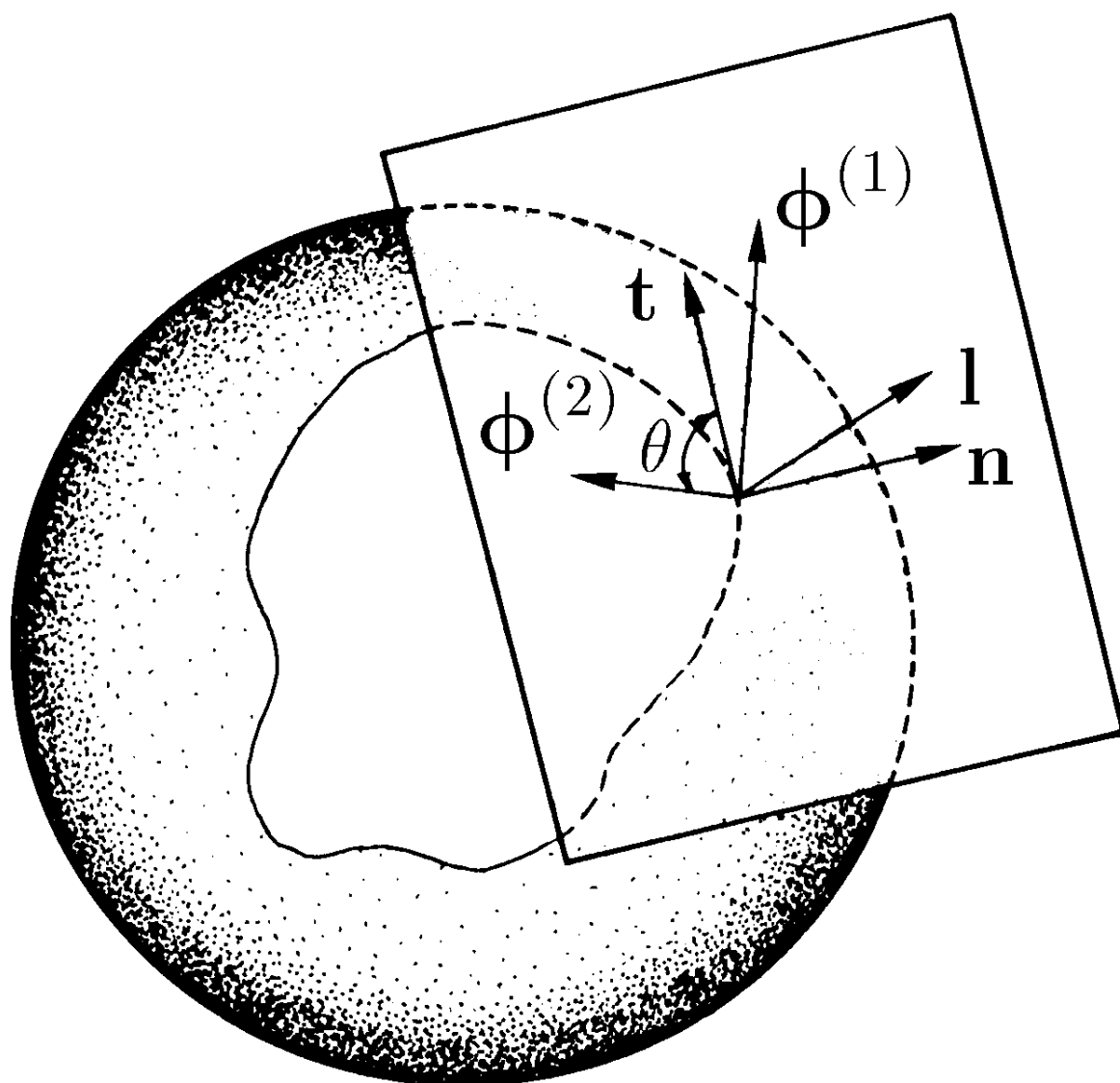


Collective Classical and Quantum Fields

in Plasmas, Superconductors, Superfluid ^3He , and Liquid Crystals

Hagen Kleinert

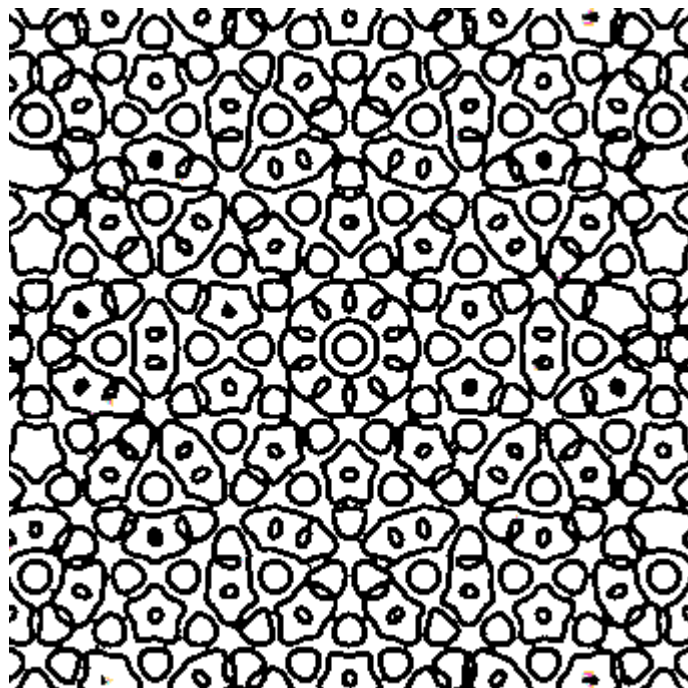


Collective Classical and Quantum Fields

in Plasmas, Superconductors,
Superfluid ^3He , and Liquid Crystals

Hagen Kleinert

*Professor of Physics
Freie Universität Berlin*



To Annemarie and Hagen II

Preface

Strongly interacting many-body systems behave often like a system of weakly interacting collective excitations. When this happens, it is theoretically advantageous to replace the original action involving the fundamental fields (electrons, nucleons, ^3He , ^4He atoms, quarks etc.) by another action in which only certain collective excitations appear as independent quantum fields. Mathematically, such replacements can be performed in many different ways without changing the physical content of the initial theory. Experimental understanding of the important processes involved can help theorists to identify the *dominant* collective excitations. If they possess only weak residual interactions, these can be treated perturbatively. The associated collective field theory greatly simplifies the approximate description of the physical system.

It is the purpose of this book to discuss some basic techniques for deriving such collective field theories. They are based on Feynman's functional integral formulation of quantum field theory. In this formulation, the transformation to collective fields amounts to mere changes of integration variables in functional integrals.

Systems of charged particles may show excitations of a type whose quanta are called *plasmons*. For their description, a real field depending on one space and one time variable is most convenient. If the particles form bound states, a complex field depending on two spacetime coordinates renders the most economic description. Such fields are *bilocal*, and are referred to as *pair fields*. If the attractive potential is of short range, the bilocal field simplifies to a local field. This has led to the field theory of superconductivity by Ginzburg and Landau. A bilocal theory of this type has been used in elementary-particle physics to explain the observable properties of strongly interacting mesons.

The change of integration variables in path integrals will be shown to correspond to an exact resummation of the perturbation series, thereby accounting for phenomena which cannot be described perturbatively in terms of fundamental particles. The path formulation has the great advantage of translating all quantum effects among the fundamental particles completely into the field language of collective excitations. All fluctuation corrections may be computed using only propagators and interaction vertices of the collective fields.

The method becomes unreliable if several collective effects compete with each other. An example is a gas of electrons and protons at low density where the attractive forces can produce hydrogen atoms. They are absent in a description involving a plasmon field. A mixture of plasmon and pair effects is needed to describe these.

Another example is superfluid ^3He , where pairing forces are necessary to produce the superfluid phase transition. Here plasma-like magnetic excitations called *paramagnons* provide strong corrections. In particular, they are necessary to obtain the pairing in the first place. If we want to tackle such mixed phenomena, another technique must be used called *variational perturbation theory*.

In Chapter 1, I explain the mathematical method of changing from one field description to another by going over to *collective fields* representing the dominant collective excitations. In Chapters 2 and 3, I illustrate this method by discussing simple systems such as an electron gas or a superconductor. At the end of Chapter 3, I had good help from my collaborator S.-S. Xue, with whom I wrote the basic strong-coupling paper ([arxiv:cond-mat/1708.04023](https://arxiv.org/abs/cond-mat/1708.04023)), that is cited as Ref. [89] on page 143. In Chapter 4, I apply the technique to superfluid ^3He . In Chapter 5, I use the field theoretic methods to study physically observable phenomena in liquid crystals. In Chapter 6, finally, I illustrate the working of the theory by treating some simple solvable models.

I want to thank my wife Dr. Annemarie Kleinert for her great patience with me while writing this book. Although her field of interest is French Literature and History (her homepage <https://a.klnrt.de>), and thus completely different from mine, her careful reading detected many errors. Without her permanent reminding me of the still missing explanations of certain questions I could never have completed this work. My son Michael, who just received his PhD in experimental physics, deserves the credit of asking many relevant questions and making me improve my sometimes too formal manuscript.

Berlin, December, 2017

H. Kleinert

Contents

1	Functional Integral Techniques	1
1.1	Nonrelativistic Fields	2
1.1.1	Quantization of Free Fields	2
1.1.2	Fluctuating Free Fields	4
1.1.3	Interactions	8
1.1.4	Normal Products	10
1.1.5	Functional Formulation	14
1.1.6	Equivalence of Functional and Operator Methods	15
1.1.7	Grand-Canonical Ensembles at Zero Temperature	16
1.2	Relativistic Fields	22
1.2.1	Lorentz and Poincaré Invariance	22
1.2.2	Relativistic Free Scalar Fields	27
1.2.3	Electromagnetic Fields	31
1.2.4	Relativistic Free Fermi Fields	34
1.2.5	Perturbation Theory of Relativistic Fields	37
	Notes and References	39
2	Plasma Oscillations	41
2.1	General Formalism	41
2.2	Physical Consequences	45
2.2.1	Zero Temperature	46
2.2.2	Short-Range Potential	47
	Appendix 2A Fluctuations around the Plasmon	48
	Notes and References	49
3	Superconductors	50
3.1	General Formulation	52
3.2	Local Interaction and Ginzburg-Landau Equations	59
3.2.1	Inclusion of Electromagnetic Fields into the Pair Field Theory	69
3.3	Far below the Critical Temperature	72
3.3.1	The Gap	73
3.3.2	The Free Pair Field	77
3.4	From BCS to Strong-Coupling Superconductivity	91
3.5	Strong-Coupling Calculation of the Pair Field	92

3.6	From BCS Superconductivity near T_c to the onset of pseudogap behavior	100
3.7	Phase Fluctuations in Two Dimensions and Kosterlitz-Thouless Transition	105
3.8	Phase Fluctuations in Three Dimensions	111
3.9	Collective Classical Fields	112
3.9.1	Superconducting Electrons	115
3.10	Strong-Coupling Limit of Pair Formation	117
3.11	Composite Bosons	122
3.12	Composite Fermions	127
3.13	Conclusion and Remarks	129
	Appendix 3A Auxiliary Strong-Coupling Calculations	131
	Appendix 3B Propagator of the Bilocal Pair Field	133
	Appendix 3C Fluctuations Around the Composite Field	135
	Notes and References	138
4	Superfluid ^3He	145
4.1	Interatomic Potential	145
4.2	Phase Diagram	147
4.3	Preparation of Functional Integral	149
4.3.1	Action of the System	149
4.3.2	Dipole Interaction	149
4.3.3	Euclidean Action	150
4.3.4	From Particles to Quasiparticles	151
4.3.5	Approximate Quasiparticle Action	152
4.3.6	Effective Interaction	155
4.3.7	Pairing Interaction	158
4.4	Transformation from Fundamental to Collective Fields	159
4.5	General Properties of a Collective Action	164
4.6	Comparison with O(3)-Symmetric Linear σ -Model	169
4.7	Hydrodynamic Properties Close to T_c	170
4.8	Bending the Superfluid ^3He -A	178
4.8.1	Monopoles	179
4.8.2	Line Singularities	182
4.8.3	Solitons	184
4.8.4	Localized Lumps	187
4.8.5	Use of Topology in the A-Phase	188
4.8.6	Topology in the B-Phase	190
4.9	Hydrodynamic Properties at All Temperatures $T \leq T_c$	193
4.9.1	Derivation of Gap Equation	194
4.9.2	Ground State Properties	199
4.9.3	Bending Energies	208
4.9.4	Fermi-Liquid Corrections	218
4.10	Large Currents and Magnetic Fields in the Ginzburg-Landau Regime	227

4.10.1	B-Phase	228
4.10.2	A-Phase	239
4.10.3	Critical Current in Other Phases for $T \sim T_c$	240
4.11	Is $^3\text{He-A}$ a Superfluid?	248
4.11.1	Magnetic Field and Transition between A- and B-Phases	272
4.12	Large Currents at Any Temperature $T \leq T_c$	274
4.12.1	Energy at Nonzero Velocities	274
4.12.2	Gap Equations	275
4.12.3	Superfluid Densities and Currents	283
4.12.4	Critical Currents	285
4.12.5	Ground State Energy at Large Velocities	289
4.12.6	Fermi Liquid Corrections	289
4.13	Collective Modes in the Presence of Current at all Temperatures $T \leq T_c$	292
4.13.1	Quadratic Fluctuations	292
4.13.2	Time-Dependent Fluctuations at Infinite Wavelength	295
4.13.3	Normal Modes	298
4.13.4	Simple Limiting Results at Zero Gap Deformation	301
4.13.5	Static Stability	303
4.14	Fluctuation Coefficients	304
4.15	Stability of Superflow in the B-Phase under Small Fluctuations for $T \sim T_c$	307
Appendix 4A	Hydrodynamic Coefficients for $T \approx T_c$	312
Appendix 4B	Hydrodynamic Coefficients for All $T \leq T_c$	315
Appendix 4C	Generalized Ginzburg-Landau Energy	319
	Notes and References	319
5	Liquid Crystals	323
5.1	Maier-Saupe Model and Generalizations	324
5.1.1	General Properties	324
5.1.2	Landau Expansion	326
5.1.3	Tensor Form of Landau-de Gennes Expansion	327
5.2	Landau-de Gennes Description of Nematic Phase	328
5.3	Bending Energy	336
5.4	Light Scattering	338
5.5	Interfacial Tension between Nematic and Isotropic Phases	347
5.6	Cholesteric Liquid Crystals	351
5.6.1	Small Fluctuations above T_1	354
5.6.2	Some Experimental Facts	355
5.6.3	Mean-Field Description of Cholesteric Phase	357
5.7	Other Phases	362
Appendix 5A	Biaxial Maier-Saupe Model	365
	Notes and References	368

6	Exactly Solvable Field-Theoretic Models	371
6.1	Pet Model in Zero Plus One Time Dimensions	371
6.1.1	The Generalized BCS Model in a Degenerate Shell	379
6.1.2	The Hilbert Space of the Generalized BCS Model	390
6.2	Thirring Model in 1+1 Dimensions	393
6.3	Supersymmetry in Nuclear Physics	397
	Notes and References	397
	Index	399

List of Figures

1.1	Contour C in the complex z -plane	19
2.1	The pure current piece of the collective action	43
2.2	The non-polynomial self-interaction terms of plasmons	44
2.3	Free plasmon propagator	44
3.1	Time evolution of critical temperatures of superconductivity	51
3.2	Fundamental particles entering any diagram	54
3.3	Free pair field following the Bethe-Salpeter equation	56
3.4	Free pair propagator	58
3.5	Self-interaction terms of the non-polynomial pair Lagrangian	59
3.6	Free part of pair field Δ Lagrangian	62
3.7	Energy gap of a superconductor as a function of temperature	76
3.8	Temperature behavior of the superfluid density ρ_s/ρ (Yoshida function) and the gap function $\bar{\rho}_s/\rho$	87
3.9	Temperature behavior of the inverse square coherence length $\xi^{-2}(T)$	88
3.10	Gap function Δ and chemical potential μ at zero temperature as functions of the crossover parameter $\hat{\mu}$	96
3.11	Temperature dependence of the gap function in three (a) and two (b) dimensions	97
3.12	Dependence of T^* on the crossover parameter in three (a) and two (b) dimensions	98
3.13	Dependence of the pair-formation temperature T^* on the chemical potential	109
3.14	Qualitative phase diagram of the BCS-BEC crossover as a function of temperature T/ϵ_F and coupling $1/k_F a$	118
3.15	Qualitative phase diagram in the unitarity limit	126
4.1	Interatomic potential between ^3He atoms as a function of the distance r	145
4.2	Imaginary part of the susceptibility caused by repeated exchange of spin fluctuations	146
4.3	Phase diagram of ^3He plotted against temperature, pressure, and magnetic field	148
4.4	Three fundamental planar textures, splay, bend, and twist of the director field in liquid crystals	172

4.5	Sphere with one, two, or no handles and their Euler characteristics	176
4.6	Local tangential coordinate system $\mathbf{n}, \mathbf{t}, \mathbf{i}$ for an arbitrary curve on the surface of a sphere	176
4.7	The $\mathbf{l} \parallel \mathbf{d}$ -field lines in a spherical container	179
4.8	Two possible parametrizations of a sphere	180
4.9	Spectra of Goldstone bosons versus gauge bosons	181
4.10	Cylindrical container with the $\mathbf{l} \parallel \mathbf{d}$ -field lines spreading outwards when moving upwards	183
4.11	Field vectors in a composite soliton	186
4.12	Nuclear magnetic resonance frequencies of a superfluid $^3\text{He-A}$ sample in an external magnetic field	187
4.13	Vectors of orbital and spin orientation in the A-phase of superfluid ^3He	188
4.14	Parameter space of $^3\text{He-B}$ containing the parameter space of the rotation group	192
4.15	Possible path followed by the order parameter in a planar texture (soliton) when going from $z = -\infty$ to $z = +\infty$	193
4.16	Another possible class of solitons	193
4.17	Fundamental hydrodynamic quantities of superfluid $^3\text{He-B}$ and -A, shown as a function of temperature	198
4.18	Condensation energies of A- and B-phases as functions of the temperature	203
4.19	The temperature behavior of the condensation entropies in B- and A-phases	205
4.20	Specific heat of A- and B-phases as a function of temperature	207
4.21	Temperature behavior of the reduced superfluid densities in the B- and in the A-phase of superfluid ^4He	211
4.22	Superfluid stiffness functions K_t, K_b, K_s of the A-phase as functions of the temperature	218
4.23	Superfluid densities of B- and A-phase after applying Fermi liquid corrections	224
4.24	Coefficients $c = c^{\parallel}$ and their Fermi liquid corrected values in the A-phase as a function of temperature	225
4.25	Coefficient K_s for splay deformations of the fields, and its Fermi liquid corrected values in the A-phase as a function of temperature	226
4.26	Remaining hydrodynamic parameters for twist and bend deformations of superfluid $^3\text{He-A}$	227
4.27	Hydrodynamic parameters of superfluid $^3\text{He-B}$ together with their Fermi liquid corrected values, as functions of the temperature	228
4.28	Shape of potential determining stability of superflow	231
4.29	Superflow in a torus	248
4.30	In the presence of a superflow in $^3\text{He-A}$, the \mathbf{l} -vector is attracted to the direction of flow	254

4.31	Doubly connected parameter space of the rotation group corresponding to integer and half-integer spin representations	254
4.32	Helical texture in the presence of a supercurrent	255
4.33	Three different regions of equilibrium configurations of the texture at $H = 0$ (schematically)	257
4.34	Pitch values for stationary helical solutions as a function of the angle of inclination β_0	260
4.35	Regions of stable helical texture, II- and II+	262
4.36	Regions of stable helical texture (shaded areas)	263
4.37	Shrinking of the regions of stability when dipole locking is relaxed	265
4.38	As a stable helix forms in the presence of superflow in $^3\text{He-A}$	266
4.39	Angle of inclination as a function of the magnetic field at different temperatures	267
4.40	Sound attenuation parametrized in terms of three constants	269
4.41	Velocity dependence of the gap in the A- and B-phases	281
4.42	Current as a function of velocity	292
4.43	Collective frequencies of B-phase in the presence of superflow of velocity v	303
5.1	Molecular structure of PAA	323
5.2	Graphical solution of the gap equation	326
5.3	Phase diagram of general Landau expansion in the (a_3, a_2) -plane	330
5.4	Biaxial regime in the phase diagram of general Landau expansion of free energy in the (a_3, a_2) -plane	331
5.5	Jump of the order parameter φ from zero to a nonzero value $\varphi_>$ in a first-order phase transition at $T = T_1$	333
5.6	Different configurations of textures in liquid crystals	337
5.7	Experimental setup of the light-scattering experiment	343
5.8	Inverse light intensities as a function of temperature	345
5.9	Behavior of coherence length as a function of temperature	346
5.10	Relevant vectors of the director fluctuation	347
5.11	Contour plots of constant reduced free energy \tilde{f}_{ext}	354
5.12	Momentum dependence of the gradient coefficients	355
5.13	Momenta and polarization vectors of a body-centered cubic phase of a cholesteric liquid crystal	363
5.14	Regimes in the plane of α, τ , where the phases, cholesteric, hexatic, or bcc are lowest	363
5.15	Momenta and polarization vectors for an icosahedral phase of a cholesteric liquid crystal	364
5.16	Density profile with five-fold symmetry	364
5.17	Density profile with seven-fold symmetry	365
5.18	Blue phases in a cholesteric liquid crystal	365

6.1	Level scheme of the BCS model in a single degenerate shell of multiplicity $\Omega = 8$	381
-----	---	-----

List of Tables

4.1	A factor of roughly 1000 separates the characteristic length scales of superconductors and ^3He	148
4.2	Pressure dependence of Landau parameters F_1 , F_0 , and F_0^S of ^3He together with the molar volume v and the effective mass ratio m^*/m	153
4.3	Parameters of the critical currents in all theoretically known phases	243