

THE ALGEBRA OF SCALAR AND VECTOR VERTEX STRENGTHS
IN REGGE RESIDUES

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An algebra of "strong" scalar and vector currents coupled to Regge residues accounts for the composite model predictions in high-energy elastic scattering. Some new relations are given; $\sigma(K^+N) = \sigma(K^+P)$ is accounted for in a simple way.

We shall present an approach based upon the identification of an algebra of scalar and vector currents in the structure of the residue functions associated with Regge trajectories, somewhat in analogy to the way in which the weak and electromagnetic transitions of the hadrons define the system of vector and axial-vector currents. Applying the theory directly to high-energy scattering, we find we can predict some hitherto unexplained features. In so doing, we also produce a theoretical interpretation of a number of good results [1-5] whose derivation has generally been considered to imply a composite-particle structure for the hadrons.

Physical intuition based upon somewhat unrealistic models has twice before within recent years opened up new extensions of unitary symmetry; in both cases [6, 7] - non-relativistic SU(6) and too-relativistic SU(6, 6) - excellent results have been obscured at times by difficulties and dilemmas in the theoretical foundations [8]. Much clarification, a new understanding and a series of new results were each time provided by the definition of an algebraic methodology [9, 10] which was gradually improved and made consistent with relativistic quantum theory [11]. It is our contention that the present use of a "naive" quark model, - leading to a new subparticle physics with methods emulating those used

in nuclear structure and the many-body problem - should be regarded in the same light as the previous suggestive break-ins.

It is with this motivation in mind - an algebraic foundation for the high-energy results - that we make our suggestions. We deal with the simple case of forward scattering, though it is probable that the treatment can be extended to other situations. Our formulation should be regarded as a first rough definition, to be further refined extensively.

The description of high-energy baryon-baryon and baryon-meson phenomena in terms of Regge trajectories has been highly [12] successful and supplies the most appropriate framework for our treatment. In a series of recent studies [13], one finds a useful and consistent parametrization of the data, based upon the residue functions $\beta(t)$ and the pole trajectories $\alpha(t)$, where t is the square of the momentum transfer and the energy dependence is explicit. The real part of the trajectory is effectively described by its intercept $\alpha(0)$ and its slope at that point.

The factorization theorem [14] allows us to re-

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place $\beta(t)$ by a product of two *vertex strength* functions $\gamma_C^{AB}(t)$, which are analogous to form factors in quantum electrodynamics; γ_C^{AB} stands for example for an upper vertex where the trajectory C occurs as an intermediate state in the t channel scattering of incoming particles A and B and as an exchanged system (Regge pole) in the s channel scattering of A into B.

Actual high-energy phenomenology has achieved a general fit of known processes in terms of vertex strength functions coupled to two even intrinsic-parity trajectories with opposite signature, dominated by two meson unitary nonets with $j = 1^-$ and 2^+ . Some differences exist between the workers in the field, mainly with respect to the number of 2^+ unitary singlet-dominated trajectories. We shall here adopt the view that one is faced with an octet-singlet set for each signature, including the Pomeranchuk trajectory.

Observations indicate that apart from the latter (whose intercept $\alpha_{S^0}(0) = 1$, where s^i denotes an even-signature trajectory corresponding to the i th unitary index, counting from 0 to 8), all trajectories s^i and v^i (v denotes odd signature) have $\alpha(0) \sim 0.5$. Deviations from the Pomeranchuk limit $\sigma^{AB} = \sigma^{\bar{A}\bar{B}}$ should thus tend to disappear with increasing energy at some general common rate. Variegation in the "law of force" picture seems to result in the main from differences between residues.

We assume that the vertex strengths $\gamma_C^{AB}(0)$ in the limit of forward scattering are given by matrix elements of algebraic operators belonging to a U(12) algebra. We introduce a system of nine strengths S^i with scalar densities; when adjoined to a second nonet of strengths V^i with the same algebraic properties as the unitary spin generators, they close on a U(3) \times U(3) sub-algebra completely isomorphic to the [U(3) \times U(3)] $_{\beta}$ contained in the [U(6) \times U(6)] $_{\beta}$ "good" rest symmetry * defined by Dashen and Gell-Mann.

The matrix elements of the system of S^i and V^i strengths are to be identified with the $\gamma_{S^i}^{AB}(0)$ and $\gamma_{V^i}^{AB}(0)$ respectively as

* It is not clear that our strengths are really to be identified with the space integrals of the currents associated directly with weak transitions. For instance, we may be dealing with a class of source currents of strong transitions, consistently definable in terms of the Regge formalism. The complete U(12) algebra of strengths would contain pseudoscalar and axial-vector operators (corresponding to trajectories with 0^- and 1^+ exchange). These additional operators cannot represent rest symmetries.

$$\delta^3(p_A - p_B) \gamma_{S^i}^{AB}(0) = \langle A | \int \mathcal{D}(\beta \sigma^0 \lambda^i; x, 0) d^3x | B \rangle \quad (1)$$

$$\delta^3(p_A - p_B) \gamma_{V^i}^{AB}(0) = \langle A | \int \mathcal{D}(\sigma^0 \lambda^i; x, 0) d^3x | B \rangle \quad (2)$$

where the integrals are carried out in the rest frame of the incident particle **. Note that to account for spin flip we would use an entire set of U(6) \times U(6) generators. In the following we shall deal with elastic scattering only, thus using in fact σ^0 with $i = 0, 3, 8$ only. The S^i and V^i would be written in a quark representation as

$$S^i = \frac{1}{2} \int d^3x q^+ \beta \lambda^i q \quad (3)$$

$$V^i = \frac{1}{2} \int d^3x q^+ \lambda^i q \quad (4)$$

We note that

$$[S^i, S^j] = i f^{ij}_k V^k \quad (5)$$

$$[V^i, S^j] = i f^{ij}_k S^k \quad (6)$$

The effect of β in the U(12) algebra can be represented in terms of constituent (1, 0, 0) representations - mathematical quarks [15] - as additive (positively) in quark and antiquark unitary charges. S^0 , for example, has eigenvalues proportional to the number of "quark charges" plus "antiquark charges" (in opposition to V^0 which is proportional to baryon charge, i.e., to the number of "quark charges" minus "antiquark charges"). S^i adds up λ^i contributions of quarks plus λ^i contributions of antiquarks (V^i picks out the differences, since $-\lambda^i$ is the unitary spin representation of the antiquarks). It is for the above reason that using S^i densities reproduces the results of "quark-additivity" and "quark counting" applied in composite models. As a simplest example, consider this crude derivation of the $\sigma_{\pi P} / \sigma_{PP}$ ratio. Fig. 1 shows the t channel exchange of a Pomeranchuk trajectory. Assuming this to be dominant † at the highest energies, we derive for the total cross sections the Levin-Frankfurt ratio [1].

$$\frac{\sigma_{\pi P}}{\sigma_{PP}} = \frac{\gamma_{S^0}^{\pi\pi}(0) \gamma_{S^0}^{PP}(0)}{\gamma_{S^0}^{PP}(0) \gamma_{S^0}^{PP}(0)} \approx \frac{2}{3} \quad (7)$$

since $\gamma_{S^0}(0)$ is, according to its definition, just the eigenvalue n_{S^0} of the generator S^0 . We have used, of course, the [U(6) \times U(6)] $_{\beta}$ assignments

** The densities will supply form factors; however we deal with $t \neq 0$ elsewhere.

† Similar considerations with both S^0 and S^8 contributing predict $\sigma_{\pi P} > \sigma_{KP}$ as can be seen from the table below. We are indebted to Dr. J. J. J. Kokkedee for this remark.

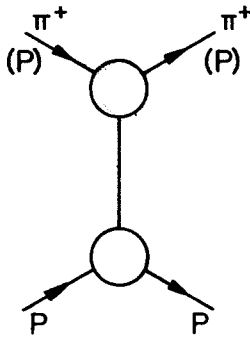


Fig. 1.

of $(56, 1)^+$ and $(6, 6)^-$ for baryons and mesons respectively. In this connection we wish to emphasize an approximation, implicit in our adoption of the Regge formalism, in which we have defined the vertex strengths at each vertex independently. It may be that a complete theory would require us to treat both vertices simultaneously and that the same classification of states in upper and lower vertices cannot be maintained as far as the S^i part of $[U(3) \times U(3)]_\beta$ goes. The possibility of an alteration in F/D ratio at one vertex should therefore be taken into account for the S^i matrix elements (this is the only allowed alteration); we discuss this later in connection with the comparison of our ideas with experiment. However, in cases such as eq. (7), where the ratio of cross sections with the same baryon-scalar vertices is computed, this possible effect cancels out completely and no approximation is involved.

Note that the commutation relations fix the relative scales of γ_{S^i} and γ_{V^i} by imposing a quadratic relation.

We assume that the elastic scattering amplitude is given by \dagger (γ_A now stands for γ_{AA} , etc.)

$$T_{AB}(\nu, t) = \sum_{i=0,3,8} \left\{ \gamma_{S^i}^A(t) \gamma_{S^i}^B(t) g(\alpha_i^S) \times \frac{1 + \exp(-i\pi\alpha_i^S)}{\sin \pi \alpha_i^S} \frac{\Gamma(\alpha_i^S + \frac{3}{2})}{\Gamma(\alpha_i^S + 1)} \left(\frac{\nu}{\nu_S}\right)^{\alpha_i^S(t)} + \gamma_{V^i}^A(t) \gamma_{V^i}^B(t) \frac{1 - \exp(-i\pi\alpha_i^V)}{\sin \pi \alpha_i^V} \frac{\Gamma(\alpha_i^V + \frac{3}{2})}{\Gamma(\alpha_i^V + 1)} \left(\frac{\nu}{\nu_V}\right)^{\alpha_i^V(t)} \right\} \quad (8)$$

where

$$\nu = (\text{total energy in c. m.}) + \frac{1}{2}t - m_A^2 - m_B^2. \quad (9)$$

$g(\alpha_i^S)$ is a ghost-killing factor which we have to separate explicitly in order to cancel out the pole at $\alpha_i^S = 0$. We shall assume that in the neighbourhood of $t = 0$, $g(\alpha_i^S) \approx 1$. Since these trajectories

Table 1
Total cross sections

$\bar{P}N$	$= 6t_0^S + 3t_8^S - t_3^S + 6t_0^V + 3t_8^V - t_3^V$
PN	$= 6t_0^S + 3t_8^S - t_3^S - 6t_0^V - 3t_8^V + t_3^V$
PP	$= 6t_0^S + 3t_8^S + t_3^S - 6t_0^V - 3t_8^V - t_3^V$
$\bar{P}P$	$= 6t_0^S + 3t_8^S + t_3^S + 6t_0^V + 3t_8^V + t_3^V$
π^+P	$= 4t_0^S + 2t_8^S - 2t_3^S - 2t_3^V$
π^-P	$= 4t_0^S + 2t_8^S + 2t_3^S + 2t_3^V$
K^+P	$= 4t_0^S - t_8^S + t_3^S - 3t_8^V - t_3^V$
K^-P	$= 4t_0^S - t_8^S + t_3^S + 3t_8^V + t_3^V$
K^+N	$= 4t_0^S - t_8^S - t_3^S - 3t_8^V + t_3^V$
K^-N	$= 4t_0^S - t_8^S - t_3^S + 3t_8^V - t_3^V$

describe the exchange of 2^+ and 1^- particles respectively, the absolute signs of their contributions are determined by the requirement that they give rise to forces between equal particles which are respectively attractive and repulsive. We assume no ambivalence is introduced by the t dependence.

The $j = 1^-$ trajectory may couple to the 1^- particles in the usual way. As to the $j = 2^+$ mesons, they cannot couple directly with scalar quantities, but our prescription is appropriate for the coupling of their trajectory at $\alpha = 0$. The problem of coupling to Regge recurrences is not special to the scalar, but occurs also for the other trajectories.

The contribution of each pole to the total cross section is equal to the product of vertex strengths defined according to eqs. (1) and (2), a factor (t_i^V or t_i^S) depending upon energy and the position of the pole, and a sign determined by the signature in eq. (8) (negative for the vector contribution). Our basic result is given in table 1, where a normalization corresponding to $\text{tr } \lambda_i^2 = 2$ is used to avoid fractional coefficients.

The coefficients actually correspond to overall F coupling for the vector trajectories and for the mesons and baryons D and F coupling, respectively, to the scalar trajectories. $SU(3)$ symmetry would imply the equality $t_3^S = t_8^S$ and $t_3^V = t_8^V$; $U(3)$ among the vector contributions would imply $t_0^V = t_3^V = t_8^V$ and $[SU(3) \times SU(3)]_\beta$ (excluding

\dagger We neglect ω - ϕ mixing since we consider the case in which their trajectories are essentially degenerate. Our identification of s^0 with the Pomernanchuk trajectory requires no $s^0 - s^8$ mixing at $\alpha = 0$.

t_0^S) would finally imply ($t_3^V = t_3^S$) that all * but the Pomernanchuk coefficients are equal. We note here that $t_3^S = t_3^V$ alone immediately implies that [5]

$$K^+P = K^+N \quad (10)$$

This relation is well satisfied between 6 and 20 GeV/c and may be interpreted in our theory as a close degeneracy between the s_3 and v_3 intercepts (both contributions become small, however at higher energies).

The parameter t_0^S is positive; it follows from our basic picture that all t_i^S and t_i^V are positive numbers and we derive the following inequalities, valid without any other restriction:

$$\begin{aligned} K^-P &> K^-N \\ \pi^-P &> \pi^+P \\ K^-P &> K^+P \\ K^-P &> K^+N \\ K^-P - K^-N &> |K^+P - K^+N| \\ K^+P + K^-P &> K^+N + K^-N \end{aligned} \quad (11)$$

The relations (11) are strikingly verified. For baryon-baryon scattering we obtain

$$\begin{aligned} \bar{P}P &> PP \\ \bar{P}P &> PN \\ \bar{P}P &> \bar{P}N \\ \bar{P}P - PP &> \bar{P}N - PN \\ \bar{P}P + PP &> \bar{P}N + PN \end{aligned} \quad (12)$$

Four identities also follow since there are 10 relations and only 6 parameters:

$$\begin{aligned} K^+P - K^+N &= PP - NP \quad [5] \\ K^-P - K^-N &= \bar{P}P - \bar{P}N \quad [5] \\ 3(\pi^+P + \pi^-P) &= \bar{P}N + PN + PP + \bar{P}P \quad [2, 4] \quad (13) \\ K^+P + \pi^-P + K^-N &= K^-P + \pi^+P + K^+N \quad [2] \end{aligned}$$

Following the procedure used for table 1 we obtain

$$\Delta P = 6t_0^S + 6t_0^V \quad (14)$$

and therefore

$$\Delta P - PP = K^-N - \pi^+P \quad [2] \quad (15)$$

The additional "antisymmetric" (t_i^S cancelling) relations follow from SU(3) (the Johnson-Treiman

relation [16]) and U(3), for the Freund [3] relation,

$$PP - \bar{P}P = \frac{5}{4}(\bar{P}N - PN) = 5(\pi^-P - \pi^+P) \quad (16)$$

As to the other "symmetric" (t_i^V cancelling) relations of Lipkin and Scheck [2]

$$PP + \bar{P}P = 2[\pi^+P + \pi^-P] - \frac{1}{2}[K^+P + K^-P] \quad (17)$$

$$K^+P + K^-P = \frac{1}{2}[\pi^+P + \pi^-P + K^+N + K^-N]$$

they require SU(3) among the scalar trajectories, which is a stronger condition and seems to be less well satisfied.

Imposing the $[SU(2) \times SU(2)]_\beta$ relation $t_3^S = t_3^V$ [leading immediately to (10)] as well as SU(3) among the vector trajectories ($t_3^V = t_3^S$) we obtain

$$K^-N = \frac{1}{2}(K^-P + K^+P) \quad [5], \quad (18)$$

which is well satisfied. However, setting also $t_3^S = t_3^S$ (imposing SU(3) among the scalar trajectories) one obtains [5]

$$\begin{aligned} \pi^-P &= K^-P \\ \pi^+P &= K^-N \end{aligned} \quad (19)$$

which are not as good. As mentioned above we should expect some mixing of the (56, 1) with other $[U(6) \times U(6)]_\beta$ states, which would generate some D coupling at the scalar trajectory nucleon vertex, affecting mainly t_0^S . Adjoining a small negative admixture of D coupling to this vertex, the experimental meson-nucleon data can be fit to within a millibarn, taking common t_i^V and t_j^S ($j \neq 0$). Alternatively, solving for the coefficients, one notices that t_0^S is rather large.

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A REMARK ABOUT THE GENERAL METHODS IN CURRENT ALGEBRA

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Two general methods used in the framework of the algebra of currents to obtain sum rules have been extensively studied in a previous paper [1] to which we refer for details and notations. The aim of this note is to compare both methods in a particular case from which the Adler-Weisberger [2, 3] sum rule follows.

¹ The basis quantity of interest is a causal amplitude

$$T_{\rho\sigma}^{\alpha\beta} = i \int d_4x \exp(-iq_2 \cdot x) \theta(x) \langle p_2 | [J_\rho^\alpha(x), J_\sigma^\beta(0)] | p_1 \rangle \quad (1)$$

where α and β are unitary spin indices.

We define q_1 as $q_1 = p_2 + q_2 - p_1$ and for simplicity we consider only the case where $p_1^2 = p_2^2 = -M^2$. As usual, we introduce convenient four vectors

$$P = \frac{1}{2}(p_1 + p_2); \quad Q = \frac{1}{2}(q_1 + q_2); \quad \Delta = p_1 - p_2 = q_2 - q_1$$

and the scalar variables

$$\nu = -q_2 \cdot P = -q_1 \cdot P = Q \cdot P, \quad t = -\Delta^2.$$

We restrict ourselves, in the following, to the part of the amplitudes *skew symmetric* in the exchange of the unitary spin indices $\alpha \leftrightarrow \beta$.

² The Fubini method [4] applied twice, gives the equation [1]:

$$q_2^\rho T_{\rho\sigma}^{\alpha\beta} q_1^\sigma = V^{\alpha\beta} + c_\gamma^{\alpha\beta} \langle p_2 | Q \cdot J^\gamma(0) | p_1 \rangle \quad (2)$$

where $V^{\alpha\beta}$ is defined by the commutator of the current divergences:

$$V^{\alpha\beta} = i \int d_4x \exp(-iq_2 \cdot x) \theta(x) \langle p_2 | [J_\rho^\alpha(x), J_\sigma^\beta(0)] | p_1 \rangle \quad (3)$$

The $c_\gamma^{\alpha\beta}$ are the U(3) structure constants and the second term of the right hand side of eq. (2) is called the equal time commutator contribution. Such an equality is obtained replacing the momenta q_1 and q_2 by the divergence operator and integrating by parts. The divergence term - also usually called the surface term - is assumed to vanish at infinity.

The Adler method [5] gives the general equation [1]:

$$\nu T_{\rho\sigma}^{\alpha\beta} = U_{\rho\sigma}^{\alpha\beta} - c_\gamma^{\alpha\beta} \times \langle p_2 | P_\rho J_\sigma^\gamma(0) + P_\sigma J_\rho^\gamma(0) - g_{\rho\sigma} P \cdot J^\gamma(0) | p_1 \rangle \quad (4)$$

where

$$U_{\rho\sigma}^{\alpha\beta} = -\frac{1}{2} P_\mu \int d_4x \exp(-iq_2 \cdot x) \theta(x) \times \langle p_2 | [\partial^\mu J_\rho^\alpha(x), J_\sigma^\beta(0)] - [J_\rho^\alpha(x), \partial^\mu J_\sigma^\beta(0)] | p_1 \rangle. \quad (5)$$

Using the definition of ν : $\nu = -\frac{1}{2}(q_1 + q_2) \cdot P$ the same technique of integration by parts is applied for the Adler method.

The scalar quantity $q_2^\rho T_{\rho\sigma}^{\alpha\beta} q_1^\sigma$ can be computed

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