New Dynamical Origin of Higgs Fields: Singular Fluctuation Loops in Pure Gauge Theories.

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Summary. – We argue that local Higgs fields may correspond neither to pointlike particles nor to bound states of some more fundamental objects. Rather, they arise naturally in *pure gauge theories* as a convenient technical device for parametrizing line-like disorder.

Our reasoning is based on the observation that nonlinear gauge field fluctuations which are singular on a random set of loops are responsible for many phase transitions of three-dimensional systems such as solid-liquid, superfluid-normal and nematic-normal. They probably are the origin of permanent confinement in QCD. We propose their explicit inclusion into the partition function by developing a field theory of random loops in minimal interaction with the gauge fields which then are free of these singular fluctuations. The result is a Higgs-like gauge theory which permits a simple study of phase transitions. In particular, the residual gauge fluctuations may sometimes be approximated as harmonic such that many difficulties of non-Abelian theories disappear.

Perturbation theory is an efficient tool for summing up field fluctuations as long as these are approximately harmonic. As interaction terms become sizable, there are, in general, important contributions, to the partition function, of macroscopic fluctuations in which the field is singular on points, lines, or surfaces. The singularities are usually damped on some short-distance length scale. Pointlike structures seem to be the least relevant, due to their high energy. Linelike singularities, on the other hand, can take many possible shapes such that at some high enough temperature T_c their entropy always over-compensates their core energy leading to their proliferation. Interactions between the lines may favour this process and depress the actual transition temperature to such low values that this becomes the dominant mechanism for many phase transitions such as melting of solids (1,2), superfluid-normal of He II (3), and

(1) F. R. N. NABARRO: *Theory of Crystal Dislocations* (Oxford, 1967).
(2) S. F. EDWARDS and M. WARNER: *Philos. Mag.*, **40**, 257 (1979).
(3) R. P. FEYNMAN: in *Progress in Low-Temperature Physics*, edited by C. J. GORTER, Vol. **1** (Amsterdam, 1955). See also E. BYCKLING: *Ann. Phys.* (*N. Y.*), **32**, 367 (1965); and V. N. POPOV: *Sov. Phys. JETP*, **37**, 341 (1973).

clearing of nematic crystals (4). Their omnipresence seems also to be responsible for the confining phase of QCD (5).

Apart from that, they are interesting objects on their own capable of forming highly regular networks in solids (6) and liquid crystals (7).

A linelike random chain in 3-space of length L has an end-to-end probability distribution (8)

$$P(\boldsymbol{x}-\boldsymbol{x}', L) = (2\pi l L/3)^{-\frac{3}{2}} \exp\left[-\frac{3}{2lL}\right](\boldsymbol{x}-\boldsymbol{x}')^2 , \tag{1}$$

where l is the lattice spacing. It may, therefore, be seen as the propagator of a free nonrelativistic field of mass $M = 3/l$ with time continued to an imaginary value $-iL$:

$$P(\boldsymbol{x}-\boldsymbol{x}', L) = \langle T\psi(\boldsymbol{x}, t)\psi^\dagger(\boldsymbol{x}, t')\rangle|_{t-t'=-iL} . \tag{2}$$

Thus the Lagrangian

$$\mathscr{L} = \psi^\dagger(\boldsymbol{x}, t)\left(i\partial_t + \frac{1}{2M}\,\partial^2\right)\psi(x, t) \tag{3}$$

can be used to study a grand-canonical ensemble of random chains and the current

$$\boldsymbol{j}(\boldsymbol{x}, t) = \frac{1}{2Mi}\,\psi^\dagger(\boldsymbol{x}, t)\overleftrightarrow{\partial}\psi(\boldsymbol{x}, t) \tag{4}$$

is the second quantized version of a single chain along $\boldsymbol{x}(t)$ for which it would read, in the absence of fluctuations,

$$\boldsymbol{j}(\boldsymbol{x}, t) = \frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t}\,\delta^{(3)}(\boldsymbol{x}-\boldsymbol{x}(t)) . \tag{5}$$

If the chains can have any length with a distribution $\exp[-(m^2/T)L]$, the probability becomes (9,10)

$$P(\boldsymbol{x}-\boldsymbol{x}') \propto \int\limits_0^\infty \mathrm{d}L \exp\left[-\frac{m^2}{T}L\right]P(\boldsymbol{x}-\boldsymbol{x}', L) \propto \int\frac{\mathrm{d}^3k}{(2\pi)^3}\exp[i\boldsymbol{k}\boldsymbol{x}]\frac{1}{6m^2/lT+\boldsymbol{k}^2} , \tag{6}$$

which is the correlation function of a Klein-Gordon field of mass $\mu^2 = 6m^2/lT$ and field energy (*)

$$F_0[\varphi, \varphi^\dagger] = \int\mathrm{d}^3x\, f_0(\boldsymbol{x}) = T\int\mathrm{d}^3x\left(\partial\varphi^\dagger(\boldsymbol{x})\,\partial\varphi(\boldsymbol{x}) + \mu^2\varphi^\dagger(\boldsymbol{x})\varphi(\boldsymbol{x})\right) . \tag{7}$$

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(4) W. HELFRICH: Berlin preprint (1981).
(5) R. P. FEYNMAN: *Nucl. Phys. B*, **188**, 479 (1981).
(6) S. AMELNICKX: in *Dislocations in Solids*, edited by F. R. N. NABARRO (New York, N. Y., 1979).
(7) S. MEIHBOOM, J. P. SENA, P. W. ANDERSON and W. F. BRINKMAN: *Phys. Rev. Lett.*, **46**, 1216 (1981); H. KLEINERT and K. MAKI: *Fortschr. Phys.*, **29**, 219 (1981); H. KLEINERT: *Phys. Lett. A*, **81**, 141 (1981).
(8) H. YAMAKAWA: *Modern Theory of Polymer Solutions* (New York, N. Y., 1971).
(9) H KLEINERT: *Phys. Lett. A* **89**, 294 (1982) *Lett. Nuovo Cimento* (in press).
(10) H. KLEINERT: Berlin preprint (September 1981).
(*) The factor T in front shows that $\varphi(\boldsymbol{x})$ describes fluctuations which are purely entropic in character (9,10).

Notice that the parameter t of the previous description (3) has disappeared in favour of a truly local field $\varphi(\boldsymbol{x})$. As chains come close to each other, they will, in general, interact via some nonlocal potential $V[\varphi, \varphi^\dagger]$. The partition function

$$Z_1 = \int \mathscr{D}\varphi\, \mathscr{D}\varphi^\dagger(\boldsymbol{x}) \exp\left[-\frac{1}{T}\{F_0[\varphi, \varphi^\dagger] + V[\varphi, \varphi^\dagger]\}\right] \tag{8}$$

thus describes a grand-canonical ensemble of closed interacting random chains. As a matter of fact, the Feynman diagrams arising in a perturbation expansion of Z can be viewed as a direct image of all possible configurations included in the sum.

The chemical potential has two terms:

$$m^2 = m_c^2 - wT\,. \tag{9}$$

The first accounts for the core energy, the second for the entropy ($w \approx \log 6$) per unit length. Obviously, at some temperature $T_c = m_c^2/w$, the chemical potential always changes sign thereby destabilizing $F_0[\varphi, \varphi^\dagger]$. Usually, $V[\varphi, \varphi^\dagger]$ prevents a catastrophe and the field settles at a nonvanishing value $\langle\varphi(\boldsymbol{x})\rangle = \varphi_0 \exp[i\gamma_0]$. This is observed as a phase transition. The disorder phase is characterized by long-range fluctuations associated with field configurations $\varphi_0 \exp[i(\gamma_0 + \delta\gamma(\boldsymbol{x}))]$, since their energy goes as

$$F_0 \propto \sum_{\boldsymbol{k}} \boldsymbol{k}^2(\delta\gamma(\boldsymbol{k}))^2\,. \tag{10}$$

Such Nambu-Goldstone modes may be called « hot sound » [9,10], since they are precisely the disorder analogue, at high temperature, of « zero sound » seen in ordered systems at low temperature.

Suppose now that a self-interacting pure gauge theory is suspected of having important large anharmonic fluctuations singular on a random set of loops and consider the partition function $Z = \int \mathscr{D}\boldsymbol{A} \exp[-(1/T)F[\boldsymbol{A}]]$. The functional integral comprises both regular and singular gauge field configurations $\boldsymbol{A}(\boldsymbol{x})$. Our proposal is to evaluate this integral by simply adding to the gauge field energy the random chain expression $F_0[\varphi, \varphi^\dagger] + V[\varphi, \varphi^\dagger]$ in which the chain field is coupled minimally to the gauge field $\boldsymbol{A}$ via the usual covariant derivative

$$\partial\varphi(x) \to \boldsymbol{D}\varphi(\boldsymbol{x}) \equiv (\partial - i\boldsymbol{A})\varphi(\boldsymbol{x}) \tag{11}$$

and integrating instead

$$Z = \int \mathscr{D}\boldsymbol{A}^{\text{reg}}(\boldsymbol{x}) \int \mathscr{D}\varphi\, \mathscr{D}\varphi^\dagger(\boldsymbol{x}) \exp\left[-\frac{1}{T}\{F[\boldsymbol{A}^{\text{reg}}] + (F_0[\varphi, \varphi^\dagger] + V[\varphi, \varphi^\dagger])\}\right]_{\partial\to\boldsymbol{D}}, \tag{12}$$

but with $\mathscr{D}\boldsymbol{A}^{\text{reg}}$ now running only over gauge fields *free* of line-like singularities (*), all of these having been transferred into the explicit $\varphi(\boldsymbol{x})$ field. This new Higgs-like representation of a pure gauge theory has the formal advantage that phase transitions can be studied in analogy with the Ginzburg-Landau theory of superconductivity with

(*) This statement can be made more precise by giving the Fourier expansion of these fields $A^{\text{reg}}(\boldsymbol{k})$ an ultraviole cut-off Λ with Λ^{-1} larger than the size of the cores of the singularities.

the only difference that $\varphi(\boldsymbol{x})$ describes disorder rather than order and destabilizes for large rather than small temperature.

Apart from this, our representation has an important property which we believe will render non-Abelian gauge theories solvable in simple terms: If a phenomenon happens to be dominated by linelike fluctuations (*), the remaining gauge fluctuations in $\boldsymbol{A}^{\text{reg}}(x)$, which now are free of such singular field configurations, can be approximated as harmonic. Thus, with respect to such phenomena, non-Abelian pure gauge theories do not differ much from their *linearized* version in our representation (12) (**). But this is Abelian in character and displays a well-understood Ginzburg-Landau–type behaviour.

In order to convince the reader, we illustrate this property with a few examples.

1) An elastic continuum has a free energy [11]

$$F_{\text{el}}=\int \mathrm{d}^3x\,\frac{1}{4\mu}\left\{\sigma_{ij}(\boldsymbol{x})^2-\frac{\nu}{1+\nu}\,\sigma_{ii}(\boldsymbol{x})^2\right\}, \tag{13}$$

where μ and ν are elastic constants and $\sigma_{ij}(\boldsymbol{x})$ is the symmetric stress tensor which satisfied $\partial_i\sigma_{ij}=0$ such that we may represent it as a double curl.

$$\sigma_{ij}(\boldsymbol{x})=\varepsilon_{jkl}\,\partial_k\,\varepsilon_{imn}\,\partial_m\,h_{en} \tag{14}$$

with the obvious invariance

$$h_{ln}(\boldsymbol{x})\rightarrow h_{ln}(\boldsymbol{x})+\partial_l\,\xi_n(\boldsymbol{x})+\partial_n\,\xi_l(\boldsymbol{x})\,.$$

It will be convenient to continue the discussion not with $h_{ln}(\boldsymbol{x})$, but to introduce a stress potential $A^i_l\equiv\varepsilon_{imn}\partial_m h_{ln}$ which transforms as

$$A^i_l(\boldsymbol{x})\rightarrow A^i_l(\boldsymbol{x})+\partial_l A^i(\boldsymbol{x}) \tag{15}$$

with $A^i(\boldsymbol{x})\equiv\varepsilon_{imn}\partial_m\xi_n(\boldsymbol{x})$ being arbitrary, purely transversal functions (*i.e.* $\partial_i A^i=0$). In terms of A^i_l, the elastic energy becomes

$$F_{\text{el}}[A]=\int \mathrm{d}^3x\,\frac{1}{4\mu}\,\boldsymbol{\partial}A^i_l(\boldsymbol{x})\left\{P^{(2,2)}+P^{(2,-2)}+\frac{1-\nu}{1+\nu}\,P^{(1,0)}\right\}_{il,i'l'}\boldsymbol{\partial}A^{i'}_{l'}(\boldsymbol{x}) \tag{16}$$

with $P^{(s,\lambda)}$ projecting out the spin s helicity λ content of A^i_l.

Thus the correlation function of the A^i_l field becomes

$$\langle A^i_l(\boldsymbol{k})\,A^{i'}_{l'}(\boldsymbol{k})\rangle=\frac{2\mu T}{\boldsymbol{k}^2}\left\{P^{(2,2)}(\hat{\boldsymbol{k}})+P^{(2,-2)}(\hat{\boldsymbol{k}})+\frac{1+\nu}{1-\nu}\,P^{(1,0)}(\hat{\boldsymbol{k}})\right\}_{il,i'l'} \tag{17}$$

in the gauge $\partial_l A^i_l(\boldsymbol{x})=0$ (notice that $\partial_i A^i_l=A^l_l=0$ by definition).

(*) Notice that magnetic monopoles which are supposed to be responsible for quark confinement are running along world-lines.

(**) The nonlinearities are also necessary to incorporate the effect of the omitted non-line–like singular field configurations.

(11) L. D. LANDAU and E. M. LIFSHITZ: *Theory of Elasticity* (New York, N. Y., 1959); μ = modulus of rigidity, ν = Poisson number.

What we have written down is an Abelian gauge field theory for linear elasticity. For large fluctuations this is certainly wrong. In fact, nonlinear versions may be constructed by introducing a metric $g_{ij}(\boldsymbol{x}) = \delta_{ij} + 2h_{ij}(\boldsymbol{x})$ and viewing $\sigma_{ij} = \varepsilon_{jkl}\varepsilon_{imn}\partial_k\partial_m h_{ln}$ as the linearized version of the divergenceless Einstein tensor $\sigma_{ij} = R_{ij} - \frac{1}{2}g_{ij}R_{ll}$ in three dimensions. Then (13) corresponds to Weyl's gravitational theory which may be used as a non-Abelian gauge theory for nonlinear elasticity [12]. In general, there will also be higher powers of R. Such nonlinear theories have large fluctuations concentrated around points, lines and surfaces. In a real crystal, these are observed as defects.

Suppose now we want to study the process of melting in an ideal infinitely extended simple cubic crystal. This process is widely believed to be caused by the explosive proliferation of linelike defects of the translational type, called dislocations [1,2,9]. They are characterized by the so-called Burger's vectors $\boldsymbol{b}^{(\alpha)}$ which are conserved quantities along each line and across branch points of lines just as though $\boldsymbol{b}^{(\alpha)}$ were triplets of currents flowing along the lines.

In the absence of elastic long-range forces the dislocations may be described as interacting random flight chains with a free energy

$$F = \sum_{\alpha} \int d^3x \{\partial\varphi_\alpha(\boldsymbol{x})\,\partial\varphi_\alpha(\boldsymbol{x}) + \mu_\alpha^2 \varphi_\alpha^\dagger(x)\varphi_\alpha(x)\} + V[\varphi_\alpha, \varphi_\alpha^\dagger]\,, \tag{18}$$

where the interaction conserves the Burger's vectors and is, therefore, invariant under phase changes

$$\varphi_\alpha(\boldsymbol{x}) \to \exp[-i\gamma\boldsymbol{b}^{(\alpha)}]\varphi_\alpha(\boldsymbol{x})\,. \tag{19}$$

We can now incorporate those nonlinear aspects of elasticity which are governed by dislocation lines by simply adding F and F_{el} and replacing $\partial_l\varphi_\alpha$ by the covariant form $(\partial_l - (i/T)b_i^{(\alpha)}A_l^i)\varphi_\alpha(\boldsymbol{x})$ making (19) a local invariance. In this way we have approximated the non-Abelian gauge theory with large fluctuations by an Abelian one with harmonic fluctuations plus a Higgs field. In order to be sure that this describes the correct physics we may calculate the energy between two dislocation lines via A_l^i exchange and find the well-known expression derived many years ago by BLIN [9,10].

The energy (18) can be simplified by noting that dislocation lines with larger Burger's vectors can be generated as bound states or resonances of a few fundamental ones (usually three) such that the sum in (18) may be restricted to the fundamental Burger's vectors only. The discussion of the phase transition solid-liquid follows the same pattern as in the Ginzburg-Landau theory of superconductivity. For $T > T_c$, the disorder parameter $\varphi_\alpha(\boldsymbol{x})$ has a nonvanishing expectation value, the sea-gull term $|b_i^{(\alpha)}A_l^i\varphi_\alpha|^2$ makes the elastic field massive which thus ceases to propagate in the liquid state. The only difference with respect to the superconductor is the following: While there the phase oscillations (« zero sound ») all disappear, being equivalent to a gauge transformation, it is not so here for all the three « hot sound » waves. Their longitudinal combination survives, since it cannot be gauged away by the purely transversal gauge functions $A^i(\boldsymbol{x})$. In fact, longitudinal « hot sound » describes the physical sound waves in the liquid state [9].

2) Another example is superfluid He II where the energy is

$$F_s = \int d^3x \frac{\varrho_s}{2} \boldsymbol{v}_s^2\,, \tag{20}$$

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[12] H. KLEINERT: *Phys. Lett. B* (in press).

which is the analogue of the linear elastic energy (13). Here ϱ_s is the superfluid density and $\boldsymbol{v}_s$ the superfluid velocity which satisfies $\partial \boldsymbol{v}_s = 0$ (incompressibility) everywhere such that we may represent $\boldsymbol{v}_s$ in terms of a gauge field

$$\boldsymbol{v}_s = \partial \times \boldsymbol{A} . \tag{21}$$

In a proper nonlinear extension of (20) there are vortex lines, around which

$$\oint \mathrm{d}x\, \boldsymbol{v}_s = nh/m \equiv K_n$$

with $n = 1, 2, \ldots$; $h/m \approx 10^{-3}\,\mathrm{cm}^2/\mathrm{s}$. These may be considered as large fluctuations in $\boldsymbol{v}_s$ and may be incorporated into the linearized theory by adding the Higgs field energy $(F_0[\varphi_n, \varphi_n^\dagger] + V[\varphi_n, \varphi_n^\dagger])|_{\partial\varphi_n \to (\partial - (i/T)K_n A)\varphi_n}$. As in the crystal lines with higher n arise as resonances of the fundamental field φ_1 such that it is sufficient to include explicitly only this.

Again it may be verified that this coupling reproduces the correct forces between vortex lines (which are the same as the magnetic forces between current loops, apart from an opposite sign). As before, the superfluid-normal transition can be described as a Higgs effect with the disorder fields taking a nonzero expectation value at some temperature T_c (9,10).

A similar discussion can be given for nematic and smectic liquid crystals, pion condensation (13) and magnetic superconductors (14) where transitions are caused by defect loops (13).

Thus it appears convincing that local Higgs fields may just be an alternative way of describing line-like defects inherent in the nonlinear aspects of pure gauge fields. If all types of relevant defects are included explicitly, the remaining field fluctuations may become harmonic and simplify so much that physical phenomena can be studied by hand rather than by imitating immense multitudes of fluctuation patterns one by one in Monte Carlo computer calculations.

The implications of these considerations for QCD will be presented elsewhere.

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(13) H. KLEINERT: *Lett. Nuovo Cimento* **34**, 103 (1982).
(14) H. KLEINERT: *Phys. Lett. A* (in press).