EXISTENCE OF HELICAL TEXTURE AROUND SUPERFLOW IN ^{3}He-A.

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Résumé.- On analyse la stabilité d'une texture de superfluide ^{3}He-A en présence d'un courant superfluide J. Se limitant aux configurations dépendantes seulement de z, on trouve trois régimes stables; deux d'entre eux montrent la briseur de la symétrie axiale où le vecteur $\hat{\ell}$ forme une hélice autour du courant.

Abstract.- We analyze the stability of textures in superfluid ^{3}He-A in the presence of a superflow J. Specializing to purely z dependent configurations, we find three regions in parameter space, which are stable. Two of them exhibit the breakdown of azimuthal symetry where $\hat{\ell}$ is no longer aligned with J_z but winds around it in a form of hélix.

For textures which depend only on z, the direction of superfluid velocity, the free energy of He-A in the dipole-locked limit can be written as

$$f = \frac{1}{2}\int dz\ \{(\rho_s - \rho_0\cos^2\beta)\ (\alpha_z + \cos\beta\ \gamma_z)^2 - 2\ c_0\ (\alpha_z + \cos\beta\ \gamma_z)\cos\beta\ \sin^2\beta\ \gamma_z + (K_b\cos^2\beta + K_t\sin^2\beta)\ \sin^2\beta\ \gamma_z^2 + (K_b\cos^2\beta + K_s\sin^2\beta)\ \beta_z^2\} \qquad (1)$$

where ρ_s, ρ_0 etc., are coefficients used by Bhattacharyya et al. /1.2/. Here $\hat{\ell}$ and $\hat{\Delta}$ are parameterized as

$$\hat{\ell} = (\sin\beta\ \cos\gamma,\ \sin\beta\ \sin\gamma,\ \cos\beta)$$
$$\hat{\Delta} = e^{-i\alpha}(-\sin\gamma - i\cos\beta\ \cos\gamma,\ \cos\gamma - i\cos\beta\ \sin\gamma,\ i\sin\beta) \qquad (2)$$

Since α is a cyclic coordinate, the z component of superflow J is completely uniform and can be used to eliminate α_z from f, by

$$\frac{\partial f}{\partial \alpha_z} = J \qquad (3)$$

with

$$g = f - J\alpha_z = \frac{1}{2}\int dz\ \{B(s)\ \beta_z^2 + G(s)\ \gamma_z^2 - A(s)^{-1}J^2 + 2JH(s)\ \gamma_z\} \qquad (4)$$

where

$$A(s) = \rho_s'' + \rho_0 s,\quad B(s) = K_b\ (1-s) + K_s s,$$
$$G(s) = [K_b\ (1-s) + K_t s - c_0^2\ s\ (1-s)\ A^{-1}]s$$
$$H(s) = (1 - c_0 s\ A^{-1})\sqrt{1-s},\quad \text{and } s = \sin^2\beta \qquad (5)$$

The dynamics of $\hat{\ell}$ is determined by the Cross-Anderson equation /3/

$$-\mu\sin^2\beta\gamma_t = (\frac{\delta g}{\delta\gamma}),\ -\mu\beta_t = \frac{\delta g}{\delta\beta} \qquad (6)$$

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where μ is the orbital viscosity.

We find that there are four types of stationary solutions (i.e. $\gamma_t = \beta_t = 0$), which satisfy equation (6);
I ($\beta = 0$), II_+ ($\beta_+ < \beta < \beta_1$), II_- ($\beta_- < \beta < \beta_1$) and III ($\beta_2 < \beta \leq \frac{\pi}{2}$). The analysis of stability against small oscillations yields (compare figure 1).

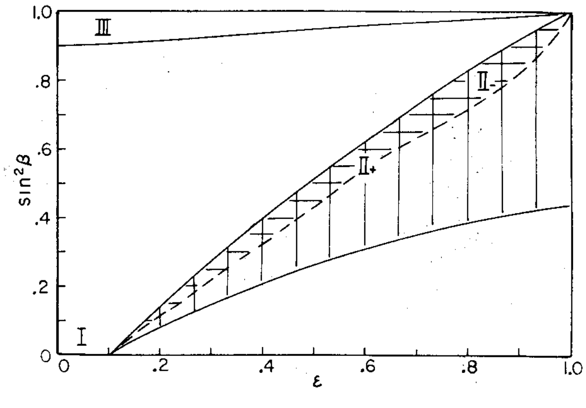

Fig. 1 : The three stable regions of the z dependent textures in the presence of a superflow are shown. The region I ($\hat{\ell}$ parallel to J) is stable only for $0 < \varepsilon < .1$. For $0.1 < \varepsilon < 1$, there are two stable regions II_+ and II_- with nonvanishing β and γ_z corresponding to helical $\hat{\ell}$ textures. Finally the region III is unstable for all ε.

1) I is stable for $c > 1$ and unstable for $c < 1$.

2) II_+ and II_- are stable only for $c < 1$ with γ_z covering limited ranges depending on β.

3) III is always unstable.

Here

$$c = K_b \rho_0 \left(\frac{1}{2}\rho_s'' + c_0\right)^{-2} \quad (7)$$

The figure displays the result of our calculation under the simplifying assumption that only ρ_0 deviates from the GL values;

$$(1-\varepsilon)^{-1}\rho_0 = \rho_s'' = c_0 = \frac{2}{5} K_{t,b,s} \quad (8)$$

The result (1) confirms earlier analyses /1,4/. The new regions II_+ and II_- correspond to the $\hat{\ell}$ vector winding around the z axis in a form of helix. The boundary curves β_1 and β_2 are given by

$$\sin^2\beta_{1,2}(\varepsilon) = \{7 + 2\varepsilon \mp (1-\varepsilon)\sqrt{17-10\varepsilon}\}(8+\varepsilon^2)^{-1} \quad (9)$$

The lower boundaries $\beta_+(\varepsilon)$ and $\beta_-(\varepsilon)$ have been calculated numerically. In the limit $c \to 1$, the stability region shrinks linearly, both $\beta_+(\varepsilon)$ and $\beta_-(\varepsilon)$, converging towards $\beta_1(\varepsilon)$. In the limit $c \to 1$, the corresponding unique value of stable $\beta(\varepsilon)$ can be calculated exactly without simplifying assumption (8) with the result

$$\sin^2\beta = (1-c)/(2D) + O(1-c)^2$$

where

$$D = \rho_0/\rho_s'' - 1 + K_t/K_b - c_0^2/(\rho_s'' K_b) - \frac{1}{4}(\rho_s'' - 4(c_0 - \rho_0))\left(\frac{1}{2}\rho_s'' + c_0\right)^{-1} \quad (10)$$

For the values (8), $D = \frac{2}{5} + O(c-1)$ implying $\sin^2\beta \simeq \frac{5}{4}(1-c)$.

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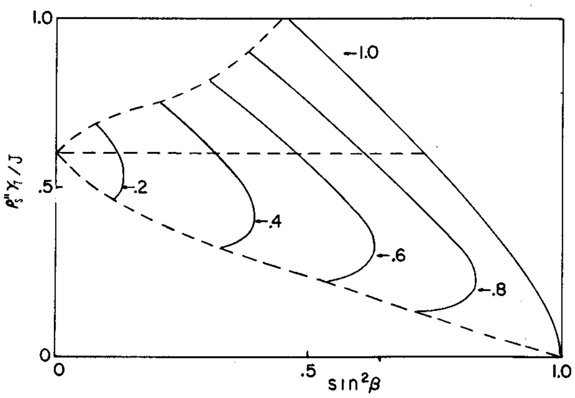

Fig. 2 : The equilibrium positions of $\frac{\rho_s'' \gamma_z}{J}$ in the stability regions II_+ and II_- are displayed as functions of $\sin^2\beta$ of different values of ε (ε = corresponds to T = Tc, ε = 1 to T = 0). Adiabatic cooling proceeds along horizontal lines of fixed $\left[\frac{\rho_s'' \gamma_z}{J}\right]$.

References

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