Fachbereich Physik | SoSe 02
Freie Universität Berlin | Set 9
Theoretische Physik | 6/19/02

Theorie der Wärme – Statistical Physics
(Prof. E. Frey)

Problem set 9

Problem 9.2 *Bogolons* (4 pts)

In a superconductor at low temperatures one finds fermionic quasi-particles excitations called Bogolons after Bogoliubov. The excitation energy $\epsilon_{\mathbf{k}}$ for such a quasi-particle is given by the relation

$$\epsilon_{\mathbf{k}}^2 = \left[\frac{\hbar^2}{2m}(k^2 - k_F^2)\right]^2 + \Delta^2 , \qquad k = |\mathbf{k}| ,$$

where $\mathbf{k}$ is the wave vector of the excitation and Δ the gap energy. The Bogolons shall be regarded as approximatively non-interacting with two independent spin states. The number of quasi-particles is not conserved, which implies that the chemical potential is zero. Convince yourself that at low temperatures the dominant contribution to the grand canonical potential (free energy) comes from excitations corresponding to wave numbers close to k_F. Expand the excitation energies in powers of $k - k_F$ and evaluate the number of excitations, their contribution to the energy and the specific heat in leading order at low temperatures.
Hint: The Bogolons are the fermionic analog to the rotons of Problem 8.4.

Problem 9.2 *Magnons* (6 pts)

For a ferromagnetic system the low-temperature excitations can approximately be described in terms of the effective hamiltonian

$$\mathcal{H} = -gHNS + \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} n_{\mathbf{k}} .$$

Here H denotes the external magnetic field, $n_{\mathbf{k}} = 0, 1, ..$ is the occupation of a magnon state with energy $\hbar\omega_{\mathbf{k}}$ and wave vector $\mathbf{k}$ and N is the number of spins (not the number of magnons!). The magnons are to be considered as non-interacting bosons. Since the number of magnons is non-conserved the chemical potential is zero $\mu = 0$ as in the case of photons. Give formal expressions for the free energy, entropy, specific heat and for the magnetization. Answer for the energy:

$$E = -gNHS + \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} \bar{n}_{\mathbf{k}} , \qquad \bar{n}_{\mathbf{k}} = [\exp(\beta\hbar\omega_{\mathbf{k}}) - 1]^{-1}$$

Replace the sum $\sum_{\mathbf{k}}$ by an appropiate integral and use the mangon dispersion relation

$$\hbar\omega_{\mathbf{k}} = gH + Ia^2k^2 ,$$

to derive the leading low-temperature behavior of the spontaneous magnetization $M_s = -(\partial F/\partial H)_T$ for $H \searrow 0$ and the zero-field magnon contribution to the specific heat for low temperatures.
Hint: You might encounter integrals that are difficult to evaluate analytically. Use dimensionless variables and show that the integrals yield just numerical prefactors.

Problem 9.3 *Capillary waves* (4 pts)

Surface waves on liquid helium have a dispersion relation given by

$$\omega_{\mathbf{k}} = \sqrt{\frac{\sigma k^3}{\rho}}\,, \qquad k = |\mathbf{k}|\,,$$

where k is the wavenumber of the surface wave, σ the surface tension, and ρ the mass density of the liquid. Treating the excitations as bosons with no number conservation, find the internal energy per unit area as a function of temperature.

Termine für Übungsgruppen:
Do 12-14 in T3, 1.3.48 (Franosch)
Do 16-18 in T1, 1.3.21 (Parmeggiani, Lattanzi) in englischer Sprache
Fr 12-14 in E2, 1.1.53 (Falcke)
Abgabe: In der Vorlesung vom 26.6.02