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ON THE BOUND STATE PROBLEM IN 1+1 DIMENSIONAL FIELD THEORIES

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In the framework of factorizing S-matrices in $1+1$ dimensions, further restrictions for the construction of S-matrices are discussed. A relation between residues of S-matrix poles and the parities of corresponding bound states is derived.

1. Introduction

In theoretical elementary particle physics quantum field theory has gained renewed interest in the last years. Non-Abelian gauge theories unify weak and electromagnetic interactions and QCD seems to be a good candidate for the description of strong interactions. Since these theories in four dimensions are very complicated it is useful to study simpler models in two space-time dimensions with similar properties e.g., "asymptotic freedom", "confinement", non-trivial topological structure, θ-vacua, etc. There are models possessing some of these properties which have a chance to be explicitly solvable. This class of 2-dimensional field theories, the so-called soliton field theories, is characterized by an infinite set of conservation laws which imply the factorization of the S-matrix. It is amazing that the procedure used to solve these models is just the old analytic S-matrix program. First, by constraints due to unitarity, crossing, internal symmetries, and the special property of factorization, the S-matrix can be determined [1a]*, then matrix elements of local operators [2], and finally the correlation functions. The whole program has been carried out until now only in a very simple soliton field theory, the Ising model in the scaling limit [3].

The procedure is at several stages non-unique but minimality assumptions are necessary. Under the constraints mentioned above, the S-matrix is unique up to CDD-like singularities. It is the purpose of this paper to give more restrictions in order to select allowed S-matrices. We shall give a necessary condition for a CDD-like pole in a two-particle S-matrix to be connected with bound states. Otherwise the pole has to be redundant [4]. This condition is based on the positivity of the

* For reviews see ref. [1b] and references therein.

state space metric. The restrictions may be useful for the derivation of S-matrices in more models, such as the chiral SU(N) model [5], the CP^n model [6], etc.

In sect. 2 we present, for the case of bosons, the framework of factorizing S-matrices. The S-matrix for the scattering of bound states with fundamental particles is constructed in sect. 3. In appendix A we discuss the general case including supersymmetric models. In appendix B the general methods are applied to an U(2) S-matrix.

2. Factorizing S-matrix

We consider an S-matrix describing the scattering of fundamental particles of various kinds labeled by α with mass m. For simplicity we take the case of bosons, the general case is discussed in appendix A. Factorization means that for a scattering process the sets of incoming and outgoing momenta are equal:

$$\{p_1, ..., p_n\}^{\text{in}} = \{p'_1, ..., p'_{n'}\}^{\text{out}}, \tag{1}$$

and the n-particle S-matrix is a product of two-particle ones in a special order (e.g., for $p_1^1 > \cdots > p_n^1$) [7]

$$S^{(n)}(p_1, ..., p_n) = \prod_{i=1}^{n-1} \left(\prod_{j=1}^{n} S^{(2)}(p_i, p_j) \right), \tag{2}$$

where $S^{(2)}(p_i, p_j) = S_{ij}$ is given by

$$S_{ij} | ... \alpha(p_i) ... \alpha(p_j) ... \rangle = | ... \alpha'(p_i) ... \alpha'(p_j) ... \rangle_{\alpha'\beta'} S^{(2)}_{\alpha\beta}(p_i, p_j) . \tag{3}$$

The factors in eq. (2) do not commute in general but they have to fulfil a special commutation rule, the factorization equation

$$S^{(3)}(p_1, p_2, p_3) = S_{12}S_{13}S_{23} = S_{23}S_{13}S_{12} . \tag{4}$$

For convenience we introduce the rapidity difference variable by $p_1 p_2 = m^2 \text{ch}\theta$. Real analyticity, unitarity and crossing imply

$$S^+(\theta) = S(-\theta^*) , \tag{5}$$

$$S(-\theta)S(\theta) = 1 , \tag{6}$$

$${}_{\alpha\beta}S_{\gamma\delta}(\theta) = {}_{\alpha\bar{\delta}}S_{\gamma\bar{\beta}}(i\pi - \theta) , \tag{7}$$

where $\bar{\alpha}$ denotes the antiparticle of α. PT invariance means for the n-particle S-matrix:

$$S^{(n)} = (\eta^* \xi^* S^{(n)} \eta \xi)^{\text{T}} ,$$

where ξ and η are diagonal matrices with

$${}_{\alpha'\beta'...}\xi_{\alpha\beta...} = \delta_{\alpha'\alpha}\xi_\alpha \delta_{\beta'\beta}\xi_\beta ..., \qquad T|\alpha(p)\rangle = \xi_\alpha |\alpha(-p)\rangle , \tag{8a}$$

$$_{\alpha'\beta'\ldots}\eta_{\alpha\beta\ldots} = \delta_{\alpha'\alpha}\eta_\alpha\delta_{\beta'\beta}\eta_\beta \cdots , \qquad P|\alpha(p)\rangle = \eta_\alpha|\alpha(-p)\rangle . \tag{8b}$$

(It is convenient to take phases and use conventions such that $\xi_\alpha = 1$ and $\eta_\alpha = \eta_{\bar\alpha} = \pm 1$ for bosons and $\eta_\alpha = \eta_{\bar\alpha} = \pm i$ for fermions.) The two-particle S-matrix can be written as

$$S^{(2)} = \sum_{\mathrm{a}} S_{\mathrm{a}}(\theta) P_{\mathrm{a}} , \tag{9}$$

where $S_{\mathrm{a}}(\theta)$ are the eigenvalues and P_{a} the projectors on the corresponding eigenstates

$$|a(p_1+p_2, \theta)\rangle = |\alpha(p_1)\beta(p_2)\rangle_{\alpha\beta}\phi_{\mathrm{a}}(\theta) . \tag{10}$$

3. Bound states

Let us assume that some of the eigenvalues of $S^{(2)}$ have a pole in the physical sheet at $\theta = i\pi\alpha$ $(0<\alpha<1)$ corresponding to bound states b with parities η_{b} and the same mass

$$m_{\mathrm{b}} = 2m \cos \tfrac{1}{2}\pi\alpha . \tag{11}$$

We are of course not able to construct the bound states (i.e., the wave functions of the states b) rigorously from the fundamental particles α, since we only know the theory on-shell. We even do not know whether they exist. The support of the b wave function intersects the α-particle mass-shell only at two points in the Euclidean region:

$$p_{1,2} = \begin{pmatrix} \sqrt{m^2+q^2} \\ \pm q \end{pmatrix}, \qquad q = i\sqrt{m^2 - \tfrac{1}{4}m_{\mathrm{b}}^2} \tag{12}$$

(in the c.m.s.).

Formally we identify the bound states with the corresponding eigenstates of $S^{(2)}$ at rapidity difference $\theta = \frac{1}{2}\pi\alpha$

$$|b(p_1+p_2)\rangle \equiv |a(p_1+p_2, i\pi\alpha)\rangle . \tag{13}$$

Let R_{a} be the residues of S_{a} (which are real) and P_{b} the projectors on $|b\rangle$ and

$$\operatorname*{Res}_{(p_1+p_2)^2 = m_{\mathrm{b}}^2} S_{12}(\theta) = R_{12} \equiv \sum_{\mathrm{b}} R_{\mathrm{b}} P_{\mathrm{b}} . \tag{14}$$

Then from the factorization equation (4) we derive:

$$R_{12}S_{13}S_{23} = S_{23}S_{13}R_{12} , \tag{15}$$

$$\Big(1 - \sum_{\mathrm{b}} P_{\mathrm{b}}\Big) S_{23}S_{13} \sum_{\mathrm{b}} P_{\mathrm{b}} = 0 . \tag{15a}$$

We now construct the two particle S-matrix for the scattering of bound states b with fundamental particles α by means of the conditions of factorization and unitarity. We make the ansatz

$$S_{1+2,3}(\theta) \equiv A \operatorname*{Res}_{(p_1+p_2)^2=m_b^2} S^{(3)}(p_1, p_2, p_3) B\,, \tag{16}$$

where the matrices A and B (which act only on the constituents of b) are to be determined and the rapidity differences are $\theta_{13} = \theta + \frac{1}{2}i\pi\alpha$, $\theta_{23} = \theta - \frac{1}{2}i\pi\alpha$. The factorization equation (4) now reads

$$S_{1+2,3}S_{1+2,4}S_{34} = S_{34}S_{1+2,4}S_{1+2,3}\,. \tag{17}$$

It is easy to see from eqs. (15), (15a) that this commutation relation holds true if*

$$BAR_{12} = \sum_b P_b. \tag{18}$$

Unitarity for the bound state S-matrix means

$$\begin{aligned} 1 &= S^+_{1+2,3}(\theta) S_{1+2,3}(\theta) \\ &= B^+ S^+_{23} S^+_{13} R^+_{12} A^+ A S_{23} S_{13} R_{12} B \\ &= B^+ E_{12} S^{-1}_{13} S^{-1}_{23} E_{12} R^+_{12} A^+ A S_{23} S_{13} R_{12} B\,, \end{aligned} \tag{19}$$

where E_{12} is the "exchange operator" defined by

$$E_{12}|\alpha(p_1)\beta(p_2)\ldots\rangle = |\beta(p_1)\alpha(p_2)\ldots\rangle\,. \tag{20}$$

In eq. (19) the fact has been used that

$$S^+_{13}(\theta_{13}) = S_{13}(-\theta^*_{13}) = S_{13}(-\theta_{23}) = E_{12}S_{23}(-\theta_{23})E_{12} = E_{12}S^{-1}_{23}(\theta_{23})E_{12}\,. \tag{21}$$

Eqs. (19) and (15a) show that S_{1+2+3} is unitary if*

$$E_{12}R^+_{12}A^+A \text{ const} = \sum_b P_b\,. \tag{22}$$

From eqs. (2), (10) and (8b) we obtain the action of E_{12} on an S-matrix eigenstate

$$E_{12}|a\rangle = \eta^{-1}|a\rangle\eta_a\,. \tag{23}$$

where $\eta^{-1} = 1$ for a boson-antiboson state. Since A^+A is a positive operator we derive from eq. (22) the condition for the residues and the bound state parities: $R_b\eta_b \text{ const} > 0$ for all bound states b corresponding to the pole of $S^{(2)}$ at $\theta = i\pi\alpha$. In potential scattering the number $R_b\eta_b$ can be shown to be always negative, which is also true for the sine-Gordon model. Therefore the condition

$$R_b\eta_b < 0 \tag{24}$$

should hold in general. From eqs. (16), (18), (22) and (23) we finally obtain the

* Solutions of factorization equations and unitarity are unique up to CDD-like singularities [1]. The solution given by eqs. (18), (22) is a minimal one.

S-matrix for the scattering of a bound state and a fundamental particle:

$$S_{1+2,3}(\theta)=\sum_{b'}|R_{b'}|^{-1/2}P_{b'}S_{23}S_{13}\sum_{b}|R_{b}|^{1/2}P_{b}\,. \tag{25}$$

Note, that if there exist "wrong" bound states with $R_b\eta_b>0$ and there are transitions between "wrong" and "right" states (with $R_b\eta_b<0$)

$$b^{\text{wrong}}+\alpha\to b^{\text{right}}+\beta\,,$$

the "wrong" ones would appear as intermediate states in the unitarity equation (19) with a minus sign. This means they have negative norm. If we want to consider an S-matrix defined in a positive definite state space, we have the following conclusion: a pole of a two-particle S-matrix can only have a physical meaning, if all residues of the S-matrix eigenvalues R_b and the eigenstate parities η_b corresponding to this pole fulfil the condition $R_b\eta_b<0$, or "wrong" states with $R_b\eta_b>0$ decouple from the "right" ones; otherwise this pole has to be redundant [4]. This condition gives a strong restriction for introducing CDD-like poles in an S-matrix by multiplication of a minimal one by a factor $\prod \mathrm{sh}(\theta+\theta_i)/\mathrm{sh}(\theta-\theta_i)$ and interpreting these poles as physical ones corresponding to physical bound states.

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Appendix A

In this appendix we discuss a general factorizing S-matrix where transitions are also allowed like fermion-antifermion→boson-antiboson, typical for supersymmetric models. The general n-particle S-matrix is given by

$$S^{(n)}=\sigma^{1\ldots n}\prod_{i<j}(\sigma S)_{ij}=\prod_{i<j}(\sigma S)_{ij}\sigma^{1\ldots n}\,, \tag{A.1}$$

where the matrices σ take into account the statistics of the particles. They are defined by

$$_{\alpha'\beta'}\sigma_{\alpha\beta}=\sigma_{\alpha\beta}\delta_{\alpha'\alpha}\delta_{\beta'\beta}\,, \tag{A.2}$$

with $\sigma_{\alpha\beta}=\pm1$ for commuting or anticommuting particles α and β, respectively, and

$$_{\alpha_1'\ldots\alpha_n'}\sigma^{1\ldots n}_{\alpha_1\ldots\alpha_n}=\delta_{\alpha_1'\alpha_1}\ldots\delta_{\alpha_n'\alpha_n}\prod_{i<j}\sigma_{\alpha_i\alpha_j}. \tag{A.3}$$

If there are no supersymmetric like transitions from fermions to bosons, the signs given by the σ's cancel and we get back formula (2). The factorization equations read

$$S^{(3)}=\sigma^{123}(\sigma S)_{12}(\sigma S)_{13}(\sigma S)_{23}=\sigma^{123}(\sigma S)_{23}(\sigma S)_{13}(\sigma S)_{12}\,. \tag{A.4}$$

Eigenstates of $S^{(2)}$ given by eq. (10) which are dominated at low energy by particles α, β which commute or anticommute fulfil

$$ {}_{\alpha\beta}\phi_{\mathrm{a}}(-\theta) = {}_{\alpha\beta}(\sigma\phi_{\mathrm{a}}(\theta))\sigma_{\mathrm{a}} , \tag{A.5}$$

with $\sigma_{\mathrm{a}} = +1$ or -1, respectively.

PT invariance implies for real θ

$$ {}_{\alpha\beta}\phi_{\mathrm{a}}(\theta)\eta_{\mathrm{a}}\xi_\alpha = \eta_\alpha\eta_\beta\xi_\alpha\xi_\beta\phi^*_{\mathrm{a}}(\theta)\sigma_{\mathrm{a}} .$$

Hence we have

$$ {}_{\alpha'\beta'}S_{\alpha\beta}(\theta) = \sum_{\mathrm{a}} S_{\mathrm{a}}(\theta)\, {}_{\alpha'\beta'}\phi_{\mathrm{a}}(\theta)\, {}_{\alpha\beta}\phi^*_{\mathrm{a}}(\theta^*) ,$$

and for $\theta \to i\pi\alpha$ we obtain the generalization of eq. (14):

$$ \operatorname*{Res}_{(p_1+p_2)^2=m^2} S_{12} = R_{12} = \sum_{\mathrm{b}} R_{\mathrm{b}}\sigma_{\mathrm{b}}P_{\mathrm{b}}\sigma . \tag{A.6}$$

If we make the same ansatz (10) for the S-matrix $S_{1+2,3}$ the factorization equation reads (with $\sigma_{1+2,3} = \sigma_{13}\sigma_{23}$)

$$\begin{aligned} &\sigma_{1+2,3}A\sigma^{123}(\sigma R)_{12}(\sigma S)_{13}(\sigma S)_{23}B\sigma_{1+2,3}A\sigma^{124}(\sigma R)_{12}(\sigma S)_{14}(\sigma S)_{24}B(\sigma S)_{34} \\ &\quad = (\sigma S)_{34}\sigma_{1+2,3}A\sigma^{124}(\sigma R)_{12}(\sigma S)_{14}(\sigma S)_{24}B\sigma_{1+2,3} \\ &\quad \times A\sigma^{123}(\sigma R)_{12}(\sigma S)_{13}(\sigma S)_{23}B , \end{aligned} \tag{A.7}$$

which is a consequence of eq. (A.4) if

$$ B\sigma_{1+2,3}A\sigma_{1+2,3}R_{12} = \sum_{\mathrm{b}} P_{\mathrm{b}} . \tag{A.8}$$

Similarly we derive a constraint from unitarity

$$\begin{aligned} 1 &= S^+_{1+2,3}S_{1+2,3} \\ &= B^+(\sigma S)^+_{23}(\sigma S)^+_{13}(\sigma R_{12})^+\sigma^{123}A^+A\sigma^{123}(\sigma S)_{23}(\sigma S)_{13}(\sigma R)_{12}B , \end{aligned}$$

which is fulfilled, by arguments analogous to the boson case, if

$$ E_{12}(\sigma R)^+_{12}\sigma^{123}A^+A\sigma^{123} \cdot \mathrm{const} = \sigma^{123}\sum_{\mathrm{b}} P_{\mathrm{b}}\sigma^{123} . \tag{A.9}$$

The consequence is, as above, that the operator

$$ E_{12}\sigma_{12}\sum_{\mathrm{b}} P_{\mathrm{b}}R_{\mathrm{b}}\sigma_{12} \cdot \mathrm{const} = \sum_{\mathrm{b}} \sigma_{\mathrm{b}}R_{\mathrm{b}}\eta_{\mathrm{b}}(\eta^{-1}\sigma)_{12}\sigma_{12}P_{\mathrm{b}}\sigma_{12} \cdot \mathrm{const}$$

has to be positive. To be in agreement with potential scattering we demand that

$$ \sigma_{\mathrm{b}}R_{\mathrm{b}}\eta_{\mathrm{b}}\sigma_{\alpha\beta}/\eta_\alpha\eta_\beta < 0 , \tag{A.10}$$

for all bound states b of mass m_{b} built up by the constituents α and β. Note that $\sigma_{\mathrm{b}} = \pm 1$ for boson-antiboson and fermion-antifermion states, respectively, and $\sigma_{\alpha\beta}/\eta_\alpha\eta_\beta = 1$ for both cases. Finally we obtain the S-matrix for the scattering of a bound state with an elementary particle:

$$S_{1+2,3}(\theta) = \sum_{\mathrm{b}'} |R_{\mathrm{b}'}|^{-1/2} P_{\mathrm{b}'} \sigma^{123} (\sigma S)_{23} (\sigma S)_{13} \sum_{\mathrm{b}} |R_{\mathrm{b}}|^{1/2} P_{\mathrm{b}} \,. \tag{A.11}$$

If there are no supersymmetric like transitions, the signs given by the σ's cancel again and we get back formula (25).

Appendix B

This appendix contains an application of the general framework developed in this paper. We consider an U(2) symmetric factorizing S-matrix for the scattering of a doublet of fermions and antifermions. There exist five classes of non-trivial S-matrices [8]. Here we consider the class II, which is characterized by the absence of particle-antiparticle reflection

$$\begin{aligned} S^{(2)}|\alpha\beta\rangle &= |\alpha\beta\rangle u_1 + |\beta\alpha\rangle u_2 \,, \\ S^{(2)}|\alpha\bar{\beta}\rangle &= |\alpha\bar{\beta}\rangle t_1 + |\gamma\bar{\gamma}\rangle \delta_{\alpha\beta} t_2 \,. \end{aligned} \tag{B.1}$$

The amplitudes u_1, u_2 and t_2 are related to t_1 due to the factorization equation and crossing as follows

$$t_2(\varphi) = \frac{1}{\varphi - 1} t_1(\varphi), \qquad u_2(\varphi) = -\frac{1}{\varphi} u_1(\varphi), \qquad u_1(\varphi) = t_1(1-\varphi) \,, \tag{B.2}$$

where we have introduced the variable $\varphi = \theta / i\pi$. The minimal solution of eqs. (B.2) which has no poles (nor zeroes) in the physical sheet together with unitarity is [8]:

$$t_1^{\min}(\varphi) = \frac{\Gamma(\frac{1}{2} + \frac{1}{2}\varphi)\Gamma(1 - \frac{1}{2}\varphi)}{\Gamma(\frac{1}{2} - \frac{1}{2}\varphi)\Gamma(1 + \frac{1}{2}\varphi)} \,. \tag{B.3}$$

A non-minimal solution with a pole at $\varphi = \alpha$ (and $\varphi = 1 - \alpha$) for $0 < \alpha < 1$ is

$$t_1(\varphi) = t_1^{\min}(\varphi) \frac{\sin \pi\varphi + \sin \pi\alpha}{\sin \pi\varphi - \sin \pi\alpha} \,. \tag{B.4}$$

This pole appears in the triplet amplitude $S_\pi = t_1$ and the singlet amplitude $S_\eta = t_1 + 2t_2$, corresponding to states π^i and η with positive as well as negative parity

$$\begin{aligned} |\pi^i_\pm\rangle &= \tfrac{1}{2}(|\alpha(p_1)\bar{\beta}(p_2)\rangle \pm |\bar{\beta}(p_1)\alpha(p_2)\rangle)\tau^i_{\alpha\beta} \,, \\ |\eta_\pm\rangle &= \tfrac{1}{2}(|\alpha(p_1)\bar{\alpha}(p_2)\rangle \pm |\bar{\alpha}(p_1)\alpha(p_2)\rangle) \,. \end{aligned} \tag{B.5}$$

The residues at $(p_1+p_2)^2 = m_b^2 = 4m^2 \cos^2 \frac{1}{2}\pi\alpha$ fulfil

$$R_\pi < 0, \qquad R_\eta > 0, \qquad R_\eta/R_\pi = -\frac{1-\alpha}{1+\alpha}. \tag{B.6}$$

From the general condition (A.10) we know that the π^i_+ and the η_- are "wrong" states with negative norms. But it can easily be shown by explicit calculation that the "wrong" states decouple from the "right" ones, e.g., $\langle \eta_-\gamma'|S|\eta_+\gamma\rangle \equiv 0$ etc. (Note that this would not be true if we replace the pole factor in eq. (B.4) by the simpler one

$$\frac{\sin \frac{1}{2}\pi(\varphi+\alpha)}{\sin \frac{1}{2}\pi(\varphi-\alpha)} .$$

From eq. (A.11) we derive the S-matrix for scattering of bound states π^i and η with the fundamental particles α and $\bar{\alpha}$. The amplitudes defined by

$$\begin{aligned}
&\langle \pi^i\alpha|S|\pi^j\beta\rangle = \langle \pi^i\bar{\beta}|S|\pi^j\bar{\alpha}\rangle = \delta_{ij}\delta_{\alpha\beta}a + i\epsilon_{ijk}\tau^k_{\alpha\beta}b\,,\\
&\langle \eta\alpha|S|\pi^i\beta\rangle = -\langle \eta\bar{\beta}|S|\pi^i\bar{\alpha}\rangle = \tau^i_{\alpha\beta}c\,,\\
&\langle \eta\alpha|S|\eta\beta\rangle = \langle \eta\bar{\beta}|S|\eta\bar{\alpha}\rangle = \delta_{\alpha\beta}d\,,
\end{aligned} \tag{B.7}$$

are then given by

$$\begin{pmatrix} a\\ b\\ c\\ d\end{pmatrix} = -\tfrac{1}{2}\begin{pmatrix} t_1t_1+u_1u_1\\ -t_1t_1+u_1u_1\\ \sqrt{1-\alpha^2}\,u_2u_2\\ t_1t_1+u_1u_1-2u_2u_2\end{pmatrix}, \tag{B.7a}$$

where the arguments on the r.h.s. are $\varphi-\frac{1}{2}\alpha$ and $\varphi+\frac{1}{2}\alpha$. Applying formula (A.11) again we obtain the bound state S-matrix elements

$$\begin{aligned}
&\langle \pi^i\pi^j|S|\pi^k\pi^l\rangle = \delta_{ij}\delta_{kl}\sigma_1 + \delta_{ik}\delta_{jl}\sigma_2 + \delta_{il}\delta_{jk}\sigma_3\,,\\
&\langle \eta\eta|S|\pi^i\pi^j\rangle = \delta_{ij}\tau\,,\\
&\langle \eta\eta|S|\eta\eta\rangle = \rho\,,
\end{aligned} \tag{B.8}$$

where

$$\begin{aligned}
&\sigma_1 = ab+ba+bb-cc, \qquad \sigma_2 = aa+cc,\\
&\sigma_3 = -ab-ba+bb-cc,\\
&\tau = -\sqrt{\frac{1-\alpha}{1+\alpha}}(ca+2cb+dc), \qquad \rho = dd-3cc\,,
\end{aligned} \tag{B.8a}$$

and the arguments are to be taken again at $\varphi-\frac{1}{2}\alpha$ and $\varphi+\frac{1}{2}\alpha$.

Note that in the limit $\alpha\to 1$ where $m/m_b\to\infty$ the amplitudes c and τ vanish, which means that the triplet π^i decouples from the singlet η. The triplet S-matrix in this limit is the minimal O(3) symmetric one, which is the S-matrix of the O(3) non-linear σ model [9]. In a recent paper [10] this fact was interpreted as the confinement property of the CP^1 model [6] in S-matrix language.

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