Theorie der Wärme – Statistical Physics
(Prof. E. Frey)

Problem set 8

Problem 8.1 *Joule-Thomson process* (4 pts)
The Joule-Thomson process is commonly used to cool or liquify gases. Here a gas is allowed to seep through a porous barrier from a region of high pressure P_i to region of low pressure P_f, and no heat is provided to or extracted from the combined system. The two pressures are kept constant by pistons. Use the first law of thermodynamics to show that the enthalpy $H = E + PV$ remains unchanged for this process. The efficiency is quantified in terms of the Joule-Thomson coefficient $(\partial T/\partial P)_H$. Show that the following relation holds

$$\left(\frac{\partial T}{\partial P}\right)_H = \frac{V}{C_P}\left(T\alpha - 1\right) , \qquad \alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

Convince yourself that there is no change in temperature for an ideal gas. Evaluate the Joule-Thomson coefficient for a van der Waals gas and determine the inversion curve in the T-v plane, i.e. the temperature T_{inv} where the Joule-Thomson coefficient changes sign.

Problem 8.2 *Bose gas in two dimensions* (3 pts)
Show that for an ideal Bose gas in two dimensions no Bose-Einstein condensation occurs.

Problem 8.3 *Pauli para-magnetism* (4 pts)
Consider the effect of spin on the magnetic susceptibility on a non-interacting electron gas. The hamiltonian is given by

$$\mathcal{H} = \sum_{s\mathbf{k}} \epsilon_{s\mathbf{k}} n_{s\mathbf{k}} , \qquad s = \uparrow, \downarrow , \quad n_{s\mathbf{k}} = 0, 1.$$

The electrons are fermions with a dispersion relation

$$\epsilon_{\uparrow\mathbf{k}} = \frac{\hbar^2 k^2}{2m} - gH , \quad \epsilon_{\downarrow\mathbf{k}} = \frac{\hbar^2 k^2}{2m} + gH$$

Find the free energy F, the magnetization $M = -(\partial F/\partial H)_T$ and the susceptibility $\chi_T = (\partial M/\partial H)_T$ of the system for small fields H. Since the Fermi energy (chemical potential) is large compared to the temperatures of interest, restrict the discussion to $T = 0$. Then the problem reduces to finding the ground state energy compatible with the Pauli principle.

Problem 8.4 *Rotons* (4 pts)
In superfluid ^{4}He the dispersion relation $\omega_\mathbf{k}$ of collective density fluctuations exhibits a minimum at some finite wave number k_0. The elementary excitations to wave numbers close to k_o can approximately regarded

as non-interacing bosons called rotons, and there the dispersion relation is given by

$$\hbar\omega_{\mathbf{k}} = \Delta + \frac{\hbar^2(k-k_0)^2}{2m^*}\,, \qquad \Delta > 0\,, m^* > 0$$

Use the effective hamiltonian

$$\mathcal{H} = \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} n_{\mathbf{k}}\,, \qquad n_{\mathbf{k}} = 0, 1, ...$$

and calculate the mean number of rotons $N_{rot} = \sum_{\mathbf{k}} \bar{n}_{\mathbf{k}}$ in leading order at low temperatures. Convince yourself that the mean occupation numbers $\bar{n}_{\mathbf{k}} = [\exp(\beta\hbar\omega_{\mathbf{k}}) - 1]^{-1}$ are small so that one can replace them by the Boltzmann value $\bar{n}_{\mathbf{k}} \approx e^{-\beta\hbar\omega_{\mathbf{k}}}$. Evaluate the mean energy $E = \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}}\bar{n}_{\mathbf{k}}$ and find the roton contribution to the specific heat.

Problem 8.5 *relativistic electrons* (4 pts)

Calculate the energy and pressure of a relativistic degenerate Fermi gas, i.e. temperature $T = 0$. Discuss the cross-over from the non-relativistic to the ultra-relativistic case.

Termine für Übungsgruppen:
Do 12-14 in T3, 1.3.48 (Franosch)
Do 16-18 in T1, 1.3.21 (Parmeggiani, Lattanzi) in englischer Sprache
Fr 12-14 in E2, 1.1.53 (Falcke)
Abgabe: In der Vorlesung vom 19.6.02