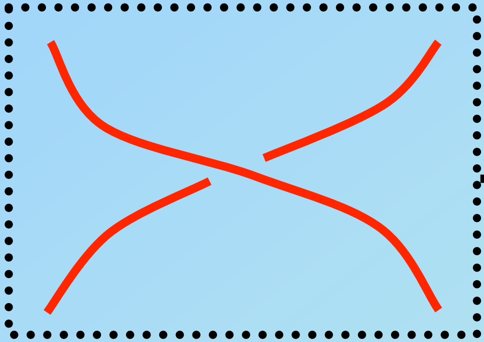
The Reidemeister moves

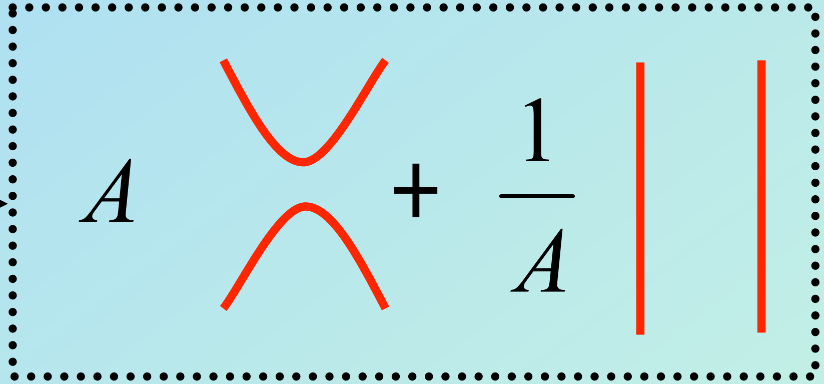
Theorem:

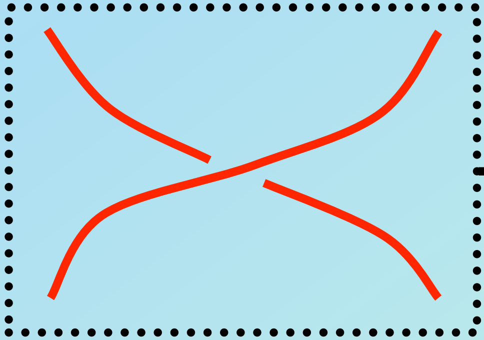
Two knots can be **deformed continuously** one into the other iff the diagram of one knot can be transformed into the diagram of the other via the sequence of the following **local moves**:

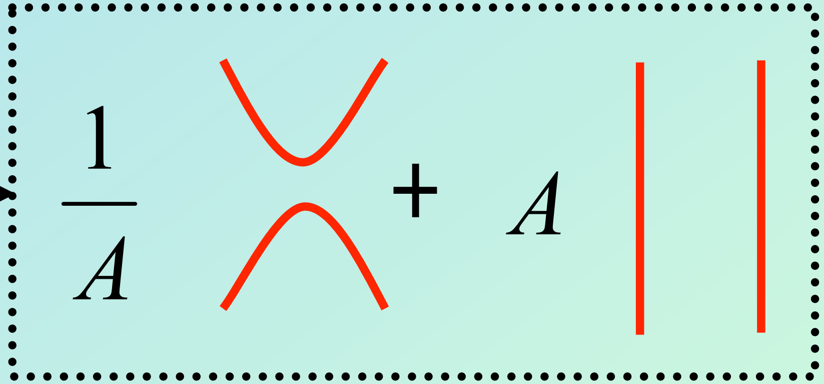


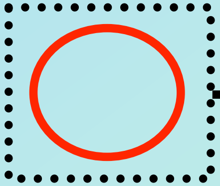
Skein relations





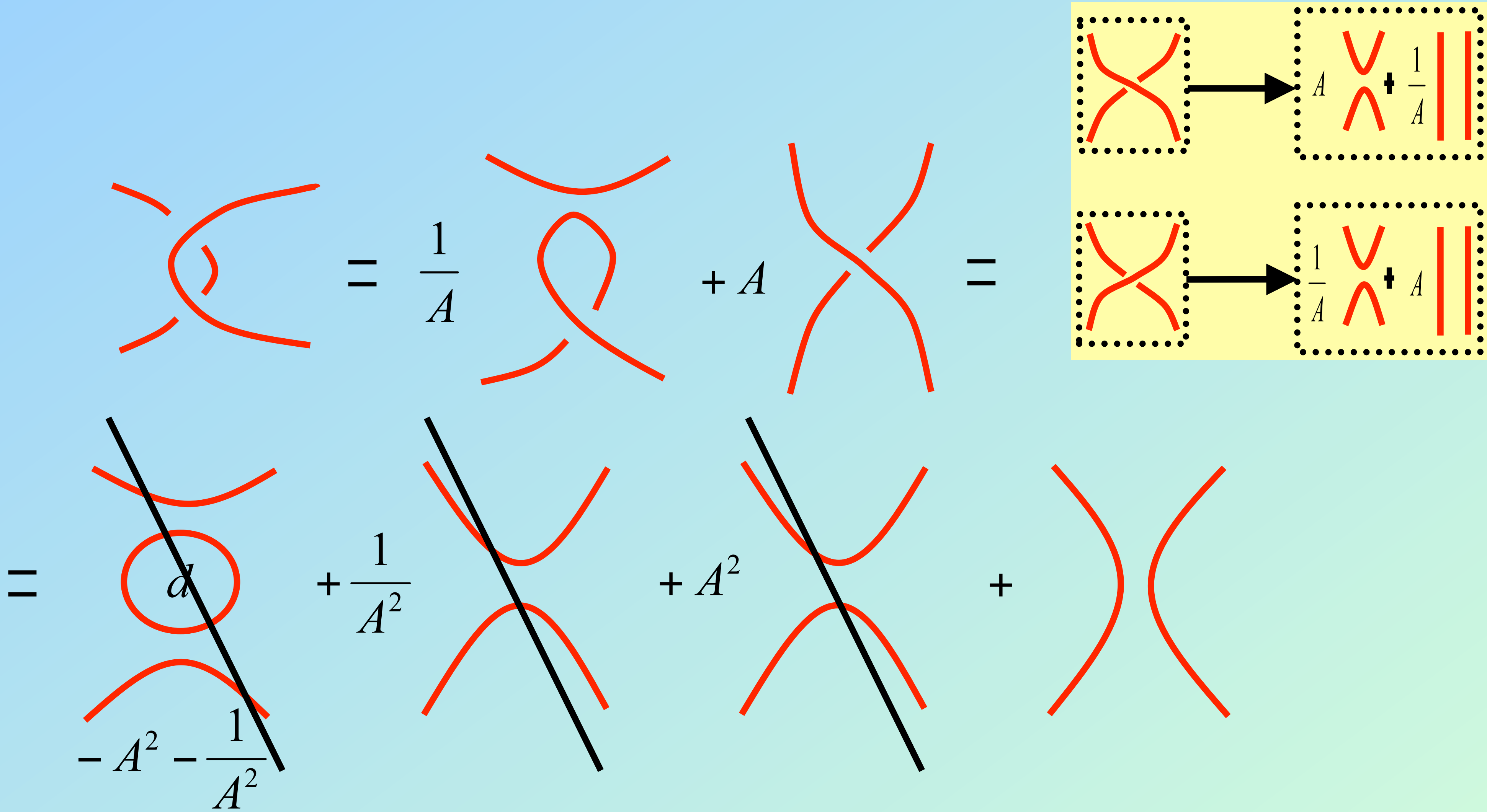






$$-A^2 - \frac{1}{A^2} = d$$

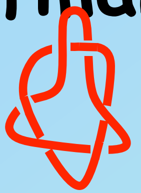
Skein and Reidemeister



Reidemeister move (II) is satisfied. Similarly (III).

Kauffman bracket

The Skein relations give rise to the **Kauffman bracket**:

Skein()= $\langle L\rangle(A) = \sum_{\text{all components } \sigma} \sigma(L, A)$

Jones polynomial

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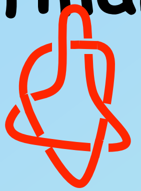
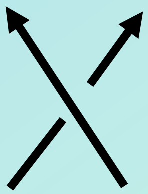
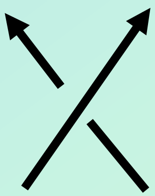
Skein() $= \langle L \rangle (A) = \sum_{\text{all components } \sigma} \sigma(L, A)$

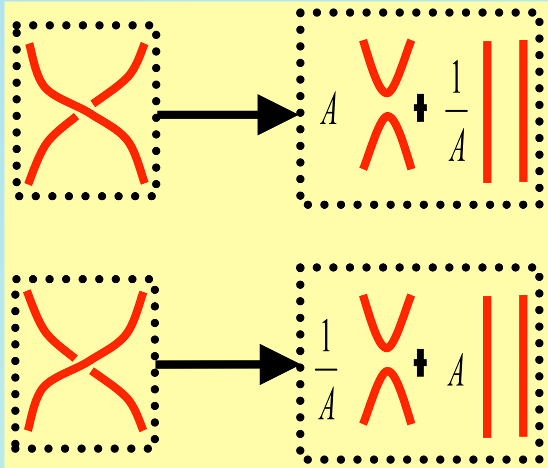
To satisfy **move (I)** one needs to define **Jones polynomial**:

$$V_L(A) = (-A)^{3w(L)} \langle L \rangle (A)$$

$w(L)$ is the twist or writhe of link. For an oriented link it is the sum of the signs for all crossings

 $= +1$  $= -1$

Jones polynomial



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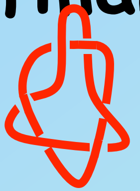
Skein()= $\langle L\rangle(A) = \sum_{\text{all components } \sigma} \sigma(L,A)$

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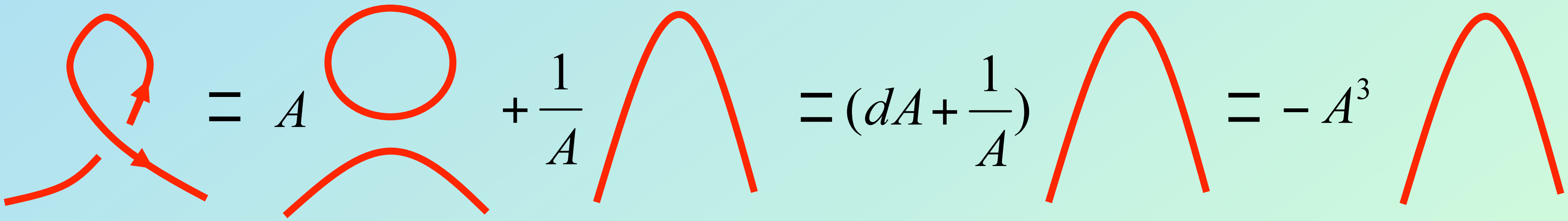
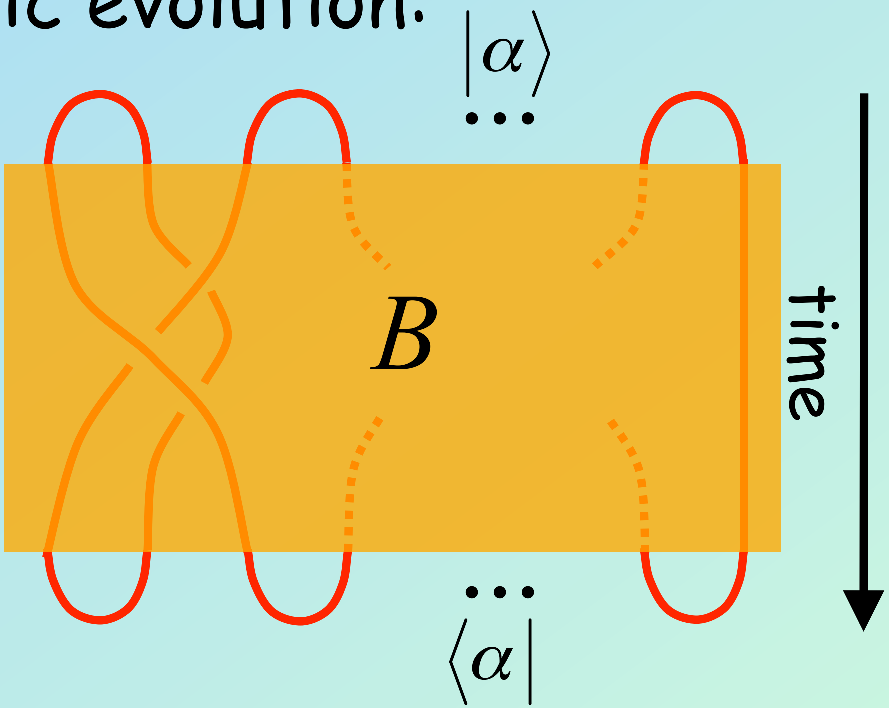


$w = -1$

Jones polynomial from anyons

Consider the following anyonic evolution:



1) Create n anyons from the vacuum state $|\alpha\rangle$
2) Perform B braiding
3) Pairwise fuse them

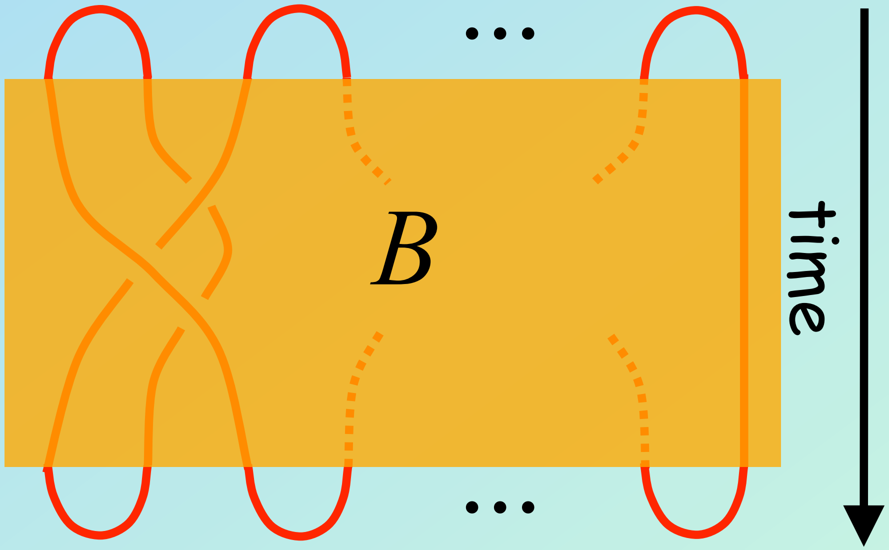
The **probability** of ending with the vacuum fusion state $\langle\alpha|$ is given by:

$$\langle\alpha|B|\alpha\rangle = \frac{1}{d^{n/2-1}}\left\langle (B)^{\text{Plat}} \right\rangle$$

Complexity Jones polynomial

The **probability** is given by:

$$\langle \alpha | B | \alpha \rangle = \frac{1}{d^{n/2-1}} \left\langle (B)^{\text{Plat}} \right\rangle$$



#P-hard problem (generally)

Two theorems:

1) [Freedman, Kitaev, Wang]: If A^4 is a root of unity then QC can "**additively**" approximate Jones polys in poly time.
2) [Freedman, Larsen, Wang]: If $A = e^{\frac{\pi i}{2r}}, r = 5, r \geq 7$ then approximation of Jones is **Universal**.

[Kuperberg]

Summary

Simulate the knot with anyonic braiding

One can translate the anyonic evolution to a **quantum algorithm** that estimates traces of matrices.

A new quantum algorithm

With quantum computers it is **polynomially** easy to *approximate*:

Jones poly is BQP –*computable with bounded error, given quantum resources, in poly time.*

[Freedman, Kitaev, Larsen, Wang (2002); Aharonov, Jones, Landau (2005); *et al.* Glaser (2009)]