GAUGE DEPENDENCE OF THE RENORMALIZATION GROUP PARAMETERS IN PURE YANG-MILLS THEORY

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The β-function of a massless Yang-Mills theory is gauge independent in lowest order for a general class of gauges. For a subclass of gauges including the Landau and the Coulomb gauges the effective gauge approaches the Coulomb gauge in the ultraviolet limit.

The asymptotic behaviour of Green's functions or Wilson coefficient functions has recently been discussed for non abelian gauge theories using the renormalization group equation [1]. For these theories which are believed to be relevant in a field theoretical discussion of Bjorken scaling the gauge dependence of the renormalization group parameters [1, 2] must not destroy their "asymptotic freedom" [3] in the ultraviolet region.

In this note a pure Yang-Mills theory is considered and the gauge dependence of the renormalization group parameters is given for a more general class of gauges including the Landau and the Coulomb gauge. The Coulomb gauge may be useful for the discussion of infrared problems [4].

The Lagrangian for the massless vector fields is given by

$$\mathcal{L} = -\tfrac{1}{4} G^a_{\mu\nu} G^{a\nu} + \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{FP}} + \text{counterterms}$$

with

$$G^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g f^{abc} B^b_\mu B^c_\nu; \qquad f_{acd} f_{bcd} = 2C_1 \delta_{ab}, \tag{1}$$

while $\mathcal{L}_{\mathrm{G}}$ is a gauge fixing term and $\mathcal{L}_{\mathrm{FP}}$ is the gauge dependent Faddeev-Popov ghost term. In the following

$$\mathcal{L}_{\mathrm{G}} = \frac{1}{2\alpha} (\partial_\mu \bar{B}^{c\mu})^2 \text{ with } \bar{B}^c_\mu = B^c_\mu - a\eta_\mu (\eta B^c)$$

where η_μ is an arbitrary vector with $1 - a\eta^2 \geqslant 0$. This includes all renormalizable non singular [5] gauges and implies for $\alpha = 0$ the propagator

$$D^{ab}_{\mu\nu}(p) = -\frac{\mathrm{i}\delta_{ab}}{p^2} \left\{ g_{\mu\nu} + \frac{\bar{p}^2}{(p\bar{p})^2} p_\mu p_\nu - \frac{\bar{p}_\mu p_\nu + p_\mu \bar{p}_\nu}{p\bar{p}} \right\}$$

with $\bar{p}_\mu = p_\mu - a\eta_\mu(\eta p)$.

As usual in all the gauges the Lagrangian (1) is multiplicatively renormalizable, but there are two different wave-function renormalization constants z_η and z_τ corresponding to the parallel respectively transverse part of the vector field with respect to the vector η;

The 1PI-Greens functions obey the renormalization group equation

$$\left(\mu \frac{\partial}{\partial\mu} + \beta(g,a,\alpha) \frac{\partial}{\partial g} - n_\eta \gamma_\eta(g,a,\alpha) - n_\tau \gamma_\tau(g,a,\alpha) + \delta_1(g,a,\alpha) \frac{\partial}{\partial\alpha} + \delta_2(g,a,\alpha) \frac{\partial}{\partial a} \right) \Gamma^{(n_\eta, n_\tau)}(p_1, \dots p_n, \mu, g, a, \alpha) = 0 \tag{2}$$

where μ is a normalization mass and $\eta_\eta + n_\tau = n$. For $\alpha = 0$ and $\eta^2 = 1$ lowest order of perturbation theory gives

$$\gamma_\eta(g,a) = -\frac{g^2 C_1}{16\pi^2}\,\tfrac{1}{3}(22-9\sqrt{1-a} + O(g^3), \qquad \gamma_\tau(g,a) = -\frac{g^2 C_1}{16\pi^2}\,\tfrac{1}{3}\left(21-\frac{16-a}{1+\sqrt{1-a}}\right) + O(g^3),$$

$$\beta(g,a) = -\frac{g^3 C_1}{16\pi^2}\,\tfrac{22}{3} + O(g^5), \qquad \delta_1(g,a) = 0, \qquad \delta_2(g,a) = \frac{g^2 C_1}{16\pi^2}\,\tfrac{8}{3}(1-a)\left(\frac{a+2}{1+\sqrt{1-a}}-1\right) + O(g^3) \tag{3}$$

In the Landau gauge ($a = 0$) one recovers the known values of $\gamma_\eta = \gamma_\tau = \gamma_\nu$ [3]

The solution of the renormalization group equation is expressed in terms of "effective parameters" $\bar{g}(\lambda,g)$ and $\bar{a}(\lambda,g)$ using the method of characteristics with the scaling parameter λ: $\bar{g}(\lambda,g)$ and $\bar{a}(\lambda,g)$ satisfy the following differential equations

$$\frac{d\bar{g}}{d\ln\lambda} = \beta(\bar{g}); \qquad \frac{d\bar{a}}{d\ln\lambda} = \delta_2(\bar{g},\bar{a}) \tag{4}$$

with boundary conditions $\bar{g}(1) = g$ and $\bar{a}(1) = a$.

Eqs. (3) for β and δ_2 show that for $a > \upsilon$ the point $\bar{g} = 0, \bar{a} = 1$ is a stable fix point in the limit $\lambda \to \infty$. This means that for high energies the Greens functions behave like in the Coulomb gauge; in particular

$$\Gamma^{(n_\eta,n_\tau)}(\lambda_{p_1},\ldots\lambda_{p_n},\mu,g,a) \xrightarrow{\lambda\to\infty} \lambda^{4-n_\eta-n_\tau}(\ln\lambda)^{\frac{n_\eta C_\eta + n_\tau C_\tau}{D_0}}\,\Gamma^{(n_\eta,n_\tau)}(p_1,\ldots p_n,\mu,0,1) \tag{5}$$

with $C_\eta = -\frac{22}{3}$, $C_\tau = -2$, $D_0 = -\frac{22}{3}$.

It would be interesting to calculate the fixpoint (g^*, a^*, α^*) for $\alpha \neq 0$, since this point will determine the asymptotic behaviour in any gauge and presumably also the behaviour of the gauge invariant physical quantities‡.

The result obtained is that the β- function is gauge independent in lowest order for the above class of renormalizable non singular gauges. This emphasizes the dynamical meaning of the sign of the β-function. For $\alpha = 0$ the effective gauge approaches the Coulomb gauge in the ultraviolet limit.

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