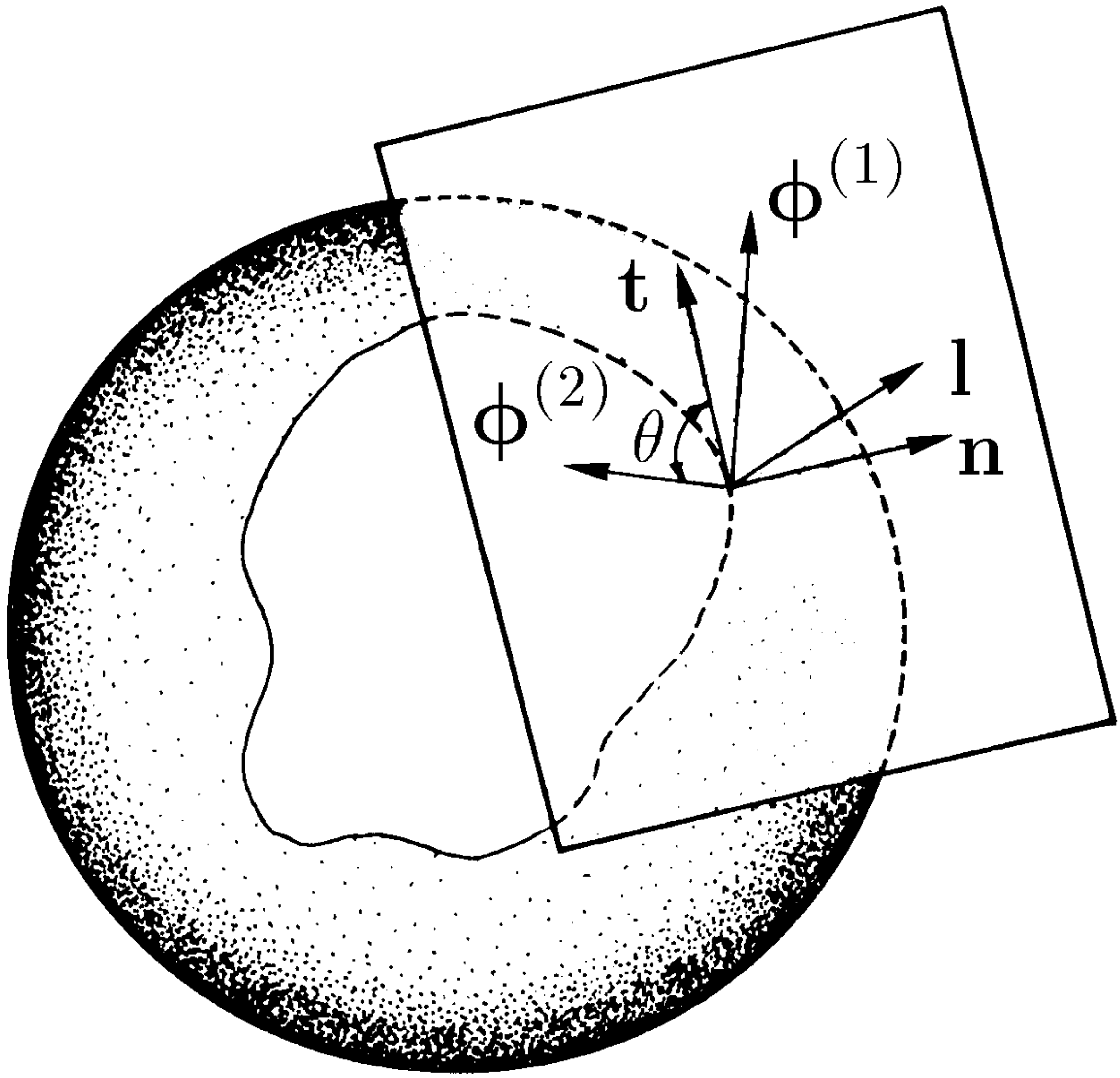
Collective Classical and Quantum Fields

in Plasmas, Superconductors, Superfluid ^{3}He, and Liquid Crystals

Hagen Kleinert



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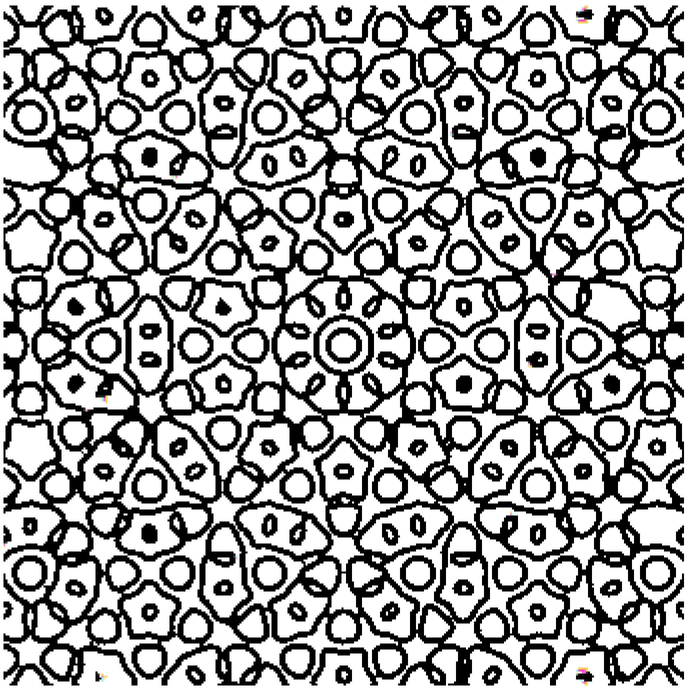

To Annemarie and Hagen II

Preface

Strongly interacting many-body systems behave often like a system of weakly interacting collective excitations. When this happens, it is theoretically advantageous to replace the original action involving the fundamental fields (electrons, nucleons, ^{3}He, ^{4}He atoms, quarks etc.) by another action in which only certain collective excitations appear as independent quantum fields. Mathematically, such replacements can be performed in many different ways without changing the physical content of the initial theory. Experimental understanding of the important processes involved can help theorists to identify the *dominant* collective excitations. If they possess only weak residual interactions, these can be treated perturbatively. The associated collective field theory greatly simplifies the approximate description of the physical system.

It is the purpose of this book to discuss some basic techniques for deriving such collective field theories. They are based on Feynman's functional integral formulation of quantum field theory. In this formulation, the transformation to collective fields amounts to mere changes of integration variables in functional integrals.

Systems of charged particles may show excitations of a type whose quanta are called *plasmons*. For their description, a real field depending on one space and one time variable is most convenient. If the particles form bound states, a complex field depending on two spacetime coordinates renders the most economic description. Such fields are *bilocal*, and are referred to as *pair fields*. If the attractive potential is of short range, the bilocal field simplifies to a local field. This has led to the field theory of superconductivity by Ginzburg and Landau. A bilocal theory of this type has been used in elementary-particle physics to explain the observable properties of strongly interacting mesons.

The change of integration variables in path integrals will be shown to correspond to an exact resummation of the perturbation series, thereby accounting for phenomena which cannot be described perturbatively in terms of fundamental particles. The path formulation has the great advantage of translating all quantum effects among the fundamental particles completely into the field language of collective excitations. All fluctuation corrections may be computed using only propagators and interaction vertices of the collective fields.

The method becomes unreliable if several collective effects compete with each other. An example is a gas of electrons and protons at low density where the attractive forces can produce hydrogen atoms. They are absent in a description involving a plasmon field. A mixture of plasmon and pair effects is needed to describe these.

Another example is superfluid ^{3}He, where pairing forces are necessary to produce the superfluid phase transition. Here plasma-like magnetic excitations called *paramagnons* provide strong corrections. In particular, they are necessary to obtain the pairing in the first place. If we want to tackle such mixed phenomena, another technique must be used called *variational perturbation theory*.

In Chapter 1, I explain the mathematical method of changing from one field description to another by going over to *collective fields* representing the dominant collective excitations. In Chapters 2 and 3, I illustrate this method by discussing simple systems such as an electron gas or a superconductor. At the end of Chapter 3, I had good help from my collaborator S.-S. Xue, with whom I wrote the basic strong-coupling paper (`arxiv:cond-mat/1708.04023`), that is cited as Ref. [89] on page 143. In Chapter 4, I apply the technique to superfluid ^{3}He. In Chapter 5, I use the field theoretic methods to study physically observable phenomena in liquid crystals. In Chapter 6, finally, I illustrate the working of the theory by treating some simple solvable models.

I want to thank my wife Dr. Annemarie Kleinert for her great patience with me while writing this book. Although her field of interest is French Literature and History (her homepage https://a.klnrt.de), and thus completely different from mine, her careful reading detected many errors. Without her permanent reminding me of the still missing explanations of certain questions I could never have completed this work. My son Michael, who just received his PhD in experimental physics, deserves the credit of asking many relevant questions and making me improve my sometimes too formal manuscript.

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H. Kleinert

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