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HADRONIZATION OF QUARK THEORIES

Hagen Kleinert

Institut für Theoretische Physik

Freie Universität Berlin, Berlin, Germany

ABSTRACT

Local quark gluon theories are converted into bilocal field theories via functional techniques. The new field quanta consist of all quark antiquark bound states in the ladder approximation. They are called "bare hadrons". Hadronic Feynman graphs are developed which strongly resemble dual diagrams. QED is a special case with the "bare hadrons" being positronium atoms. Photons couple to hadrons via intermediate vector mesons in a current-field identity . The new theory accommodates naturally bilocal currents measured in deep-inelastic e p scattering Also these couple via intermediate mesons.

In the limit of heavy gluon masses, the hadron fields become local and describe π, ρ, A_1, σ mesons in a chirally invariant Lagrangian (the "σ model"). Many interesting new relations are found between meson and quark properties such as $m_\rho^2 \approx 6M^2$ where M is the "true" non-strange quark mass after spontaneous breakdown of chiral symmetry. There is a simple formula linking these quark masses with the small "bare masses" of the Lagrangian. The quark masses also determine the vacuum expectations of scalar densities. These show an SU(3) breaking in the vacuum of $\approx$ - 16 %.

I. INTRODUCTION

In the attempt to understand strong interaction, two basically different theoretical approaches have been developed in the past years. One of them, the dual approach, is based on complete democracy among all strongly interacting particles. Within this approach, an elaborate set of rules assures the construction of certain lowest order vertex functions for any number of hadrons[1]. The other approach assumes the existence of a local field equation involving fundamental quarks bound together by vector gluons[2] Here strong interaction effects on electomagnetic and weak currents of hadrons can be analyzed in a straight-forward fashion without detailed dynamical computations[3]. Either approach has its weakness where the other is powerful. Dual models have, until now, given no access to currents while quark theories have left the problem of hadronic vertex function intractable. Not even an approximate bound state calculation is avaiable (except in 1 + 1 dimensions[4] or by substituting the field couplings by simple ad-hoc forces[5]).

At present there is hope that the problems connected with quark models are of a purely technical nature. A Lagrangian field theory of Yang-Mills type seems to have a good chance of defining a true fundamental theory of elementary particles. Dual models, on the other hand, seem to be of a more phenomenological character. Once the fundamental vertices are determined, it is difficult to find next corrections and to extend the prescriptions to what might be called a complete theory. If this could be done it would certainly have to be phrased in terms of local infinite-component or multi-local fields[6].

It would be very pleasing if both models were, in fact, essentially equivalent both being different languages for one and the same underlying dynamics. In this case one could use one or the other depending on whether one wants to answer short-or long-distance questions concerning quarks.

In order to learn how a translation between the different languages might operate we shall consider, in these lectures, the simplified field theory in which quarks are colorless, have N flavours, and are held together by vector gluons of arbitrary mass μ. This theory incorporates several realistic features of strong interactions, for example current algebra and PCAC. Moreover, the case $N=1$ and $\mu=0$ includes ordinary quantum electrodynamics (Q.E.D.). This will provide a good deal of intuition as well as the possibility of a detailed test of our results.

We shall demonstrate how functional methods can be employed to transform the local quark gluon theory into a new completely equivalent field theory involving only bilocal fields. The new free field quanta coincide with quark-antiquark bound states when calculated by ladder exchanges only. They may be considered as "bare hadrons". Accordingly, the transition from the local quark- to the bilocal hadron-theory will be named "hadronization". In the special case of Q.E.D., "bare hadrons" are positronium atoms in ladder approximation.

The functional technique will ensure that bare hadrons have exactly the correct interactions among each other in order that hadronization preserves the equivalence to the original quark gluon theory. It is simple to establish the connection between classes of Feynman graphs involving quarks and gluons with single graphs involving hadrons. The topology of hadron graphs is the same as that of dual diagrams. It is interesting to observe the appearence of a current-hadron field identity for photons just as employed in phenomenological discussions of vector meson dominance. Moreover, since the theory is bilocal, this identity can be extended to bilocal currents which are measured in deeply inelastic electromagnetic and weak interactions.

The limit of a very heavy gluon mass can be hadronized most simply. Here the bilocal fields become local and describe only a few hadrons with the quantum numbers of σ, π, ρ, A_1 mesons. The Lagrangian coincides with the standard chirally invariant σ model which is known to account quite well for the low-energy aspects of meson physics. Here hadronization renders additional connection between quark and meson properties. It also makes transparent the connection between the very small bare quark masses (which describe the explicit breakdown of chiral symmetry) and the mechanical quark masses (which include the dynamic effects due to spontaneous symmetry violations).

II TECHNICAL PRELIMINARIES

Before we embark in our program we have to recall certain functional techniques[7]. They are generalizations of the basic Fresnel integrals, valid for all real A=0

$$\int_{-\infty}^{\infty} \frac{d\xi}{\sqrt{2\pi i}}\, e^{\frac{i}{2}\xi A \xi} = A^{-\frac{1}{2}} \qquad (2.1)$$

$$\int \frac{d\xi}{\sqrt{2\pi i}}\frac{d\xi^*}{\sqrt{2\pi i}}\, e^{i\xi^* A \xi} = A^{-1} \qquad (2.2)$$

where the complex integral $\int d\xi\, d\xi^*$ stands symbolic for $\int_{-\infty}^{\infty} d\,\frac{\xi+\xi^*}{\sqrt{2}} \int_{-\infty}^{\infty} d\,\frac{\xi-\xi^*}{\sqrt{2}\,i}$. Quadratic completion in the exponent yields:

$$\int_{-\infty}^{\infty} \frac{d\xi}{\sqrt{2\pi i}}\, e^{i\left(\frac{1}{2}\xi A \xi + j\xi\right)} = A^{-\frac{1}{2}} e^{-\frac{i}{2} j A^{-1} j} \qquad (2.3)$$

$$\int \frac{d\xi}{\sqrt{2\pi i}}\frac{d\xi^*}{\sqrt{2\pi i}}\, e^{i\left(\xi^* A \xi + j^*\xi + \xi^* j\right)} = A^{-1} e^{-i j^* A^{-1} j} \qquad (2.4)$$

If ξ is a vector $\xi = (\xi_1, \ldots, \xi_n)$ and $\int \mathcal{D}\xi$ stands short for $\prod_k \int \frac{d\xi_k}{\sqrt{2\pi i}}$, then for non-singular symmetric or hermitian matrices A these formulas become

$$\int \mathcal{D}\xi\, e^{i\left(\frac{1}{2}\xi A \xi + j\xi\right)} = (\det A)^{-\frac{1}{2}} e^{-\frac{i}{2} j A^{-1} j} \qquad (2.5)$$

$$\int \mathcal{D}\xi \mathcal{D}\xi \; e^{i(\xi^{+}A\xi + j^{+}\xi + \xi^{+}j)} = (\det A)^{-1} e^{-ij^{+}A^{-1}j} \tag{2.6}$$

respectively, as can be seen immediately by diagonalizing A via an orthogonal or unitary transformation (which leaves the measures $\mathcal{D}\xi$ or $\mathcal{D}\xi\,\mathcal{D}\xi^{+}$ invariant). If finally $\xi(x)$ is a function $\xi(x)$ and $\xi^{+}A\xi$ is understood in the functional sense as

$$\xi^{+}A\xi \equiv \int dx\,dy\; \xi^{*}(x)\, A(x,y)\, \xi(y) \tag{2.7}$$

these integrals may still be defined by grating the x axis into finer and finer lattices of points $x_k = k\varepsilon$ with $k=0,\pm 1,\pm 2,\ldots$ and reducing the problem to the previous case via the vector components $\xi_k = \sqrt{\varepsilon}\,\xi(x_k)$.
For large matrices A, the calculation of the determinant is performed most simply by expanding

$$\det A = \exp \ln \det A = \exp \operatorname{tr} \ln A \tag{2.8}$$

$$= \exp \operatorname{tr} \ln [1 + (A-1)] = \exp \operatorname{tr} \left\{ A - 1 - \tfrac{1}{2}(A-1)^2 + \tfrac{1}{3}(A-1)^3 \pm \ldots \right\}$$

This formula is directly applicable in the functional case if all sums over intermediate indices are replaced by the corresponding integrals, for example:

$$\begin{aligned} \mathrm{tr}\, A &= \sum_k A_{kk} \longrightarrow \int dx\, A(x,x) \\ \mathrm{tr}\, A^2 &= \sum_{k,\ell} A_{k\ell} A_{\ell k} \rightarrow \int dx\,dy\, A(x,y)A(y,x) \end{aligned} \tag{2.9}$$

The integrals (2.5) and (2.6) can also be extended to functions $\xi(x)$ with values in an anticommuting algebra (i.e. $\{\xi(x), \xi(y)\} = 0$). In this case the (det A) in equs. (2.5),(2.6)appears in the inverse forms $(\det A)^{1/2}$ and $(\det A)^{1}$, respectively.

With these preparations consider now a Lagrangian composed of fermion and boson fields and split into free and interacting part $\mathcal{L}(\psi,\bar\psi,\varphi) = \mathcal{L}_0 + \mathcal{L}_{int}$. All time ordered Green's functions can be obtained from the derivatives with respect to the external sources of the generating functional

$$Z[\eta,\bar\eta,j] = \mathrm{const}\, \langle 0 | T e^{i\int dx\, (\mathcal{L}_{int} + \bar\eta\psi + \bar\psi\eta + j\varphi)} | 0 \rangle \tag{2.10}$$

The fields in the exponent follow free equations of motion and $|0\rangle$ is the free-field vacuum. The constant is conventionally chosen to make $Z[0,0,0] = 1$, i.e.

$$\mathrm{const} = \left[\langle 0 | T e^{i\int dx\, \mathcal{L}_{int}(\psi,\bar\psi,\varphi)} | 0 \rangle \right]^{-1} \tag{2.11}$$

This normalization may always be enforced at the very end of any calculation such that $Z[\eta,\bar\eta,j]$ is only interesting as far as its functional dependence is concerned, modulo the irrelevant constant in front.

It is then straight-forward to show that $Z[\eta,\bar{\eta},j]$ can alternatively be computed via the Feynman path integral formula

$$Z[\eta,\bar{\eta},j] \propto \int \mathcal{D}\psi\,\mathcal{D}\bar{\psi}\,\mathcal{D}\varphi\; e^{i\int dx\left(\mathcal{L}_0(\psi,\bar{\psi},\varphi)+\mathcal{L}_{int}+\bar{\eta}\psi+\bar{\psi}\eta+j\varphi\right)} \tag{2.12}$$

Here the fields are no more operators but classical functions (with the mental reservation that classical Fermi fields are anticommuting objects). Notice that contrary to the operator formula (2.10) the <u>full</u> action appears in the exponent.

For simplicity, we demonstrate the equivalence only for one real scalar field $\varphi(x)$. The extension to other fields is immediate [7,8]. First note that it is sufficient to givethe proof for the free field case, i.e.

$$\begin{aligned} Z_0[j] &= \langle 0|\,T e^{i\int dx\, j(x)\varphi(x)}\,|0\rangle \\ &\propto \int \mathcal{D}\varphi\; e^{i\int dx\left(\frac{1}{2}\varphi(x)(-\Box_x-\mu^2)\varphi(x)+j(x)\varphi(x)\right)} \end{aligned} \tag{2.13}$$

For if it holds there a simple multiplication on both sides of (2.13) by the differential operator

$$e^{i\int dx\,\mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta j(x)}\right)} \tag{2.14}$$

would extend it to the interacting functionals (2.10) or (2.12). But (2.13) follows directly from Wick's theorem according to which any time ordered product of a free field can be expanded into a sum of normal products with all possible time ordered contractions. This statemen can be summarized in operator form valid for any functional $F[\varphi]$ of a free field $\varphi(x)$:

$$T\,F[\varphi] = e^{\frac{1}{2}\int dx\,dy\,\frac{\delta}{\delta\varphi(x)}D(x-y)\frac{\delta}{\delta\varphi(y)}} :F[\varphi]: \tag{2.15}$$

where $D(x-y)$ is the free-field propagator

$$D(x-y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} \frac{i}{q^2-\mu^2+i\varepsilon} = \frac{i}{-\Box_x-\mu^2+i\varepsilon}\,\delta(x-y) \tag{2.16}$$

Applying this to (2.13) gives

$$\begin{aligned} Z_0[j] &= e^{\frac{1}{2}\int dx\,dy\,\frac{\delta}{\delta\varphi(x)}D(x-y)\frac{\delta}{\delta\varphi(y)}} \langle 0|:e^{i\int dx\, j(x)\varphi(x)}:|0\rangle \\ &= e^{-\frac{1}{2}\int dx\,dy\, j(x)D(x-y)j(y)} \langle 0|:e^{i\int dx\, j(x)\varphi(x)}:|0\rangle \\ &= e^{-\frac{1}{2}\int dx\,dy\, j(x)D(x-y)j(y)} \end{aligned} \tag{2.17}$$

The last part of the equation follows from the vanishing of all normal products of $\varphi(x)$ between vacuum states.

Now exactly the same result is obtained by performing the functional integral in (2.13) and using (2.5). The matrix A is in this case $A(x,y) = (-\Box_x - \mu^2)\delta(x-y)$, such that its inverse becomes the propagator $D(x-y)$:

$$A^{-1}(x,y) = \frac{1}{-\Box_x - \mu^2 + i\varepsilon}\,\delta(x-y) = -i\,D(x-y) \qquad (2.18)$$

yielding again (2.17).

Notice that it is Wick's expansion which supplies the free part of the Lagrangian when going from the operator form (2.15) to the functional version (2.12).

III QUARK GLUON THEORY

Consider now a system of N quarks $\psi(x)$ held together by one gluon field $G^{\nu}(x)$ of mass μ via a Lagrangian

$$\mathcal{L}(x) = \bar{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x) + g\,\bar{\psi}(x)\gamma_{\nu}\psi(x)G^{\nu}(x) \qquad (3.1)$$
$$- \frac{1}{4}F_{\mu\nu}^{2}(x) + \frac{\mu^2}{2}G_{\nu}^{2}$$

Here $F_{\mu\nu}$ is the usual curl $\partial_\mu G_\nu - \partial_\nu G_\mu$. In the special case in which N = 1, $\mu = 0$, and $g^2 = 4\pi\alpha$, this Lagrangian describes quantum electrodynamics. In other cases it may be considered as a model field theory which carries many interesting properties of strong interactions, for example approximate SU(3) symmetry, chiral SU(3) xSU(3) current algebra, PCAC, and scaling up to small corrections. Certainly this model will never be able to confine quarks, give symmetric baryon wave functions, and explain infinitely rising hadron trajectories. For this it would have to contain an additional, exactly conserved, color symmetry with $G^\nu(x)$ being its non-abelian gauge mesons. Before attempting to deal with this far more complicated situation we shall develop[10] our tools for the less realistic but much simpler model (3.1) without color.

The generating functional of all time ordered Green's function is

$$Z[\eta,\bar{\eta},j^\nu] \propto \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}G\, e^{i\int dx\left(\mathcal{L}+\bar{\psi}\eta+\bar{\eta}\psi+g\, j^\nu G_\nu\right)} \tag{3.2}$$

The exponent is quadratic in $G^\nu(x)$, such that the functional integration over the gluon field can be performed[7,8] (using (2,5)) leaving:

$$Z[\eta,\bar{\eta},j^\nu] \propto \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\, e^{i\mathcal{A}[\psi,\bar{\psi},\eta,\bar{\eta},j^\nu]} \tag{3.3}$$

where the action is

$$\mathcal{A}[\psi,\bar{\psi},\eta,\bar{\eta},j^\nu] = \int dx\,dy \Big\{\left(\mathcal{L}(x)+\bar{\psi}(x)\eta(x)+\bar{\eta}(x)\psi(x)\right) \times\delta(x-y) - \frac{i}{2}g^2 D(x-y)\left(\bar{\psi}(x)\gamma^\nu\psi(x)+j^\nu(x)\right)\left(\bar{\psi}(y)\gamma_\nu\psi(y)+j_\nu(y)\right)\Big\} \tag{3.4}$$

By employing the Fierz identity:

$$\gamma^{\nu}_{\alpha\beta} \otimes \gamma_{\nu\,\gamma\delta} = 1_{\alpha\delta} \otimes 1_{\gamma\beta} + (i\gamma_5)_{\alpha\delta} \otimes (i\gamma_5)_{\gamma\beta}$$
$$- \frac{1}{2}\gamma^{\nu}_{\alpha\delta} \otimes \gamma_{\nu\,\gamma\beta} - \frac{1}{2}(\gamma^{\nu}\gamma_5)_{\alpha\delta} \otimes (\gamma_\nu\gamma_5)_{\gamma\beta} \tag{3.5}$$

the quartic quark interaction term can be written in a different fashion

$$\frac{i}{2} g^2 D(x-y)\{\bar\psi(x)\psi(y)\bar\psi(y)\psi(x) + \bar\psi(x)i\gamma_5\psi(y)\bar\psi(y)i\gamma_5\psi(x)$$
$$-\frac{1}{2}\bar\psi(x)\gamma^{\nu}\psi(y)\bar\psi(y)\gamma_\nu\psi(x) - \frac{1}{2}\bar\psi(x)\gamma^{\nu}\gamma_5\psi(y)\bar\psi(y)\gamma_\nu\gamma_5\psi(x)\}$$
$$\equiv \frac{i}{2} g^2 D(x-y)\,\bar\psi_\alpha(x)\psi_\delta(y)\,\xi_{\alpha\delta,\gamma\beta}\,\bar\psi_\gamma(y)\psi_\beta(x) \tag{3.6}$$

This is the point where our elimination of quark fields in favor of new bilocal fields can set in.

Let $S(x,y), P(x,y), V^{\nu}(x,y), A^{\nu}(x,y)$ be a set of hermitian auxiliary fields, i.e.

$$S(x,y) = S(y,x)\ ,\ P(x,y) = P^{*}(y,x)\ ,\ etc \tag{3.7}$$

With these fields one can certainly construct the following functional identities[9]

$$\int \mathcal{D}S(x,y)\, e^{-\frac{i}{2}|S(x,y)+ig^2 D(x-y)\bar{\Psi}(y)\Psi(x)|^2/ig^2D(x-y)} = \text{const}$$

$$\int \mathcal{D}P(x,y)\, e^{-\frac{i}{2}|P(x,y)+ig^2 D(x-y)\bar{\Psi}(y)i\gamma_5\Psi(x)|^2/ig^2D(x-y)} = \text{const}$$

$$\int \mathcal{D}V(x,y)\, e^{i|V^\nu(x,y)-\frac{i}{2}g^2 D(x-y)\bar{\Psi}(y)\gamma^\nu\Psi(x)|^2/ig^2D(x-y)} = \text{const}$$

$$\int \mathcal{D}A(x,y)\, e^{i|A^\nu(x,y)-\frac{i}{2}g^2 D(x-y)\bar{\Psi}(y)\gamma^\nu\gamma_5\Psi(x)|^2/ig^2D(x-y)} = \text{const} \tag{3.8}$$

which are independent of the fields $\Psi(x)$. If we now multiply $Z[\eta,\bar{\eta},j^\nu]$ in (3.3) by these constants and make use of (3.6), all quartic quark terms are seen to cancel. The generating functional becomes

$$Z[\eta,\bar{\eta},j^\nu] \propto \int \mathcal{D}\Psi\mathcal{D}\bar{\Psi}\mathcal{D}S\mathcal{D}P\mathcal{D}V\mathcal{D}A\, e^{i\tilde{\mathcal{A}}} \tag{3.9}$$

where the new action $\tilde{\mathcal{A}}$ denotes the integral

$$\tilde{\mathcal{A}}[\Psi,\bar{\Psi},S,P,V,A,\eta,\bar{\eta},j^\nu] = \int dx\,dy\, \mathcal{L}(x,y) \tag{3.10}$$

with the bilocal Lagrangian

$$\begin{aligned}\mathcal{L}(x,y) \equiv \{&\bar{\Psi}(x)(i\partial\!\!\!/-m)\Psi(x)+\bar{\Psi}(x)\eta(x)+\bar{\eta}(x)\Psi(x)\}\,\delta^{(4)}(x-y)\\ &-\bar{\Psi}(x)m(x,y)\Psi(y) - \tfrac{i}{2}g^2 D(x-y)j^\nu(x)j_\nu(y)\\ &-\{\tfrac{1}{2}|S|^2+\tfrac{1}{2}|P|^2-|V|^2-|A|^2\}/ig^2D(x-y)\end{aligned} \tag{3.11}$$

Here m(x,y) has been introduced as an abbreviation for the combined field

$$m(x,y) \equiv S(x,y) + P(x,y) i\gamma_5 \tag{3.12}$$
$$+ \left(V^{\nu}(x,y) + \delta^{(4)}(x-y) \int dz \, ig^2 D(x-z) j^{\nu}(z)\right)\gamma_\nu + A^{\nu}(x,y)\gamma_\nu\gamma_5$$

Due to (3.7), the matrix m(x,y) is self-adjoint in the sense

$$\left(\overline{m(x,y)}\right)_{\alpha\beta} \equiv \gamma_{0\,\alpha\alpha'}\left(m^{*}(x,y)^{T}\right)_{\alpha'\beta'} \gamma_{0\,\beta'\beta} = m_{\alpha\beta}(\ ,x) \tag{3.13}$$

At this place it is worth remarking that the Lagrangian (3.11) shows its equivalence to the previous form (3.4) also quite directly. By virtue of the Euler Lagrange equations the fields S, P, V, A are seen to be dependent fields coinciding with the corresponding bilocal quark expressions

$$S(x,y) = -ig^2 D(x-y) \bar{\psi}(y) \psi(x)$$
$$P(x,y) = -ig^2 D(x-y) \bar{\psi}(y) i\gamma_5 \psi(x)$$
$$V^{\nu}(x,y) = \frac{i}{2} g^2 D(x-y) \bar{\psi}(y) \gamma^{\nu} \psi(x) \tag{3.14}$$
$$A^{\nu}(x,y) = \frac{i}{2} g^2 D(x-y) \bar{\psi}(y) \gamma^{\nu} \gamma_5 \psi(x)$$

Inserting these relations back into (3.11) reproduces (3.4). In the action (3.10), quark fields enter only in quadratic form such that they can be integrated according to formula (2.6) (in its fermionic version). The matrix A is in this case

$$A(x,y) = (i\partial\!\!\!/ - m)\,\delta^{(4)}(x-y) - m(x,y)$$

Hence $A^{-1}(x,y) \equiv -iG(x,y)$ becomes simply the Green's function of the equation

$$\int dy\, \left[(i\partial\!\!\!/ - m)\,\delta^{(4)}(x-y) - m(x,y)\right] G(y,z) = i\delta^{(4)}(x-z) \qquad (3.15)$$

With this notation, the quark integration brings the functional (3.9) to the form

$$Z[\eta,\bar\eta,j^\nu] \propto \int \mathcal{D}m(x,y)\, e^{i\mathcal{A}[m,\eta,\bar\eta,j^\nu]} \qquad (3.16)$$

with

$$\mathcal{A}[m,\eta,\bar\eta,j^\nu] = \int dx dy \Big\{ -i\,\mathrm{tr}\left(\ln iG^{-1}\right)(x,y)\delta(x-y) \qquad (3.17)$$

$$-\frac{1}{2}\,\mathrm{tr}\left(m(x,y)\,\xi^{-1}\, m(y,x)\right)/ig^2 D(x-y) + i\bar\eta(x)G(x,y)\eta(y)$$

$$-\frac{2}{D(0)} V^\nu(x,x) D(x-y) j_\nu(y) - ig^2 \frac{\delta^{(4)2}(x-y)}{D(0)} \int dz dz'\, D(z-x) D(y-z') j^\nu(z) j_\nu(z') \Big\}$$

Here we have introduced, for brevity, the notation $\mathcal{D}m(x,y) \equiv \mathcal{D}S\mathcal{D}P\mathcal{D}V\mathcal{D}A$. Notice that the effect of the matrix ξ^{-1} defined in equ. (3.6) is simply to devide the projections into S, P, V, A by 4, −4, −2, 2, respectively+). The trace refers only to Dirac indices.

+) Since $\frac{1}{4}1\otimes 1$, $-\frac{1}{4}(\gamma_5)\otimes(\gamma_5)$, $\frac{1}{4}\gamma^\nu\otimes\gamma_\nu$, $-\frac{1}{4}\gamma^\nu\gamma_5\otimes\gamma_\nu\gamma_5$ are the correcponding projection operators.

The new functional (3.16) is identical to the original one in equ. (3.2). As a consequence, a quantum theory based on the action (3.17) must be completely equivalent to the original quantized quark gluon theory.

A word is in order concerning the internal symmetry SU(N) among the N quarks (i=1,...N) under consideration. Since the gluon is an SU(N) singlet, the interaction in equ. (3.1) is $g\sum_{i=1}^{N} \bar\psi_i \gamma^\nu \psi_i G_\nu(x)$. In the Fierz transformed version (3.6) the indices i and j appear separated

$$\frac{i}{2} g^2 D(x-y)\, \bar\psi^j(x)\, \psi_i(y)\, \xi\, \bar\psi^i(y)\, \psi_j(x)$$

Hence in the presence of N quarks, the fields m(x,y) have to be thought of a matrices in SU(N) space $m(x,y)_i{}^j$. This carries over to the action with the traces including Dirac as well as SU(N) indices.

Let us now develop a quantum theory for the new action. In general, the field m(x,y) may oscillate around some constant non-zero vacuum expectation value $m_0 \delta^{(4)}(x-y)$. It is convenient to subtract such a value from m(x,y) and introduce the field

$$m'(x,y) \equiv m(x,y) - m_0 \delta^{(4)}(x,y) \tag{3.18}$$

With this and the definition

$$M \equiv \mathfrak{M} + m_0 , \tag{3.19}$$

equ. (3.15) can be rewritten as

$$\int dy \left[(i\partial\!\!\!/ - M)\delta^{(4)}(x-y) - m'(x,y) \right] G(y,z) = i\delta^{(4)}(x-z) \tag{3.20}$$

Now let us assume that the oscillations m'(x,y) are sufficiently small as to permit a perturbation expansion for $G(x,y)$:

$$G(x,y) = G_M(x,y) - i\left(G_M m' G_M\right)(x,y) - \left(G_M m' G_M m' G_M\right)(x,y) + \ldots \tag{3.21}$$

where $G_M(x,y)$ are the usual propagators of a free fermion of mass M.

$$G_M(x,y) \equiv G_M(x-y) \equiv \int \frac{d^4p}{(2\pi)^4}\, e^{-ipx} \frac{i}{p\!\!\!/ - M}$$

Using this expansion, the action (3.17) takes the form+)

$$\mathcal{A}[m', \eta, \bar{\eta}, j^\nu] = \mathcal{A}_1[m'] + \mathcal{A}_2[m'] + \mathcal{A}_{int}[m'] + \mathcal{A}_{ext}[m', \eta, \bar{\eta}, j^\nu] \tag{3.22}$$

with $\mathcal{A}_1$ denoting the term linear in the field m'(x,y)

$$\mathcal{A}_1[m'] \equiv \int dx\,dy\; \mathrm{tr}\left[G_M(x-y)m'(x,y) - \xi^{-1} m'(x,y)\, m_0 \delta(x-y)/ig^2 D(x-y) \right] \tag{3.23}$$

+) A trivial additive constant has been dropped.

and $\mathcal{A}_2$ being quadratic in m'

$$\mathcal{A}_2[m'] \equiv \int dx\,dy\, \mathrm{tr}\left[\tfrac{i}{2} G_M m\, G_M m')(x,y) - \tfrac{1}{2}\xi^{-1} m'(x,y)\, m'(y,x)/ig^2 D(x-y)\right] \tag{3.24}$$

The term $\mathcal{A}_{int}[m']$ collects all remaining powers in m'

$$\mathcal{A}_{int}[m'] \equiv \int dx\, \mathrm{tr}\left[-\sum_{n=3}^{\infty} \frac{(-i)^{n+1}}{n} (G_M m')^n (x,x)\right] \tag{3.25}$$

The last piece $\mathcal{A}_{ext}$, finally, contains all interactions with the external sources

$$\mathcal{A}_{ext}[m', \eta, \eta, j^\nu] \equiv \int dx\,dy \Big\{ i\bar{\eta}(x) G(x,y) \eta(y)$$

$$-\frac{i}{2} g^2 D(x-y)\, j^\nu(x)\, j_\nu(y) \tag{3.26}$$

$$-\frac{2}{D(0)} V^\nu(x,x) D(x-y)\, j_\nu(y) - ig^2 \frac{\delta^{(4)}(0)\delta^{(4)}(x-y)}{D(0)} \int dz\,dz'\, D(z-x) D(y-z')\, j^\nu(z)\, j_\nu(z') \Big\}$$

For the quantization we shall adopt an interaction picture. As usual, the quadratic part of the action, $\mathcal{A}_2[m']$, serves for the construction of free-particle Hilbert space. According to the least action principle, the free equation of motion are obtained from $\delta\mathcal{A}_2[m']/\delta m'(x,y) = 0$ rendering

$$m'(x,y) = g^2 \xi\, D(x-y) (G_M\, m' G_M)(x,y) \tag{3.27}$$

Going to momentum space

$$m''(p_2,p_1) \equiv \int dx_2 dx_1 \, e^{i(x_2 p_2 - x_1 p_1)} m'(x_2,x_1)$$

and introducing relative and total momenta

$$P \equiv (p_2+p_1)/2 \;, \; q \equiv (p_2-p_1)/2$$

together with the notation

$$m'(P|q) \equiv m''(p_2,p_1)$$

the field equation becomes

$$m'(P|q) = \xi g^2 \int \frac{d^4P'}{(2\pi)^4} D(P'-P) G_M(P'+\tfrac{q}{2}) m'(P'|q) G_M(P'-\tfrac{q}{2}) \tag{3.28}$$

In this form we easily recognize the Bethe Salpeter equation[11] in ladder approximation for the vertex functions of quark-antiquark bound states

$$\Gamma^H(P|q) \equiv N_H G_M(P+\tfrac{q}{2}) \int dz \, e^{iPz} \langle 0|T\,\psi(\tfrac{z}{2})\bar{\psi}(-\tfrac{z}{2})|0\rangle G_M(P-\tfrac{q}{2}) \tag{3.29}$$

where N_H is some normalization factor. As a consequance our free field m'(x,y) can be expanded in a complete set of ladder bound state solutions. These are the bare quanta spanning the Hilbert space of the interaction picture. Because of their bound quark-antiquark nature, they will be called "bare hadrons". In the special case of QED, "quarks" are electrons and the bare hadrons" are positronium atoms.

For mathematical reasons it is convenient to solve (3.28) for fixed $q^2 \in (0, 4M^2)$ and all possible coupling constants g^2, to be called $g_H^2(q^2)$, i.e.

$$\Gamma^H(P|q) = \xi g_H^2(q^2) \int \frac{d^4P'}{(2\pi)^4} D(P-P') G_M(P'+\tfrac{q}{2}) \Gamma^H(P'|q) G_M(P'-\tfrac{q}{2}) \tag{3.30}$$

A useful normalization condition is

$$-i \int \frac{d^4P}{(2\pi)^4} \mathrm{tr}\left[G_M(P+\tfrac{q}{2}) \Gamma^{H'}(P|q) G_M(P-\tfrac{q}{2}) \bar{\Gamma}^{H'}(P|q)\right] = \varepsilon^H \delta^{HH'} \tag{3.31}$$

Here we have allowed for a sign factor $\varepsilon^H(q)$ which cannot be absorbed in the normalization N_H of (3.29). It may take the values +1, -1 or zero. Then the expansion of the free field m'(x,y) in terms of hadron creation and annihilation operators $a_H^\dagger(q), a_H(q)$ can be written as

$$m'_{\alpha\beta}(x,y) = \int \frac{d^4q}{(2\pi)^4} \sum_H \delta^{(+)}(g_H^2(q^2) - g^2) \int \frac{d^4P}{(2\pi)^4} \tag{3.32}$$
$$\left\{ e^{-i(q(x+y)/2 + P(x-y))} \Gamma^H(P|q) n_H a_H(q) \right.$$
$$\left. e^{i(q(x+y)/2 - P(x-y))} \bar{\Gamma}^H(P|-q) n_H^* a_H^\dagger(q) \right\}$$

where n_H are appropriate factors giving $a_H(q)$ the standard normalization

$$\left[a_H(q), a_{H'}^\dagger(q')\right] = (2\pi)^3 \delta^{(3)}(\mathbf{q}-\mathbf{q}') 2\omega^H(q) \varepsilon^H(q) \tag{3.33}$$

Now the sign factor $\varepsilon^{H}(q)$ appears at the norm of the hadronic state $a_H^{+}|0\rangle \equiv |H\rangle$. In general there will be many states with unphysical norms since the "bare hadrons" are produced by ladder diagrams only and may not be directly related to physical particles. This situation presents no fundamental difficulty. There are many interactions among bare hadrons which are capable of excluding unphysical states from the S-matrix. In fact, the equivalence of the hadronized theory to the healthy original quark gluon version is a guarantee for physical results (on shell).

The propagator of the free field m'(x,y) can be found most directly by adding an external disturbance to the free action

$$\mathcal{A}_2[m'] \to \mathcal{A}_2[m'] - \int dx\,dy\ \mathrm{tr}\left[m'(x,y)J(y,x)\right] \quad (3.34)$$

This current enters the equation of motion as

$$\begin{aligned} m'(x,y) &= \xi g^2 D(x-y)\left(G_M m' G_M\right)(x,y) \\ &\quad - \xi i g^2 D(x-y) J(x,y) \end{aligned} \quad (3.35)$$

The propagator $G_{\alpha\beta,\alpha'\beta'}(xy,x'y') = \overbrace{m'_{\alpha\beta}(x,y)\, m'_{\alpha'\beta'}}(x',y')$ is then defined as the solution of (3.35) for the δ-function disturbance

$$J_{\alpha\beta}(x,y) = i\,\delta(x-x')\,\delta(y-y')\,\delta_{\alpha\alpha'}\delta_{\beta\beta'} \quad (3.36)$$

It satisfies the inhomogeneous Bethe-Salpeter equation

$$\mathcal{G}_{\alpha,\beta,\alpha'\beta'}(x,y;x',y') = \xi_{\alpha\beta,\alpha'\beta'}\,D(x-y)\int d\bar{x}\,d\bar{y}\,G_M(x-\bar{x})_{\alpha\bar{\alpha}}$$

$$\mathcal{G}_{\bar{\alpha}\bar{\beta},\alpha'\beta'}(\bar{x},\bar{y};x',y')\,G_M(\bar{y}-y)_{\bar{\beta}\beta} + \xi_{\alpha\beta,\beta'\alpha'}\,g^2 D(x-y)\delta(x-x')\delta(y-y') \tag{3.37}$$

This is immediately recognized as the equation for the two-quark trasition matrix in ladder approximation (see equ. (A.19) in App. A .

An explicite representation of the Green's function in terms of the solutions $\Gamma^H(P|q)$ of the homogeneous equation (3.30) can now be given.

If $\mathcal{G}_{\alpha\beta,\alpha'\beta'}(P,P'|q)$ denotes the Fourier transform

$$(2\pi)^4\,\delta^{(4)}(q-q')\,\mathcal{G}_{\alpha\beta,\alpha'\beta'}(P,P'|q) \tag{3.38}$$

$$\equiv \int dx\,dy\,dx'\,dy'\,e^{i[P(x-y)+q(x+y)/2-P'(x'-y')-q'(x'+y')/2]}\,\mathcal{G}_{\alpha\beta,\alpha'\beta'}(x,y;x',y')$$

it can be written as the sum over all hadron solutions:

$$\mathcal{G}_{\alpha\beta,\alpha'\beta'}(P,P'|q) = -iq^2\sum_H \varepsilon_H(q)\,\frac{\Gamma^H_{\alpha\beta}(P|q)\,\bar{\Gamma}^H_{\beta'\alpha'}(P'|-q)}{q_H^2(q^2)-q^2} \tag{3.39}$$

where the sum comprises possible integrals over a continuous set of solutions. If quarks and gluons were scalars, the sum would be discrete for $q^2 \in (0, 4M^2)$ since the kernel of the integral equation (3.30) would be of the Fredholm type. A more detailed discussion is given in Appendix A. Here we only note that a power series expansion of the denominator

$$\mathcal{G}_{\alpha\beta,\alpha'\beta'}(P,P'|q) = -i \sum_{n=1}^{\infty} \sum_{H} \left(\frac{q^2}{q_H^2(q^2)}\right)^n \varepsilon_H(q)\, \Gamma^H_{\alpha\beta}(P|q)\, \bar{\Gamma}^H_{\beta'\alpha'}(P'|-q) \qquad (3.40)$$

renders explicit the exchange of one, two, three etc. gluons. Hence one additional gluon can be inserted (or removed) by multiplying (or dividing) (3.39) by a factor $q^2/q_H^2(q^2)$. This fact will be of use later on.

Seen microscopically in terms of quarks and gluons, the free hadron propagator (3.39) is given by the sum of ladders (see Fig. 1)

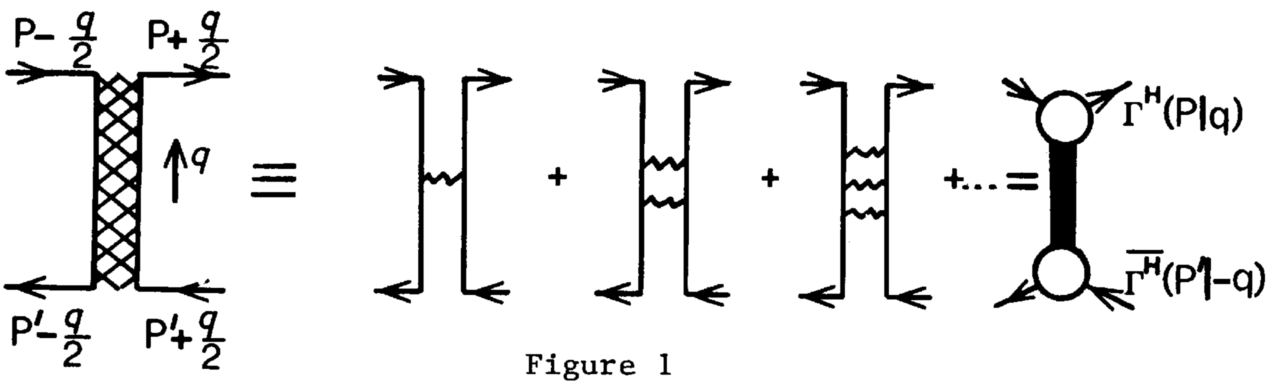

Figure 1

Graphically it will be represented by a wide band. In the last term of Fig 1 we have also given a visualisation of the expansion (3.39). Here the fat line denotes the propagator

$$\Delta_H(q) = -i\,\varepsilon_H(q)\,\frac{g^2}{g_H^2(q^2)-g^2} \tag{3.41}$$

while upper and lower bubbles stand for the Bethe Salpeter vertices $\Gamma^H(P|q)$ and $\bar{\Gamma}^H(P'|-q)$, respectively. This picture suggests another way of representing the new bilocal theory in terms of an <u>infinite component</u> hadron field depending only on the average position $X=(x+y)/2$. For this we simply expand the <u>interacting</u> field $m'(P|q)$ in terms of the complete set of free vertex function

$$m'(P|q) = \sum_H \Gamma^H(P|q)\, m_H(q) \tag{3.42}$$

Inserting this expansion into (3.22), the free action becomes directly

$$\mathcal{A}_2[m'] = \frac{1}{2}\int dX\, m_H(X)\left(1-g_H^2(q^2)/g^2\right)m_H(X) \tag{3.43}$$

implying the free propagator (3.41) for the field $m_H(X)$. With this understanding of the free part of the action we are now prepared to interprete the remaining pieces.

Consider first the linear part $\mathcal{A}_1[m']$. The first term in it can graphically be represented as whown in Fig. 2 . When attached to other hadrons it produces a tadpole correction.

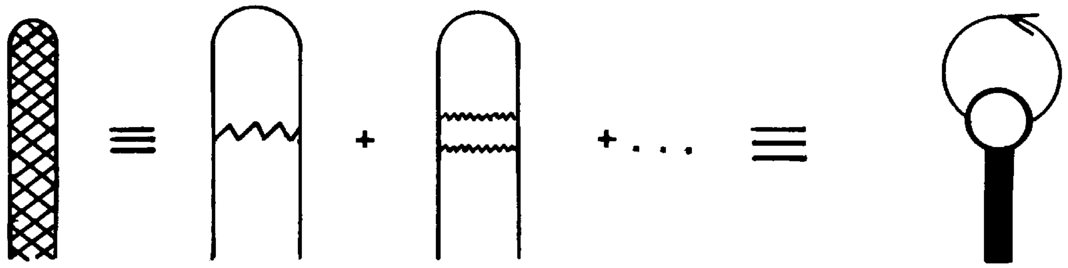

Figure 2

When interpreted within the underlying quark gluon picture, such a correction sums up all rainbow contributions to the quark propagator. Also the second term in $\mathcal{A}_1[m']$ has a straight-forward interpretation. First of all, the division by $\xi i g^2 D(x-y)$ has the effect of removing one rung from the ladder sum (such that the ladder starts with no rung, one rung, etc.) and creating two open quark legs. This can be seen directly from (3.30) and (3.39): Suppose a hadron line ends at the interaction $-\int dx dy \, \mathrm{tr}\,[m'(x,y)\xi^{-1} m_0]/ig^2 D(x-y)\delta(x-y)$

Then the factor $[\xi i g^2 D(x-y)]^{-1}$ applied to $\mathcal{G}(P,P'|q)$

gives (leaving out irrelevant indices)

$$[\xi g^2 D]^{-1}\mathcal{G} = -ig^2 \sum_H \varepsilon_H \frac{[\xi g^2 D]^{-1}\Gamma^H \bar{\Gamma}^H}{g_H{}^2(q^2) - g^2} \quad (3.43)$$

Using (3.30) this yields

$$= -ig^2 \sum_H \varepsilon_H \frac{g_H{}^2(q^2)}{g^2} \frac{(G_M \Gamma^H G_M)\bar{\Gamma}^H}{g_H{}^2(q^2) - g^2} \quad (3.44)$$

As discussed before, the factor $g_H{}^2(q^2)/g^2$ amounts to the removal of one rung. Multiplication by $-m_0$ and integration over $\int dP/(2\pi)^4$ yields the total contribution of this hadron graph

$$i m_o \sum_H \frac{g_H^2(q^2)}{g_H^2(q^2)-g^2} \int \frac{d^4P}{(2\pi)^4} \mathrm{tr}\left[G_M(P+\tfrac{q}{2})\Gamma^H(P|q)G_M(P-\tfrac{q}{2})\right]$$
$$\times \varepsilon_H(q)\bar{\Gamma}^H(P'|-q) \quad (3.45)$$

As far as quarks and gluons are concerned, this amounts to the insertion of a mass term m_o on top of a ladder graph with one rung removed (this being indicated by a slash in Fig. 3 .

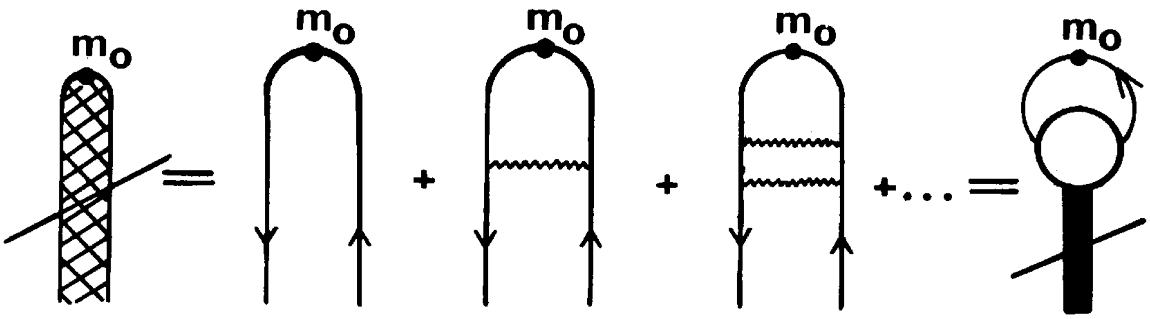

Figure 3

The quark gluon picture leads us to expect that m_o must be a cutoff dependent quantity cancelling the logarithmic divergence in every upper loop of the ladder sum of Fig. 2 . Numerically, m_o is most easily calculated by cancelling the infinite contributed by $\mathcal{A}_1[m']$ to the equation of motion (3.28). If we include $\mathcal{A}_1[m']$, this equation reads

$$m_o(2\pi)^4\delta^{(4)}(q) + m'(P|q) = \left[\xi i g^2 \int \frac{d^4P'}{(2\pi)^4} D(P-P')G(P')\right](2\pi)^4\delta^{(4)}(q) \quad (3.46)$$
$$+ \xi g^2 \int \frac{d^4P'}{(2\pi)^4} D(P-P')G_M(P'+\tfrac{q}{2})m'(P'|q)G_M(P'-\tfrac{q}{2})$$

The first term on the right-hand side is exactly the usual self energy $\Sigma(P)(2\pi)^4\delta^{(4)}(q)$ in second order

$$\Sigma_{\alpha\beta}(P) \equiv -\xi_{\alpha\beta,\gamma\delta}\, i\int \frac{d^4P'}{(2\pi)^4}\frac{1}{(P-P')^2-\mu^2}\,\frac{1}{\not{P}'-M} \tag{3.47}$$

Normalizing $\Sigma(P)$ on mass shell one finds the usual expression

$$\Sigma(P) = \Sigma_0 + \Sigma_1(\not{P}-M) + \Sigma_R(\not{P}) \tag{3.48}$$

where Σ_R is the regularized self-energy.
The cutoff dependent term

$$\Sigma_0 = \frac{3}{4\pi}\,\frac{g^2}{4\pi}\,M\left(\log \Lambda^2/M^2 + \tfrac{1}{2}\right) \tag{3.49}$$

must be balanced by choosing $m_0 = -\Sigma_0$ on the left had side of (3.46). Also the second term Σ_1 is cutoff dependent:

$$\Sigma_1 = \frac{1}{4\pi}\,\frac{g^2}{4\pi}\left(\log \Lambda^2/M^2 + \frac{9}{2} + 2\log \mu^2/M^2\right) \tag{3.50}$$

and a renormalization is necessary to cancel this infinity. Most economic is the introduction of an appropriate wave function counter term $(Z_2-1)\bar{\Psi}(i\not{\partial}-M)$ in the original Lagrangian (3.1). Such a term would enter equ. (3.15) as

$$\int dy\,\left\{(i\not{\partial}-\mathcal{M})\,\delta^{(4)}(x-y) + (Z_2-1)(i\not{\partial}-M)\,\delta^{(4)}(x-y) - m(x,y)\right\}G(y,z) = i\,\delta^{(4)}(x-z) \tag{3.51}$$

Instead of (3.18), m(x,y) should now be assumed to oscillate around $[m_0 + (Z_2^{-1}-1)(i\partial\!\!\!/ - M)]\,\delta(x-y)$.
By defining a new $m'(x,y)$ via

$$m'(x,y) \equiv m(x,y) - \left[m_0 + \left(Z_2^{-1} - 1\right)(i\partial\!\!\!/ - M)\right]\delta^{(4)}(x-y), \tag{3.52}$$

the full action (3.22) is obtained exactly as before except for the linear part $\mathcal{A}$ in which the new wave function renormalization term enters together with m_0:

$$\begin{aligned}\mathcal{A}_1[m'] = \int dx\,dy\ \mathrm{tr}\Big\{ & G_M(x-y)\,m'(x,y) \\ & - \xi^{-1}\,m'(x,y)\left[m_0 + \left(Z_2^{-1} - 1\right)(i\partial\!\!\!/ - M)\right]\delta(x-y)/ig^2 D(x-y)\Big\}\end{aligned} \tag{3.53}$$

By choosing

$$Z_2^{-1} - 1 = -\Sigma_1 \tag{3.54}$$

the cutoff dependent term Σ_1 is exactly compensated in the equation of motion (3.46). After this renormalization procedure, only the finite term $\Sigma_R(P)$ is left. The regularized action is

$$\mathcal{A}_1[m']_R = -\int dx\,dy\ \Sigma_R(x-y)\,m'(x,y)/ig^2 D(x-y) \tag{3.55}$$

Using the expansion (3.42), this can be rewritten as

$$\mathcal{A}_1[m']_R = \sum_H \int dX\, f_H(-\Box)\, m_H(X) \tag{3.56}$$

with

$$f_H(q^2) = i \int \frac{d^4P}{(2\pi)^4} \, \mathrm{tr}\left[\Sigma_R(P) G_M(P+\tfrac{q}{2}) \Gamma^H(P|q) G_M(P-\tfrac{q}{2})\right] \frac{g_H^2(q^2)}{q^2} \tag{3.57}$$

By momentum conservation, the tadpole momentum always vanishes such that only $f_H(0)$ is needed eventually.

Let us now proceed to the discussion of the interaction part $\mathcal{A}_{int}[m']$ of equ. (3.25). Take as an example the term of the third order in m'. If a hadron line ends at every m', it can be represented graphically as shown in Fig. 4

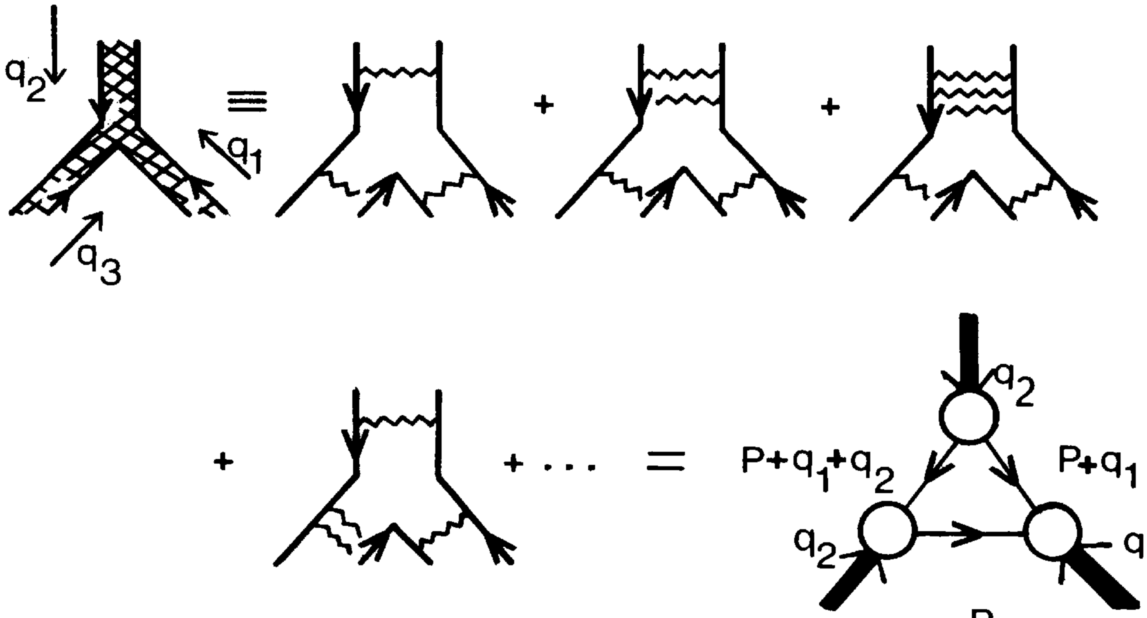

Figure 4

Employing the expansion (3.42), this interaction term can be rewritten as

$$\mathcal{A}_{int}^{3\,hadr}[m'] = -\frac{1}{3}\sum_{H_1 H_2 H_3}\int \frac{d^4q_3}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{d^4q_1}{(2\pi)^4}(2\pi)^4\delta(q_1+q_2+q_3)$$

$$\times\int\frac{d^4P}{(2\pi)^4}\,\mathrm{tr}\left[\Gamma^{H_3}(P-\tfrac{q_3}{2}|q_3)G_M(P+q_1+q_2)\Gamma^{H_2}(P+q_1+\tfrac{q_2}{2}|q_2)\right. \tag{3.58}$$

$$\left.G_M(P+q_1)\Gamma^{H_1}(P+\tfrac{q_1}{2}|q_1)G_M(P)\right]m_{H_3}(q_3)m_{H_2}(q_2)m_{H_1}(q_1)$$

$$=\frac{1}{3}\sum_{H_1H_2H_3}\int dX\, v_{H_3H_2H_1}(i\partial_X^{H_3}, i\partial_X^{H_2}, i\partial_X^{H_1})\, m_{H_3}(X)m_{H_2}(X)m_{H_1}(X)$$

with a vertex function $v_{H_3H_2H_1}(i\partial_X^{H_3}, i\partial_X^{H_2}, i\partial_X^{H_1})$ whose derivatives $\partial_X^{H_i}$ are to be applied only to the argument of the corresponding field $m_{H_i}(X)$. A corresponding formula holds for every power of m'.

Notice that the flow of the quark lines in every interaction is anticlockwise. When drawing up hadronic Feynman graphs it may sometimes be more convenient to draw a clockwise flow. A simple identity helps to write down directly the corresponding Feynman rules. Consider a graph for a three hadron interaction and cross the upper band downwards (see Fig. 5). The interaction appears now with the hadronic bands in anticyclic order, and the fermion lines in the hadron vertex flowing clockwise. This is topologically compensated by twisting every band once. Mathematically, this deformation displays the

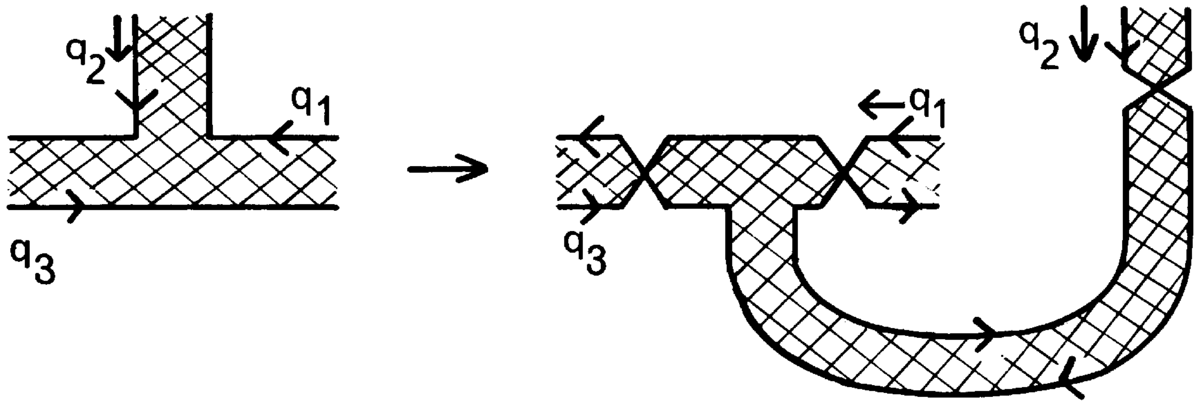

Figure 5

following identity of the vertex functions

$$v_{H_3 H_2 H_1}(q_3, q_2, q_1) = \eta_{H_3} \eta_{H_2} \eta_{H_1} v_{H_1 H_2 H_3}(q_1 q_2 q_3) \quad (3.59)$$

where the phase η_H denotes the charge parity of the hadron H. This phase may be absorbed in the propagator characterizing the twisted band.

The proof of this identity (3.59) is quite simple. Let C be the charge conjugation matrix. Then the vertices satisfy:

$$C\Gamma^H(P|q)C^{-1} = \eta_H \Gamma^H(-P|q)^T \quad (3.60)$$

Inserting now CC^{-1} between all factors in (3.58) and observing $C\gamma^\mu C^{-1} = -\gamma^{\mu T}$ one has

$$v_{H_3 H_2 H_1}(q_3,q_2,q_1) = -\eta_{H_3}\eta_{H_2}\eta_{H_1} \int \frac{dP}{(2\pi)^4} \operatorname{tr}\left\{ \Gamma^{H_3}\left(-P+\frac{q_3}{2}\middle|q_3\right)^T \right.$$
$$\left. \left(\frac{i}{-\not{P}-\not{q}_1-\not{q}_2-M}\right)^T \Gamma^{H_2}\left(-P-q_1-\frac{q_2}{2}\middle|q_2\right)^T \left(\frac{i}{-\not{P}-\not{q}_1-M}\right)^T \Gamma^{H_1}\left(-P-\frac{q_1}{2}\middle|q_1\right)\left(\frac{i}{\not{P}-M}\right)^T \right\}$$

Taking the transpose inside the trace and changing the dummy variable P to $-P$, the vertices appear in anti-cyclic order and the right hand side coincides indeed with $\eta_{H_3}\eta_{H_2}\eta_{H_1} v_{H_1 H_2 H_3}(q_1,q_2,q_3)$. Twisted propagators are physically very important. They describe the strong rearrangement collisions of quarks and certain classes of cross-over gluon lines. Fig. 6 shows some twisted graphs together with thei quark gluon contents. In meson scattering rearrangement collisions (Fig 6a) have roughly the same coupling strength as direct (untwisted) exchanges. In QED, on the other hand, they provide for the main molecular binding forces.

The exchange of two twisted hadron lines (Fig 6b) seems to be an important part of diffraction scattering (Pomeron).

Two more examples are shown in Fig. 7. Notice that in the pseudoscalar channel these graphs incorporate the effect of the Adler triangle anomaly.

In this connection it is worth pointing out that all fundamental hadron vertices are planar graphs as far as the quark lines are concerned. Non-planar graphs are generated by building up loops involving twisted

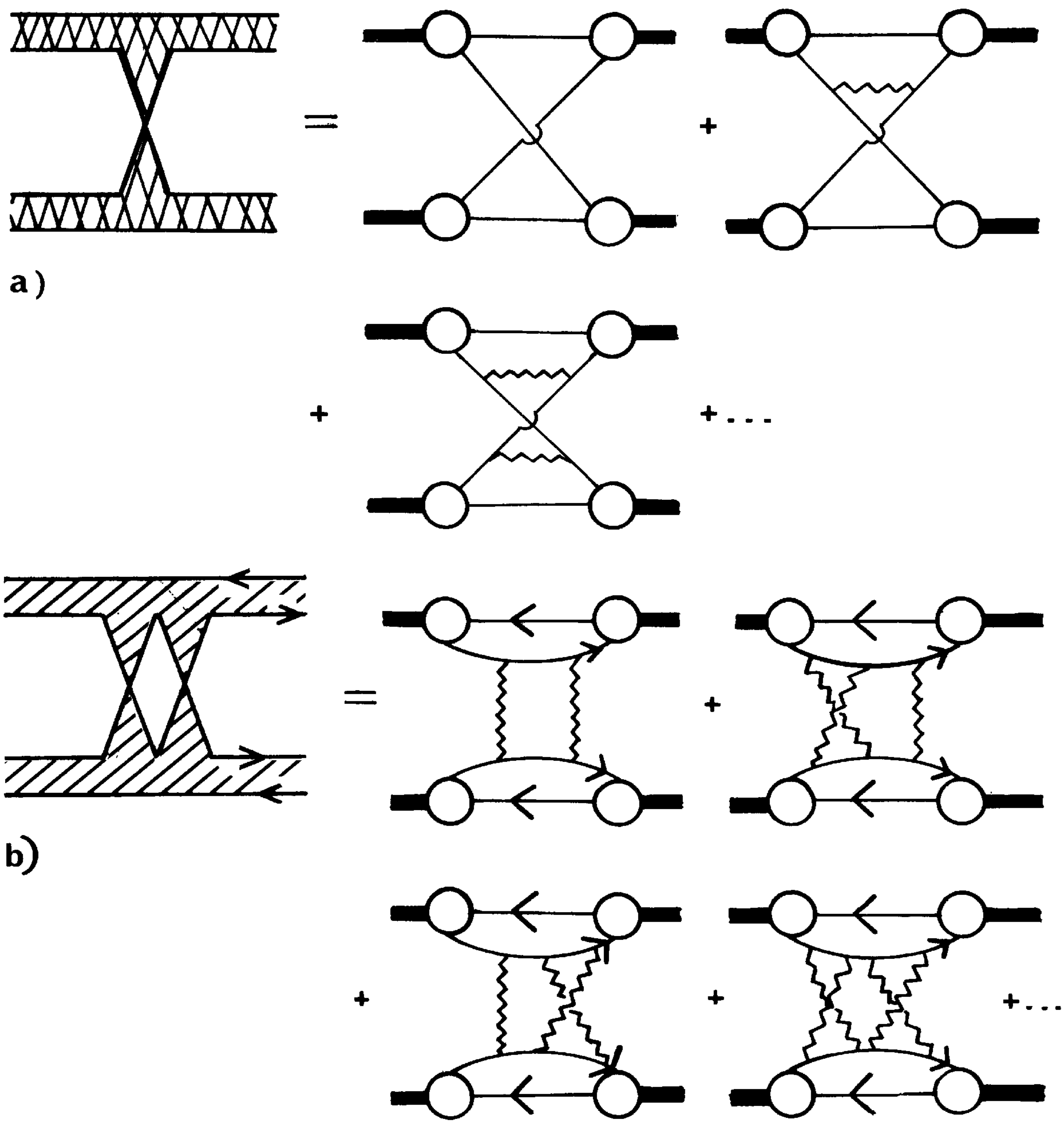

Figure 6

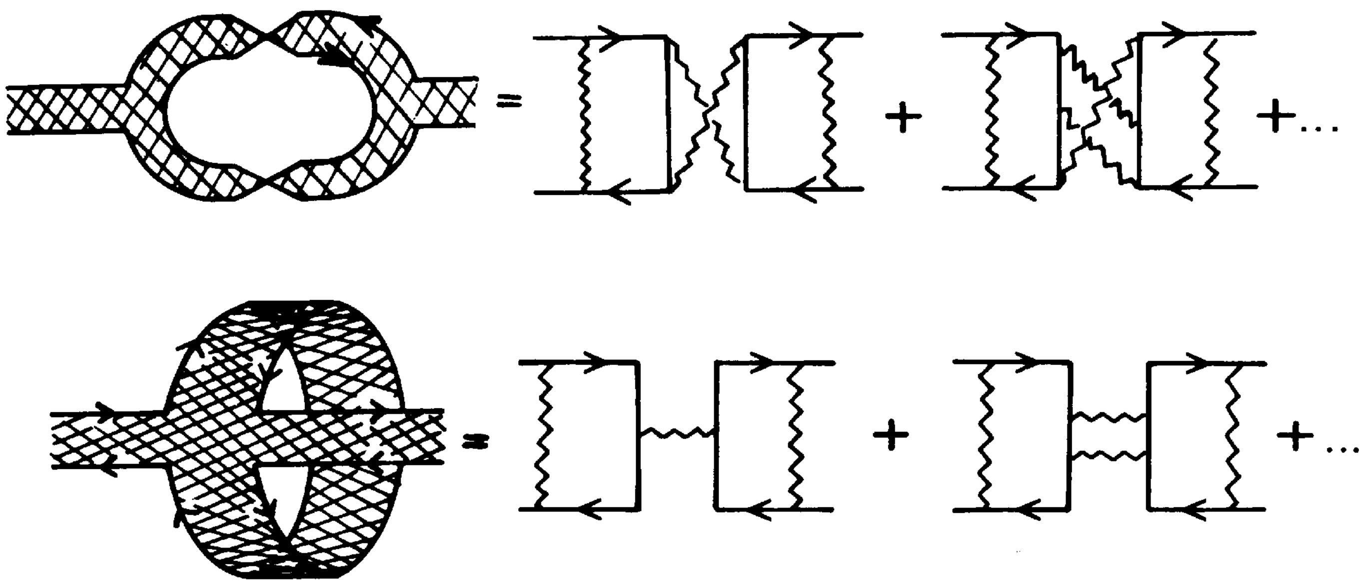

Figure 7

propagators.

With propagator bands, their twisted modifications and planar fundamental couplings hadron graphs are seen to possess exactly the same topology as the graphs used in dual models[12)] except for the stringent dynamical property of duality itself: In the present hadronized theory one still must sum s and t channel exchanges and they are by no means the same. Only after introduction of color and the ensuing linearly rising mass spectra one can hope to account also for this particular aspect of strong interactions.

The similarity in topology should be exploited for a model study of an important phenomenon of strong interactions: the Okubo, Zweig, and Iizuka rule. Obviously all hadron couplings derived by hadronization exactly respect this rule. All violations have to come from graphs of the so called cylinder type[13] (for example Fig 6b). If it is true that the topological expansion[12] is the correct basis for explaining this rule[+)],it may also provide the appropriate systematics for organizing the hadronized perturbation expansion.

Let us finally discuss the external souces. From $\mathcal{L}_{ext}$ in (3.36) we see that external fermion lines are connected via the full propagator G wich after expansion in powers of m' amounts to radiation of any number of hadrons (see Fig. 8)

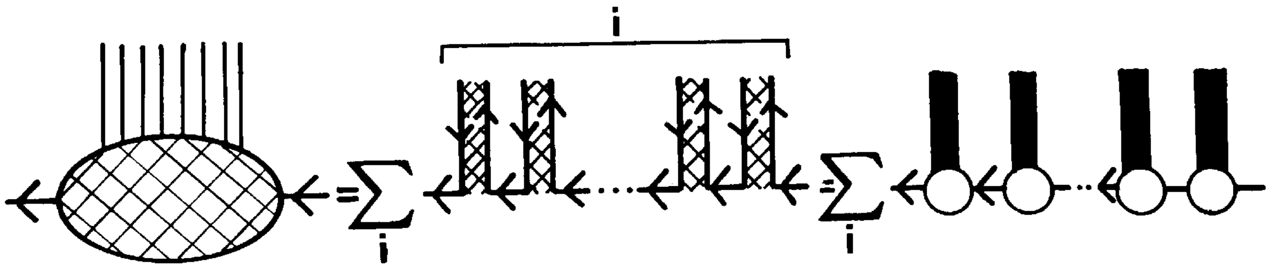

Figure 8

These hadrons then interact among each other as quantum fields. Diagrammatically, every bubble carries again a factor $\Gamma^{H}(P|q)$.

It has to be watched out that hadrons are always emitted to the right of each line. For example, the lowest order quark-quark scattering amplitude should initially be drawn as shown in Fig 9 in order to avoid phase errors due to twisted bands. Then the graphical rules yield directly the expression (3.39) as they should. Afterwards, arbitrary deformations can be performed if all twisted factors η_H are respected.

+) See the forth of Ref. 14).

External gluons interact with hadrons according to the third term in equ. (3.26)

$$-\frac{2}{g^2 D(0)}\int dx\,dy\; V^{\nu}(x,x)\, g^2 D(x-y)\, j_{\nu}(y) \tag{3.63}$$

Hence every external gluon enters the hadronic world only via an intermediate vector particle and there is a current field identity as has been postulated in phenomenological treatments of vector mesons (VMD). Here one finds a non-trivial coupling between the gluon and the vector mesons: As discussed before, the division by $g^2 D$ amounts to a removal of one rung from the ladder of the incoming hadron propagator and takes care of the direct coupling of the gluon to the quarks <u>without</u> the ladder corrections. This effect was shown to be accounted for a factor $g_H^2(q^2)/g^2$ in the propagator sum (3.39). Thus the direct coupling of the vector meson field $m_H(x)$ to an external gluon field $G^{ext}_{\nu}(x)$ can be written as:

$$g \sum_H \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4P}{(2\pi)^4}\, \mathrm{tr}\left\{\gamma^{\nu} G_M(P+\tfrac{q}{2})\Gamma^H(P|q) G_M(P-\tfrac{q}{2})\right\} \times \frac{g_H^2(q^2)}{q^2}\, m_H(q) G^{ext}_{\nu}(-q) \tag{3.64}$$

In a hadronic graph, the removal of one rung will be indicated by a slash. As an example, the lowest order contribution to the quark gluon form factor is illustrated in Fig 10. The slash guarantees the presence of the direct coupling. The free propagator of external gluon is given by the second term of equ. (3.26). The lowest radiative corrections consist in an intermediate slashed vector mesons (see Fig 11).

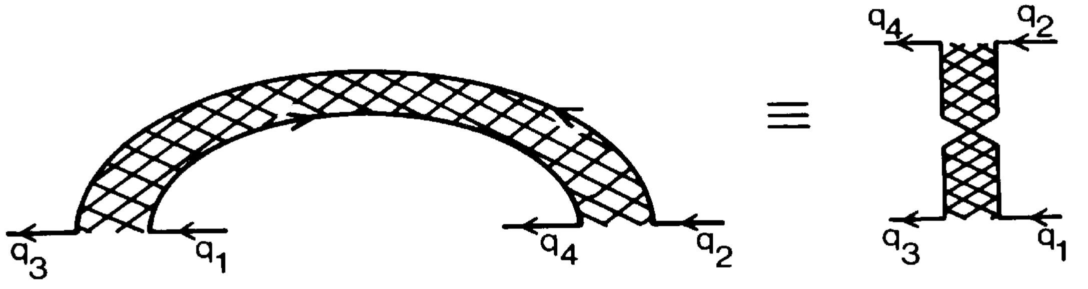

Figure 9

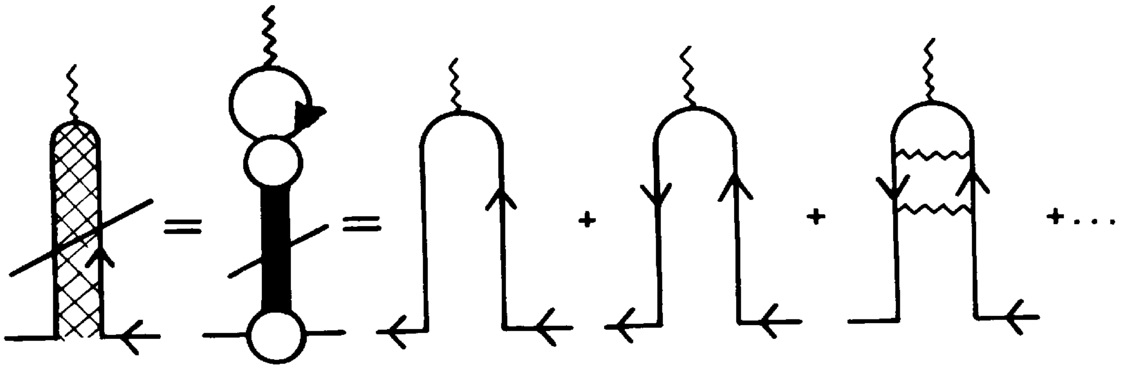

Figure 10

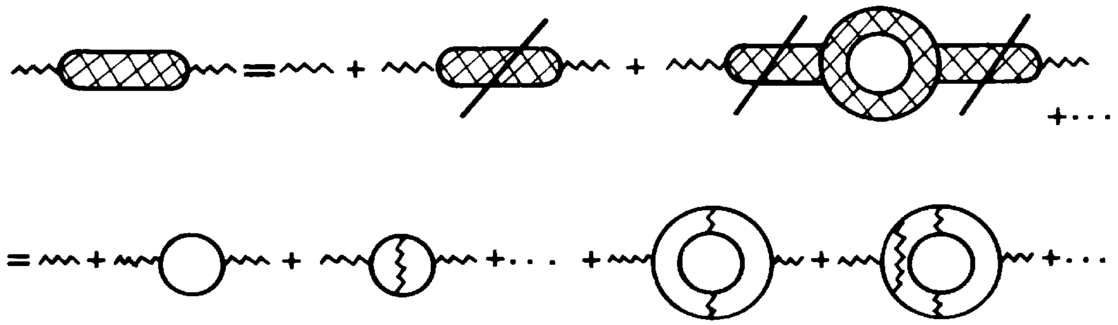

Figure 11

Here the slash is important to ensure the presence of one single quark loop.

The divergent last term in the external action (3.26) has no physical significance since it contributes only to the external gluon mass and can be cancelled by an appropriate counter term.

A final remark concerns the bilocal currents as measured in deep inelastic electron and neutrino scattering. These are vector currents of the type

$$j^{\nu}(x,y) = \bar{\Psi}(x)\gamma^{\nu}\Psi(y) \qquad (3.65)$$

It is obvious, that also for bilocal currents there is a current-field identity with the bilocal field $V^{\mu}(x,y)$. In fact, if one would have added an external source term $C_{\nu}(x,y)$ in the quark action :

$$\Delta \mathcal{A}_{ext} \equiv \int dx dy\, \bar{\Psi}(x)\gamma^{\nu}\Psi(y)\, C_{\nu}(x,y) \qquad (3.66)$$

this would appear in the hadronized version in the form

$$\Delta \mathcal{A}_{ext} = \int dx dy \frac{1}{ig^2 D(x-y)} V^{\nu}(x,y)\, C_{\nu}(x,y) \qquad (3.67)$$

which proves our statement. Again, a rung has to be removed in order to allow for the pure quark contribution (see Fig 12)

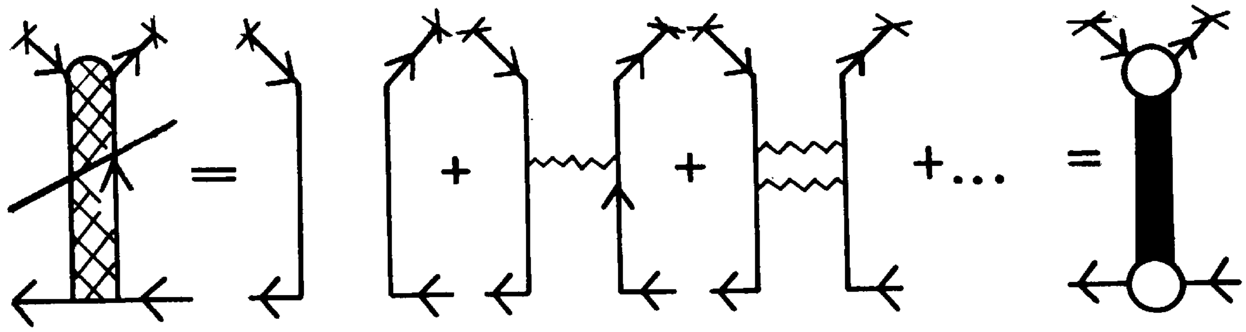

Figure 12

Bilocal currents carry direct information on the properties of Regge trajectories[15]. Therefore the present bilocal field theory seems to be the appropriate tool for the construction of a complete field theory of Reggeons[16], which is again equivalent to the original quark gluon theory. Technically, such a construction would proceed via analytic continuation of the propagators (3.39) in the angular momentum (and the principal quantum number) of the hadrons H. The result would be a "reggeonized" quark gluon theory. The corresponding Feynman graphs would guarantee unitary in all channels. Present attempts at such a theory enforces t channel unitarity only[17]. Also, they are asymptotically valid by construction and apparently have a chance of approximating nature only at energies unaccessible in the near future[+)].

IV. THE LIMIT OF HEAVY GLUONS

As an illustration of the hadronization procedure we now discuss in detail the limit of very heavy gluons[18,34]. Apart from its simplicity, this limit is quite attractive on physical grounds since it may yield a reasonable approximation to low energy meson interactions. This is suggested by the following arguments:

Suppose hadrodynamics follows a colored quark gluon theory. In this theory the color degree of freedom is very important for generating a potential between quarks rising at long distances which can explain the observed great number of high mass resonances. However, as far as low-energy interactions among the lowest lying mesons

+) See, for example, D. Amati and R. Jengo, Physics Letters B 54 (1974).

are concerned, color seems to be a rather superfluous luxury:

First, many fundamental aspects of strong interaction dynamics such as chiral SU(3) x SU(3) current algebra, PCAC (together with the low-energy theorems derived from both) and the approximate light cone algebra are independent of color.

Second, there is no statistics argument concerning the symmetry of the meson wave functions as there is for baryons[19].

Third, high-lying resonances are known to contribute very little in most dispersion relations of low-energy amplitudes. For example, the low-energy value of the isospin odd $\pi\pi$ scattering amplitude is given by a dispersion integral over the mesons ρ and σ with $\approx$ 90% accuracy[20]. Similarly, $\pi\rho$ scattering is saturated by the intermediate mesons π and A_1. By looking at all scattering combinations one can easily convince oneself that the resonances π, ρ, σ, A_1, form an approximately closed "subworld" of hadrons as far as dispersion relations are concerned. As a consequence, it would not at all be astonishing if the neglect of color in a quark gluon theory would not change the dynamics when restricting the attention to this hadronic "subworld"+)

+) There is one estimate concerning the electromagnetic decay of $\pi^0 \to \gamma\gamma$ which is based on short distance arguments and therefore depends on color[21]. However, the same decay can be estimated also via intermediate distance arguments, namely by using the coupling $\rho\omega\pi$ and vector meson dominance such that color does not come in.

The point is now that in the limit of a large gluon mass $\mu \to \infty$, exactly this restricted set of mesons appears as particles in the hadronized quark gluon theory (3.1) without color. Thus it might be considered as some approximation to the low-energy aspects of the colored version. Indeed, we shall see that the hadronized theory coincides exactly with the well known chirally invariant σ model. This model has proven in the past to be an appropriate tool for the rough description of low-energy meson physics[23]. Our derivation of the σ model via hadronization will render several new relations between meson and quark properties[18]. We shall at first confine ourselves to SU(2) quarks only, such that symmetry breaking may be neglected. The extension to broken SU(3) will be performed afterwards.

In order to start with the derivation observe that in the limit $\mu \to \infty$, the gluon propagator approaches a δ-function:

$$i D(x-y) \to \frac{1}{\mu^2} \delta(x-y) \tag{4.1}$$

The equation of motion (3.27) forces m'(x,y) to become a local field m'(x):

$$m'(x,y) \to m'(x)\, \delta(x-y) \tag{4.2}$$

which satisfies the free field equation

$$m'(x) = -i \frac{g^2}{\mu^2} \xi \int dy\, G_M(x-y)\, m'(y)\, G_M(y-x) \tag{4.3}$$

In the local limit, the action without external sources takes the form

$$\mathcal{A}[m'] = \int dx \, \mathrm{tr} \Big\{ G_M(x,x)\, m'(x) - \tfrac{1}{2}(G_M m' G_M m')(x,x)$$

$$+ \sum_n \frac{(-i)^{n-1}}{n} (G_M m')^n(x,x) - \frac{\mu^2}{g^2}\frac{1}{2\xi} m'(x)^2 - \frac{1}{\xi} m'(x)\, m_0 \Big\} \tag{4.4}$$

where $(G_M m' G_M m')(x,x)$ stands short for $\int dy\, G_M(x-y)\, m'(y)\, G_M(y-x)$ etc. As before in the general discussion, the constant m_0 is determined by the vanishing of the tadpole parts in (4.4) which amounts to balancing the constant contributions in the wave equation. Due to the singularity of $G_M(x-y)$ for $x \to y$ this condition has a meaning only if a cutoff is introduced such that $G_M(0)$ is finite:

$$[G_M(0)]_{\alpha\beta} = \int \frac{d^4p}{(2\pi)^4} \left[\frac{i}{\not p - M}\right]_{\alpha\beta} = M \int_0^\Lambda \frac{d^4 p_E}{(2\pi)^4} (p_E^2 + M^2)^{-1} \delta_{\alpha\beta} \tag{4.5}$$

$$= M \frac{\pi^2}{(2\pi)^4} (\Lambda^2 - M^2 \log \Lambda^2/M^2)\, \delta_{\alpha\beta} \equiv M\, Q\, \delta_{\alpha\beta}$$

Here the dP^0 integration has been Wick-rotated by 90° such that the momentum $P^\mu = (P^0, \underline{P})$ becomes $(iP^4, \underline{P})$ with $P^4 \in (-\infty, \infty)$ along the integration path. The new real momentum $(P^4, \underline{P})$ has been denoted by P_E^μ and its euclidean scalar product by $P_E^2 = P^{4\,2} + \underline{P}^2 = -P^2$. The tadpoles can now be cancelled by setting m_0 equal

$$m_0 = 4 \frac{g^2}{\mu^2} Q M \tag{4.6}$$

Remembering the relation to the bare quark mass $m_0 = M - \mathfrak{m}$, this determines the connection between the "true" quark mass M and the bare mass $\mathfrak{M}$ contained in the Lagrangian:

$$M = \mathfrak{M} + 4 \frac{g^2}{\mu^2} Q M \tag{4.7}$$

Equation (4.8) is often called "gap equation" because of its analoguous appearance in the theory of superconductivity[24].

Consider now the free part $\mathcal{A}_2[m']$ of the action. Performing again a decomposition of type (3.12) but with the local field m'(x), it can be written in the form

$$\mathcal{A}_2[m'] = \int dx \, \mathrm{tr}_{SU(2)} \Big\{ \tfrac{1}{2} m_i'(x) \, \mathcal{J}_{ij}(i\partial) \, m_j(x) - \frac{\mu^2}{2g^2} \big(S^2_{(x)} + P^2_{(x)} - 2V^2(x) - 2A^2(x) \big) \Big\} \tag{4.8}$$

where $m_i'(x)$ (i=1,2,3,4) stands short for the fields+) $S(x)$, $P(x)$, $V(x)$, $A(x)$ and the trace runs only over internal SU(2) indices. The coefficients $\mathcal{J}_{ij}(q)$ are given by the integrals

$$\mathcal{J}_{ij}(q) \equiv -4 \int \frac{d^4 P_E}{(2\pi)^4} \frac{1}{(P+\frac{q}{2})^2_E + M^2} \frac{1}{(P-\frac{q}{2})^2_E + M^2} t_{ij}(P|q) \tag{4.9}$$

where $t_{ij}(P|q)$ denotes the Dirac traces

$$t_{ij}(P|q) \equiv \frac{1}{4} \mathrm{tr}_{\mathrm{Dirac}} \Big\{ \Gamma_i \big(\not{P} + \tfrac{\not{q}}{2} + M \big) \Gamma_j \big(\not{P} - \tfrac{\not{q}}{2} + M \big) \Big\} \tag{4.10}$$

+) The Lorentz indices of V^ν and A^ν fields are suppressed.

with Γ_i (i=1,2,3,4) abbreviating the standard Dirac covariants 1, $i\gamma_5$, γ^ν, $\gamma^\nu\gamma_5$. The traces are displayed in Appendix A equ. (A.33). Some of them grow quadratically in P. The corresponding integrals $J_{ij}(q)$ are quadratically divergent for large cutoffs. The others diverge logarithmically. If one introduces the basic integral

$$L \equiv \int \frac{d^4P_E}{(2\pi)^4} \frac{1}{(P_E^2+M^2)^2} = \frac{\pi^2}{(2\pi)^4}\left(\log\frac{\Lambda^2}{M^2}-1\right) \tag{4.11}$$

the divergent parts of $J_{ij}(q)$ are (see App. A)

$$J_{SS}(q) = Q + L\left(\frac{q^2}{2} - 2M^2\right)$$

$$J_{PP}(q) = Q + L\,\frac{q^2}{2} \quad ; \quad J_{PA^\nu} = -iLMq^\nu$$

$$J_{V^\mu V^\nu}(q) = -\frac{1}{3}(q^2g^{\mu\nu} - q^\mu q^\nu)\,L \tag{4.12}$$

$$J_{A^\mu A^\nu}(q) = -\frac{1}{3}\left[(q^2g^{\mu\nu} - q^\mu q^\nu) - 6M^2g^{\mu\nu}\right]L \; ; \; J_{A^\mu P} = iLMq^\mu$$

with all other integrals vanishing.

If we neglect the finite contributions as compared with these divergent ones, the action $\mathcal{A}_2[m']$ is seen to corresponds to the local Lagrangian+)

+) Since $m(x)$ and $m'(x)$ differ only by a Dirac scalar constant $m_0 1_{\omega 3}$ there is no difference between primed and unprimed fields except for $S'(x) = S(x) - m_0$

$$\mathcal{L}(x) = \mathrm{tr}_{SU(2)} \Big\{ \tfrac{1}{2} S'(x)\Big[4Q - 2(\Box + 4M^2)L - \tfrac{\mu^2}{g^2}\Big]S'(x)$$
$$+ \tfrac{1}{2} P(x)\Big[4Q - 2\Box L - \tfrac{\mu^2}{g^2}\Big] P(x)$$
$$+ \tfrac{1}{2} V_\mu(x)\Big[\tfrac{4}{3}(\Box g^{\mu\nu} - \partial^\mu\partial^\nu)L + \tfrac{2\mu^2}{g^2}\Big]V_\nu(x)$$
$$+ \tfrac{1}{2} A_\mu(x)\Big[\tfrac{4}{3}(\Box g^{\mu\nu} - \partial^\mu\partial^\nu)L + 8M^2 g^{\mu\nu}L + \tfrac{2\mu^2}{g^2}\Big]A_\nu(x)$$
$$+ 2ML\,\big(\partial_\mu P(x) A^\mu(x) + A^\mu(x)\partial_\mu P(x)\big)\Big\} \qquad (4.13)$$

If we respect the gap equation (4.7) in this Lagrangian, the quadratically divergent terms Q can be eliminated. The mixed terms can be removed by introducing a new field $\hat{A}^\mu(x)$ via

$$A^\mu(x) = \hat{A}^\mu(x) + \lambda\,\partial^\mu P \qquad (4.14)$$

and fixing λ as

$$\lambda = -\,3M / m_A^2 \qquad (4.15)$$

where m_A^2 stands short for

$$m_A^2 = m_V^2 + 6M^2 \qquad (4.16)$$

with

$$m_V^2 = 3\mu^2/(2g^2 L) \qquad (4.17)$$

This substitution produces additional kinetic terms for the pseudoscalar fields which now appears with a factor

$$-\,\mathrm{tr}_{SU(2)}\big(P(x)\Box P(x)\big)\big(1 + \tfrac{2}{3} m_A^2\lambda^2 + 4M\lambda\big)L \qquad (4.18)$$
$$\triangleq\; \mathrm{tr}_{SU(2)}\big(\partial_\mu P(x)\partial^\mu P(x)\big)\, Z_P^{-1} L$$

Using (4.15), this renormalization factor becomes

$$Z_p^{-1} = 1 - 6M^2/m_A^2 \tag{4.19}$$

After this diagonalization, the Lagrangian reads

$$\begin{aligned} \mathcal{L}(x) = \mathrm{tr}_{SU(2)} \{ & \partial_\mu S' \partial^\mu S' - (4M^2 + \tfrac{1}{3} m_V^2 \mathfrak{M}/M) S'^2 \\ & + \partial_\mu P \partial^\mu P \, Z_p^{-1} - \tfrac{1}{3} m_V^2 \mathfrak{M}/M) P^2 \\ & - \tfrac{1}{3} F_V^{\mu\nu} F^V_{\mu\nu} + \tfrac{2}{3} m_V^2 V_\mu^2 \\ & - \tfrac{1}{3} F_{\hat{A}}^{\mu\nu} F^{\hat{A}}_{\mu\nu} + \tfrac{2}{3} m_A^2 \hat{A}_\mu^2 \} \times L \end{aligned} \tag{4.20}$$

where $F^{\mu\nu}_{V,\hat{A}}$ are the usual field tensors of vector and axial vector fields. The particle content of this free Lagrangian is now obvious. There are vector mesons of mass m_V^2, axial-vector mesons of mass m_A^2 and scalar and pseudoscalar mesons of mass

$$m_s^2 = 4M^2 + \tfrac{1}{3} m_V^2 \mathfrak{M}/M \tag{4.21}$$

$$m_p^2 = \qquad \tfrac{1}{3} m_V^2 \mathfrak{M}/M \; Z_p \tag{4.22}$$

With (4.17), the constant (4.19) can also be written as

$$Z_p^{-1} = m_V^2/m_A^2 \tag{4.23}$$

As we have argued before, there is a good chance that the fields P, V, S, A describe approximately the lowest lying mesons π, ρ, σ, A_1. Let us test this hypothesis as far as the masses are concerned. Since experimentally $m_{A_1}^2 \approx 2m_\rho^2$ the factor Z_π becomes ≈ 2.

Forthermore, equ. (4.16) determines the quark mass as:

$$6M^2 = m_{A_1}^2 - m_\rho^2 \quad ; \quad M \approx 310 \text{ MeV} \tag{4.24}$$

in good agreement with other estimates.[25] The small pion mass yields via (4.22)

$$\mathfrak{m} \approx 15 \text{ MeV} \tag{4.25}$$

Thus the bare quark mass has to be extremely small. Aslo this result has been obtained by many authors[26]. It is common to all models in which the smallness of the pion mass is related to the approximate conservation of the axial current (PCAC).

The scalar meson finally is predicted from (4.21) to have a mass

$$m_\sigma \approx 2M \sim 620 \text{ MeV} \tag{4.26}$$

This agrees well with the observed broad resonance in $\pi\pi$ scattering.[27][20]

One disagreement with experiment appears in connection with the SU(2) singlet pseudoscalar mass (the η meson). According to (4.22) it should be degenerate with the pion. The resolution of this problem will be discussed later when the theory has been extended to SU(3).

After these first encouraging results we shall rename the fields P , V , S , A by the corresponding particle symbols

$$\sqrt{L}\, P \equiv \pi, \quad \sqrt{L}\, S' \equiv \sigma', \quad \sqrt{\tfrac{2}{3}L}\, V^\mu \equiv \rho^\mu, \quad \sqrt{\tfrac{2}{3}L}\, A^\mu \equiv A_1^\mu \tag{4.27}$$

where a normalization factor has been introduced in order to bring the kinetic terms in the Lagrangian to a conventional form.

A comment is in order concerning the appearance of a quadratic divergence in equations (4.7), (4.12). Such a strong divergence indicates, that the limiting procedure $\mu \to \infty$ of equ. (4.1) has been performed too carelessly. In fact, if one inserts (4.1) into the action (3.4), the theory becomes of the $(\bar{\psi}\psi)^2$ type and thus non-renormalizable. In order to keep the renormalizability while dealing with a large gluon mass $\mu^2 \gg M^2$ we actually have to watch out that the gluon mass stays always far below the cutoff: $\mu^2 \ll \Lambda^2$. Then the quadratic divergence becomes actually of the logarithmic type (compare (3.47)):

$$Q = \int \frac{d^4 P_E}{(2\pi)^4} \frac{1}{P_E^2 + M^2} \frac{1}{P_E^2 + \mu^2} = \frac{\pi^2}{(2\pi)^4} \frac{\mu^2}{\mu^2 - M^2} \left[\mu^2 \log\left(\frac{\Lambda^2}{\mu^2} + 1\right) - M^2 \log\left(\frac{\Lambda^2}{M^2} + 1\right) \right] \tag{4.28}$$

(which in the careless limit $\mu^2 \to \infty$ reduces again to (4.5)). The logarithmic divergence (4.11) on the other hand becomes in this more careful treatment independent of the cutoff which is replaced by the large gluon mass

$$L = \int \frac{d^4 P_E}{(2\pi)^4} \frac{1}{(P_E^2 + M^2)^2} \frac{1}{(P_E^2 + \mu^2)} = \frac{\pi^2}{(2\pi)^4} \frac{\mu^2}{(\mu^2 - M^2)^2} \tag{4.29}$$

$$\times \left[\mu^2 \log \frac{\mu^2}{M} + M^2 - \mu^2 \right] \approx \frac{\pi^2}{(2\pi)^4} \log \frac{\mu^2}{M^2}$$

Hence all our results refer to a renormalizable theory if one reads both Q and L as logarithmic expression once in the cutoff and once in the gluon mass, respectively.

Let us now proceed to study the interaction terms. The n'th order contribution to the action is given by

$$\mathcal{A}_n[m'] = \frac{(-i)^{n-1}}{n} \int dx \, \mathrm{tr}\left(G_M m'\right)^n \tag{4.30}$$

In momentum space this can be written as the one loop integral

$$\mathcal{A}_n[m'] = 4\,\frac{(-1)^{n-1}}{n} \int \frac{dq_n}{(2\pi)^4} \cdots \frac{dq_1}{(2\pi)^4} (2\pi)^4 \delta(q_n + \ldots + q_1)$$

$$\int \frac{d^4P_E}{(2\pi)^4} \frac{1}{(P+q_n+\ldots+q_1)_E^2 + M^2} \cdots \frac{1}{(P+q_1)_E^2+M^2}\, t_{i_n \ldots i_1}(P|q_{n-1},\ldots,q_1) \tag{4.31}$$

$$\mathrm{tr}_{SU(2)}\left[m'_{i_n}(q_n)\cdot \ldots \cdot m'_{i_1}(q_1)\right]$$

where $t_{i_n \ldots i_1}(P|q_{n-1},\ldots,q_1)$ is the generalization of the tensor (4.10)

$$t_{i_n \ldots i_1}(P|q_{n-1},\ldots,q_1) \equiv \frac{1}{4}\,\mathrm{tr}\left[\Gamma_{i_n}(\not{P}+\not{q}_{n-1}+\ldots+\not{q}_1+M)\Gamma_{i_{n-1}}\cdots \right. \tag{4.32}$$

$$\left. \Gamma_{i_2}(\not{P}+\not{q}_1+M)\Gamma_{i_1}(\not{P}+M)\right]$$

The result is hard to evaluate in general (except in a 1 + 1 dimensional space). With the approximation of a large cutoff one may however, neglect again all contributions which do not diverge. This considerably simplifies the results. Since $t_{i_n \dots i_1}(P|q_{n-1},\dots,q_1)$ are polynomials in P of order n, the integral is seen to converge for $n > 4$. For $n = 4$ there is a logarithmic divergence with only the leading momentum behaviour of $t_{i_n \dots i_1}$ contributing. For $n \leq 3$ also lower powers in momentum P of $t_{i_n \dots i_1}(P|q_{n-1},\dots,q_1)$ diverge logarithmically. A simple but somewhat tedious calculation of all the integrals (see App. B) yields the remaining terms in the Lagrangian. They can be written down in a most symmetric fashion by employing the unshifted fields+) $S(x) \equiv M + S'(x)$ rather than S', or in renormalized form

$$\sigma(x) = -\sqrt{L}\, M + \sigma'(x) \tag{4.33}$$

Then the Lagrangian reads

$$\begin{aligned} \mathcal{L}(x) = \mathrm{Tr}_{SU(2)} \Big\{ & [(D_\mu\sigma)^2 + (D_\mu\pi)^2] + 2M_0^2(\sigma^2+\pi^2) \\ & - \tfrac{2}{3}\gamma^2 [\sigma^4 + \pi^4 + 4\sigma^2\pi^2 - 2\sigma\pi\sigma\pi] - \tfrac{1}{2}F_{\mu\nu}^{V\,2} - \tfrac{1}{2}F_{\mu\nu}^{A\,2} \\ & + m_V^2(V_\mu^2 + A_\mu^2) - \tfrac{2}{3} m_V^2 \sqrt{L}\, \mathfrak{M}\sigma \Big\} \end{aligned} \tag{4.34}$$

Here $D_\mu\sigma$ and $D_\mu\pi$ are the usual covariant derivatives:

$$\begin{aligned} D_\mu\sigma &= \partial_\mu\sigma - i\gamma[V_\mu\sigma] - \gamma\{A_\mu\pi\} \\ D_\mu\pi &= \partial_\mu\pi - i\gamma[V_\mu\pi] + \gamma\{A_\mu\sigma\} \end{aligned} \tag{4.35}$$

and $F_{\mu\nu}^V$, $F_{\mu\nu}^A$ are the covariant curls

+) Notice that with this notation $m(x) = m_0 + m'(x)$ $= (M - \mathfrak{M}) + m'(x) = -\mathfrak{M} + S + Pi\gamma_5 + V^\mu\gamma_\mu + A^\mu\gamma_\mu\gamma_5$

$$F_{\mu\nu}^{V} = \partial_\mu V_\nu - \partial_\nu V_\mu - i\gamma\,[V_\mu, V_\nu] - i\gamma\,[A_\mu, A_\nu]$$

$$F_{\mu\nu}^{A} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\gamma\,[V_\mu, A_\nu] - i\gamma\,[A_\mu, V_\nu] \qquad (4.36)$$

The constant γ denotes

$$\gamma = \sqrt{\frac{3}{2L}} \qquad (4.37)$$

It describes the direct coupling of the vector mesons to the currents, i.e. it coincides with the coupling conventionally denoted by γ_ρ . Here γ has its origin in the renormalization of the fields. The mass term stands short for

$$2M_0^2 \equiv 2M^2 - \frac{1}{3} m_V^2\, \mathfrak{m}/M \qquad (4.38)$$

Actually, the so defined mass quantity has an intrinsic significance. This can be seen by deriving the Lagrangian in a different fashion from the beginning. consider the tadpole terms of the action

$$\mathcal{A}_1[m'] = \int dx\, \mathrm{tr}\left\{ G_M(x,x)\, m'(x) - \frac{1}{g}\, m'(x) m_0 \right\} \qquad (4.39)$$

In the former treatment we have eliminated m_0 completely by giving the quarks a mass M satisfying the gap equation

$$M - \mathfrak{m} \equiv m_0 = 4 \frac{g^2}{\mu^2} Q\, M \qquad (4.40)$$

Instead, we could have introduced an auxiliary mass M_0 satisfying the equation

$$M_0 = 4\frac{g^2}{\mu^2} Q_0 M_0 \tag{4.41}$$

where Q_0 is the same function of M_0 as Q is of M. The connection between this M_0 and the other masses is obtained by inserting $M = M_0 + \delta M$ into Q:

$$Q = Q_0 - 2M_0\delta M(1 + \delta M/2M_0)L \tag{4.42}$$

which holds exactly in δM with only small corrections for large cutoffs (notice that at this accuracy $L_0 = L$). Inserting this into (4.40) we find

$$\mathfrak{M} = 4\frac{g^2}{\mu^2} L\, 2M_0 M(1 + \delta M/2M_0)\delta M \tag{4.43}$$

and using m_V^2 from

$$\mathfrak{M} = \frac{12}{m_V^2} M_0 M(1 + \delta M/2M_0)\delta M \tag{4.44}$$

If now $m(x)$ is split in a different fashion

$$m(x) = \tilde{m}_0 + m''(x) \tag{4.45}$$

with a new $\tilde{m}_0 = M_0 - \mathfrak{M}$ then the propagator $G(x,y)$ would have an expansion

$$G(x,y) = G_{M_0}(x-y) - i(G_{M_0} m'' G_{M_0})(x,y) + \tag{4.46}$$

For this reason, the derivation of all Lagrangian terms yields exactly the same results as before only with m'', M_0, L_0, and Q_0 occurring rather than m', M, L and Q, respectively. There are only two differences: First, due to the gap equation (4.41), the scalar and pseudoscalar mass terms become $4M_0^2$ and 0 rather than (4.21),(4.22) second, the tadpole terms in this derivation do <u>not</u> cancel completely. Instead one finds from (4.39)

$$\mathcal{A}_1[m'] = \int dx \, \mathrm{tr}\left\{\left(4Q_0 M_0 - \frac{M_0 - \mathfrak{m}}{g^2/\mu^2}\right) m''(x)\right\}$$

$$= \int dx \, \frac{\mu^2}{g^2} \, \mathrm{tr}\{\mathfrak{m} \, m''(x)\} = \frac{3}{3} m_V^2 \int dx \, \mathrm{tr}\{\mathfrak{m} \, m''(x)\} \tag{4.47}$$

These tad pole terms provide exactly the necessary additional shifts in the fields which are needed in order to bring the scalar and pseudoscalar masses from $4M_0^2$ and 0 to their correct values m_σ^2 and m_π^2. The symmetric form (4.34) of the Lagrangian is again reached by introducing the original unprimed fields

$$S(x) \equiv M_0 + S''(x) \quad , \quad \sigma = -\sqrt{L} \, M_0 + \sigma''$$

Then the mass term appears as an SU(3) x SU(3) invariant

$$2M_0^2(\sigma^2 + \pi^2).$$

+) With this substitution, the unprimed field S really coincides with the formerly introduced field S since now

$$m(x) = (M_0 - \mathfrak{m}) + S''(x) + P(x) i\gamma_5 + \ldots$$
$$= -\mathfrak{m} + S(x) + P(x) i\gamma_5 + \ldots$$

while before

$$m(x) = (M - \mathfrak{m}) + S'(x) + P(x) i\gamma_5 + \cdots$$
$$= -\mathfrak{m} + S(x) + P(x) i\gamma_5 + \cdots$$

Notice now that this coincides exactly with the former calculation which rendered (see (4.38)

$$\left(2M^2 - \tfrac{1}{3}\, m_V^2\, \mathfrak{m}/M\right)\left(\sigma^2+\pi^2\right)$$

Inserting here $M = M_0 + \delta M$ and (4.44) gives

$$\begin{aligned} 2M^2 - \tfrac{1}{3} m_V^2 \mathfrak{m}/M &= 2M_0^2 + 4M_0\delta M + 2(\delta M)^2 - 4M_0\left(1+\tfrac{\delta M}{M_0}\right)\delta M \\ &= 2M_0^2 \end{aligned} \tag{4.48}$$

Hence the SU(3) symmetric mass M_0 defined by the gap equations (4.41) coincides with the mass M_0 introduced as an abbreviation to the mass combination (4.38).

The Lagrangian (4.34) is recognized as the standard chirally invariant σ model. Its symmetry transformations are for isospin

$$\begin{aligned} \delta\sigma &= i\,[\alpha,\sigma] \quad ; \quad \delta\pi = i\,[\alpha,\pi] \\ \delta V^\mu &= i\,[\alpha, V^\mu] + \tfrac{1}{\gamma}\partial^\mu\alpha \quad ; \quad \delta A^\mu = i\,[\alpha, A^\mu] \end{aligned} \tag{4.49}$$

For axial transformations the fields change according to

$$\begin{aligned} \bar\delta\sigma &= -\{\bar\alpha,\pi\} \quad ; \quad \bar\delta\pi = \{\bar\alpha,\sigma\} \\ \bar\delta V^\mu &= i\,[\bar\alpha, A^\mu] \quad ; \quad \bar\delta A^\mu = i\,[\bar\alpha, V^\mu] + \tfrac{1}{\gamma}\partial^\mu\bar\alpha \end{aligned} \tag{4.50}$$

The only term in the Lagrangian which is not invariant is the last linear term. In fact from

$$\bar\delta\mathcal{L} = i\tfrac{2}{3} m_V^2\, \mathfrak{m}\, \sqrt{}\, \{\bar\alpha\pi\} \equiv i\{\bar\alpha, \partial A\} \tag{4.51}$$

one finds

$$\partial A(x) = f_\pi\, m_\pi^2\, Z_\pi^{-1/2}\, \pi(x) \tag{4.52}$$

Introducing the conventional pion decay constant via

$$\partial A(x) \equiv f_\pi\, m_\pi^2\, Z_\pi^{-1/2}\, \pi(x) \tag{4.53}$$

one can read off

$$f_\pi\, m_\pi^2 = Z_\pi^{1/2}\, \frac{2}{3} m_V^2 \sqrt{L}\, \delta\sigma \tag{4.54}$$

Inserting m_π^2 from (4.22) this gives

$$f_\pi = Z_\pi^{-1/2}\, 2M\sqrt{L} \tag{4.55}$$

By squaring this and using $\gamma = \sqrt{\frac{3}{2L}}$, one obtains

$$f_\pi^2 = \frac{6M^2}{Z_\pi}\frac{1}{\gamma^2} = \frac{m_\rho^2}{\gamma^2}\;\frac{m_A^2 - m_\rho^2}{m_A^2} \tag{4.56}$$

which for $m_A^2 \approx 2m_\rho^2$ renders the well known KSFR relation. The model has the usual predictions

$$g_{\rho\pi\pi} = \gamma_\rho\left(1 - \frac{m_A^2 - m_\rho^2}{2m_A^2}\right) \approx \frac{3}{4}\gamma_\rho \tag{4.57}$$

and

$$\begin{aligned} g_{A_1\rho\pi} &= \frac{1}{2m_\rho}\,\gamma^2 Z_\pi f_\pi \approx \frac{m_\rho}{2f_\pi} \approx 4 \quad ; \; h_{A_1\rho\pi} = 0 \\ g_{A_1\sigma\pi} &= \gamma\, Z_\pi^{1/2} \approx \frac{m_\rho}{f_\pi} \approx 8 \\ g_{\sigma\pi\pi} &= \frac{2}{m_\sigma}\,\frac{1}{3}\gamma^2 f_\pi Z_\pi^{3/2} \approx \frac{m_\sigma}{f_\pi}\sqrt{2} \approx 9 \end{aligned} \tag{4.58}$$

When compared with experiment, the only real defect consists in the d-wave $A_1 \rho \pi$ coupling, $h_{A_1 \rho \pi}$, being absent. additional chirally invariant terms are needed in the Lagrangian, for example, the so called δ-term:

$$\delta \, \mathrm{tr} \left[\left(F^A_{\mu\nu} + F^V_{\mu\nu} \right) D^\mu (\sigma + i\pi) D^\nu (\sigma - i\pi) - (V \to -V, \, \pi \to -\pi) \right] \tag{4.59}$$

Such terms appear in our derivation if the approximation of large μ^2 is improved by terms which do not grow lagarithmically in μ .

Let us now determine the couplings of π , σ , ρ , A_1 A_1 to external quark fields. The external propagation proceeds via

$$i\bar{\eta} G \eta = i\bar{\eta} G_{M_0} \eta + \bar{\eta} G_{M_0} M'' G_{M_0} \eta - i \dots \tag{4.60}$$

If one defines the couplings by

$$\begin{aligned} \mathcal{L} \propto \; & g_{\pi QQ} \bar{\psi} i \gamma_5 \tau_a \psi \, \pi^a + g_{\sigma QQ} \bar{\psi} \tau_a \psi \, \sigma^a \\ & + g_{VQQ} \bar{\psi} \gamma^\mu \frac{\tau_a}{2} \psi \, V^a_\mu + g_{AQQ} \bar{\psi} \gamma^\mu \gamma_5 \frac{\tau_a}{2} \psi \, A^a_\mu \end{aligned} \tag{4.61}$$

and can read off

$$g_{\pi QQ} = \frac{1}{2\sqrt{L}} Z_\pi^{\frac{1}{2}} = \frac{M}{f_\pi} \; ; \; g_{\sigma QQ} = \frac{1}{2\sqrt{L}} = \frac{M}{f_\pi Z_\pi^{1/2}} \tag{4.62}$$

$$g_{\rho QQ} = g_{A_1 QQ} = \gamma$$

We see the vector coupling to quarks agree with vector-meson dominance. Due to PCAC also the Goldberger Treiman relation is respected

$$g_{\pi QQ} = g_A \frac{M}{f_\pi} \tag{4.63}$$

since the axial charge of the quark is $g_A = 1$. Since the quark mass is $M \sim m_N/3$, the pionic coupling to quarks is considerably smaller than to nucleons. Numerically

$$\frac{g^2_{\pi QQ}}{4\pi} \approx \frac{1}{17} \frac{g^2_{\pi NN}}{4\pi} \approx .86 \tag{4.64}$$

The σ meson couples even weaker

$$g^2_{\sigma QQ}/4\pi \approx .43$$

Vector- and axial-vector mesons, on the other hand, couple as strongly as to nucleons which is an expression of universality:

$$\frac{g^2_{\rho QQ}}{4\pi} = \frac{g^2_{AQQ}}{4\pi} = \frac{\gamma^2}{4\pi} \approx 2.6 \left(= \frac{g^2_{\rho NN}}{4\pi} \right) \tag{4.65}$$

We are now ready to extend our consideration to SU(3) (and higher groups). In this case the explicit symmetry breaking in the Lagrangian is too large to be neglected. Thus the bare masses $\mathfrak{m}$ of the quarks have to be considered as a matrix

$$\mathfrak{m} \approx \begin{pmatrix} \mathfrak{m}^u & & \\ & \mathfrak{m}^d & \\ & & \mathfrak{m}^s \end{pmatrix} \tag{4.66}$$

The derivation of the Lagrangian presented above (via the gap equation (4.41)) has shown the complete SU(3) symmetry of M_0^2. Hence when extending from SU(2) to SU(3), no change occurs except in the last symmetry breaking term of (4.34). As a consequence, the mass expressions for m_P^2 and m_S^2 remain as they are only that the renormalization constants Z_P become more complicated SU(3) dependent quantities due to the involved mixing of pseudoscalar and axial-vector mesons. For a complete discussion of this SU(3) x SU(3) invariant chiral Lagrangian the reader is referred to the review articles[23]. Here we only give a few results:

A best fit to π and K meson masses requires[+)]

$$\mathfrak{M} \approx \begin{pmatrix} 15 & & \\ & 15 & \\ & & 435 \end{pmatrix} \text{MeV} \tag{4.67}$$

Thus the explicit symmetry breakdown of SU(3) caused by the bare masses is quite large. The standard parameter[28] c characterizes this:

$$c \equiv \frac{\mathfrak{M}^8}{\mathfrak{M}^0} = \frac{\frac{1}{\sqrt{3}}\left(\mathfrak{M}^u+\mathfrak{M}^d-2\mathfrak{M}^s\right)}{\sqrt{\frac{2}{3}}\left(\mathfrak{M}^u+\mathfrak{M}^d+\mathfrak{M}^s\right)} \approx -1.28 \quad (\approx -\sqrt{2}) \tag{4.68}$$

Inserting into (4.44) we find the shifts in the quark masses caused by dynamics

$$\delta M = \begin{pmatrix} 7 & & \\ & 7 & \\ & & 127 \end{pmatrix} \text{MeV} \tag{4.69}$$

+) For other determinations of see Ref. 26).

and hence for the "physical" quark masses

$$M \approx \begin{pmatrix} 312 & & \\ & 312 & \\ & & 432 \end{pmatrix} \text{MeV} \tag{4.70}$$

Thus contrary to the large explicit SU(3) violation the bare masses $\mathfrak{M}$, the physical quark masses M show only the moderate violation

$$c' \equiv \frac{M^8}{M^0} = \frac{\frac{1}{\sqrt{3}}(M^u+M^d-2M^s)}{\sqrt{\frac{2}{3}}(M^u+M^d+M^s)} \approx -16\% \tag{4.71}$$

Since the quark masses M are produced almost completely by dynamical effects we expect some symmetry breakdown to appear also in the vacuum. A measure of this is provided by the expectation values of the scalar quark densities

$$\langle 0|\tilde{u}^a|0\rangle = \langle 0|\bar{\psi}(x)\frac{\lambda^a}{2}\psi(x)|0\rangle \tag{4.72}$$

In the hadronized theory, the scalar densities are identical with the scalar fields , up to a factor:

$$S^i_j(x) = -\frac{g^2}{\mu^2}\bar{\psi}^i(x)\psi_j(x) \tag{4.73}$$

as can be seen most easily by considering the equations of constraint (3.14) following from the Lagrangian (3.11) in the large-μ limit. Hence

$$\begin{aligned} \langle 0|\tilde{u}^a|0\rangle &= -\frac{\mu^2}{g^2}\sum_{i,j}\lambda^a{}_i{}^j\langle 0|S^i_j|0\rangle \\ &= -\frac{\mu^2}{2g^2}\operatorname{tr}(M\lambda^a) = -\frac{1}{2}\frac{\mu^2}{g^2}M^a \end{aligned} \tag{4.74}$$

Inserting (4.17) and (4.55) the factor becomes simply

$$\frac{1}{2}\frac{\mu^2}{g^2} = \frac{1}{3}\frac{1}{\frac{2}{3}\frac{g^2}{\mu^2}L}L = \frac{1}{3}m_V^2 Z_\pi \frac{f_\pi^2}{4M^2} \approx f_\pi^2$$

such that

$$\langle 0|\tilde{u}^0|0\rangle \approx -f_\pi^2 M^0 = -f_\pi^2\sqrt{\tfrac{2}{3}}(M^u+M^d+M^s) \approx -8\times10^{-3}\,\mathrm{GeV}^3$$
$$\langle 0|\tilde{u}^8|0\rangle \approx -f_\pi^2 M^8 = c'\langle 0|\tilde{u}^0|0\rangle \tag{4.75}$$

This shows that the SU(3) violation in the vacuum equals that in the quark masses $\approx -16\%$ +). Notice that the three results (4.22) (4.44) and (4.73) are in complete agreement with what one obtains by very general considerations using only chiral symmetry and PCAC (see App. C).

The extension of the Lagrangian to SU(3) produces additional defects which are well known from general discussions of chiral SU(3) x SU(3) symmetry[23)]. For example the vector mesons ω_1, φ are not mixed (almost) ideally as they should but φ remains close to an SU(3) singlet. In general discussions, additional terms have been added to chiral Lagrangian in order to account for this. There are the so called "current mixing terms":

$$\mathrm{tr}\left\{\left(F_{\mu\nu}^V + F_{\mu\nu}^A\right)^2(\sigma+i\pi)(\sigma-i\pi) + (A\to -A,\ \pi\to-\pi)\right\} \tag{4.76}$$

as well as "mass mixing terms"

+) In Ref. 30), SU(3) breaking in the vacuum was neglected. For a more general discussion and earlier references see Ref. 29).

$$\mathrm{tr}\left\{ (V^\mu + A^\mu)^2 (\sigma + i\pi)(\sigma - i\pi) + (V^\mu - A^\mu)^2 (\sigma - i\pi)(\sigma + i\pi) \right\} \tag{4.77}$$

In our derivation these arise as a next correction to the $\mu^2 \to \infty$ limit. Another problem is the degeneracy of the ideally mixed isosinglet pseudoscalar meson η'_{ideal}, with the pion. In order to account for the fact that the $\eta'\,(X_0)$ meson is almost a pure SU(3) singlet and much heavier than the other pseudoscalar mesons one needs some chirally symmetric term

$$\det(\sigma + i\pi) + \det(\sigma - i\pi). \tag{4.78}$$

Such a term breaks PCAC for the ninth axial current. It is well known[30)] that the quark gluon triangle anomaly operates in the singlet channel and might be capable of producing such a PCAC violation. In fact, if this was not true, quantum electrodynamics would possess an exactly massless Goldstone boson [31)] with η quantum numbers. Also the term (4.78) will appear when μ is not any more very large.

It is obvious that corrections to the $\mu^2 \to \infty$ approximation will become even more important if one tries to extend the consideration to SU(4) since then vector and pseudoscalar masses are quite heavy. In addition, the narrow width of the SU(4) vector meson ψ/J seems to indicate that short-distance parts of the gluon propagator are being probed. Thus the colorless quark gluon theory itself cannot be considered any more a realistic approximation to the colored theory.

At this place we should remark that present explanations of electromagnetic mass differences require also an breakdown of SU(2) symmetry in $\mathfrak{M}$[32]. This is conventionally parametrized by

$$\alpha \equiv \frac{\mathfrak{M}^3}{\mathfrak{M}^0} = \frac{\mathfrak{M}^u - \mathfrak{M}^d}{\sqrt{\frac{2}{3}}\,(\mathfrak{M}^u + \mathfrak{M}^d + \mathfrak{M}^s)} \tag{4.79}$$

From meson masses (as well as from the electromagnetic $\eta \to 3\pi$ decay) one finds[33]

$$\alpha \approx -3\% \tag{4.80}$$

This amounts to the bare quark masses

$$\mathfrak{M} \approx \begin{pmatrix} 10 & & \\ & 20 & \\ & & 435 \end{pmatrix} \mathrm{MeV} \tag{4.81}$$

giving the "true" masses

$$M \approx \begin{pmatrix} 310 & & \\ & 315 & \\ & & 432 \end{pmatrix} \mathrm{MeV} \tag{4.82}$$

Thus the SU(2) breaking of the vacuum is very small

$$\alpha' \equiv \frac{\langle 0 | \tilde{u}^3 | 0 \rangle}{\langle 0 | \tilde{u}^0 | 0 \rangle} = \frac{M^3}{M^0} \approx -.6\% \tag{4.83}$$

With all parameters fixed numerically we should finally check whether the approximation of a large gluon mass is self consistent. From (4.55) we have

$$L \approx \frac{1}{2}\frac{f_\pi^2}{M^2} \approx .046. \tag{4.84}$$

Inserting this into (4.29) we calculate

$$\log \frac{\mu^2}{M^2} \sim (4\pi)^2 \times 0.046 \approx 7.3 \tag{4.85}$$

and hence

$$\mu^2 \approx 1500\, M^2 \gg M^2 \tag{4.86}$$

or

$$\mu \approx 12\ \mathrm{BeV} \tag{4.87}$$

It is gratifying to note that this value is much larger than the mass of the vector mesons. In this way it is assured that higher powers of $q^2/(P_E^2+M^2)$ which were neglected in the derivation of the Lagrangian remain really small as compared to unity for all mesons of the theory (see app. B).

We should point out that the quark gluon theory in the limit $\mu^2 \to \infty$ coincides with the well-known Nambu-Jona Lasinio[24)] model which has proven in the past to be a convenient tool of studying the spontaneous breakdown of chiral symmetry and the dynamical generation of PCAC. Those authors have demonstrated the close analogy of the dynamic structure of this model with that of superconductivity. As we have mentioned before the equation (4.40) removing the tadpoles in the action is analogous to the gap equation for superconductors.

A similar analogy to super-conductors exists also for the hadronized theory. The classical version of it corresponds exactly to the classical Ginzburg-Landau equation for type II super-conductors in which the gap is allowed to be space time dependent. In fact, the classical hadronized theory can be derived alternatively by assuming such a dependence in the gap equation[34,18]. The advantage of our functional derivation is that the hadronized theory is not merely some classical approximation but becomes upon quantization completely equivalent to the original quark gluon theory.

A final comment concerns the Okubo-Zweig-Iizuka rule. As argued in the general section, the meson Lagrangian exactly respects this rule. This can be checked directly for all interaction terms in (4.34). Violations of this rule are all coming from meson loops. The calculation of some important loop diagrams leads to straight-forward estimates for the size of such violations.

V OUTLOOK

We have shown that in the absence of color, quark gluon theories can successfully be hadronized. The resulting quantum field theory incorporates correctly many features of strong interactions. It's basic fields are bilocal and the Feynman rules are topologically similar to dual diagrams. Our considerations have taken place at a rather formal level. Certainly, there are many problems which have been left open. For example, there is need for an understanding of the non-trivial gauge properties of the bilocal theory. Also, a consistent renormalization procedure will have to be developed in future investigations.

The inclusion of color is the challenging problem left open by this investigation. If color quark gluon theory is really equivalent to some kind of dual model the corresponding hadronization program should not produce bilocal but multilocal fields which are characterized by the position of a whole string rather that just its end points. A field theory should by constructed for gauge invariant objects like

$$\bar{\Psi}(x)\exp\left(ig\int_x^y G^\nu(z)dz_\nu\right)\Psi(y)$$

which depend on the whole path from x to y.

The difficulty in a direct generalization of the previous procedure is the self-interaction of the gluons. Only after the infrared behaviour of gluon propagators will be known, bare hadrons can be constructed inside the corresponding potential well and the "hadronization" methods can serve for the determination of the complete residual interactions.

It is hoped that hadronic Feynman rules in the presence of color will follow a pattern similar to that found here for the non-abelian theory.

Let us finally mention that an interesting field of applications of our methods lies in solid-state physics. Semi-conductors in which conduction and valence band have only small separations may show a phasetransition to what is called excitonic insulator. The critical phenomena taking place inside such an exciton system will find their most appropriate description by studying the scaling properties of the bilocal field theory.

APPENDIX A: Remarks on the Fermion Bose-Salpeter Equation

Consider the four Fermion Green's function

$$G^{(4)}_{\alpha\beta,\alpha'\beta'}(x,y;x'y') \equiv \langle 0|T(\psi_\alpha(x)\bar\psi_\beta(y)\bar\psi_{\alpha'}(x')\psi_{\beta'}(y'))|0\rangle \tag{A.1}$$

which becomes in the interaction picture

$$G^{(4)}_{\alpha\beta,\alpha'\beta'}(x,y;x',y') = \left[\langle 0|T e^{ig\int d^4z\,\bar\psi(z)\gamma^\mu\psi(z)G_\mu(z)}|0\rangle\right]^{-1}$$
$$\times \langle 0|T(e^{ig\int d^4z\,\bar\psi(z)\gamma^\mu\psi(z)G_\mu(z)}\psi_\alpha(x)\bar\psi_\beta(y)\psi_{\alpha'}(x')\psi_{\beta'}(y'))|0\rangle \tag{A.2}$$

Expanding the exponential and keeping only the ladder exchanges corresponding to the Feynman graph in Fig 13,

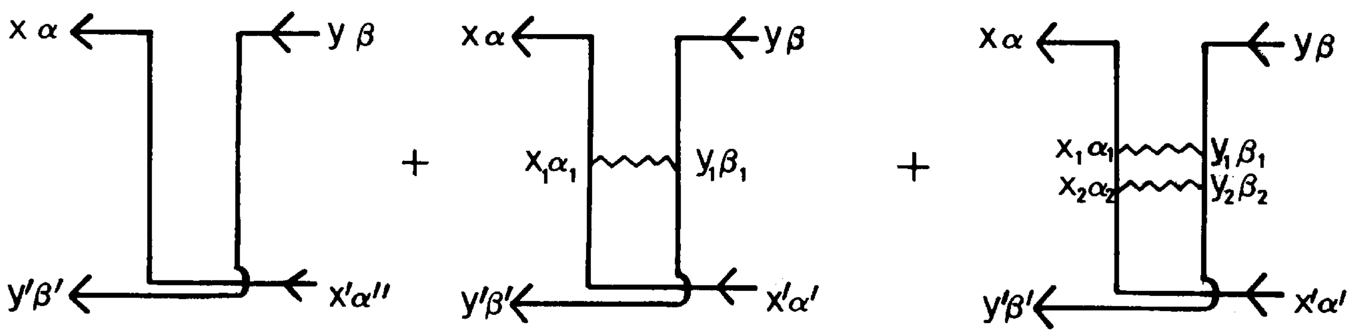

Figure 13

we obtain

$$G^{(4)}_{\alpha\beta,\alpha'\beta'}(xy,x'y') = G_{\alpha\alpha'}(x-x')G_{\beta'\beta}(y'-y) \tag{A.3}$$

$$+g^2\int dx_1dy_1\, G_{\alpha\alpha_1}(x-x_1)\gamma^\mu_{\alpha_1\alpha_1'}\, G_{\alpha_1'\alpha'}(x_1-x')G_{\beta'\beta_1'}(y'-y_1)\gamma_{\mu\,\beta_1'\beta_1}G_{\beta_1\beta}(y_1-y)D(x_1-y_1)$$

$$+g^4\int dx_1dy_1dx_2dy_2\, G_{\alpha\alpha_1}(x-x_1)\gamma^{\mu_1}_{\alpha_1\alpha_1'}G_{\alpha_1'\alpha_2}(x_1-x_2)\gamma^{\mu_2}_{\alpha_2\alpha_2'}G_{\alpha_2'\alpha'}(x_2-x')$$

$$D(x_1-y_1)\, D(x_2-y_2)\, G_{\beta'\beta_2'}(y'-y_2)\gamma_{\mu_2\,\beta_2'\beta_2}G_{\beta_2\beta_1'}(y_2-y_1)\gamma_{\mu_1\,\beta_1'\beta_1}G_{\beta_1\beta}(y_1-y)$$

$+ \dots$

The series can be summed to the integral equation

$$G^{(4)}_{\alpha\beta,\alpha'\beta'}(xy,x'y') = G_{\alpha\alpha'}(x-x')\, G_{\beta'\beta}(y'-y)$$

$$+g^2\int dx_1dy_1\; G_{\alpha\alpha_1}(x-x_1)\,\gamma^\mu_{\alpha_1\alpha_1'}\, G^{(4)}_{\alpha_1'\beta_1',\alpha'\beta'}(x_1y_1,x'y') \tag{A.4}$$

$$\gamma_{\mu\,\beta_1'\beta_1}\, G_{\beta_1\beta}(y_1-y)\; D(x_1-y_1)$$

With the abbreviation

$$\xi_{\alpha_1\beta_1,\beta_1'\alpha_1'} = \gamma^\mu_{\alpha_1\alpha_1'}\,\gamma_{\mu\,\beta_1'\beta_1} = 1_{\alpha_1\beta_1}\,1_{\beta_1'\alpha_1'} + (i\gamma_5)_{\alpha_1\beta_1}(i\gamma_5)_{\beta_1'\alpha_1'} \tag{A.5}$$

$$-\tfrac{1}{2}\gamma^\mu_{\alpha_1\beta_1}\;\gamma_{\mu\,\beta_1'\alpha_1'} - \tfrac{1}{2}(\gamma^\mu\gamma_5)_{\alpha_1\beta_1}(\gamma_\mu\gamma_5)_{\beta_1'\alpha_1'}$$

This can be written as

$$G^{(4)}_{\alpha\beta,\alpha'\beta'}(xy,x'y') = G_{\alpha\alpha'}(x-x')\,G_{\beta'\beta}(y'-y)$$

$$+g^2\int dx_1dy_1\, G_{\alpha\alpha_1}(x-x_1)\,\xi_{\alpha_1\beta_1,\beta_1\alpha_1'}\,D(x_1-y_1)G^{(4)}_{\alpha_1'\beta_1',\alpha'\beta'}(x_1y_1,x'y')\,G_{\beta_1\beta}(y_1-y) \tag{A.6}$$

or , symbolically:

$$G^{(4)} = GG^T + GG^T \xi g^2 D\, G^{(4)} \tag{A.7}$$

The transition matrix T is defined by removing the external particle poles in the connected part of $G^{(4)}$

$$\begin{aligned} G^{(4)}_{\alpha\beta,\alpha'\beta'}(xy,x'y') &\equiv G_{\alpha\alpha'}(x-x')G_{\beta'\beta}(y'-y) \\ &+ \int dx_1 dy_1 dx_2 dy_2\, G_{\alpha\alpha_1}(x-x_1)G_{\beta'\beta_2}(y'-y_2)\, T_{\alpha_1\beta_1,\alpha_2\beta_2}(x_1y_1,x_2y_2) \\ &\qquad G_{\alpha_2\alpha}(x_2-x')G_{\beta_1\beta}(y_1-y) \end{aligned} \tag{A.8}$$

which may be abbreviated by

$$G^{(4)} = GG^T + GG^T\, T\, GG^T \tag{A.9}$$

From (A.6) and (A.8) the transition matrix satisfies the integral equation

$$\begin{aligned} T_{\alpha_1\beta_1,\alpha_2\beta_2}(x_1y_1,x_2y_2) &= \xi_{\alpha_1\beta_1,\beta_2\alpha_2}\, g^2 D(x_1-y_1)\delta(x_1-x_2)\delta(y_1-y_2) \\ &+ \xi_{\alpha_1\beta_1,\bar\beta_1\bar\alpha_1}\, g^2 D(x_1-y_1)\int dx_1' dy_1' G_{\bar\alpha_1\alpha_1'}(x_1-x_1')T_{\alpha_1'\beta_1',\alpha_2\beta_2}(x_1'y_1',x_2y_2)G_{\beta_1'\bar\beta_1}(y_1'-y_1) \end{aligned} \tag{A.10}$$

which is seen to coincide with the equation (3.37) for the propagator of the bilocal field. In a short notation, this equation can be written as

$$T = \xi g^2 D + \xi g^2 D G G^T T \tag{A.11}$$

The perturbation expansion

$$T = \xi g^2 D + \xi g^2 D G G^T \xi g^2 D + \dots \tag{A.12}$$

reveals the one, two etc. photon exchanges of the ladder diagrams. In momentum space the four particle Green's function is defined by

$$(2\pi)^4 \delta^4(q'-q)\, G^{(4)}_{\alpha\beta,\alpha'\beta'}(P,P'|q)$$
$$\equiv \int dx\,dy\,dx'dy'\; e^{i\left[(P+\frac{q}{2})x+(P'-\frac{q'}{2})y'-(P-\frac{q}{2})y-(P'+\frac{q'}{2})x'\right]}\, G^{(4)}_{\alpha\beta\alpha'\beta'}(x\,y,x'y') \tag{A.13}$$

where the momenta are indicated in Fig. 14.

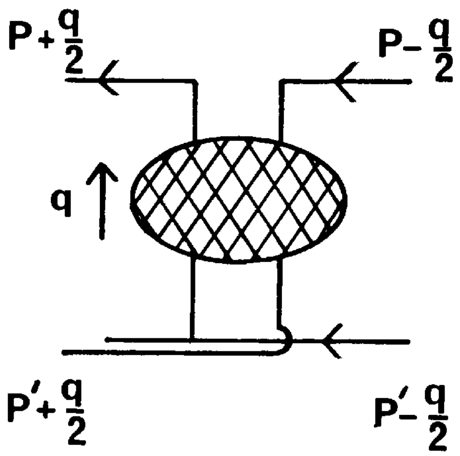

Figure 14

The corresponding scattering matrix $T(P,P'|q)$ satisfies the integral equation

$$T(P,P'|q) = \xi g^2 D(P-P') + \xi g^2 \int \frac{dP''}{(2\pi)^4} D(P-P'') G(P''+\tfrac{q}{2}) T(P'',P'|q) G(P''-\tfrac{q}{2}) \tag{A.14}$$

The ladder exchange is in general expected to produce quark anti-quark bound states. Suppose $|H(q)\rangle$ is one of them. Inserting it into (A.1) as an intermediate state gives for $x_0, y_0 > x_0', y_0'$ a contribution

$$G^{(4)}_{\alpha\beta,\alpha'\beta'}(x,y;x'y') = \int \frac{d^3q}{2E_q(2\pi)^3}\, \Theta\left(R_0 - R_0' - \tfrac{1}{2}(|z_0| + |z_0'|)\right)$$
$$\times \langle 0|T(\psi_\alpha(x)\bar{\psi}_\beta(y))|H(q)\rangle\langle H(q)|T(\bar{\psi}_{\alpha'}(x')\psi_{\beta'}(y'))|0\rangle \tag{A.15}$$

where $R \equiv (x+y)/2$ and $z = x-y$. The Θ function is non-zero if

$$\min(x_0, y_0) > \max(x_0' y_0')$$

Using the integral representation

$$\Theta(x_0) = \frac{i}{2\pi}\int da\, e^{-a x_0} \frac{1}{a+i\varepsilon}$$

we have

$$\Theta\left(R_0-R_0'-\tfrac{1}{2}(|z_0|+|z_0'|)\right)=\frac{i}{2\pi}\int dq_0\, e^{-i(q_0-E_q)(R_0-R_0')}$$

$$e^{i(q_0-E_q)\frac{1}{2}(|z_0|+|z_0'|)}\ \frac{1}{q_0-E_q+i\varepsilon} \qquad \text{(A.16)}$$

Introducing Bethe-Salpeter wave functions

$$\phi_{\alpha\beta}(x,y|q)\equiv\langle 0|T(\psi_\alpha(x)\bar\psi_\beta(y))|H(q)\rangle$$

$$\bar\Phi_{\beta'\alpha'}(y'x'|q)=\langle H(q)|T(\psi_{\beta'}(y')\bar\psi_{\alpha'}(x'))|0\rangle \qquad \text{(A.17)}$$

and their momentum space versions

$$\phi_{\alpha\beta}(x,y|q)=e^{-i(E_qR_0-\mathbf{q}\mathbf{R})}\int\frac{d^4P}{(2\pi)^4}e^{-iP(x-y)}\ \phi_{\alpha\beta}(P|q)$$

$$\bar\Phi_{\beta'\alpha'}(y'x'|-q)=e^{i(E_qR_0'-\mathbf{q}\mathbf{R}')}\int\frac{d^4P}{(2\pi)^4}e^{iP'(x'-y')}\ \bar\Phi_{\beta'\alpha'}(P'|-q) \qquad \text{(A.18)}$$

the four-particle Green's function in momentum space is seen to exhibits a pole at $q_0\approx E_q$

$$G^{(4)}_{\alpha\beta\alpha'\beta'}(P,P'|q)\approx\frac{-i}{2E_q(q_0-E_q+i\varepsilon)}\ \phi_{\alpha\beta}(P|q)\bar\Phi_{\beta'\alpha'}(P'|-q) \qquad \text{(A.19)}$$

The opposite time ordering $x_0, y_0 < x_0', y_0'$ contributes a pole at $q_0 = -E_q$. Both poles can be collected by writing in (A.19) the factor

$$-\frac{i}{q^2 - M_H^2 + i\varepsilon} .$$

This factorization is consistent with the integral equation only for a specific normalization of the Bethe-Salpeter wave functions. In order to see this write (A.7) in the form

$$\begin{aligned} G^{(4)} &= GG^T + GG^T \xi g^2 D G^{(4)} \\ &= (1 - GG^T \xi g^2 D)^{-1} GG^T \\ &= GG^T (1 - \xi g^2 D GG^T)^{-1} \end{aligned} \tag{A.20}$$

Suppose now that a solution is found for different values of the coupling constant g^2. Then the variation of $G^{(4)}$ for small changes of g^2 is

$$\begin{aligned} \frac{\partial G^{(4)}}{\partial g^2} &= (1 - GG^T \xi g^2 D)^{-1} GG^T \xi D \\ &\quad \times GG^T (1 - \xi g^2 D)^{-1} \\ &= G^{(4)} \xi D G^{(4)} \end{aligned} \tag{A.21}$$

If one goes in the vicinity of the pole $q^2 \approx M_H^2(g^2)$ this becomes

$$-\frac{\partial}{\partial g^2}\,\frac{i}{s-M_H^2(g^2)}\,\Phi_H^q(P|q)\bar{\Phi}_H^q(P|-q)$$

$$=\frac{i}{s-M_H^2(g^2)}\,\Phi_H^q(P|q)\int\frac{d\bar{P}d\bar{P}'}{(2\pi)^8}\,\bar{\Phi}_H^q(\bar{P}|-q)\,\xi D(\bar{P}-\bar{P}')\,\Phi_H^q(\bar{P}'|q)$$
$$\bar{\Phi}_H^q(P'|-q)\,\frac{i}{s-M_H^2(g^2)} \tag{A.22}$$

This can be true at the double pole at $q^2 = M_H^2(g^2)$ only if

$$\frac{\partial M_H^2(g^2)}{\partial g^2} = -i\int\frac{dP\,dP'}{(2\pi)^4(2\pi)^4}\,\Phi_H(P|q)\,\xi D(P-P')\,\bar{\Phi}_H(P'|-q) \tag{A.23}$$

If we go over to the Bethe Salpeter vertex function (3.29)

$$\Gamma^H(P|q) = N_H\, G_M^{-1}(P+\tfrac{q}{2})\,\Phi^H(P|q)\,G_M^{-1}(P-\tfrac{q}{2}) \tag{A.24}$$

$$\bar{\Gamma}^H(P|-q) = N_H^*\, G_M^{-1}(P-\tfrac{q}{2})\,\bar{\Phi}^H(P|-q)\,G_M^{-1}(P+\tfrac{q}{2})$$

this amounts to

$$g_H^2(q^2)\frac{\partial M_H^2(q^2)}{\partial q^2} = i|N_H|^2\int\frac{dP\,dP'}{(2\pi)^4(2\pi)^4}\,\mathrm{tr}\,\Big\{G_M(P+\tfrac{q}{2})\Gamma^H(P|q)$$

$$G_M(P-\tfrac{q}{2})\,g_H^2(q^2)\,D(P-P')\,G_M(P'-\tfrac{q}{2})\,\bar{\Gamma}^H(P'|-q)\,G_M(P'+\tfrac{q}{2})\Big\} \tag{A.25}$$

Using the integral equation (3.30) this reduces to the normalization

$$\frac{g_H^2(q^2)}{\dfrac{\partial g_H^2(q)}{\partial q^2}} = -i|N_H|^2\int\frac{d^4P}{(2\pi)^4} \tag{A.26}$$

$$\times\,\mathrm{tr}\left[G_M(P+\tfrac{q}{2})\Gamma^H(P|q)G_M(P-\tfrac{q}{2})\bar{\Gamma}^H(P|-q)\right]$$

This determines $|N_H|^2$ as

$$|N_H|^2 = g_H^2(q^2)\Big/\frac{\partial g_H^2(q^2)}{\partial q^2} \tag{A.27}$$

Notice that this normalization is defined for all q^2 with some $N_H(q^2)$. For real $\Gamma(P|q)$ one may choose $N_H(q^2)$ real such that

$$\bar{\Gamma}(P|-q) = \Gamma(P|-q)$$

(Both satisfy the same integral equation).
The orthogonality of $\Gamma^H(P|q)$ and $\bar{\Gamma}^{H'}(P|-q)$ for different hadrons is proved as usual by considering (3.30) once for $(\xi g^2 D)^{-1}\Gamma^H$ and once for $(\xi g^2 D)^{-1}\bar{\Gamma}^{H'}$,

multiplying the first by $\bar{\Gamma}^{H'}$ and the second by Γ^{H}, taking the trace and subtracting the results from each other (assuming no degeneracy of $g_H(q^2)$ and $g_{H'}(q^2)$). The normalization (A.27) is seen to be consistent with the expansion of the T matrix given in (3.39)

$$T_{\alpha\beta,\alpha'\beta'}(P,P'|q) = -ig^2 \sum_H \frac{\Gamma^H_{\alpha\beta}(P|q)\bar{\Gamma}^H_{\beta'\alpha'}(P'|-q)}{g_H^2(q^2) - g^2} \tag{A.28}$$

If q^2 runs into a pole M_H^2 this expression is singular as

$$T_{\alpha\beta,\alpha'\beta'}(P,P'|q) \approx -ig^2 \frac{1}{(q^2-M_H^2)\frac{\partial g_H^2(q^2)}{\partial q^2}} \Gamma^H_{\alpha\beta}(P|q)\bar{\Gamma}^H_{\beta'\alpha'}(P'|-q) \tag{A.29}$$

According to (A.9) this produces a singularity in $G^{(4)}$ (in short notation)

$$\begin{aligned} G^{(4)} &\approx -i \frac{g^2}{\frac{\partial g_H^2(q^2)}{\partial q^2}} \frac{1}{q^2-M_H^2} (G\Gamma^H G)(G\bar{\Gamma}^H G) \\ &= -i \frac{g^2}{\frac{\partial g_H^2(q^2)}{\partial q^2}} \frac{1}{q^2-M_H^2} |N_H|^2 \Phi^H \bar{\Phi}^H \end{aligned} \tag{A.30}$$

which coincides with (A.19) by virtue of (A.27).

For completeness we now give the Bethe-Salpeter equation (3.27) the form projected into the different covariants:

$$m'(P|q) = S(P|q) + P(P|q)\, i\gamma_5 + V^\mu(P|q)\,\gamma_\mu + A^\mu(P|q)\,\gamma_\mu\gamma_5 \tag{A.31}$$

If $m_i(P|q)$ $(i=1,2,3,4)$ abbreviates S,P,V,A, one has

$$m_i(P|q) = -4\,\xi_i\, g^2 \sum_{j=1}^{4} \frac{d^4P'}{(2\pi)^4} \frac{1}{(P-P')_E^2+\mu^2}\, \frac{1}{(P'+\frac{q}{2})_E^2+M^2}\, \frac{1}{(P'-\frac{q}{2})_E^2+M^2} \tag{A.32}$$
$$\times\, t_{ij}(P'|q)\, m_j(P'|q)$$

with $t_{ij}(P|q)$ being the traces defined in (4.10) and $\xi_i = (4, +4, -2, -2)$. Explicitly one finds

$$t_{SS}(P|q) = P^2 - \frac{q^2}{4} + M^2 \quad , t_{SV}(P|q) = t_{VS} = 2MP^\mu$$

$$t_{PP}(P|q) = P^2 - \frac{q^2}{4} - M^2 \quad , t_{PA^\mu}(P|q) = -t_{AP} = iMq^\mu$$

$$t_{V^\mu V^\nu}(P|q) = -\left(P^2 - \frac{q^2}{4} - M^2\right)g^{\mu\nu} + 2P^\mu P^\nu - \frac{1}{2}q^\mu q^\nu$$

$$t_{A^\mu A^\nu}(P|q) = -\left(P^2 - \frac{q^2}{4} - M^2\right)g^{\mu\nu} + 2P^\mu P^\nu - \frac{1}{2}q^\mu q^\nu - 2M^2 g^{\mu\nu}$$

with all other traces vanishing. Notice that in the Bethe-Salpeter equation for m' there is no tensor contribution due to the absence of such a term in the Fierz transform of $\gamma^\mu \otimes \gamma_\mu$. The integrals in (A.31) go directly over into (4.9) for large gluon mass μ.

APPENDIX B: The vertices for Heavy Gluons

Here we present the calculation of the vertices $\mathcal{A}_2$, $\mathcal{A}_3$, $\mathcal{A}_4$ for large μ. As discussed in the text, all higher vertices remain finite when the cutoff and μ go to infinity in the order $\Lambda^2 \gg \mu^2 \gg M^2$ and will consequently be neglected.

Consider first $\mathcal{A}_2$ as described in (4.10) and (A.33). The integrals $\mathcal{J}_{ij}(q)$ are evaluated by expanding

$$\left[(P+\tfrac{q}{2})_E^2 + M^2\right]^{-1}\left[(P-\tfrac{q}{2})_E^2 + M^2\right]^{-1}$$

$$= \left[P_E^2 + M^2\right]^{-2}\left\{1 + \frac{q^2/2}{P_E^2+M^2} + \frac{(Pq)^2}{(P_E^2+M^2)^2} + \mathcal{O}\left(\frac{M^4}{(P_E^2+M^2)^2}, \frac{q^4}{(P_E^2+M^2)^2}\right)\right\} \tag{B.1}$$

Since $t_{ij}(P|q)$ grow at most like P_E^2 (see (A.33)) the terms $\mathcal{O}(M^4/P_E^4, q^4/P_E^4)$ contribute finite amounts upon integration and will be neglected. At this place we have assumed q^2 to remain of the same order of M^2. Actually this is not true for vector and axial-vector meson fields[+)] but since numerically $m_\rho^2, m_{A_1}^2 \lesssim \frac{1}{100}\mu^2$, the neglected terms are indeed very small.

The following integrals are needed in addition to (4.5), (4.11) (neglecting finite amounts)

+) $m_\rho^2 = 6M^2$; $m_{A_1}^2 \approx 12 M^2$; $\mu^2 \approx 144\,\mathrm{GeV}^2$

$$\int \frac{d^4P_E}{(2\pi)^4} \frac{1}{(P_E^2+M^2)^2} P^2 = -(Q-M^2L)$$

$$\int \frac{d^4P_E}{(2\pi)^4} \frac{1}{(P_E^2+M^2)^2} P_\mu P_\nu = -(Q-M^2L) \frac{g_{\mu\nu}}{4}$$

$$\int \frac{d^4P_E}{(2\pi)^4} \frac{1}{(P_E^2+M^2)^3} P_\mu P_\nu = -L \frac{g_{\mu\nu}}{4} \qquad \text{(B.2)}$$

$$\int \frac{d^4P_E}{(2\pi)^4} \frac{1}{(P_E^2+M^2)^4} P_\mu P_\nu P_\lambda P_\varkappa = \frac{L}{24}(g_{\mu\nu}g_{\lambda\varkappa}+g_{\mu\lambda}g_{\nu\varkappa}+g_{\mu\varkappa}g_{\nu\lambda})$$

The results are displayed in equ. (4.12).

There is one subtlety connected with gauge invariance when evaluating the integrals $J_{VV}(q)$ and $J_{AA}(q)$. In fact, the first of these integrals coincides with the standard photon self-energy graph in quantum electro-dynamics. There the cutoff procedure is known to produce a non-gauge invariant result. The cutoff calculation yields:

$$J_{V^\mu V^\nu}(q) = -\frac{1}{3}(q^2 g_{\mu\nu} - q_\mu q_\nu)L - \frac{1}{2}(Q+M^2L)g_{\mu\nu}$$

$$\text{(B.3)}$$

$$J_{A^\mu A^\nu}(q) = -\frac{1}{3}((q^2-6M^2)g_{\mu\nu} - q_\mu q_\nu)L - \frac{1}{2}(Q+M^2L)g_{\mu\nu}$$

There are many equivalent ways to enforce gauge invariance. The simplest one proceeds via dimensional regularization If one evaluates the integrals Q and L in D = 4 $-\varepsilon$ dimensions with a small $\varepsilon > 0$, then

$$L = \int \frac{d^D p}{(2\pi)^4} \frac{1}{(P_E^2+M^2)^2} = \frac{\pi^{D/2}}{(2\pi)^4} \frac{1}{(M^2)^{2-\frac{D}{2}}} \frac{\Gamma(2-\frac{D}{2})}{\Gamma(2)}$$

$$= \frac{\pi^2}{(2\pi)^4} \frac{2}{\varepsilon} + O(\varepsilon)$$

$$Q = \int \frac{d^D p_E}{(2\pi)^4} \frac{1}{P_E^2+M^2} = \frac{\pi^{D/2}}{(2\pi)^4} \frac{1}{(M^2)^{1-\frac{D}{2}}} \frac{\Gamma(1-\frac{D}{2})}{\Gamma(2)}$$

$$= \frac{\pi^2}{(2\pi)^4} M^2 \left(-\frac{2}{\varepsilon}\right) + O(\varepsilon)$$

Hence at the pole $\varepsilon = 0$, Q and L become related such that $Q + M^2 L = 0$, cancelling the last terms in (B.3). Notice that when dealing with the renormalizable theory with large gluon mass $\mu^2 \ll \Lambda^2$, this cancellation is still present while the other Q integrals in $J_{ij}(q)$ become unrelated with the L integrals, the first being essentially $\mu^2 \log \Lambda^2/\mu^2$, the other $\log \mu^2/M^2$.

Consider now the interaction terms $\mathcal{A}_3$. Here the traces grow at most as P_E^3. Thus as far as the divergent contributions are concerned, the denominators in the integrals (4.27) can be approximated as

$$\frac{1}{(P+q_2+q_1)_E^2+M^2} \frac{1}{(P+q_1)_E^2+M^2} \frac{1}{P_E^2+M^2} \tag{B.4}$$

$$= \frac{1}{(P_E^2+M^2)^3} \left\{ 1 + \frac{P(2q_1+q_2)}{P_E^2+M^2} + O\left(\frac{M^2}{P_E^2+M^2}, \frac{q_i^2}{P_E^2+M^2}\right) \right\}$$

Since this expression decreases at least as $1/p_E^6$ the traces have to be known only with respect to their leading p_E^3 and p_E^2 behaviours. These are

$$
\begin{aligned}
& t_{SSS}(P|q_2q_1) \cong 3P^2M \quad , t_{SPP} \\
& t_{SPA^\mu}(P|q_2q_1) \cong iP^2P^\mu + 2iP^\mu[P(q_1+q_2)] - iP^2q_2^\mu \\
& t_{SA^\mu P}(P|q_2q_1) \cong -iP^2P^\mu - iP^2(2q_1+q_2)^\mu \\
& t_{SSV^\mu}(P|q_2q_1) \cong P^2P^\mu - P^2q_2^\mu + 2P^\mu[P(q_1+q_2)] = t_{PPV^\mu}(P|q_2q_1) \qquad \text{(B.5)} \\
& t_{SV^\mu S}(P|q_2q_1) \cong P^2P^\mu + P^2(2q_1+q_2)^\mu = t_{PV^\mu P}(P|q_2q_1) \\
& t_{SV^\mu V^\nu}(P|q_2q_1) \cong 4MP^\mu P^\nu - MP^2g^{\mu\nu} \\
& t_{SA^\mu A^\nu}(P|q_2q_1) \cong 4MP^\mu P^\nu - 3MP^2g^{\mu\nu} \\
& t_{V^\mu V^\lambda V^\kappa}(P|q_1q_2) \cong 4P^\mu P^\lambda P^\kappa - P^2(P^\mu g^{\lambda\kappa} + P^\lambda g^{\mu\kappa} + P^\kappa g^{\mu\lambda}) \\
& \qquad + 2P^\mu P^\lambda q_1^\kappa + 2P^\lambda P^\kappa(q_1+q_2)^\mu + 2P^\mu P^\kappa(2q_1+q_2)^\lambda \\
& - 2P^\kappa P(q_1+q_2)g^{\mu\lambda} - 2P^\mu Pq_1 g^{\lambda\kappa} - P^2(-g^{\mu\lambda}q_2 + g^{\lambda\kappa}q_2 + g^{\mu\kappa}(2q_1+q_2))
\end{aligned}
$$

Using (B.2) one obtains exactly the third order terms in the Lagrangian equ. (4.29) (if this is written in the σ 'form).

The fourth order couplings in $\mathcal{L}_4$ are the simplest to evaluate. Here only the leading P^4 behaviour of $t_{ij}(P|q_3,q_2,q_1)$ contributes proportional to L and the propagator can diractly be used in the form $[P_E^2+M^2]^{-4}$.

$$t_{SSSS}(P|q_3 q_2 q_1) \cong t_{SSPP} \cong t_{PPPP} \cong P^4$$

$$\begin{aligned} t_{SSV^\mu V^\nu}(P|q_3 q_2 q_1) &\cong t_{PPV^\mu V^\nu} \cong t_{SSA^\mu A^\nu} \cong \\ &\cong t_{PPA^\mu A^\nu} \cong -it_{SPA^\mu V^\nu} \cong it_{PSA^\mu V^\nu} \\ &\cong 2P^2 P^\mu P^\nu - P^4 g^{\mu\nu} \end{aligned} \quad (B.6)$$

etc.

APPENDIX C: Some Algebraic Derivations

Here we want to compare some of our results with traditional derivations[28,29] obtained by purely algebraic considerations together with PCAC.
The vector and axial-vector currents

$$V_\mu^a(x) = \bar{\psi}(x)\gamma_\mu^a \frac{\lambda^a}{2}\psi(x) \; ; \; A_\mu^a(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\frac{\lambda^a}{2}\psi(x) \qquad (C.1)$$

generate chiral SU(3) x SU(3) under which the quark gluon Lagrangian transforms as

$$\mathcal{L} = \mathcal{L}_{\text{chiral invariant}} - u^0 - cu^8 - du^3$$

where

$$\begin{aligned} u^0 + cu^8 + du^3 = \bar{\psi}m\psi &\equiv \sum_a m^a \bar{\psi}\frac{\lambda^a}{2}\psi \\ &= \sqrt{\tfrac{2}{3}}(m^u + m^d + m^s)\bar{\psi}\frac{\lambda^0}{2}\psi \qquad (C.2) \\ &+ \frac{1}{\sqrt{3}}(m^u + m^d - 2m^s)\bar{\psi}\frac{\lambda^8}{2}\psi \\ &+ (m^u - m^d)\bar{\psi}\frac{\lambda^3}{2}\psi \end{aligned}$$

Hence

$$u^a \equiv m^0\bar{\psi}\frac{\lambda^a}{2}\psi \quad (= m^0 \hat{u}^a) \qquad (C.3)$$

$$c \equiv \frac{m^8}{m^0} \; , \; d = \frac{m^3}{m^0} \qquad (C.4)$$

Defining also the pseudoscalar densities

$$v^a \equiv m^0 \bar{\psi} i\gamma_5 \frac{\lambda^a}{2}\psi \qquad (C.5)$$

then u^a and v^a form the $(3\bar{3})\pm(\bar{3}3)$ representation of SU(3) x SU(3):

$$[Q_5^a, u^a] = i d^{abc} v^c \quad , [Q_5^a, v^b] = -i d^{abc} u^c \tag{C.6}$$

From the equation of motion one finds the conservation law[+)]

$$\partial^\mu V_\mu^a(x) = \partial^\mu \bar{\Psi}\gamma_\mu \frac{\lambda^a}{2}\Psi = \bar{\Psi}\left[\frac{\lambda^a}{2}\mathcal{M}\right]\Psi \tag{C.7}$$

$$= i f^{abc}\mathcal{M}^b \; \bar{\Psi}\frac{\lambda^c}{2}\Psi \qquad a=0,1,\dots,8$$

$$\partial^\mu A_\mu^a(x) = \partial^\mu \bar{\Psi}\gamma_\mu\gamma_5 \frac{\lambda^a}{2}\Psi = -\bar{\Psi}\left\{\frac{\lambda^a}{2}\mathcal{M}\right\}\gamma_5\Psi$$

$$= i d^{abc}\mathcal{M}^b \bar{\Psi} i\gamma_5 \frac{\lambda^c}{2}\Psi \qquad a=1,\dots,8 \tag{C.8}$$

Let us neglect SU(2) breaking in $\mathcal{M}$. By taking (C.8) between vacuum and pseudoscalar meson states one finds

$$f_\pi m_\pi^2 = \frac{1}{\sqrt{3}}(\sqrt{2}\mathcal{M}^0 + \mathcal{M}^8)\,\frac{m_V^2}{3}\sqrt{L}\; Z_\pi^{1/2}$$

$$f_K m_K^2 = \frac{1}{\sqrt{3}}(\sqrt{2}\mathcal{M}^0 - \tfrac{1}{2}\mathcal{M}^8)\,\frac{m_V^2}{3}\sqrt{L}\; Z_K^{1/2} \tag{C.9}$$

etc. for the other members of the multiplet, where one has used (see the pseudoscalar version of (4.73)):

$$\langle 0|\partial^\mu A_\mu^a|\pi\rangle \equiv f_\pi m_\pi^2 \; ; \; \langle 0|\bar{\Psi} i\gamma_5 \frac{\lambda^\pi}{2}\Psi|\pi\rangle = \frac{\mu^2}{2g^2}\frac{1}{\sqrt{L}}\langle 0|\pi|\pi\rangle$$

etc.

$$= \frac{\mu^2}{2g^2}\frac{Z_\pi^{1/2}}{\sqrt{L}} = \frac{m_V^2}{3}\sqrt{L}\, Z_\pi^{1/2}$$

By writing $\mathfrak{M}$ as

$$\mathfrak{M} = \begin{pmatrix} \mathfrak{M}^u & & \\ & \mathfrak{M}^d & \\ & & \mathfrak{M}^s \end{pmatrix} = \frac{1}{2\sqrt{3}} \begin{pmatrix} \sqrt{2}\,\mathfrak{M}^0 + \mathfrak{M}^8 & & \\ & \sqrt{2}\,\mathfrak{M}^0 + \mathfrak{M}^8 & \\ & & \sqrt{2}\,\mathfrak{M}^0 - 2\mathfrak{M}^8 \end{pmatrix} \tag{C.10}$$

equ. (C.9) takes the form

$$f_\pi^2 m_\pi^2 = (\mathfrak{M}^u + \mathfrak{M}^d)/2 \quad Z_\pi^{1/2} \frac{2}{3} m_V^2 \sqrt{L}$$
$$f_K^2 m_K^2 = (\mathfrak{M}^u + \mathfrak{M}^s)/2 \quad Z^{1/2} \frac{2}{3} m_V^2 \sqrt{L} \tag{C.11}$$

which agrees with (4.54)

By evaluating (C.6) between vacuum states and saturating the commutator with pseudoscalar intermediate state one finds

$$f_\pi \mathfrak{M}_0 \frac{\mu^2}{2g^2} \frac{Z_\pi^{1/2}}{\sqrt{L}} = \frac{1}{\sqrt{3}} \left(\sqrt{2} \langle 0 | u^0 | 0 \rangle + \langle 0 | u^8 | 0 \rangle \right)$$
$$f_K \mathfrak{M}_0 \frac{\mu^2}{2g^2} \frac{Z_K^{1/2}}{\sqrt{L}} = \frac{1}{\sqrt{3}} \left(\sqrt{2} \langle 0 | u^0 | 0 \rangle - \frac{1}{2} \langle 0 | u^8 | 0 \rangle \right) \tag{C.12}$$

and similar for the other partners of the multiplet.

Inserting the result of equ.(4.73)

$$\langle 0 | u^a | 0 \rangle = \mathfrak{M}^0 \langle 0 | \tilde{u}^a | 0 \rangle = -\frac{1}{2} \frac{\mu^2}{g^2} \mathfrak{M}^0 M^a \tag{C.13}$$

and writing M in the same way as $\mathfrak{M}$ in (C.10) brings (C.12) to the form

$$f_\pi Z_\pi^{1/2} = \sqrt{L}\,(M^u + M^d)$$
$$f_K Z_K^{1/2} = \sqrt{L}\,(M^u + M^s) \qquad (C.14)$$

which agrees exactly with (4.54) (written there in SU(3) matrix form). Considerations of this type have led to the determination[28,29,33)]

$$c \approx -1.28$$

or (C.15)

$$\frac{(m^u + m^d)/2}{m^s} \approx \frac{1}{29}$$

Including also SU(2) violation in such a consideration gives[33)]

$$d \approx -.03$$

or (C.16)

$$\frac{m_u - m_d}{m_u + m_d} \approx -\frac{1}{4}$$

There are numerous extensions to SU(4)[35)] but they have to be viewed with great caution since it is hard to see how the large pseudoscalar and vector masses occuring there can dominate the divergence of the axial current and the vector current, respectively.

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D I S C U S S I O N S

CHAIRMAN: Prof. H. Kleinert

Scientific Secretaries: B. Freedman and B. Mathis

DISCUSSION 1

- *GOURDIN:*

Why is colour so difficult to introduce into your formalism?

- *KLEINERT:*

When you have colour, a gluon self-interaction coming from $G^2_{\mu\nu} = [\partial_\mu G_\nu - \partial_\nu G_\mu - g(G_\mu, G_\nu)]^2$ will be present which cannot be directly integrated out in the generating functional.

- *GOURDIN:*

Is what you have done just a formal transformation?

- *KLEINERT:*

No, this is a re-summation of the perturbation series that yields results which cannot be obtained by a perturbation expansion in the coupling. This we know from solid-state physics where one reaches a new phase by such a technique; to describe superconductivity, one sums bubblewise, while for a collective effect, such as plasmons, one sums ringwise. Here, in hadron physics, we sum ladder by ladder.

It is important to note that one does not expand in powers of the quark-gluon coupling constant. Let me remind you that in electron-positron scattering with two-photon exchange, the parameter in the expansion is not g, but becomes something like $g^4 \log(s - 4m^2)$. When s is near threshold, the effective expansion parameter becomes large. Thus, we have to sum the whole ladder in order to get finite results, and the ladder can be a good approximation even close to the threshold.

- *ROSSI:*

Turning the heavy lines in your diagrams into Preparata's double lines, one obtains the same graphs describing the same phenomena. Did you find a theoretical framework into which Preparata's point of view can be inserted and justified, at least as far as quark-antiquark bound states are concerned? Did you find anything like vector-meson dominance and the direct-coupling term? What about multihadron production in fire sausages?

- *KLEINERT:*

My rules contain all of Preparata's rules but the reverse is not true. For simplicity, Preparata only keeps three- and four-point couplings among hadrons. However, in order to have a hadronic theory that is equivalent to the original quark-gluon theory, one must must keep *all* n-point couplings. The photon couples via vector mesons, and there is an additional direct-coupling term as Preparata wants it. The direct coupling is taken care of by slashing the propagator as I have explained in the lecture.

- *BUCCELLA:*

Preparata's unconventional model incorporates many hadronic features, such as confinement; these are put in so as to agree with known experimental facts. Your formulation provides a natural framework for explaining current algebra, PCAC, etc. How is confinement and the colour-singlet nature of hadrons introduced into your functional integration method?

- *KLEINERT:*

The quark theory that I have hadronized is not yet unconventional because there is no confinement in it. The whole structure that I have shown works for QED for electrons and positrons. The success of Preparata's model indicates that this new structure can be generalized to the confined situation more easily than the original quark-gluon theory. The graphical rules for hadrons may be independent of whether or not there is confinement in the original quark theory.

- *GARCIA:*

In connection with the introduction of colour, you would like to have infrared slavery? What happens to the gluon mass at long distances?

- *KLEINERT:*

This is difficult to answer within my framework because the gluon field has disappeared from the theory upon hadronization. In the colour quark-gluon version, there will be a massless gluon providing the long-range force to confine the quarks. However, this will be true only within the confinement region, the hadron. Outside, the gluon should acquire a dynamically generated effective mass, $\mu \to \to \infty$, to prevent the quark-gluon system from escaping.

- *FREEDMAN:*

Ultimately, when you look for the bound states in the SU(3) × × SU(3) theory, you want to ensure that the poles correspond to hadrons and not to quarks since that is what one means by confinement. Would you care to speculate about how you want to achieve this in the context of your program?

- *KLEINERT:*

There has been a very exciting talk by Migdal at the Tbilisi conference which will be very helpful here. Migdal has *assumed* confinement and has calculated the hadronic spectrum following from quantum chromodynamics. He looked at the vacuum expectation value of operators like $T[\bar{\psi}(x)\psi(x)\bar{\psi}(0)\psi(0)]$, etc., Fourier-transformed to momentum space, and used the behaviour in the far space-like region known from asymptotic freedom. He then assumed a simple pole structure for q^2 positive and found a unique meromorphic function that fits the required asymptotic behaviour: $[\log(-q^2)]^{\gamma}$, with exponential accuracy.

- *FREEDMAN:*

This new development seems very interesting; however, the absence of coloured states from the pole structure as used by Migdal is still an input to the calculation.

- *KLEINERT:*

He never looked at coloured currents.

- *JONES:*

Could you amplify your claim that a colourless theory forms a reasonable description of meson interactions at intermediate energies?

- *KLEINERT:*

I would claim that the low-mass mesons, the π, ρ, σ, and A_1, and their low-energy, $\lesssim$ 1 GeV, scattering amplitudes form a closed set of physical phenomena. This approximation may work to within 20% accuracy. The fact that higher mass resonances do not interfere here shows that colour, which is responsible for the existence of these higher-mass states, can play no role at this level.

- *ORZALESI:*

Your hadronization corresponds roughly to treating hadrons as bound states in a ladder approximation. My first question is whether your approximation is a first step in an iterative approximation scheme and, if so, what is the prescription for calculating hadronic amplitudes to arbitrary order? My second question has to do with gauge invariance: in QED, the ladder approximation is not gauge invariant, and the position of bound-state poles depends on the gauge chosen; furthermore, if one only keeps ladders and rainbow type graphs, the theory is not even renormalizable. How do you deal with such difficulties?

- *KLEINERT:*

My *bare* hadrons are certainly a first approximation, as is any bare field in an interacting field theory. However, after taking into account all hadronic interactions specified in my Lagrangian, there is no more approximation but a complete equivalence to the original quark theory. This also answers your second question: gauge invariance in the original theory has a counterpart in the bilocal theory causing relations among hadron graphs.

- *FREEDMAN:*

I propose that, for gauge theories, it would be useful to work in an arbitrary covariant gauge and let the gauge parameter keep track of the cancellation between gauge-dependent term when calculating physical quantities. I think this is especially useful here where one does not have an intuitive picture of the cancellations occurring in the perturbation series in the effective hadronic coupling.

- *KLEINERT:*

Yes, you will always find the correct family of hadronic diagrams which throws out the gauge dependence.

- *PAULI:*

Today, you wrote down an equation: $m' = \xi g^2 D G_0 m' G_0$, which is essentially a Bethe-Salpeter equation for quark-hadronic vertices. Can this, or should this, give results similar to the constitutent-interchange model of Blancenbecler, Brodsky, Farrar, and Gunion for hadron-hadron scattering?

- *KLEINERT:*

I think you are referring to those rules that apply to rearrangement collisions. An example of this is electron exchange in molecular collisions which give rise to the Van der Waals forces. Certainly the rules given by Brodsky et al. can be rephrased in terms of my ladder re-summation of scattering graphs.

- *POSNER:*

Could you please reassure us that the expansion of $\log(1 + iG_0 m')$ converges?

- *KLEINERT:*

The convergence has not been studied on rigorous grounds. I can only assure you that my re-summation will converge better near thresholds and bound states where normal perturbation theory certainly fails.

- *POSNER:*

In your process of functional integration, you first eliminated the gluons and then the quarks. If you integrate out the fields in the reverse order, another equivalent theory is obtained, which is very different in appearance. What is the theory like? Is it worth studying?

- *KLEINERT:*

If you do it in the other order, you will get what may be called "plasmonization". By first integrating out the fermion fields, you will leave only a dressed photon field, which is now a very complicated object. Plasmonization occurs by eating up all the fermion degrees of freedom bubblewise. Although this theory plays an important role in solid-state physics and in some two-dimensional field theories, e.g. the Schwinger model, I have not explored it further because it seems uninteresting for hadron physics.

DISCUSSION 2

- *MARCIANO:*

Can we understand how infrared divergences emerge from your hadronized version of QED? Their role will be very important in providing confinement in QCD.

- *KLEINERT:*

I have not examined the infrared properties of my formalism.

- *PHAM QUANG HUNG:*

This morning you mentioned the $\eta \to 3\pi$ problem and the neutron-proton mass difference. Do you have any idea how to solve these problems?

- *KLEINERT:*

For these two problems one usually introduced an explicit SU(2) breaking term into the quark mass matrix. Taking the d quark to be 10 MeV heavier than the u-quark yields the correct $(m_n - m_p)$ value. This choice also gives the correct $\eta \to 3\pi$ decay rate. I do find this an unsatisfactory procedure, and I hope a more natural explanation will be found.

- *ALVAREZ:*

How do you propose to recover the sigma model in the colour non-Abelian theory?

- *KLEINERT:*

The sigma model can be recovered by neglecting the effect of higher resonances. Within quantum flavour dynamics, the same thing was achieved by sending μ^2 to infinity. By this trick, quantum flavour dynamics reduces exactly to the sigma model.

- *FREEDMAN:*

One cannot introduce a gluon mass μ due to renormalization requirements. What mass parameter do you plan to use to implement confinement?

- *KLEINERT:*

The only mass parameter available in the colour gauge gluon-quark model, other than the quark mass, is the infrared cut-off that one needs to define the Green's functions. Many people believe that this parameter is related to the momentum cut-off of the quarks inside the confinement region.

- *FREEDMAN:*

But this parameter is arbitrary. How can it determine the hadron masses?

- *KLEINERT:*

I think that eventually one will get an infinite family of solutions, and that the infrared cut-off will be fixed by the lowest hadron mass in the spectrum. All other masses are then determined.

- *WILKIE:*

Are you going to have difficulties forming Regge poles, which are long-distance effects in a $\mu \to \infty$ limit?

- *KLEINERT:*

The $\mu \to \infty$ limit of quantum flavour dynamics gives the sigma model which corresponds to the zero-slope approximation in Regge theory where all higher-mass states have moved to infinity. I assume that when colour is introduced and we make contact with the dual aspect of hadron dynamics, the correct Regge behaviour will be regained.

- *WILKIE:*

Is it obvious in your treatment of the sigma model that the ρ is a $\pi\pi$ resonance?

- *KLEINERT:*

The ρ appears here as a fundamental field as well as a resonance. This is similar to the situation encountered in N/D calculations; one gets only a narrow contribution to the ρ when treating it as a pure $\pi\pi$ system. One has to put in an elementary ρ as well.

- *BERLAD:*

In performing the functional integration over the gluon field, there should appear not only ladder exchanges but also crossed ladders. Where do these appear?

- *KLEINERT:*

This is a point I was trying to explain by showing some examples. The cross-ladder diagrams can be obtained by topological rearrangements of ladder diagrams.

- *PAULI:*

In order to derive the sigma model, the mass of the gluon must be $\geq$ 12 GeV. This would result in point-like hadrons, yet we know that the hadronic size is roughly the Compton wavelength of the pion. Could you elaborate?

- *KLEINERT:*

In this model, mesons are fundamental fields after the large μ-mass limit has been taken. Obviously, the form factor for the mesons will be point-like. Radiative corrections due to heavy gluons should broaden the form factor by a small amount only. However, for this purpose, this is not a realistic model of hadrons.

- *VON DARDEL:*

What does your model give the for magnetic moments of the baryons? Do small bare quark masses make it difficult to obtain small enough magnetic moments?

- *KLEINERT:*

No. The magnetic moments are determined by the dynamical quark masses $M \approx 312$ MeV. Hence

$$\mu_Q \approx \frac{1}{2M} \approx 3\frac{1}{2M_N}$$

becomes thrice the nucleon Bohr magnetic moment, thus coinciding with the magnetic moment of the proton, as it should.

- *POSNER:*

When you consider the three-point hadronic interaction, a mass relationship involving m' is obtained. Would you give a physical explanation of why the four-point function depends only on m', m'^2, and m'^3?

- *KLEINERT:*

I only took leading order in $\mu \to \infty$ into account. The higher powers of m'^n ($n \geq 4$) are lower order corrections. They certainly exist, but remember: M^2/μ^2 is very small (< 1%).

- *POSNER:*

The quantity $C' = M_8/M_0$ ($\approx -16\%$) plays a role in πN and KN scattering. Please elaborate on its significance in these interactions.

- *KLEINERT:*

In the exact $SU(3) \times SU(3)$ symmetry limit, the πN and KN cross-sections would be equal, but experiment gives $(\sigma_{\pi N} - \sigma_{KN}) \approx 6$ mb. One would expect this difference in the elastic scattering amplitude, governed by the Pomeron, to be caused by the breakdown of the symmetry in the vacuum of the underlying quark dynamics, specifically in the mass matrix. This is only a rough argument, not a quantitative one.