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UNIVERSALITY IN THE ALGEBRA OF VERTEX STRENGTHS AS GENERATED BY BILOCAL CURRENTS *

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Abstract We review the derivation of the algebra of vertex strengths (AVS) from the algebra of current bilocals on the null plane in the deep Regge scaling region (DRSR) We discuss the difficulties and present a mathematical procedure yielding AVS through a smooth removal of the divergent ω^{α} dependences which differ on both sides of the infinite "ω-local" algebra of scaling functions. We suggest that the S_a charges in AVS (the 2^{++} trajectories' vertex strengths) may be given by the antisymmetrized matrix elements of the vector bilocals

1. Introduction

The application of the "independent quark model" (an impulse approximation with additivity of quark-quark amplitudes) to high energy hadron-hadron forward and near-forward scattering [1] was given an *ad hoc* algebraic formulation in the algebra of vertex strengths [2] (AVS) $[SU(3) \times SU(3)]_R$. This was later modified [3] so as to comply with two-component duality [4] (i.e. with a segregated diffractive contribution); it did indeed then reproduce the entire set of graphical duality [5] results. AVS was further generalized [6] to $[U(6) \times U(6)]_R$ so as to include the action of two sets of spin-flip exchanges, beyond the original charge-like contributions of reggeized vector and tensor exchanges. A comparison with experiment will be provided elsewhere. In this article, we discuss the attempt to provide the ad hoc phenomenological AVS with a theoretical foundation. This appears to have become

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possible after the phenomenological discovery of hadron scaling in deep inelastic lepton-hadron scattering, and the indications thus acquired about the nature of hadron current-commutators [7] on the light cone. The possible existence of a "deep" Regge-cum-scaling region (DRSR) represents a hypothetical overlap between physical operators belonging to the system of currents and their Regge analogs.

The theoretical laboratory we use is derived from the SLAC scaling experiments. Scaling occurs for large values of $-q^2 \to \infty$ the squared momentum transfer from the lepton to the nucleon, and large energy values $\nu \to \infty$, $\nu = p \cdot q$ (p is the nucleon 4-momentum), keeping the ratio $\omega = \nu/-q^2$ fixed. In that regime, only ω-dependence is left in the dimensionless invariant matrix elements. The hadron part of this cross section can be reinterpreted as the spin-averaged absorptive part of a Compton-like amplitude (with negative q^2, though we may also generalize and take the limit $q^2 \to \infty$). The current commutator is then dominated by its light cone behavior.

Taking the further limit $\omega \to \infty$, the relevant amplitude represents the asymptotic semi-elastic scattering of a hadron ($\nu \to \infty$) on a "heavy" current (with mass $-q^2 \to \infty$). Regge behavior can then be assumed in this DRSR, being realized in powers of $\omega \sim \nu$. The kinematics now correspond to large values of $x_- = x_0 - x_3$, on a null plane $x_+ = 0$, besides $x^2 = 0$. One is not limited to vector currents in the commutators.

2. Bilocals

The absorptive part of a model high-energy scattering amplitude in DRSR is written as

$$M_{ab}^{uv} = \int \mathrm{d}^4x\, \mathrm{e}^{iqx} \langle N(p)|[j_a^u(x), j_b^v(0)]|N(p)\rangle_{\substack{x^2=0\\ x_+=0}} , \tag{1}$$

$$a, b = 0, 1, 2, \ldots 8 , \quad u, v = 0, 1, \ldots 15 .$$

(a,b) are SU(3) indices, (u,v) denote SU(4) matrices in Dirac spinor space. The right-hand side is written in terms of the light-cone expansion [7, 8]. The simplest answers are given by the free quark field model [8] and only the dominant singularities on the light cone are kept (this is denoted by $\simeq$):

$$M_{ab}^{uv} \underset{\substack{|q^2| \to \infty\\ \omega \to \infty}}{\simeq} \int \mathrm{d}^4x\, \mathrm{e}^{iqx} g_{tc(\pm)}^{ua,vb} D_t(x^2, x_0) \langle N(p)|b_c^{t(\pm)}(x,0)|N(p)\rangle_{\substack{x^2 \to 0\\ x_+ \to 0}} . \tag{2}$$

Here D_t contains the singularities (such as $\partial_0 [\epsilon(x_0)\, \delta(x^2)]$) and $b_c^{t(\pm)}(x,0)$ is a bilocal operator, such that

$$b_c^{t(\pm)}(x,0) = \tfrac{1}{2}\{b_c^t(x,0) \pm b_c^t(0,x)\} . \tag{3}$$

$g_{tc(\pm)}^{ua,vb}$ is the appropriate structure constant (including f_{abc} or d_{abc} of SU(3)). The matrix element $\langle N(p)|b_c^{t\pm}(x,0)|N(p)\rangle$ can be analyzed through the Wigner-Eckart theorem for the bilocal U(12) null-plane algebra [9]:

$$\langle N(p)|b_c^{t(\pm)}(x,0)|N(p)\rangle_{\substack{x^2\to 0\\ x_+\to 0}} C_{cNN}^{t(\pm)} f^{(\pm)}(x_-\cdot p_+)\, h^t\ . \tag{4}$$

$h(p^0)$ is a kinematical factor such as p^μ, C_{cNN}^{tn} is a Clebsch-Gordan coefficient and $f_c^n(x_-p_+)$ is an even or odd real function. Using the Fourier transform

$$f(x\cdot p) = \int \mathrm{d}\xi\ \mathrm{e}^{i\xi(x\cdot p)} F(\xi)\ , \tag{5}$$

one finds in the Bjorken limit, (n denotes $\pm$, a some constant factors)

$$M_{ab}^{uv} \simeq a g_{tnc}^{ua,vb} C_{cNN}^{tn}\, F^n\left(\frac{1}{\omega}\right) h^t\ . \tag{6}$$

At this stage one takes the $\omega\to\infty$ limit, i.e. DRSR. The $F_c^{t(\pm)}$ is expanded in powers of $1/\omega$, and after a Sommerfeld-Watson transformation one finds,

$$F^n\left(\frac{1}{\omega}\right) = \lim_{\omega\to\infty} G^n\, \omega^{\alpha^{tn}(0)}\ . \tag{7}$$

Now returning to the AVS, we find that M_{ab}^{uv} is given by a sum of terms with the appropriate signature, charge conjugation etc.,

$$M_{ab}^{uv} = \gamma_{tnc}^{ua,vb} \gamma_{tnc}^{NN}\, \omega^{\alpha^{tn}(0)}\ , \tag{8}$$

to be compared with

$$M_{ab}^{uv} \simeq a g_{tnc}^{ua,vb} C_{tnc}^{NN}\, \omega^{\alpha^{tn}(0)}\ . \tag{9}$$

It is straightforward to identify $g_{tnc}^{ua,vb}$ with $\gamma_{tnc}^{ua,vb}$ up to a constant. This is a U(12) Clebsch-Gordan coefficient determined by the quantum numbers of the "target" currents,

$$\gamma_{tnc}^{ua,vb} = a g_{tnc}^{ua,vb}\ . \tag{10}$$

Similarly,

$$\gamma_{tnc}^{NN} = C_{tnc}^{NN}\ . \tag{11}$$

It is the latter identification which we require. To derive AVS, the coefficient C_{tnc}^{NN} now has to be shown to represent a structure constant of U(12). This is indeed so, since the $b_c^{tn}(x,y)$ are elements of the system of bilocal densities defining the algebra. The U(12) generators are recovered at the limit $x-y\to 0$ by integrating over d^3x.

3. Problems

Notwithstanding the apparent straightforwardness of the above derivation [11–15], we are faced with a number of difficulties.

(A) Difficulties arising from the "current" nature of the bilocals.

(A1) In using the Wigner-Eckart theorem, we appeal to the U(12) generators. However, most of these, and their densities, suffer from Coleman's theorems [10]. They cannot be considered as symmetries except on the light-cone. One can thus not use the rest-classification in eq. (4) and it is hard to understand why this seems to fit on the light-cone.

(A2) Several of the densities on the l.h.s., i.e. eq. (1), are either "bad" or "terrible" in the sense of ref. [9], destroying (1). The same is true for some of the bilocals on the r.h.s., eq. (2). In particular, the bilocal $s_a^{(+)}(x,y)$ corresponding to the quark-counting even-signatured half of $[SU(3) \times SU(3)]_R$ will be affected. This is one of the most important elements in AVS. This also holds for its local limit, the $u_a(x)$ used in some proofs [12].

(A3) To the extent that one is ready to limit the application of the Wigner-Eckart theorem to SU(3) itself [11, 12] (because of (A1)), universality has to be proved directly, since there is then no common scale to contributions of different trajectories.

(A4) The quantum numbers of the states as read by the operators will be of the "current" type [16], whereas our quark model results are due to "constituent" representations such as **56**. We would thus not preserve the good D/F ratio corresponding to the $\langle \mathbf{56}|\mathbf{35}|\mathbf{56}\rangle$ couplings for the $(\sigma\lambda)$ subset of densities, for instance.

(B) Comments relating to the Regge region features.

(B1) There is no room for a "dominance" argument. The Regge couplings arise for very large $x_- \to \infty$, and cannot go over to $x_- \to 0$, which is the limit begetting local currents. Couplings to $u_a(x)$ are thus not an approximation of Regge strengths in $s_a^{(+)}(x,y)$.

(B2) We get on the r.h.s., i.e. in eq. (2), operators $s_a^{(-)}(x,y)$, $v_{a+}^{(-)}(x,y)$ etc., with vanishing local limits. On the other hand they are required [11] in the amplitude contributions [17].

(B3) We thus have 288 bilocals, instead of 144. This introduces a question of identification. $v_{a+}^{(-)}(x,y)$ and $s_a^{(+)}(x,y)$ have the same eigenvalues of charge conjugation etc. As suggested in [17], $v_{a+}^{(+)}$ and $v_{a+}^{(-)}$ might be sufficient to describe the entire $[SU(3) \times SU(3)]_R$.

(B4) The derivation in eq. (4) does not yet yield a physical definition of the charge-like (strength) operator whose eigenvalue is C_{cNN}^{tn}. We still have to supply such a definition for the strengths and their Lie product [17–20]. The difficulties due to the "current" nature of the bilocals, as listed in (A1–A4) have led to a scheme in which the currents in eqs. (1, 2) are replaced by "rotated" currents

$$W j^\mu(x)\, W^{-1} = \hat{j}^\mu((x))\,,$$

and "rotated" bilocals, where W is a special Foldy-Wouthuysen transformation preserving some of the properties of the null-plane frame. If one stays within the subsystem of "good" currents and "good" bilocals, W may be just the Melosh transformation [16]. If one is to exploit the full U(12) system, and especially the $s_a^+(x,y)$ set, the transformation is the one described in ref. [14, 15]. This subject has been discussed in great detail in ref. [15] and we shall not deal with it here.

4. Universality

In the following, we study an example. We do not deal with the "constituent" versus "current" issue, and use "current" operators throughout. We shall derive directly the non-linear relation guaranteeing universality, thus disposing of the difficulties in defining the group for which the Wigner-Eckhardt theorem is used. The algebra of vector bilocals on the light-cone is given by [9]:

$$[v_{a\mu}(x,u), v_{b\nu}(y,v)] \simeq \frac{1}{4\pi}\partial_\rho\{\epsilon(x_0-v_0)\,\delta((x-v)^2)\}$$

$$\times\,(if_{abc}-d_{abc})\,(S_{\mu\nu\rho\sigma}\,v_{c\sigma}(y,u)+i\epsilon_{\mu\nu\rho\sigma}\,a_{c\sigma}(y,u))$$

$$+\frac{1}{4\pi}\partial^\rho\{\epsilon(u_0-y_0)\,\delta((u-y)^2)\}\,(if_{abc}+d_{abc})$$

$$\times\,(S_{\mu\nu\rho\sigma}\,v_{c\sigma}(x,v)-i\epsilon_{\mu\nu\rho\sigma}\,a_{c\sigma}(x,v))\,. \qquad (12)$$

We are interested in the algebra of symmetric and antisymmetric bilocals, so forming as in (3) symmetric and antisymmetric combinations

$$2v^{(+)}(x,u) = v(x,u) + v(u,x)\,,$$

$$2v^{(-)}(x,u) = v(x,u) - v(u,x)\,.$$

Note that $n^\mu(1,0,0,-1)$ is a null-vector, and

$$S_{\mu\nu\rho\sigma} = g_{\mu\rho}\,g_{\nu\sigma} + g_{\nu\rho}\,g_{\mu\sigma} - g_{\mu\nu}\,g_{\rho\sigma}\,.$$

We get for their algebra,

$$n^\mu n^\nu\,[v^{(+)}_{a\mu}(x,u), v^{(+)}_{b\nu}(y,v)]$$

$$\simeq\frac{n^\rho n^\sigma}{4\pi}\,[\partial^x_\rho\{\epsilon(x_0-v_0)\,\delta((x-v)^2)\}\{if_{abc}\,v^{(+)}_{c\sigma}(y,u) - d_{abc}\,v^{(-)}_{c\sigma}(y,u)\}$$

$$+\,\partial^y_\rho\{\epsilon(y_0-u_0)\,\delta((y-u)^2)\}\{if_{abc}\,v^{(+)}_{c\sigma}(x,v) + d_{abc}\,v^{(-)}_{c\sigma}(x,v)\}$$

$$+\,\partial^x_\rho\{\epsilon(x_0-y_0)\,\delta((x-y)^2)\}\{if_{abc}\,v^{(+)}_{c\sigma}(u,v) + d_{abc}\,v^{(-)}_{c\sigma}(u,v)\}$$

$$+\,\partial^u_\rho\{\epsilon(u_0-v_0)\,\delta((u-v)^2)\}\{if_{abc}\,v^{(+)}_{c\sigma}(x,y) + d_{abc}\,v^{(-)}_{c\sigma}(x,y)\}]\,, \qquad (13)$$

$$n^\mu n^\nu [v_{a\mu}^{(-)}(x,u), v_{b\nu}^{(-)}(y,v)]$$

$$\simeq \frac{n^\rho n^\sigma}{4\pi} [\partial_\rho^x \{\epsilon(x_0-v_0)\,\delta((x-v)^2)\} \{if_{abc}\, v_{c\sigma}^{(+)}(y,u) - d_{abc}\, v_{c\sigma}^{(-)}(y,u)\}$$

$$+ \partial_\rho^u \{\epsilon(y_0-u_0)\,\delta((y-u)^2)\} \{if_{abc}\, v_{c\sigma}^{(+)}(x,v) + d_{abc}\, v_{c\sigma}^{(-)}(x,v)\}$$

$$- \partial_\rho^x \{\epsilon(x_0-u_0)\,\delta((x-y)^2)\} \{if_{abc}\, v_{c\sigma}^{(+)}(x,v) + d_{abc}\, v_{c\sigma}^{(-)}(u,v)\}$$

$$- \partial_\rho^u \{\epsilon(u_0-v_0)\,\delta((u-v)^2)\} \{if_{abc}\, v_{c\sigma}^{(+)}(x,y) + d_{abc}\, v_{c\sigma}^{(-)}(x,y)\}] \,, \qquad (14)$$

$$n^\mu n^\nu [v_{a\mu}^{(+)}(x,u), v_{b\nu}^{(-)}(y,v)]$$

$$\simeq \frac{n^\rho n^\sigma}{4\pi} [\partial_\rho^x \{\epsilon(x_0-v_0)\,\delta((x-v)^2)\} \{if_{abc}\, v_{c\sigma}^{(-)}(y,u) - d_{abc}\, v_{c\sigma}^{(+)}(y,u)\}$$

$$+ \partial_\rho^y \{\epsilon(y_0-u_0)\,\delta((y-u)^2)\} \{if_{abc}\, v_{c\sigma}^{(-)}(x,v) + d_{abc}\, v_{c\sigma}^{(+)}(x,v)\}$$

$$+ \partial_\rho^x \{\epsilon(x_0-y_0)\,\delta((x-y)^2)\} \{if_{abc}\, v_{c\sigma}^{(-)}(u,v) + d_{abc}\, v_{c\sigma}^{(-)}(u,v)\}$$

$$+ \partial_\rho^u \{\epsilon(u_0-v_0)\,\delta((u-v)^2)\} \{if_{abc}\, v_{c\sigma}^{(-)}(x,y) + d_{abc}\, v_{c\sigma}^{(-)}(x,y)\}] \,. \qquad (15)$$

In eqs. (12) to (15) $\simeq$ means as before, that the right-hand sides include only the most singular, connected terms. The space-time points x, y, u, v should all be on a single light ray for the above algebra to be valid

If we should now take one of the above sets of relations, say (13), and try to derive an algebra of strengths, i.e. factorized Regge residues, by taking the Fourier transforms (7) of matrix elements on both sides, we would find that as $1/\omega \to 0$ (DRSR limit) the two sides would diverge at different rates. This was the difficulty encountered by Kislinger and Young [13] and Kleinert [19] in their attempt to derive an algebra of residues. Kislinger and Young could derive only those commutation relations for which the right-hand sides vanished, Kleinert derived a cut-off dependent commutator [20]. We shall show that by linking the approach to the DRSR limit with the "geometric" approach to the light-cone, we can consistently extract an algebra of strengths.

We start with the commutator of symmetric bilocals (12), take its matrix element between octet states A and B (spin averaged if they are nucleons) and consider the following integral

$$L = 2\int_0^{2\epsilon'} \mathrm{d}\omega \int \mathrm{d}^2x_\perp \int_{-\infty}^{\infty} \mathrm{d}a \int_{-\infty}^{\infty} \mathrm{d}b \int_{-\infty}^{\infty} \mathrm{d}c \int_{-\infty}^{\infty} \mathrm{d}d \int \mathrm{d}p_+^B$$

$$\times \exp i[(\omega+\epsilon')p_+^A(a+b-c-d)] \exp i[(\omega^s-\epsilon^m)(d-c)]_{\epsilon\to 0}$$

$$\times n^\mu n^\nu \langle A|[v_{a\mu}^{(+)}(x,u), v_{b\nu}^{(+)}(y,v)|B\rangle\,, \tag{16}$$

where

$$x^\mu = an^\mu + x_\perp^\mu\,, \quad y^\mu = bn^\mu\,, \quad u^\mu = cn^\mu + x_\perp^\mu\,,$$

$$v^\mu = dn^\mu\,, \quad n^\mu \equiv (1,0,0,-1)\,, \quad x_\perp \equiv (0,x_1,x_2,0)\,,$$

$$p_\perp^A = p_\perp^B = 0\,, \quad \text{and} \quad \epsilon' = \epsilon^{m/s} m/s < 1\,. \tag{17}$$

Here again for simplicity in presentation, we assume mass degenerate octet states but we could carry out the derivation with mass differences within the octets as shown in appendix D of ref. [18]. Even though we are integrating over $x_\perp$ in (16), the commutator vanishes for $x_\perp \neq 0$ and we can still use the commutation relation (13) which is valid on a light ray.

We expand the commutator over intermediate states

$$\langle A|[v_{a\mu}^{(+)}(x,u), v_{b\nu}^{(+)}(y,v)]|B\rangle n^\mu n^\nu$$

$$= n^\mu n^\nu \int \mathrm{d}^4p'\delta(p'^2-m^2)\Big\{\sum_n \langle A|v_{a\mu}^{(+)}((a-c)n,0)|p',r\rangle$$

$$\times \langle r,p'|v_{b\nu}^{(+)}((b-d)n,0)|B\rangle \exp i[c(p^A-p')\cdot n + d(p'-p^B)\cdot n - (p^A-p')_\perp \cdot x_\perp]$$

$$- \langle A|v_{b\nu}^{(+)}((b-d)n,0)|p',r\rangle\langle p',r|v_{a\mu}^{(+)}((a-c)n,0)|B\rangle$$

$$\times \exp i[c(p'-p^B)\cdot n + d(p^A-p')\cdot n - (p'-p^B)_\perp \cdot x_\perp]\Big\} + \langle A|R_{ab}|B\rangle\,. \tag{18}$$

$\sum_r$ is the sum over single particle octet states and $\langle A|R_{ab}|B\rangle$ is the contribution of all other intermediate states. As we shall observe, universality of AVS only occurs when the R_{ab} can be ignored.

Substituting (18) in (16), defining a new set of variables $[b-d=b', a-c=a', c$ and $d]$ and carrying out the integrations over c, d and $x_\perp$, we get

$$L = \underset{\epsilon\to 0}{K} \int_0^{2\epsilon'} \mathrm{d}\omega \int \mathrm{d}p_+^B \int \mathrm{d}p'_+ \mathrm{d}p'_- \,\mathrm{d}^2p'_\perp\, \delta(p'_+p'_- - p'^2_\perp - m^2)$$

$$\int \mathrm{d}a' \int \mathrm{d}b' \delta^2(p'_\perp) \exp i[(\omega+\epsilon')p_+^A(a'+b')]\; n^\mu n^\nu\{\langle A|v_{a\mu}^{(+)}(a'n,0)|p'r\rangle\langle p'r|v_{b\nu}^{(+)}(b'n,0)|B\rangle$$

$$\times \delta(p_+^A - p'_+ - \omega^s + \epsilon^m)\,\delta(p'_+ - p_+^B + \omega^s - \epsilon^m) - \langle A|v_{b\nu}^{(+)}(b'n,0)|p'r\rangle\langle p'r|v_{a\mu}^{(+)}(a'n,0)|B\rangle$$

$$\times \delta(p_+^A - p'_+ + \omega^s - \epsilon^m)\,\delta(p'_+ - p_+^B - \omega^s + \epsilon^m)\} + \langle A|R_{ab}|B\rangle\,. \tag{19}$$

K is an overall positive real constant which absorbs all the Jacobians, 2π's etc. In the exponent we have set $p_+^A = p_+^B$ because of the δ functions.

Now we carry out the integrations over p'_+ and $p'_\perp$ and get

$$L = \underset{\epsilon\to 0}{K} \int_0^{2\epsilon'} \mathrm{d}\omega \int \mathrm{d}p_+^B \delta(p_+^B - p_+^A) \int \mathrm{d}p'_-$$

$$\times\, \delta(p'_-(p_+^A - \omega^s + \epsilon^m) - m^2) \int \mathrm{d}a'\mathrm{d}b' \exp i[(\omega+\epsilon')p_+^A(a'+b')]$$

$$\times\, n^\mu n^\nu \{\langle A|v_{a\mu}^{(+)}(a'n, 0), v_{b\nu}^{(+)}(b'n, 0)]|B\rangle\}_{\mathrm{L.C}} + \langle A|R_{ab}|B\rangle\,, \tag{20}$$

where L.C. means that in the intermediate states we set $p'_+ = p_+^A$, $p'_\perp = p_\perp^A = 0$. For degenerate octet masses, this, together with the δ functions in (20) implies $p^A = p^B = p'$ but we do not need this.

Using the δ functions, we integrate over p_+^B and p'_- and get

$$L = \underset{\epsilon\to 0}{K} \int_0^{2\epsilon'} \mathrm{d}\omega \int \mathrm{d}a' \int \mathrm{d}b' \exp i[(\omega+\epsilon')p_+(a'+b')]$$

$$\times\, n^\mu n^\nu \{\langle A|v_{a\mu}^{(+)}(a'n, 0), v_{b\nu}^{(+)}(b'n, 0)]|B\rangle\}_{\mathrm{L.C.}} + \langle A|R_{ab}|B\rangle\,. \tag{21}$$

Now we substitute for the Fourier transforms of the matrix elements of the bilocals, following the inverse transforms defined in eq. (5):

$$F_a^{(+)}(\zeta) = \frac{1}{2\pi} n^\mu \int_{-\infty}^{\infty} \mathrm{d}a' \langle A|v_{a\mu}^{(+)}(a'n, 0)|B\rangle\, \mathrm{e}^{i\zeta(p\cdot n)a'}$$

$$= \frac{1}{2\pi} n^\mu p_\mu \int_{-\infty}^{\infty} \mathrm{d}a' f_a^{(+)}(p\cdot a'n)\mathrm{e}^{i\zeta(p\cdot a'n)}\,, \tag{22}$$

and following eq. (7), we put the Regge behavior assumption, collecting the $C_{ctn}^{NN}\cdot G^n$ into one constant matrix element,

$$F_a^{(+)}(\zeta) = \left(\frac{1}{\zeta}\right)^{\alpha(+)} \langle A|C_a^{(+)}|B\rangle\,. \tag{23}$$

We assume α is SU(3) degenerate. We thus get, recognizing in C_n^+ the factorized residue, the l.h.s.

$$L = K\langle A|[C_a^{(+)}(0), C_b^{(+)}(0)]|B\rangle \int_{\substack{0\\ \epsilon\to 0}}^{2\epsilon'} \frac{1}{(\omega+\epsilon')^{2\alpha}}\mathrm{d}\omega + \langle A|R_{ab}|B\rangle\,. \tag{24}$$

From (23) we get $C_a(t=0)$ because $p_+^A = p'_+ = p_+^B$ and $p_\perp^A = p_\perp^B = p_\perp^n = 0$ and the matrix elements of the bilocals are between A or B and n.

On the right-hand side of (13) we have four terms. The first two give identical contributions, except for the difference in sign between the antisymmetric bilocals. Thus only the symmetric term survives from these first two terms. The third and fourth terms can be shown [18] to vanish upon integration.

We consider the first term on the right-hand side of (13)

$$R_1 = \int_{\substack{0\\ \epsilon\to 0}}^{2\epsilon'} \mathrm{d}\omega \mathrm{d}^2 x_\perp \int \mathrm{d}p_+^B \int_{-\infty}^{\infty} \mathrm{d}a \int_{-\infty}^{\infty} \mathrm{d}b \int_{-\infty}^{\infty} \mathrm{d}c \int_{-\infty}^{\infty} \mathrm{d}d\, i f_{abc}$$

$$\times\, n^\rho \partial_\rho \{\epsilon(x_0 - v_0)\, \delta((x-v)^2)\}\, n^\sigma \langle A | \bar{v}_{c\sigma}^{(+)}(y,u) | B\rangle$$

$$\times\, \exp i[(\omega+\epsilon') p_+^A (a+b-c-d)]\ \exp i[(\omega^s - \epsilon^m)(d-c)]\ . \qquad (25)$$

Now

$$n^\sigma \langle A | v_{c\sigma}^{(+)}(y,u) | B\rangle = n^\sigma \langle A | v_c^{(+)}(bn, cn + x_\perp) | B\rangle$$

$$= \exp i[(p^A - p^B)\cdot nc - (p^A - p^B)_\perp \cdot x_\perp]\, n^\sigma \langle A | v_{c\sigma}^{(+)}((b-c)n - x_\perp, 0) | B\rangle\ . \quad (26)$$

We can set $x_\perp = 0$ in (26) because of the δ function

$$\delta((x-v)^2) = \delta(x_\perp^2)\ .$$

Defining new variables $b' = b - c$, $a' = a - d$, c, d, we get for the right-hand side

$$R_1 = \underset{\epsilon\to 0}{K'} \int_0^{2\epsilon'} \mathrm{d}\omega \int \mathrm{d}^2 x_\perp\, \delta(x_\perp^2) \int_{-\infty}^{\infty} \mathrm{d}a' n^\rho \partial_\rho^{a'} (a'n)_0)$$

$$\times\, \exp i[(\omega+\epsilon') a' p_0] \int_{-\infty}^{\infty} \mathrm{d}d\, \exp i[(\omega^s - \epsilon^m) d]$$

$$\times \int \mathrm{d}p_+^B \int_{-\infty}^{\infty} \mathrm{d}c\, \exp i[\{p_+^A - p_+^B - (\omega^s - \epsilon^m)\} c]\, i f_{abc} \int \mathrm{d}b'\, \exp i[(\omega+\epsilon') b' p_+^A]$$

$$n^\sigma \langle A | v_{c\sigma}^{(+)}(b'n, 0) | B\rangle\ . \qquad (27)$$

Now, from (22) and (23) we have

$$\int \mathrm{d}b'\, \mathrm{e}^{i(\omega+\epsilon')b'} n^\sigma \langle A | v_{c\sigma}^{(+)}(b'n, 0) | B\rangle = \left(\frac{1}{\omega+\epsilon'}\right)^\alpha \langle A | C_c^{(+)}(0) | B\rangle\ .$$

We obtain the residue at $t = 0$ in (27) because $p_+^A = p_+^B$ from $\delta(p_+^A - p_+^B)$ obtained from the c integration. The $x_\perp$ integration can be done using $\delta(x_\perp^2)$ and the a' integration with the help of $\delta(a')$ obtained by differentiating the step function. Thus we get

$$R_1 = \underset{\epsilon\to 0}{K'} \int_0^{2\epsilon'} \mathrm{d}\omega \, \frac{1}{(\omega+\epsilon')^\alpha} \, \delta(\omega^s - \epsilon^m) \, if_{abc} \langle A | C_c^{(+)}(0) | B \rangle \tag{28}$$

$$= \underset{\epsilon\to 0}{K'} \left[\int_0^{2\epsilon'} \mathrm{d}\omega \, \frac{1}{(\omega+\epsilon')^\alpha} \, \frac{\delta(\omega - \epsilon^{m/s})}{s(\epsilon^{m/s})^{s-1}} \right] if_{abc} \langle A | C_c^{(+)}(0) | B \rangle \tag{29}$$

$$= \underset{\epsilon\to 0}{K'} \, [\epsilon^{-m(1+\alpha/s-1/s)}] \, if_{abc} \langle A | C_c^{(+)}(0) | B \rangle \, . \tag{30}$$

We have used $\epsilon' = \epsilon^{m/s}$ and K' absorbs all the positive real constants. Now putting the right- and left-hand sides together from (29) and (30) we have

$$\langle A | R_{ab} | B \rangle + \underset{\epsilon\to 0}{K} \langle A | [C_a^{(+)}(0), C_b^{(+)}(0)] | B \rangle \int_0^{2\epsilon'} \left[\frac{\mathrm{d}\omega}{(\omega+\epsilon')^{2\alpha}} \right]$$

$$= \underset{\epsilon\to 0}{K'} \langle A | C_c^{(+)}(0) | B \rangle \, if_{abc} \, \epsilon^{-m(1+\alpha/s-1/s)} \, . \tag{31}$$

We note again that K, K' are positive real constants, independent of state labels A and B and trajectory labels a, b, c.

We now show that if the contribution of the nonoctet intermediate states is small and can be ignored, we recover AVS. m and s are parameters which are so far unspecified, except that $m/s < 1$. We will show that irrespective of the value of α, the $t = 0$ intercept of the trajectories (we have assumed exchange degenerate trajectories for simplicity but we could carry through the arguments with different α for different trajectories), we can choose m and s such that the two sides of (31) have the same 'ϵ' dependence, as $\epsilon \to 0$. We can then compare coefficients and get AVS universality explicitly.

Case 1. $\alpha > \frac{1}{2}$ or $\alpha < \frac{1}{2}$.

$$\text{l.h.s.} \sim \int_0^{2\epsilon^{m/s}} \left(\frac{1}{\omega+\epsilon'} \right)^{2\alpha} \mathrm{d}\omega \sim \epsilon^{m/s(1-2\alpha)} \, ;$$

$$\text{r.h.s.} \sim \epsilon^{-m(1+\alpha/s-1/s)} = \epsilon^{+m/s(1-\alpha-s)} \, .$$

We need to choose

$$m/s(1-2\alpha) = m/s - m/s\alpha - m \, ,$$

i.e. $m = \alpha(m/s)$.

We choose $m/s < 1$ and $s = \alpha$, to have the same ϵ functional dependence on the two sides. A choice of m is required in principle for the definition of the integration boundary $2\epsilon^{m/s}$.

Case 2. $\alpha = \frac{1}{2}$.

$$\text{l.h.s.} \sim \int_0^{2\epsilon^{m/s}} \left(\frac{1}{\omega+\epsilon'}\right) d\omega \sim \log(3\epsilon^{m/s}) - \log \epsilon^{m/s} \sim \log 3 \,.$$

Thus we need to choose

$$m/s - m/s\alpha - m = 0 \,,$$

i.e. $\frac{1}{2}m/s = m$, $s = \frac{1}{2} = \alpha$.

With the appropriate choice of m and s, we get from (31), upon ignoring the non-octet contribution,

$$\langle A | [C_a^{(+)}(0), C_b^{(+)}(0)] | B \rangle = i f_{abc} \langle A | C_c^{(+)}(0) | B \rangle \,, \tag{32}$$

for the vertex strengths, i.e. the residues of the nonet of 1^{--} trajectories. We obtained (32) by considering the commutator of two symmetric bilocals (13). If we now look at (14), the commutator of antisymmetric bilocals, we notice that the right-hand side differs from the previous case only in the third and fourth terms, which can be shown to vanish upon integration [18]. Hence we get from (14), upon following the same procedure as above

$$\langle A | [C_a^{(-)}(0), C_b^{(-)}(0)] | B \rangle = i f_{abc} \langle A | C_c^{(+)}(0) | B \rangle \,. \tag{33}$$

From (13), we get, by inspection

$$\langle A | [C_a^{(+)}(0), C_b^{(-)}(0)] | B \rangle = i f_{abc} \langle A | C_c^{(-)}(0) | B \rangle \,. \tag{34}$$

These are the commutation relations of $[\mathrm{SU}(3) \times \mathrm{SU}(3)]_\mathrm{R}$ in AVS. The $C_a^{(-)}$ has the quantum numbers (η_c and U(3)) and signature of the 2^{++} trajectories, corresponding to the S_a operators in AVS.

We have absorbed K, K' in a uniform redefinition of the residues. This redefinition which amounts to multiplying all the residues by a constant positive factor does not affect the results.

In our above derivation of the "charges" to obtain the finite Lie algebra, we have used a mathematical procedure which can be extended to other cases [17], and can be used even if the pole behavior $(1/\omega)^{\alpha(t)}$ is replaced by cuts or some other function of ω.

Actually, the $F_a^{(+)}(\zeta)$ fulfill [17] an infinite algebra resembling the expressions for current algebra,

$$[F_a^{(+)}(\zeta_1), F_a^{(+)}(\zeta_2)] = i f_{abc}\, \delta(\zeta_1 - \zeta_2) F_c^{(+)}(\zeta_1) + i f_{abc}\, \delta(\zeta_1 + \zeta_2) F_c^{(+)}(\zeta_1) \,, \tag{35}$$

except that the divergent behavior in terms of the ζ variable here ($\sim \zeta^{-\alpha}$ for $\zeta \to 0$) differs entirely from that of the local currents in terms of x^μ (generally assumed to go $j \underset{x \to \infty}{<} x^{-2}$). Our specific prescription for taking the limits $\zeta_i \to 0$ of the several ζ_i variables in a correlated way enables us to remove smoothly the ζ_i dependence and define the charges. It holds directly as a formal cancellation procedure for (35) as distributions.

After the above study was made [17, 18] we have learned of similar results in recent work by Kleinert [20*, 21]. Beyond (35), Kleinert's method for getting "charges" now consists in projecting out of $F_a^{(+)}(\zeta)$ and $F_a^{(-)}(\zeta)$ the coefficients $F_a^{(\pm)J}(0)$ in a Taylor expansion in powers of ζ^{-J}

$$F_a^{(\pm)}(\zeta) = \sum_J \zeta^{-J} F_a^{\pm J}(0) .$$

The new Kleinert charges are thus identical to (32)–(34) only if $J = \alpha, J' = \alpha'$. For $\alpha \neq 0, 1$ the Kleinert charges project out only pieces of (32)–(34). Both sets contain all "daughters", as explicitly shown by Kleinert.

Summing up, we note the following conclusions of our analysis:

(a) The vertex strengths (factorized residues at zero momentum-transfer) do form the postulated Lie algebra, as derived in (32)–(34). This involves the assumptions of scaling and of analyticity in the J-plane. It would hold true for more general functions of $1/\zeta$ than the usual power behavior. The Lie product can always be assumed to exist.

(b) If the function of $1/\zeta$ is known, one can extract the vertex-strength directly, through (23). This is then the "charge".

(c) We are allowed to assume an idealized picture in which there is no leakage from the relevant multiplets in the intermediate states, since there are no Coleman theorems on the null plane-light-cone intersection where we have worked. On the other hand, we can also estimate the actual leakage, by observing departures from universality in the experimental results.

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* See the cut-off dependent commutator (4.24), the $\zeta = 1/\omega \equiv \xi$ "local" algebra (2.24), the expression (2 18) and the J-projected charges (2.26). Note that Kleinert's notation is inverted relatively to ours, ± denoting signatures.

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