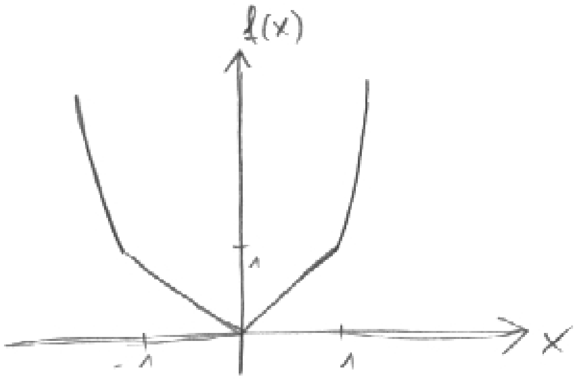
Daniel Sachse 10.5.05

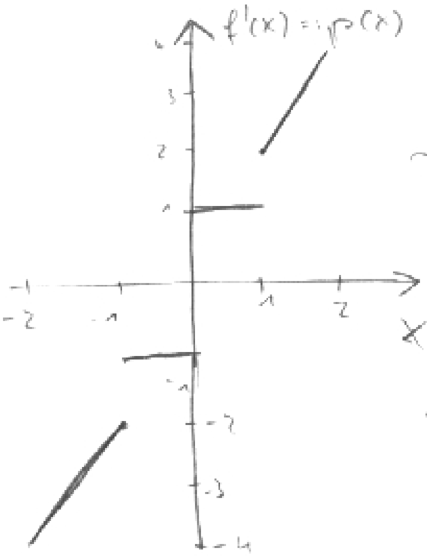
Wärme: Blatt 3

Aufgabe 1)

Gegeben sei $f(x) = \begin{cases} |x| & : -1 < x < 1 \\ x^2 & : |x| > 1 \end{cases}$,

definiere $p(x) := f'(x) = \begin{cases} -1 & : -1 < x < 0 \\ 1 & : 0 < x < 1 \\ 2x & : |x| > 1 \end{cases}$.

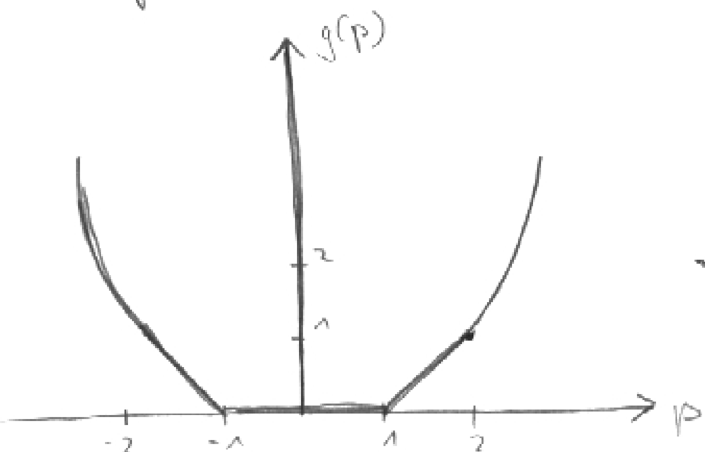




Umkehrfunktion x(p) unbestimmt

Wir wählen die Umkehrfunktion $x(p)$ so, dass $g(p) := xp - f(x) = x(p)\cdot p - f(x(p))$ konvex ist:

$$x(p) = \begin{cases} p/2 & : |p| > 2 \\ -1 & : -2 < p < -1 \\ 1 & : 1 < p < 2 \\ 0 & : |p| < 1 \end{cases} \Rightarrow g(p) = \begin{cases} (p/2)^2 & : |p| > 2 \\ |p| - 1 & : 1 < |p| < 2 \\ 0 & 0 < |p| < 1 \end{cases}$$



Nochmaliges Anwenden der Transformation:

$$q(p) := g'(p) = \begin{cases} p/2 & : |p| > 2 \\ -1 & : -2 < p < -1 \\ 1 & : 1 < p < 2 \\ 0 & : 0 < |p| < 1 \end{cases}$$, finde wieder

$$p(q) = \begin{cases} 2q & : |q| > 1 \quad \text{(zwingend)} \\ -1 & : q < 0 \\ 1 & : q > 0 \quad \text{(ergänzt)} \end{cases}$$

Damit wird $f(q) := pq - g(p) = p(q) \cdot q - g(p(q))$,

$$f(q) = \begin{cases} 2q^2 - q^2 = q^2 & : |q| > 1 \\ -q & : -1 < q < 0 \\ q & : 0 < q < 1 \end{cases}$$

$$= \begin{cases} q^2 & : |q| > 1 \\ |q| & : -1 < q < 1 \end{cases}$$ ... entspricht $f(x)$!

r

Aufgabe 2)

(a) $$-\left(\frac{\partial \ln T}{\partial \ln V}\right)_S = -\frac{V}{T}\left(\frac{\partial T}{\partial V}\right)_S \overset{\text{Ü1}}{=} \frac{V}{T}\left(\frac{\partial T}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_T$$

$$\overset{\text{MW}}{=} \frac{V}{T}\left(\frac{\partial T}{\partial S}\right)_V \left(\frac{\partial p}{\partial T}\right)_V \overset{\text{Ü1}}{=} -\frac{V}{T}\left(\frac{\partial T}{\partial S}\right)_V \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p$$

$$\overset{\text{erweitern}}{=} \frac{\frac{1}{V}}{\frac{1}{V}} \cdot \frac{\left(\frac{\partial V}{\partial T}\right)_p}{-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T T\left(\frac{\partial S}{\partial T}\right)_V} = \frac{\alpha_p}{\varkappa_T \left(\frac{C_V}{V}\right)}$$

$(pV = RT)$ ...

$$\boxed{T\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V}$$

(b) ... $\alpha = \frac{1}{V}\frac{R}{p} = \frac{1}{T}$, $C_V = \frac{f}{2}R$, $\varkappa_T = -\frac{1}{V}\frac{\partial}{\partial p}\frac{RT}{p} = \frac{RT}{Vp^2} = \frac{1}{p}$

$$\Rightarrow \Gamma = \frac{\alpha}{\varkappa \frac{C_V}{V}} = \frac{2}{f}$$

experimentell über Adiabaten-koeff. bestimmbar.

(b) besser: $-\left(\frac{\partial \ln T}{\partial \ln V}\right)_S$ $\Rightarrow$ Adiabatenglg. verwenden:

$$pV^{\frac{c_p}{c_V}} = \text{konst}$$

$$TV^{\frac{c_p - c_V}{c_V}} = TV^{\frac{R}{c_V}} = \text{konst}$$

$$\Rightarrow \boxed{\ln T = -\frac{R}{c_V}\ln V + \text{konst}}$$

$$\frac{R}{c_V} = \Gamma$$