From Landau's Order Parameter to Modern Disorder Fields

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Landau's work was crucial for the development of the modern theory of phase transitions. He showed that such transitions can be classified by an order parameter, which in the low-temperature phase becomes nonzero. Together with Ginzburg he made this order parameter a spacetime-dependent order field and introduced a local energy functional whose extrema yield field equations and whose fluctuations determine the universal critical behavior of second-oder transitions. In the same spirit, but from a dual point of view, I have developed in the last twenty years a disorder field theory that describes phase transitions via the statistical mechanics of grand-canonical ensembles of vortex lines in superfluids and superconductors, or of defect lines in crystals. The Feynman diagrams of the disorder fields are pictures of the vortex or defect lines. A nonzero ground state expectation value of the disorder field at high temperature signalizes the proliferation of line like excitations in the ordered phase. It was this description of the superconductor that led in 1982 to a first understanding of the order of the superconducting phase transition. Recent experimental progress in the critical regime of high-T_c superconductors will be able to verify the predicted tricritical point of the Ginzburg parameter $\kappa \approx 0.8/\sqrt{2}$ where the second-order transition becomes first-order.

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1. Introduction

The critical regime of old-fashioned superconductors can be described extremely well by the Ginzburg-Landau theory [1] in mean-field approximation [2,3]. The reason is the smallness of the Ginzburg temperature interval $\Delta T_{\rm G}$ around the mean-field critical temperature $T_c^{\rm MF}$ where fluctuation become important [4]. A first discussions of the order of the superconductive phase transition by Halperin, Lubensky, and Ma in 1972 [5] did initially not seem experimentally relevant.

The situation has changed with the advent of modern high-T_c superconductors. In these the Ginzburg temperature interval is large enough to observe field fluctuations and see critical properties beyond mean-field. Several experiments have found a critical point of the XY universality class [6]. In addition, there seems to be recent evidence for an additional critical behavior associated with the so-called charged fixed point [7]. In view of future experiments, it is important to understand precisely the nature of critical fluctuations.

2. Ginzburg-Landau Theory

The Ginzburg-Landau theory [1] describes a superconductor with the help of an energy density

$$\mathcal{H}(\psi, \nabla\psi, \mathbf{A}, \nabla\mathbf{A}) = \frac{1}{2}\Big\{[(\nabla - ie\mathbf{A})\psi]^2 + \tau|\psi|^2 + \frac{g}{2}|\psi|^4\Big\} + \frac{1}{2}\left(\nabla \times \mathbf{A}\right)^2, \tag{1}$$

where $\psi(x), \mathbf{A}(x)$ are pair field and vector potential, respectively, and e is the charge of the Cooper pairs. The parameter $\tau \equiv T/T_c^{\rm MF} - 1$ is a reduced temperature measuring the distance from the characteristic temperature $T_c^{\rm MF}$ at which the $|\psi|^2$-term changes sign. The theory needs gauge fixing, which is usually done by setting $\psi(x) = \rho(x)e^{i\theta(x)}$, rewriting the covariant derivative of ψ as

$$D\psi = [i(\nabla\theta - e\mathbf{A})\rho + \nabla\rho]e^{i\theta}, \tag{2}$$

and eliminating the phase variable $\theta(x)$ by a local gauge transformation $\mathbf{A} \to \mathbf{A} + \nabla\theta/e$. This brings $\mathcal{H}(\psi, \nabla\psi, \mathbf{A}, \nabla\mathbf{A})$ to the form

$$\mathcal{H}_1 = \frac{1}{2}(\nabla\rho)^2 + V(\rho) + \frac{1}{2}\left(\nabla \times \mathbf{A}\right)^2 + \frac{\rho^2 e^2}{2}\mathbf{A}^2, \tag{3}$$

where $V(\rho)$ is the potential of the ρ-field:

$$V(\rho) = \frac{\tau}{2}\rho^2 + \frac{g}{4}\rho^4. \tag{4}$$

The last term in (3) is the famous Meissner-Higgs mass $m_A = \rho e$ [2,3] of the vector potential $\mathbf{A}$. An analogous mass in the gauge theory of electroweak interactions explains why interactions are so much weaker than electromagnetic interactions.

At the mean-field level, the energy density (3) describes a second order phase transition. It takes place if τ drops below zero where the pair field $\psi(x)$ acquires the nonzero expectation value $\langle\psi(x)\rangle = \rho_0 = \sqrt{-\tau/g}$, the order parameter of the system. The ρ-fluctuations around this value have a *coherence length* $\xi = 1/\sqrt{-2\tau}$. The Meissner-Higgs mass term in (3) gives rise to a finite *penetration depth* of the magnetic field $\lambda = 1/m_A = 1/\rho_0 e$. The ratio of the two length scales $\kappa \equiv \lambda/\sqrt{2}\xi$, which for historic reasons carries a factor $\sqrt{2}$, is the Ginzburg parameter whose mean field value is $\kappa_{\rm MF} \equiv \sqrt{g/e^2}$. Type I superconductors have small values of κ, type II superconductors have large values. At the mean-field level, the dividing line is lies at $\kappa = 1/\sqrt{2}$.

3. Fluctuation Corrections

In high-T_c superconductors, field fluctuations become important. These can be taken into account by calculating the partition function and field correlation functions from the functional integral

$$Z = \int \mathcal{D}\rho \, \rho \, \mathcal{D}\mathbf{A} \, e^{-\int d^3x \, \mathcal{H}_1} \tag{5}$$

(in natural units with $k_B T = 1$). So far, all analytic approximations to Z pursued since the initial work [5] have had notorious difficulties in accounting for the order of the superconductive phase transition. In [5], simple renormalization group arguments [8] in $4-\epsilon$ dimensions suggested that the transition should be of first order. The technical signal for this was the nonexistence of an infrared-stable fixed point in the renormalization group flow of the coupling constants e and g as a function of the renormalization scale. However, due to the smallness of the Ginzburg interval $\Delta T_{\rm G}$, the first order was never verified experimentally. Since then, there has been much work [9] trying to find an infrared-stable fixed point in higher loop orders or by different resummations of the divergent perturbations expansions, with little success.

Recall the simplest argument suggesting a first-order nature of the transition arises at the mean-field level of the pair field ρ as follows: The fluctuations of the vector potential are Gaussian and can be integrated out in (5). Assuming ρ to be smooth, this may be done in the Thomas-Fermi approximation [10], leading to an additional cubic term in the potential (4), changing it to

$$V(\rho) = \frac{\tau}{2}\rho^2 + \frac{g}{4}\rho^4 - \frac{c}{3}\rho^3, \qquad c \equiv \frac{e^3}{2\pi}. \tag{6}$$

The cubic term generates, for $\tau < c^2/4g$, a second minimum at

$$\tilde{\rho}_0 = \frac{c}{2g}\left(1 + \sqrt{1 - \frac{4\tau g}{c^2}}\right). \tag{7}$$

If τ decreases below

$$\tau_1 = 2c^2/9g. \tag{8}$$

the new minimum lies *lower* than the one at the origin (see Fig. 1), so that the order parameter jumps from zero to

$$\rho_1 = 2c/3g \tag{9}$$

in a phase transition. At this point, the coherence length of the ρ-fluctuations $\xi = 1/\sqrt{\tau + 3g\rho^2 - 2c\rho}$ has the finite value (the same as the fluctuations around $\rho = 0$)

$$\xi_1 = \frac{3}{c}\sqrt{\frac{g}{2}}. \tag{10}$$

The phase transition is therefore of first-order [11].

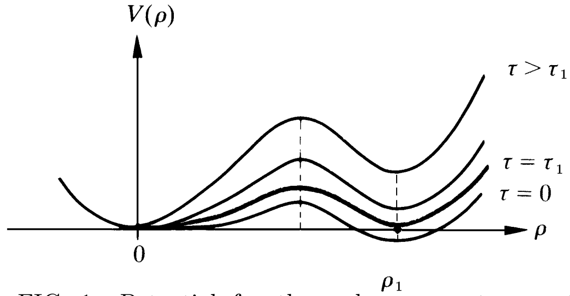

FIG. 1. Potential for the order parameter ρ with cubic term. A new minimum develops around ρ_1 causing a first-order transition for $\tau = \tau_1$.

4. Vortex Corrections

The above conclusion is reliable only if the jump of ρ_0 is sufficiently large. For small jumps, the mean-field discussion of the energy density (6) cannot be trusted. The mistake in the above discussion of (1) lies in the neglect of vortex fluctuations. In fact, the transformation of the covariant derivative $D\psi$ to the ρ-θ expression in Eq. (2) is false. Since $\theta(\mathbf{x})$ and $\theta(\mathbf{x}) + 2\pi$ are physically indistinguishable — the complex field $\psi(\mathbf{x})$ is the same for both — the correct substitution is

$$D\psi = [i(\boldsymbol{\nabla}\theta - 2\pi\boldsymbol{\theta}^v - e\mathbf{A})\rho + \boldsymbol{\nabla}\rho]e^{i\theta}, \tag{11}$$

The cyclic nature of the scalar field $\theta(\mathbf{x})$ requires the presence of a vector field $\boldsymbol{\theta}^v(\mathbf{x})$ called *vortex gauge field*. This field is a sum of δ-functions on Volterra surfaces across which $\theta(\mathbf{x})$ has jumps by 2π. The boundary lines of the surfaces are vortex lines. They are found from the vortex gauge field $\boldsymbol{\theta}^v(\mathbf{x})$ by forming the curl

$$\boldsymbol{\nabla} \times \boldsymbol{\theta}^v(\mathbf{x}) = \mathbf{j}^v(\mathbf{x}), \tag{12}$$

where $\mathbf{j}^v(\mathbf{x})$ is the *vortex density*, a sum over δ-functions along the vortex lines $\boldsymbol{\delta}(L;\mathbf{x}) \equiv \int_L d\bar{\mathbf{x}}\, \delta(\mathbf{x} - \bar{\mathbf{x}})$. Vortex gauge transformations are deformations of the surfaces at fixed boundary lines which add to $\boldsymbol{\theta}^v(\mathbf{x})$ pure gradients of the form $\boldsymbol{\nabla}\delta(V;\mathbf{x})$, where $\delta(V;\mathbf{x}) \equiv \int_V d^3\bar{x}\, \delta(\mathbf{x} - \bar{\mathbf{x}})$ are δ-functions on the volumes V over which the surfaces have swept. The theory of these fields has been developed in the textbook [13] and the Cambridge lectures [14]. Being a gauge field, $\boldsymbol{\theta}^v(\mathbf{x})$ may be modified by a further gradient of a smooth function to make it purely transverse, $\boldsymbol{\nabla} \cdot \boldsymbol{\theta}^v_T(\mathbf{x}) = 0$, as indicated by the subscript T. Since the vortex gauge field is not a gradient, it cannot be absorbed into the vector potential by a gauge transformation. Hence it survives in the last term in Eq. (3), and the correct partition function is

$$Z \approx \int \mathcal{D}\boldsymbol{\theta}^v_T \int \mathcal{D}\rho\rho\,\mathcal{D}\mathbf{A} \exp\left[-\frac{1}{2}(\boldsymbol{\nabla}\rho)^2 - \frac{\tau}{2}\rho^2 - \frac{g}{4}\rho^4 - \frac{1}{2}(\boldsymbol{\nabla}\times\mathbf{A})^2 - \frac{\rho^2 e^2}{2}(\mathbf{A} - 2\pi\boldsymbol{\theta}^v_T/e)^2\right]. \tag{13}$$

The symbol $\int \mathcal{D}\boldsymbol{\theta}^v_T$ does not denote an ordinary functional integral. It is defined as a sum over all numbers and shapes of Volterra surfaces S in $\boldsymbol{\theta}_T^v$, across which the phase jumps by 2π [14].

The important observation is now that the partial partition function of the A-field contained in (13)

$$Z_1[\rho] \equiv \int \mathcal{D}\boldsymbol{\theta}_T^v \mathcal{D}\mathbf{A} \exp\left\{-\frac{1}{2}\int d^3x(\boldsymbol{\nabla}\times\mathbf{A})^2 -\frac{\rho^2}{2}\int d^3x[e\mathbf{A}-2\pi\boldsymbol{\theta}_T^v]^2\right\} \quad (14)$$

can give rise to a second-order transition of the XY-model type if the Ginzburg parameter κ is sufficiently large. To see this we integrate out the $\mathbf{A}$-field and obtain

$$Z_1[\rho] = \exp\left[\int d^3x \frac{e^3\rho^3}{6\pi}\right]\int \mathcal{D}\boldsymbol{\theta}_T^v \quad (15)$$
$$\times \exp\left[\frac{4\pi^2\rho^2}{2}\int d^3x\left(\frac{1}{2}\boldsymbol{\theta}_T^{v\,2} - \boldsymbol{\theta}_T^v\frac{\rho^2e^2}{-\boldsymbol{\nabla}^2+\rho^2e^2}\boldsymbol{\theta}_T^v\right)\right].$$

The second integral can be simplified to

$$\frac{4\pi^2\rho^2}{2}\int d^3x\left(\boldsymbol{\theta}_T^v\frac{-\boldsymbol{\nabla}^2}{-\boldsymbol{\nabla}^2+\rho^2e^2}\boldsymbol{\theta}_T^v\right). \quad (16)$$

Integrating this by parts, and replacing $\nabla_i\boldsymbol{\theta}_T^v\,\nabla_i\boldsymbol{\theta}_T^v$ by $(\boldsymbol{\nabla}\times\boldsymbol{\theta}_T^v)^2 = \mathbf{j}^{v\,2}$, since $\boldsymbol{\nabla}\dot{\boldsymbol{\theta}}_T^v = 0$, the partition function (15) without the prefactor takes the form

$$Z_2[\rho] = \int \mathcal{D}\boldsymbol{\theta}_T^v \exp\left[-\frac{4\pi^2\rho^2}{2}\int d^3x\left(\mathbf{j}^v\frac{1}{-\boldsymbol{\nabla}^2+\rho^2e^2}\mathbf{j}^v\right)\right]. \quad (17)$$

This is the partition function of a grand-canonical ensemble of closed fluctuating vortex lines. The interaction between them has a finite range equal to the penetration depth $\lambda = 1/\rho e$.

It is well-known how to compute pair and magnetic fields of the Ginzburg-Landau theory for a single straight vortex line from the extrema of the energy density [2]. In an external magnetic field, there exist triangular and various other regular arrays of vortex lattices and various phase transitions. In the core of each vortex line, the pair field ρ goes to zero over a distance ξ. If we want to sum over grand-canonical ensemble of fluctuating vortex lines of any shape in the partition function (13), the space dependence of ρ causes complications. These can be avoided by an approximation, in which the system is placed on a simple-cubic lattice of spacing $a = \alpha\,\xi$, with α of the order of unity, and a *fixed* value $\rho = \tilde{\rho}_0$ given by Eq. (7). Thus we replace the partial partition function (17) approximately by

$$Z_2[\tilde{\rho}_0] = \sum_{\{\mathbf{l};\boldsymbol{\nabla}\cdot\mathbf{l}=0\}} \exp\left[-\frac{4\pi^2\tilde{\rho}_0^2 a}{2}\sum_{\mathbf{x}}\mathbf{l}(\mathbf{x})v_{\tilde{\rho}_0 e}(\mathbf{x}-\mathbf{x}')\mathbf{l}(\mathbf{x}')\right]. \quad (18)$$

The sum runs over the discrete versions of the vortex density in (12). These are integer-valued vectors $\mathbf{l}(\mathbf{x}) = (l_1(\mathbf{x}), l_2(\mathbf{x}), l_3(\mathbf{x}))$ which satisfy $\boldsymbol{\nabla}\cdot\mathbf{l}(\mathbf{x}) = 0$, where $\boldsymbol{\nabla}$ denotes the lattice derivative. This condition restricts the sum over all $\mathbf{l}(\mathbf{x})$-configurations in (18) to all non-selfbacktracking integer-valued closed loops. The function

$$v_m(\mathbf{x}) = \prod_{i=1}^{3}\int\frac{d^3(ak_i)}{(2\pi)^3}\frac{e^{i(k_1x_1+k_2x_2+k_3x_3)}}{2\sum_{i=1}^3(1-\cos ak_i)+a^2m^2}$$
$$= \int ds e^{-(6+m^2)s} I_{x_1}(2s)I_{x_2}(2s)I_{x_3}(2s). \quad (19)$$

is the lattice Yukawa potential [15].

The lattice partition function (18) is known to have a second-oder phase transition in the universality class of the XY-model. This can be seen by a comparison with the Villain approximation [16] to the XY model, whose partition function is a lattice version of

$$Z_V[\rho] = \int\mathcal{D}\theta\int\mathcal{D}\boldsymbol{\theta}_T^v \exp\left[-\frac{b}{2}\int d^3x\,(\boldsymbol{\nabla}\theta-\boldsymbol{\theta}_T^v)^2\right].$$

After integrating out $\theta(\mathbf{x})$, this becomes

$$Z_V[\rho] = \mathrm{Det}^{-1/2}(-\boldsymbol{\nabla}^2)\int\mathcal{D}\boldsymbol{\theta}_T^v\exp\left(-\frac{b}{2}\int d^3x\,\boldsymbol{\theta}_T^{v\,2}\right), \quad (20)$$

and we can replace $\boldsymbol{\theta}_T^{v\,2}$ by $(\boldsymbol{\nabla}\times\boldsymbol{\theta}_T^v)(-\boldsymbol{\nabla}^2)^{-1}(\boldsymbol{\nabla}\times\boldsymbol{\theta}_T^v) = \mathbf{j}^v(-\boldsymbol{\nabla}^2)^{-1}\mathbf{j}^v$. By taking this expression to a simple-cubic lattice we obtain the partition function (18), but with $\tilde{\rho}_0^2 a$ replaced by $\beta_V \equiv ba$, and the Yukawa potential $v_{\tilde{\rho}_0 e}(\mathbf{x})$ replaced by the Coulomb potential $v_0(\mathbf{x})$.

The partition function (18) has the same transition at roughly the same place as its local approximation

$$Z_2[\tilde{\rho}_0] \approx \sum_{\{\mathbf{l};\boldsymbol{\nabla}\cdot\mathbf{l}=0\}}\exp\left[-\frac{4\pi^2\tilde{\rho}_0^2a}{2}v_{\tilde{\rho}_0 e}(\mathbf{0})\sum_{\mathbf{x}}\mathbf{l}^2(\mathbf{x})\right]. \quad (21)$$

A similar approximation holds for the Villain model with $v_0(\mathbf{x})$ instead of $v_{\tilde{\rho}_0 e}(\mathbf{x})$, and $\tilde{\rho}_0^2a$ replaced by $\beta_V \equiv ba$.

The Villain model is known to undergo a second-order phase transition of the XY-model type at $\beta_V = r/3$ with $r \approx 1$, where the vortex lines become infinitely long [16,17]. Thus we conclude that also the partition function (21) has a second-order phase transition of the XY-model type at $\tilde{\rho}^2 v_{\tilde{\rho}_0 e}(\mathbf{0})a \approx v_0(\mathbf{0})/3$. The potential (19) at the origin has the hopping expansion [18]

$$v_m(\mathbf{0}) = \sum_{n=0,2,4}\frac{H_n}{(a^2m^2+6)^{n+1}},\ H_0 = 1, H_2 = 6, \dots . \quad (22)$$

To lowest order, this yields the ratio $v_m(\mathbf{0})/v_0(\mathbf{0}) \equiv 1/(m^2/6+1)$. A more accurate numerical fit to the ratio $v_m(\mathbf{0})/v_0(\mathbf{0})$ which is good up to $m^2 \approx 10$ (thus comprising all interesting κ-values since m^2 is of the order of $3/\kappa^2$) is $1/(\sigma\,m^2/6+1)$ with $\sigma \approx 1.38$. Hence the transition takes place at

$$\frac{\tilde\rho_0^2 a}{(\sigma\, a^2\tilde\rho_0^2 e^2/6+1)} \approx \frac{r}{3} \;\text{ or }\; \tilde\rho_0 \approx \frac{1}{\sqrt{3a}}\sqrt{\frac{r}{1-\sigma r a e^2/18}}. \tag{23}$$

The important point is now that this transition can occur only until $\tilde\rho_0$ reaches the value $\rho_1 = 2c/3g$ of Eq. (9). From there on, the transition will no longer be of the XY-model type but occur discontinuously as a first-order transition.

Replacing in (23) a by $\alpha\xi_1$ of Eq. (10), and $\tilde\rho_0$ by ρ_1, we find the equation for the mean-field Ginzburg parameter $\kappa_{\rm MF} = \sqrt{g/e^2}$:

$$\kappa_{\rm MF}^3 + \alpha^2\sigma\frac{\kappa_{\rm MF}}{3} - \frac{\sqrt{2}\alpha}{\pi r} = 0. \tag{24}$$

Inserting $\sigma \approx 1.38$ and choosing $\alpha \approx r \approx 1$, the solution of this equation yields the tricritical value

$$\kappa_{\rm MF}^{\rm tric} \approx 0.81/\sqrt{2}. \tag{25}$$

The approximation has three uncertainties. First, the identification of the effective lattice spacing $a = \alpha\xi$ with $\alpha \approx 1$; second the associated neglect of the $\mathbf{x}$-dependence of ρ and its fluctuations, and third the localization of the critical point of the XY-model type transition in Eq. (23).

Thus we have shown the existence of a tricritical point in a superconductor within the fluctuating Ginzburg-Landau theory [19]. For this is was crucial to take the vortex fluctuations into account. This became possible after correcting the covariant derivative (2) of $\psi = \rho e^{i\theta}$ to (11). For $\kappa > 0.81/\sqrt{2}$, vortex fluctuations give rise to an XY-model type second-order transition before the cubic term becomes relevant. This happens for $\kappa < 0.81/\sqrt{2}$ where the cubic term causes a discontinuous transition.

5. Disorder Field Theory

In the spirit of Landau is is suggestive to describe the second-order phase transition caused by the proliferation of vortex lines again by a Ginzburg-Landau like field theory of its own. This theory will have the property that its diagrammatic representation displays directly the pictures of the vortex lines in a grand-canonical ensemble. Thus it may rightfully be called *disorder field theory* [13]. Since the vortex lines proliferate in the high-temperature phase, their disorder field acquires a nonzero expectation value in that phase, in contrast to the order field in the original Ginzburg-Landau energy density (1). It can easily be shown that if $\varphi(\mathbf{x})$ denotes the disorder field, the partition function (14) can be replaced by a field partition function

$$Z_2 = \int \mathcal{D}\varphi\mathcal{D}\varphi^*\, e^{-\int d^3x\,\mathcal{H}_2} \tag{26}$$

with the energy density

$$\mathcal{H}_2(\varphi, \boldsymbol{\nabla}\varphi) = \frac{1}{2}\left\{[\boldsymbol{\nabla}\varphi]^2 + \tau_2|\varphi|^2 + \frac{g_2}{2}|\varphi|^4\right\}. \tag{27}$$

The parameter τ_2 becomes negative *above* T_c, where $\langle\varphi\rangle$ takes a nonzero expectation value. The disorder theory can be studied with the usual techniques developed by Landau. In this way we find once more the result In spite of the roughness of the approximations, this result is very close to the value $0.8/\sqrt{2}$ derived from the dual theory in [12]. The details are given in Ref. [13].

6. Outlook

The disorder field exhibited here can be used to describe many different phase transitions as long as they are caused by the proliferation of line-like excitations. In some systems, the relevant excitations are surface- or volume-like. For instance, the transition of a magnetic system in three dimensions to a nonmagnetic phase may be viewed as a consequence of a proliferation of boundary surfaces of Weiss domains. For surfaces or volumes there exists, unfortunately, no fluctuating field which is capable of describing their grand-canonical ensembles. This is why the theory of random surfaces, which is equivalent to string theory, is always studied in the first-quantized description, in which a functional integral is performed explicitly over any number of fluctuating surfaces or volumes. The construction of such a field theory is one of the major problems of string and brane theory. Once this will be solved, we shall be able to study also the associated phase transitions with Landau's techniques via a disorder field theory.

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[1] L.D. Landau, Zh. Elsp. Teor. Fiz. *7*, 627 (1937); V.L. Ginzburg and L.D. Landau, ibid. *20*, 1064 (1950).

[2] D. Saint-James, G. Sarma, and E.J. Thomas, *Type II Superconductivity*, Pergamon, Oxford, 1969; M. Tinkham, *Introduction to Superconductivity*, McGraw-Hill, New York, 1996.

[3] M. Tinkham, *Introduction to Superconductivity*, 2nd ed., Dover, New York, 1996.

[4] V.L. Ginzburg, Fiz. Twerd. Tela **2**, 2031 (1960) [Sov. Phys. Solid State **2**, 1824 (1961)]. See also the detailed discussion in Chapter 13 of the textbook L.D. Landau and E.M. Lifshitz, *Statistical Physics*, 3rd edition, Pergamon Press, London, 1968. The Ginzburg criterion is derived for and applicable to systems with only a real order parameter. If the order parameter is complex, the important fluctuations take place in the phase of the order parameter: H. Kleinert, Phys. Rev. Lett. **84** , 286 (2000) (cond-mat/9908239).

[5] B.I. Halperin, T.C. Lubensky, and S. Ma, Phys. Rev. Lett. **32**, 292 (1972).

[6] T. Schneider, J. M. Singer, *A Phase Transition Approach to High Temperature Superconductivity: Universal Properties of Cuprate Superconductors*, World Scientific, Singapore, 2000.

[7] T. Schneider, R. Khasanov, and H. Keller, Phys. Rev. Lett. 94, 77002 (2005), T. Schneider, R. Khasanov, K. Conder, E. Pomjakushina, R. Bruetsch, and H. Keller, J. Phys. Condens. Matter **16**, L1 (2004) (cond-mat/0406691).

[8] L.P. Kadanoff, Physics *2*, 263 (1966); K.G. Wilson, Phys. Rev. B *4*, 3174, 3184 (1971); K.G. Wilson and M.E. Fisher, Phys. Rev. Lett. *28*, 240 (1972); and references therein.

[9] A small selection is: J. Tessmann, MS thesis 1984 (the pdf file is available on the internet at (`kl/MS-Tessmann.pdf`), where kl is short for `www.physik.fuberlin.de/~kleinert`; S. Kolnberger and R. Folk, Phys. Rev. B **41**, 4083 (1990); R. Folk and Y. Holovatch, J. Phys. A **29**, 3409 (1996); I.F. Herbut and Z. Tešanović, Phys. Rev. Lett. **76**, 4588 (1996); H. Kleinert and F.S. Nogueira Nucl. Phys. B **651**, 361 (2003) (cond-mat/0104573).

[10] H. Kleinert, *Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets*, World Scientific, Singapore 2004, Third extended edition, pp. 1–1450 (`kl/b5`).

[11] See Eq. (3.111) in Part II of Ref. [13] (`kl/b1/gifs/v1-337s.html`).

[12] H. Kleinert, Lett. Nuovo Cimento **37**, 295 (1982) (`kl/108`). The tricritical value $\kappa \approx 0.8/\sqrt{2}$ derived in this paper was confirmed only recently by Monte Carlo simulations: J. Hove, S. Mo, and A. Sudbø, Phys. Rev. B 66, 64524 (2002).

[13] H. Kleinert, *Gauge Fields in Condensed Matter*, Vol. I Superflow and Vortex Lines, World Scientific, Singapore 1989, pp. 1–756 (`kl/b1`).

[14] H. Kleinert, *Theory of Fluctuating Nonholonomic Fields and Applications: Statistical Mechanics of Vortices and Defects and New Physical Laws in Spaces with Curvature and Torsion*, in *Proceedings of NATO Advanced Study Institute on Formation and Interaction of Topological Defects* at the University of Cambridge, Plenum Press, New York, 1995, p. 201–232 (`kl/227`).

[15] See Eq. (6.110) in Part I of Ref. [13] (`kl/b1/gifs/v1-165s.html`).

[16] J. Villain, J. Phys. (Paris) **36**, 581 (1977). See also `kl/b1/gifs/v1-489s.html`.

[17] See the figure on p. 501 of Ref. [13] (`kl/b1/gifs/v1-501s.html`).

[18] See Eq. (6.122) in Part I of Ref. [13] (`kl/b1/gifs/v1-168s.html`).

[19] H. Kleinert, *Vortex Origin of Tricritical Point in Ginzburg-Landau Theory*, Europh. Letters **74**, 889 (2006) (cond-mat/0509430).