Fachbereich Physik
Freie Universität Berlin
Theoretische Physik

SoSe 02
Set 7
6/5/02

Theorie der Wärme – Statistical Physics
(Prof. E. Frey)

Problem set 7

Problem 7.1 *Thermodynamic identities* (3 pts)
Prove the following thermodynamic relations

$$C_P - C_V = \frac{TV\alpha^2}{\kappa_T}, \qquad \kappa_T - \kappa_S = \frac{TV\alpha^2}{C_P}.$$

Here C_P, C_V denote the specific heat at constant pressure and volume, respectively, and κ_T, κ_S are the isothermal and adiabatic compressibilities and $\alpha = V^{-1}(\partial V/\partial T)_P$ the thermal expansion coefficient. Can you think of a system, where the two specific heats or compressibilities coincide?

Problem 7.2 *Diesel engine* (3 pts)
The Diesel cycle can approximately be regarded as a sequence of four steps: 1. adiabatic compression, 2. isobaric expansion (ignition), 3. adiabatic expansion, 4. isochoric cooling. Sketch the cycle in a S–V and a P–V diagram. Calculate the heat and the corresponding work for each of the steps for an ideal gas. Determine the efficiency of the Diesel engine.
[For an ideal gas the equation of state reads $PV = Nk_BT$. The specific heats C_P and C_V are independent of pressure and temperature. Adiabats fulfill $PV^\gamma = const.$ with $\gamma = C_P/C_V, C_P - C_V = Nk_B$.]

Problem 7.3 (3 pts)
For a particular system it is found that if the volume is kept constant at the value V_0 and the pressure is changed from P_0 to an arbitrary pressure P', the heat transfer to the system is

$$Q' = A(P' - P_0), \qquad A > 0.$$

In addition it is known that the adiabates of the system are of the form

$$PV^\gamma = const., \qquad \gamma > 0.$$

Find the energy $E(P,V)$ for an arbitrary point in the P–V plane, expressing $E(P,V)$ in terms of $P_0, V_0, A, E_0 \equiv E(P_0, V_0)$ and γ (as well as P and V).
Answer:

$$E - E_0 = A(Pr^\gamma - P_0) + PV(1 - r^{\gamma-1})/(\gamma - 1), \qquad r = V/V_0$$

Problem 7.4 (3 pts)
Show that if a single-component system is such that PV^k is constant in the adiabatic process (k is a positive constant) the energy is

$$E = \frac{1}{k-1}PV + Nf(PV^k/N^k)$$

where f is an undetermined function.
Hint: Write the adiabat in terms of intensive variables. The constant depends only on the entropy density.

Problem 7.5 *van der Waals fluid* (5 pts)

Use the empirical equation of state for a van der Waals fluid

$$\left(P + \frac{N^2 a}{V^2}\right)(V - Nb) = N k_B T$$

to find the dependence of the free energy density $f(T, v) = F(T, V, N)/N$ on the volume per particle $v = V/N$. Here a and b are some constants with appropiate units. Determine the free energy density completely by matching to the free energy density

$$f_{id}(T, v) = -k_B T \ln(\lambda^{-3} v)\,, \qquad \lambda = \sqrt{\frac{h^2}{2\pi M k_B T}}$$

of an ideal monatomic gas at large v [Answer: $f(T, v) = -a/v - k_B T \ln[(v - b)\lambda^{-3}]$]. Determine the thermal expansion coefficient $\alpha = V^{-1}(\partial V/\partial T)_P$, the heat capacities C_V and C_P and the compressibilities κ_T, κ_S.

Termine für Übungsgruppen:
Do 12-14 in T3, 1.3.48 (Franosch)
Do 16-18 in T1, 1.3.21 (Parmeggiani, Lattanzi) in englischer Sprache
Fr 12-14 in E2, 1.1.53 (Falcke)
Abgabe: In der Vorlesung vom 12.6.02