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XIII. Exact S-Matrices and Form Factors in 1 + 1 Dimensional Field Theoretic Models with Soliton Behaviour

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Abstract.

A review is given of the derivation of exact S-matrices in field theoretic models with soliton behaviour, that means models obeying infinitely many conservation laws which imply the factorization of the S-matrix. Form factors of various operators are calculated exactly by means of Watson's theorem. The exact value of the finite Sine–Gordon wave function renormalization constant is determined.

1. Introduction

1.1. The results

i) For some field theoretic models we give the onshell solution. By means of analytic S-matrix methods [1, 2] we calculate exactly S-matrix elements like

$$^{\text{out}}\langle p'_1, \ldots, p'_{n'} | p_1, \ldots, p_n \rangle^{\text{in}}$$

for the scattering of any number and arbitrary kinds of particles appearing in the models.

ii) We want to determine offshell quantities for these models like the two-point function

$$\langle 0 | \phi(x)\, \phi(y)\, | 0 \rangle = \sum_n \langle 0 | \phi(x)\, | n \rangle^{\text{in}}\ {}^{\text{in}}\langle n | \phi(y)\, | 0 \rangle.$$

Thus we try to calculate "generalized form factors" [3, 4] like

$$^{\text{out}}\langle p_1, \ldots, p_m | \, O(x)\, | p_{m+1}, \ldots, p_n \rangle^{\text{in}},$$

where $O(x)$ is a local operator. The problem is solved for the cases

a) $n = 2$ and $N =$ (number of kinds of particles) arbitrary

b) $n = 3$ and $N = 1$.

1.2. The models

We will consider some field theoretic models in two space-time dimensions with special common properties which we call soliton behaviour.

i) The Sine–Gordon (SG) model, defined by the Lagrangian

$$\mathscr{L}^{\text{SG}} = \tfrac{1}{2}(\partial_\mu \phi)^2 + \frac{\alpha}{\beta^2}(\cos \beta\phi - 1), \tag{1}$$

is on the classical level completely integrable by means of the inverse scattering method [5]. There are localized classical solutions, the soliton and the breathers, which are soliton–antisoliton bound states. The semiclassical breather spectrum is [6]

$$m_k = 2m \sin(k\pi/2\lambda), \qquad k = 1, 2, \ldots < \lambda, \tag{2}$$

where m is the soliton mass and the parameter λ is related to the SG-coupling constant β by

$$\lambda = 8\pi/\beta^2 - 1. \tag{3}$$

ii) The massive Thirring (MT) model

$$\mathscr{L}^{\text{MT}} = \bar{\psi}(\mathrm{i}\partial\!\!\!/ - m)\psi - \tfrac{1}{2}g(\bar{\psi}\gamma_\mu\psi)^2 \tag{4}$$

describes the interaction of fermions f, $\bar{\text{f}}$. For $g > 0$ there exist fermion–antifermion bound states b_k. The famous equivalence of the quantum Sine–Gordon and the massive Thirring model due to Coleman [7] says: identify the SG-soliton and breathers with the MT-fermion and bound states, respectively, relate the SG-field to the MT-current by

$$\beta\varepsilon^{\mu\nu}\partial_\nu\phi = -2\pi j^\mu \tag{5}$$

and the coupling constants by

$$\lambda = 8\pi/\beta^2 - 1 = 1 + 2g/\pi. \tag{6}$$

iii) The nonlinear σ (NLS) model [8], defined by

$$\mathcal{L}^{\mathrm{NLS}} = \tfrac{1}{2} \sum_{i=1}^{N} (\partial_\mu n_i)^2 \quad \text{with} \quad g \sum_{i=1}^{N} n_i^2 = 1, \tag{7}$$

describes an O(N) symmetric interaction of N bosons which get a dynamically generated mass. There are no two-particle bound states in this model.

iv) The Gross–Neveu (GN) model [9], with the Lagrangian

$$\mathcal{L}^{\mathrm{GN}} = \sum_{\alpha=1}^{N} \bar{\psi}_\alpha \mathrm{i}\partial\!\!\!/ \psi_\alpha + \tfrac{1}{2}g^2\left(\sum_{\alpha=1}^{N} \bar{\psi}_\alpha \psi_\alpha\right)^2, \tag{8}$$

describes an U(N)-symmetric interaction of N fermions f_α and N antifermions $\bar{\mathrm{f}}_\alpha$ or, more precise, an O($2N$)-symmetric interaction of $2N$ selfconjugate fermions f_i with a dynamically generated mass. There are bound states b and b_{ik} which transform like an O($2N$) scalar and antisymmetric tensor, respectively, with the semiclassical mass formula [6]

$$m_{\mathrm{b}} = m_{\mathrm{b}_{ik}} = m \sin\frac{\pi}{N-1} \Big/ \sin\frac{\pi}{2(N-1)}. \tag{9}$$

2. Infinitely many conservation laws and S-matrix factorization

The models presented in the last section are in some sense simple because they possess infinitely many conservation laws.

2.1. The classical case

In the SG [10] and the MT [11] model there exist infinitely many local conservation laws:

$$\partial_\mu J_n^\mu(x) = 0, \qquad n = 1, 3, 5, \ldots, \tag{10}$$

where the currents J_n^μ are local functions of the fields. In the NLS- and GN-model there exist infinitely many nonlocal conservation laws [12].

2.2. Quantization

The BPHZ-quantization [13] of the conservation laws eq. (10) apparently produce anomalies which cancel after redefinition of the currents J_n^μ [14]. The nonlocal charges of the NLS- and GN-models are also conserved in the quantized models [15]. Moreover, there exist also local conservation laws in the quantum NLS-model [16]. In summary we can say the infinitely many conservation laws are valid in the quantum models.

2.3. Consequences

The conservation laws eq. (10) imply for a scattering process that

i) the set of incoming and outgoing momenta are equal

$$\{p_1, \ldots, p_n\}^{\text{in}} = \{p'_1, \ldots, p'_{n'}\}^{\text{out}}. \tag{11}$$

This means absence of particle production and only momentum exchange.

ii) Furthermore one can show [17] that the n-particle S-matrix factorized into two-particle ones (in a special order, e.g. for $p_1^1 > \ldots > p_n^1$) [18]

$$S^{(n)}(p_1, \ldots, p_n) = \prod_{i=1}^{n-1} \left(\prod_{j=i+1}^{n} S^{(2)}(p_i, p_j) \right), \tag{12}$$

where the matrix elements of $S^{(n)}$ are defined by

$$S^{(n)} |\alpha_1(p_1), \ldots\rangle^{\text{in}} = |\alpha'_1(p_1), \ldots\rangle^{\text{in}}{}_{\alpha'_1} \ldots S^{(n)}_{\alpha_1} \ldots$$

and ${}_{\alpha_1\alpha_2}S^{(2)}_{\alpha_1\alpha_2} = t_{\alpha_1\alpha_2}$, ${}_{\alpha_2\alpha_1}S^{(2)}_{\alpha_1\alpha_2} = r_{\alpha_1\alpha_2}$ are the transmission and reflection amplitudes, respectively, for the scattering of two-particles of kind $\alpha_1 \neq \alpha_2$, $\alpha_i = 1, \ldots, N$. The factors in eq. (12) do not commute in general, but they have to fulfill the rule (with $S^{(2)}(p_i, p_j) = S_{ij}$) [18]

$$S_{12}S_{13}S_{23} = S_{23}S_{13}S_{12}, \tag{13}$$

which means that $S^{(3)}$ is symmetric, i.e. time reflection invariant. The commutation relation eq. (13) gives constraints for the two-particle scattering amplitudes called "factorization equations", which allow to calculate the S-matrix exactly [1].

3. S-matrix and bound state spectrum

We calculate the S-matrix from the assumptions:

i) Factorization.

ii) Qualitative knowledge of the bound state spectrum, i.e. we assume:
for the SG $\equiv$ MT-model, the existence of a coupling region with no bound states,
for the NLS-model, the absence of bound states,
for the GN-model, the existence of a bound state in the O(2N)-isoscalar and antisymmetric tensor channel and absence of bound states in the traceless symmetric tensor channel.

iii) Absence of redundant poles (which do not correspond to bound states) and zeros in the physical sheet for the transmission amplitudes (which can be proved for one-dimensional potential scattering [19]).

Theorem 1: Let a so-called minimal two-particle S-matrix $S^{\min}$ fulfill:

a) O(N) or U(N) symmetry,

b) factorization,

c) unitarity and crossing,

d) $S^{\min}$ analytic and not zero in the physical sheet and

$$S^{\min} = o\{\exp(p_1 p_2/m)\} \qquad \text{for} \quad p_1 p_2 \to \infty,$$

then $S^{\min}$ is "uniquely" determined or, more precise, we have

for $U(1) \cong O(2)$ a one-parametric set of solutions [1]
for $U(N)$, $N > 1$ five solutions [20]
for $O(N)$, $N > 2$ one solution [2].
Remark: If we modify d) and allow poles, the general solution is

$$S(\theta) = \prod_{k=1}^{L} \frac{\operatorname{sh}\theta + \mathrm{i}\sin\alpha_k}{\operatorname{sh}\theta - \mathrm{i}\sin\alpha_k} S^{\min}(\theta), \tag{14}$$

where $\theta = |\theta_1 - \theta_2|$ is the rapidity difference of $p_i = m(\operatorname{ch}\theta_i, \operatorname{sh}\theta_i)$.

A sketch of the proof [1, 2, 20]: The factorization eq. (13) imply a functional equation for the ratio of transmission and reflection

$$h(\theta) = t(\theta)/r(\theta).$$

For U(1) we obtain [1]: $h(\alpha + \beta) = h(\mathrm{i}\pi + \alpha)h(\beta) + h(\alpha)h(\mathrm{i}\pi - \beta)$ with the solution $h(\theta) = \operatorname{sh}(\lambda\theta)/\operatorname{sh}(\lambda\mathrm{i}\pi)$ where λ is a free parameter.

For $O(N)$ $(N > 2)$ we obtain [2] $h(\alpha + \beta) = h(\alpha) + h(\beta)$ with the solution $h(\theta) = \text{const.}\,\theta$ where the const. is determined by unitarity const. $= -(N-2)/2\pi\mathrm{i}$.

The $U(N)$ case is more complicated, see ref. [20].

Together with unitarity and crossing $S^{\min}(\theta)$ can be calculated from $h(\theta)$.

3.1. *The* SG ≡ MT *S-matrix*

The minimal U(1) symmetric two-particle S-matric for $0 < \lambda < 1$ which describes the scattering of fermions f (antifermions $\bar{\mathrm{f}}$) is in diagonalized form

$$S^{\min}(\theta, \lambda) = \begin{pmatrix} S_{\mathrm{ff}} & & & 0 \\ & S^{(+)}_{\mathrm{f}\bar{\mathrm{f}}} & & \\ & & S^{(-)}_{\mathrm{f}\bar{\mathrm{f}}} & \\ 0 & & & S_{\bar{\mathrm{f}}\bar{\mathrm{f}}} \end{pmatrix}^{\min} = \begin{pmatrix} 1 & & & 0 \\ & -\dfrac{\operatorname{sh}\frac{1}{2}\lambda(\theta + \mathrm{i}\pi)}{\operatorname{sh}\frac{1}{2}\lambda(\theta - \mathrm{i}\pi)} & & \\ & & -\dfrac{\operatorname{ch}\frac{1}{2}\lambda(\theta + \mathrm{i}\pi)}{\operatorname{ch}\frac{1}{2}\lambda(\theta - \mathrm{i}\pi)} & \\ & 0 & & 1 \end{pmatrix} S^{\min}_{\mathrm{ff}}(\theta, \lambda),$$

where

$$S^{\min}_{\mathrm{ff}}(\theta, \lambda) = \exp \int_0^\infty \frac{\mathrm{d}x}{x} \frac{\operatorname{sh}\frac{1}{2}x(1 - 1/\lambda)}{\operatorname{sh}(x/2\lambda)\operatorname{ch}\frac{1}{2}x} \operatorname{sh} x \frac{\theta}{\mathrm{i}\pi} = \frac{g(-\theta)}{g(\theta)} \tag{15}$$

and

$$g(\theta) = \prod_{l=0}^{\infty} \prod_{k=1}^{\infty} \frac{(2l + 1 + k/\lambda + \theta/\mathrm{i}\pi)\{2l + 1 + (k-1)/\lambda + \theta/\mathrm{i}\pi\}}{(2l + k/\lambda + \theta/\mathrm{i}\pi)\{2l + 2 + (k-1)/\lambda + \theta/\mathrm{i}\pi\}}.$$

Since $S^{\min}$ has no poles in the physical sheet for $0 < \lambda < 1$ and the MT-model has no bound states for $g < 0$, we propose in this coupling region

$$S^{\mathrm{MT}}(\theta) = S^{\min}(\theta, \lambda). \tag{16}$$

For $\lambda > 1$ we take the analytic continuation. There are poles in $0 < \mathrm{Im}\,\theta < \pi$ at

$$\theta_k = \mathrm{i}\pi(1 - k/\lambda), \qquad k = 1, 2, \ldots < \lambda,$$

corresponding to bound states with masses

$$m_k = 2m \sin(k\pi/2\lambda)$$

in agreement with the WKB-spectrum eq. (2), if we relate the parameter λ to the coupling constants g and β by eq. (6). The S-matrix given by eqs. (16) and (15) was first proposed by Zamolodchikov [21] who used results of Korepin and Faddeev [22]. All these results were checked in perturbation theory at $g \to 0$ and $\beta \to 0$ [22].

3.2. *The* NLS- *and the* GN–*S-matrix* [2]

The O(N)-symmetric minimal S-matrix eigenvalues corresponding to the scalar, traceless symmetric and antisymmetric channel, respectively, are given by

$$S^{\min}(\theta, N) = \begin{pmatrix} S_0 & & 0 \\ & S_+ & \\ 0 & & S_- \end{pmatrix}^{\min} = \begin{pmatrix} \dfrac{\theta + \mathrm{i}\pi}{\theta - \mathrm{i}\pi} & & 0 \\ & \dfrac{\theta - 2\pi\mathrm{i}/(N-2)}{\theta + 2\pi\mathrm{i}/(N-2)} & \\ 0 & & 1 \end{pmatrix} S_-^{\min}(\theta, N), \tag{17}$$

where

$$S_-^{\min}(\theta, N) = \exp 2\int_0^\infty \frac{\mathrm{d}x}{x}\,\frac{\mathrm{e}^{-2x/(N-2)} - 1}{\mathrm{e}^x + 1}\,\mathrm{sh}\, x\frac{\theta}{\mathrm{i}\pi}\ .$$

Since the O(N)–NLS-model has no bound states, one proposes [2]

$$S^{\mathrm{NLS}}(\theta, N) = S^{\min}(\theta, N). \tag{18}$$

Since for the O($2N$)–GN-model S_0 and S_- should have a pole and S_+ not, one proposes [2]

$$S^{\mathrm{GN}}(\theta, 2N) = \frac{\mathrm{sh}\,\theta + \mathrm{i}\sin\{\pi/(N-1)\}}{\mathrm{sh}\,\theta - \mathrm{i}\sin\{\pi/(N-1)\}}\, S^{\min}(\theta, 2N), \tag{19}$$

with bound state masses in agreement with eq. (9). All formulas were checked in $1/N$-expansion up to $1/N^2$, [2, 24].

The scattering of bound states can be calculated by considering the residue of three-particle S-matrices [18]

$$\operatorname*{Res}_{(p_1+p_2)^2 = m_b^2} S^{(3)}(p_1, p_2, p_3) \to S^{(2)}(p_1 + p_2, p_3),$$

where m_b is the mass of the bound state.

4. Form factors

We want to calculate "generalized form factors" like ${}^{\text{out}}\langle \alpha_1(p_1)\ldots|O(x)|\ldots \alpha_n(p_n)\rangle^{\text{in}}$ where $\alpha_i = 1,\ldots,N$ denote the kinds of the particles.

4.1. Watson's theorem [25]

For simplicity we only consider the case $n = 2$, $N = 1$, and $O^+ = O$, for the general case see [4]. If we define (with $p_1p_2 = m^2 \operatorname{ch}\theta$)

$$\langle 0|\, O(0)\,|p_1p_2\rangle^{\text{in}} = F(\theta),$$

it follows from

a) CPT-invariance $\langle 0|\, O(0)\,|p_1p_2\rangle^{\text{out}} = F(-\theta)$

b) unitarity $F(\theta) = \sum_{n'} \langle 0|\, O(0)\,|n'\rangle^{\text{out}}\, {}^{\text{out}}\langle n'|\, p_1p_2\rangle^{\text{in}}$

c) factorization $F(\theta) = \langle 0|\, O(0)\,|p_1p_2\rangle^{\text{out}}\, S(\theta)$

d) crossing $\langle p_1|\, O(0)\,|p_2\rangle = F(\mathrm{i}\pi - \theta)$ (remark $|p\rangle = |p\rangle^{\text{in}} = |p\rangle^{\text{out}}$).

From a)–d) we obtain Watson's equations

$$F(\theta) = F(-\theta)\, S(\theta), \qquad F(\mathrm{i}\pi - \theta) = F(\mathrm{i}\pi + \theta). \tag{20}$$

Theorem 2 [4]: $F(\theta)$ fulfilling eqs. (20) is uniquely (up to a normalization) determined by the poles at $\theta = \mathrm{i}a_k$ in the physical strip $0 < \operatorname{Im}\theta < \pi$ (and zeros):

$$F(\theta) = K(\theta) F^{\min}(\theta), \tag{21}$$

where

$$K(\theta) = \text{const} \prod_{k=1}^{L} \left(\operatorname{sh}\tfrac{1}{2}(\theta - \mathrm{i}a_k)\operatorname{sh}\tfrac{1}{2}(\theta + \mathrm{i}a_k)\right)^{-1}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}\theta}[\ln F^{\min}(\theta)] = \frac{1}{8\pi \mathrm{i}} \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{\operatorname{sh}^2 \frac{1}{2}(z-\theta)} \ln S(z).$$

Remarks: The poles of $F(\theta)$ are determined by one-particle states in the channel given by $O(x)$. We assume absence of redundant poles and zeros in $0 < \operatorname{Im}\theta < \pi$.

4.2. Examples

i) The electromagnetic SG-soliton form factor [3] is defined by

$$\langle f(p_1)|\, j^{\mu}(0)\,|f(p_2)\rangle = \bar{u}(p_1)\gamma^{\mu} u(p_2) F_{-}^{\text{MT}}(\mathrm{i}\pi - \theta), \tag{22}$$

where $p_1p_2 = m^2 \operatorname{ch}\theta$. Since there are no bound states for $g < 0$, i.e. $\lambda < 1$, we propose F_{-}^{MT} to be the minimal solution of Watson's equations (20) with the negative C-parity S-matrix eigenvalue given by eq. (15):

$$F_{-}^{\text{MT}}(\mathrm{i}\pi - \theta) = \frac{\operatorname{ch}\frac{1}{2}\theta}{\operatorname{ch}\frac{1}{2}\lambda\theta} \exp \int_{0}^{\infty} \frac{\mathrm{d}x}{x} \frac{\operatorname{sh}\frac{1}{2}x(1 - 1/\lambda)}{\operatorname{sh}(x/2\lambda)\operatorname{ch}\frac{1}{2}x} \frac{\sin^2(x\theta/2\pi)}{\operatorname{sh} x}. \tag{23}$$

This formula was checked in perturbation theory. The asymptotic behaviour of the form factor is $F_-^{\mathrm{MT}} \sim (-t)^{g/2\pi}$ for large momentum transfer $t = (p_1 + p_2)^2 \to -\infty$.

ii) The O(N) current $J^\mu_{ij} = n_i \overleftrightarrow{\partial} n_j$ form factor in the NLS-model [4], defined by

$$\langle b_k(p_1)|\, J^\mu_{ij}(0)\, |b_l(p_2)\rangle = \mathrm{i}(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})(p_1 + p_2)^\mu F_-^{\mathrm{NLS}}(\mathrm{i}\pi - \theta), \tag{24}$$

is proposed to be the minimal solution of Watson's equations (20) with the S-matrix eigenvalue S_-^{NLS} given by eqs. (17, 18):

$$F_-^{\mathrm{NLS}}(\mathrm{i}\pi - \theta) = \exp\left\{2\int_0^\infty \frac{\mathrm{d}x}{x}\,\frac{\mathrm{e}^{-2x/(N-2)} - 1}{\mathrm{e}^x + 1}\,\frac{\sin^2(x\theta/2\pi)}{\operatorname{sh} x}\right\}. \tag{25}$$

The asymptotic behaviour $F_-^{\mathrm{NLS}} \sim (\ln(-t))^{-1/N-2}$ is to be anticipated from the asymptotic freedom of the model. Formula (25) was checked in $1/N$-expansion [4].

iii) The exact value of the SG-wave function renormalization constant defined by $\langle 0|\,\phi(0)|\,\mathrm{b}_1\rangle = \sqrt{Z}$ can be calculated from

$$\langle\phi(x)\,\phi(y)\rangle \doteq \sum \langle\phi(x)|\, f(p_1)\bar{f}(p_2)\rangle^{\mathrm{in}}\ {}^{\mathrm{in}}\langle f(p_1)\bar{f}(p_2)\,|\phi(y)\rangle + \ldots$$

since the elementary SG-boson b_1 can be built up only by an $f\bar{f}$ pair. Using eqs. (5, 6, 22, 23) one obtains

$$Z = (1 + \tfrac{1}{\lambda})\left(\frac{2\lambda}{\pi}\sin\frac{\pi}{2\lambda}\right)^{-1} \exp\left[-\frac{1}{\pi}\int_0^{\pi/\lambda} \mathrm{d}x\,\frac{x}{\sin x}\right],$$

$$= \mathrm{O}(g) \text{ for } g \to 0, \text{ where the MT-model becomes free,}$$

$$= 1 - \left(\frac{\beta^2}{8\pi}\right)^2\left(\frac{1}{2} - \frac{\pi}{24}\right) + \mathrm{O}(\beta^6) \text{ for } \beta \to 0, \text{ where the SG-model becomes free.}$$

The last equation can be checked in SG-perturbation theory [4]. For $1 < \lambda < \infty$ where the state b_1 exist we have $0 < Z < 1$ in agreement with a general theorem [26].

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