Fachbereich Physik
Freie Universität Berlin
Theoretische Physik

SoSe 02
Set 3
5/8/02

Theorie der Wärme – Statistical Physics
(Prof. E. Frey)

Problem set 3

Problem 3.1 *Liouville's theorem* (3 pts)
Show that the time evolution for a classical system in terms of Newton's equations leaves the phase space density for the microcanonical/canonical ensemble invariant.

Problem 3.2 *variational principle* (3 pts)
Find the distribution function of momenta $f(\mathbf{p})$ that maximizes the functional

$$H = -\int d^3p f(\mathbf{p}) \ln f(\mathbf{p}) \,.$$

with the constraint that $f(\mathbf{p})$ is normalized to $\int d^3p f(\mathbf{p}) = n$ and the kinetic energy is fixed to $n^{-1} \int d^3p f(\mathbf{p}) p^2/2m = \epsilon$.

Problem 3.3 *Paramagnet* (4 pts)
The Hamilton operator of a paramagnet with N particles in an external magnetic field H is

$$\mathcal{H} \;=\; -H \sum_{i=1}^{N} \sigma_i \,, \qquad \sigma_i = \pm 1$$

Write down the possible energy levels of the system and calculate the number of states for such energies using a combinatorial approach. Evaluate the extensive part of the entropy as $N \to \infty$.

Problem 3.4 *Harmonic oscillators* (4 pts)
For a system of N identical uncoupled oscillators the energy eigenvalues read

$$E = \sum_{k=1}^{N} \hbar\omega (n_k + \frac{1}{2}) \,, \qquad n_k = 0, 1, 2, ...$$

In how many ways can the given energy be distributed on the oscillators? Calculate the extensive part of the entropy as $N \to \infty$.

Problem 3.5 *Kinetic energy* (4 pts)

For a fluid of N structureless classical particles the Hamilton function reads

$$\mathcal{H} = \sum_{k=1}^{N} \frac{\mathbf{p}_k^2}{2M} + \mathcal{V}\,.$$

Here the potential energy $\mathcal{V}$ depends only on the coordinates $\mathbf{r}_i$. Calculate the probability distribution of the kinetic energy in the canonical ensemble, i.e with a phase space density

$$\rho(\{\mathbf{p}_i, \mathbf{r}_i\}) = Z^{-1} \exp(-\beta\mathcal{H})\,.$$

Note that the phase space integrals factorize into a kinetic and potential part.

Termine für Übungsgruppen:
Do 12-14 in T3, 1.3.48 (Franosch)
Do 16-18 in T1, 1.3.21 (Parmeggiani, Lattanzi) in englischer Sprache
Fr 12-14 in E2, 1.1.53 (Falcke)
Abgabe: In der Vorlesung vom 15.5.02