COLLECTIVE EXCITATIONS OF ^{3}He B IN THE PRESENCE OF SUPERFLOW

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The energy levels of collective excitations of ^{3}He B are given in the presence of superflow. There is level splitting due to the distortion of the energy gap which should be observable experimentally.

The recently observed BCS-like behavior of the critical currents of superfluid ^{3}He B at zero pressure [1] has led to a renewed theoretical interest in the flow properties of this phase. A complete Ginzburg–Landau type of study close to T_c [2] and a BCS calculation for all T neglecting gap distortion have been available [3] for some time. The results have now been improved by calculating all BCS properties including the effects of gap distortion for all temperatures [4]. It turns out that the maximal distortion of transverse to longitudinal gap $r^2 \equiv 1 - c^2 \equiv 1 - \Delta_\parallel^2/\Delta_\perp^2$, which is 4/9 for $T \lesssim T_c$ at the critical velocity $v_c/v_0 = \frac{1}{3}(5/3)^{1/2}(1 - T/T_c)^{1/2}$ ($v_0 \equiv 1/2m^* \xi_0 \approx 6.3$ cm/s at zero pressure, m^* = effective mass, ξ_0 = coherence length), gradually decreases to zero for $T \to 0$.

It is the purpose of this note to point out how the distortion of the gap manifests itself in the energy spectrum of collective excitations. We find that the well-known $J = 2$ levels $\omega^2 = \frac{8}{5}\Delta_\perp^2$ and $\omega^2 = \frac{12}{5}\Delta_\perp^2$ split up into three branches each, the $\pm J_3$ levels remaining degenerate. The results should be observable in sound experiments [5], in particular at zero pressure where the line widths are small.

Consider the collective action [6,7] in the weak coupling limit

$$\mathcal{A} = -\tfrac{1}{2}\mathrm{i}\,\mathrm{Tr}\log\begin{pmatrix} \mathrm{i}\partial_t - \xi + \boldsymbol{p}\boldsymbol{v} & \sigma_a A_{ai}\mathrm{i}\overleftrightarrow{\partial}_i/2p_\mathrm{F} \\ \sigma_a A_{ai}^\dagger \mathrm{i}\overleftrightarrow{\partial}_i/2p_\mathrm{F} & \mathrm{i}\partial_t + \xi + \boldsymbol{p}\boldsymbol{v} \end{pmatrix} - \frac{1}{3g}\int \mathrm{d}\boldsymbol{x}\,\mathrm{d}t\,|A_{ai}|^2 , \tag{1}$$

where ξ stands short for $-\nabla^2/2m - \mu$ and σ_a are the Pauli matrices. The trace runs over 4×4 matrix indices as well as space–time variables. A constant flow velocity is enforced on the average by means of an external source coupling to the particle current which appears in the diagonal terms as $\boldsymbol{p}\boldsymbol{v}$ [8]. Apart from the full quantum mechanics, the partition function obtained by summing over all fluctuating field configurations $Z = \Sigma_{A_{ai}} \exp(\mathrm{i}\mathcal{A}[A_{ai}])$ describes the thermodynamics at a fixed temperature T by Wick rotating all energy integrations and grating them into Matsubara frequencies $\omega_n = (2n + 1)\pi T$. For $T < T_c$, there is equilibrium at a non-zero constant value ($l^2 = 1$) $A_{ai} = \Delta_\perp[\delta_{ai} + (c - 1)l_a l_i]$ which gives rise to an energy gap in the quasiparticle spectrum: $E = (\xi^2 + |A_{ai}\hat{p}_i|^2)^{1/2} \equiv [\xi^2 + \Delta_\perp^2(1 - r^2 z^2)]^{1/2}$. The two gap values $\Delta_\perp$, $\Delta_\parallel \equiv \Delta_\perp(1 - r^2)^{1/2}$ are found from the simultaneous solution of a longitudinal and a transversal gap equation [4]

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$$\log \frac{T}{T_c} = \int_{-1}^{1} \frac{dz}{2} \begin{Bmatrix} 3z^2 \\ \frac{3}{2}(1-z^2) \end{Bmatrix} \gamma , \tag{2}$$

where γ denotes the function

$$\gamma = \int_{-\infty}^{\infty} d\xi \frac{1}{4E} \{ [\mathrm{th}[(E + p_F vz)/2T] + (z \to -z)] - [\Delta_\perp = 0] \} . \tag{3}$$

For $T \lesssim T_c$ one finds [4] $r^2 = 2(p_F v/\Delta_\perp)^2 = 3v^2/v_0^2$. For $T = 0$, r stays at zero up to $p_F v/\Delta_\perp = 1$, i.e. practically up to the critical current [4].

Consider now small time dependent fluctuations in the complex order parameter $A_{ai} = A_{ai}^0 + \Delta_\perp d_{ai}(\tau) = A_{ai}^0 + \Delta_\perp(r_{ai} + i_{ai})(\tau)$. Expanding the action $\mathcal{A}$ up to quadratic order we find with $d(\tau) = \Sigma_{\nu_n} \exp(-i\nu_n \tau) d(\nu_n)$:

$$\begin{aligned} i\mathcal{A} = & -\frac{F_{\mathrm{free}}}{T} - \frac{F_{\mathrm{cond}}}{T} - \frac{\Delta_\perp^2}{4p_F^2} \rho \frac{V}{T} \sum_{\nu_n} \{\lambda^\perp(\nu_n) |d(\nu_n)|^2 - [\lambda^\perp - \lambda^\parallel(\nu_n)] \, |d(\nu_n) l|^2\} \\ & + \mathrm{Re}\{\sigma_2 [d_{aa} d_{bb} + d_{ai} d_{ia}] - (c^2\sigma_1 + \sigma_2) d_{ai} d_{ai} + (2c^2\sigma_1 - \sigma_2)(l^T d)_i (l^T d)_i \\ & + (\sigma_1 + \sigma_2 - c^2\sigma_3)(dl)_a (dl)_a + 2(2\sigma_1 - \sigma_2)(dl)_a (l^T d)_a + [-(1 + 4c + c^2)\sigma_1 + \tfrac{3}{2}\sigma_2 + c^2\sigma_3](l^T dl)^2\} . \end{aligned} \tag{4}$$

Here $\sigma_{1,2,3}(\nu_n)$ are angular projections of the dynamical generalization of Yoshida's function ϕ:

$$\sigma_{1,2,3} \equiv \frac{1}{2} \int_{-1}^{1} \frac{dz}{2} \{3z^2(1-z^2), \tfrac{3}{2}(1-z^2)^2, 3z^4\} \phi , \tag{5}$$

with

$$\phi \equiv \frac{\Delta_\perp^2}{2} \int_{-\infty}^{\infty} d\xi \frac{1}{E[E^2 - (\omega + i\epsilon)^2/4]} \{\mathrm{th}[(E + p_F vz)/2T] + (z \to -z)\} . \tag{6}$$

The functions $\lambda_\perp, \lambda_\parallel$ are the integrals

$$\lambda_{\parallel \atop \perp} = \int_{-1}^{1} \frac{dz}{2} \begin{Bmatrix} 3z^2 \\ \frac{3}{2}(1-z^2) \end{Bmatrix} (1 - r^2 z^2 - \omega^2/4\Delta_\perp^2)\phi , \tag{7}$$

and can be expressed in terms of $\sigma_{1,2,3}$ as

$$\lambda_\parallel = 2\sigma_1 + c^2\sigma_3 - 2(\omega^2/4\Delta_\perp^2)(2\sigma_1 + \sigma_3) , \quad \lambda_\perp = c^2\sigma_1 + 2\sigma_2 - 2(\omega^2/4\Delta_\perp^2)(\sigma_1 + 2\sigma_2) .$$

In writing eq. (6) we have continued analytically to physical frequencies ω by merely replacing the Matsubara frequencies ν_n^2 by $-(\omega + i\epsilon)^2$. Notice that all these functions depend on ω only via $w^2 \equiv \omega^2/4\Delta_\perp^2$ which will henceforth be used as natural variable.

The expression (4) can be diagonalized in the spaces (r_{11}, r_{22}, r_{33}), (r_{12}, r_{21}), (r_{13}, r_{31}), (r_{23}, r_{32}) and the corresponding imaginary parts. In these spaces, the sum in eq. (4) is composed of the following matrices:

$$R = \begin{pmatrix} 3\sigma_2 - 2w^2(\sigma_1 + 2\sigma_2) & \sigma_2 & 2c\sigma_1 \\ \sigma_2 & 3\sigma_2 - 2w^2(\sigma_1 + 2\sigma_2) & 2c\sigma_1 \\ 2c\sigma_1 & 2c\sigma_1 & 2c^2\sigma_3 - 2w^2(2\sigma_1 + \sigma_3) \end{pmatrix} ,$$

$$R^{12} = \begin{pmatrix} \sigma_2 - 2w^2(\sigma_1 + 2\sigma_2) & \sigma_2 \\ \sigma_2 & \sigma_2 - 2w^2(\sigma_1 + 2\sigma_2) \end{pmatrix}, \quad R^{\substack{13\\23}} = \begin{pmatrix} 2\sigma_1 - 2w^2(2\sigma_1 + \sigma_3) & 2c\sigma_1 \\ 2c\sigma_1 & 2c^2\sigma_1 - 2w^2(\sigma_1 + 2\sigma_2) \end{pmatrix},$$

$$I = \begin{pmatrix} 2c^2\sigma_1 + \sigma_2 - 2w^2(\sigma_1 + 2\sigma_2) & -\sigma_2 & -2c\sigma_1 \\ -\sigma_2 & 2c^2\sigma_1 + \sigma_2 - 2w^2(\sigma_1 + 2\sigma_2) & -2c\sigma_1 \\ -2c\sigma_1 & -2c\sigma_1 & 4\sigma_1 - 2w^2(2\sigma_1 + \sigma_3) \end{pmatrix},$$

$$I^{12} = \begin{pmatrix} 2c^2\sigma_1 + 3\sigma_2 - 2w^2(\sigma_1 + 2\sigma_2) & -\sigma_2 \\ -\sigma_2 & 2c^2\sigma_1 + 3\sigma_2 - 2w^2(\sigma_1 + 2\sigma_2) \end{pmatrix},$$

$$I^{\substack{13\\23}} = \begin{pmatrix} 2(\sigma_1 + c^2\sigma_3) - 2w^2(2\sigma_1 + \sigma_3) & -2c\sigma_1 \\ -2c\sigma_1 & 4\sigma_2 - 2w^2(\sigma_1 + 2\sigma_3) \end{pmatrix}.$$

The collective frequencies are determined by the values w at which the determinants vanish. They are found as

$$R: \quad w_1^2 = \sigma_2/(\sigma_1 + 2\sigma_2), \qquad (1,-1,0) \hat{=} |22\rangle + |2\,-2\rangle,$$

$$w_{\substack{2\\3}}^2 = \frac{\sigma_2}{\sigma_1 + 2\sigma_2} + \frac{1}{2}\frac{c^2\sigma_3}{2\sigma_1 + \sigma_3} \pm \frac{1}{2}\frac{\alpha}{(\sigma_1 + 2\sigma_2)(2\sigma_1 + \sigma_3)}, \qquad \hat{=} \begin{matrix} |00\rangle + \ldots, \\ |20\rangle + \ldots, \end{matrix}$$

$$\alpha \equiv \{[(\sigma_1 + 2\sigma_2)c^2\sigma_3 - (2\sigma_1 + \sigma_3)2\sigma_2]^2 + 8c^2\sigma_1^2(\sigma_1 + 2\sigma_2)(2\sigma_1 + \sigma_3)\}^{1/2},$$

$$R^{12}: \quad w_{\substack{1\\2}}^2 = \begin{Bmatrix} 0 \\ \sigma_2/(\sigma_1 + 2\sigma_2) \end{Bmatrix}, \qquad \begin{matrix} (1,-1) & \hat{=} |10\rangle, \\ (1,1) & \hat{=} |22\rangle - |2\,-2\rangle, \end{matrix}$$

$$R^{\substack{13\\23}}: \quad w_{\substack{1\\2}}^2 = \begin{cases} 0, & (1,-1/c) \hat{=} |11\rangle \pm |1\,-1\rangle, \\ \dfrac{\sigma_1}{(\sigma_1 + 2\sigma_2)(2\sigma_1 + \sigma_3)}[(2c^2+1)\sigma_1 + 2\sigma_2 + c^2\sigma_3], & \hat{=} |21\rangle \pm |2\,-1\rangle + \ldots, \end{cases}$$

$$I: \quad w_{\substack{1\\2\\3}}^2 = \begin{cases} 0, & (1,1,c) \hat{=} |00\rangle + \ldots, \\ \dfrac{c^2\sigma_1 + \sigma_2}{\sigma_1 + 2\sigma_2}, & (1,-1,0) \hat{=} |22\rangle + |2\,-2\rangle, \\ \dfrac{(2+c^2)\sigma_1}{2\sigma_1 + \sigma_3}, & \hat{=} |20\rangle, \end{cases} \tag{8}$$

$$I^{12}: \quad w_{\substack{1\\2}}^2 = \begin{cases} \dfrac{c^2\sigma_1 + 2\sigma_2}{\sigma_1 + 2\sigma_2}, & (1,-1) \hat{=} |10\rangle, \\ \dfrac{c^2\sigma_1 + \sigma_2}{\sigma_1 + 2\sigma_2}, & (1,1) \hat{=} |22\rangle - |2\,-2\rangle, \end{cases}$$

$$I^{\left\{\substack{13\\23}\right\}}: \quad w_{\substack{1\\2}}^2 = \frac{\sigma_2}{\sigma_1 + 2\sigma_2} + \frac{1}{2}\frac{\sigma_1 + c^2\sigma_3}{2\sigma_1 + \sigma_3} \pm \frac{1}{2}\frac{\alpha}{(\sigma_1 + 2\sigma_2)(2\sigma_1 + \sigma_3)}, \qquad \hat{=} \begin{matrix} |11\rangle \{\pm\} |1\,-1\rangle + \ldots, \\ |21\rangle \{\pm\} |2\,-1\rangle + \ldots, \end{matrix}$$

$$\alpha \equiv \{[(2\sigma_1 + \sigma_3)2\sigma_2 - (\sigma_1 + c^2\sigma_3)(\sigma_1 + 2\sigma_2)]^2 + 4c^2\sigma_1^2(2\sigma_1 + \sigma_3)(\sigma_1 + 2\sigma_2)\}^{1/2}.$$

Behind each eigenvalue we have written down the eigenvector if it is simple as well as the $|JJ_3\rangle$ content. The symbol + ... indicates that for $c \neq 1$ there is mixing among states with $\pm J_3$. For small current or for all $vp_F/\Delta_\perp \leqslant 1$ at $T = 0$, the distortion parameter r is zero, F is independent of z, and we see from eq. (10) that $\sigma_1 : \sigma_2 : \sigma_3 = 1 : 2 : 3$. Consequently, we recover the well-known frequencies

$$\omega/\Delta_\perp = (\sqrt{\tfrac{8}{5}}, \sqrt{\tfrac{8}{5}}, 1)^R, \quad (0, \sqrt{\tfrac{8}{5}})^{R^{12}}, \quad (0, \sqrt{\tfrac{8}{5}})^{R^{\substack{13\\23}}}, \quad (0, \sqrt{\tfrac{12}{5}}, \sqrt{\tfrac{12}{5}})^I, \quad (2, \sqrt{\tfrac{12}{5}})^{I^{12}}, \quad (2, \sqrt{\tfrac{12}{5}})^{I^{\substack{13\\23}}}, \tag{9}$$

now valid also in the presence of currents at $T = 0$.

Since gap distortion is strongest close to T_c, let us now consider this regime. Here ϕ has the simple form $\phi = (\pi\Delta_\perp/4T)(1 - r^2z^2 - w^2)^{-1/2}$, such that $\sigma_{1,2,3}$ can be calculated, apart from the factor $\pi\Delta_\perp/4T$ as

$$\begin{aligned}
\sigma_1 &= (3/4r^5)\{[-\tfrac{3}{4}(1 - w^2)^2 + r^2(1 - w^2)]\,l + [\tfrac{3}{4}(1 - w^2) - \tfrac{1}{2}r^2]\,r(1 - w^2 - r^2)^{1/2}\}, \\
\sigma_2 &= \tfrac{3}{8}(3/4r^5)\{[(1 - w^2)^2 - \tfrac{8}{3}r^2(1 - w^2 - r^2)]\,l + [-(1 - w^2) + 2r^2]\,r(1 - w^2 - r^2)^{1/2}\}, \\
\sigma_3 &= \tfrac{3}{4}(3/4r^5)\{(1 - w^2)^2 l - [(1 - w^2) + \tfrac{2}{3}r^2]\,r(1 - w^2 - r^2)^{1/2}\}.
\end{aligned} \tag{10}$$

Here, l denotes the function $r\int_{-1}^{1}(\mathrm{d}z/2)(1 - r^2z^2 - w^2)^{-1/2} = \arcsin\,[r(1 - w^2)^{-1/2}]$. By inserting the values (9) into the right-hand side of eqs. (10) and iterating eqs. (8) a few times, the values of w^2 converge rapidly against the correct eigenvalue at any $r^2 \leqslant r_c^2 = 5/9$. The results are displayed in fig. 1, together with the $T = 0$ lines.

It is quite simple to include Fermi liquid corrections [10]: in all equations one has to use, instead of v, the local velocity v^*, which is v modified by the molecular field $\tfrac{1}{3}F_1^s\rho_n^\parallel$, i.e., $[1 + \tfrac{1}{3}F_1^s\rho_n^\parallel(v^*)]\,v^* = v$, where $\rho_n^\parallel$ is the density of the normal component and F_1^s is the standard Landau parameter of current–current coupling [10].

It is hoped that the splitting between the $\sqrt{\tfrac{12}{5}}\,\Delta_\perp$ modes will soon be detected in sound experiments. A similar discussion of the collective modes in the A phase is not yet possible since no field configuration is known, in the

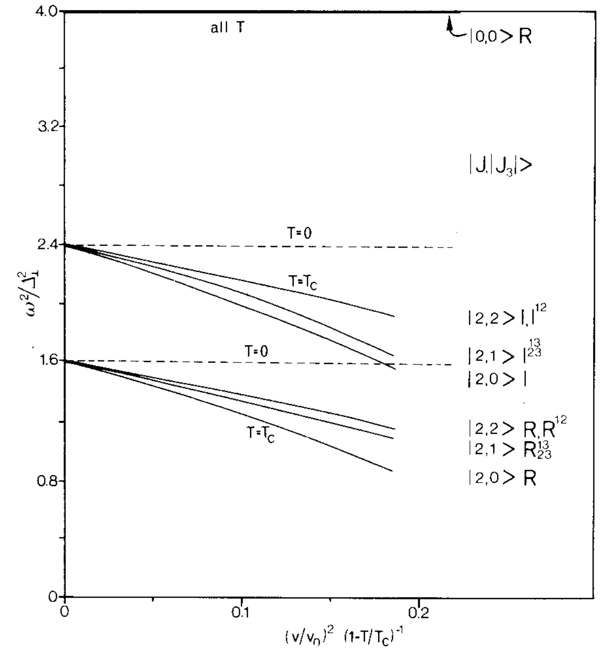

Fig. 1. The collective frequencies are shown, at infinite wavelength, as a function of $(v^2/v_0^2)(1 - T/T_c)^{-1}$. The dashed lines are the limits $T \to 0$. For $T \lesssim T_c$, there is appreciable splitting between levels of different $|J_3|$. The angular momenta are displayed on the right-hand side of each curve. For $T \approx T_c$, the gap distortion $r^2 \equiv 1 - \Delta_\parallel^2/\Delta_\perp^2$ is related to the superfluid velocity by $r^2 = 3 \times (v^2/v_0^2)(1 - T/T_c)^{-1}$ ($v_0 \equiv 1/2m^*\xi_0 \approx 6.3$ cm/s at zero pressure). The curves are drawn up to the critical velocity.

presence of a current, which is stable under infinitesimal static fluctuations, except for very small velocities limited by the dipole interaction ($v \lesssim 0.1$ mm/s) [11,12]. Only in this very restricted region are there two types of stable helical textures around which dynamic fluctuations may be studied.

More details will be presented elsewhere [13].

Note added. After completion of this work a paper appeared by Tewordt and Schopohl [14], which discusses the line splitting due to a strong magnetic field.

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