Disorder Version of the Abelian Higgs Model and the Order of the Superconductive Phase Transition.

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Summary. We transform the Abelian Higgs model in three dimensions (*i.e.* the Ginzburg-Landau theory) to a disorder field theory. The new fields describe the grand canonical ensemble of vortex lines in the superconductor and take a nonzero expectation in the normal state (signalizing their proliferation). The disorder theory allows for a simple determination of the tricritical point where the second-order phase transition changes to first. This happens for a Ginzburg-Landau parameter (= ratio of penetration depth to coherence length) $K < 0.8/\sqrt{2}$.

Some years ago, stimulated by a field-theoretic discussion in 4 dimensions (1), it was suggested that the superconductive phase transition should be weakly first order (2). There were two arguments: First, due to the Meissner effect, the magnetic-field fluctuations are massive with a mass m proportional to the size of the order parameter, $m \propto |\varphi|$. The associated black-body radiation energy has a mass dependence $am^2 - bm^3$. But the cubic term $|\varphi|^3$ should lead to a first-order transition. This simple conclusion was apparently confirmed by the second argument that, at the one-loop level of a $4 - \varepsilon$ expansion, there exists no infra-red stable fixed point, unless the complex-order parameter is artificially extended to a n-vector with $n > 365$ (2).

That something must be wrong with such reasoning became clear when the same conclusions, derived from the De Gennes theory for the smectic A to nematic phase transition, was disproven experimentally (3,4). Theoretically, trouble is certainly expected for deep type-II superconductors, for it is easy to see that the first-order jump occurs in the regime which, according to the Ginzburg criterion, is characterized by strong $|\varphi|$ fluctuations (5). Thus the black-body formula derived for constant $|\varphi| \propto m$ is no longer applicable. The $4 - \varepsilon$ argument is dubious for related reasons: In a type-II

(1) S. COLEMAN and E. WEINBERG: *Phys. Rev. D*, **7**, 1888 (1973).
(2) B. I. HALPERIN, T. C. LUBENSKY and S. K. MA: *Phys. Rev. Lett.*, **32**, 292 (1974).
(3) B. I. HALPERIN and T. C. LUBENSKY: *Solid State Commun.*, **14**, 997 (1974).
(4) For a review, see J. D. LITSTER, R. J. BIRGENEAU, M. KAPLAN, C. P. SATINYA and J. ALS-NIELSEN, in *Ordering in Strongly Fluctuating Condensed Matter Systems*, edited by T. RISTE (New York, N. Y., 1980).
(5) V. L. GINZBURG: *Fiz. Twerd. Tela*, **2**, 2031 (1960) (*Sov. Phys. Solid State*, **2**, 1824 (1961)).

superconductor, vortex fluctuations are expected to produce nonperturbative terms in the β-functions $\beta(g, e^2)$, whose imaginary part behaves as $\exp[-1/|g|]$ for $g < 0$. These escape the power series, but become important when continuing to such large a value as $\varepsilon = 1$.

A clarification of the situation is particularly important as far as recently developed gauge theories of defect-mediated phase transitions are concerned (6). They all have a Ginzburg-Landau form, but some are of first order (dislocation melting (7)) others of second order (vortex-induced λ-transition in superfluid ^{4}He (8), spiral to disordered transition in magnetic superconductors (9) and pion condensates (10), dislocation-line–induced smectic-A–to–nematic phase transition (11)). Thus the Ginzburg-Landau description should be able to accommodate both orders of transition.

A free parameter for doing this is available: It is the parameter K which denotes the ratio of penetration depth $\xi_{\rm mg}$ of magnetic field and $\sqrt{2}$ times the coherence length ξ of the $|\varphi|$ fluctuations in the $|\varphi| \neq 0$ phase. When this ratio is $\gtrless 1/\sqrt{2}$ (*i.e.* $\xi_{\rm mg} \gtrless \xi$) one speaks of type-I and type-II superconductors, respectively. These have quite distinct properties as far as parallel vortex lines are concerned which repell and attract each other in type II and I, respectively.

In this note we demonstrate that there does indeed exist a point $K_{1,2} \approx 0.55 \approx \approx 0.77/\sqrt{2}$, *i.e.* slightly within the type-I regime, where the second-order transition changes to first. We shall first establish an identity between a XY model coupled to a massive vector field W (to be called XYW model) and a Ginzburg-Landau type of theory of *any* K. After this, we shall transform the XYW model to a $|\psi|^4$ field theory with a Landau expansion of the type

$$f \approx \frac{1}{24}|\partial\psi|^2 - \frac{9}{8\pi^2}(T - T_c)|\psi|^2 + \frac{1}{64}\frac{3}{2}\sqrt{3}\left(\frac{3}{\pi}\right)^3 (K - K_{1,2})|\psi|^4 + \dots$$

for $T \sim T_c$ and $K \sim K_{1,2}$.

Then it is obvious that for $K < K_{1,2}$ the transition becomes first order (as can be checked by a simple mean-field calculation).

Physically, the ψ-fields play the role of vortex lines of the Ginzburg-Landau theory. Conversely, the complex fields of the latter can be interpreted as the vortex lines of the XYW model. The vector potential generates the long-range « elastic » interaction between these.

Our starting point is the XYW model on a simple cubic lattice with sites $\boldsymbol{x}$ and oriented links $\boldsymbol{i} = a(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}})$ for $i = 1, 2, 3$:

$$(1) \qquad Z_{XYW} = \prod_{\boldsymbol{x},i} \int_{-\infty}^{\infty} \sqrt{\frac{2\pi}{t^2 T}}\, \mathrm{d}W_i(\boldsymbol{x}) \prod_{\boldsymbol{x}} \int_{-\pi}^{\pi} \frac{\mathrm{d}\gamma(\boldsymbol{x})}{2\pi} \cdot$$

$$\cdot \exp\left[\sum_{\boldsymbol{x},i}\left[\frac{1}{t}\cos\left(\nabla_i\gamma(\boldsymbol{x}) - (\boldsymbol{\nabla}\times\boldsymbol{W})_i\right) + \frac{(2\pi)^2}{2t^2 T} W_i^2(\boldsymbol{x}) + \frac{1}{2t}(\boldsymbol{\nabla}\times\boldsymbol{W})_i^2\right]\right],$$

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(6) H. KLEINERT: *Gauge Theory of Defects* (New York, N. Y., 1983).
(7) H. KLEINERT: *Phys. Lett. A*, **89**, 294 (1982); *Lett. Nuovo Cimento* **34**, 464 (1982); **34**, 471 (1982).
(8) H. KLEINERT: *Phys. Lett. A*, in press.
(9) H. KLEINERT: *Phys. Lett. A*, **90**, 259 (1982).
(10) H. KLEINERT: *Lett. Nuovo Cimento*, **34**, 103 (1982).
(11) H. KLEINERT: *J. Phys. (Paris)* (in press).

where the lattice curl is defined as $(\nabla\times\boldsymbol{W})_i \equiv \varepsilon_{ijk}\overset{*}{\nabla}_j W_k(\boldsymbol{x}-k)$ with $\nabla_i\gamma(\boldsymbol{x}) \equiv \gamma(\boldsymbol{x}+\boldsymbol{i})-\gamma(\boldsymbol{x})$, $\overset{*}{\nabla}_i\gamma(\boldsymbol{x}) = \gamma(\boldsymbol{x})-\gamma(\boldsymbol{x}-\boldsymbol{i})$, being the usual lattice derivatives, and $T<0$ and $t>0$ are two parameters. Using a Villain approximation ([12]) this can be rewritten as

$$(2)\qquad Z_{XYW}\sim Z_{XYVW} \equiv \prod_{\boldsymbol{x},i}\int_{-\infty}^{\infty}\sqrt{\frac{2\pi}{t^2T}}\,\mathrm{d}W_i(\boldsymbol{x})\prod_{\boldsymbol{x}}\int_{-\pi}^{\pi}\frac{\mathrm{d}\gamma(\boldsymbol{x})}{2\pi}\cdot$$
$$\cdot\sum_{\{n_i(\boldsymbol{x})\}}\exp\left[-\frac{1}{2t}\sum_{\boldsymbol{x}}(\nabla_i\gamma(\boldsymbol{x})-(\nabla\times\boldsymbol{W})_i-2\pi n_i(\boldsymbol{x}))^2\right]\exp\left[\frac{(2\pi)^2}{2t^2T}\sum_{\boldsymbol{x}}W_i^2+\frac{1}{2t}\sum_{\boldsymbol{x}}(\nabla\times\boldsymbol{W})_i^2\right].$$

An auxiliary vector field may be used to bring the first part of the exponent to the form

$$\exp\left[-\frac{t}{2}\sum_{\boldsymbol{x}}b_i(\boldsymbol{x})^2+i\sum_{\boldsymbol{x}}b_i(\boldsymbol{x})(\nabla_i\gamma(\boldsymbol{x})-(\nabla\times\boldsymbol{W})_i-2\pi n_i(\boldsymbol{x}))\right]$$

with an integral over all $b_i(\boldsymbol{x})$'s ensuring that (2) has not been changed. The sum over all $n_i(\boldsymbol{x})$ can then be executed squeezing the $b_i(\boldsymbol{x})$-integrals into a sum over-integer-valued $b_i(\boldsymbol{x})$, whereupon the integrals over $\gamma(\boldsymbol{x})$'s enforce a vanishing lattice divergence of $b_i(\boldsymbol{x})$, *i.e.* $\overset{*}{\nabla}_i b_i(\boldsymbol{x})=0$. Thus (2) becomes

$$(3)\qquad Z_{XYVW}=\prod_{\boldsymbol{x},i}\int_{-\infty}^{\infty}\sqrt{\frac{2\pi}{t^2T}}\,\mathrm{d}W_i(\boldsymbol{x})\sqrt{\frac{t}{2\pi}}\sum_{\{b_i(\boldsymbol{x})\}}\delta_{\overset{*}{\nabla}_i b_i,0}\cdot$$
$$\cdot\exp\left[-\frac{t}{2}\sum_{\boldsymbol{x}}b_i^2-i\sum_{\boldsymbol{x}}b_i(\nabla\times\boldsymbol{W})_i+\frac{(2\pi)^2}{2t^2T}\sum_{\boldsymbol{x}}W_i^2+\frac{1}{2t}\sum_{\boldsymbol{x}}(\nabla\times\boldsymbol{W})_i^2\right].$$

The constraint $\overset{*}{\nabla}_i b_i=0$ is trivially enforced by introducing an integer-valued vector potential $a_i(\boldsymbol{x})$ with $b_i\equiv(\nabla\times\boldsymbol{a})_i$, a decomposition which is invariant under the gauge transformations $a_i(\boldsymbol{x})\to a_i(\boldsymbol{x})+\nabla_i A(\boldsymbol{x})$. Moreover, the sums over the fields a_i can be turned into integrals via another auxiliary integer vector field $l_i(\boldsymbol{x})$ and (2) takes the form

$$(4)\qquad Z_{XYVW}=\prod_{\boldsymbol{x},i}\left[\int_{-\infty}^{\infty}\sqrt{\frac{2\pi}{t^2T}}\,\mathrm{d}W_i(\boldsymbol{x})\int_{-\infty}^{\infty}\frac{\mathrm{d}A_i(\boldsymbol{x})}{\sqrt{2\pi/t}}\right]\sum_{\{l_i(\boldsymbol{x})\}}\delta_{\overset{*}{\nabla}_i l_i,0}\cdot$$
$$\cdot\exp\left[-\frac{t}{2}\sum_{\boldsymbol{x}}(\nabla\times\boldsymbol{A})_i^2-i\sum_{\boldsymbol{x}}(\nabla\times\boldsymbol{A})_i+2\pi i\sum_{\boldsymbol{x}}l_iA_i+\frac{(2\pi)^2}{2t^2T}\sum_{\boldsymbol{x}}W_i^2+\frac{1}{2t}(\nabla\times\boldsymbol{W})_i^2\right].$$

The divergenceless condition on $l_i(\boldsymbol{x})$ is required by gauge invariance. Changing the field variables A_i to $A_i'=A_i+(i/t)W_i$ and dropping the primes, the exponent becomes

$$(5)\qquad -\frac{t}{2}\sum_{\boldsymbol{x}}(\nabla\times\boldsymbol{A})_i^2+2\pi i\sum_{\boldsymbol{x}}l_iA_i+\frac{(2\pi)^2}{2t^2T}\sum_{\boldsymbol{x}}W_i^2+\frac{2\pi}{t}\sum_{\boldsymbol{x}}l_iW_i\,.$$

([12]) J. Villain: *J. Phys. (Paris)*, **36**, 581 (1975).

Integrating out the W_i's and changing $A_i \to (1/\sqrt{t})A_i$ leaves

$$(6)\qquad Z_{XYVW} = \prod_{x,i} \int \frac{dA_i}{\sqrt{2\pi}} \sum_{\{l_i(x)\}} \delta_{\overset{*}{\nabla}_i l_i,0} \exp\left[-\frac{1}{2}\sum_x (\nabla \times A)_i^2 + \frac{2\pi i}{\sqrt{t}}\sum_x l_i A_i - \frac{T}{2}\sum_x l_i^2\right].$$

At this point it is preferable to use A fields which have no expectation $\langle A_i^2(x)\rangle$. This is achieved by introducing a subtracted Coulomb potential $v'(x) = v(x) - v(0)\,\delta_{x,0}$ with $v(0) \sim 0.253$, and denoting by $(\nabla' \times A)^2$ the gradient energy which leads to $v'(x)$ (*). Then (6) takes the form

$$(7)\qquad Z_{\mathrm{LSC}} = \prod_{x,i} \int \frac{dA_i(x)}{\sqrt{2\pi}} \sum_{\{l_i(x)\}} \delta_{\nabla_i l_i(x),0} \cdot \exp\left[-\frac{1}{2}\sum_x (\nabla' \times A)_i^2 + ie\sum_x l_i A_i - \frac{T + e^2 v(0)}{2}\sum_x l_i^2\right],$$

It is well known [13] that this partition function can also be seen as a Villain approximation to a lattice superconductor of charge $e^2 = (2\pi)^2/t$ and temperature $T' = T + e^2 v(0) > 0$:

$$(8)\qquad Z_{\mathrm{LSC}} = \prod_{x,i} \int_{-\infty}^{\infty} \frac{dA_i(x)}{\sqrt{2\pi}} \prod_x \int_{-\pi}^{\pi} \frac{d\theta(x)}{2\pi} \exp \frac{1}{T'} \sum_{x,i} \cos(\nabla_i \theta(x) - eA_i) - \sum_x \frac{1}{2}(\nabla' \times A)_i^2 .$$

The proof involves the same steps as from (1) to (3) (with l_i instead of b_i).

The equality of (1) and (8), up to Villain approximations, form the basis of our discussions.

First, we observe that both XY model parts can be transformed into a complex field theory with fields $\varphi = \varphi_1 + i\varphi_2$, using the identity valid in D dimensions [8]:

$$(9)\qquad \prod_x \int_{-\pi}^{\pi} \frac{d\theta(x)}{2\pi} \exp \frac{1}{T} \sum_{x,i} \cos(\nabla_i \theta)\Bigg] = \prod_x \iint_{-\infty}^{\infty} \frac{d\varphi_1\, d\varphi_2(x)}{\sqrt{4\pi D/t}} \exp\left[-\left(\frac{T}{4D}\sum_x \varphi_a^2 - \sum \log I_0(\sqrt{\tilde{\varphi}_a^2})\right)\right],$$

where $\tilde{\varphi}_a = [1 + \overset{*}{\nabla}_i \nabla_i / 2D]^{\frac{1}{2}} \varphi_a$ with $a = 1, 2$ and $I_0(z)$ is the modified Bessel function. This follows directly from rewriting

$$(10)\qquad \sum_{x,i} \cos(\nabla_i \theta(x)) = D\sum_x (\cos\theta(x), \sin\theta(x))(1 + \overset{*}{\nabla}_i \nabla_i / 2D)(\cos\theta(x), \sin\theta(x))$$

(*) This amounts to replacing $\overset{*}{\nabla}_i \nabla_i$ by $\overset{*}{\nabla}_i \nabla_i / (1 + v(0)\overset{*}{\nabla}_i \nabla_i)$.

[13] M. E. PESKIN: *Ann. Phys.*, **113**, 122 (1978).

performing a quadratic completion

$$-\frac{T}{4D}\sum_{x}\varphi_a^2+\sum_{x}\left(\tilde{\varphi}_1\cos\theta(\boldsymbol{x})+\tilde{\varphi}_2\sin\theta(x)\right)$$

and integrating out the $\theta(\boldsymbol{x})$ variables. It is now easy to see that the field $eA_i(\boldsymbol{x})$ can be coupled to $\theta(\boldsymbol{x})$ via the gauge-invariant replacement

$$(11)\qquad \begin{cases}\nabla_i\varphi(\boldsymbol{x})\to D_i\varphi(\boldsymbol{x})=\varphi(\boldsymbol{x}+\boldsymbol{i})\exp[-ieA_i(\boldsymbol{x})]-\varphi(\boldsymbol{x}),\\ \overset{*}{\nabla}_i\varphi(\boldsymbol{x})\to D_i^*\varphi(\boldsymbol{x})\equiv\varphi(\boldsymbol{x})-\varphi(\boldsymbol{x}-\boldsymbol{i})\exp[ieA_i(\boldsymbol{x}-\boldsymbol{i})].\end{cases}$$

In this way we arrive at the following exponent for the partition function of the lattice superconductor (8):

$$(12)\qquad S_{\text{LSC}}[\varphi,\varphi^+,\boldsymbol{A}]=-\tfrac{1}{2}\sum_{x}(\boldsymbol{\nabla}'\times\boldsymbol{A})_i^2+S^{eA}[\varphi,\varphi^+]$$

where for $D=3$

$$(13)\qquad -S^{eA}[\varphi,\varphi^+]=\sum_{x}\left(\frac{T+e^2v(0)}{12}|\varphi|^2-\log I_0\left(\sqrt{\tilde{\varphi}_a^2}\right)\right).$$

The fluctuation integrals over A and φ are obvious and need not be written down. In the critical regime, this has the Landau expansion

$$(14)\qquad -S^{eA}\sim\sum_{x}\left(\frac{T+e^2v(0)-3}{12}|\varphi|^2+\frac{1}{64}|\varphi|^4+\frac{1}{24}|D_i\varphi|^2\right),$$

where for small enough e $D_i\sim\nabla_i-ieA_i$ such that (12) becomes a standard Ginzburg-Landau expression. It is now trivial to identify the parameter $K=(1/\sqrt{2})\xi_{\text{mg}}/\xi$ as

$$(15)\qquad K=\frac{3}{e}=3\sqrt{t}/2\pi,$$

which at $1/\sqrt{2}$ separates type-I and type-II superconductivity. The phase transition at the mean field level can be read off (14) to follow a straight line $e^2=(1/v(0))(3-T)$ which is shown on fig. 1 as a dotted line and agrees roughly with Monte Carlo calculations ([14]) for $T\geqslant 0$. For $T\geqslant 0$, $K_0\leqslant 1.17/\sqrt{2}$, where (14) can be trusted to lie in the type-II regime.

A similar treatment can now be given to the partition function (1). This leads to an exponent

$$(16)\qquad S_{XYVW}[\psi,\psi^+,\boldsymbol{W}]=\frac{(2\pi)^2}{2t^2T}\sum_{x}W_i^2+\frac{1}{2t}\sum_{x}(\boldsymbol{\nabla}\times\boldsymbol{W})_i^2+S^{\boldsymbol{\nabla}\times\boldsymbol{W}}[\psi,\psi^+]$$

with

$$(17)\qquad -S^{\Delta\times\boldsymbol{W}}[\psi,\psi^+]=\sum_{x}\left(\frac{t}{12}|\psi|^2-\log I(\tilde{\varphi}_a^2)\right),$$

([14]) C. Dasgupta and B. I. Halperin: *Phys. Rev. Lett.*, **21**, 1556 (1981).

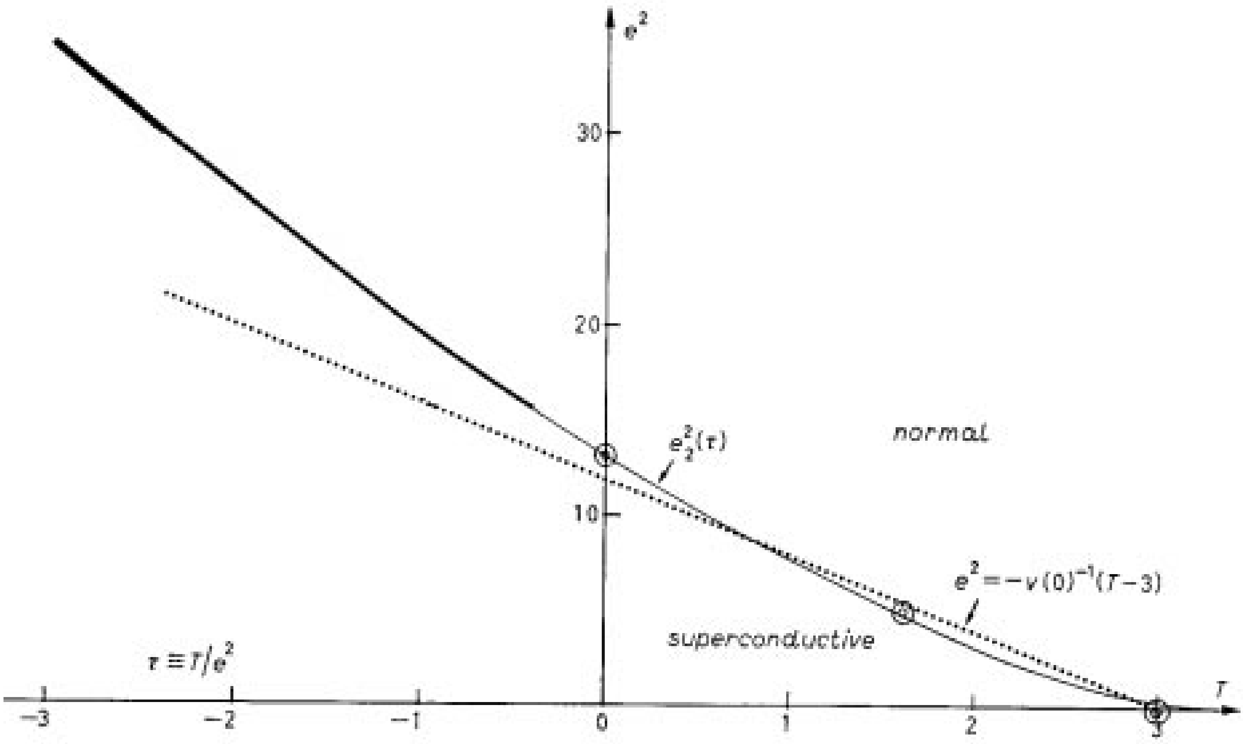

Fig. 1. – The phase diagram of the lattice superconductor as calculated in this work. The three circles are Monte Carlo data ([14]). The dotted line gives the transition which would follow at the mean field level from the Ginzburg-Landau theory directly, without going to the disorder version.

which has a Landau expansion

$$-S^{\nabla\times W}\sim\sum_x\left(\frac{t-3}{12}|\psi|^2+\frac{1}{64}|\psi|^4+\frac{1}{24}|D_i\psi|^2\right), \tag{18}$$

where for small T $D_i\sim\nabla_i-i(\nabla\times W)_i$.

For small negative T, the W fluctuations are very massive and small and can be integrated out, giving a pure ψ field theory $S^0[\psi,\psi^+]$ with an additional black-body term

$$-\Delta S_0[\psi,\psi^+]=\sum_k\log\left[1+\frac{T}{e^2}K^2\left(1-\frac{t}{12}|\psi|^2\right)\right] \tag{19}$$

with $K^2=\sum_i 2(1-\cos k_i)\leqslant 12$. For $-T/e^2<1/12$, this can be expanded in powers of $|\psi|^2$. The only relevant terms are

$$-\sum_k\frac{K^2}{1+(T/e^2)K^2}\frac{tT}{12e^2}|\psi|^2-\frac{1}{2}\sum_k\frac{K^4}{(1+(T/e^2)K^2)^2}\left(\frac{tT}{12e^2}\right)^2|\psi|^4\dots . \tag{20}$$

The first term shifts the transition from $t=3$ to $t=3+\alpha_2(\tau)\tau t$ or $e^2=e_2^2(\tau)=(4\pi^2/3)(1-\alpha_2(\tau)\tau)$, where $\tau=T/e^2$ and

$$\alpha_2(\tau)=\sum_k\frac{K^2}{1+\tau K^2}=+\tau^{-1}-\tau^{-1}(1+6\tau)^{-1}\left[1+\left(\frac{2\tau}{1+6\tau}\right)^2\frac{3}{2}+\left(\frac{2\tau}{1+6\tau}\right)^3\frac{45}{8}+\dots\right]$$

The resulting curve is displayed in the (T, e^2) diagram in fig. 1. It is in excellent agreement with the Monte Carlo points ([14]), one at $e^2 = 13.16$ and the other at $e^2 = 5$ (*). This shows that for $e^2 > 5$ the ψ field is much less modified by fluctuations than the Ginzburg-Landau field φ, due to the absence of the long-range magnetic field. Therefore, it is better suited for studying the critical behaviour. The good agreement also justifies *a posteriori* the integrating out of the massive W_i fields.

The formal manipulations have a simple physical interpretation, as stated shortly in the introduction and can now be seen directly: The Ginzburg-Landau theory (12) contains vortex lines with short-range interactions. These are manifestly displayed in the form (3) of the partition function with the sum over $b_i(\boldsymbol{x})$ with $\bar{\nabla}_i b_i(\boldsymbol{x}) = 0$ being a sum over nonbacktracking random walks of vortex lines. The coupling to the massive field $\boldsymbol{W}$ produces the correct short-range interactions between these. When bringing (3) to the form (16), this is the lattice equivalent to the usual passage from a first quantized quantum mechanics of orbits to a second quantized field theory. The field ψ is the disorder field representing the grand-canonical ensemble of fluctuating vortex lines ([6-8]). The term $|\psi|^4$ is the local approximation to the repulsive interaction, relevant for long-range phenomena, which stabilizes the second-order phase transition. For $K < K_{1,2}$, the respulsion turns into an attraction and this marks the change from second- to first-order phase transition.

Conversely, the XYW model allows for closed vortex lines with long-range elastic interactions and an additional core energy, as displayed in (6). The Ginzburg-Landau theory involving φ is the « second quantized » way of describing this ensemble.

Being in possession of these two equivalent representations of one and the same theory we are finally prepared to derive the desired conclusions on the order of the superconductive phase transition. The critical behaviour of the ψ field theory is well known and experimentally confirmed in superfluid ^{4}He. There is a second-order phase transition and we can therefore conclude that also the Ginzburg-Landau theory of superconductivity has such a transition in the region $T \geqslant 0$. In this region, the value K increases from $1.17/\sqrt{2}$ to ∞.

The possibility of a first-order transition can arise only for smaller K. At first sight it appears as though a simple decrease of T to negative values could be used to invade into this regime. This is, however, not true, since we can easily verify that the lattice superconductor (6) has the symmetry $(T_1 e^2) \to (-T, e^2 + 4TD)$ such that each point for negative T has an equivalent point for positive T on the phase diagram. Thus formula (15) cannot simply be continued to larger e values. There is the breakdown of the approximation $D_i \sim \nabla_i - ieA_i$ (which may have a 10% error already at $T = 0$.) Thus we shall trust the equivalence only down to $T \geqslant 0$ and must find another way of invading into the regime $K < 1.17/\sqrt{2}$.

For this we observe that for $T \sim 0$ both theories have the same quartic terms $(1/64)|\varphi|^2$, $(1/64)|\psi|^2$. The quadratic terms, on the other hand, are

$$\frac{1}{12}(T + e^2 v(0) - 3)|\varphi|^2 \quad \text{and} \quad \frac{1}{12}(t - 3 - \alpha_2(\tau)\tau t)|\psi|^2 \sim \frac{1}{12}\left(t - 3 - \frac{18}{e_0^2}\right)|\psi|^2,$$

where $e_0^2 = 4\pi^2/3$ is the value of e^2 at $T = 0$. Thus the quadratic terms have dif-

(*) The third point ([14]) $e^2 = 0$, $T \sim 3$ disagrees with the ψ mean field theory (while being in excellent agreement with the φ mean field theory). It is no wonder, since for large $t = (2\pi)^2/e^2$ the Villain approximation (1) $\to$ (2) breaks down.

ferent sensitivities upon variations of T, the second

$$\frac{18}{e_0^2} \sim \frac{27}{2\pi^2} \sim 1.3678 \tag{21}$$

times as much as the first. It is this property which we can take advantage of.

Let us modify both theories by assuming the temperature to perform local Gaussian random fluctuations with a distribution $\exp\left(-(1/2a)\sum_x T(\boldsymbol{x})^2\right)$. Integrating these out in the partition function produces additional quartic terms and changes the original ones to

$$\left(\frac{1}{64}-\frac{a}{288}\right)|\varphi|^4, \quad \left(\frac{1}{64}-\frac{a}{288}\left(\frac{18}{e_0^2}\right)^2\right)|\psi|^4.$$

Now, because of the enhancement factor (21) it is possible to make the $|\psi|^4$ term vanish with the $|\varphi|^4$ term having the reduced size $(1/64)(1-(4/9)(\pi/3)^4)|\varphi|^4$. As a consequence, the magnetic-penetration length decreases and the $T=0$ value of K drops from $K_0 = 3/e_0$ to $K_{1,2} = K_0\sqrt{1-(4/9)(\pi/3)^4}$. Near it, the quartic term has the form $(1/64)(3/2)\sqrt{2}(3/4)^3(K-K_{1,2})|\psi|^4$ as stated in the beginning.

It is expected that this result can still be modified by $\sim 10\%$ by radiative corrections which should be calculated before quantitative comparison with experiment.

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