NO CRYPTOFERROMAGNETIC STATE DUE TO FLUCTUATIONS

H. KLEINERT
Institut für Theoretische Physik, D-1000 Berlin 33, Germany

Received 13 March 1981

It is shown that the cryptoferromagnetic state which arises in a mean-field treatment of magnetic superconductors cannot form due to strong fluctuations. The Bragg-like neutron reflection is due to these fluctuations rather than a helical texture.

Field configurations with non-vanishing momentum \( q_0 \neq 0 \) occur, at the mean-field level, in several physical systems; examples are cholesteric liquid crystals [1], pion condensates [2], and magnetic superconductors [3]. Fluctuations, however, drastically change the picture. In the first example, the phase transition is strongly shifted to lower temperatures and occurs only thanks to a cubic term in the free energy which changes the order from second to first [3]. Moreover, the transition does not proceed directly from the disordered to the chiral state but there is an intermediate blue phase in which the texture forms a lattice [5].

In pion condensates, where such a first-order mechanism is absent, the phase transition is apparently prevented completely [6].

It is the purpose of this note to show that in the magnetic superconductors the situation is similar to pion condensation: No long-range order can develop, all susceptibilities remain finite, and there are no Goldstone bosons. All phenomena attributed to the \( q_0 \neq 0 \) state [7] are really due to fluctuations and "pretransitional" in character.

Consider the free-energy density of the magnetic superconductor [8] which we write in natural units as

\[
2f = \gamma |\Delta|^2 + \frac{1}{2} |\Delta|^4 + \frac{1}{2} |\Delta|^2 + \frac{1}{2} \beta M^4 + \frac{1}{4} |\nabla \times \mathbf{M}|^2.
\]

where \( \gamma = \gamma_0 - 2 \alpha A \Delta \Delta^2 + \frac{1}{2} \beta M^4 + \frac{1}{4} |\nabla \times \mathbf{M}|^2 \).

The partition function is given by

\[
Z = \int D\Delta D\Delta^* D\mathbf{M} D\mathbf{A}
\times \exp \left[ - \int d^2 \mathbf{x} \int_0^{1/T} dt \left( \frac{1}{2} \mathbf{A}^2 + f \right) \right],
\]

where we have included only static fluctuations except for the magnetic potential \( \mathbf{A} \) where the time dependence is relevant [3] (\( t = \text{imaginary time}, T = \text{temperature} \)). Since \( \mathbf{A} \) appears quadratically, it is integrat-ed out and the exponent becomes

\[
F[\Delta, \mathbf{M}] = \frac{1}{2} \int d^2 \mathbf{x} \left[ \tau \Delta^2 + \frac{1}{2} \Delta^4 + (\partial \Delta)^2 \right]
\]

\[
+ \left( \tau_M + 1 \right) M^2 + \frac{1}{2} \beta M^4 + \frac{1}{4} |\nabla \times \mathbf{M}|^2
\]

\[
- \frac{1}{2} (\nabla \times \mathbf{M}) G(\nabla \times \mathbf{M}) + \frac{1}{2} \text{tr} \log G^{-1},
\]

where

\[
G_{ij}(\mathbf{x}, \mathbf{x}'; t') = \int_0^{1/T} dt \tilde{G}_{ij}(\mathbf{x}, t; \mathbf{x}', t')
\]

is the correlation function of the field \( A_i \) whose mass term \( \mu \) depends on \( \Delta(x) \) as
\[ \mu(x) = 2e\Delta(x), \]  

thereby accounting for the Meissner effect. Before integration we have chosen a gauge such that \( \Delta \) becomes real. The trace of the logarithm collects the "black body" energy of the massive photons. It can be neglected except in the immediate vicinity of \( \tau = 0 \) where \( \mu = 0 \). Notice that if it is expanded in powers of \( \Delta \), it generates a cubic term. This changes the superconducting phase transition from second to first order \([9]\), but since there is a factor \( e^3 \) the effect is so weak that it has never been seen \([3]\).

The important feature of eq. (3) is the new bending energy for the magnetization. Assuming, for a moment, a constant order parameter \( \Delta \), we may invert

\[ G_{ij}^{-1} = (\delta_{ij} - \alpha q_i q_j/\mu^2)/(q^2 + \mu^2), \]

separate longitudinal and transverse components of \( M \), write the quadratic piece as

\[ (r_M + 1 + \xi_M^2 q^2)M_\perp^2 \]

and realize that for

\[ \gamma \equiv \xi_M \mu < 1 \]

the coefficient of the transverse part has a minimum at

\[ q_0^2 = \mu^2 (1 - \gamma)/\gamma. \]

Close to it, the bending energy can be expanded as

\[ [\tau_s + \alpha(q - q_0)^2]M_\perp^2, \]

where

\[ \tau_s = r_M + 1 + (\gamma - 1)^2 \equiv r_0^2(T/T_S - 1), \]

\[ \alpha = \gamma(4 - \gamma). \]

The temperature \( T_S \) at which \( \tau_s = 0 \) marks the point below which a helical magnetic configuration \( M_\perp = M_\perp^0 \exp(iq_0^2t) \) may form a stable ground state at the mean-field level.

We shall now show that this solution is an illusion. Fluctuations prevent the field from settling down.

In order to study this problem we may neglect \( M_\perp \) and the fluctuations in the gap parameter \( \Delta \) since they remain hard close to \( \tau_s = 0 \). Only the fluctuations of \( M_\perp \) are of a severe nature. In the critical regime, they are controlled by the free energy

\[ 2f_{M_\perp} = [\tau_s + \alpha(q - q_0)^2]M_\perp^2 + \frac{1}{2}BM_\perp^4 \]

+ irrelevant terms.

From here on the conclusion follows precisely in the same way as in ref. \([6]\). It will suffice to repeat only the physics behind the formal argument: The bare correlation function

\[ \langle M_\perp^2 \rangle = \int \frac{d^3q}{(2\pi)^3} [\tau_s + \alpha(q - q_0)^2]^{-1} \]

has gigantic directional fluctuations of the wave vector \( q \) over a whole spherical shell and this leads to a divergence

\[ \langle M_\perp^2 \rangle \sim 1/\tau_s \]

for \( \tau_s \to 0 \). If this is inserted into Dyson's equation the renormalized \( \tau_s^\text{ren} \) satisfies

\[ \tau_s^\text{ren} \sim \tau_s + \text{const.}/\sqrt{\tau_s}, \]

such that the transition can never take place. The expectation of \( \langle M_\perp \rangle \) always remains zero.

The non-perturbative fluctuation effects are most easily accounted for by using higher effective actions as employed recently by the author \([10]\).

The non-existence of the phase transition does not ruin many of the observational characteristics of the mean-field phase (see ref. \([6]\)). The large fluctuations for \( q \approx q_0 \) can reflect neutrons very similarly to a stationary helical texture but the line width is increased and shows a typical "pretransitional" behaviour except that the renormalized \( \tau_s^\text{ren} \) keeps decreasing as \( \tau_s^{-2} \) for \( \tau_s \to -\infty \), i.e. \( \Delta q \sim (\tau_s^\text{ren}/\alpha)^{1/2} \to 1/\sqrt{\tau_s} \).

The idea to this note was conceived during an interesting lecture of Professor J. Keller on the problems of magnetic superconductors which I happened to attend just after finishing the related problem in pion condensates \([6]\). I am grateful to him for sending me his review with P. Fulde and for several critical discussions.

References


S.-O. Bäckman and W. Weise, in: Mesons in nuclei, eds.
M. Rho and D.H. Wilkinson (North-Holland, Amsterdam,
1979);
G. Baym and D. Cambell, ibid., p. 1031;
R.F. Sawyer, ibid., p. 991;
B. Banerjee, N.K. Glendenning and M. Gyalass, LBL
preprint-10979 (1980);
M. Gyalass, LBL preprint 10883 (1980).


therein.

219;
H. Kleinert, Phys. Lett. 81A (1981) 141, and references
therein.


1079;
R.A. Ferrell, J.K. Bhattacharjee and A. Bagchi, Phys.
Rev. Lett. 43 (1979) 154;
H. Matsumoto, H. Umezawa and M. Tachiki, Solid State
Commun. 31 (1979) 157.


A, to be published.