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Introduction

Systems containing a large number of particles exhibit a great variety of phase transitions. Most common are *first-* and *second-order transitions*. A transition is said to be first-order if the internal energy changes discontinuously at a certain temperature. Such a transition is accompanied by the release or absorption of *latent heat*. Important examples are melting and evaporation processes. Second-order transitions involve no latent heat, and the internal energy changes continuously with temperature. The derivative of the internal energy with respect to the temperature diverges at the transition temperature T_c, which is also called the *critical temperature*. The most important examples for materials undergoing second-order transitions are ferromagnets, superfluids, and superconductors.

There also exist phase transitions of higher order in which the first appearance of a divergence occurs in some higher derivative of the internal energy with respect to the temperature. A famous extreme example is the *Kosterlitz-Thouless transition* [1] of a Coulomb gas in two space dimensions. The same type of transition is also found in thin films of ^{4}He at temperatures of a few degrees Kelvin where the films become superfluid. In this transition, the internal energy may be differentiated any number of times with respect to the temperature and does not show any divergence. Instead, the temperature behavior exhibits an essential singularity of the form $e^{\text{const}\times(T-T_c)^{-1/2}}$.

The present text is devoted to a field-theoretic description of second-order transitions. Transitions of the first and higher than second order will not be considered.

1.1 Second-Order Phase Transitions

An important property of second-order phase transitions is the divergence, at the critical temperature T_c, of the length scale, over which the system behaves coherently. This is accompanied by a divergence of the size of thermal fluctuations. As a consequence, many physical observables show, near T_c, a power behavior in the temperature difference $|T - T_c|$ from the critical point, i.e., they behave like $|T - T_c|^p$. The power p is called a *critical exponent* of the associated observable.

We shall focus our attention on those physical systems whose relevant thermal fluctuations near the transition temperature can be described by some N-component *order field* $\boldsymbol{\phi}(\mathbf{x}) = (\phi_1(\mathbf{x}) \cdots \phi_N(\mathbf{x}))$. An order field is the space-dependent generalization of Landau's famous *order parameter*, which characterizes all second-order transitions in a molecular field approximation. The energy of a field configuration is described by some functional of the order field $E = E[\boldsymbol{\phi}(\mathbf{x})]$. To limit the number of possible interaction terms, certain symmetry properties will be assumed for the energy in the N-dimensional field space.

The thermal expectation value of the order field will be denoted by

$$\boldsymbol{\Phi}(\mathbf{x}) \equiv \langle \boldsymbol{\phi}(\mathbf{x}) \rangle. \tag{1.1}$$